

**“NATURE ISN'T CLASSICAL, DAMMIT, AND IF YOU WANT TO MAKE A SIMULATION OF NATURE, YOU'D BETTER MAKE IT QUANTUM MECHANICAL, AND BY GOLLY, IT'S A WONDERFUL PROBLEM, BECAUSE IT DOESN'T LOOK SO EASY.”**

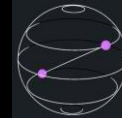
**RICHARD P. FEYNMAN**

# Running Shor's Algorithm on IBM Quantum Experience

*Presentation By:*  
**Pinakin Padalia**  
*Student No. 4743644*

&

*Amitabh Yadav*  
*Student No. 4715020*

 IBM Q

QISKit  
Quantum Information Software Kit

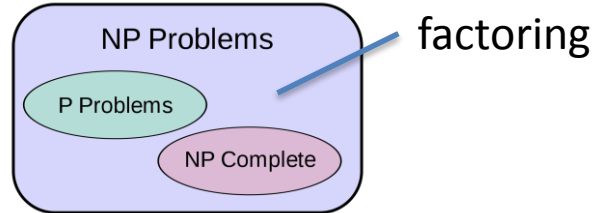
**Developed By: Pinakin Padalia & Amitabh Yadav**

# Agenda

- The Factoring Problem
- Shor's Algorithm for Factoring Integers
- Quantum Circuit for Period Finding
- IBM Quantum Experience
- Running Shor's Algorithm on IBM-Q:  
Methodology and Challenges
- Design Space Exploration
- Results & Conclusion

# The Factoring Problem

- RSA uses a public key  $N$  which is the product of two large prime numbers.
- One way to crack RSA encryption is by factoring  $N$ , but with classical algorithms, factoring becomes increasingly time-consuming as  $N$  grows large.
- **RSA-1024** has 1,024 bits (309 decimal digits), and has not been factored so far.



- No classical algorithm is known that can factor in polynomial time.
- Shor's (Quantum) Algorithm can crack RSA in polynomial time.

# Shor's Algorithm



Peter Shor

Shor's algorithm is a quantum algorithm for factoring a number  $N$  in  $O(n^3)$  time, named after Peter Shor.

Factor a number into primes:

$$M = p * q$$

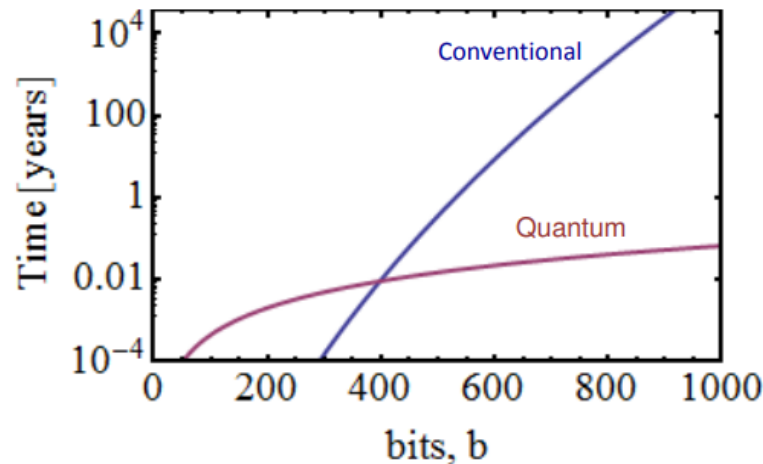
How long will it take ? (t)

Classical

$$t \sim O(2^{n^{1/2}})$$

Quantum

$$t \sim O(n^3)$$



Source: <http://www.ibm.com/>

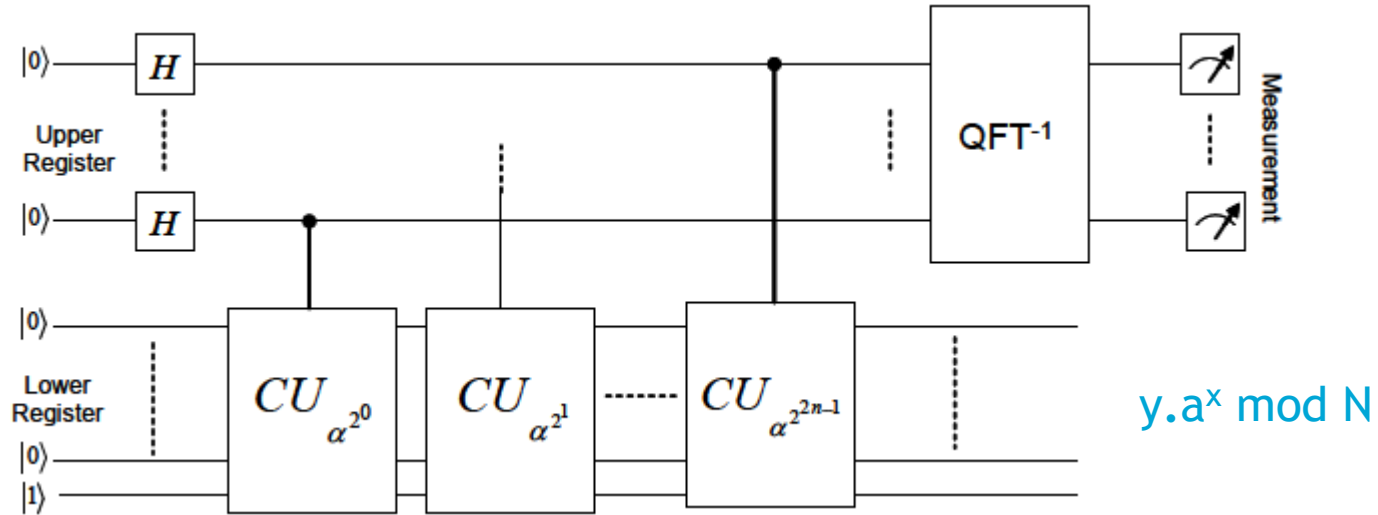
# Shor's Algorithm: Implementation

Number to be factored (let),  $N = 15$ .

1. If  $N$  is even/integer power of prime number  $\rightarrow$  can be factored classically.
2. Choose a random integer  $a \in [2, \dots, N-1]$ , (let)  $a = 11$  and compute  $t = \text{GCD}(a, N)$ .
3. If  $t > 1$ ,  $t$  is a factor; else if  $t = 1$  (here  $t = 1$ )
4. Find  $r = \text{period}(a^x \bmod N)$  [Using Shor's Algorithm] (here  $r = 2$ )
5. If  $r$  is odd  $\rightarrow a^{r/2} + 1 = 0 \bmod N \rightarrow$  Cannot Infer Factors. Go To Step 2.
6. Else If  $r$  is even,  
factor\_1 =  $\text{gcd}(a^{r/2} + 1, N) = 3$   
factor\_2 =  $\text{gcd}(a^{r/2} - 1, N) = 5$

Challenge: Period Finding!

# Quantum Circuit for Period Finding (Shor's Algorithm)

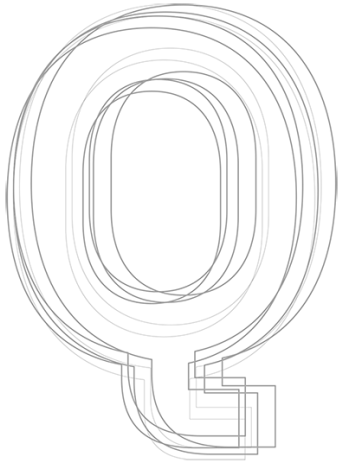


$$a^x \bmod N = (a^{2^0} \bmod N)^{x_0} \cdot (a^{2^1} \bmod N)^{x_1} \cdots (a^{2^{2n-1}} \bmod N)^{x_{2n-1}} \bmod N$$

Ref: Fast Quantum Modular Exponentiation Architecture for Shor's Factorization Algorithm [4]

# IBM Quantum Experience

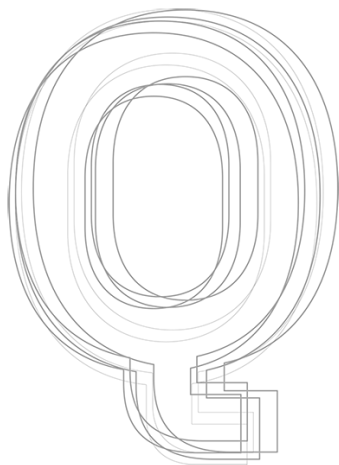
<http://research.ibm.com/ibm-q/>



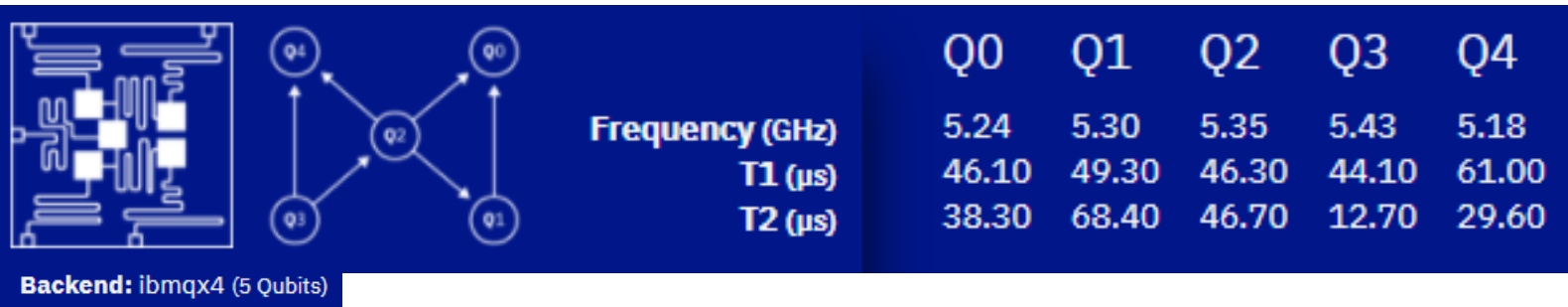
- First commercially available Quantum Computer and Developer Ecosystem
- Simulation Tools
- Quantum Experiments through cloud
- QASM Programming
- QISKit SDK  
(simulation and quantum execution using Python API)
- Active User Community  
([qiskit.slack.com](https://qiskit.slack.com))



# QISKit



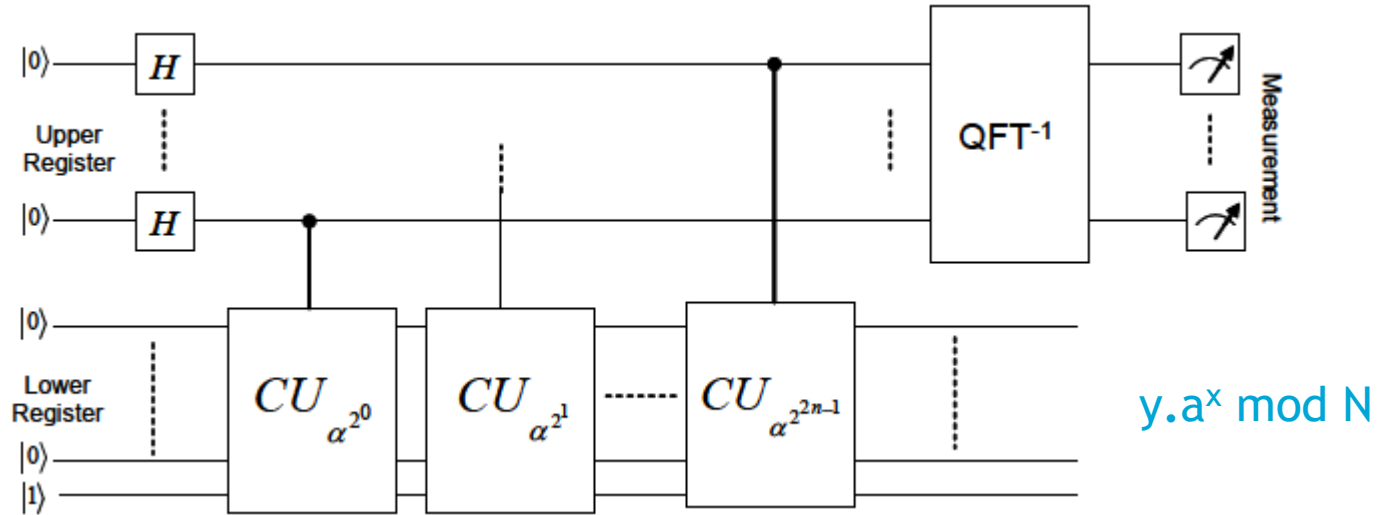
- Python API and SDK
- Contains:
  - QISKit SDK** : Python Interface for programming
  - QISKit API** : Python Wrapper to connect to IBM's Quantum Chip.
  - QISKit OpenQASM** : Execute OPENQASM code from Python
- Enables working with quantum circuits
- Quantum Processor: **Raven (ibmqx4)**



# Shor's Algorithm on IBM Q: Design Space Exploration

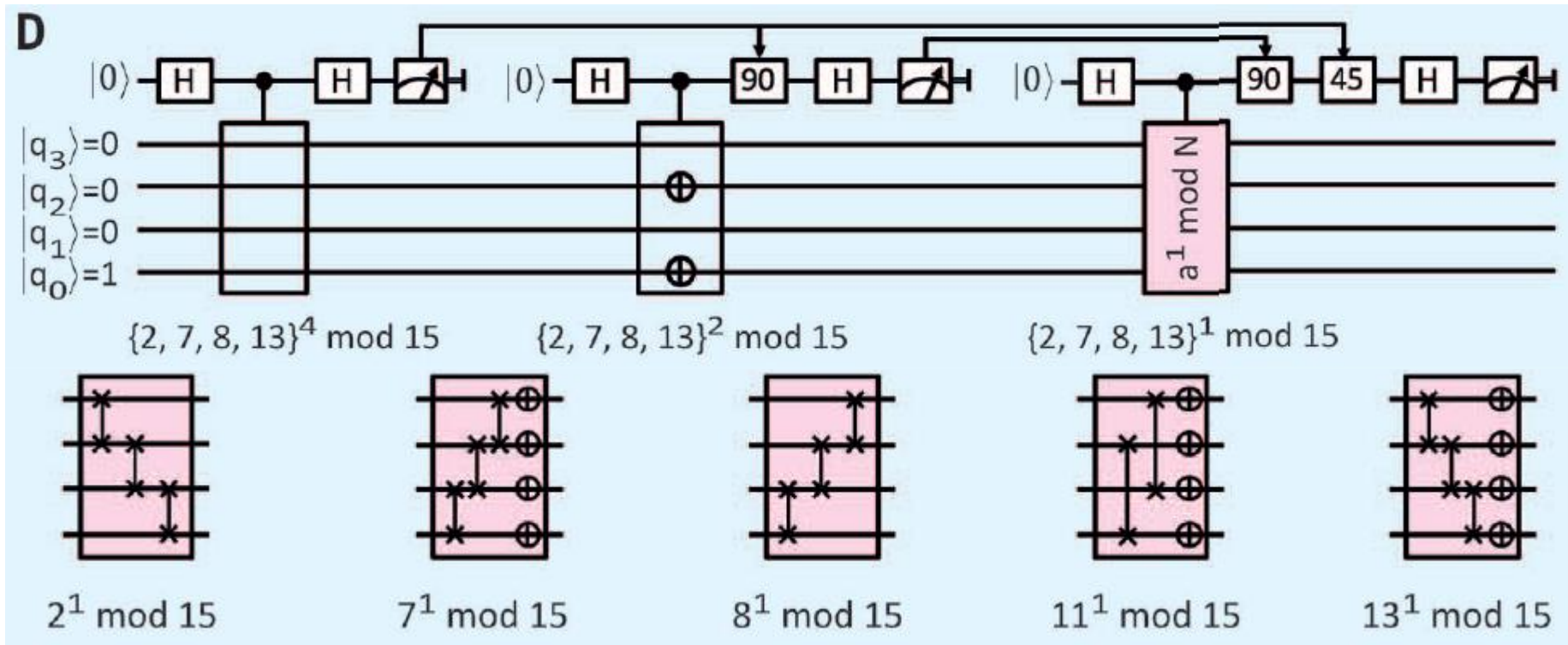
- 12 qubit simulation
- 5 qubit simulation
- 5 qubit hardware - general and particular ( $a = 11$ )

# Quantum Circuit for Period Finding (Shor's Algorithm)



$$a^x \bmod N = (a^{2^0} \bmod N)^{x_0} \cdot (a^{2^1} \bmod N)^{x_1} \cdot \dots \cdot (a^{2^{2n-1}} \bmod N)^{x_{2n-1}} \bmod N$$

# Running Shor's Algorithm on IBM Quantum Experience

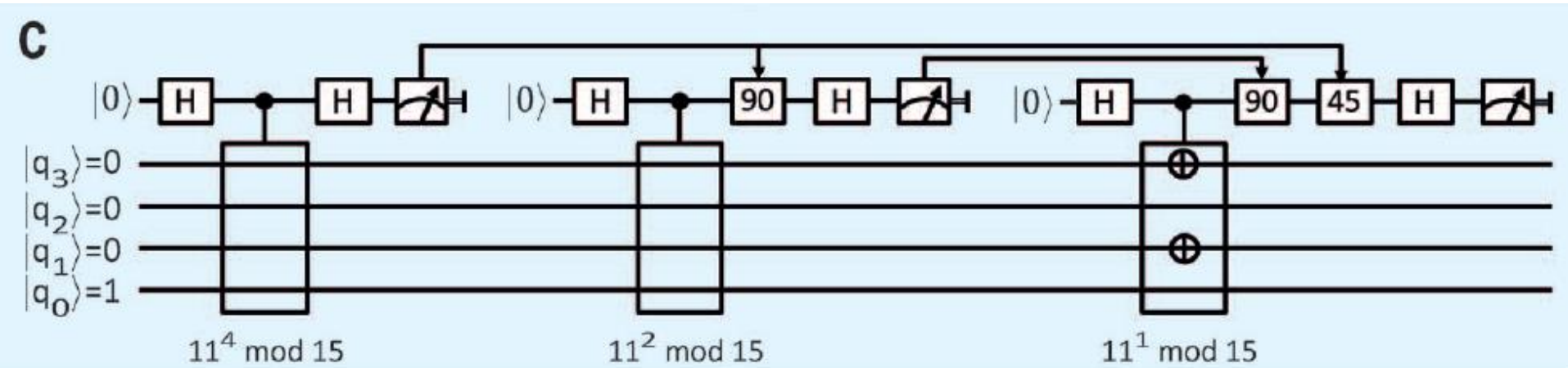


Quantum Circuit for Period Finding (Qubit Recycling) [2]

Realization of a scalable Shor's algorithm

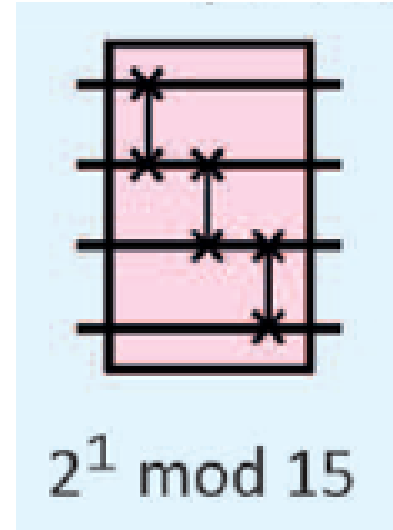
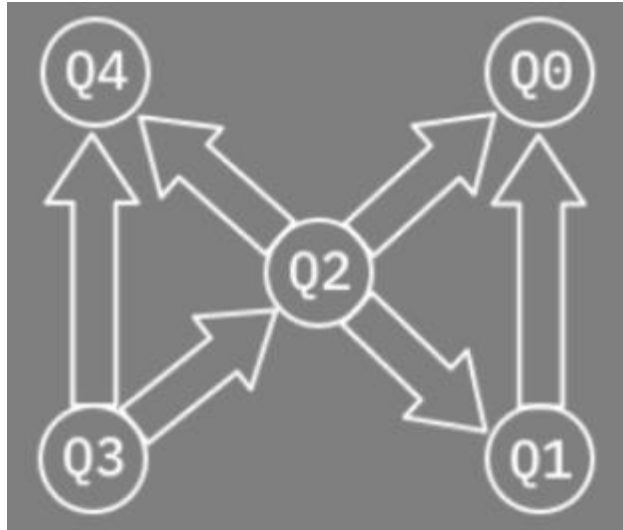
Developed By: Pinakin Padalia & Amitabh Yadav

# Shor's Algorithm on IBM Q (ibmqx4)

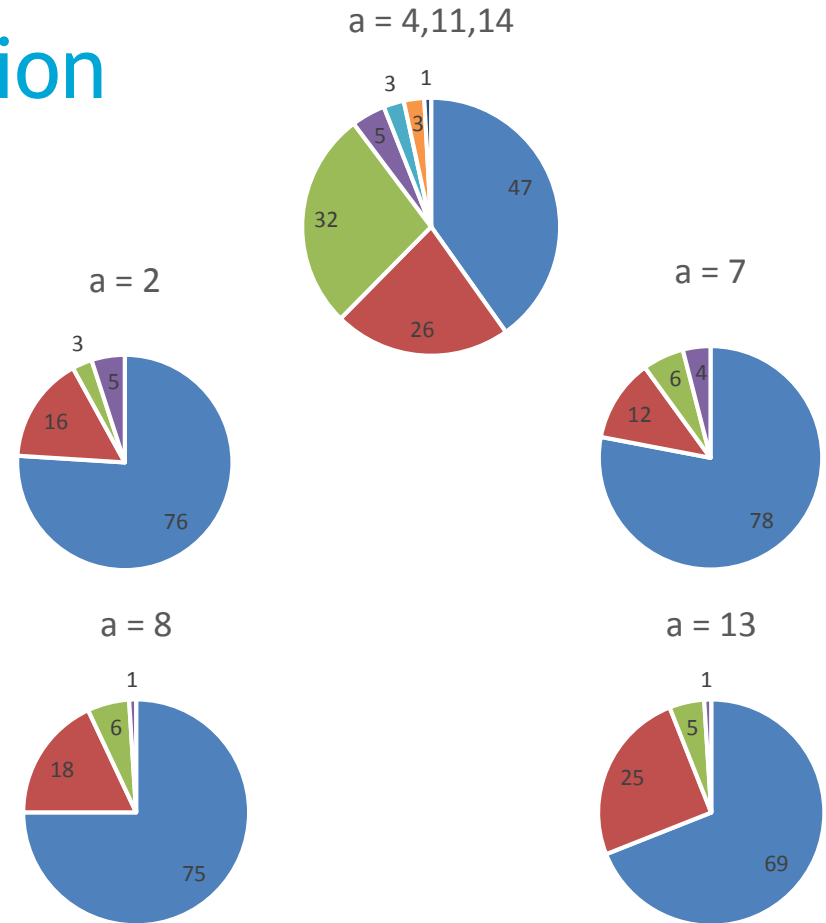
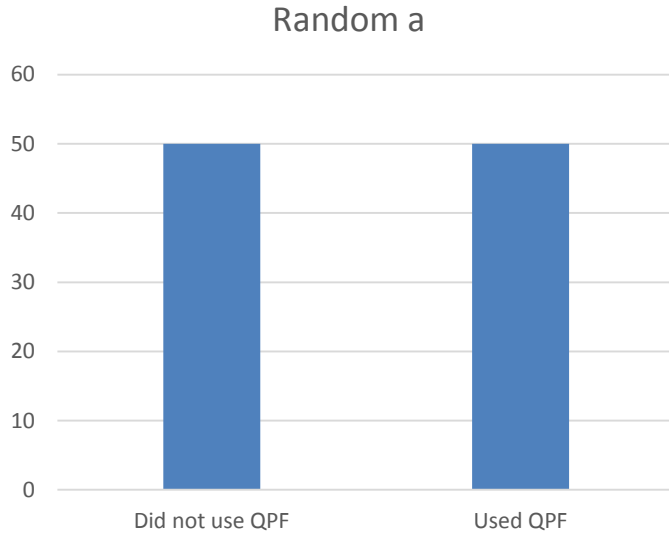


# Shor's Algorithm on IBM Q: Challenges

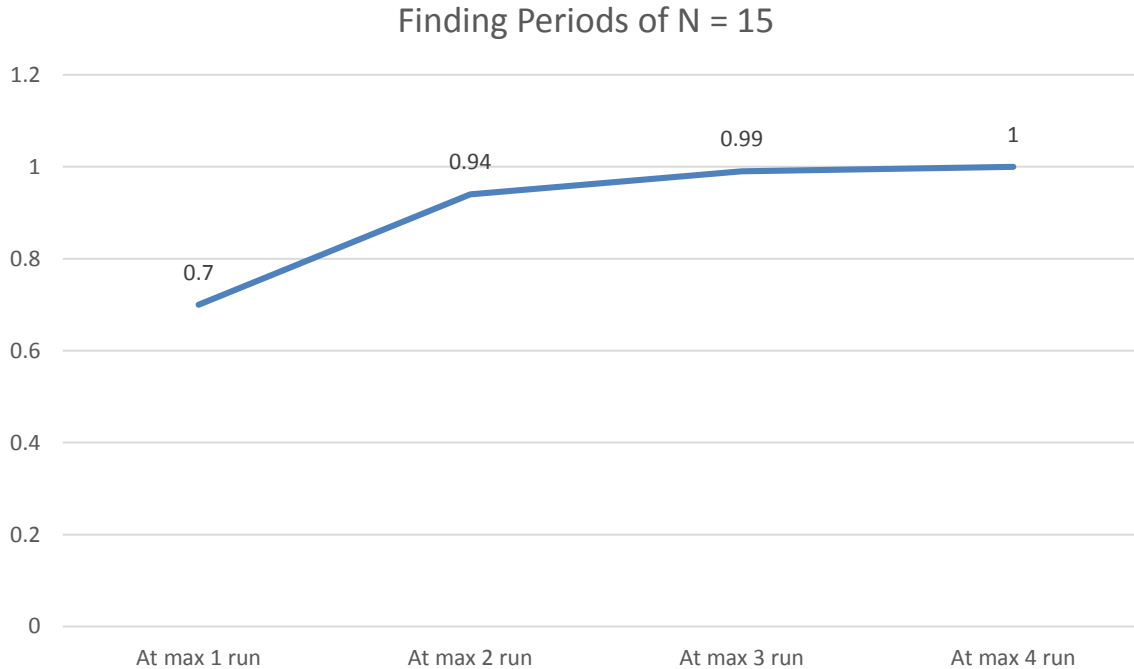
- Installation!
- Coupling Map!



# Results - 5 qubit simulation

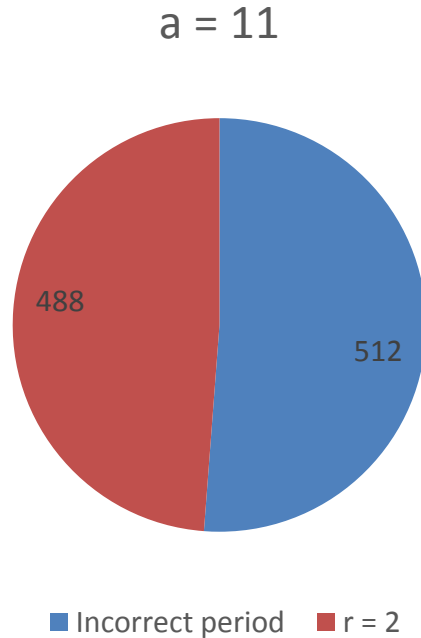


# Results - 5 qubit simulation





# Results - 5 qubit Hardware ( $a = 11$ )



# Conclusion

- Algorithm runs - 5 qubit hardware and simulation.
- Challenges -
  - scalability
  - fidelity
  - generality



# References

- 1) Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer. arXiv:quant-ph/9508027
- 2) Monz, T., Nigg, D., Martinez, E.A., Brandl, M.F., Schindler, P., Rines, R., Wang, S.X., Chuang, I.L. and Blatt, R., 2016. Realization of a scalable Shor algorithm. Science, 351(6277), pp.1068-1070.
- 3) A. Y. Kitaev, <http://arxiv.org/abs/quant-ph/9511026> (1995).
- 4) Fast Quantum Modular Exponentiation Architecture for Shor's Factorization Algorithm. arXiv:1207.0511
- 5) Circuit for Shor's algorithm using  $2n+3$  qubits. arXiv:quant-ph/0205095
- 6) IBM Quantum Experience Documentation/Full User Guide. <https://quantumexperience.ng.bluemix.net/qx/tutorial?sectionId=full-user-guide&page=introduction>
- 7) <https://github.com/QISKit/qiskit-tutorial>
- 8) <https://github.com/QISKit/qiskit-sdk-py>