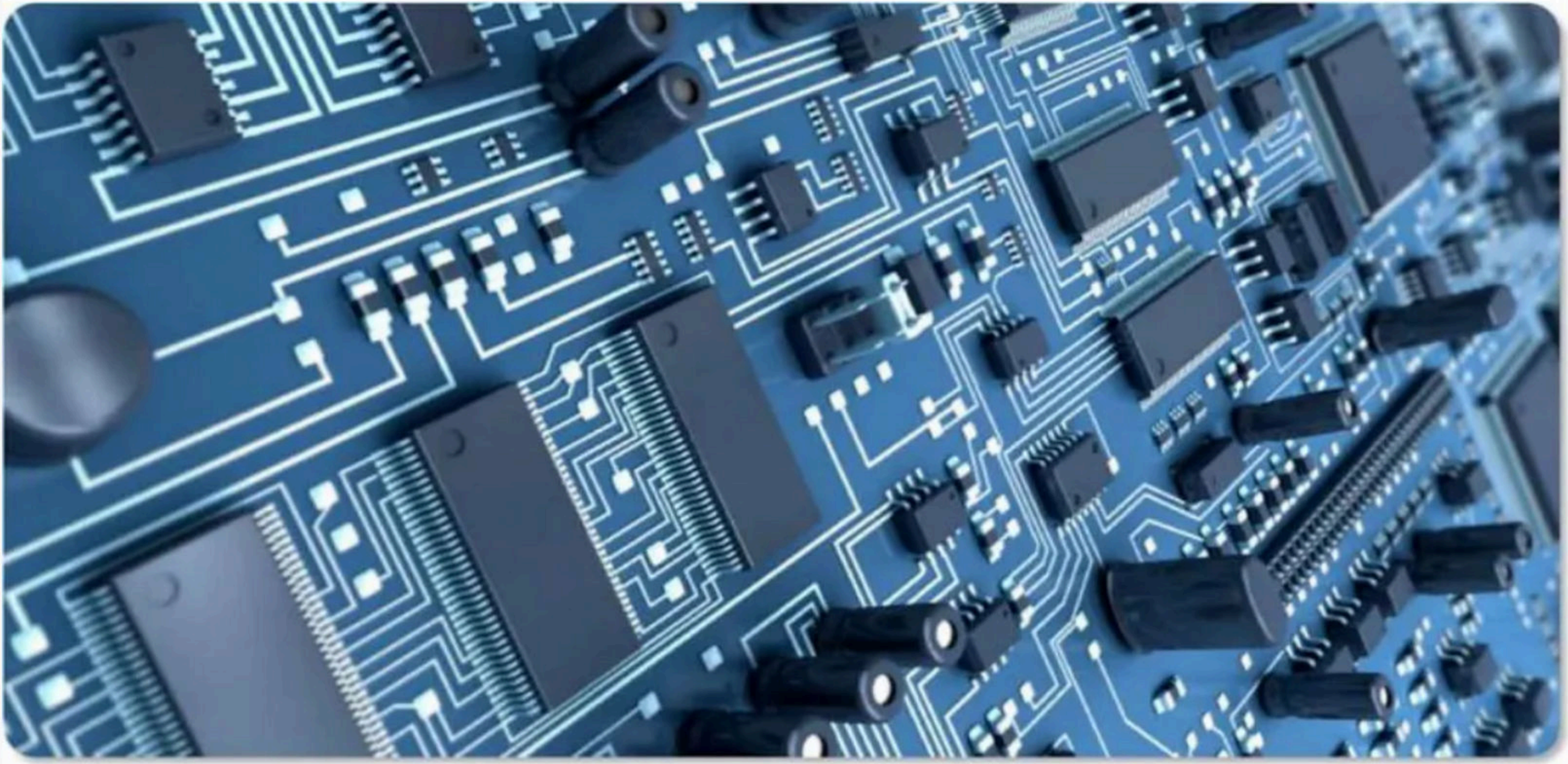
Boolean Algebra & Expression - Part I

Complete Course on Digital Electronics for GATE

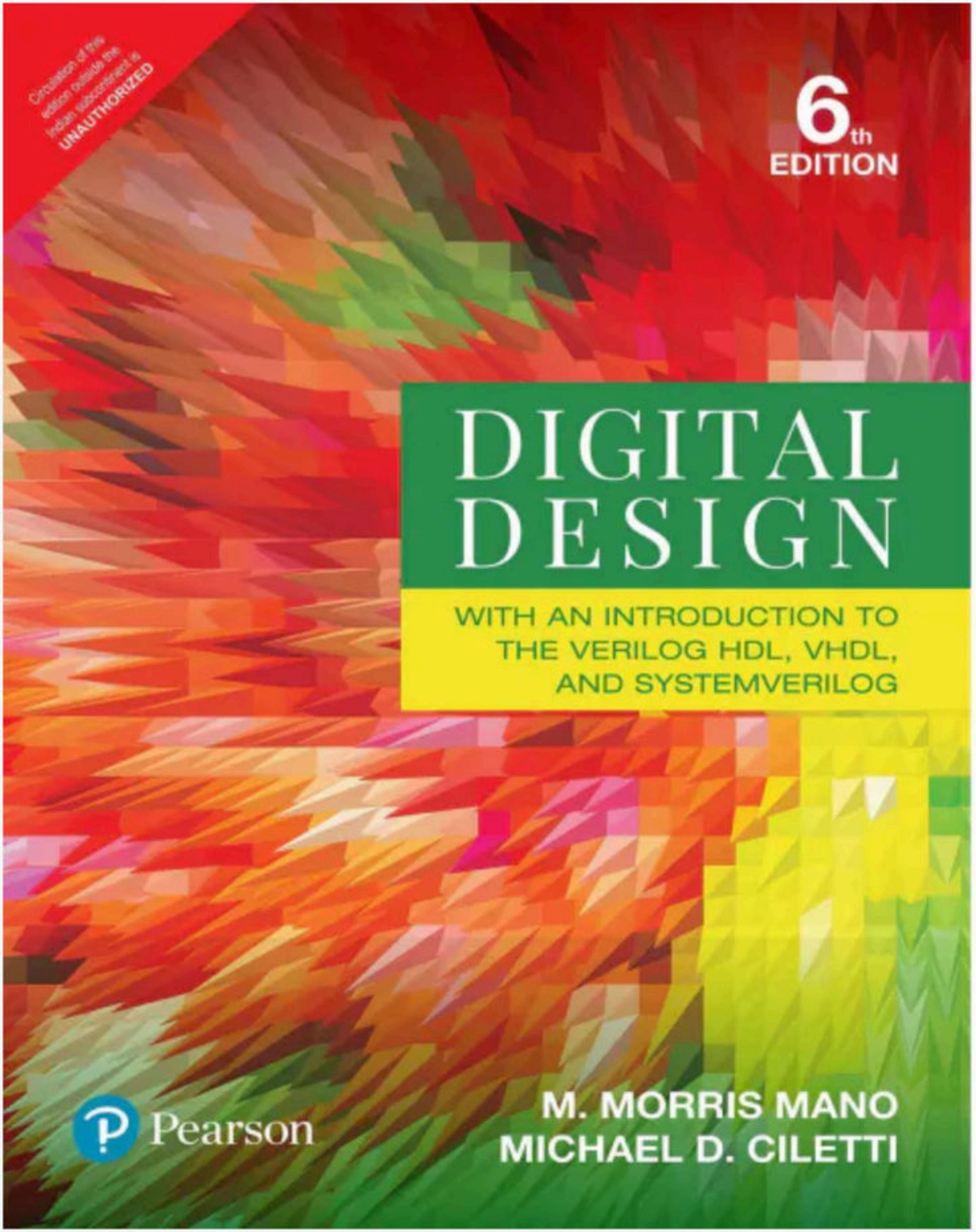
Sanchit Jain • Lesson 1 • Nov 3, 2021

Digital Electronics

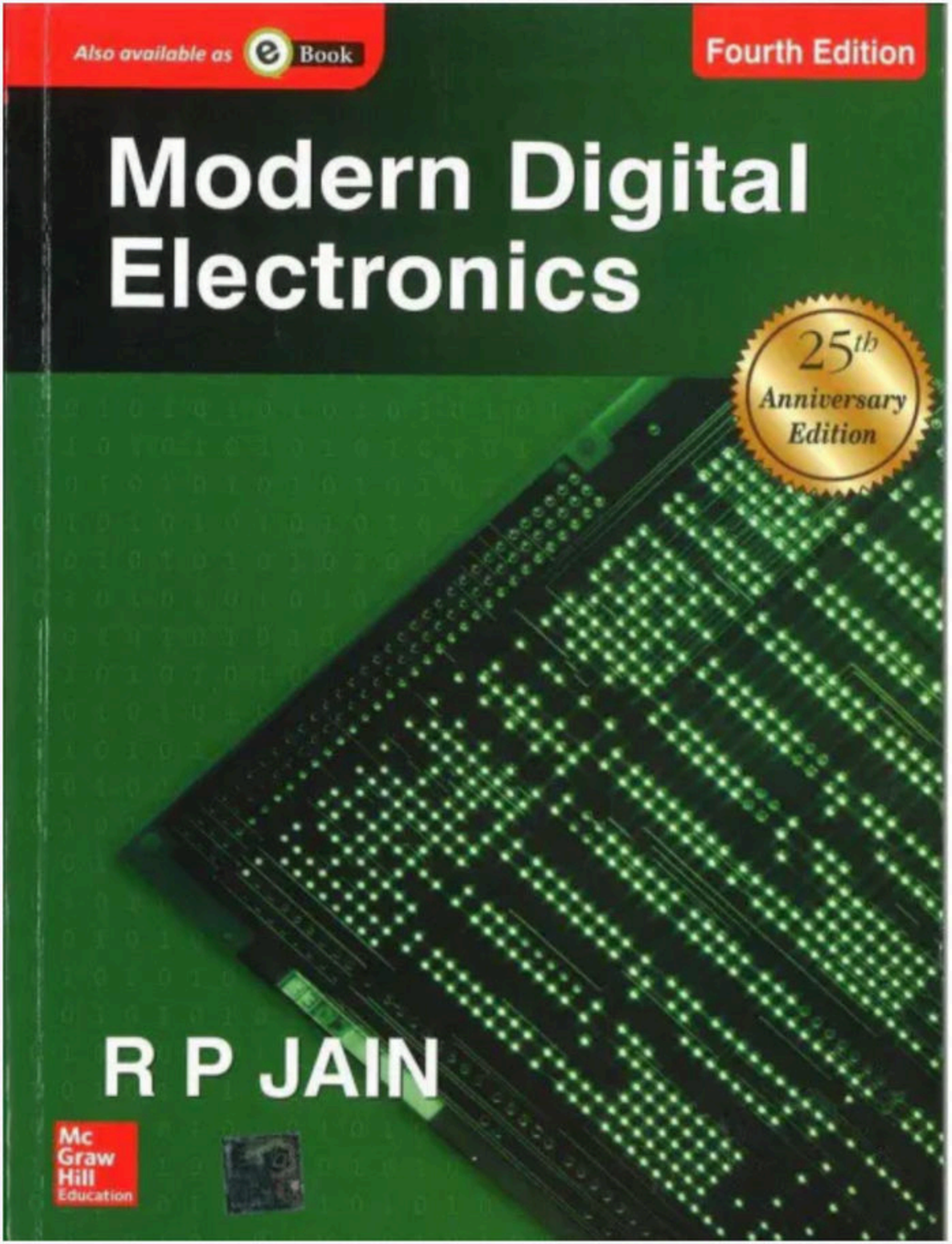


Digital Electronics

- Core subjects for CS/IT Students
- In GATE 7-8 Marks out of 100 Marks, and 5-6 questions on an average
- In NET 20-22 Marks out of 200 marks and 10-12 questions
- Most questions are Numerical
- Needs less time, good scoring
- Not asked in Industry



- **Book Name** - Digital Design
- **Writers** -
  - M. Morris Mano
  - Michel D. Ciletti
- **Publisher** – Pearson
- **Edition** – 6th



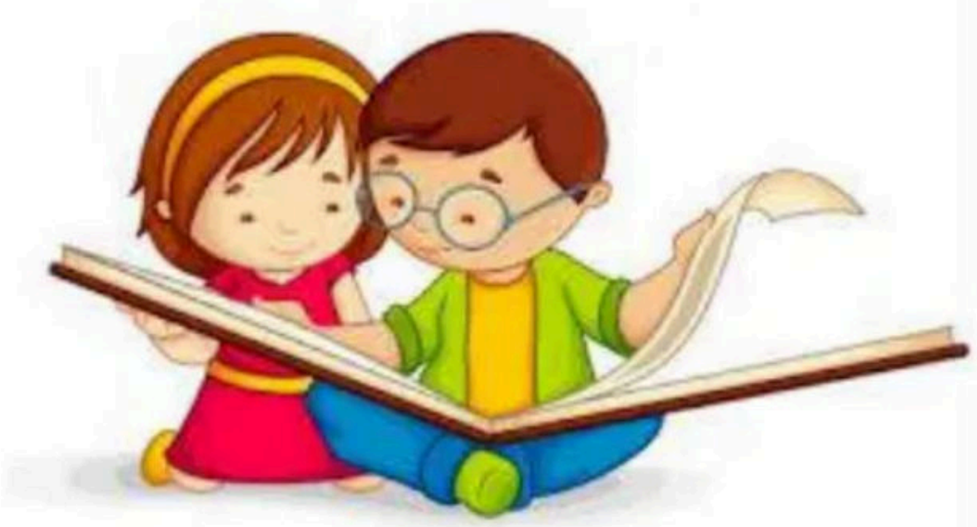
- Book Name – Modern Digital Electronics
- Writers – R P Jain
- Publisher – McGraw Hill
- Edition – 4th

Syllabus

- Understanding Digital Electronics (History and Motive)
- Boolean Algebra Laws and Logic Gates (1-2)
- Boolean Expression (SOP and POS) (Minimization) (1-2) 78%
- Combinational Circuit (1-2)
- Sequential Circuit (1-2)
  important
- Number System and Number Representation (1-2)

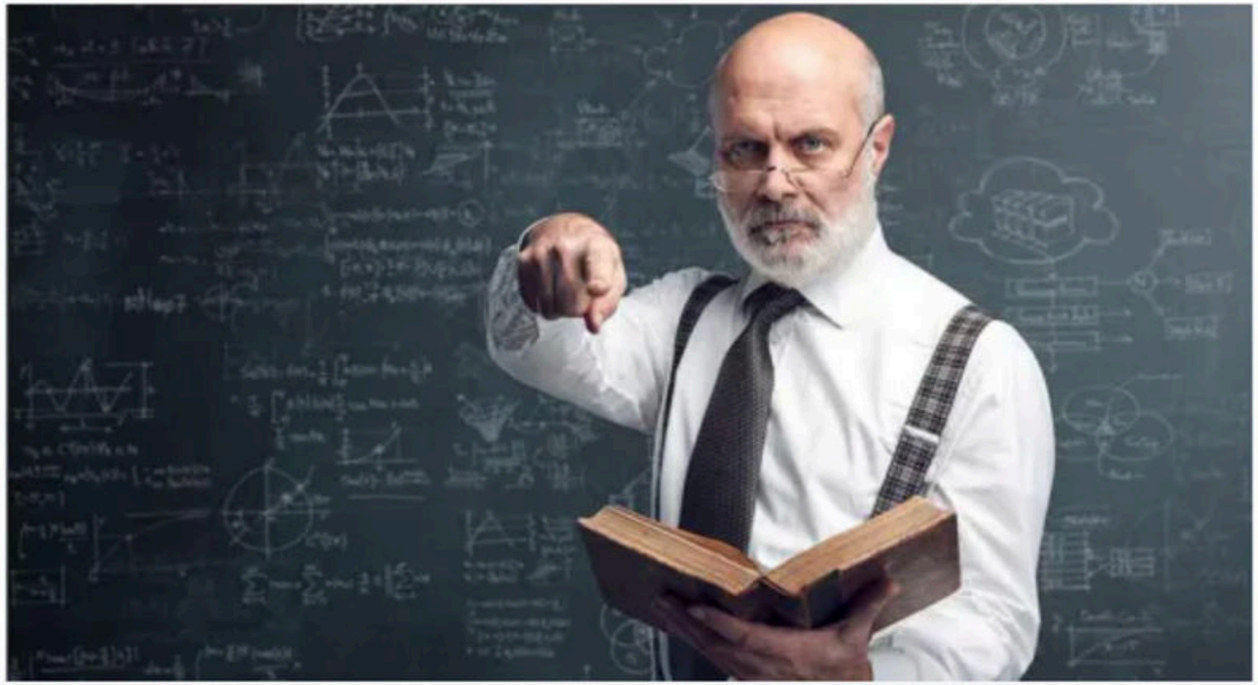
What you can expect from me

- Will take care of theory and numerical both
- Will give more weightage to the topics that are asked more frequently in GATE
- Will not emphasize more than required, on a topic
- Will provide PDF of related books



What i expect from you

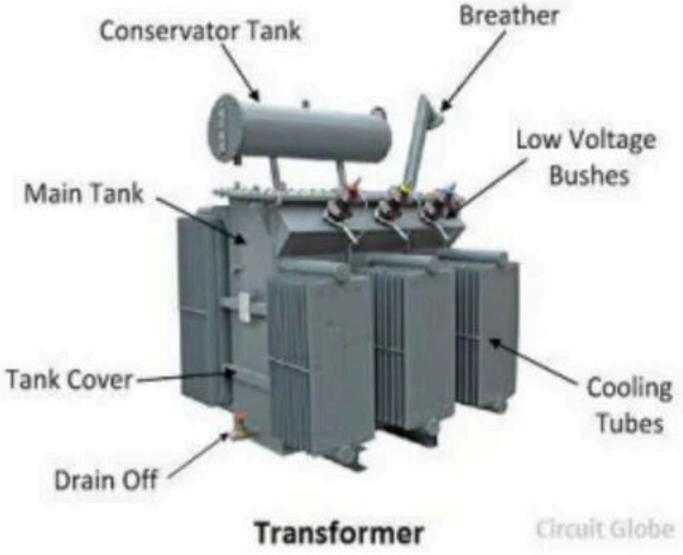
- There is no hurry, feel free to ask questions any time through out the class, but first listen
- Please revise the entire lecture before and after the class
- Be regular, Consistency is most important
- More you practice, more clarity you will get
- If we follow all the above specified points Success in guaranteed

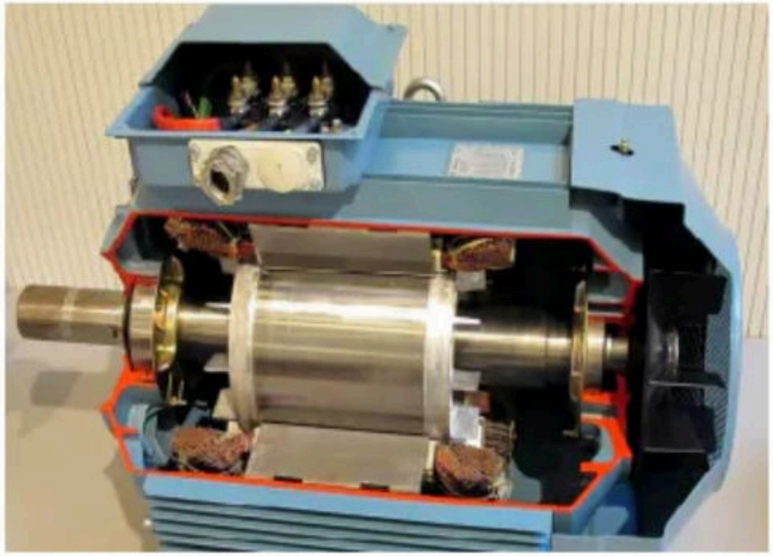


Break

Basics

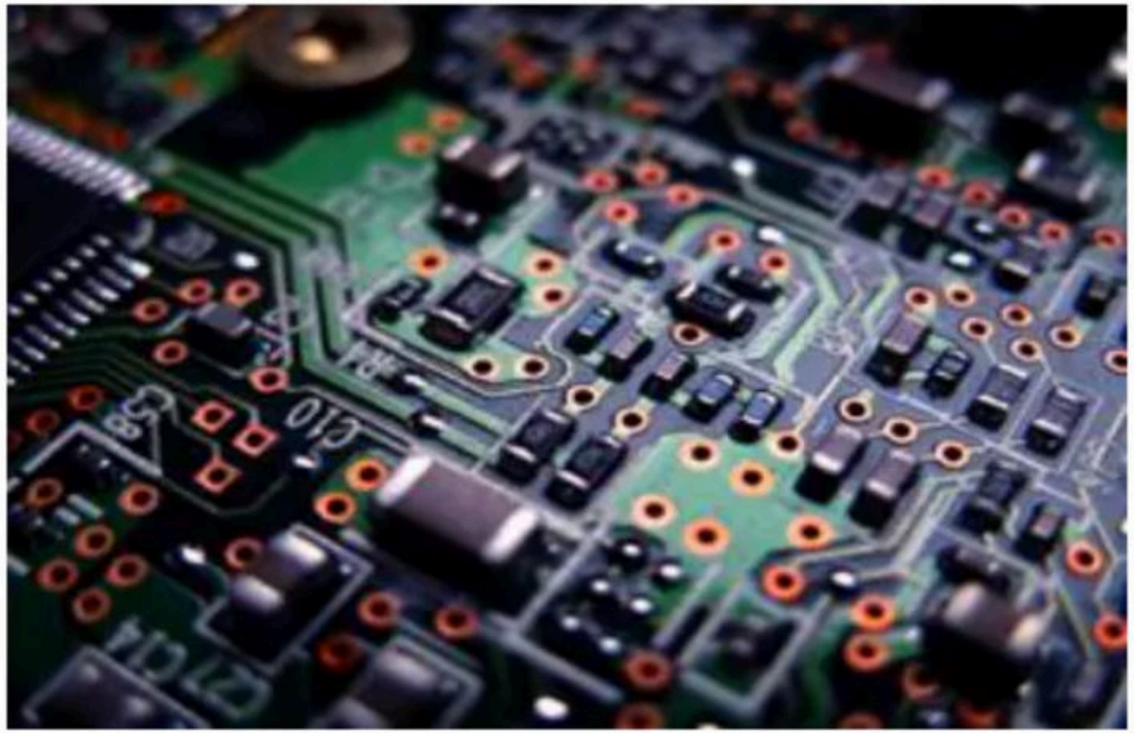
- **Electrical engineering** is a professional engineering discipline that generally deals with the study and application of electricity, electronics and electromagnetism and heavy voltage devices like transformers, motors etc.



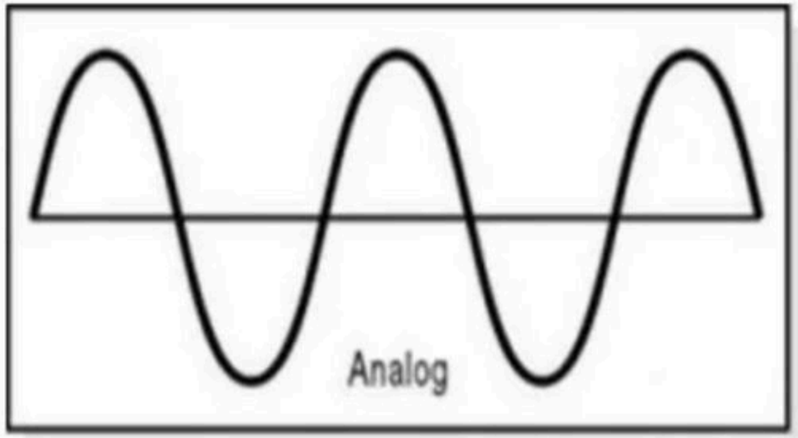


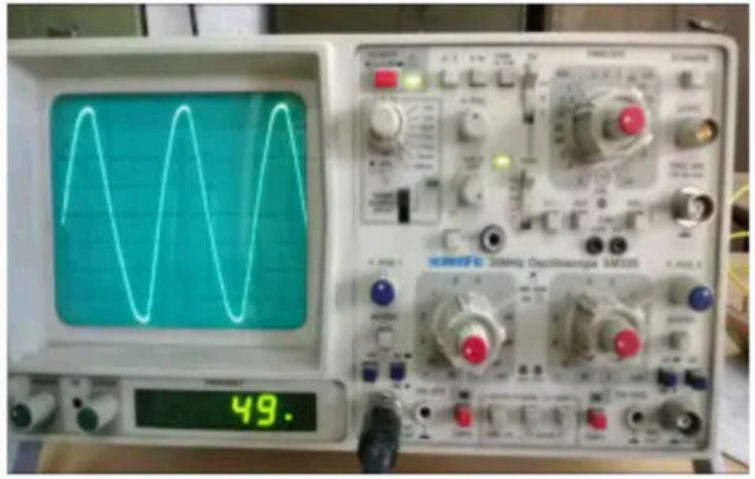
- **Electronic engineering** is an electrical engineering discipline where we work on low voltage devices (such as semiconductor devices, especially transistor, diodes and integrated circuits) to design electronic circuits, VLSI devices and systems.

EE → EC → CS → IT ??

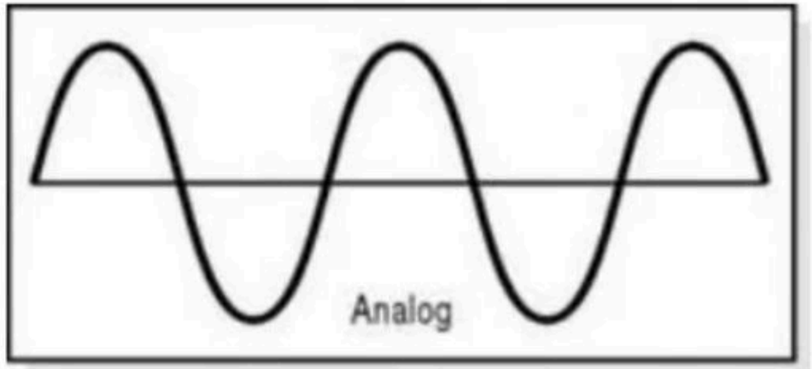


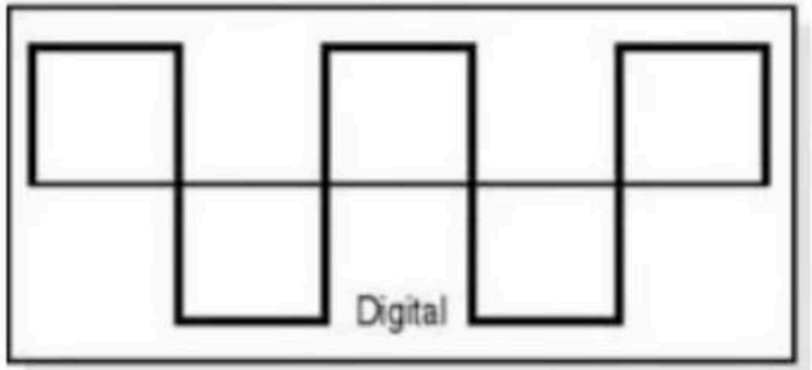
- Electronic systems are generally of two types –
  - **Analog system** in an analog representation, a continuous value is used to denote the information. Analog signal is defined as any physical quantity which varies continuously with respect to time. e.g. amplifier, cro, ecg. Watch, radio.





- **Digital system** – the information is denoted by a finite sequence of discrete value or digits. Digital signal is defined as any physical quantity having discrete values. E.g. digital watch, calculator, bp machine, thermometer etc.





160 AD 110 AD 150 AD

0.3 → Mp

VGA

66..

1.3 mp

Advantage of Digital system

1) Digital system are easy to design because they do not require sound engineering and mathematical knowledge, uses only switching circuit.

2) Storage for long time and processing of information is easy.

3) They provide better accuracy and precision compared to analog devices, as digital devices are less affected by noise, sound, electric or magnetic field etc.

4) Better modularity and easy fabrication. It means the of digital devices are very small compared to analog devices. E.g. integrated circuits.

5) Less cost.

Disadvantage of Digital system

- Only analog signal is available in the real world, so an extra hardware is required to convert this analog signal to digital signal by using analog to digital converter.

Conclusion

- Digital systems have such a prominent role in everyday life that we refer to the present technological period as digital age.

- Digital system are used in communication, business, transactions, traffic control, space guidance, medical treatment, weather forecasting, the internet, and many other commercial, industrial and scientific enterprises.

Digital India
Power To Empower

Note

- Early digital computers were used for numeric computation. The discrete elements were the digits, from this application, the term digital computer emerged.

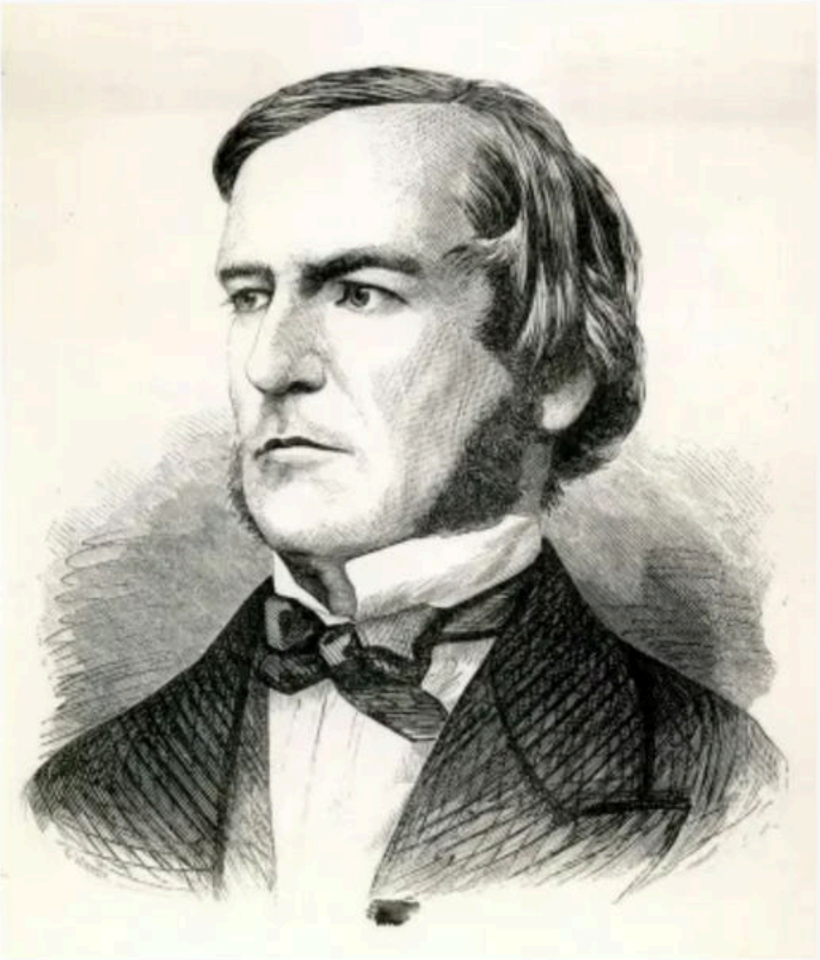
Break

Boolean algebra

- The signal in most present day electronic digital system uses just two discrete values and are therefore said to be binary.

- Boolean algebra was introduced by George Boole in his first book The Mathematical Analysis of Logic (1847), and set forth more fully in his An Investigation of the Laws of Thought (1854).

- George boole introduced the concept of binary number system in the studies of the mathematical theory of logic and developed its algebra known as Boolean algebra.



Boolean algebra

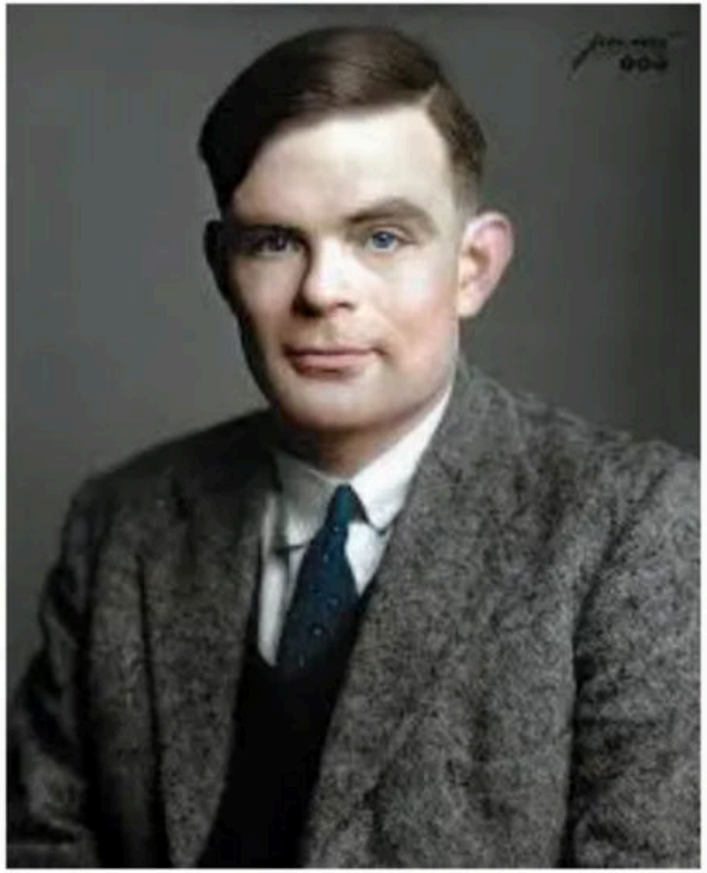
1. In mathematics and mathematical logic, Boolean algebra is the branch of algebra in which the values of the variables are the truth values true and false, usually denoted 1 and 0 respectively.

2. Instead of elementary algebra where the values of the variables are numbers, and the prime operations are addition and multiplication. The main operations of Boolean algebra are
   - The conjunction and denoted as $\wedge$
   - The disjunction or denoted as $\vee$
   - The negation not denoted as $\neg$

Turing Machine

- *The Church-Turing thesis states that any algorithmic procedure that can be carried out by human beings/computer can be carried out by a Turing machine.(1936)*

- It has been universally accepted by computer scientists that the Turing machine provides an ideal theoretical model of a computer.

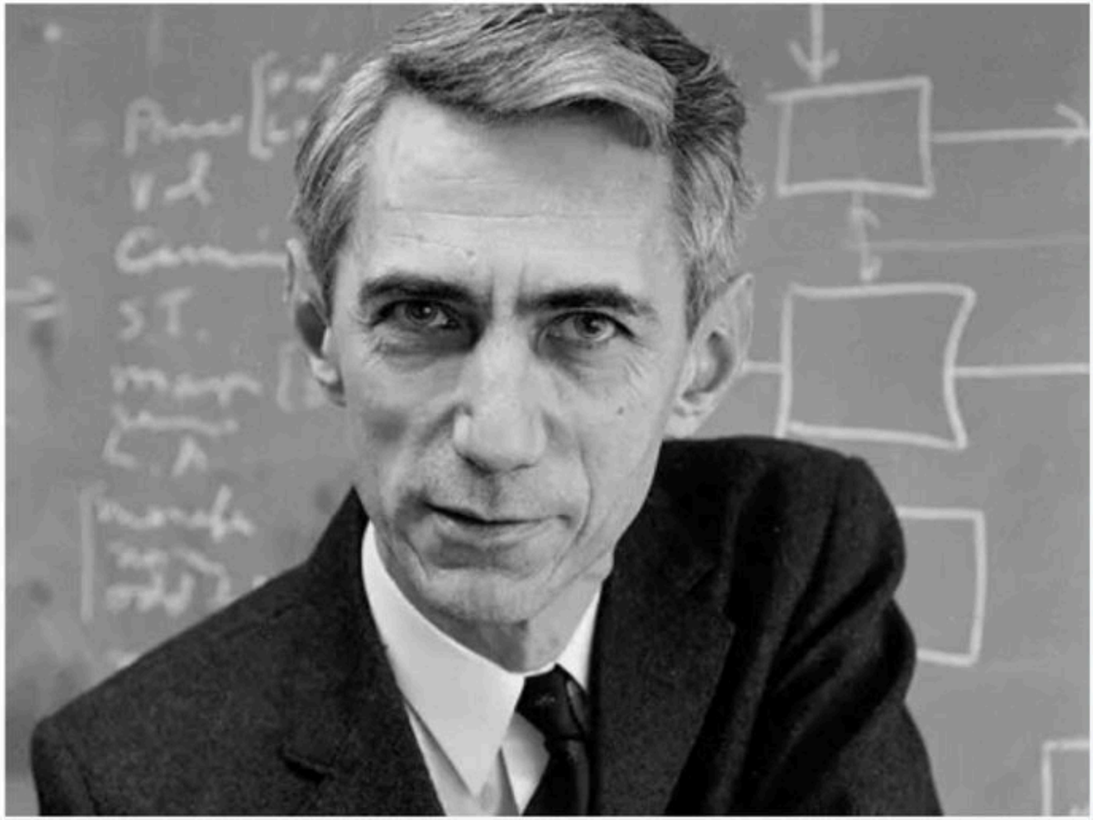


Digital System

1930

- In the 1930s, while studying switching circuits, Claude Shannon observed that one could also apply the rules of Boole's algebra in this setting, and he introduced switching algebra as a way to analyze and design circuits by algebraic means in terms of logic gates.

- These logic concepts have been adapted for the design of digital hardware since 1938 Claude Shannon (father of information theory), organized and systematized Boole's work.



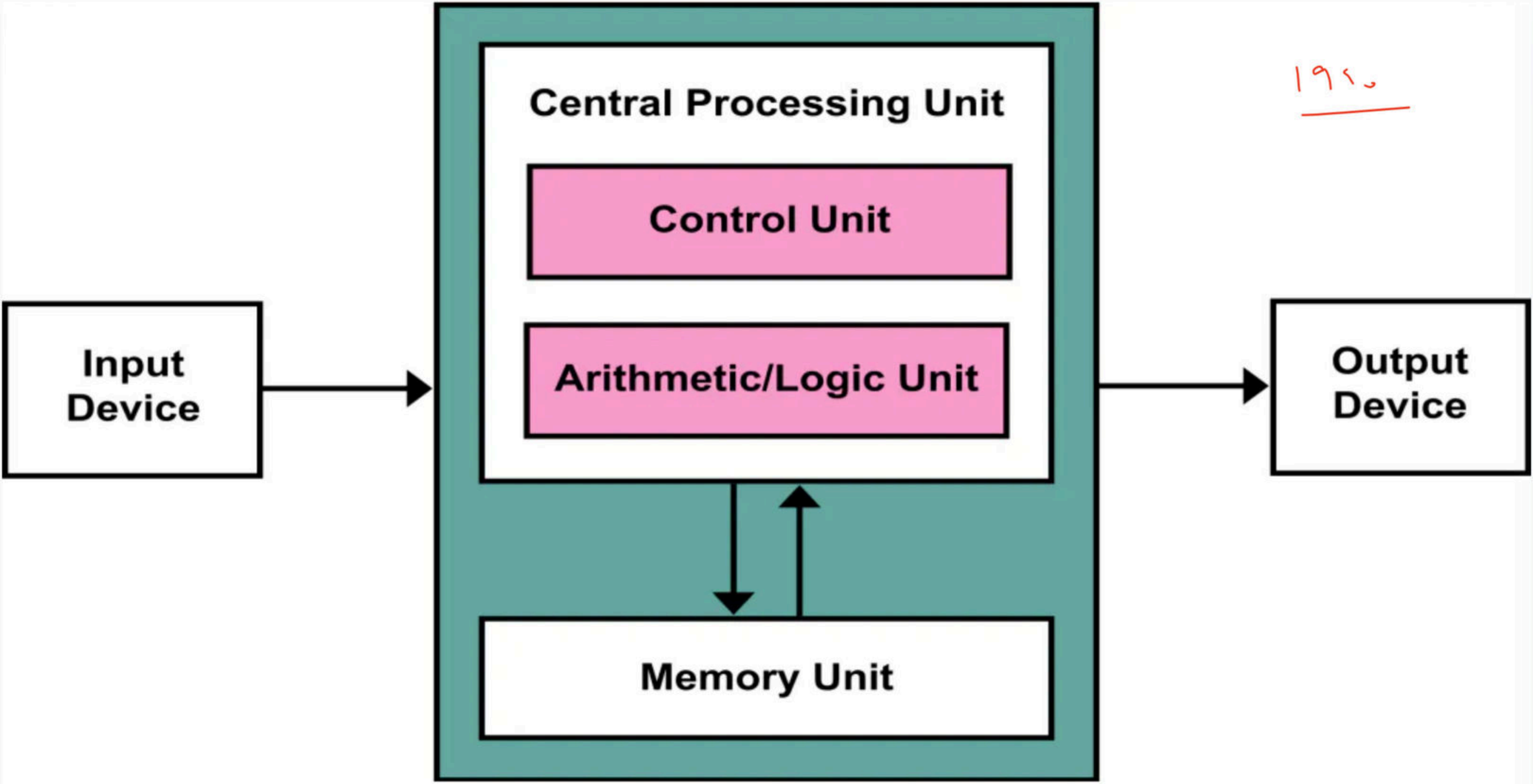
1) Shannon already had at his disposal the Boolean algebra, thus he cast his switching algebra as the two-element Boolean algebra. Efficient implementation of Boolean functions is a fundamental problem in the design of combinational logic circuits.

2) Boolean constants are denoted by 0 or 1. Boolean variables are quantities that can take different values at different times. They may represent input, output or intermediate signals.

3) Here we can have n number of variables usually written as a, b, c…. (lower case) and it satisfy all Boolean laws, which will be discussed later.

von Neumann architecture

- The **von Neumann architecture** is a computer architecture based on a 1945 description by John von Neumann. That document describes a design architecture for an electronic digital computer with these components:
  - A processing unit that contains an arithmetic logic unit and processor registers
  - A control unit that contains an instruction register and program counter
  - Memory that stores data and instructions
  - External mass storage
  - Input and output mechanisms


Central Processing Unit
Control Unit
Arithmetic/Logic Unit
Input Device
Output Device
Memory Unit

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Break

- Digital systems have two logic levels
  - The lower voltage level is called a logic low or logic 0 (0-1 v), which represent digit 0.
  - The higher voltage level is called a logic high or logic 1 (3.5-5 v), which represent digit 1.

Digital system designing

- Let’s try an example of designing a digital device, using this example, we will understand step by step process to design a digital system starting from problem statement.

Q Design a digital system for a car manufacturing company, where we want to design a warning signal for a car, there are three inputs, lights of the car(L), day or night(D), ignition (on/off).?

Solution

1) Understand the problem- here we will Understand the definition of the problem

Design the truth table –

Light	Day	Engine	Warning
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

2) Write the Boolean expression

- $W(L, D, E) = \sum_m (1, 4, 6, 7)$

SOP

1 – $\bar{L}\bar{D}E$

4 – $L\bar{D}\bar{E}$

6 – $LD\bar{E}$

7 – LDE

- $W(L, D, E) = \prod_M (0, 2, 3, 5)$

POS

	Light	Day	Engine	Warning
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

3) Minimize Boolean expression

$$W = \cancel{a'}\,\cancel{b'}\,\cancel{c} + ab'c' + abc' + abc$$

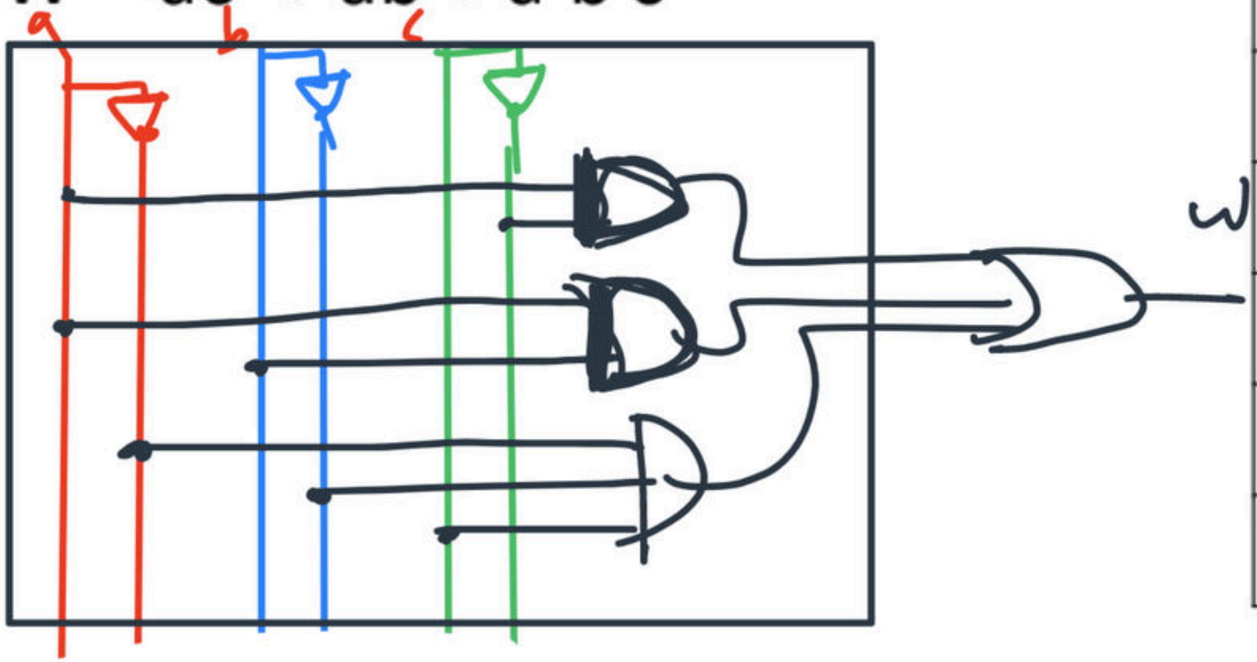
	ab	a'b'	a'b	ab	ab'
c		00	01	11	10
c'	0	0	2	1 6	1 4
c	1	1 1	3	1 7	5

$$W = \bar{a}\bar{b}c + ab + a\bar{c}$$

Light a	Day b	Engine c	Warning
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

4) Implement the expression using logic gates, draw the implementation using logic gates

$W = ac' + ab + a'b'c$



Light	Day	Engine	Warning
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Break

Boolean Algebra Laws

Idempotent Law

Associative law

Commutative law

Distributive law

Boolean Algebra Laws

Idempotent Law

$a \cdot a = a$

$a + a = a$

Associative law

$a. (b. c) = (a. b). c$

$a + (b + c) = (a + b) + c$

Commutative law

$a. b = b. a$

$a + b = b + a$

Distributive law

$a. (b + c) = a. b + a. c$

$a + (b. c) = (a + b). (a + c)$

De-Morgan law

Identity law

Complementation law

Involution law

De-Morgan law

$(a + b)' = a' . b'$

$(a. b)' = a' + b'$

Identity law

$a + 0 = a$

$a. 0 = 0$

$a + 1 = 1$

$a. 1 = a$

Complementation law

$0' = 1$

$1' = 0$

$a. a' = 0$

$a + a' = 1$

Involution law

$(a')' = a$

Break

Q The idempotent law in Boolean algebra says that: **(NET-JUNE-2008)**

(A) $\sim(\sim x) = x$

(B) $x + x = x$

(C) $x + xy = x$

(D) $x(x + y) = x$

Q A V A = A is called: **(NET-DEC-2004)**

(A) Identity law

(B) De Morgan's law

(C) Idempotent law

(D) Complement law