# The Art of Active Listening

1[st] Zvi Lotker
*Faculty of Engineering*
*Bar Ilan University*
Ramt-Gan 52900, Israel
email:zvi.lotker@gmail.com, https://orcid.org/0000-0002-3759-5584

***Abstract*—This paper initiates the study of the art of listening from a social network perspective. A conversation between several people is a significant human activity. We propose a novel structure, the directed conversation hypergraph, to capture the dynamic interplay between a speaker and a listener and further explore its evolution over time.**

**We validate our framework using drama scripts written in the Moliere format, converting them into evolving directed conversation hypergraphs. A key element of our research introduces the degree martingale for directed hypergraphs, an innovative function that helps gauge listening skills.**

***Index Terms*—social network, martingale, literature theory, hypergraphs, psychology, active listening.**

## I. Introduction

Listening is a key element in many fields: medical care, sales, psychology, management, etc. There are many pieces of advice, courses, and books about the art of listening. For example, J. Brownell described the importance of active listening for managers, [9], and Wolvin & Coakley wrote about listening training for directors of the Fortune 500 industrial and the Fortune 500 service corporations [21]. They all claim that improving your listening skills is crucial for your social life.

In this paper, we initiate the study of listening skills from a social network perspective. We suggest two new terms for a directed hypergraph for a node $i$: the "listening degree" ($ld_i$) and the "speaking degree" ($sd_i$). These terms extend the concepts of in-degree $d_i^{in}$ and out-degree $d_i^{out}$, respectively, from traditional directed graph theory to directed hypergraph contexts. We then explain combining those two functions into the degree martingale $dm_i$. We show that the degree martingale is a good measure of the ability of nodes to listen in the social network hypergraph. To demonstrate the use of the listening martingale, we introduce the who-spoke-to-whom directed hypergraph and compute the degree martingale on the four main characters of "The Misanthrope" by Moliere, see [17].



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## II. Previous Work

In recent years, some papers suggested looking at the social network through the lens of a social hypergraph. For a general introduction and overview of the hypergraph theory, see [19]. The study of simple random walks on hypergraphs was initiated by [5]. From the perspective of social networks, the Barabasi model of preferential attachment was extended in analysis on hypergraph [4], [6], [12].

The idea of using algorithms to study literature on a corpus of massive narratives was popularised by Franco Moretti. For example, in the paper "Conjectures on World Literature" [18], he poses questions and thoughts about the dynamics of the global literary sphere. Orthogonal research direction for studying a single literature work with social network tools, such as conflict function and subjective clock, was developed in [14]–[16]. This paper continues this approach by suggesting studying the directed conversation hypergraph.

Bolelli et al. used graphs to analyze scientific literature by linking terms and separating distinct groups of documents with an improved clustering technique, see [8]. The problem of the rapidly growing amount of scientific publications was also discussed by Chen and Luo, who proposed to use graphs to analyze the current situation and trends of a specific research field [10]. Erten et al. presented a system for visualization of the evolution of the computing literature using a novel graph drawing technique for visualization of large graphs with a temporal component [11].

Beck et al. devoted their paper to an interactive visual analytics system SurVis that was designed as a versatile tool for authors to structure and analyze the references of their survey [7]. Andrews and Macé used graph analysis to build an empirical model of scribal text variation [3]. Thornton and Lee described the opportunities of using the funnel graphs to demonstrate publication bias [20]. For more examples, interested readers can refer to an article by Janicke et al., who presented an overview of the research conducted since 2005 on supporting text analysis tasks with close and distant reading visualizations in the digital humanities, see [13].

## III. The Model

We follow the notation from the book [16]. In this paper, we will assume that we are given a script for a drama in the format of Moliere's. In Moliere's works, all the characters appear at the beginning of a play. Following Moliere, we denote a set of characters in the play to be $V$.

### A. Moliere Scripts

After introducing the set of characters $V$, each scene starts with the list of characters that are on the stage and then the text itself. This means that a script, according to Moliere

$$MSc = (l_i)_{i=1}^{\tau} \tag{1}$$

is a finite sequence of lines $l_i$, where $\tau$ is the number of responses in a complete play. Each line

$$l_i = (c_i, h_i, w_i) \tag{2}$$

is a triple, composed of the character $c_i \in V$ that speaks the line, the set of characters $h_i \subset V$ that on the stage and hear the line, and the words $w_i \in english$, where "english" is a set of all paragraphs that can be spoken in English up to one million characters, since no play of Moliere contains more than a million characters.

For the paper to be more precise, we select Act III, Scene II from Moliere's "Misanthrope" as a running example. Therefore, we will use this scene as an example of Moliere's script $MSc$. Since our example has a single Scene, the set of characters is

$$V_{3,2} = \{ACASTE, CELIMENE, CLITANDRE, \}. \tag{3}$$

In this Scene, there are three responses, so in this example

$$MSc = \begin{pmatrix} \text{CELIMENE} & V_{3,2} & \text{Still here?} \\ \text{CLITANDRE} & V_{3,2} & \text{Love stays our steps.} \\ \text{CELIMENE} & V_{3,2} & \text{I heard just now...} \end{pmatrix} \tag{4}$$

### B. Hypergraph

According to the Encyclopedia of Mathematics in Springer, see [1], the hypergraph is a generalization of the concept of a Graph. A hypergraph is defined by a set $V$, whose elements are known as vertices, and by a family $\mathcal{E}$ of subsets of $V$, known as edges or hyperedges. A hypergraph is denoted by $(V, \mathcal{E})$.

Two vertices of a hypergraph are said to be adjacent if there exists an edge containing these vertices. A vertex $v$ and an edge $e \in \mathcal{E}$ of a hypergraph are said to be incident if $v \in e$. A hypergraph $H$ with $n$ vertices and $m = |\mathcal{E}|$ edges may be defined by an incidence matrix $Ic = [ic_{i,j}]$, i.e. by the matrix of dimension $n \times m$ in which the columns correspond to the edges $(E_j)_{i=1}^{m}$, while the rows correspond to the vertices of the hypergraph, and where for all $i = 1 \ldots n,\ j = 1, \ldots, m$;

$$ic_{i,j} = \begin{cases} 1 & \text{if } v_i \in E_j, \\ 0 & \text{if } v_i \notin E_j, \end{cases} \tag{5}$$

Similar to the graph, there are versions of a directed hypergraph. Formally, a directed hypergraph is a pair $\mathcal{D} = (V, \mathcal{E})$ where, $V$ is a set of elements called nodes, vertices, points, or elements and $\mathcal{E}$ is a set of pairs of subsets of $V$. Each of these pairs $(S, T) \in \mathcal{E}$, is called an edge or hyperedge; the vertex subset $S$ is known as its source or speaker, and $T$ is called a target audience.

We can also represent a directed hypergraph as an $Ic = [ic_{i,j}]$ incident matrix, where the out-going edges will be denoted as $-1$ and the in-going will be $+1$ formally: A vertex $v$ and an directed edge $h = (S, T)$ of a directed hypergraph $D(V, \mathcal{E})$ are said to be incident if $v \in S \cup T$. A hypergraph $H$ with $n$ vertices and $m$ directed edges may be defined by an incidence matrix, i.e. by the matrix $[ic_{i,j}]$ of dimension $n \times m$ in which the columns correspond to the directed edges while the rows correspond to the vertices of the hypergraph, and where for all $i = 1 \ldots n,\ j = 1, \ldots, m$;

$$ic_{i,j} = \begin{cases} -1 & \text{if } v_i \in S_j, \\ 0 & \text{if } v_i \notin S_j \cup T_j, \\ +1 & \text{if } v_i \in T_j, \end{cases} \tag{6}$$

Note that we assume that the sets $S_j, T_j$ are disjoint. Finally, we defined direct weighted hypergraphs to be

$$Hw = (V, \mathcal{E}, W), \tag{7}$$

where $V$ is a set of nodes of $Hw$, $\mathcal{E}$ is a set of directed edges of $Hw$, and the weight function $W$ defined by

$$W : \mathcal{E} \to \mathbb{R} \tag{8}$$

Sometimes direct weighted hypergraphs are called multi-direct hypergraphs, where edges are allowed to appear many times. This is very useful when analyzing conversation in general. However, the mathematical framework of a weighted hypergraph is more robust, and we adopt a weighted hypergraph.

### C. From Script to the Directed Conversation Hypergraph

Next, we explain how to transform the Moliere script $MSc$ into a *directed weighted conversation hypergraph*

$$\mathcal{D}(MSc) = (V, \mathcal{E}, W). \tag{9}$$

The set of all Nodes of $\mathcal{D}$ is denoted by $V$ and is equal to the set of all characters in the play. Formally,

$$V = \cup_{i=1}^{\tau} c_i \cup h_i \tag{10}$$

For each line $l_i$ we add a single directed edges $(c_i, h_i)$ to the directed conversation hypergraph $\mathcal{H}$, formally

$$\mathcal{E} = \cup_{i=1}^{\tau} (c_i, h_i). \tag{11}$$

Therefore, the rows of the incident matrix of the conversation hypergraph in our running example represent characters according to alphabetical order and columns are the three hyperedges in the Scene.

$$[ic_{i,j}] = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \tag{12}$$

### D. Hypergraphs and Matrices

In general, weighted graphs and matrices are similar objects. Since there is a one-to-one correspondence between the graphs, it is a weighted adjacency matrix. There are advantages to working with graphs, and there are advantages to working with matrices. We will consider them as equivalents and mainly work with representations since one can use linear algebra, and calculus of different variables, which is more

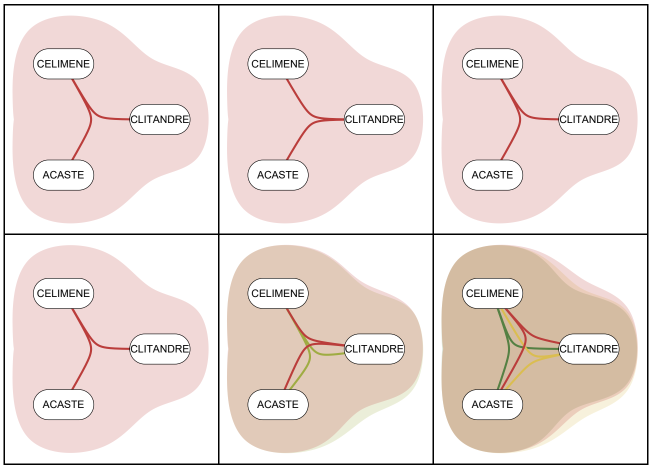


Fig. 1. Figure represents the conversation hypergraph of Act III, scene 2 in Misanthrope. This scene contains three responses: the first is by Celimene, the second is by Clitandre, and the third is by Celimene again. Acaste is a listener who doesn't speak during the scene. The speaker is connected by different red lines to each of the listeners. In the second row, we show the evolution of the accumulated hypergraph. The first hypergraph from the left contains the single hyperedge, the second hypergraph contains two hyperedges, and the last one contains three.

natural to do in matrices. When given a weighted graph $G$, it is a weighted adjacency matrix, and it will be denoted by $[G]$.

In general, graphs, hypergraphs, and matrices are similar objects when considering conversation graphs since one can transform any graph/(hypergraph) to its matrix representation either as incidents matrices or its adjacency matrices and vice versa. Therefore, whenever we do not care about the specific representation of the conversation hypergraph, we denote the space $\mathbb{G}$. By using $\mathbb{G}$, we mean that the reader can substitute $\mathbb{G}$ by the space of all square matrices $\mathbb{M}_{n,n}[\mathbb{R}]$ or the space of all hypergraphs, direct hypergraphs, and so on.

## IV. From Hypergraph to Evolving Hypergraph

Drama $Msc$, in particular, and conversation, in general, are embedded in time. Therefore there is a natural process for moving from the static model to the dynamic model, where edges arrive according to their order in the conversation. Mathematically, the evolving hypergraph $\gamma$ is a sequence of hypergraphs. Formally:

$$\gamma = (\gamma_1, \gamma_2 ..., \gamma_\tau) \tag{13}$$

where $\gamma_i = (V, \mathcal{E}_i, W_i)$ is an hypergraphs. The set of nodes $V$ is

$$V = \cup_{i=1}^{\tau} c_i \tag{14}$$

The sets of hyperedge $\mathcal{E}_i$ is:

$$\mathcal{E}_i = \{(c_1, h_1), ..., (c_i, h_i)\} \tag{15}$$

and the weights of the hyperedge $h \in \mathcal{E}_i$ is

$$W_i(e) = |\{j \in \mathbb{N} : 1 \leq j \leq i \text{ and } (c_j, h_j) = e\}| \tag{16}$$

To summarize, finite evolving hypergraph/(direct hypergraph) $\gamma$ is a sequence or function from the ordered set $T$ to the space of all hypergraph/(square matrices of dimension $n$). In this paper, we will assume $T = \{1, 2, ..., \tau\}$, where $\tau \in \mathbb{N}$ is the end of the evolution of the story. Formally in the algebraic language an evolving hypergraph

$$\gamma : T \to \mathbb{G} \tag{17}$$

Clearly, a sequence is a function, so the definition at the end of this section is the extension of the definition at the beginning of the section. However, the language of composition is better defined by function than by sequence. So, when we use the composition of a function, we use the function definition.

### A. Degree Martingales

It is well-known that the sum of degrees in the graph is even since each edge in the graph is counted twice. On the directed graph we have the sum of all out-degrees being equal to the sum of all in-degrees or, $\sum_{i \in V} d_{in}(i) - d_{out}(i) = 0$

However, when working with hypergraphs we need to balance the in-degree and out-degree by the size of the sets to do this we use the delta function.

$$\mathbb{1}_S(i) = \begin{cases} 0 \text{ if } i \notin S \\ 1 \text{ if } i \in S \end{cases} \tag{18}$$

Using the delta function $\mathbb{1}S(i)$, we define the out-degree martingale $dm_i^{out}(D)$ of the direct hypergraph $D$ as follows:

$$dm_i^{out}(D) = -\sum_{e=(S,T)\in\mathcal{E}} \frac{\mathbb{1}_S(i)}{|S|} \tag{19}$$

Next, we define the in-degree martingale $dm_i^{in}(D)$ of the node $i$ in the directed hypergraph $D$ to be the number of hyperedges node $i$ in the source of the edge, formally,

$$dm_i^{in}(D) = \sum_{e=(S,T)\in\mathcal{E}} \frac{\mathbb{1}_T(i)}{|T|} \tag{20}$$

Since each Hyperedge contrbute -1 to $dm_i^{out}$ and +1 to $dm_i^{in}$ it follwos that

$$\sum_{i\in V} dm_i^{in} + \sum_{i \in V} dm_i^{out} = 0 \tag{21}$$

The above equation explains why we use martingale in the definition of degree martingale.

Finally, let

$$dm_i(D) = dm_i^{in}(D) - dm_i^{out}(D). \tag{22}$$

The degree function of our running example is summarized in the matrix where columns represent time and rows represent characters according to alphabetical order.

$$[dm_{i,t}] = \begin{bmatrix} 0.5 & 1 & 1.5 \\ -1 & -0.5 & -1.5 \\ 0.5 & -0.5 & 0. \end{bmatrix} \tag{23}$$

The next section explains how to start Moliers's script and compute the evolving conversation hypergraph from it. One of the problems of evolving hypergraph is that it captures the full narrative, which is a high-dimensional complex object. Sometimes we wish to decompose the narrative into many simpler objects. This will allow us to concentrate on single characters and shade their behavior. In the next section, we will describe a general technique to do so.

## V. From Evolving Hypergraph to Real Functions

We use the language of function and decomposition to describe the general method. We already defined finite evolving hypergraph as a function $\gamma$ from the set $\{1, 2, ..., \tau\}$ to the space $\mathbb{G}$ of all complex objects. Our next ingredient is a general function $F$,

$$F : \mathbb{G} \to \mathbb{R}. \tag{24}$$

Once we have those two functions, $\gamma, F$, we can compose them. See the function diagram (25) where $f = F(\gamma)$.

$$\begin{tikzcd} \mathbb{T} \arrow[r, "\gamma"] \arrow[dr, "f"'] & \mathbb{G} \arrow[d, "F"] \\ & \mathbb{R} \end{tikzcd} \tag{25}$$

In the next section, we will give an example of a real function, that captures the ability to listen versus the ability to talk. We call this function the degree martingale. Formally,

$$dm_i(t) = dm_i(\gamma_t) \tag{26}$$

Note that the degree martingale function $dm_i(t)$ can be used to compute the ability to listen in an evolving conversation hypergraph $\gamma$ at time $t$.

In general, if $dm_i(t) < 0$, then the character $i$ speaks more than he listens at time $t$. On the other hand, if $dm_i(t) > 0$ the character $i$ listens more than he speaks. Therefore, the degree martingale function describes the ability to listen through the evolution of the conversation. We summarize the ability to listen as follows:

| |
|---|
| The positivity of the of function $dm_i(t)$ determines the ability to listen. |

## VI. Degree Martingale in Misanthrope

To demonstrate the meanings of the listening criteria, we examine the function $dm_i(t)$ of evolving hypergraph generated by the Misanthrope script of Moliere. The original translation of Moliere can be found in Wiki-source; see [2].

*The Misanthrope* discusses the question of honesty. Molière argues that completely honest people cannot live in society. The main characters of the play are two couples. The main couple is Alceste and Célimène. The second couple is Alceste's best friend, Philinte, and his partner (at the end of the play), Éliante. Alceste and Célimène eventually separate in the play. Perhaps the reason for their separation is that Alceste is too honest, and Célimène is too dishonest. To compare these lovebirds, we are given another couple who eventually find themselves together.

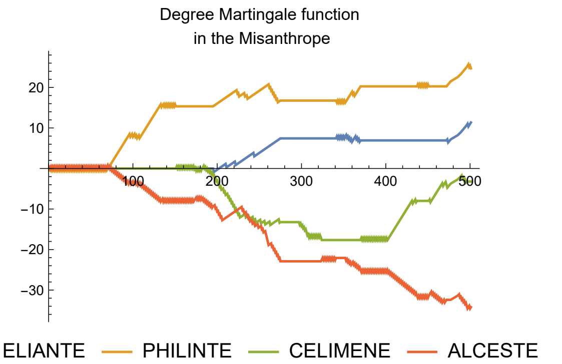


Fig. 2. This figure shows the bounded degree function of time of the two main couples in *The Misanthrope*. The first couple (Alceste and Célimène) is the upper (positive) function, and the second couple (Éliante and Philinte) is the lower (negative) couple.

Surprisingly, as the function $dm$ shows, the main couple Alceste and Célimène do not have the ability to listen to each other since $dm < 0$ is negative, and therefore one can argue that this is the main reason they split at the end of the play from each other. On the other hand, the second couple Philinte, and his partner (at the end of the play), Éliante can listen to each other since their degree martingale function is positive $dm > 0$, see figure 2.

## VII. Conclusion

This paper defines the Degree Martingale function, and shows how to use it in analyzing conversations between several characters. We show that the balance degree martingale can distinguish people who speak more from people who listen more. We believe that the art of listening is an important subject and has many applications, from management to human resources, customer service, psychology, and literature analyses.

In listening theory, there are many different kinds of listening, such as Informational, Discriminative, Biased, Sympathetic, Comprehensive, Empathetic or Therapeutic, and Critical. Developing mathematical tools distinguishing between these different subcategories of listening is an open problem and a big challenge, suitable for machine learning.

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