A multi-class centrality for transportation networks with heterogeneous agents

William F. A. Weber[1] and Mahendra Piraveenan[1]

School of Computer Science, Faculty of Engineering, University of Sydney, Sydney, Australia
wweb0052@uni.sydney.edu.au
mahendrarajah.piraveenan@sydney.edu.au

Abstract. This paper applies graph-theoretic measures to anticipate traffic dynamics in transportation networks. We extend the Transportation Centrality metric of Piraveenan and Saripada (2023) by incorporating toll pricing and heterogeneous traveler preferences into its cost model. Path costs combine class-specific distance and toll weights, distributing flows according to a logit-based allocation. We prove the value of this multi-class cost model for toll policy evaluation and robust network design through applications to sample networks. Under high price sensitivity, incorporating tolls causes toll corridor junctions to lose centrality while toll-free alternatives gain prominence.

Keywords: centrality · heterogeneous agents · transportation network · transportation centrality

1 Introduction

Transportation networks comprise the arterial system of modern societies, shaping the flow of labor, goods, and services [4]. Planners have long recognized the value of identifying nodes whose congestion or failure might disproportionately degrade network performance [6, 15], fragmenting connectivity and causing cascading disruptions. [13, 12]. Effective modeling requires not only the identification of these critical nodes, but also assessment of how policy levers such as tolling and new construction can redirect traffic and redistribute utilization.

Empirical studies corroborate the theoretical link between topological centrality and traffic. Luo et al. [10] found that betweenness centrality explained most of the variance in tram network ridership, while Kopsidas et al. [8] reported similar findings for metro stations passenger volumes. However, while network topology might provide a meaningful proxy to estimate the traffic serviced by a node, these classical centrality measures assume uniform travel cost and homogeneous user behavior, without consideration of the influence of tolls and individual cost sensitivities. In contrast, traffic-assignment and modal-split models account for such heterogeneity but are rarely integrated with centrality analysis. Recently, Transportation Centrality [11] was introduced to bridge this gap by incorporating bounded rationality into route choice. We extend this framework with a richer cost model that produces node-level TC scores reflecting the expected traversing proportion of multi-class traffic. Graph topology, toll assignments, and class-profiles are user-configurable inputs, enabling scenario analysis.

2 Measuring Centrality

We represent a transportation or logistics system by an undirected graph $G = (V, E)$ with $|V| = n$. The classical *Betweenness Centrality* (BC) of a node $v \in V$ quantifies its relative frequency as an intermediary on shortest paths between other nodes [5]. Let $\sigma_{s,t}$ denote the number of shortest paths between distinct nodes s and t, and let $\sigma_{s,t}(v)$ be the number of those paths passing through v. Then

$$BC(v) = \tfrac{1}{(n-1)(n-2)} \sum_{s,t \in V\setminus\{v\}} \frac{\sigma_{s,t}(v)}{\sigma_{s,t}}. \tag{1}$$

While BC is used to estimate node importance in traffic, it assumes travelers follow only shortest paths. In practice, route choice exhibits dispersion, with some travelers choosing less topologically efficient alternatives regardless. To capture this, Piraveenan and Saripada [11] introduced *Transportation Centrality* (TC), an "all-paths" metric that logit-weights each origin-destination path similarly to discrete choice models. Let $P_{s,t}$ be the set of all simple paths between s and t, and $P^v_{s,t} \subset P_{s,t}$ those passing through v. If $C^p_{s,t}$ is the cost of path p, and sensitivity parameter $\beta > 0$ governs dispersion, then

$$TC(v) = \tfrac{1}{(n-1)(n-2)} \sum_{s,t \in V\setminus\{v\}} \left(\frac{\sum_{p \in P^v_{s,t}} e^{-\beta C^p_{s,t}}}{\sum_{p \in P_{s,t}} e^{-\beta C^p_{s,t}}} \right). \tag{2}$$

As $\beta \to \infty$, weights concentrate on minimal-cost paths and $TC(v)$ converges to $BC(v)$. We note that the original TC formulation assumes a uniform cost function across travelers. In practice, costs often include fares and tolling is a key congestion management tool; Dong et al. [3] find that differentiated tolling substantially affect routing. Subpopulations experience costs differently, and toll effectiveness varies with traveler profile. Studies of value-of-time (VOT) heterogeneity find that high-VOT travelers prefer tolled expressways, while low-VOT users tend to divert to slower, cheaper alternatives [9]. Thus, shortest-path-based measures may overstate the importance of tolled links that many travelers avoid.

These observations motivate two key extensions to the TC framework: (i) incorporate tolls or other monetary charges into the path cost $C^p_{s,t}$; and (ii) partition the population into classes, each with distinct cost sensitivity, toll weighting, and population share, to compute both disaggregate and aggregate centralities.

3 Cost Model

Our objective is to assign each node $v \in V$ a centrality score that reflects traffic flows influenced by both distance and tolling. Unlike classical centralities, the proposed measure is explicitly behavioral: it accounts for the routing decisions of heterogeneous agents. This cost-sensitive formulation enables planners to evaluate how toll policies or new links shift node prominence.

3.1 Path Cost

Let each simple path p between origin and destination incur a generalized cost $C(p) = d(p) + \alpha\,\tau(p)$, where $d(p)$ is the sum of edge lengths, $\tau(p)$ the total tolls along the path, and $\alpha \geq 0$ a toll-weight reflecting a traveler's valuation of tolls relative to distance. In networks without physical length data, we set $d(p) = \#\text{hops}$, treating all edges as unit-length as in the original TC model. More generally, in networks traversed at substantial differences in speed along different links (as in the case of a multi-modal network), edge lengths will reflect normalized travel time.

3.2 Agent Heterogeneity

We partition the population into m classes each indexed by c, defined by parameters $(\alpha_c, \beta_c, \omega_c)$, where α_c is the toll-weight, β_c the cost sensitivity, and $\omega_c \geq 0$ the population share of class c, with $\sum_{c=1}^{m} \omega_c = 1$. Let

$$TC_{\alpha_c,\beta_c}(v) = \frac{1}{(n-1)(n-2)} \sum_{s,t \in V \setminus \{v\}} \frac{\sum_{p \in P^v_{s,t}} e^{-\beta_c\,[d(p)+\alpha_c\,\tau(p)]}}{\sum_{p \in P_{s,t}} e^{-\beta_c\,[d(p)+\alpha_c\,\tau(p)]}} \tag{3}$$

be the class-c transportation centrality of node v. This expression captures the share of flow from class c that passes through v. Class-specific centrality enables disaggregation of flows: Planners can identify not only which nodes are important, but also which segments of the population contribute to that importance. The aggregate multi-class centrality is defined by

$$TC_{\text{MC}}(v) = \sum_{c=1}^{m} \omega_c\, TC_{\alpha_c,\beta_c}(v). \tag{4}$$

As an example, suppose one class represents high-VOT travelers ($\alpha = 0.1$) and another highly toll-averse users ($\alpha = 5.0$), with a 60–40 population split. Then $(\alpha_1, \beta_1, \omega_1) = (0.1, 1.0, 0.6)$, $(\alpha_2, \beta_2, \omega_2) = (5.0, 1.0, 0.4)$, and node v has aggregate centrality

$$TC_{\text{MC}}(v) = 0.6\, TC_{0.1,1.0}(v) + 0.4\, TC_{5.0,1.0}(v). \tag{5}$$

Heterogeneous centrality isolates enables planners to target interventions to the behavioral origin of congestion, not merely its location. If a bottleneck is used mainly by toll-averse traffic, even a modest toll may redirect flows. If dominated by toll-insensitive users, pricing may be ineffective and capacity expansion more appropriate.

4 Analysis and Results

4.1 Implications for Transportation Management

A planner with a relatively under or over-utilized junction may choose between constructing an additional link or setting a new toll price as interventions.

Link Construction One may verify that $TC_{\mathrm{MC}}(v)$is non-decreasing and submodular in the set S of added edges: each new link opens simple paths that increase centrality with diminishing returns. This supports a greedy routine that iteratively adds the edge (v, u) offering the largest marginal gain. Given an undirected graph $G = (V, E)$ and a budget of up to k new links incident to v, the planner selects

$$S \subseteq \{(v, u) : u \in V \setminus \{v\}\}, \qquad |S| \leq k, \tag{6}$$

to maximize

$$\max_{S\colon |S| \leq k} \left[TC_{\mathrm{MC}}(G \cup S)(v) - TC_{\mathrm{MC}}(G)(v)\right]. \tag{7}$$

The process iterates:
1. Compute baseline $TC_{\mathrm{MC}}(G)(v)$.
2. For each candidate edge $(v, u) \notin E$, compute its marginal gain

$$\Delta_u = TC_{\mathrm{MC}}(G \cup \{(v, u)\})(v) - TC_{\mathrm{MC}}(G)(v). \tag{8}$$

3. Add the edge with largest Δ_u, update G, and repeat until $|S| = k$.

Toll Adjustment Tolling congested links can redirect traffic to underutilized routes, approximating system-optimal flow [9]. Let $TC_{\mathrm{MC}}(\tau)$ be the centrality of node v when edge e carries toll $\tau \in [\tau_{\min}, \tau_{\max}]$, with all other costs fixed. The goal is to select τ so that $TC_{\mathrm{MC}}(\tau) \approx C_{\mathrm{target}}$, for a target centrality (e.g., a capacity threshold). Since higher τ decreases the probability of choosing paths through e, $TC_{\mathrm{MC}}(\tau)$ is strictly decreasing and continuous. A bisection search solves

$$\min_{\tau \in [\tau_{\min}, \tau_{\max}]} |TC_{\mathrm{MC}}(\tau) - C_{\mathrm{target}}| \tag{9}$$

in $I = O(\log(\tau_{\max} - \tau_{\min})/\varepsilon)$ steps to precision ε. This gives planners a direct method to tune tolls without simulating all scenarios.

Transport Operators Some links may be operated by firms (e.g., airlines, toll-road companies, or ferries) that aim to maximize revenue, not system-wide efficiency. With unlimited service capacity, the operator seeks

$$\max_{\tau_e \geq 0} \quad \tau_e \cdot TC_{\mathrm{MC}}(e). \tag{10}$$

With constrained capacity we add the constraint

$$TC_{\mathrm{MC}}(e) \leq \mathrm{Capacity}_e. \tag{11}$$

This one-dimensional optimization is again solvable by bisection.

4.2 Computational Complexity

TC is significantly more computationally demanding than BC [11]. It requires enumerating all simple s–t paths for each node pair, giving worst-case time complexity $O(N^2 \cdot P \cdot L)$, where P is the number of paths and L their average length. While P remains small in sparse networks, worst-case growth is exponential in N. In contrast, BC is computed in $O(N \cdot E)$ (unweighted) or $O(NE + N^2 \log N)$ (weighted) [1]. This complexity increases the cost of computing link or toll optimizations. For link additions, each greedy step requires one TC evaluation, yielding $O(k \cdot n \cdot T_{\text{TC}})$ overall. For toll setting, complexity is $O(I \cdot T_{\text{TC}})$.

Mitigations Several strategies improve tractability: (i) restrict path length to avoid unrealistic detours; (ii) limit candidate links to those within a hop or distance radius of v; (iii) constrain the set of tollable links; (iv) use a cost-aware BC variant (omitting β) as a TC proxy. These limits will need to be set at a 'goldilocks' level with respect to a particular network.

4.3 Synthetic Example

To illustrate how the toll-sensitive TC metric can be used to identify non-topological reductions of congestion risk, we use a 19-node undirected network (Fig. 1) comparing node 1 to nodes $(2, 3, 4)$ and $(5, 6, 7)$. All edges have unit cost, and we assume a single traveler class ($\beta = 1$, $\alpha = 1$). Table 1 reports BC and TC under two settings: toll-free and toll $\tau = 0.65$ on all links incident to node 1.

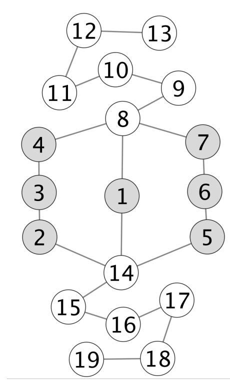

Fig. 1: Three paths in a constructed network ($|V| = 19$, $|E| = 20$).

Without tolls, node 1 dominates shortest paths, recording the highest BC and TC. It constitutes a potential point of congestion, while other routes might be underutilized. With tolls, node 1's TC drops sharply, while nodes 2-7 gain importance. As $\beta \to \infty$, TC approaches BC and the equalizing τ value tends toward 0.5. Any positive sensitivity admits a toll profile balancing route centrality. This is an enriched decision framework for organizing traffic, with purely topological centrality measures only simulating the impacts of modifying the graph itself. In practical terms graph alterations represent a capital expenditure as opposed to a fare change and may therefore be less desirable.

Node	BC	TC – No Toll	TC – $\tau = 0.65$
1	0.318	0.291	0.158
2	0.074	0.109	0.155
3	0.050	0.085	0.141
4	0.074	0.109	0.155
5	0.074	0.109	0.155
6	0.050	0.085	0.141
7	0.074	0.109	0.155

Table 1: Effect of tolling node 1's links on centrality.

4.4 Road Network Example

Figure 2 presents a topological representation of selected major roads and junctions in Houston and Harris County, simplified from TxDOT GIS data [14], omitting local connectors and setting nominal tolls. Only intracity transit is modeled. Peripheral nodes appear less central than in full-scale traffic due to boundary effects.

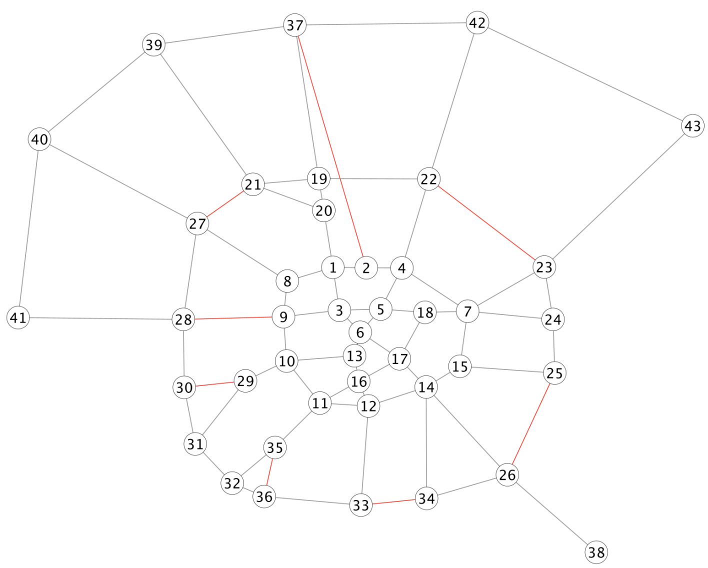

Fig. 2: Houston road network. Toll roads in red. ($|V| = 43$, $|E| = 74$).

We compare two scenarios:

1. **Baseline:** Every edge has unit length and zero toll, yielding a uniform, unweighted network (original TC).
2. **Multi-Class TC:** Three traveler classes represent heterogeneous preferences:

- Time-sensitive: $\beta = 5$, $\alpha = 0$ (25%), sensitive to path efficiency.
- Balanced: $\beta = 1$, $\alpha = 1$ (50%), equal weight on distance and toll, lower cost sensitivity broadly.
- Toll-averse: $\beta = 5$, $\alpha = 5$ (25%), highly price-sensitive.

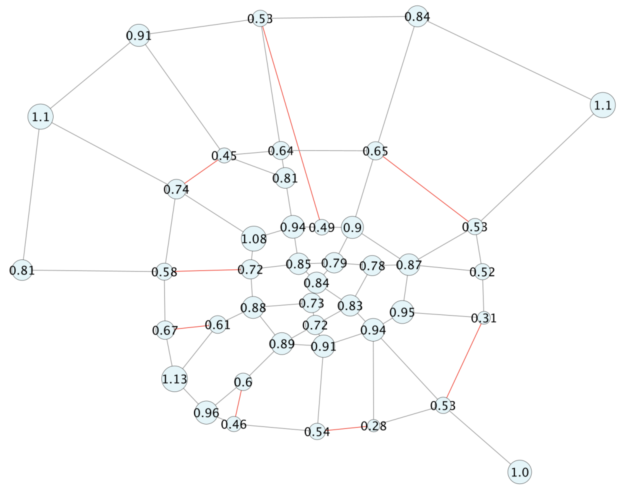

Fig. 3: Ratio of multi-class TC to baseline TC for each Houston node. Values greater than 1 denote relative gain in importance.

Node	TC Ratio
34	0.28
25	0.31
21	0.45
36	0.46
2	0.49

Table 2: Nodes with greatest TC losses (cf. Fig. 3).

Node	TC_{time}	TC_{toll}	Ratio
34	0.132	0.002	65.6
25	0.073	0.011	6.7
21	0.049	0.016	3.0
43	0.028	0.093	0.30
20	0.045	0.131	0.34
8	0.056	0.141	0.40

Table 3: Nodes with highest TC disparity between traveler profiles.

Nodes situated along toll roads record reductions in centrality. This reflects a behavioral shift in route choice: cost-averse travelers increasingly avoid these

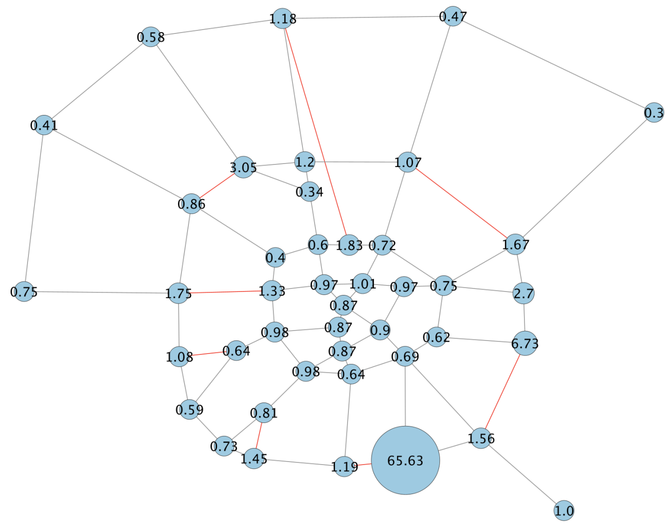

Fig. 4: Ratio of time-sensitive TC to toll-averse TC across Houston nodes. Values greater than 1 indicate relative preference by high-VOT travellers.

corridors, diminishing the strategic importance of nodes relative to purely topological or toll-free assumptions. Complementarily, nodes like 8 and 20 gain prominence as detours for toll-sensitive users. Surrounded by toll roads, Node 34 is nearly entirely avoided by toll-averse drivers, a population class obscured in baseline or aggregate metrics. Cutting the toll on the 34-33 link revives its throughput.

5 Conclusion

This study introduces a toll-aware, multi-class extension of Transportation Centrality and demonstrates its practical utility. The framework supports robust scenario analysis: altering toll levels, modifying network structure, and adjusting traveler-class parameters all yield updated centrality distributions. This provides planners with a principled tool for infrastructure investment and policy design. Projects that offer the greatest improvement in connectivity or congestion relief can be prioritized by quantifying how each added link or toll adjustment influences node utilization. By disaggregating flow by traveler class, the model captures disparate impacts across user groups. Applications extend beyond urban mobility to logistics networks, where tolls analogize to intermediate processing costs in freight, rail, air, or pipeline systems. In each case, submodular greedy optimization ensures decisions can be made efficiently. We illustrate how toll sensitivity alters network centrality: nodes prominent only due to toll-free shortcuts lose importance as toll-averse classes divert to cheaper paths, while alternative routes and previously marginal nodes may gain centrality, highlighting latent bottlenecks. Future extensions might incorporate capacity constraints, generalize the approach to multi-modal networks, or address the mechanism-design problem of optimal toll segmentation. Empirical calibration from revealed-preference data can further enhance the model's realism and applicability.

References

1. U. Brandes, "A faster algorithm for betweenness centrality," *J. Math. Sociol.*, vol. 25, no. 2, pp. 163–177, 2001.
2. M. Castaldo, C. Catalano, G. Como, and F. Fagnani, "On a centrality maximization game," *IFAC-PapersOnLine*, vol. 53, no. 2, pp. 2844–2849, 2020.
3. X. Dong, Y. Zeng, R. Luo, N. Lyu, D. Xu, and X. Zhou, "Influence of differentiated tolling strategies on route choice behavior of heterogeneous highway users," *Future Transp.*, vol. 5, no. 2, article 41, 2025.
4. R. Engström, "The Roads' Role in the Freight Transport System," *Transp. Res. Procedia*, vol. 14, pp. 1443–1452, 2016.
5. L. C. Freeman, "Centrality in social networks: Conceptual clarification," *Soc. Networks*, vol. 1, no. 3, pp. 215–239, 1979.
6. E. Jenelius, T. Petersen, and L.-G. Mattsson, "Importance and exposure in road network vulnerability analysis," *Transp. Res. Part A*, vol. 40, no. 7, pp. 537–560, 2006.
7. A. Karduni, A. Kermanshah, and S. Derrible, "A protocol to convert spatial polyline data to network formats and applications to world urban road networks," *Sci. Data*, vol. 3, no. 1, pp. 1–7, 2016.
8. A. Kopsidas, A. Douvaras, and K. Kepaptsoglou, "Exploring the association between network centralities and passenger flows in metro systems," *Appl. Netw. Sci.*, vol. 8, article 69, 2023.
9. L. J. Lehe, "Winners and losers from road pricing with heterogeneous travelers and a mixed-traffic bus alternative," *Transp. Res. Part B*, vol. 139, pp. 432–446, 2020.
10. D. Luo, O. Cats, and H. Lint, "Can passenger flow distribution be estimated solely based on network properties in public transport systems?," *Transportation*, vol. 47, no. 6, pp. 2757–2776, 2020.
11. M. Piraveenan and N. B. Saripada, "Transportation centrality: Quantifying the relative importance of nodes in transportation networks based on traffic modeling," *IEEE Access*, vol. 11, pp. 142214–142234, 2023.
12. A. Strzelczyk and S. Guze, "An approach to the analysis of critical elements of transport and logistics networks using graph theory," *TransNav*, vol. 18, pp. 535–544, 2024.
13. M. A. P. Taylor and G. M. D'Este, "Transport network vulnerability: A method for diagnosis of critical locations in transport infrastructure systems," In: A. T. Murray and T. H. Grubesic (Eds.), *Critical Infrastructure*, Advances in Spatial Science, Springer, Berlin, pp. 9–30, 2007.
14. Texas Department of Transportation, "Texas Highway Freight Network" [Dataset], Feature Layer; Published 5 Apr 2016; Updated 22 Mar 2023; Accessed 26 Mar 2025; Texas Department of Transportation. https://gis-txdot.opendata.arcgis.com/datasets/TXDOT::texas-highway-freight-network/.
15. L. Wang, S. Zhang, G. Szűcs, and Y. Wang, "Identifying the critical nodes in multi-modal transportation networks with a traffic demand-based computational method," *Reliab. Eng. Syst. Saf.*, vol. 244, article 109956, 2024.