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Perturbation Analysis of Centrality Measures

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***Abstract*—In recent decades, a large number of centrality measures have been proposed to assess the importance of nodes in complex networks. The choice of the most appropriate centrality index for specific applications is one of the biggest challenges. This paper performs the perturbation analysis of 8 centrality measures. Since most real networks are incomplete and prone to bias, we compare centrality measures in order to evaluate their sensitivity to small changes in a graph structure. Our experiments are performed on 8 classical graph structures ranging from a simple path graph to a Watts-Strogatz graph model. As a result, we provide a sensitivity of centrality measures on different graph structures.**

***Index Terms*—network, centrality, perturbation analysis**

I. Introduction

Centrality is a powerful concept for identifying important actors in a network. Currently, there are more than 400 centrally measures [1], ranging from classical centralities (e.g.: degree, eigenvector, PageRank, betweenness, closeness [2]) to the measures that take into account specific features of a network, such as the group influence of nodes (e.g.: Myerson value [3]), individual attributes of nodes (e.g.: diffusion centrality [4]) or both of them (e.g.: LRIC [5]).

Identification of central elements in a network is an ill-defined problem. There is no universal definition of the most important objects that one should be looking for. Hence, there is no general approach on how to compare various centrality measures and select the best one. In most cases, there is no ground truth of the real importance of nodes. The choice of the most appropriate centrality measure also depends on the type of a network and the interpretation of important elements. Some indices, which are based on the paths calculation (betweenness, closeness, etc.), are more suitable for transportation or telecommunication networks. Other indices, such as PageRank or Hubs and Authorities, were developed to estimate the importance in cross-reference graphs (webpages, citation networks, etc.). As a result, the researcher should understand the idea behind each applied index instead of considering all possible centrality measures.

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Centrality measures can be compared to each other using the axiomatic approach [6]–[8]. The list of axioms includes the monotonicity axiom [6], the endpoint increase axiom [6], the density axiom [7], the cut-vertex additivity axiom [8], etc. The extended list of axioms is provided in [9]. Axiomatic verification aims to better understand the behavior of centrality measures in different situations. If the centrality does not satisfy a certain property, it does not mean that this centrality is deficient. Moreover, most axioms mainly focus on the change in the numerical values of centrality measures rather than on the change in their ranks; consequently, the relative node positions in a graph are not taken into account.

Some studies aim to select the most appropriate centrality measure for a specific network. Vignery and Laurier [10] proposed a methodology that compares and validates centrality indices for a friendship student network. Ashtiani et al. [11] apply 27 centrality measures to protein-protein interaction networks and choose the most informative measures using a principal component analysis (PCA) and a hierarchical clustering.

An important aspect of centrality evaluation is a perturbation analysis. Many real networks are incomplete due to the presence of missing values [12], [13]. The effect of missing data has been studied for criminal networks [14] and the global trade network [15]. Therefore, it is reasonable to examine the stability or the robustness of the centrality measures to fluctuations in data.

Centrality measures have been examined through the changes in a network structure. Borgatti et al. [16] have studied the robustness of 4 centralities (degree, betweenness, closeness and eigenvector) toward edge/node removal/addition. They calculated a square of the Pearson correlation coefficient as well as the proportion of times the central nodes keep their top positions in a modified network. Frantz et al. [17] have examined the robustness of these measures in the Erdős–Rényi (ER) random network, the small-world, the scale-free, the core-periphery, and the cellular networks. Segarra and Ribeiro [18] introduced the concepts of stability and continuity of centrality measures for weighed networks and examined the same centralities. Martin and Niemeyer [19] have applied several measurement error mechanisms to ER and Barabási–Albert graphs and proposed a method to estimate the impact of errors on the reliability of 5 classical centrality measures including PageRank. The sensitivity of several centralities to edge removal is studied from a theoretical point of view in

[20]. Overall, the existing studies are limited to the analysis of 4-5 classic centrality measures.

This paper provides a perturbation analysis of 8 centrality measures. We study the sensitivity of centralities to small changes in network structure (edge/node removal/addition). The experiments are performed on 8 graph configurations: Erdős–Rényi graph, Barabási–Albert graph, small world graph, path graph, balanced tree, star graph, regular graph and square lattice. Our main contribution is the extensive analysis of the sensitivity of widely studied and recently proposed centrality measures on different graph structures.

The paper is organized as follows. In Section II, we provide some basic information about centrality measures and network structures. In Section III, we describe the methodology of our experioments and provide the results for 8 centrality measures. Section IV concludes.

II. Preliminaries

We consider a graph $G = (V, E)$, where $V = \{1, ..., n\}$ is a set of nodes, $|V| = n$, and $E \subseteq V \times V$ is a set of edges. We consider undirected unweighted graphs, i.e., $\forall i, j \in V : (i, j) \in E \Rightarrow (j, i) \in E$ without loops, i.e. $\forall i \in V (i, i) \notin E$. The graph G can be represented by $n \times n$ adjacency matrix $A = [a_{ij}]$, where $a_{ij} = 1$ if $(i, j) \in E$, otherwise $a_{ij} = 0$.

A. Centrality Measures

We describe the centrality measures, which are applied in our research.

1) Degree Centrality: the number of node neighbours

$$C_i^{deg} = \sum_{j=1}^{n} a_{ij}. \tag{1}$$

2) Eigenvector Centrality: the generalization of a degree centrality. It takes into account not only the direct connections of a node but also indirected ones. It is a solution of

$$C_i^{ev} = \frac{1}{\lambda} \cdot \sum_{j=1}^{n} a_{ij} \cdot C_j^{ev}, \tag{2}$$

or, in a matrix form,

$$A \cdot C^{ev} = \lambda \cdot C^{ev}, \tag{3}$$

where λ is the largest eigenvalue of matrix A. In basic terms, the importance of a node depends on the importance of its neighbours.

3) PageRank: PageRank centrality is based on random walks in a graph and it assigns high values to nodes that have a higher probability of being visited. In particular, the PageRank centrality of node i in undirected graph is defined as

$$C_i^{PR} = \alpha \cdot \sum_{j=1}^{n} a_{ij} \cdot \frac{C_j^{PR}}{C_i^{deg}} + \frac{1-\alpha}{n}, \tag{4}$$

where α is a teleportation coefficient.

4) Betweenness Centrality: it is based on the calculation of the shortest paths between nodes. Betweenness centrality ranks higher nodes that lie on the shortest paths between other nodes more frequently, i.e.,

$$C_i^{bet} = \sum_{j,k \in V} \frac{\sigma_{jk}(i)}{\sigma_{jk}}, \tag{5}$$

where σ_{jk} is the number of the shortest paths between nodes j and k and $\sigma_{jk}(i)$ is the number of the shortest paths between nodes j and k through node i.

5) Closeness Centrality: it is also based on the shortest paths calculation. The centrality of a node i is inversely proportional to the sum of shortest paths from node i to other nodes in a graph, i.e.,

$$C_i^{cl} = \frac{n-1}{\sum_{j=1}^{n} d_{ij}}, \tag{6}$$

where d_{ij} is a shortest path distance between nodes i and j.

6) Harmonic Centrality: it is similar to the closeness centrality but performs better when a graph has several connected components. The harmonic centrality is defined as

$$C_i^{harm} = \sum_{j=1, j \neq i}^{n} \frac{1}{d_{ij}}. \tag{7}$$

If nodes i and j are in different connected components, then the corresponding summand equals 0.

7) Subgraph Centrality: it estimates the number of closed walks (starting and ending at a particular node) of different length by

$$C_i^{subgraph} = \sum_{k=0}^{\infty} \frac{\mu_k(i)}{k!}, \tag{8}$$

where $\mu_k(i)$ is the number of closed walks of length k starting and ending at node i.

8) Long Range Interaction Centrality (LRIC): each node j has a pre-defined threshold of influence (or quota) q_j, which indicates the minimal level when this node becomes affected (=individual attributes). If the total weight of connections from a group of nodes $\Omega(j)$ to node j exceeds its threshold q_j, then $\Omega(j)$ is called critical (=group influence). Next, a node i is called pivotal if its exclusion makes the group $\Omega(j)$ non-critical. LRIC omits unimportant links by transforming the initial network into the network of direct influence by

$$c_{ij} = \max \frac{a_{ij}}{\sum_{k \in \Omega_p(j)} a_{kj}}, \tag{9}$$

where $\Omega_p(j)$ is a group of nodes where node i is pivotal for node j. Otherwise, if node i is not pivotal for node j, then $c_{ij} = 0$. After the network of direct influence is constructed, LRIC aggregate indirect influence between nodes through all possible paths of length k between nodes. By default, the LRIC is calculated for $k = 3$.

A detailed description of centrality measures is provided in [2] for degree, eigenvector, PageRank, betweenness and closeness centralities, in [21] for the subgraph centrality and [5] for the LRIC.

B. Network structures

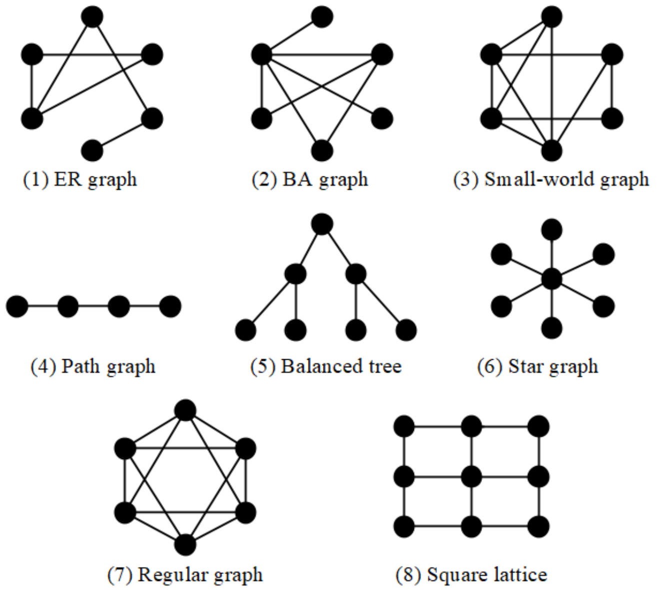

Fig. 1. Graph structures.

We examine the following graph structures (see Fig. 1):

1) *Erdős–Rényi graph (ER graph)*: a random graph model $G(n, p)$ where n in a number of nodes and p is a probability for edge creation [2]. Each edge is included in the graph with probability p, independently from every other edge.
2) *Barabási–Albert graph (BA graph)*: a random graph model where nodes are added sequentially to the existing nodes with a probability that is proportional to the degree that the existing nodes already have [2].
3) *Small-world (Watts–Strogatz) graph model*: a graph with high transitivity and short path length [2]. Small-world networks are similar to ring lattice with some edges are replaced with probability p.
4) *Path graph*: a connected graph where n nodes constitute a chain with $n-1$ links. Each node has degree 2 except two terminal nodes with degree 1.
5) *Balanced tree*: a rooted tree graph in which the depths of the subtrees below each node differ by no more than 1 [2]. The root has degree r and all other internal nodes have degree $r+1$.
6) *Star graph*: a graph with one central hub that is connected to other nodes and no other two nodes are connected with each other.
7) *Regular graph*: a graph where each node has the same degree d.
8) *Square lattice*: a graph whose nodes are the points in the Cartesian plane with coordinate $x = 1, ..., m$ and coordinate $y = 1, ..., m$. Nodes are connected if the distance between them is 1.

III. Experiments

We perform a perturbation analysis of 8 centrality measures on different graph structures. We describe the graph modification techniques as well as several approaches to assess the performance of centralities toward these modifications. We also discuss the obtained results.

A. Methodology

We consider 4 actions of the graph modification:

a) random addition of 1 edge;
b) random removal of 1 edge;
c) random addition of 1 node with degree that is equal to the average node degree (rounded down);
d) random removal of 1 node.

Intuitively, the centrality measures are more stable toward edge addition and edge removal compared to nodes operations, because edge modifications change only 1 edge in a graph while node modifications change multiple edges in a graph.

We apply 5 performance metrics to assess the stability of centralities:

- **Correlation**. We compute the Kendall rank correlation coefficient, which measures the similarity of the orderings of centrality measures [1]. Thus, we focus on the relative importance of nodes in a network.
- **TOP1**. We evaluate the percentage of nodes, which are remained in TOP-1 after modification of a network.
- **TOP3**. We evaluate the percentage of nodes, which are remained in TOP-3 after modification of a network.
- **TOP5**. We evaluate the percentage of nodes, which are remained in TOP-5 after modification of a network.
- **TOP10**. We evaluate the percentage of nodes, which are remained in TOP-10 after modification of a network.

The performance measures are averaged over the number of generations of a particular graph structure.

B. Stability of Centrality Measures

*1) **ER graph**:* we consider $n = 100$ nodes and for each p from 0.01 to 0.1 with step 0.01 we generate 100 ER graphs. For each graph we make random edge/node additions/removals and compare centralities between initial and modified graphs with respect to 5 performance measures. We repeat this procedure 100 times for each ER graph. Thus, the performance measures for ER graphs are averaged across 10000 experiments.

The general result is that the increase of a graph density lead to a higher stability of centrality measures, because any change in a dense graph is more significant than in a sparse graph. The centrality measures are more stable toward edge modification compared to nodes operations. The exception here are very sparse graphs (with $p \leq 0.02$) where some nodes have a zero degree. The addition/removal of these nodes do not affect the centralities.

[1] In the case of node addition (removal), we assume that the added (removed) node was isolated before (after) the action.

TABLE I
STABILITY OF CENTRALITY MEASURES FOR ER GRAPH MODEL*

	Measure	Centrality															
		Degree		**Eigenvector**		**PageRank**		**Betweenness**		**Closeness**		**Harmonic**		**Subgraph**		**LRIC**	
Edge addition	Corr	0.98	0.99	0.97	0.98	0.97	0.99	0.97	0.98	0.97	0.98	0.97	0.98	0.84	0.98	0.97	0.98
	TOP1	0.97	0.99	0.90	0.95	0.90	0.97	0.90	0.95	0.87	0.97	0.90	0.97	0.90	0.97	0.88	0.95
	TOP3	0.98	0.98	0.91	0.96	0.92	0.97	0.91	0.96	0.91	0.98	0.91	0.97	0.93	0.97	0.90	0.95
	TOP5	0.97	0.98	0.92	0.96	0.94	0.98	0.93	0.96	0.92	0.97	0.92	0.96	0.94	0.97	0.91	0.95
	TOP10	0.98	0.99	0.93	0.97	0.95	0.98	0.94	0.97	0.94	0.97	0.94	0.97	0.95	0.98	0.94	0.96
Edge removal	Corr	0.98	0.99	0.97	0.98	0.97	0.99	0.97	0.98	0.97	0.98	0.97	0.98	0.83	0.98	0.97	0.98
	TOP1	0.96	0.99	0.90	0.95	0.90	0.97	0.88	0.95	0.86	0.97	0.89	0.97	0.90	0.96	0.87	0.95
	TOP3	0.96	0.98	0.91	0.96	0.93	0.97	0.90	0.96	0.90	0.97	0.91	0.97	0.92	0.97	0.90	0.95
	TOP5	0.96	0.98	0.92	0.96	0.94	0.97	0.92	0.96	0.91	0.97	0.92	0.97	0.94	0.97	0.91	0.95
	TOP10	0.97	0.99	0.93	0.97	0.95	0.98	0.94	0.97	0.93	0.97	0.93	0.97	0.95	0.97	0.93	0.96
Node addition	Corr	0.98	0.97	0.96	0.95	0.96	0.95	0.99	0.95	0.98	0.95	0.98	0.95	0.83	0.95	0.98	0.93
	TOP1	0.98	0.98	0.96	0.93	0.94	0.93	0.98	0.92	0.98	0.94	0.97	0.94	0.96	0.94	0.96	0.89
	TOP3	0.99	0.96	0.97	0.93	0.95	0.93	0.98	0.93	0.98	0.94	0.97	0.93	0.97	0.94	0.97	0.89
	TOP5	0.99	0.96	0.97	0.94	0.97	0.94	0.99	0.94	0.98	0.94	0.98	0.93	0.97	0.95	0.97	0.90
	TOP10	0.99	0.97	0.97	0.95	0.97	0.95	0.98	0.95	0.98	0.94	0.98	0.94	0.98	0.96	0.98	0.92
Node removal	Corr	0.98	0.96	0.96	0.94	0.96	0.95	0.98	0.94	0.97	0.94	0.97	0.94	0.82	0.94	0.97	0.93
	TOP1	0.97	0.95	0.94	0.91	0.95	0.91	0.94	0.89	0.93	0.91	0.94	0.91	0.95	0.92	0.93	0.87
	TOP3	0.97	0.94	0.95	0.91	0.96	0.92	0.95	0.91	0.94	0.92	0.95	0.92	0.95	0.92	0.94	0.88
	TOP5	0.97	0.95	0.95	0.92	0.96	0.93	0.95	0.92	0.95	0.92	0.95	0.91	0.96	0.93	0.95	0.89
	TOP10	0.98	0.95	0.96	0.93	0.97	0.94	0.96	0.93	0.96	0.92	0.96	0.93	0.96	0.94	0.96	0.91

*White columns indicate ER graphs with $p = 0.01$ and grey columns indicate ER graphs with $p = 0.06$.
The color scale varies from green if the value is high (≥ 0.97) to red if the value in low (≤ 0.83).

Table I demonstrates the results for $G(100, 0.01)$ (white columns) and $G(100, 0.06)$ (grey columns). Overall, the degree centrality outperforms other measures and shows the most stable results by most performance metrics. The Kendall rank correlation is very strong (≥ 0.92) for most centrality measures and all the graphs modifications. The only exception is the subgraph centrality for G with $p = 0.01$ (the correlation is equal to $0.82 - 0.84$). We also note that the standard deviation (std) for the correlation coefficient does not exceed 0.06. As to set of the most central nodes (TOP1, TOP3, TOP5, TOP10), the std reaches 0.37 for TOP1, 0.24 for TOP3, 0.23 for TOP5 and 0.20 for TOP10. High values of the std indicate that the group of the most central elements can significantly change in particular graph modifications, though it is very stable in general.

*2) **BA graph**:* we generate BA graphs, where each new node with degree m is sequentially and preferentially attached to existing nodes until the number of nodes reaches $n = 100$. We consider $m = 1, .., 4$ separately[2]. We generate 100 BA graphs for each m and for each graph we make random edge/node additions/removals. We repeat the modification procedure 100 times for each graph. Thus, the performance measures for BA graphs are averaged across 10000 experiments for each $m = 1, .., 4$.

[2]For $m = 1$ the BA model generates a tree (connected graph without cycles) with a small network diameter.

The results of our experiments for the BA graphs are similar to the ER graphs. The centrality measures are more stable for edge modifications compared to node modifications. The higher the density of a graph is, the less changes occur in the ranking of nodes.

The change of 1 link in a graph does not significantly affect the centrality of nodes (> 0.97 for all performance measures). The node addition/removal does not change TOP1-TOP10 rankings (TOP measures are higher than 0.95 for all centralities). The Kendall rank correlation of rankings varies from 0.91 (LRIC) to 0.97 (closeness, eigenvector and subgraph centralities) for a node addition and from 0.92 (LRIC) to 0.96 (degree) for a node removal. Overall, all the centrality measures are very stable for BA graphs.

*3) **Small-world graph**:* we consider the Watts–Strogatz graph model that generates a ring topology, where each node is connected to 4 nearest nodes, and then rewires edges with probability $p = 0.01, ..., 0.07$. For each value of p we generate 30 small-world graphs and make 30 random edge/node additions/removals for each graph. Thus, the performance measures of the small-world graph are averaged across 900 experiments for each p.

Our experiments demonstrate similar results for different values of p. Fig. 2 illustrates the results for $p = 0.04$.

The addition of a new edge/node significantly influences all centrality measures, because the Kendall rank correlation is moderate or weak. For instance, it is ≤ 0.7 for the degree

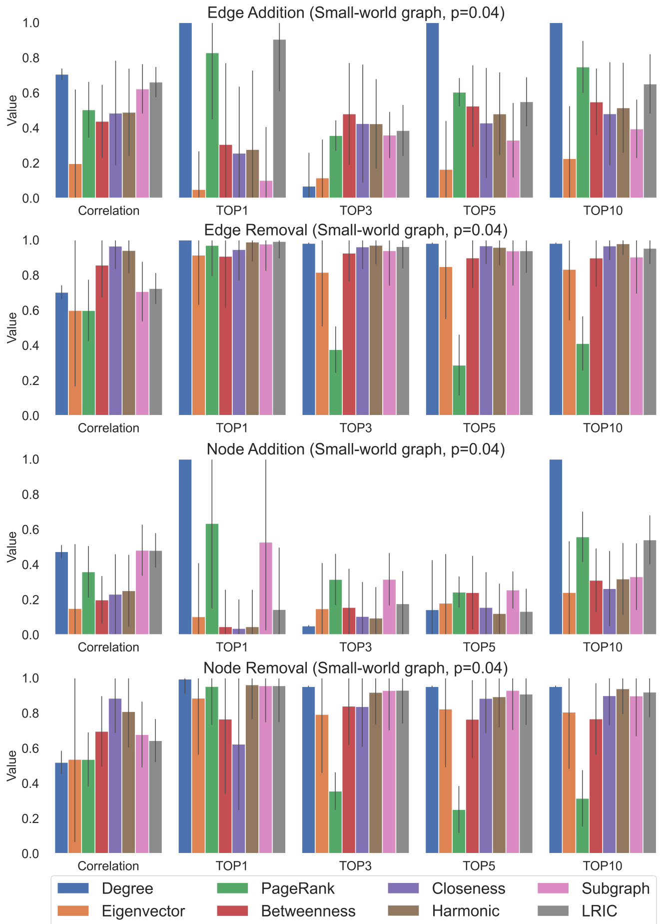

Fig. 2. Stability of centrality measures for Small-World graph with $p = 0.04$.

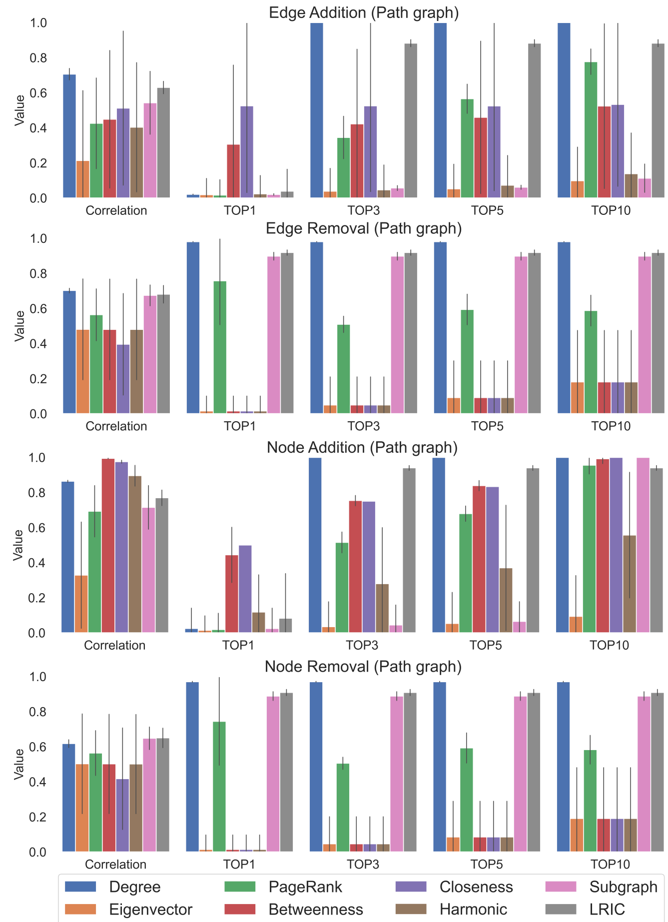

Fig. 3. Stability of centrality measures for a path graph.

centrality (edge addition) and 0.15 for eigenvector centrality (node addition). The TOP1 ranking is stable for the degree, the LRIC and the PageRank (edge addition) while the degree, the PageRank and the subgraph centrality are less sensitive to a node addition.

Centralities, which are based on the shortest paths between nodes (betweenness, closeness and harmonic measures), are more stable to the removal of a random edge/node according to the Kendall rank correlation. However, the TOP1 measure for the closeness centrality is the lowest and is equal to 0.62 for a node removal. On the contrary, the TOP1 values for other centralities are greater than 0.73 for a node removal and 0.9 for an edge removal.

Overall, most of the centralities are not vulnerable to the removal of nodes or edges while the addition of new elements has a high impact on the nodes ranking.

4) ***Path graph:*** we consider a path graph with 100 nodes and 500 random graph changes of each type (edge/node addition/removal). Fig. 3 illustrates the results for a path graph.

First, the degree centrality shows the most stable results according to the most of the metrics (except for TOP1) toward all modifications. In fact, the degree centrality considers $n-2$ nodes as the most important ones (TOP1). A node/edge change affects the centrality of only 2-3 nodes, hence, the degree centrality does not change significantly. Similarly, the node/edge removal does not affect the degree centrality of TOP1 nodes. However, addition of new nodes/edges changes the degree centrality dramatically, because, in general, TOP1 contains only 1 node instead of $n-2$. The degree centrality is also similar to the LRIC index, which does not take into account very long connections between nodes and considers many nodes in the middle of a network as the most central ones.

Second, most of the centralities, which are based on path calculation (eigenvector, betweenness, closeness, harmonic centralities), are vulnerable to the removal of nodes/edges in a graph. In general, these measures identify the middle node in a chain as the most important one while the removal operation splits the graph into two disconnected chains.

Next, the node/edge addition has the largest impact on the list of TOP1, TOP3, TOP5 central nodes for the eigenvector and the subgraph centrality. We also observe a very week correlation between two ranking for the eigenvector centrality. In contrast to the ER or BA graphs, the correlation coefficient is weak or moderate for all graph modifications except for the node addition.

Overall, all the centrality indices except for the degree, the subgraph centrality and the LRIC are very sensitive to minor changes in a path graph.

*5) **Balanced tree**:* we examine perfectly balanced trees (all leaves are at the same distance from the root) of height h 3 and 4 and consider 500 random graph changes of each type. The root has degree 4 and all other internal nodes have degree 5.

The centrality measure select distinct nodes as the most important nodes in the balanced tree. The degree centrality selects all internal nodes as the most central ones. The eigenvector, the betweenness, the closeness, the harmonic and the LRIC centralities select the root of the tree. The PageRank selects nodes, which are adjacent to the leafs. Finally, the subgraph centrality selects nodes, which are adjacent to the root.

Our experiments on graphs with $h = 3$ and $h = 4$ provide similar results for all centrality measures. All centrality measures are relatively stable as there is a strong correlation coefficient (> 0.7). We observe that degree and betweenness centralities are almost vulnerable to all graph modifications as they have the highest correlation coefficient > 0.94 for all actions. All the centralities, which are based on the shortest paths (betweenness, closeness, harmonic), never change the TOP1 node (the root) if it is not removed from the network.

Next, we observe that the node addition has no effect on the TOP1-10 for all centrality measures except for the degree, the PageRank and the subgraph centralities. In fact, the PageRank and the subgraph centralities rely on indirect paths between nodes, which change significantly after the addition/removal of nodes or edges.

Overall, we conclude that that the degree and the betweeness centralities are the most stable in terms of the correlation coefficient while the betweenness, the closeness and the harmonic centralities provides the most stable set of central nodes.

*6) **Star graph**:* we consider a star with $n = 100$ nodes. As all $n - 1$ peripheral nodes are isomorphic, it is sufficient to consider graph modifications for only 2 nodes: the center of a star and a peripheral node.

The most important node (TOP1) by all centralities is a center of a star while the remaining nodes are at the second place. Thus, the edge removal changes the centrality of only 1 peripheral node, which is dropped to last place, while the center of a star is still the most important node. As a result, all the centralities show stable results by all performance metrics (TOP1-10 ≥ 0.99. In particular, the betweenness centrality do not change after the edge removal, because all the peripheral nodes had a zero value in the initial graph. Similarly, the removal of a peripheral node only influences its own centrality. However, if we remove center of a star then all nodes become equal. As a result, all the centralities are stable by all TOP indicators.

Edge addition occurs between two peripheral nodes. Thus, these two nodes remain at the second place by all centralities while other peripheral nodes are dropped to the fourth place. Hence, TOP3 measure is equal to 0.03 (as only 3 out of 100 nodes remain in Top-3) while the other TOP measures are 1. The only exception for the TOP3 is the betweenness centrality which does not change the rank of nodes after the edge addition.

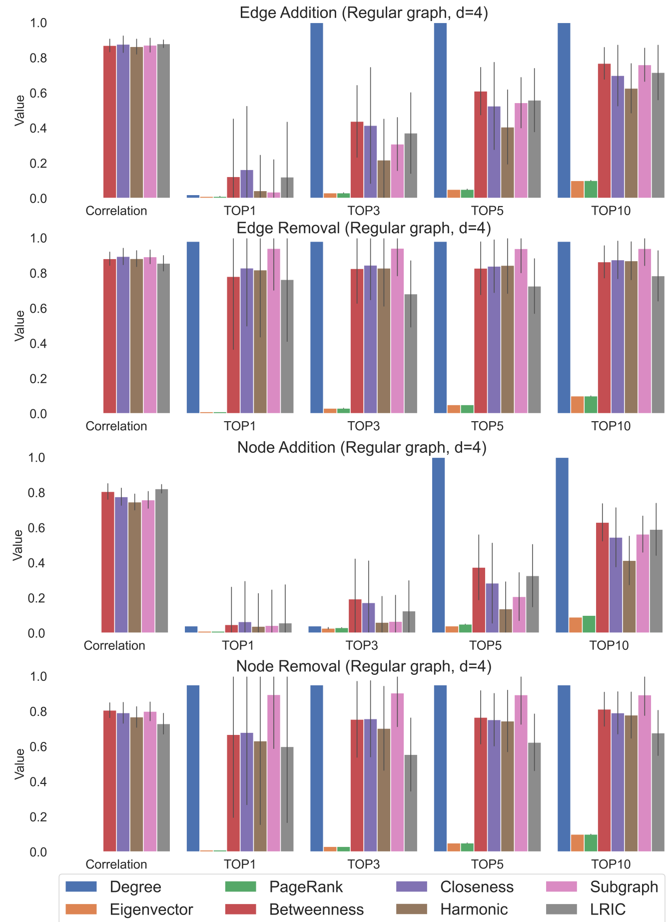

Fig. 4. Stability of centrality measures for a regular graph ($d = 4$).

Since the average degree of a star graph is strictly less than 2, the addition of a node creates only one new link in a graph. If a new node is adjacent to the center of a star, then the rank of all nodes remains. However, if we add a new node to the peripheral node, the TOP1 does not change while this peripheral node will take the second place. Interestingly, most centrality measures (degree, eigenvector, betweenness, closeness, harmonic and subgraph centralities) put the other peripheral nodes in the third place while a new node has the lower centrality. However, the PageRank and the LRIC indices put a new node in the third place while peripheral nodes have the lowest centrality. As a result, the TOP3 metric is low for the PageRank and the LRIC.

Overall, all the centrality indices are very stable to small changes in a star graph.

*7) **Regular graph**:* we examine 50 random regular graphs for $d \in \{3, 4, 5\}$ and consider 50 graph modifications. Thus, the performance measures for a regular graph with degree d are averaged across 2500 experiments. Fig. 4 illustrates the results for $d = 4$.

First, the nodes have always the same degree, eigenvector and PageRank centralities in the initial regular graph for any d. Hence, we cannot estimate the Kendall correlation coefficient for these centralities as one of the random variables is a constant. All other centralities have a strong or moderate correlation coefficient for any d and all types of graph modifications (> 0.72 for edge addition/removal and > 0.6 for node addition/removal).

Second, the addition of new nodes significantly changes the set of TOP1-10 nodes for all centrality measures. For instance, the highest value for the TOP1 is 0.25 ($d = 5$), which refers to the closeness centrality. In fact, the initial degree of each node is d while the addition of a new node may increase the centrality of d arbitrary nodes in the network. Hence, these nodes and their $d + 1$ neighbors are more likely to displace other nodes from TOP1-10. The centrality measures are also quite sensitive to the edge addition. The only exception is the degree centrality. Indeed, the addition of an edge increases the degree of 2 nodes while all other nodes have the same degree d and are included in the TOP3, TOP5 and TOP10 lists.

Finally, the degree, the betweenness, the closeness, the harmonic and the subgraph centralities are more stable to the removal operation in comparison to the eigenvector, the PageRank and the LRIC indices. However, the node addition considerably changes the TOP1-10 lists for the most centrality measures.

Overall, we observe that the increase of d from 3 to 5 does not lead to more stable results while the PageRank and eigenvector centralities are very vulnerable to the changes in the graph structure.

*8) **Square lattice**:* we examine a square lattice 10×10 and consider 10000 random modifications of each type.

Fig. 5 demonstrates that the most stable centrality to all graph modifications is a degree centrality. Indeed, all nodes can be combined into 3 groups: 64 nodes with the largest degree 4 (the middle of a lattice), 32 nodes with degree 3 (the border of a lattice) and 4 nodes with degree 2 (the corners). Therefore, the random edge addition will more likely place the incident nodes among TOP1 and at least 62 nodes will not be the most important. However, the edge addition does not change the degree of TOP3, TOP5, TOP10 nodes. Similarly, the node addition with degree 3 (the average degree of the 10×10 lattice is $\frac{4\cdot 64+3\cdot 32+2\cdot 4}{100} = 3.6$) changes the TOP1 while TOP3, TOP5, TOP10 remain the same. Finally, the edge/node removal does not change the degree of at least 59 nodes (out of 64). Therefore, these nodes ($> 92\%$) are still the most central one with respect to the degree centrality.

Next, the betweenness, the closeness and the harmonic centralities are very sensitive to the random addition of new nodes/edges. In fact, the addition of new edges produces a large number of new shortest paths between nodes, which, in turn, affects these centrality measures. However, the total ranking of nodes is relatively stable, which can be proved a strong correlation coefficient for most of the measures.

The removal of nodes/edges has the largest impact on eigenvector, PageRank and subgraph centralities. On the other hand, the removal operation still produces a strong correlation coefficient (> 0.85) for all centrality measures.

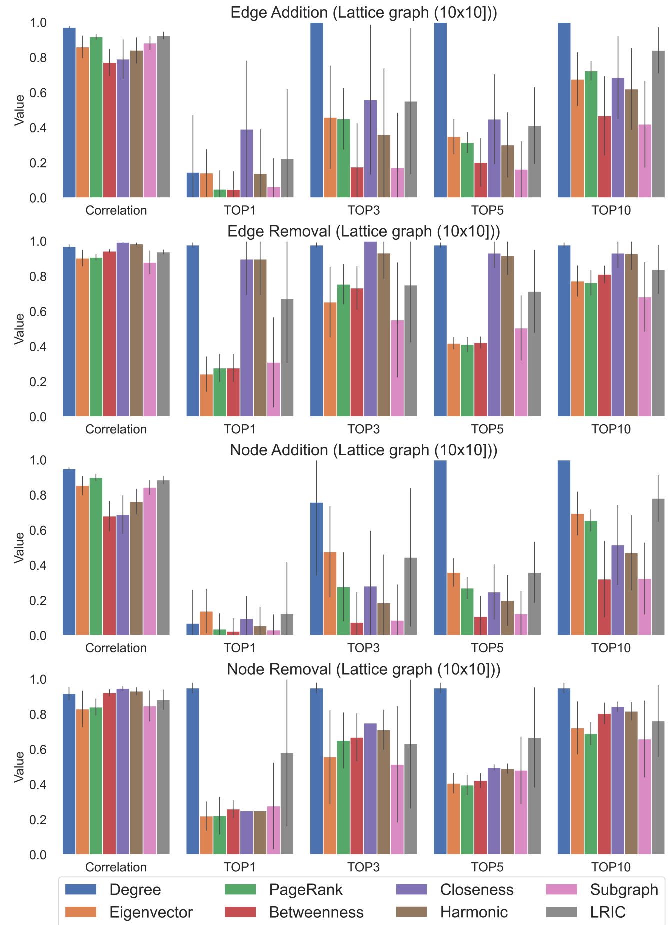

Fig. 5. Stability of centrality measures for a square lattice 10×10.

Overall, although the ranking of nodes for all centrality measures is stable (> 0.7 correlation) on a square lattice., the list of the most central nodes may significantly change for some measures.

IV. Conclusion

In this paper we have studied the sensitivity of 8 centrality measures to the presence of errors or missing information in the data. Out analysis is performed on 8 classical graph structures while the performance of the measures is estimated using the correlation coefficient and TOP1, TOP3, TOP5 and TOP10 lists of central elements.

Overall, the removal of a random edge has the lowest impact on centrality measures while the addition of a new node has the largest effect. The sensitivity of centrality measures to inaccuracy in the data highly depends on the graph structure.

Our experiments demonstrate that the centrality measures are stable on ER and BA graph models and do not change a lot if we add/delete 1 random link/edge. The general results depend on the probability p while the stability of centralities increases with a higher graph density. All centrality measures (especially, betweenness, closeness, harmonic) are not sensitive to the removal operation on the small-world graphs.

The results on a path and a lattice graphs illustrate that centrality measures (especially TOP1) are very sensitive to small modification in a graph structure. The degree, the subgraph and the LRIC centralities provide the most robust output in comparison to other centrality measures. The PageRank and the eigenvector centralities are very sensitive on regular graphs. The centralities, which are based on the shortest paths (in particular, the betweenness centrality), provide robust rankings on the balanced trees. Finally, all the measures are very stable to minor changes in a star graph.

Finally, we remark that our study is limited to the analysis of only 1 change in a graph structure. Hence, the analysis of k changes could be included in the future research. Possible future directions of the research also include the study of other centrality measures as well as the perturbation analysis on real networks.

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