Modularity-Based Fairness in Community Detection

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***Abstract*—In this paper, we study the fairness of community structures in networks from a group-based perspective. Specifically, we assume that individuals in a social network belong to different groups based on the value of one of their sensitive attributes, such as their age, gender, or race, and we consider community fairness towards the protected group. Most previous work has focused on a balanced-based definition of fairness that seeks for an appropriate representation of the members of the protected group in each community. We introduce a novel form of community fairness, termed modularity-based fairness, that asks that the members of the protected group are well connected in their respective communities. We present results of the balanced-based and modularity-based fairness of several real and synthetic networks.**

***Index Terms*—community detection, fairness, fair clustering, modularity, fair communities**

I. Introduction

Social networks play a pivotal role in shaping opinions and influencing decision-making processes. However, despite the extensive research on algorithmic fairness, the fairness issues stemming from the interconnection between individuals in a network have received comparatively less attention [1]. In this paper, we examine the community structures formed within social networks through the lens of fairness.

We take a group-based approach in which we assume that individuals in a social network belong to one of two groups based on the value of one of their attributes, for example their gender, age, or race. Most previous work on group-based fairness in clustering takes a representation based approach that asks that a sufficient percentage of nodes in each cluster belongs to the protected group [2]–[4].

In this paper, we introduce a novel fairness notion for communities, termed *modularity fairness*. Modularity is a characterization of the quality of communities based on the connectivity of each community. Specifically, high values of modularity indicate that there are many edges within communities and few edges between them [5]. The proposed modularity fairness asks that the nodes of the protected group in each community are well-connected, that is, in each community, the nodes that belong to the protected group have many intra-community edges and few inter-community ones.

Then, we provide insights of the balance and modularity fairness present in real networks by evaluating the balance and modularity fairness of several real world networks. In addition, we seek to explore the confounding factors that may lead to unfairness. Previous research has shown that the relative size of the groups and homophily (i.e., the tendency of nodes to connect with similar nodes) affect various properties in the network, such as the degree and Pagerank distribution of the groups [6], [7]. To study the effect of these parameters on community fairness, we propose a new extension of the stochastic block model [8] and use it to create various synthetic networks. We report findings from experiments conducted on these synthetic networks, aiming at evaluating the impact of size imbalance, homophily, and connectivity on balance and modularity fairness.

The remainder of this paper is structured as follows. In Section II, we define the two types of community fairness, while in Section III, we present our model for generating synthetic graphs and an evaluation of the community fairness of both real and synthetic networks. In Section IV, we present related work and in Section V, we offer our conclusions.

II. Fairness in Community Detection

Let $G = (V, E)$ be an undirected graph, where V is the set of nodes and $E \subseteq V \times V$ the set of edges. We assume that nodes in V belong to one of two groups based on the value of one of their sensitive attributes. Let us call the two groups, blue group, denoted as B, and red group, denoted as R, where $B \cup R = V$ and $B \cap R = \emptyset$. Let us also assume without loss of generality that the red group is the *protected* group. We will use ϕ to denote the ratio of the red nodes in the overall population, that is, $\phi = \frac{|R|}{|V|}$.

We assume that the nodes in the graph are partitioned into a set $\mathcal{C} = \{C_1, C_2, \ldots C_k\}$ of k communities. For a community C_i, abusing slightly the notation, we use $B(C_i)$ and $R(C_i)$, for respectively the blue and red nodes that belong to C_i. We assess the fairness of each community from two distinct perspectives. Firstly, we analyze whether each group is adequately represented within each community. Secondly, we

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evaluate whether the members of each group are sufficiently well-connected within their respective communities.

A. Balanced-based Community Fairness

Our first notion of fairness is based on balance. For a community $C_i \in \mathcal{C}$, its balance, $balance(C_i)$, is defined as [2]:

$$balance(C_i) = min\left(\frac{|R(C_i)|}{|B(C_i)|}, \frac{|B(C_i)|}{|R(C_i)|}\right).$$

Balance focus on the representation of each group in each community. A community with an equal number of red and blue nodes has balance 0.5 (perfectly balanced), while a monochromatic community has balance 1 (fully unbalanced). Balance encapsulates a specific notion of fairness.

We adopt a demographic parity approach [9] to balanced-based fairness. For a community C_i to be fair towards the protected group R, we ask that the percentage of red nodes in the community is at least equal to ϕ, i.e., the percentage of red nodes in the overall population.

Definition 1: For a community $C_i \in \mathcal{C}$, the balance fairness of C_i, $f_{balance}(C_i)$, is defined as:

$$f_{balance}(C_i) = \frac{|R(C_i)|}{|C_i|} - \phi.$$

Negative values of $f_{balance}(C_i)$ indicate unfairness towards the red group, that is, the fact that the red nodes are less well-represented in the community than the blue ones. Positive values indicate the opposite.

B. Modularity-based Community Fairness

Our second notion of fairness evaluates for each community how well the members of each group are connected within the community emphasizing the importance of strong connections. Our definition is based on modularity [5]. Modularity measures the divergence between the number of intra-communities edges from the expected such number assuming random connections. Specifically, the modularity of community C_i, Q_{C_i}, is defined as:

$$Q_{C_i} = \frac{1}{2m}\sum_{u\in C_i}\sum_{v\in C_i}(A_{uv} - \frac{d_u\,d_v}{2m})$$

where A is the adjacency matrix of G, m the number of edges in G and d_u, d_v the degree of node u, and v respectively.

Modularity provides a measure of how well all nodes in a community are connected with each other. We are interested in the connectivity of the red nodes in particular. To this end, for each red node u in C_i we take the difference between the actual number of its intra-community edges and the expected such number. We call this measure *red modularity* ($Q^R_{C_i}$):

$$Q^R_{C_i} = \frac{1}{2m}\sum_{u\in R(C_i)}\sum_{v\in C_i}(A_{uv} - \frac{d_u\,d_v}{2m}).$$

We define similarly the *blue modularity* ($Q^B_{C_i}$) as:

$$Q^B_{C_i} = \frac{1}{2m}\sum_{u\in B(C_i)}\sum_{v\in C_i}(A_{uv} - \frac{d_u\,d_v}{2m}).$$

An equivalent more efficient way to express modularity was derived in [10]:

$$Q_{C_i} = \frac{L_{C_i}}{m} - \left(\frac{d_{C_i}}{2m}\right)^2$$

where L_{C_i} is the number of intra-community edges in C_i, and d_{C_i} the sum of the degrees of the nodes in C_i.

Following a similar approach, we derive the following more efficient formulas for the red and blue modularity:

$$Q^R_{C_i} = \frac{L^B_{C_i}}{m} - \frac{d_{C_i}\,d^R_{C_i}}{(2m)^2}$$

$$Q^B_{C_i} = \frac{L^B_{C_i}}{m} - \frac{d_{C_i}\,d^B_{C_i}}{(2m)^2}$$

where $L^R_{C_i}$ ($L^B_{C_i}$) is the number of intra-community edges with at least one red (blue) endpoint and $d^R_{C_i}$ ($d^B_{C_i}$) is the sum of the degrees of the red (blue) nodes in C_i.

We now define *modularity fairness* by comparing the red and the blue modularity.

Definition 2: For a community $C_i \in \mathcal{C}$, the modularity fairness of C_i, $f_{modularity}(C_i)$, is defined as:

$$f_{modularity}(C_i) = \frac{Q^R_{C_i} - Q^B_{C_i}}{abs(Q_{C_i})}.$$

Negative values of $f_{modularity}(C_i)$ indicate unfairness towards the red group, that is, the fact that the red nodes are less well-connected within the community than the blue ones. Positive values indicate the opposite.

III. Evaluation of Community Fairness

In this section, we study the balance and modularity fairness of several real and synthetic networks. To detect communities, we use the Louvain algorithm, a greedy algorithm that optimizes modularity [11].

A. Are Real Networks Fair?

We study the following real datasets:

- **Pokec**[1]: Nodes are the users of the Pokec social network and edges are friendship relationships between them. We study both the gender attribute (**Pokec-g**) and the age attribute (**Pokec-a**). For the age attribute, we remove nodes that have no value for this attribute, or the value is not a possible value for age. We use the median value of the remaining nodes for splitting the nodes into two (almost) equal-sized groups.
- **Deezer**[2]: Nodes are Deezer users from European countries and edges are mutual follow relationships between them.

[1] https://snap.stanford.edu/data/soc-Pokec.html
[2] https://snap.stanford.edu/data/feather-deezer-social.html

TABLE I: Network characteristics. AvRd (AvBd): average degree of the red (blue) nodes, Rh (Bh): red (blue) homophily.

Network	# Nodes	# Edges	Attribute	Protected	# Blue nodes	# Red nodes	AvRd	AvBd	Rh	Bh	ϕ
Pokec-g	1,632,803	22,301,964	Gender	Female	804,474	828,289	28.28	26.32	0.97	1.02	0.51
Pokec-a	1,095,590	10,779,932	Age	Younger	546,212	549,381	14.20	25.20	0.99	1.01	0.50
Deezer	28,281	92,752	Gender	-	12,535	15,738	6.73	6.34	1.07	0.97	0.57
Facebook	4,039	88,234	Gender	-	1,532	2,507	42.09	46.30	1.01	1.23	0.62
Twitch Gamers	168,114	6,797,557	Maturity	Low	79,033	89,081	74.31	88.26	0.90	1.11	0.53

TABLE II: Detected communities.

Network	Number	Avg. Size	Avg. Mod
Pokec-g	38	42,967	0.71
Pokec-a	47	23,309	0.72
Deezer	88	348	0.69
Facebook	16	252	0.83
Twitch Gamers	21	8,005	0.42

TABLE III: Synthetic dataset characteristics.

Parameter	Meaning	Default
N	number of nodes	4000
p_R	ratio of red nodes	0.5
l	edges per new node	4
k	initial number of communities	5
p_h	homophily	0.5
p_c	prob. of intra-cluster edge	0.7

- **Facebook**[3]: The dataset consists of friends list from Facebook.
- **Twitch Gamers**[4]: Nodes are twitch users and edges are mutual follow relationship between them.

The characteristics of the real datasets are summarized in Table I. Note that for Deeezer and Facebook the actual values of the sensitive attribute are hidden in the datasets. We also report homophily values that indicate the tendency of nodes to connect with nodes with similar attribute values, in our case, with nodes of the same color. We report separately the homophily of the red (Rh) and the homophily of the blue nodes (Bh). Rh is computed as the ratio of the number of the actual edges connecting two red nodes and the expected number of such edges (estimated as ϕ^2). $Rh > 1$ indicates homophily, while $Rh < 1$ heterophily (tendency to connect with nodes of the opposite color). Similarly, we compute Bh as the ratio of the number of the actual edges between two blue nodes and the expected such number (estimated as $(1-\phi)^2$).

In Table II, we report results about the communities detected in the real networks using the Louvain algorithm, specifically, we report the number of communities detected, their average size, and their average modularity.

In Figures 1 and 2, we plot respectively the distribution of $f_{balance}$ and $f_{modularity}$ of the communities found in the real networks. Negative values correspond to communities unfair towards the red group, while positive values to communities unfair towards the blue group. Unfair communities exist in all networks both in the case of $f_{balance}$ and $f_{modularity}$, as indicated by the large number of communities having non-zero $f_{balance}$ and $f_{modularity}$ values. In terms of $f_{balance}$, Pokec is almost gender balanced (Pokec-g), while it is unfair towards the younger individuals (Pokec-a). Pokec is also modularity fair for gender (Pokec-g) and modularity unfair towards the younger individuals (Pokec-a). Deeezer is almost gender balanced, but the members of the red group are more connected in their communities as indicated by the majority of positive $f_{modularity}$ values. Facebook is slightly balanced unfair towards the red group, which is also the largest group, but heavily modularity unfair towards the blue group. Finally, Twitch is both balanced and modularity unfair towards the more mature users.

[3]http://snap.stanford.edu/data/ego-Facebook.html

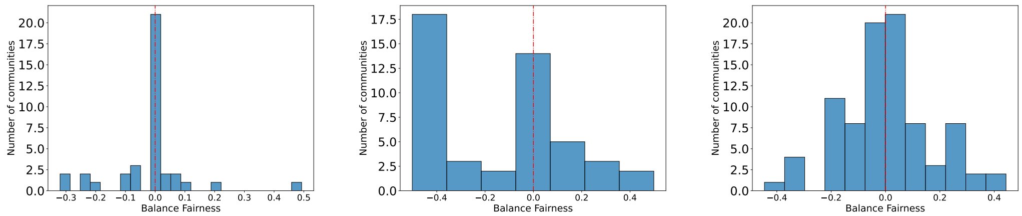
[4]https://snap.stanford.edu/data/twitch_gamers.html

B. What are the Factors that Lead to Unfairness?

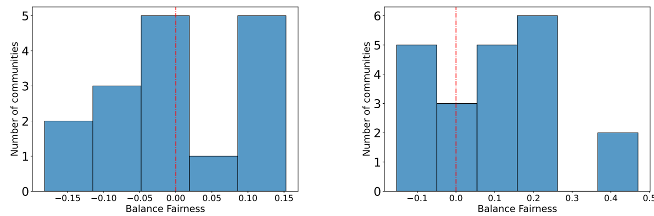
To study the factors that may lead to unfairness, we introduce a new model based on the stochastic block model [8] to create networks with nodes of different colors and connectivity behavior. The model has three important parameters:

- p_R: the probability that a node belongs to the red group. This parameter controls the relative size of the two groups. By setting $p_R = 0.5$, we get groups of equal size, while when $p_R < 0.5$, the red group is the minority group.
- p_h: the probability that a node connects with a a node that has the same color with it. This parameter controls homophily. With $p_h = 1$, we have perfect homophily, with $p_h = 0$, nodes connect only with nodes of the opposite color and we get heterophily, while $p_h = 0.5$ results in neutral behavior.
- p_c: the probability that a node connects with a a node that belongs to the same community with it. This parameter controls modularity. Large values of p_c lead to well-connected communities, while when $p_c \leq 0.5$, there is no community structure.

We start by an initial assignment of nodes in k communities and then generate edges between the nodes. Note that the actual number of communities created differs from k, depending on the values of the other parameters. First, we assign an equal number of nodes to each community. Then, for each of the nodes, we use p_R to determine the color of the node. Finally, we generate edges as follows. For each node u, we create l edges on average. To create an edge $e = (u, v)$ for u, we first select the community that v will belong to and then the color of v. Specifically, we use p_c to determine whether e will be an inter-community or an intra-community edge. Then, we use p_h to determine the color of node v. Let $X \in \{R, B\}$ be the selected color. If e is an intra-community edge, we choose

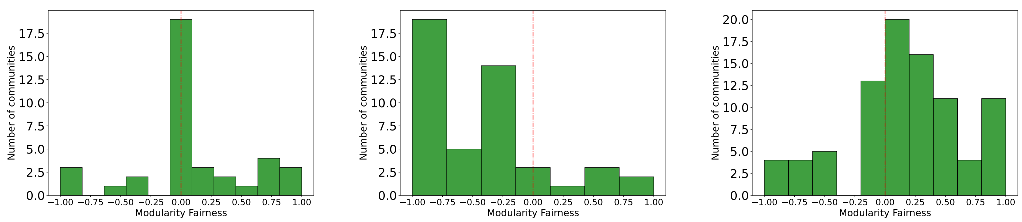


(a) Pokec-g (b) Pokec-a (c) Deezer

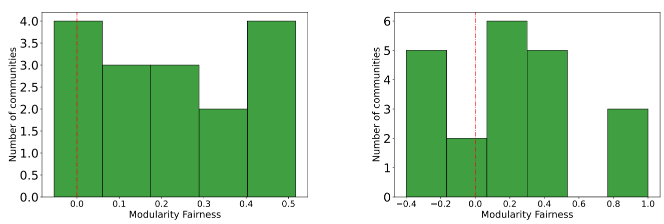


(d) Facebook (e) Twitch

Fig. 1: Distribution of $f_{balance}$ in the real datasets. The red line corresponds to 0.



(a) Pokec-g (b) Pokec-a (c) Deezer



(d) Facebook (e) Twitch

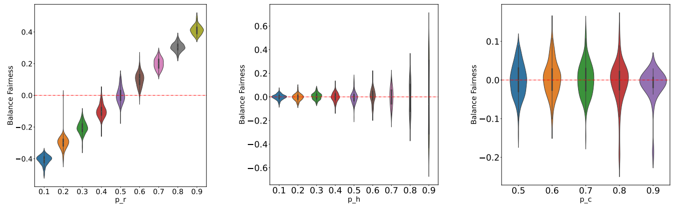
Fig. 2: Distribution of $f_{modularity}$ in the real datasets. The red line corresponds to 0.

as v uniformly at random a node of color X in the same community as u. If e is an inter-community edge, we first choose uniformly at random one of the $k-1$ communities other than the community of u. Let C be this community. We choose as v uniformly at random a node of color X in community C.

Table III summarizes the parameters. We study the influence of size imbalance (p_R), homophily (p_h) and intra-cluster connectivity (p_c) in fairness. In each case, we vary one of the three parameters and use the default values for the other. We run each experiment 5 times.

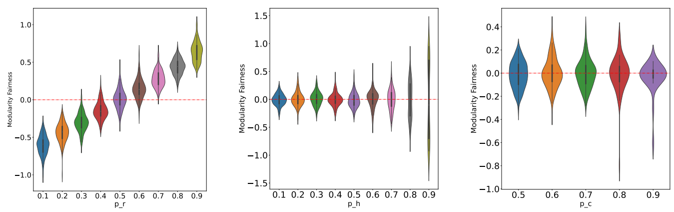
In Figure 3, we report results regarding $f_{balance}$. We use violin plots to depict the distribution of $f_{balance}$ in the communities for different values of our parameters. Again negative values indicate unfairness towards the red group, while positive values unfairness towards the blue group. Size imbalance (p_B) is directly reflected in $f_{balance}$, since there is balance unfairness towards the smaller group. In terms of homophily (p_h), when the networks have low homophily, i.e., $p_h < 0.5$, all communities are almost balanced, i.e., their $f_{balance}$ is very close to 0. As networks become homophilic (p_h increases), the $f_{balance}$ of many communities deviates from 0 towards values corresponding to monochromatic communities (recall that $\phi = p_R = 0.5$). Finally, intra-cluster connectivity (p_c) has a very small effect on $f_{balance}$. While p_c affects the quality of the communities, it does not affect fairness, indicating that quality may not have a direct influence on fairness.

In Figure 4, we report results regarding $f_{modularity}$. In terms



(a) p_R (b) p_h (c) p_c

Fig. 3: Distribution of $f_{balance}$ in synthetic datasets. The red line corresponds to 0.



(a) p_R (b) p_h (c) p_c

Fig. 4: Distribution of $f_{modularity}$ in synthetic datasets. The red line corresponds to 0.

of p_R, we get the best fairness, when the two groups have equal sizes ($p_R = 0.5$). In all other cases, the algorithm favors the larger group, creating better connected communities (positive values) for this group. Homophily (p_h) seems to also affect $f_{modularity}$, since nodes end up in communities in which they are less connected. Again p_c has a smaller effect on fairness.

IV. Related Work

There has been a lot of recent research in fairness in machine learning [12], [13], although fairness for graph data has been less explored [1]. The balanced view approach of fairness was introduced in the seminal work of fairlets [2] for the case of clustering of non-graph data. It has been extended in various directions, such as for supporting more than one protected group [3], and for improving performance [14]. A balanced view has also been considered for graph data through a spectral clustering algorithm with fairness constraints [4].

Another view of fairness is that of ensuring results of equal quality for both groups by minimizing the clustering cost. This view is taken for non-graph data in the *socially fair* k-means clustering approach that seeks to minimize the maximum of the average k-means objective applied to each group [15] and in *equitable* clustering that seeks to minimize the distance of each point to its nearest center [16]. In a sense, modularity-based clustering follows this view, since its goal is maintaining good clustering quality in terms of intra-cluster connectivity.

There is also research on individual fairness in graphs e.g., [17]. Modularity-based fairness can be also applied at the individual node level, by looking at the connectivity of individual nodes; we leave this as future work. Finally, modularity has been refined to promote *mixed links*, i.e., links connecting nodes of different color in link recommendations [18]. In this paper, we focus on the connectivity of the protected group inside each community. It would be interesting to also look into promoting mixed links inside each community.

V. Conclusions

In this short paper, we introduced a novel definition of group fairness in communities based on modularity. We evaluated the modularity-based and balanced-based fairness of communities in several real and synthetic networks. In the future, we plan to design community detection algorithms that produce both balance-based and modularity-based fair communities. We will also investigate individual notions of fairness.

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