A Model of Net Flaming Caused by News Propagation in Online Social Networks

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Abstract. Net flaming that occurs on Online Social Networks (OSNs) has become a serious problem. Net flaming occurs when some news is spread on OSNs and users react strongly to it. In OSNs, however, net flaming can occur even though there is nothing particularly wrong with the news itself that is spread. In this paper, we use a network oscillation model to analyze user dynamics caused by news propagation in OSNs, and show that net flaming may be caused by user comments added to the news during the propagation process. Based on these observations, we discuss countermeasures to suppress net flaming.

Keywords: Online flaming · Oscillation model · Spectral graph theory · Laplacian matrix

1 Introduction

The spread of social networking services (SNS) has been remarkable in recent years and has now become a major communication tool. As the influence of existing mass media such as TV and newspapers is declining, more and more people are getting most of their information from SNS, and the influence of SNS on people's thinking and behavior is becoming increasingly significant.

On the other hand, net flaming is a major problem in OSNs.Many cases of net flaming that seriously affected the social activities of individuals and companies in the real world have been reported [6].

Net flaming in OSNs occur when users react to certain news spread on OSNs. However, whether or not a given news item causes net flaming in OSNs is not necessarily determined by the news itself. For example, in the case of the advertisement posted by Marks & Spencer on Instagram [1], the intent of the ad was not controversial, but some users added critical comments based on their own interpretation, which many users agreed with, resulting in net flaming. In this case, whether or not net flaming occurs depends largely on the propagation path of the news, i.e., through which users the news is transmitted to other users. This paper focuses on net flaming caused by the propagation of news through users, which is unique to OSNs.

In order to analyze OSNs, we have proposed the oscillation model with a graphical representation of the relationship between users [2, 3]. To apply the oscillation model to OSN analysis, each OSN user is mapped to a node in the network, so that the user dynamics are represented by the oscillation of the node. By considering the net flaming as a divergence of user dynamics and relating the explosive dynamics that occur in the oscillation model to it, we aim to clarify the factors that cause net flaming and the mechanisms behind it.

Our previous study [5] has shown that imposing a periodic external force as an external stimulus to a node in the oscillation model can cause a phenomenon called resonance, which leads to explosive dynamics. Corresponding the propagation of an external stimulus to the propagation of news in OSNs, this phenomenon represents how the news spread causes net flaming. In this case, whether or not net flaming occurs depends on the angular frequency of the external stimulus, which indicates that the cause of the net flaming is the news itself.

On the other hand, in this paper, we model net flaming in which the propagation path of news affects its occurrence more than the news itself. The propagation path of news in OSNs is represented by a directed acyclic graph (DAG). Therefore, we consider the oscillation model represented by DAG and examine the behavior of nodes when an external stimulus is imposed to one node. We show that net flaming may be caused by user comments added to the news during the propagation process. Based on these observations, we discuss measures to suppress net flaming.

The rest of this paper is organized as follows. Section 2 describes related works. Section 3 provides an overview of the network oscillation model. Section 4 describes the proposed model and shows that this model represents the net flaming targeted in this paper, and Section 5 discusses countermeasures against such net flaming. Finally, we state conclusions and future work in Section 6.

2 Related Work

The phenomenon of information spreading in a short period of time in OSNs is known as the information cascade. There are many studies dealing with information cascades. For example, Zhou et al. [8] analyzed the dynamics of the information cascade on Twitter during the 2009 Iranian election. Also, Li et al. [7] analyze the information diffusion that occurred on Twitter during the Fukushima nuclear accident. Goel et al. [4] proposed a model of information cascades based on the SIR model. On the other hand, in net flaming, information changes the behavior of users, which further influences other users. This paper focuses on changes in user behavior caused by information rather than information spreading itself. While the SIR model represents information propagation by node state transitions, the oscillation model representing node states as oscillation, thereby represents not only information propagation but also the changes in node dynamics that it causes. This allows us to represent explosive dynamics such as net flaming.

3 Overview of Network Oscillation Model

3.1 Laplacian Matrix of the Network with Directed Links

Let $\mathcal{G}(V, E)$ be a directed graph representing the structure of a network with n nodes, where $V = \{1, ..., n\}$ is the set of nodes and E is the set of directed links. Also, let $w_{ij} > 0$ be the weight of the directed link $(i \to j) \in E$ from node i to node j. The (weighted) adjacency matrix $\boldsymbol{\mathcal{A}} := [\mathcal{A}_{ij}]_{1 \le i,j \le n}$ is an $n \times n$ matrix defined as

$$\mathcal{A}_{ij} := \begin{cases} w_{ij}, & (i \to j) \in E, \\ 0, & (i \to j) \notin E. \end{cases} \tag{1}$$

For the weighted out-degree $d_i := \sum_{j=1}^{n} \mathcal{A}_{ij}$ of node i, the degree matrix $\boldsymbol{\mathcal{D}}$ is an $n \times n$ matrix defined as $\boldsymbol{\mathcal{D}} := \mathrm{diag}(d_1, \ldots, d_n)$. The Laplacian matrix $\boldsymbol{\mathcal{L}}$ of the (weighted) directed graph is defined as $\boldsymbol{\mathcal{L}} := \boldsymbol{\mathcal{D}} - \boldsymbol{\mathcal{A}}$.

3.2 Oscillation Model on Directed Networks

Let $x_i(t)$ be the state of node i at time t, and each node is subjected to a force from each adjacent node. That is, node i is subjected to the restoring force that is represented as $f_{i \to j} = -w_{ij}(x_i(t) - x_j(t))$, where $f_{i \to j}$ is the force acting on node i from adjacent node j; w_{ij} is a positive constant. Note that the direction of the directed links and the direction of force propagation are opposite.

We consider the situation that we impose a periodic external stimulus with angular frequency ω and amplitude F on a certain node, s. Also, each node is subjected to a damping force that is proportional to its own velocity. The equation of motion of the node state vector $\boldsymbol{x}(\omega, t) :=^t (x_1(\omega, t), x_2(\omega, t), ..., x_n(\omega, t))$ for the forced oscillation of a directed graph can be written by using its Laplacian matrix $\boldsymbol{\mathcal{L}}$ as follows:

$$\frac{\partial^2 \boldsymbol{x}(\omega, t)}{\partial t^2} + \gamma \frac{\partial \boldsymbol{x}(\omega, t)}{\partial t} + \boldsymbol{\mathcal{L}} \boldsymbol{x}(\omega, t) = F \cos(\omega t) \mathbf{1}_{\{s\}}, \tag{2}$$

where $\gamma \ge 0$ is the damping coefficient and $\mathbf{1}_{\{s\}}$ is an n-dimensional vector whose s-th component is 1 and all others are 0.

Let $\lambda_\mu (\mu = 0, 1, \ldots, n-1)$ be the eigenvalue of $\boldsymbol{\mathcal{L}}$ and $\boldsymbol{v}_\mu$ be the eigenvector of $\boldsymbol{\mathcal{L}}$ associated with λ_μ. We assume λ_μs are different from each other. We expand $\boldsymbol{x}(\omega, t)$ and $\mathbf{1}_{\{s\}}$ by using $\boldsymbol{v}_\mu$ as follows:

$$\boldsymbol{x}(\omega, t) = \sum_{\mu=1}^{n} a_\mu(\omega, t) \boldsymbol{v}_\mu, \tag{3}$$

$$\mathbf{1}_{\{s\}} = \sum_{\mu=1}^{n} b_\mu \boldsymbol{v}_\mu. \tag{4}$$

Substituting these into equation (2), we obtain the equation of motion for the oscillation mode $a_\mu(\omega, t)$ as follows:

$$\frac{\partial^2 a_\mu(\omega, t)}{\partial t^2} + \gamma \frac{\partial a_\mu(\omega, t)}{\partial t} + \lambda_\mu a_\mu(\omega, t) = F \cos(\omega t) b_\mu. \tag{5}$$

The equation of motion (5) means that the oscillation dynamics on directed networks can be expressed by superposing the oscillations of each oscillation mode.

The solution of equation (5) is given by

$$a_\mu(\omega, t) = c_\mu e^{-\frac{\gamma}{2}t} \cos\left(\sqrt{\lambda_\mu - \left(\frac{\gamma}{2}\right)^2} t + \phi_\mu\right) + A_\mu(\omega) \cos(\omega t + \theta_\mu(\omega)), \tag{6}$$

where c_μ and ϕ_μ are constants, and $A_\mu(\omega)$ and $\theta_\mu(\omega)$ are the amplitude and the initial phase, respectively. They are expressed as

$$A_\mu(\omega) = \frac{F b_\mu}{\sqrt{(\lambda_\mu - \omega^2)^2 + (\gamma\omega)^2}}, \tag{7}$$

$$\theta_\mu(\omega) = \arctan\left(-\frac{\gamma\omega}{\lambda_\mu - \omega^2}\right). \tag{8}$$

4 Model of Net flaming Caused by News Propagation in OSNs

In this section, we model the unidirectional propagation of news in OSNs using the oscillation model and show that the path of news propagation affects the occurrence of net flaming. Since the path of news propagation in OSNs is represented by DAG that maps users to nodes, we focus the oscillation model represented by DAG. The external stimulus imposed to one node propagate to other nodes through links, which corresponds to the news propagation in an OSN. When a user receives news directly from another user, a directed link exists between the corresponding nodes. The larger the weight of the link, the more strongly the user is influenced by the other user.

4.1 Assumptions

We assume that nodes are topologically sorted and node IDs are assigned in descending order. That is, if $i < j$, then $w_{ij} = 0$ because there is no link from node i to node j. Since node 1 has no outgoing link, $d_1 = 0$. Also, assume that $d_1, \ldots, d_n$ are different from each other. Thus, since $d_i \geq 0$ for any $i \geq 2$, all nodes except node 1 have outgoing links. This also means that node 1 is reachable from all nodes. An external stimulus is imposed to node 1, which is transmitted to all other nodes.

4.2 Mapping of Oscillation Modes to Nodes

Since the Laplacian matrix of DAG is a triangular matrix, each eigenvalue of $\boldsymbol{\mathcal{L}}$ is equal to one of its diagonal components. From the definition of the Laplacian matrix described in Section 3.1, each diagonal component of $\boldsymbol{\mathcal{L}}$ is d_i of some node i. Therefore, for each node i, d_i is equal to one of the eigenvalues of $\boldsymbol{\mathcal{L}}$, which means that the oscillation mode μ such that $\lambda_\mu = d_i$ can be mapped to node i. Hereafter, the oscillation mode corresponding to node i will be referred to as oscillation mode i.

From equation (6), the solution of oscillation mode i is

$$a_i(\omega, t) = c_i e^{-\frac{\gamma}{2}t} \cos(\omega_i t + \phi_i) + A_i(\omega) \cos(\omega t + \theta_i(\omega)), \tag{9}$$

where $\omega_i \coloneqq \sqrt{d_i - \left(\frac{\gamma}{2}\right)^2}$.

4.3 Addition of Oscillation at Nodes

Let $\boldsymbol{v}_i =^t (v_{i1}, v_{i2}, \ldots, v_{in})$ be the eigenvector associated with the eigenvalue d_i of the oscillation mode i.

From the definitions of eigenvalues and eigenvectors, $\boldsymbol{\mathcal{L}}\boldsymbol{v}_i = d_i\boldsymbol{v}_i$, which becomes

$$\begin{pmatrix} d_1 v_{i1} \\ w_{21}v_{i1} + d_2 v_{i2} \\ w_{31}v_{i1} + w_{32}v_{i2} + d_3 v_{i3} \\ \vdots \\ w_{n1}v_{i1} + \cdots + w_{n(n-1)}v_{i(n-1)} + d_n v_{in} \end{pmatrix} = \begin{pmatrix} d_i v_{i1} \\ d_i v_{i2} \\ d_i v_{i3} \\ \vdots \\ d_i v_{in} \end{pmatrix}, \tag{10}$$

since $\boldsymbol{\mathcal{L}}$ is a triangular matrix.

Lemma 1. *For any $j < i$, $v_{ij} = 0$.*

Proof. We prove by mathematical induction that $v_{ij} = 0$ for any $j < i$.
When $j = 1$, the j-th component of equation (10) is $d_j v_{ij} = d_i v_{ij}$. Since $d_j = 0 \neq d_i$, $v_{ij} = 0$ holds.
Assume that $v_{ij} = 0$ holds when $j \leq k$, where $k < i - 1$.
When $j = k + 1$, using the inductive hypothesis, the j-th component of equation (10) is $d_j v_{ij} = d_i v_{ij}$. Since $d_j \neq d_i$ from $j = k + 1 \neq i$, $v_{ij} = 0$ holds.
By induction, we have shown $v_{ij} = 0$ for any $j < i$. □

Lemma 2. *For any i, $v_{ii} \neq 0$.*

Proof. Applying Lemma 1 to equation (10), $v_{ii} = 0$ implies $\boldsymbol{v}_i = \mathbf{0}$, which contradicts the fact that $\boldsymbol{v}_i$ is an eigenvector of $\boldsymbol{\mathcal{L}}$.
Therefore, $v_{ii} \neq 0$. □

Lemma 3. *If $v_{ij} \neq 0$, node i is reachable from node j.*

Proof. Let i be an arbitrary node. We show by mathematical induction that for any j, if $v_{ij} \neq 0$, node i is reachable from node j.
When $j = 1$, if $i = 1$, node i is reachable from node j, otherwise, $v_{ij} = 0$ from Lemma 1 since $j < i$.
Assume that for $j \leq k$, if $v_{ij} \neq 0$, node i is reachable from node j.
When $j = k + 1$, from the j-th component of equation (10),

$$v_{ij} = \frac{w_{j1}v_{i1} + \cdots + w_{j(j-1)}v_{i(j-1)}}{d_i - d_j}. \tag{11}$$

From equation (11), if $v_{ij} \neq 0$, there exists $l \leq j - 1$ such that $w_{jl} \neq 0 \wedge v_{il} \neq 0$. Using the inductive hypothesis, node i is reachable from node l since $v_{il} \neq 0$. Also, there is a directed link $(j \to l)$ since $w_{jl} \neq 0$. Therefore, node i is reachable from node j.
By induction, we have shown that for any node j, if $v_{ij} \neq 0$, node i is reachable from node j. ◻

Lemma 4. *At any node j, the amplitude of oscillation mode i is proportional to v_{ij} and b_i.*

Proof. From equation (3), the oscillation of node j can be expressed as a superposition of each oscillation mode as follows:

$$x_j(\omega, t) = \sum_{i=1}^{n} a_i(\omega, t)v_{ij}. \tag{12}$$

From equation (12), the amplitude of oscillation mode i is proportional to v_{ij}. From equation (7), $A_i(\omega)$ in equation (9) is proportional to b_i. Under $a_i(\omega, 0) = 0$ as an initial condition, c_i in equation (9) is

$$c_i = -\frac{A_i(\omega)\cos(\theta_i(\omega))}{\cos(\phi_i)}, \tag{13}$$

which implies that c_i is also proportional to b_i. Therefore, from equation (9), $a_i(\omega, t)$ is proportional to b_i, that is, the amplitude of oscillation mode i is proportional to b_i. ◻

From Lemma 4, oscillation mode i does not affect node j where $v_{ij} = 0$. Therefore, from Lemma 3, oscillation mode i affects only nodes those are reachable to node i. Equation (9) indicates that oscillation mode i consists of two oscillations, one with angular frequency ω_i and the other with angular frequency ω. Of these, oscillation with angular frequency ω_i is only included in oscillation mode i and is therefore observed only at nodes that are reachable to node i. This indicates that as the external stimulus propagates over the network, a new oscillation is added at each node, which then propagates to other nodes.

This new oscillation represents information that is different from the original news and corresponds to a comment added by the user to the news.

We show by simulation how new oscillations are added at each node during the propagation of an external stimulus on the network. As the response of node i to the external stimulus, we observe the oscillation energy $E_i(\omega) = \frac{1}{2}\omega^2 \sum_{k=1}^{n}(a_k(\omega))^2(v_{ki})^2$. The network used in the simulation is shown in Figure 1. In the figure, the numbers next to the links indicate the link weights.

Figure 2 shows the spectrum of oscillation energy at nodes 2,3, and 4 when the damping factor $\gamma = 0.1$ and an external stimulus with an angular frequency of 100 is imposed to node 1. The peak of angular frequency 100 seen at each node corresponds to the oscillation of the external stimulus propagated from node 1. A peak at an angular frequency of 20 is seen at node 2, which represents the addition of oscillation with angular frequency of $\omega_2 = \sqrt{d_2 - (\frac{\gamma}{2})^2}$, since the eigenvalue of node 2 is $d_2 = 400$. In addition, the peak around an angular frequency of 60 seen at node 4 is not seen at nodes 2 and 3. This indicates that this oscillation is added at node 4. The eigenvalue of node 4 is $d_4 = 3600$, which means that the angular frequency $\omega_4 \simeq 60$.

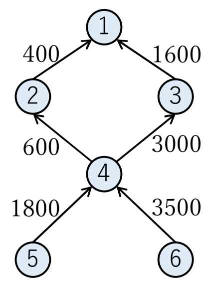

Fig. 1. A Simple Network

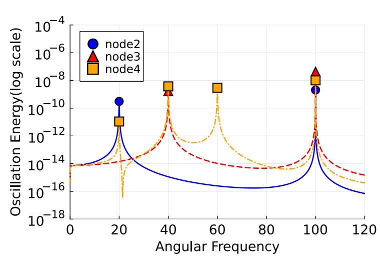

Fig. 2. Adding Oscillations at Nodes

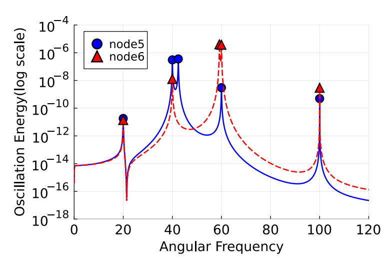

Fig. 3. Occurrence of Resonance at Nodes

4.4 Resonance Caused by Added Oscillation

We show that the phenomenon of nodes resonating with the added oscillation occurs, and that this model represents net flaming that occur when a user's comment prompts other users.

Suppose that $d_i \simeq d_j$ for two nodes i, j where $i < j$.

Since $|d_i - d_j| \simeq 0$, equation (11) implies that $|v_{ij}|$ is very large unless $v_{ij} = 0$. Therefore, the amplitude of oscillation mode i at node j is very large from Lemma 4. Equation (9) indicates that oscillation mode i consists of two oscillations, one with angular frequency ω_i and the other with angular frequency ω. The oscillation with angular frequency ω is included in all oscillation modes and may cancel each other out due to superposition. On the other hand, the oscillation with angular frequency ω_i is included only in oscillation mode i. Therefore, at node j, the amplitude of oscillation with angular frequency ω_i increases.

From Lemma 2, for any i, we can choose an eigenvector $\boldsymbol{v}_i$ so that $v_{ii} = 1$. Since the external stimulus is imposed to node 1, equation (4) becomes

$$\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_1 v_{12} + b_2 \\ \vdots \\ b_1 v_{1n} + \cdots + b_{n-1} v_{(n-1)n} + b_n \end{pmatrix}. \tag{14}$$

From the j-th component of equation (14),

$$-b_j = b_1 v_{1j} + \cdots + b_{(j-1)} v_{(j-1)j}. \tag{15}$$

Since $i < j$, the right side of equation (15) contains the term $b_i v_{ij}$, which implies that $|b_j|$ becomes very large with $|v_{ij}|$. Therefore, the amplitude of oscillation mode j at node j is very large from Lemma 4. From equation (9), oscillation mode j includes the oscillation with angular frequency ω_j, which is included only in oscillation mode j and do not cancel each other out by superposition. Therefore, at node j, the amplitude of oscillation with angular frequency ω_j increases.

Thus, when $d_i \simeq d_j$, the amplitudes of oscillations with angular frequency ω_i and ω_j increase at node j. These amplitude increases mean that the oscillation added at node i has caused resonance at node j. This resonance corresponds to the situation that a user reacts strongly to the comments added by the other user, and represents the occurrence of net flaming caused by users' comments.

On the other hand, when node i is not reachable from node j, no amplitude increase occurs because $v_{ij} = 0$ from Lemma 3. This means that user j reacts strongly only when the news arrives via user i, indicating that the occurrence of net flaming depends on the propagation path of the news.

Figure 3 shows the oscillation energy spectrum at nodes 5 and 6 in the same simulation as Figure 2. At node 5, two high peaks are seen around the angular frequency of 40. This indicates that node 5 resonates with the oscillation added at node 3 because the eigenvalue $d_5 = 1800$ of node 5 is close to the eigenvalue

d_3 = 1600 of node 3. Similarly, at node 6, there are two high peaks around the angular frequency of 60, which indicates that node 6 resonates with the oscillation added at node 4.

5 Measures to Suppress Network Resonance

5.1 Manipulating Link Weights

As mentioned in Section 4.4, whether resonance occurs at node i depends on its eigenvalue, which is equal to its weighted out-degree d_i, i.e. the sum of weights of all links from node i. Therefore, if a resonating node can be identified, a possible countermeasure is to change the eigenvalue of the node by manipulating the weights of any of the links from the node. The link weights in the oscillation model reflect the strength of the association between users in the OSN, such as the speed of information transfer and trust in other users. Manipulation of these may be able to keep resonance to a small scale and prevent net flaming. However, since it is difficult to predict the occurrence of resonance in advance, this measure can be taken only after catching the omen of resonance. Figure 4 shows the results of the simulation under the same conditions as Figure 3, except that the weight of the link ($6 \rightarrow 4$) changed from 3500 to 3000. The peak around the angular frequency of 60 is lower than that of Figure 3. This means that the resonance was suppressed because the eigenvalue of node 6 changed.

5.2 Increase in Dumping Coefficient

Equation (9) shows that the oscillation with angular frequency ω_i added at node i is damped with time by the damping coefficient γ. This implies that an increase in γ is effective in reducing the occurrence of node resonance. γ reflects the degree to which OSN user dynamics decrease over time, i.e. the fickleness of users. Although γ is difficult to artificially manipulate, it is likely to increase in the future, since users are becoming increasingly fickle with the explosive increase in information contents, and this trend is expected to further accelerate. As a result, the number of net flaming caused by node resonance may become smaller and shorter in the future. Figure 5 shows the results of a simulation of the network in Figure 1 with a damping factor of $\gamma = 0.5$ and other conditions identical to those in Figure 3. Due to the effect of damping, all peaks are lower than in Figure 3.

6 Conclusion

In this paper, we proposed a theoretical model of net flaming that occurs in the process of unidirectional propagation of news in OSNs. In the oscillation model represented by DAG, another oscillation is added at a node during the propagation of an external stimulus, which may cause other nodes to resonate with it. This phenomenon represents the mechanism by which net flaming is triggered by user comments during news propagation. Based on these observations, we discussed countermeasures to suppress net flaming.

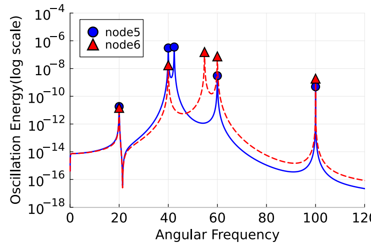

Fig. 4. Manipulating a Link Weight

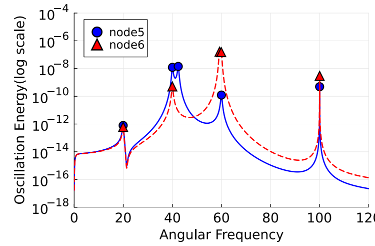

Fig. 5. Increase in Damping Coefficient

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