

OPTI 517

Image Quality

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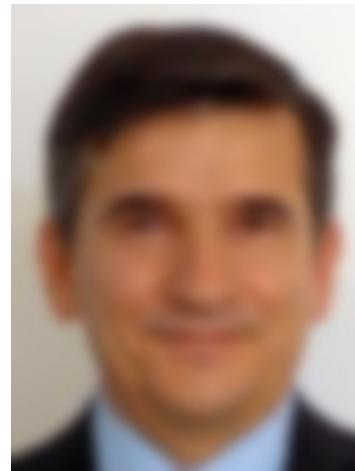
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Why is Image Quality Important?

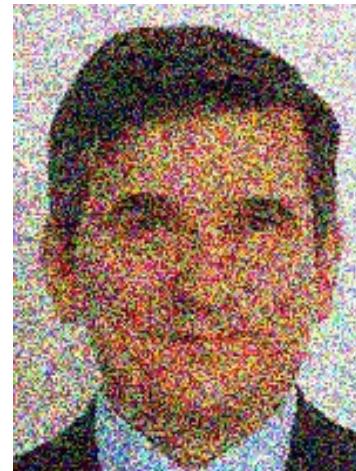
- Resolution of detail
 - Smaller blur sizes allow better reproduction of image details
 - Addition of noise can mask important image detail



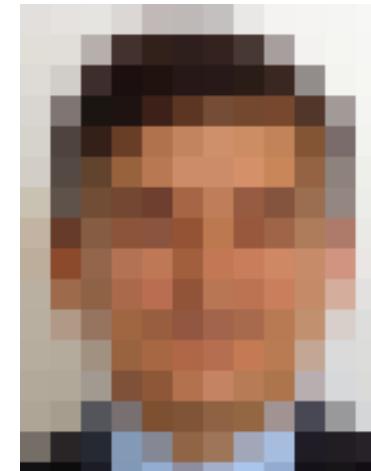
Original



Blur added



Noise added



Pixelated

Step One - What is Your Image Quality (IQ) Spec?

- There are many metrics of image quality
 - Geometrical based (e.g., spot diagrams, RMS wavefront error)
 - Diffraction based (e.g., PSF, MTF)
 - Other (F-theta linearity, uniformity of illumination, etc.)
- It is imperative that you have a specification for image quality when you are designing an optical system
 - Without it, you don't know when you are done designing!

You vs. the Customer

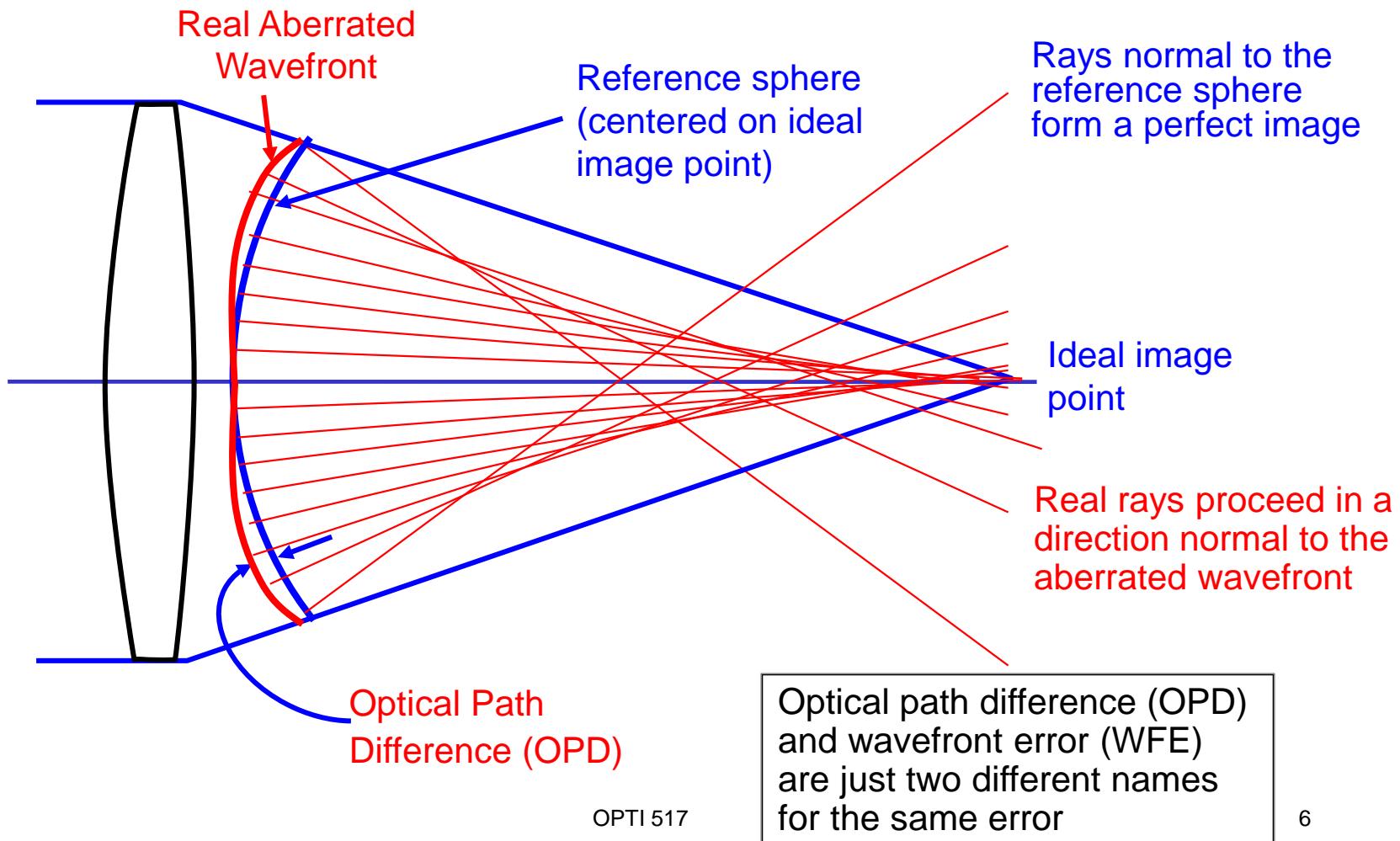
- Different kinds of image quality metrics are useful to different people
- **Customers** usually work with performance-based specifications
 - MTF, ensquared energy, distortion, etc.
- **Designers** often use IQ metrics that mean little to the customer
 - E.g., ray aberration plots and field plots
 - These are useful in the design process, but they are not end-product specs
- In general, you will be working to an end-product specification, but will probably use other IQ metrics during the design process
 - Often the end-product specification is difficult to optimize to or may be time consuming to compute
- Some customers do not express their image quality requirements in terms such as MTF or ensquared energy
 - They know what they want the optical system to do
- It is up to the optical engineer (in conjunction with the system engineer) to translate the customer's needs into a numerical specification suitable for optimization and image quality analysis

When to Use Which IQ Metric

- The choice of appropriate IQ metric usually depends on the application of the optical system
 - Long-range targets where the object is essentially a point source
 - Example might be an astronomical telescope
 - Ensquared energy or RMS wavefront error might be appropriate
 - Ground-based targets where the details of the object are needed to determine image features
 - Example is any kind of image in which you need to see detail
 - MTF would be a more appropriate metric
 - Laser scanning systems
 - A different type of IQ metric such as the variation from F-theta distortion
- The type of IQ metric may be part of the lens specification or may be a derived requirement flowed down to the optical engineer from systems engineering
 - Do not be afraid to question these requirements
 - Often the systems engineering group doesn't really understand the relationship between system performance and optical metrics

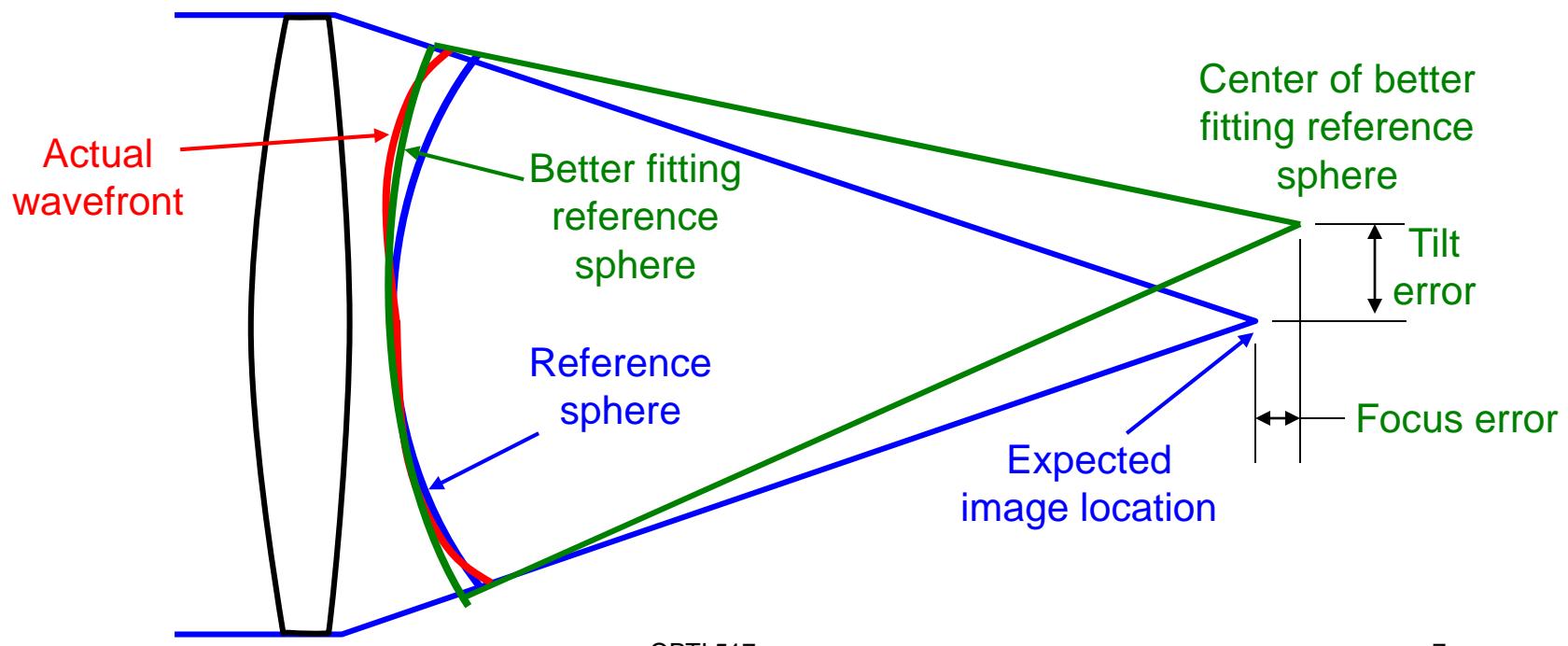
Wavefront Error

- Aberrations occur when the converging wavefront is not perfectly spherical

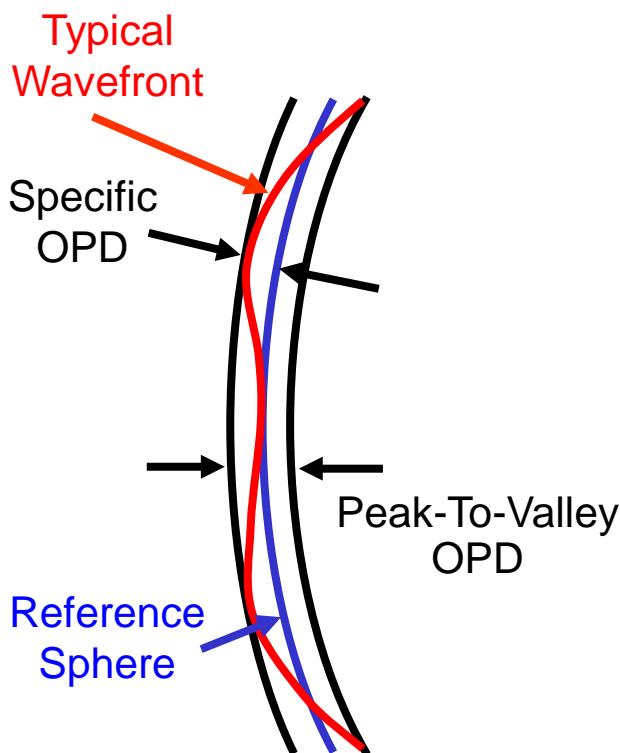


Tilt and Focus of a Wavefront

- When calculating the wavefront error, the reference sphere is centered on the "expected" image location (usually the image surface location of the chief ray)
- It may be that a different reference sphere will fit the actual wavefront better
 - If the center of the better reference sphere is at a different **axial** location than the expected image location, there is a "**focus error**" in the wavefront
 - If the center of the better reference sphere is at a different **lateral** location than the expected image location, there is a "**tilt error**" in the wavefront



Optical Path Difference



Peak-to-valley OPD is the difference between the longest and the shortest paths leading to a selected focus

RMS wavefront error is given by:

$$\langle W^n \rangle = \frac{\iint [W(x, y)]^n dx dy}{\iint dx dy}$$

$$W_{rms} = \sqrt{\langle W^2 \rangle - \langle W \rangle^2}$$

For n discrete rays across the pupil

$$RMS = \sqrt{\sum OPD^2/n}$$

This wavefront has the same P-V wavefront error as the example at the left, but it has a lower RMS

Peak-to-Valley vs. RMS

- The ratio of P-V to RMS is not a fixed quantity
- Typical ratios of P-V to RMS (from Shannon's book)
 - Defocus 3.5
 - 3rd order spherical 13.4
 - 5th order spherical 57.1
 - 3rd order coma 8.6
 - 3rd order astigmatism 5.0
 - Smooth random errors ~5
- In general, for a mixture of lower order aberrations, $P\text{-}V/\text{RMS} \approx 4.5$
- When generating wavefront error budgets, RMS errors from different sources can be added in an RSS fashion
 - P-V errors cannot be so added
- In general, Peak-to-Valley wavefront error is a poor choice to use for error budgeting
 - However, Peak-to-Valley surface error or wavefront error is still commonly used as a surface error specification for individual optical components and even for complete optical systems

Rayleigh Criterion

- Lord Rayleigh observed that when the maximum wavefront error across a wavefront did not exceed $\lambda/4$ peak-to-valley, the image quality was "not sensibly degraded"
 - This quarter-wave limit is now called the [Rayleigh Criterion](#)
- This is approximately equivalent to the RMS wavefront error being about 0.07 wave or less (using the value for defocus)
- The [Strehl Ratio](#) is a related measure of image quality
 - It can be expressed (for RMS wavefront error < 0.1 wave) as

$$\text{Strehl Ratio} = e^{-(2\pi\Phi)^2} \approx 1 - (2\pi\Phi)^2$$

where Φ is the RMS wavefront error (in waves)

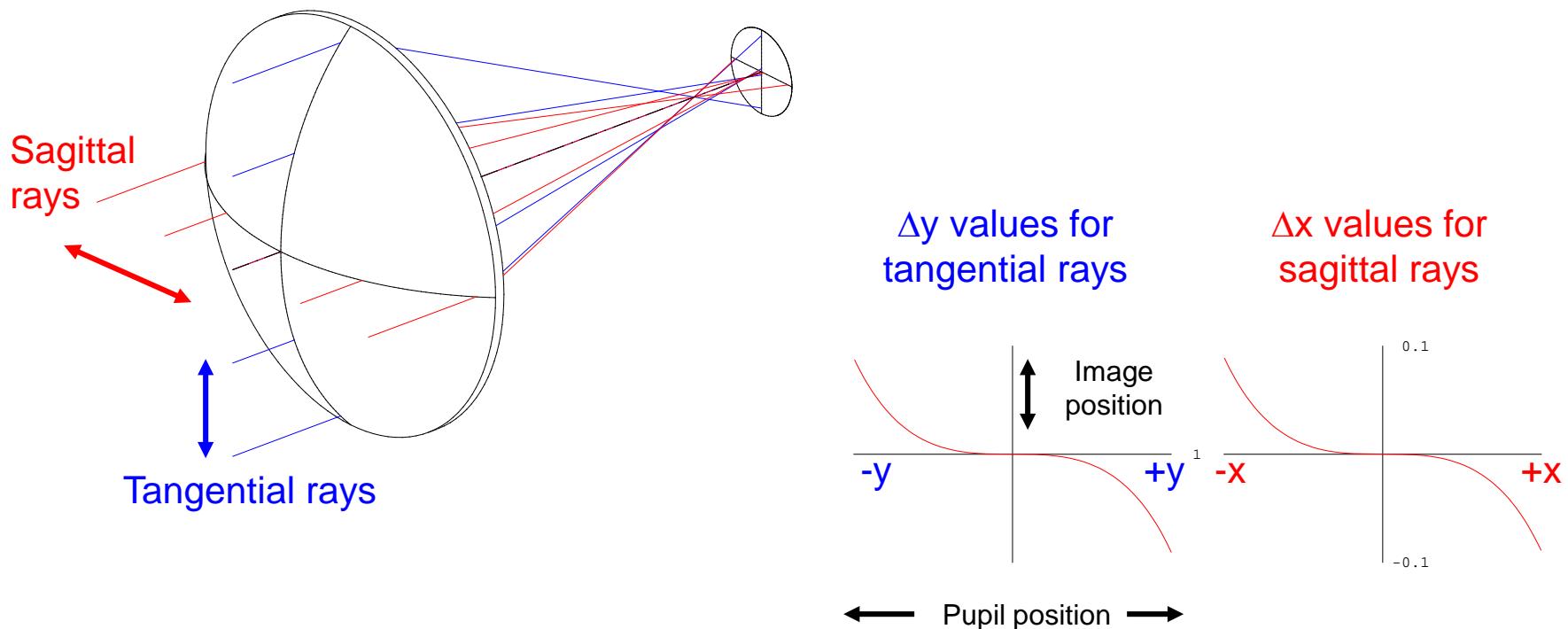
- For $\Phi = 0.07$ wave, the Strehl Ratio ≈ 0.8
- Requiring the Strehl Ratio to be 0.80 or greater for acceptable image quality is often called the [Maréchal Criterion](#)

Image Quality Metrics

- The most commonly used **geometrical-based** image quality metrics are
 - Ray aberration curves
 - Spot diagrams
 - Seidel aberrations
 - Encircled (or ensquared) energy
 - RMS wavefront error
 - Modulation transfer function (MTF)
- The most commonly used **diffraction-based** image quality metrics are
 - Point spread function (PSF)
 - Encircled (or ensquared) energy
 - MTF
 - Strehl Ratio

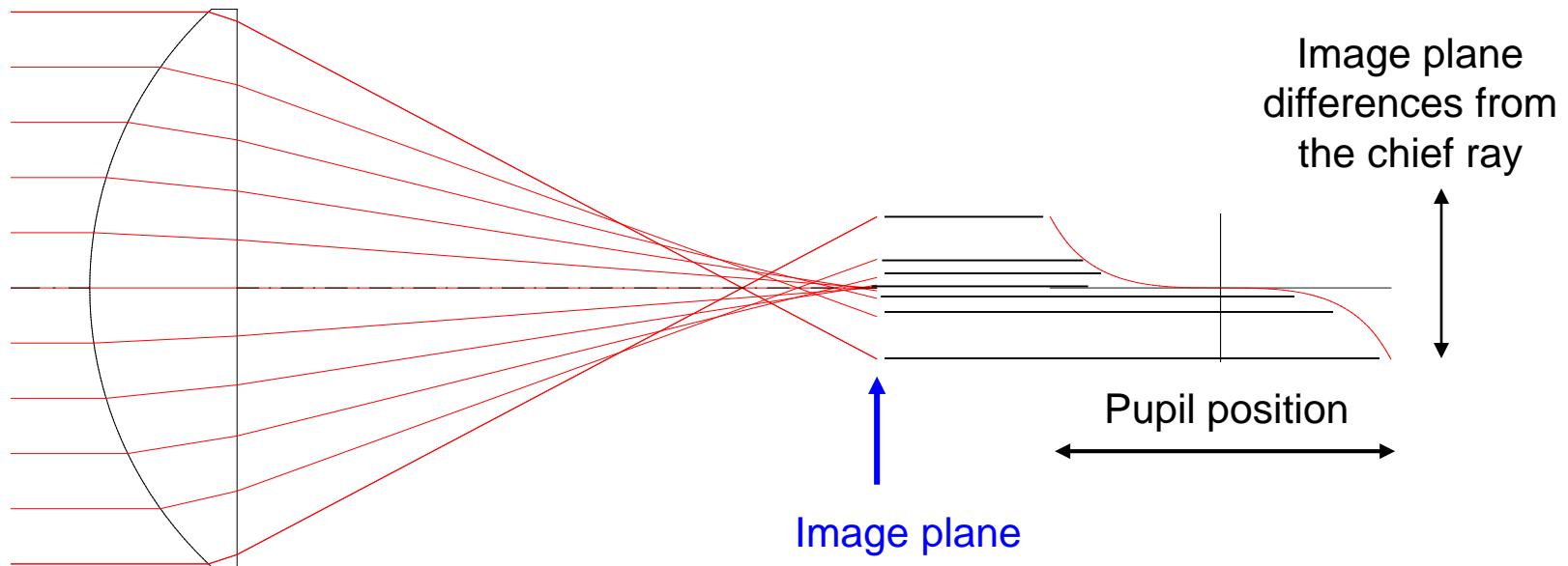
Ray Aberration Curves

- These are by far the image quality metric most commonly used by optical designers during the design process
- Ray aberration curves trace fans of rays in two orthogonal directions
 - They then map the image positions of the rays in each fan relative to the chief ray vs. the entrance pupil position of the rays



Graphical Description of Ray Aberration Curves

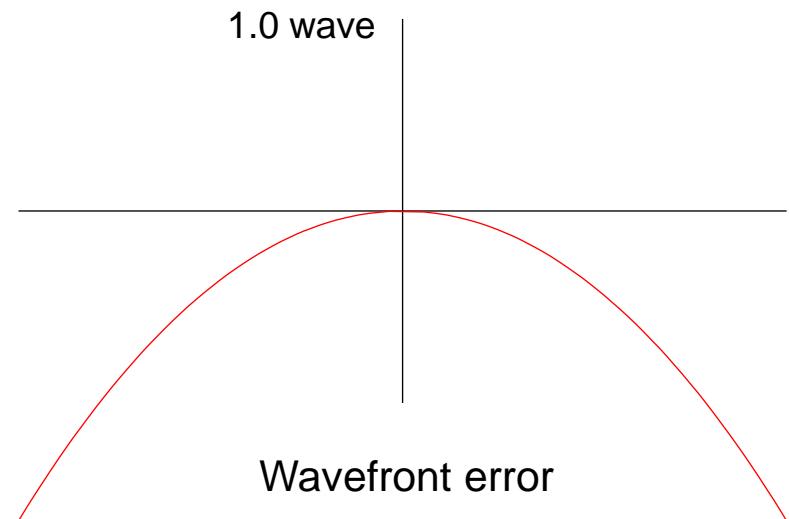
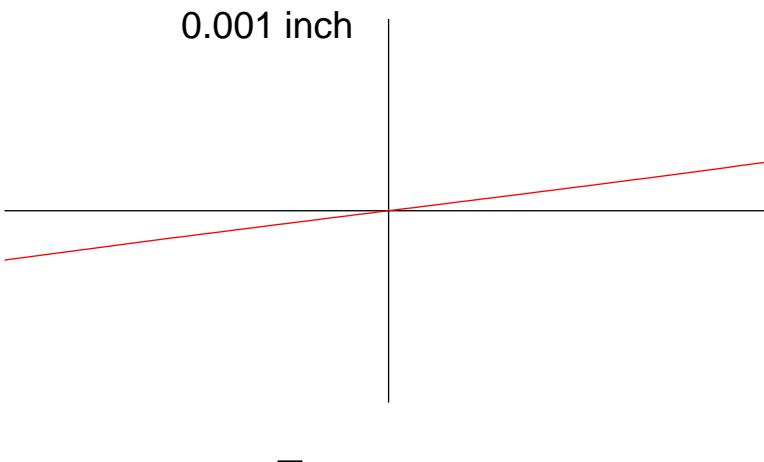
- Ray aberration curves map the image positions of the rays in a fan
 - The plot is image plane differences from the chief ray vs. position in the fan



- Ray aberration curves are generally computed for a fan in the YZ plane and a fan in the XZ plane
 - This omits skew rays in the pupil, which is a failing of this IQ metric

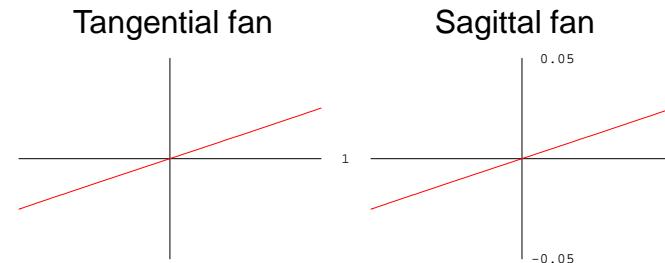
Transverse vs. Wavefront Ray Aberration Curves

- Ray aberration curves can be transverse (linear) aberrations in the image vs. pupil position or can be OPD across the exit pupil vs. pupil position
 - The transverse ray errors are related to the slope of the wavefront curve
$$\varepsilon_y(x_p, y_p) = -(R/r_p) \partial W(x_p, y_p) / \partial y_p$$
$$\varepsilon_x(x_p, y_p) = -(R/r_p) \partial W(x_p, y_p) / \partial x_p$$
$$R/r_p = -1/(n'u') \approx 2 f/\#$$
- Example curves for pure defocus:

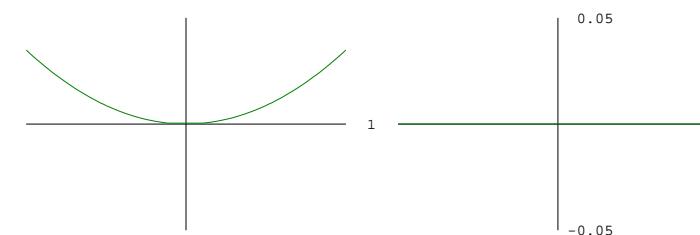


More on Ray Aberration Curves

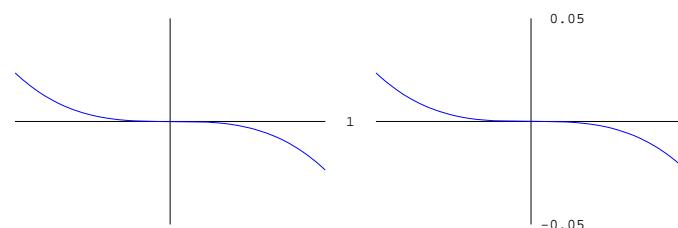
- The shape of the ray aberration curve can tell what type of aberration is present in the lens for that field point (transverse curves shown)



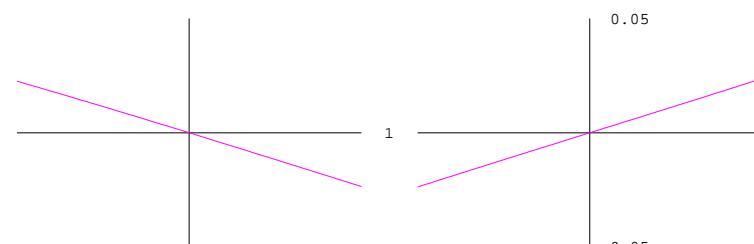
Defocus



Coma



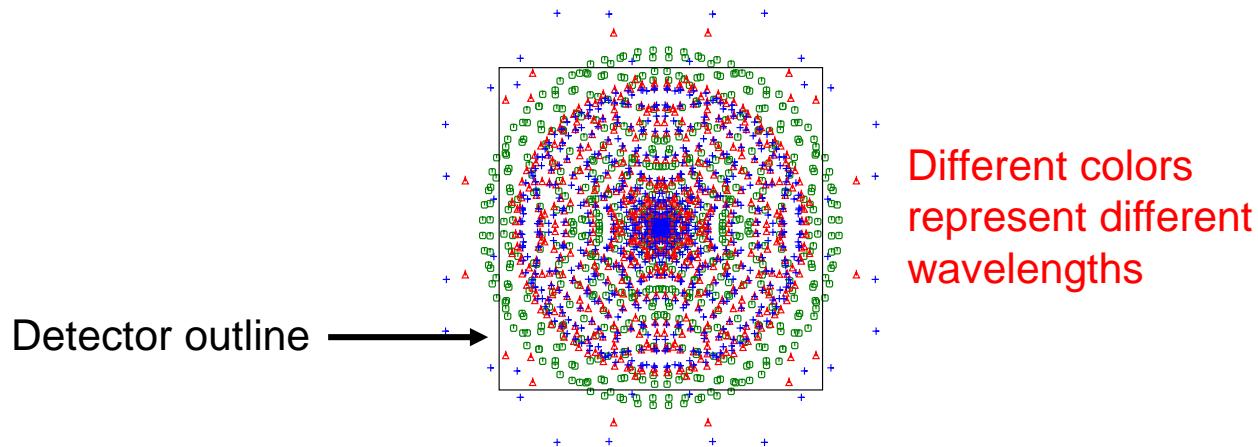
Third-order spherical



Astigmatism

The Spot Diagram

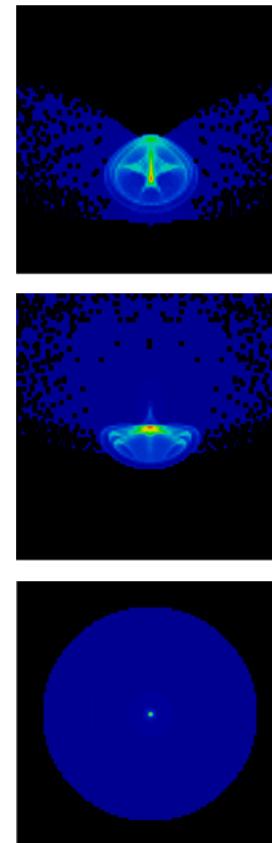
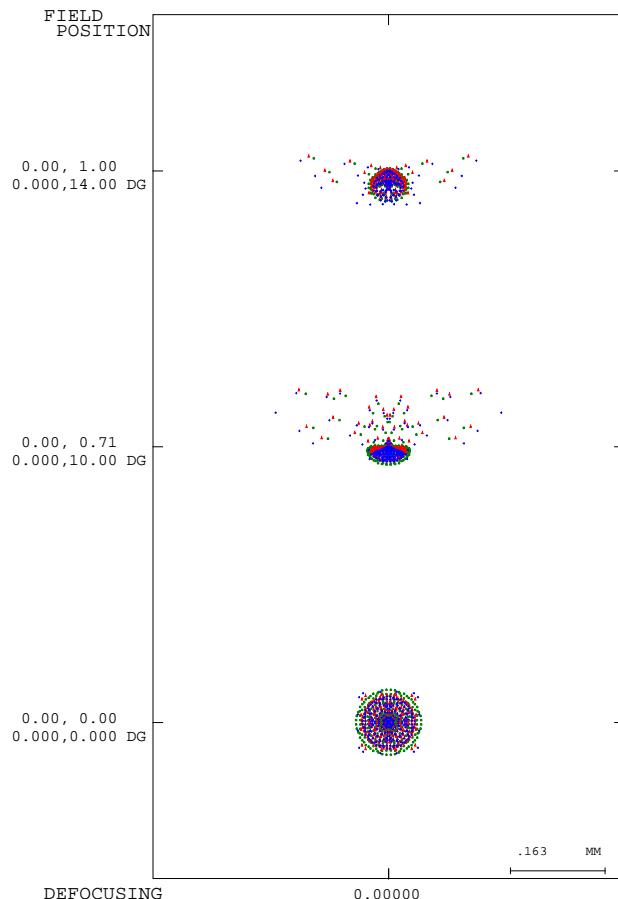
- The spot diagram is readily understood by most engineers
- It is a diagram of how spread out the rays are in the image
 - The smaller the spot diagram, the better the image
 - This is geometrical only; diffraction is ignored
- It is useful to show the detector size (and/or the Airy disk diameter) superimposed on the spot diagram



- The shape of the spot diagram can often tell what type of aberrations are present in the image

Main Problem With Spot Diagrams

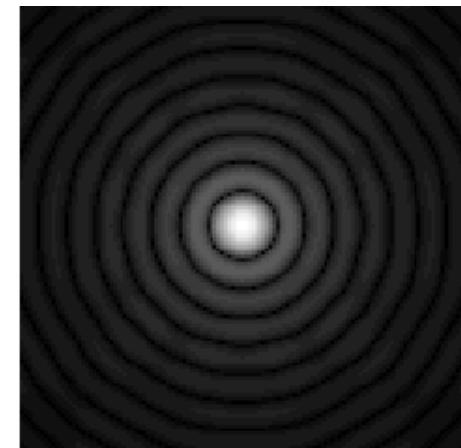
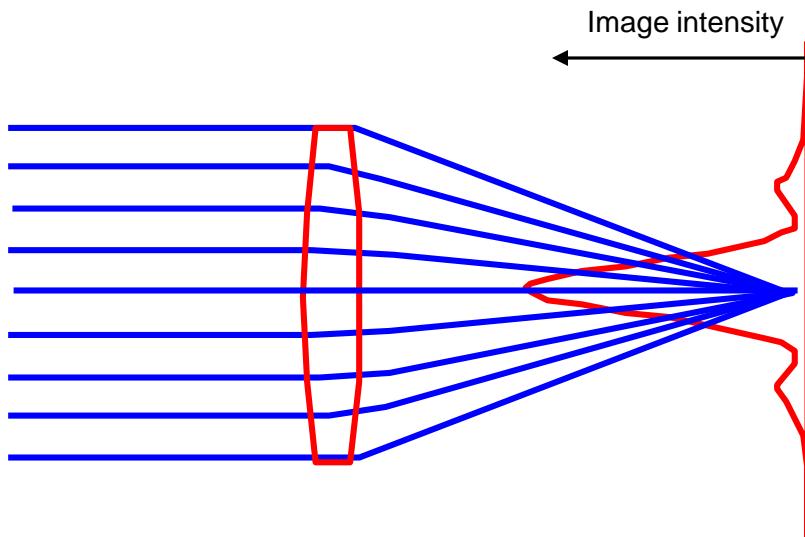
- The main problem is that spots in the spot diagram don't convey intensity
 - A ray intersection point in the diagram does not tell the intensity at that point



The on-axis image appears spread out in the spot diagram, but in reality it has a tight core with some surrounding low-intensity flare

Diffraction

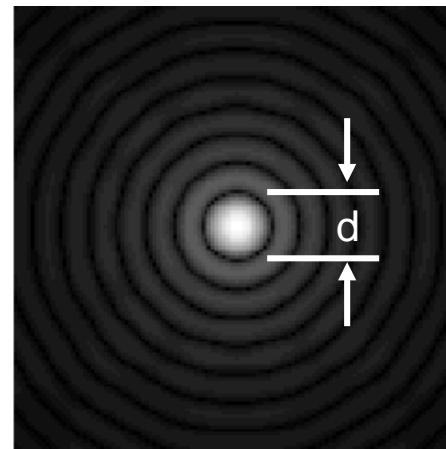
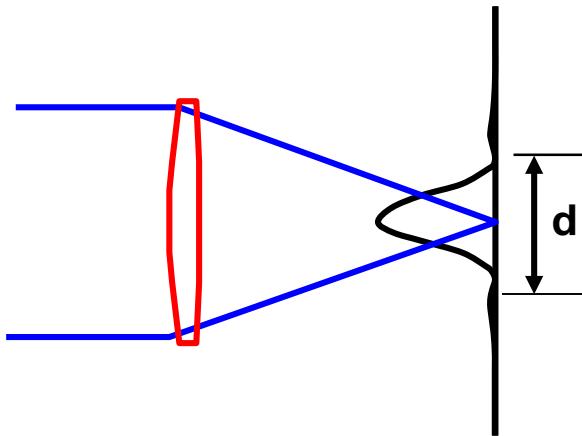
- Some optical systems give point images (or near point images) of a point object when ray traced geometrically (e.g., a parabola on-axis)
- However, there is in reality a lower limit to the size of a point image
- This lower limit is caused by diffraction
 - The diffraction pattern is usually referred to as the [Airy disk](#)



Diffraction pattern of a perfect image

Size of the Diffraction Image

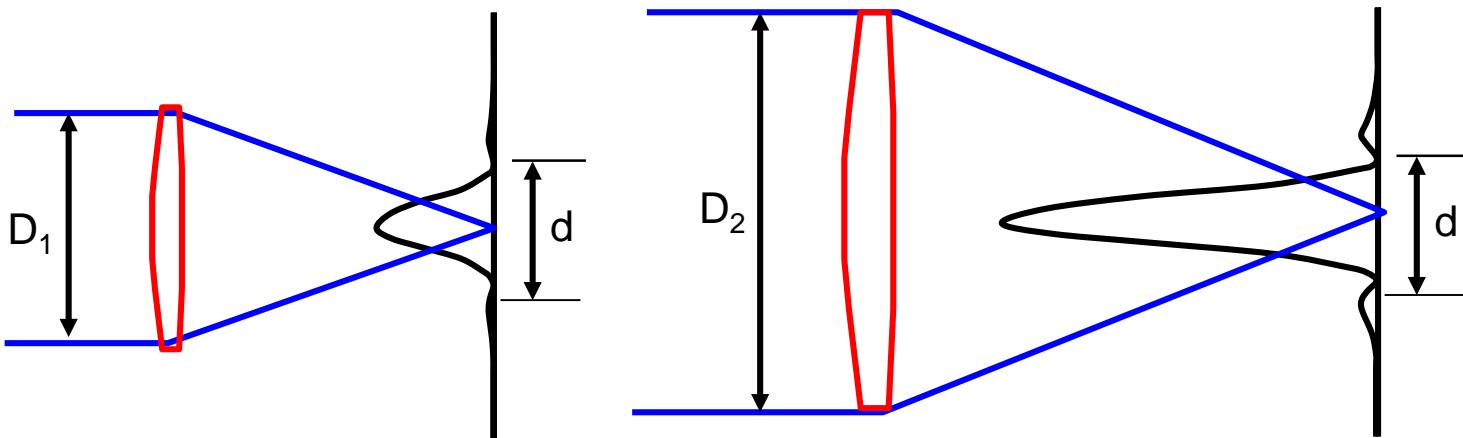
- The diffraction pattern of a perfect image has several rings
 - The center ring contains ~84% of the energy, and is usually considered to be the "size" of the diffraction image



Very important !!!!

- The diameter of the first ring is given by $d \approx 2.44 \lambda f/\#$
 - This is independent of the focal length; it is only a function of the wavelength and the f-number
 - The angular size of the first ring $\beta = d/F \approx 2.44 \lambda/D$
- When there are no aberrations and the image of a point object is given by the diffraction spread, the image is said to be diffraction-limited

Image of a Point Object and a Uniform Background

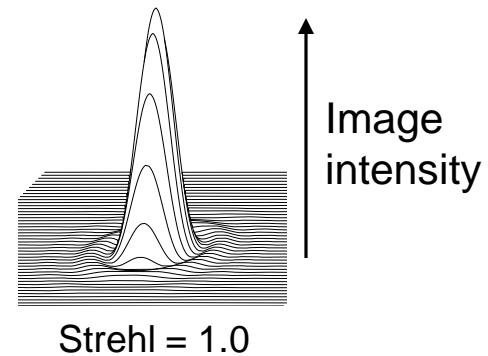
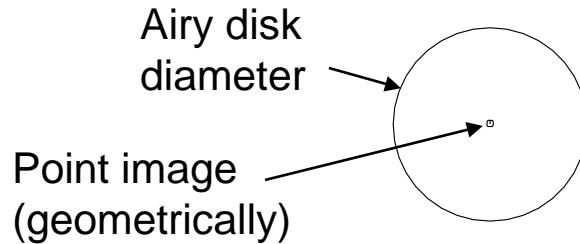


Same $f/\#$

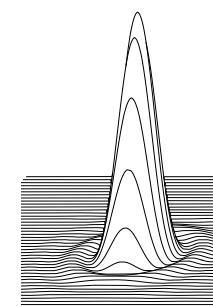
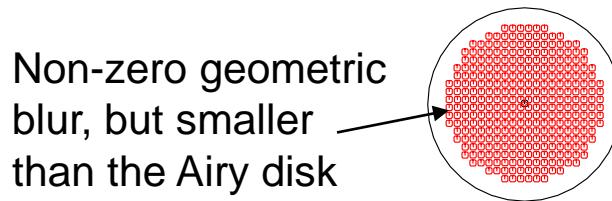
- For both systems, the Airy disk diameter is the same size
 - $d = 2.44 \lambda f/\#$
- For both systems, the irradiance of the background at the image is the same
 - $E_B = L_B(\pi/4f^2)$
- The flux forming the image from the larger system is larger by $(D_2/D_1)^2$
 - We get more energy in the image, so the signal-to-noise ratio (SNR) is increased by $(D_2/D_1)^2$
 - This is important for astronomy and other forms of point imagery

Spot Size vs. the Airy Disk

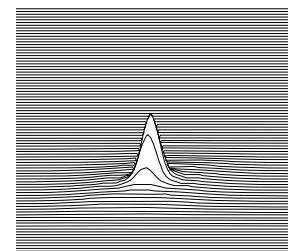
- Regime 1 – Diffraction-limited



- Regime 2 – Near diffraction-limited

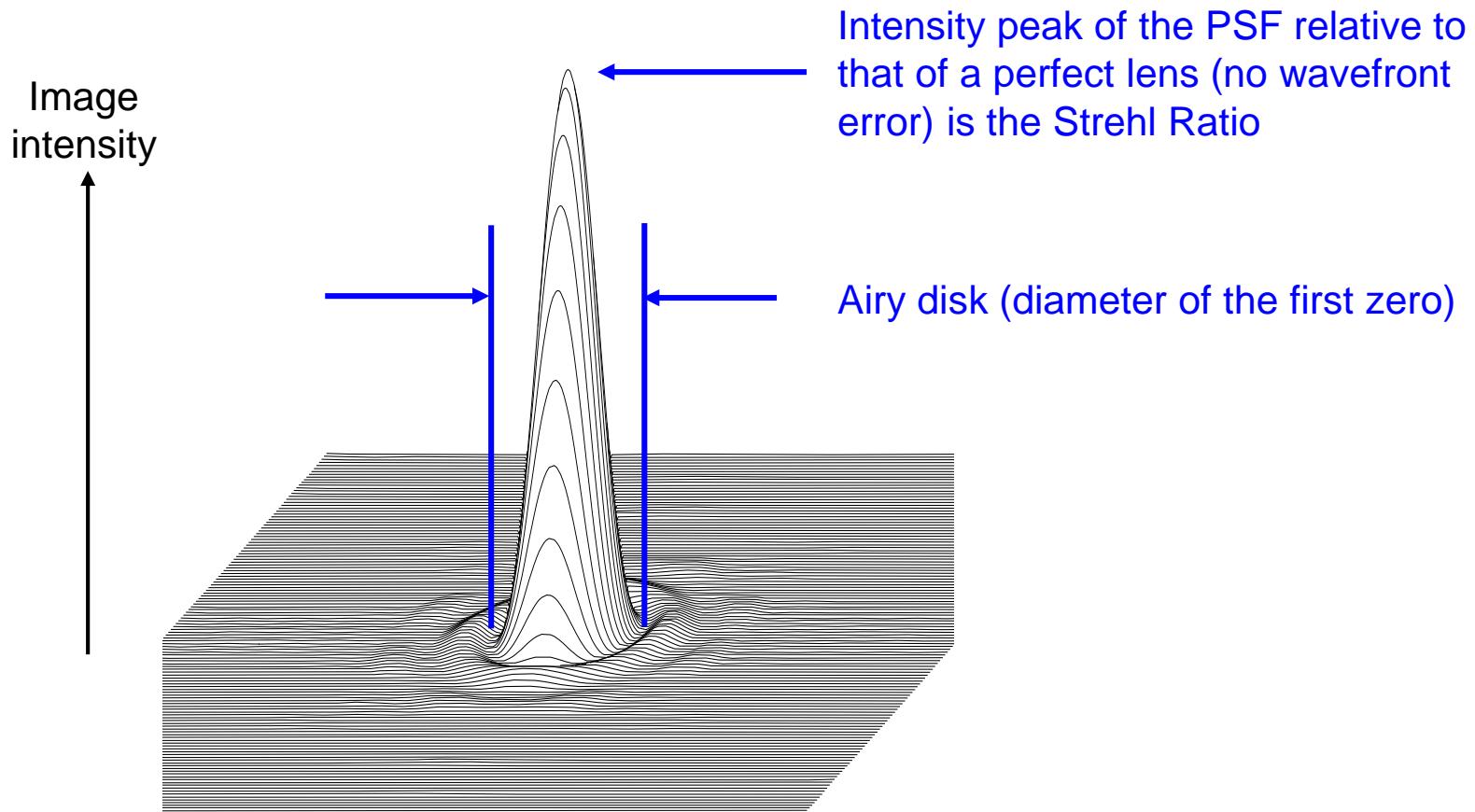


- Regime 3 – Far from diffraction-limited



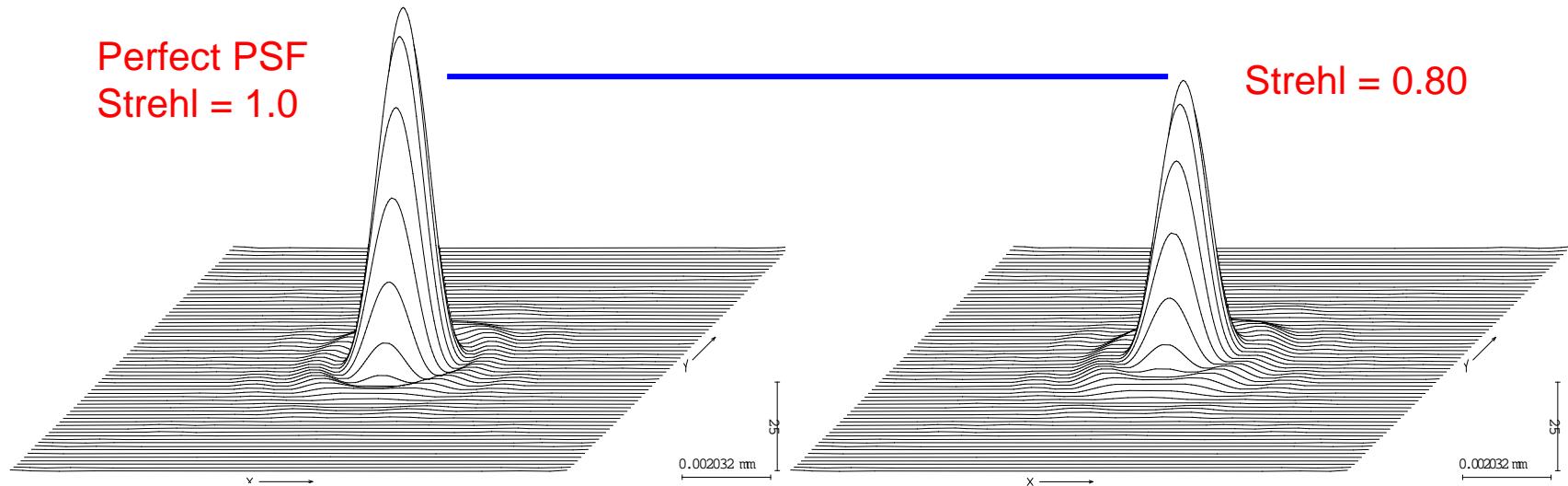
Point Spread Function (PSF)

- This is the image of a point object including the effects of diffraction and all aberrations



Diffraction Pattern of Aberrated Images

- When there is aberration present in the image, two effects occur
 - Depending on the aberration, the shape of the diffraction pattern may become skewed
 - There is less energy in the central ring and more in the outer rings



PSF vs. Defocus

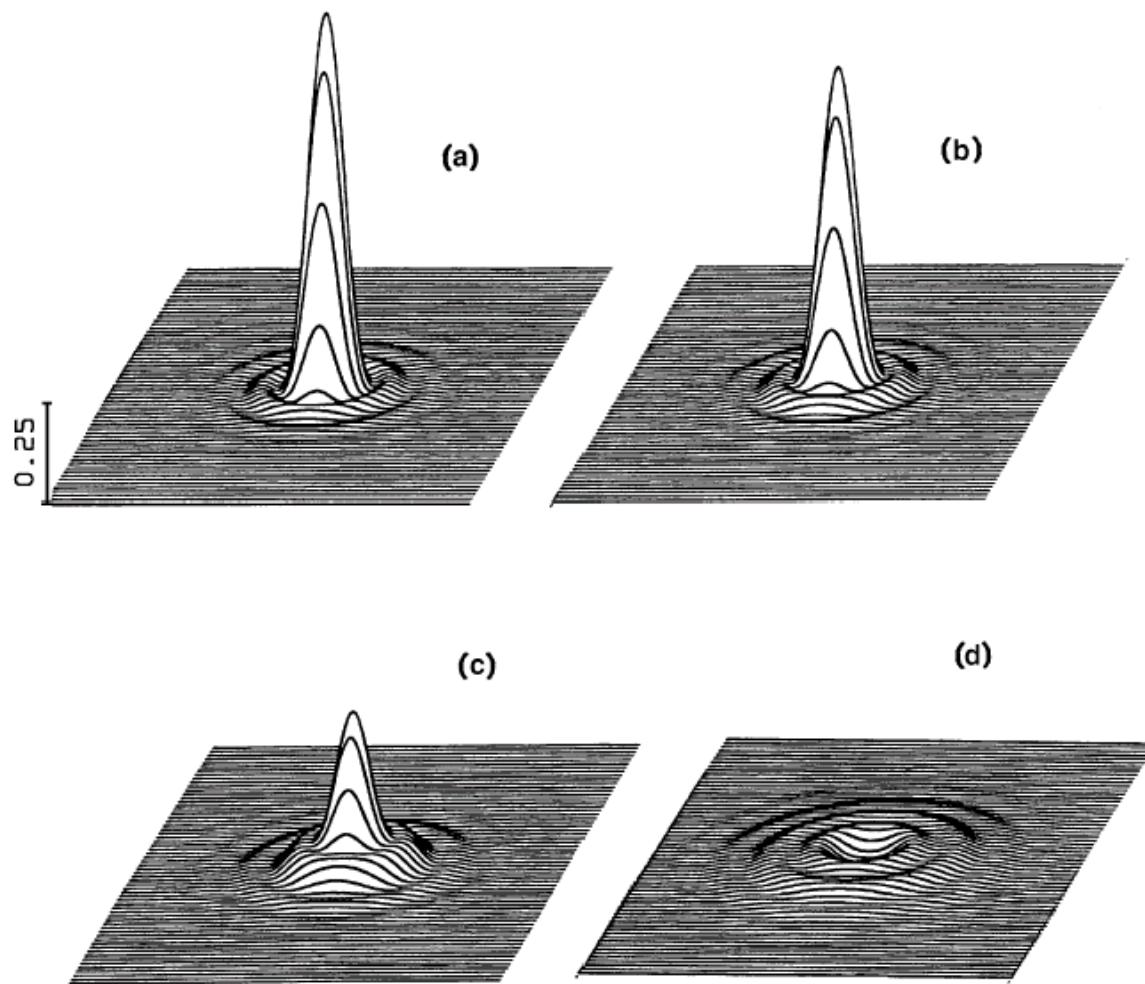


Figure 11.24 Point spread functions for different amounts of defocus. (a) 0.125 wave (P-V); 0.037 wave rms; 0.95 Strehl. (b) 0.25 wave (P-V); 0.074 wave rms; 0.80 Strehl. (c) 0.50 wave (P-V); 0.148 wave rms; 0.39 Strehl. (d) 1.00 wave (P-V); 0.297 wave rms; 0.00 Strehl.

PSF vs. Third-order Spherical Aberration

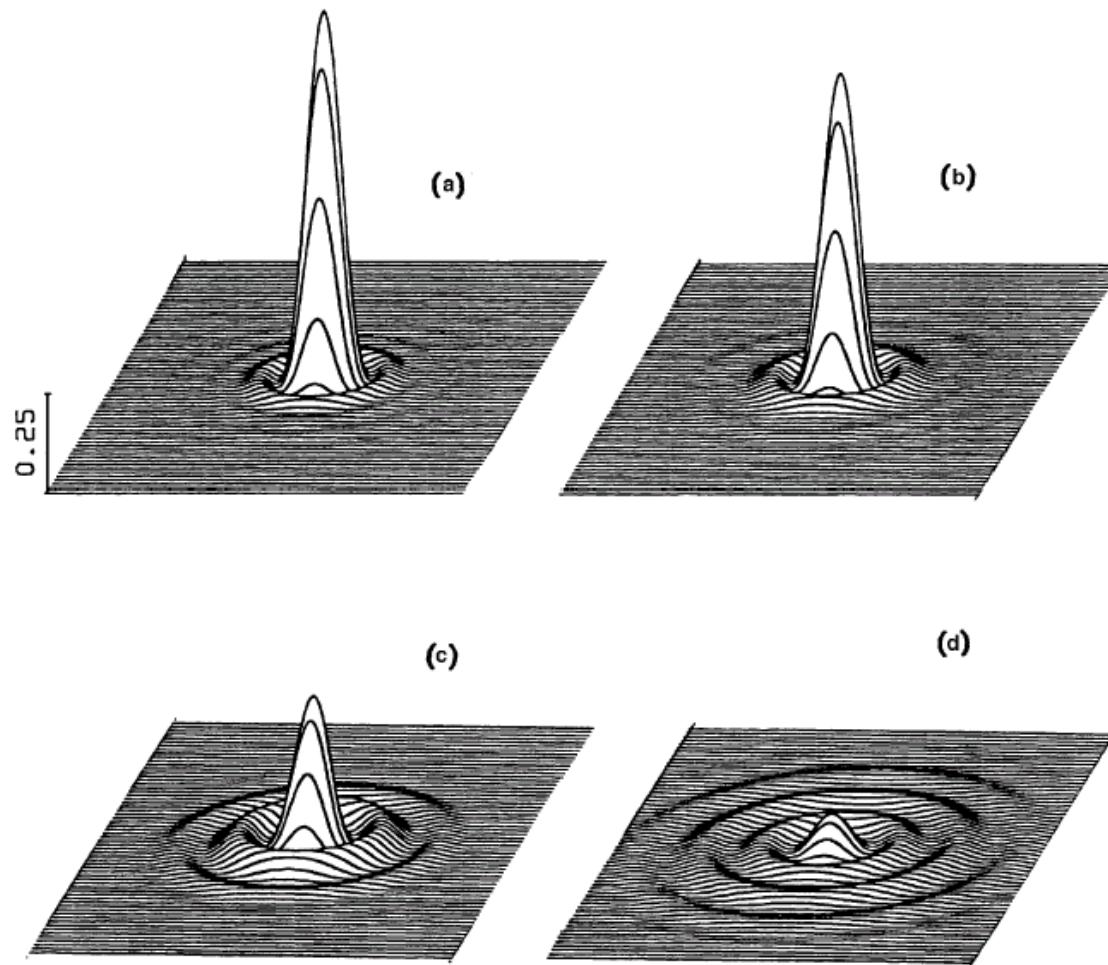


Figure 11.25 Point spread functions for different amounts of third-order spherical aberration. (a) 0.125 wave (P-V); 0.040 wave rms; 0.94 Strehl. (b) 0.25 wave (P-V); 0.080 wave rms; 0.78 Strehl. (c) 0.50 wave (P-V); 0.159 wave rms; 0.37 Strehl. (d) 1.00 wave (P-V); 0.318 wave rms; 0.08 Strehl. Note: Reference sphere centered at $0.5LA_m$ (midway between marginal and paraxial foci).

PSF vs. Third-order Coma

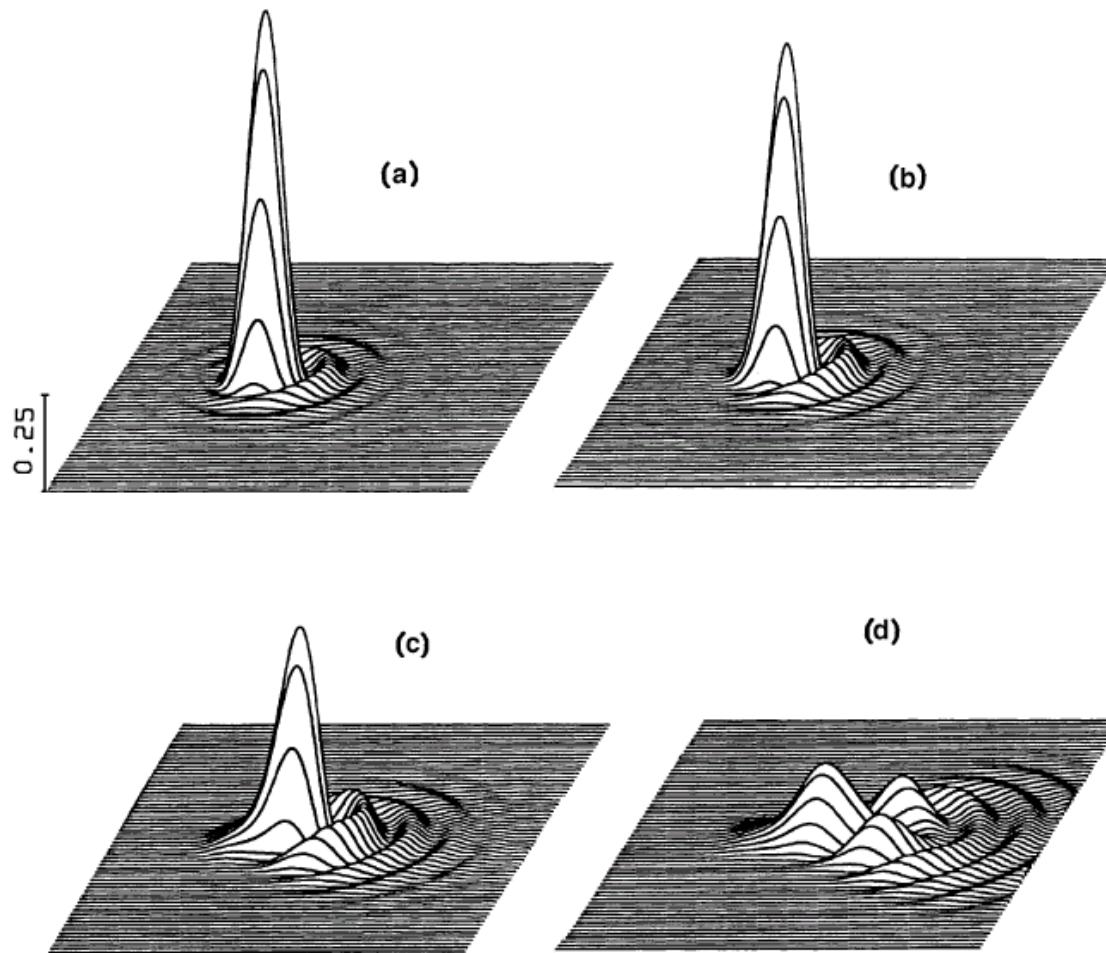


Figure 11.26 Point spread functions for different amounts of third-order coma. (a) 0.125 wave (P-V); 0.031 wave rms; 0.96 Strehl. (b) 0.25 wave (P-V); 0.061 wave rms; 0.86 Strehl. (c) 0.50 wave (P-V); 0.123 wave rms; 0.65 Strehl. (d) 1.00 wave (P-V); 0.25 wave rms; 0.18 Strehl. Note: P-V OPD reference sphere centered at 0.25Coma_T from chief ray intersection point. rms OPD reference sphere centered at 0.226Coma_T from chief ray intersection point.

PSF vs. Astigmatism

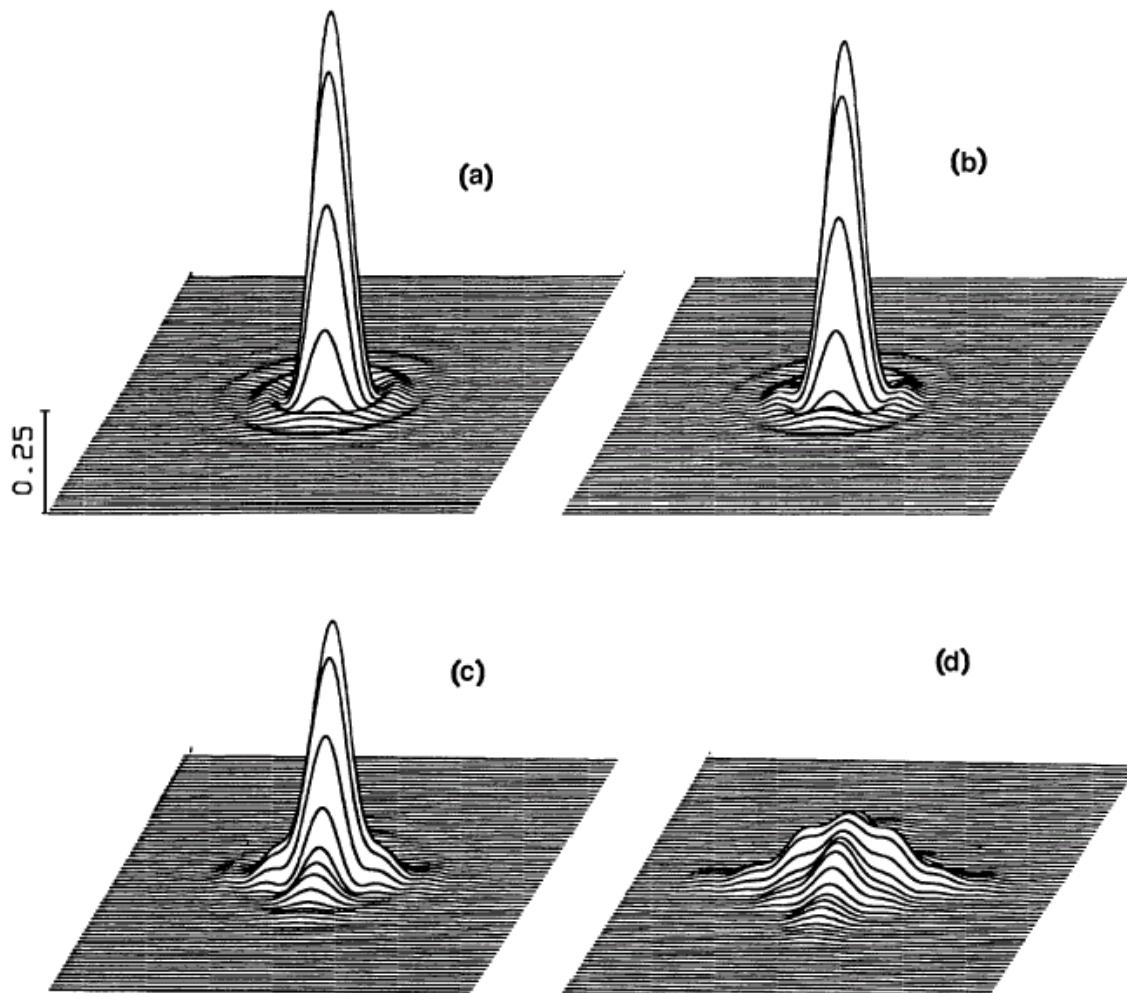


Figure 11.27 Point spread functions for different amounts of astigmatism. (a) 0.125 wave (P-V); 0.026 wave rms; 0.97 Strehl. (b) 0.25 wave (P-V); 0.052 wave rms; 0.90 Strehl. (c) 0.50 wave (P-V); 0.104 wave rms; 0.65 Strehl. (d) 1.00 wave (P-V); 0.207 wave rms; 0.18 Strehl. Note: Reference sphere centered midway between sagittal and tangential foci.

PSF for Strehl = 0.80

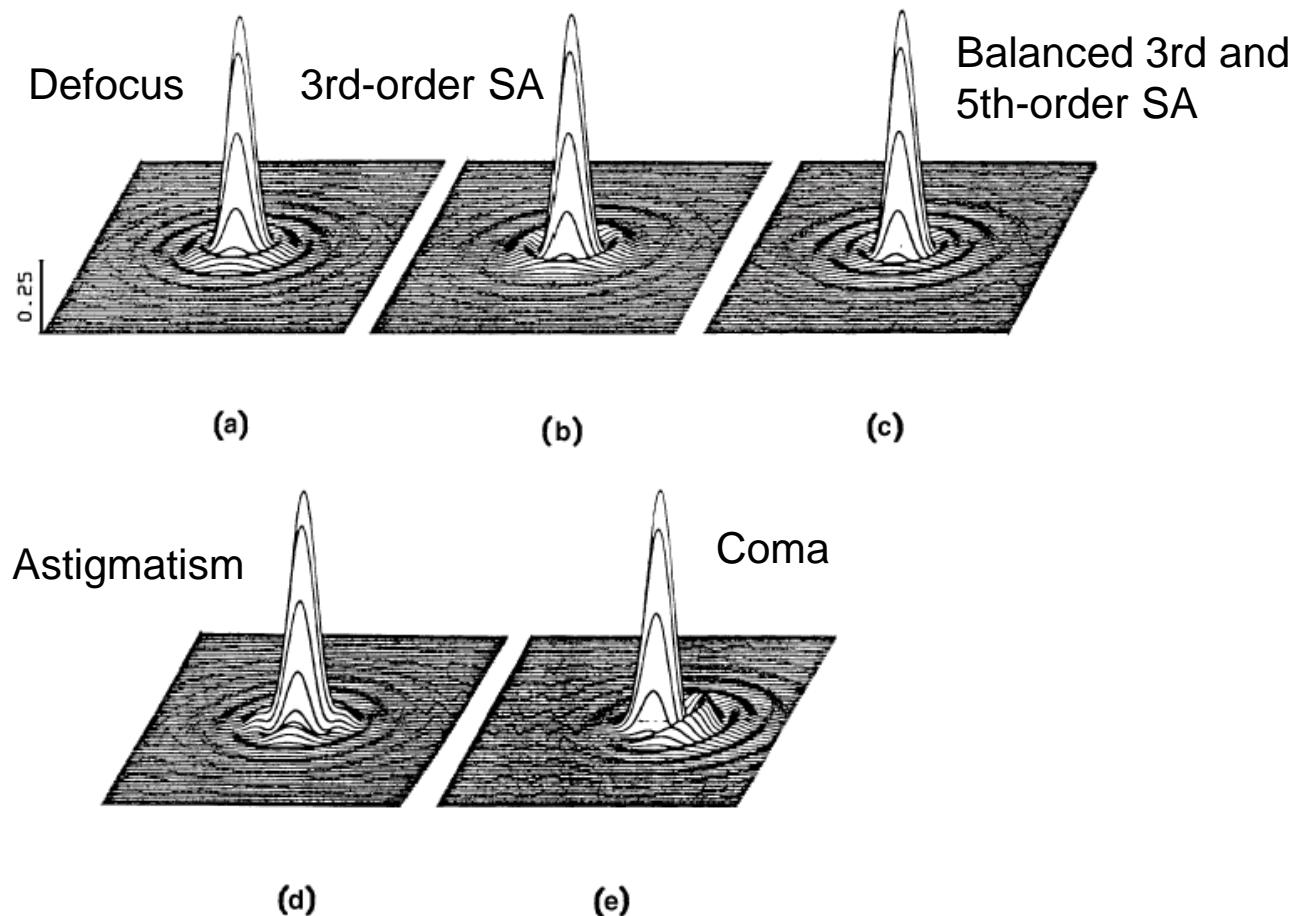


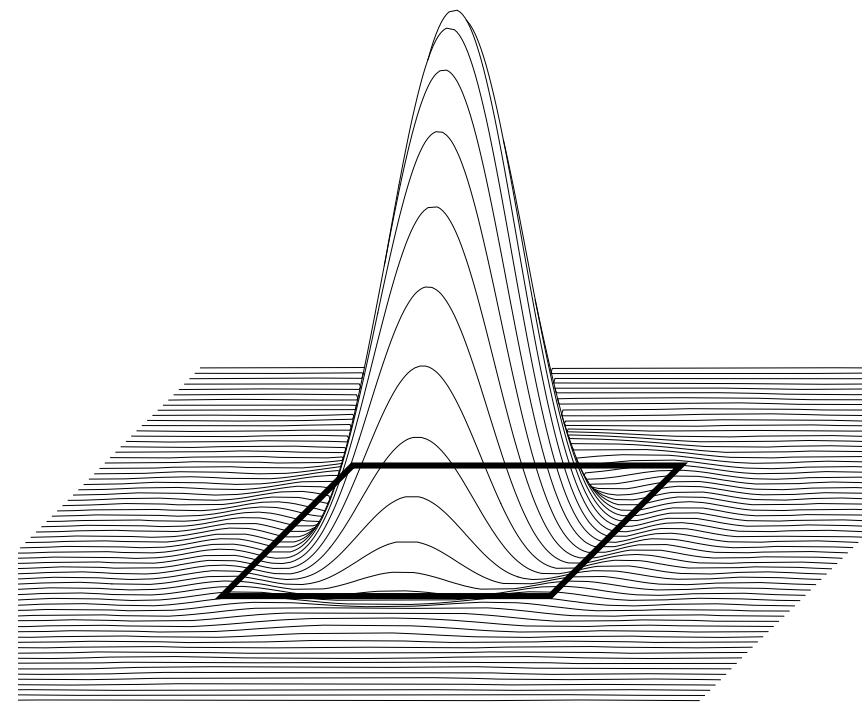
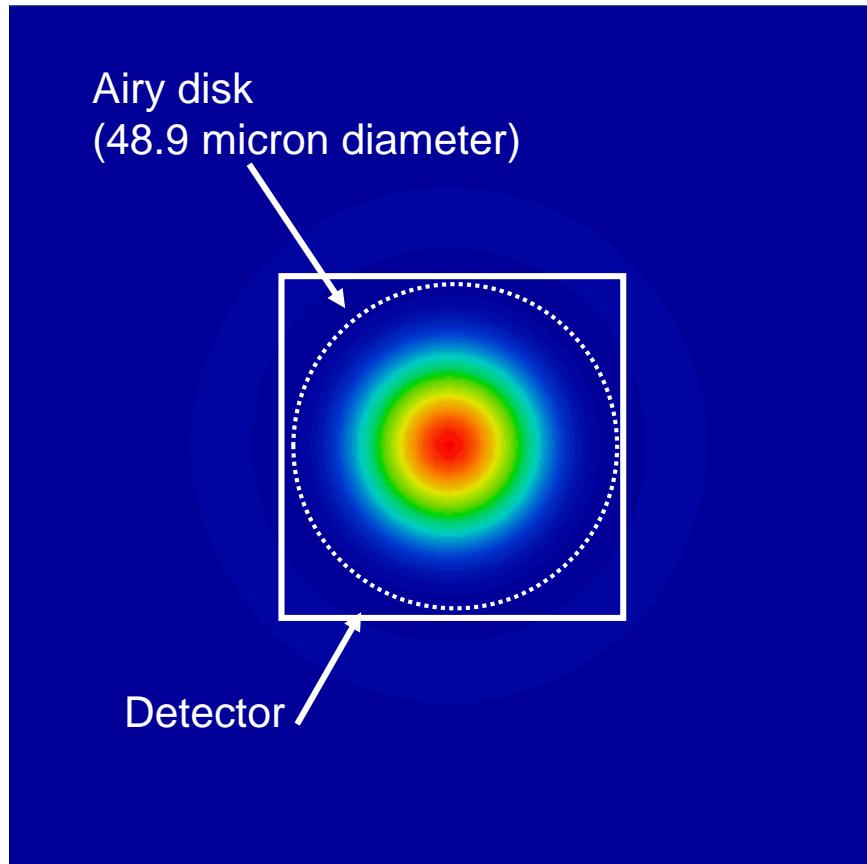
Figure 11.29 Point spread functions for five different aberrations, each with a Strehl ratio of 0.80 (the Marechal criterion). In each case the center of the reference sphere is located to minimize the rms OPD, which is 0.075 wave for all five aberrations. (a) Defocus: 0.25 wave (P-V). (b) Third-order spherical: 0.235 wave (P-V). (c) Balanced third- and fifth-order spherical: 0.221 wave (P-V). (d) Astigmatism: 0.359 wave (P-V). (e) Coma: 0.305 wave (P-V).

Encircled or Ensquared Energy

- Encircled or ensquared energy is the ratio of the energy in the PSF that is collected by a single circular or square detector to the total amount of energy that reaches the image plane from that object point
 - This is a popular metric for system engineers who, reasonably enough, want a certain amount of collected energy to fall on a single pixel
 - It is commonly used for systems with point images, especially systems which need high signal-to-noise ratios
- For %EE specifications of 50-60% this is a reasonably linear criterion
 - However, the specification is more often 80%, or even worse 90%, energy within a near diffraction-limited diameter
 - At the 80% and 90% levels, this criterion is highly non-linear and highly dependent on the aberration content of the image, which makes it a poor criterion, especially for tolerancing

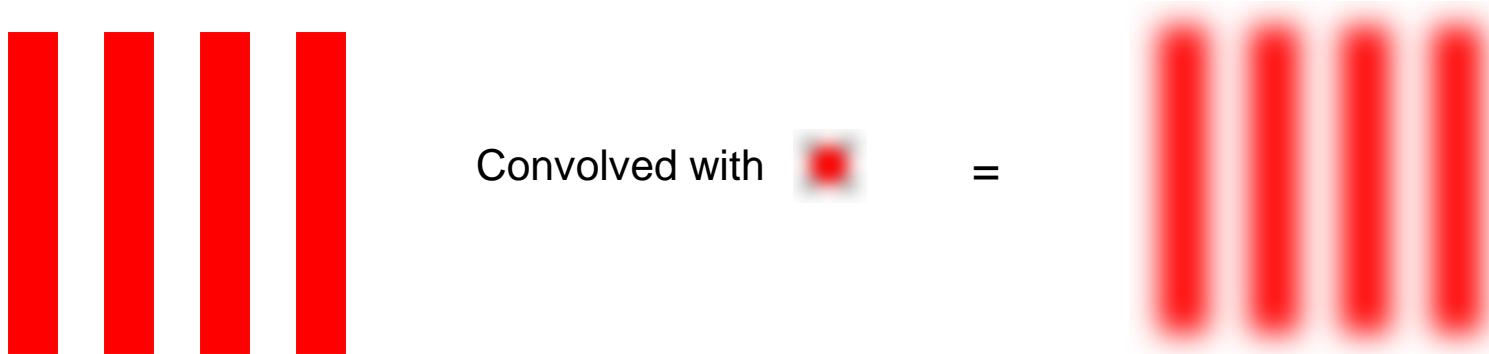
Ensquared Energy Example

Ensquared energy on a detector of same order of size as the Airy disk
Perfect lens, f/2, 10 micron wavelength, 50 micron detector



Modulation Transfer Function (MTF)

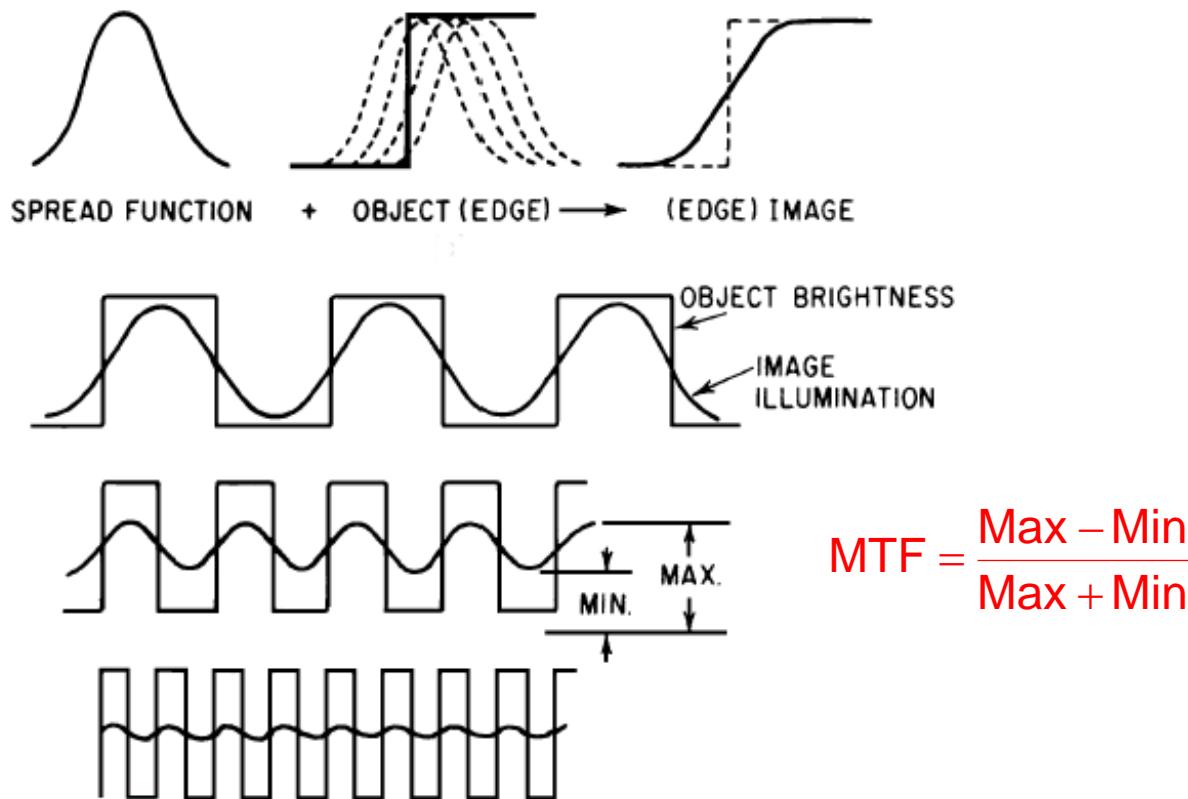
- MTF is the Modulation Transfer Function
- Measures how well the optical system images objects of different sizes
 - Size is usually expressed as spatial frequency (1/size)
- Consider a bar target imaged by a system with an optical blur
 - The image of the bar pattern is the geometrical image of the bar pattern convolved with the optical blur



- MTF is normally computed for sine wave input, and not square bars to get the response for a pure spatial frequency
- Note that MTF can be geometrical or diffraction-based

Computing MTF

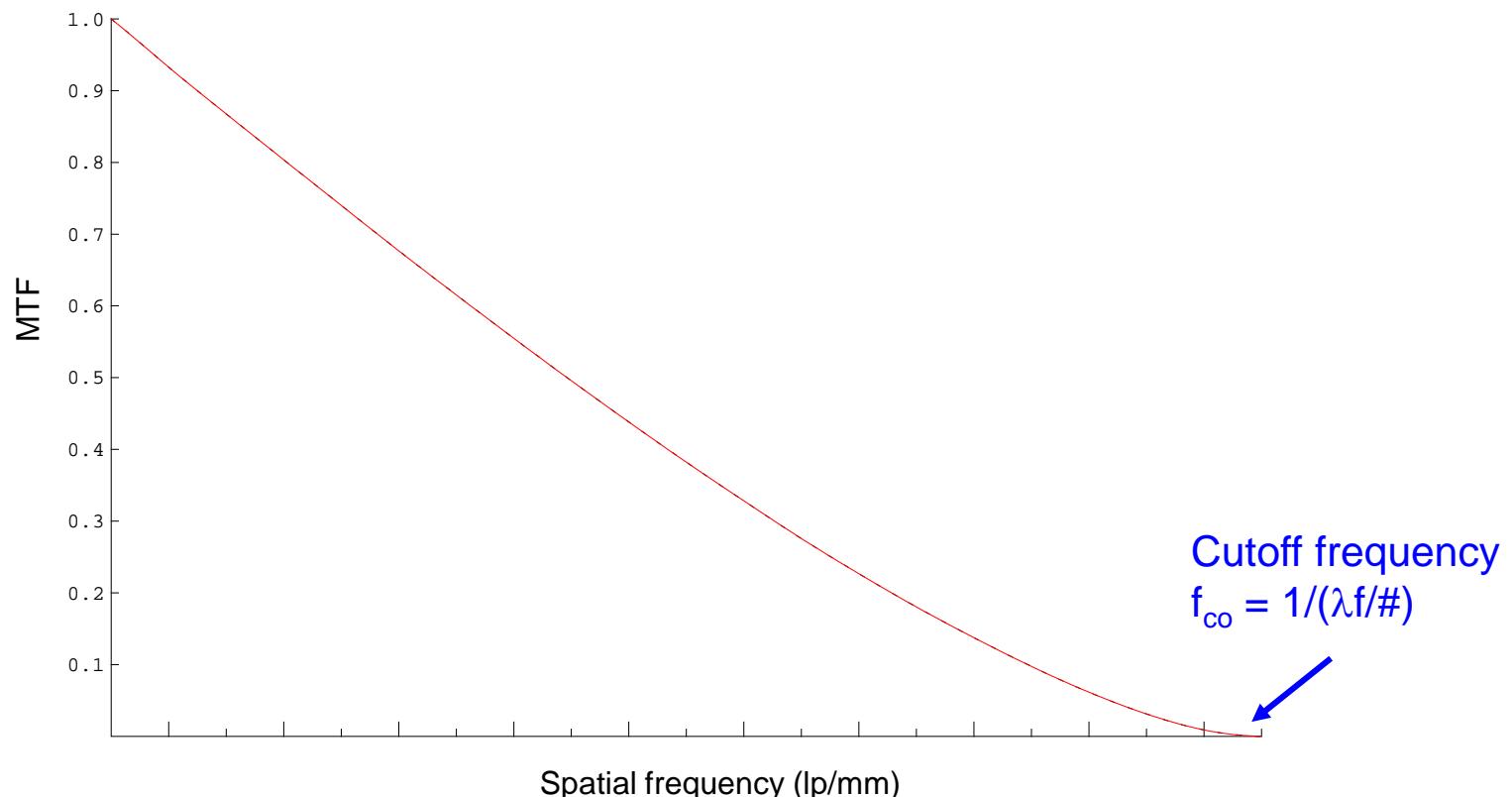
- The MTF is the amount of modulation in the image of a sine wave target
 - At the spatial frequency where the modulation goes to zero, you can no longer see details in the object of the size corresponding to that frequency
- The MTF is plotted as a function of spatial frequency (1/sine wave period)



MTF of a Perfect Image

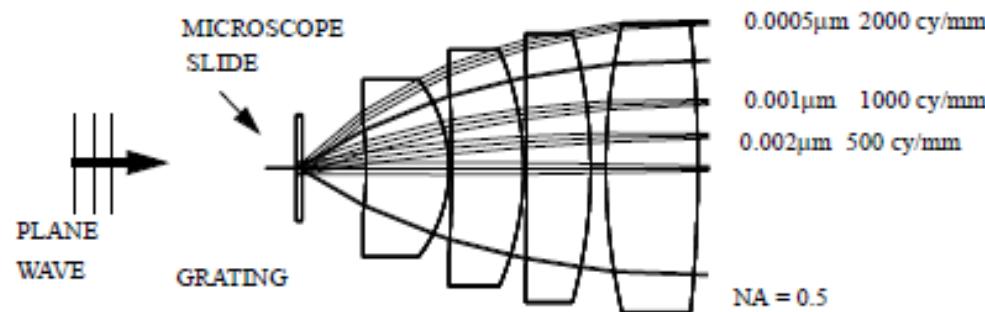
- For an aberration-free image and a round pupil, the MTF is given by

$$\text{MTF}(f) = \frac{2}{\pi} [\varphi - \cos \varphi \sin \varphi] \quad \varphi = \cos^{-1}(f / f_{co}) = \cos^{-1}\left(\frac{\lambda f}{2NA}\right)$$



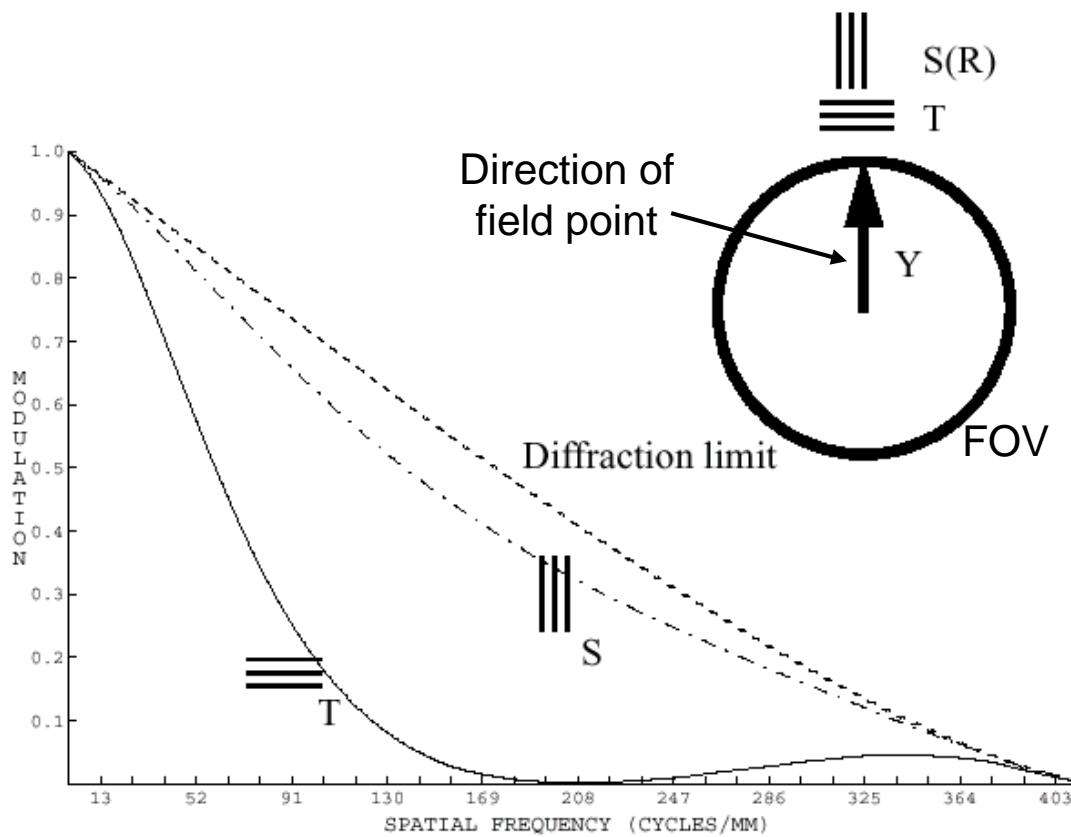
Abbe's Construct for Image Formation

- Abbe developed a useful framework from which to understand the diffraction-limiting spatial frequency and to explain image formation in microscopes



- If the first-order diffraction angle from the grating exceeds the numerical aperture ($NA = 1/(2f/\#)$), no light will enter the optical system for object features with that characteristic spatial period

Example MTF Curve



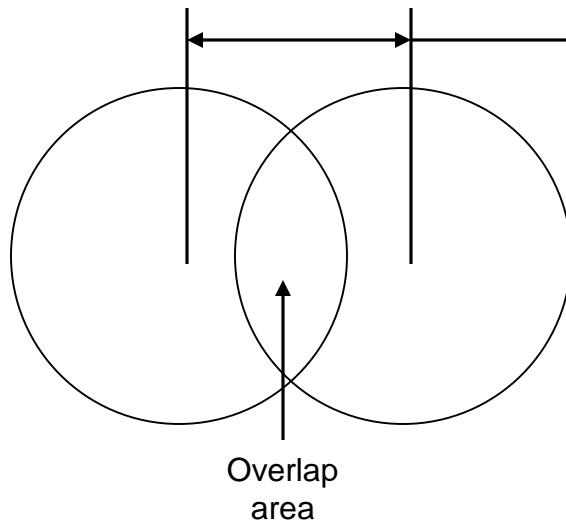
MTF depends on target orientation

S = Sagittal
R = Radial

T = Tangential

MTF as an Autocorrelation of the Pupil

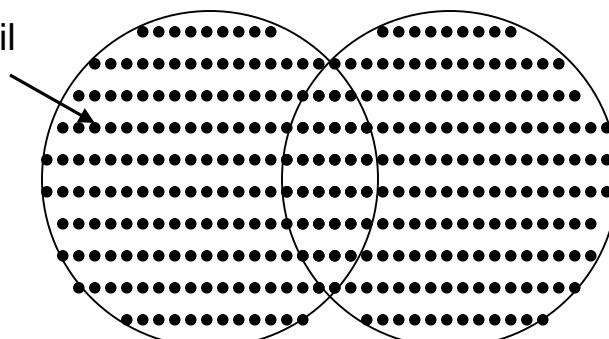
- The MTF is usually computed by lens design programs as the autocorrelation of the OPD map across the exit pupil



Relative spatial frequency = spacing
between shifted pupils
(cutoff frequency = pupil diameter)

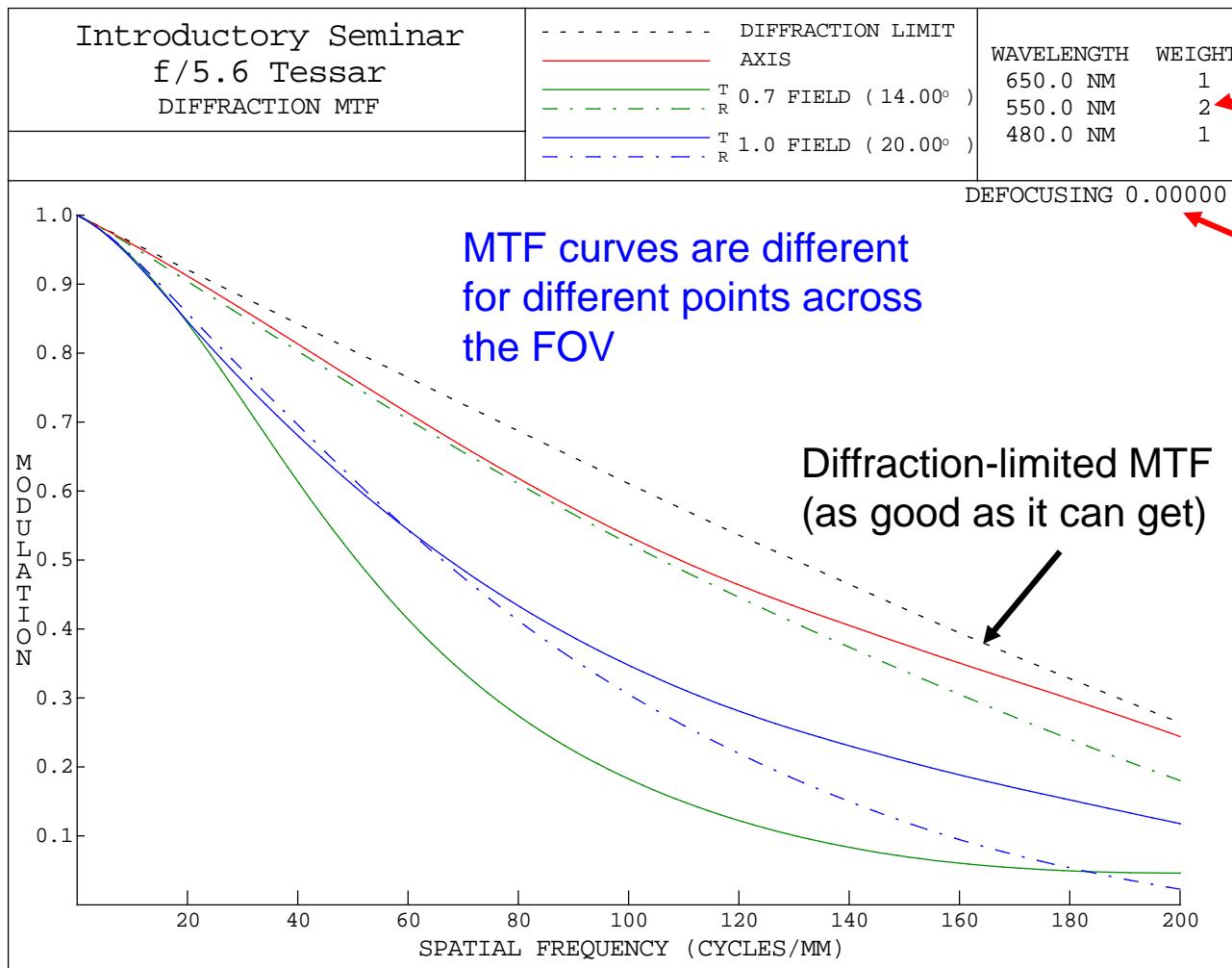
Perfect MTF = overlap area / pupil area

Complex OPD
computed for many
points across the pupil



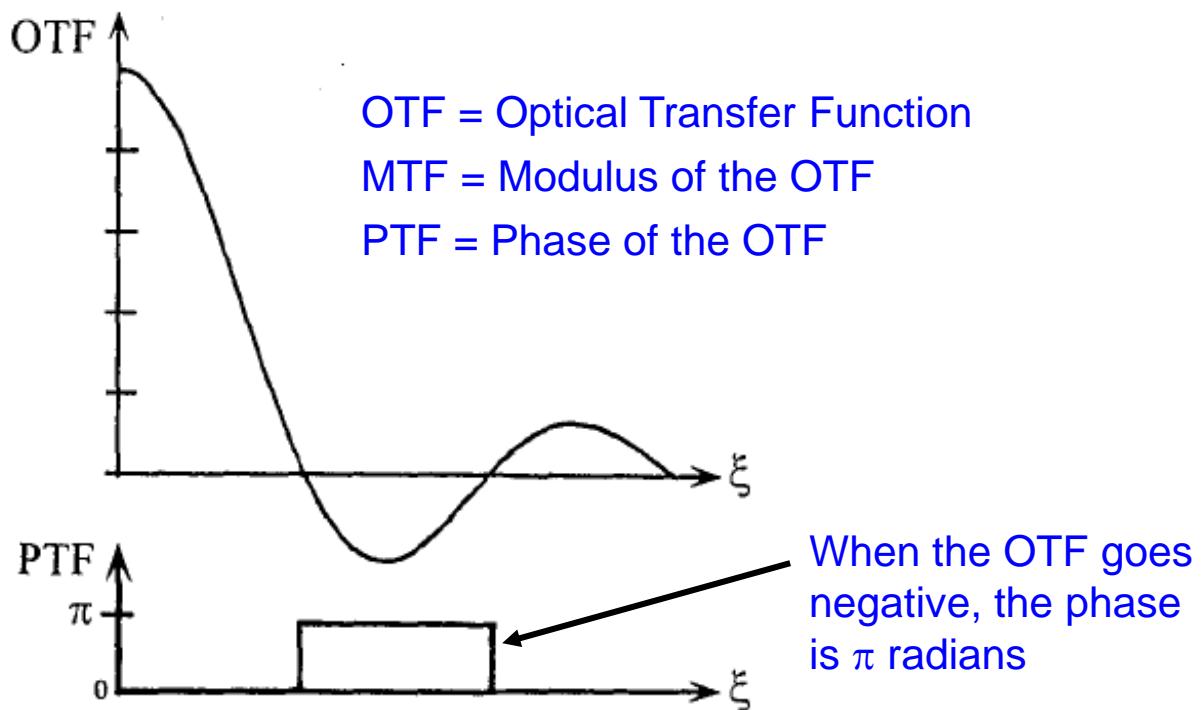
MTF is computed as the normalized integral
over the overlap region of the difference
between the OPD map and its shifted
complex conjugate

Typical MTF Curves



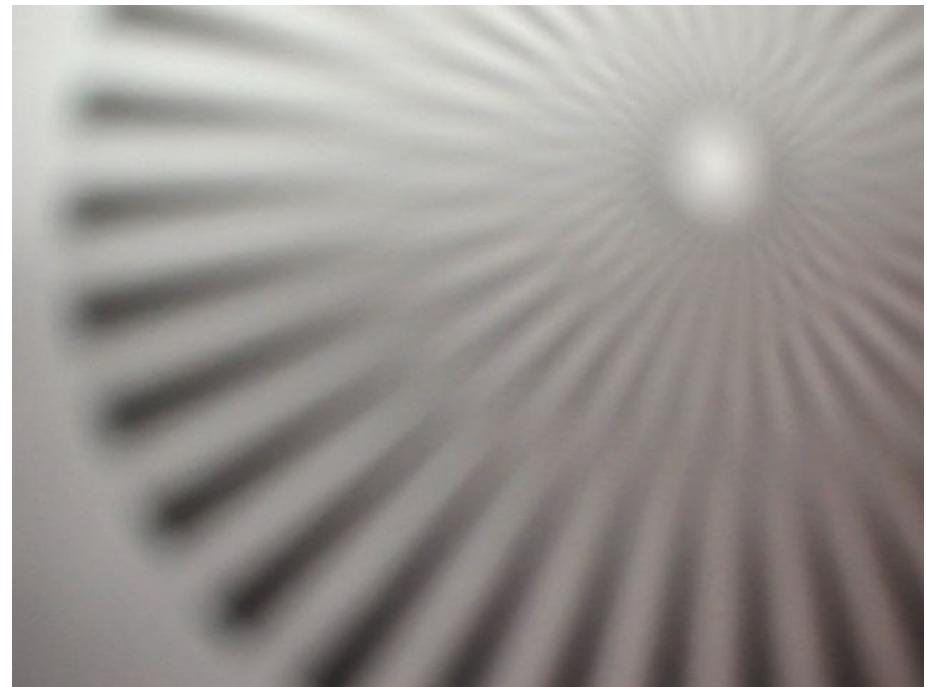
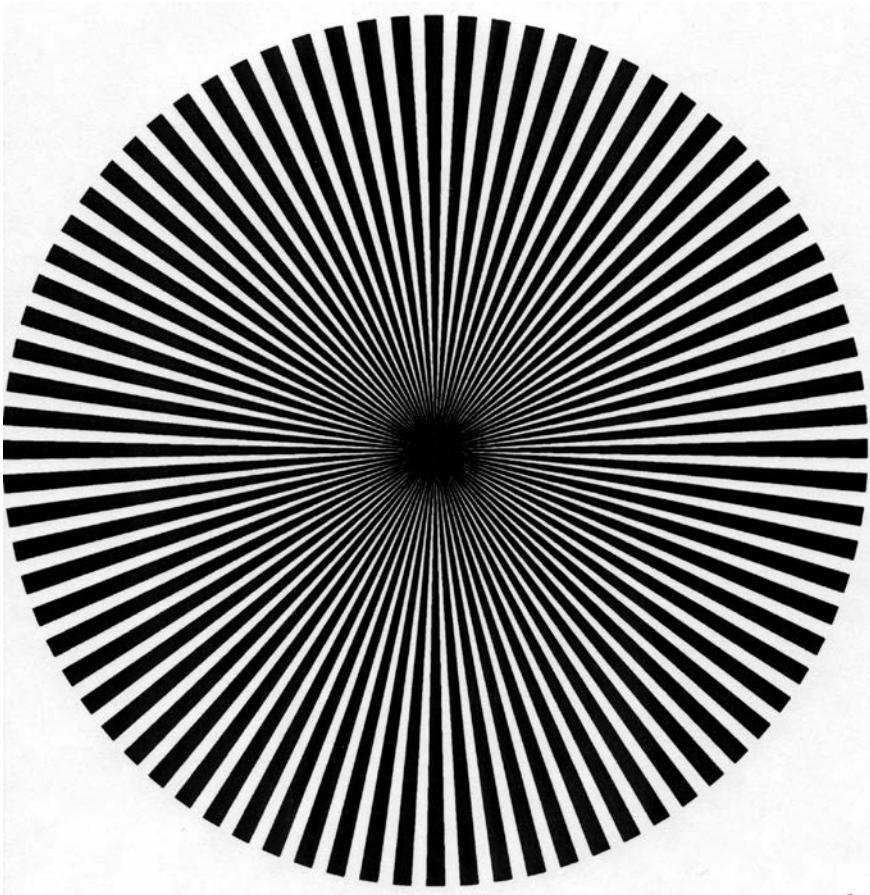
Phase Shift of the OTF

- Since OPD relates to the phase of the ray relative to the reference sphere, the pupil autocorrelation actually gives the OTF (optical transfer function), which is a complex quantity
 - MTF is the real part (modulus) of the OTF

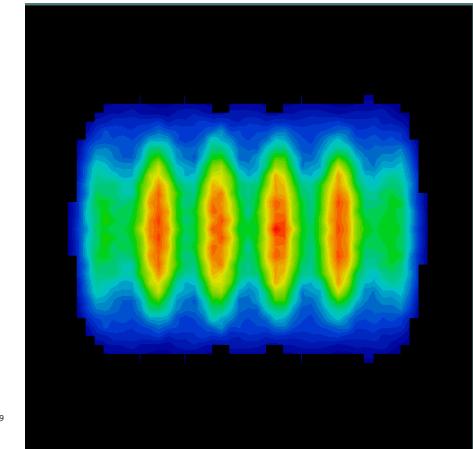
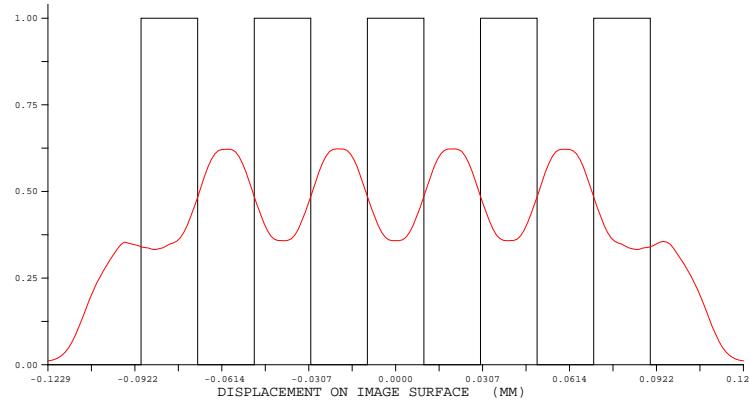
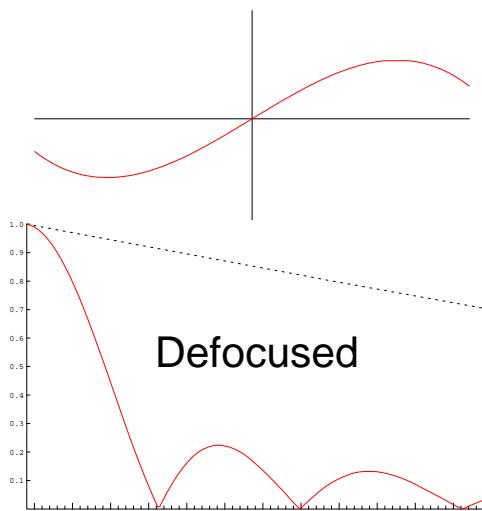
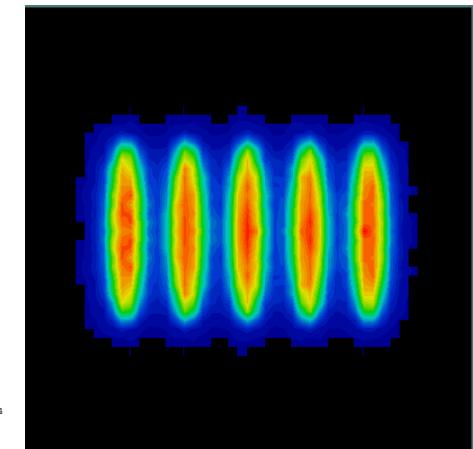
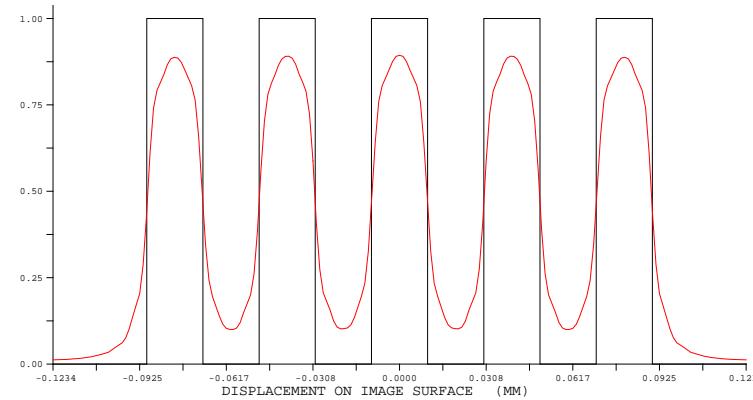
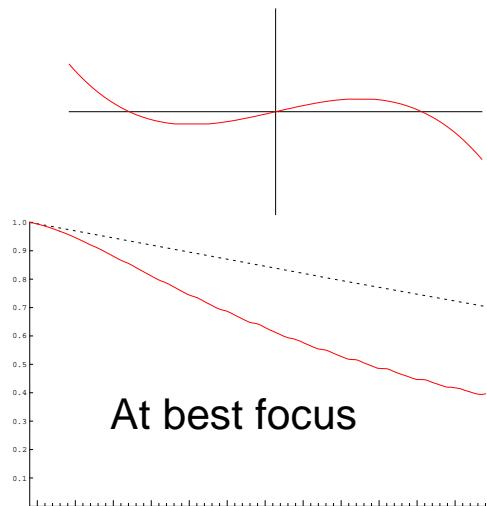


What Does OTF < 0 Mean?

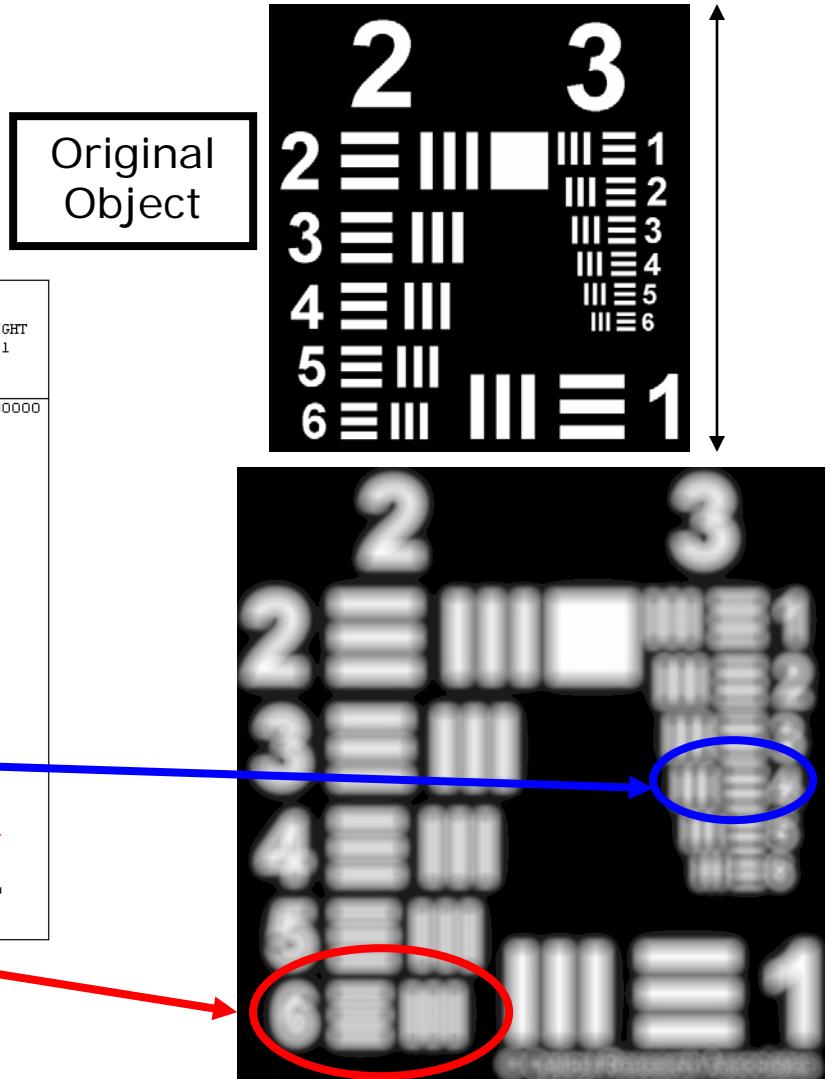
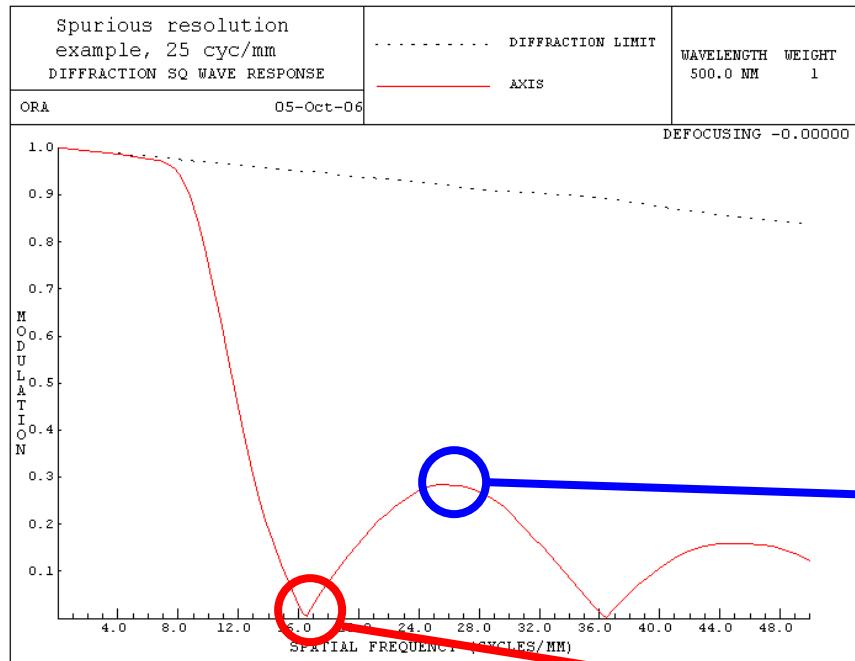
- When the OTF goes negative, it is an example of [contrast reversal](#)



Example of Contrast Reversal

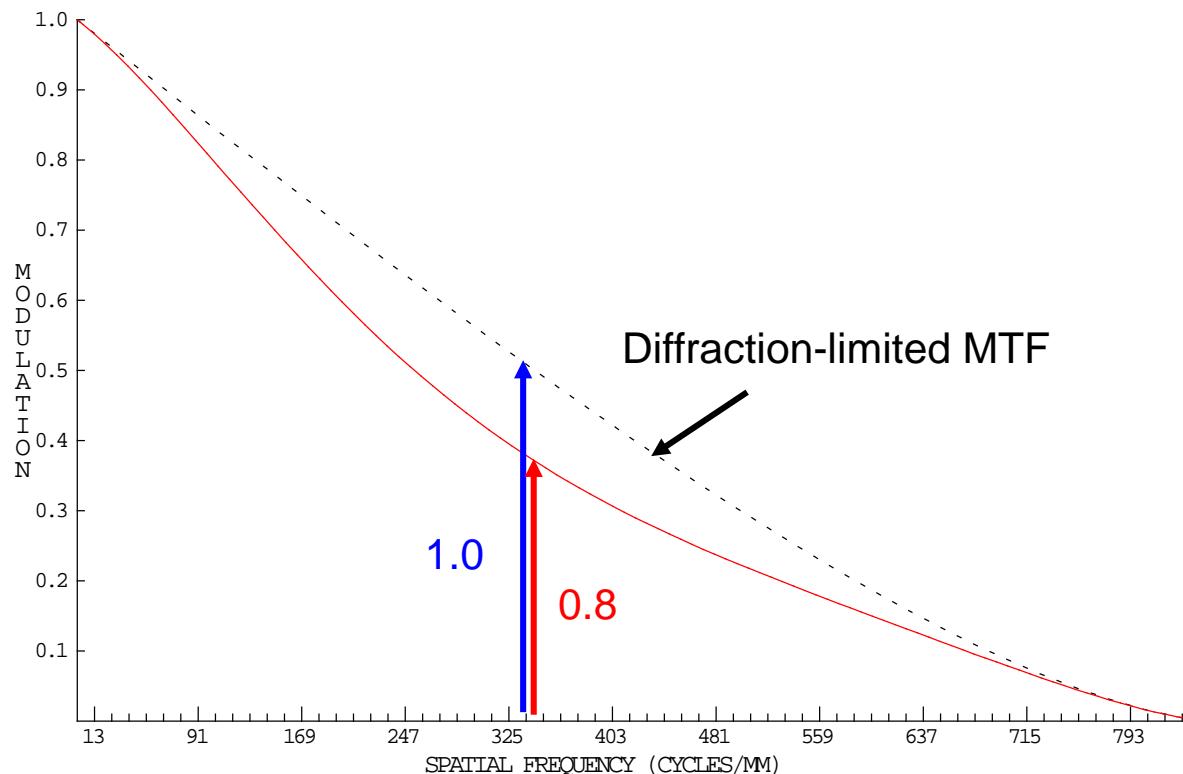


More on Contrast Reversal



Effect of Strehl = 0.80

- When the Strehl Ratio = 0.80 or higher, the image is considered to be equivalent in image quality to a diffraction-limited image
- The MTF in the mid-range spatial frequencies is reduced by the Strehl ratio



Aberration Transfer Function

- Shannon has shown that the MTF can be approximated as a product of the diffraction-limited MTF (DTF) and an aberration transfer function (ATF)

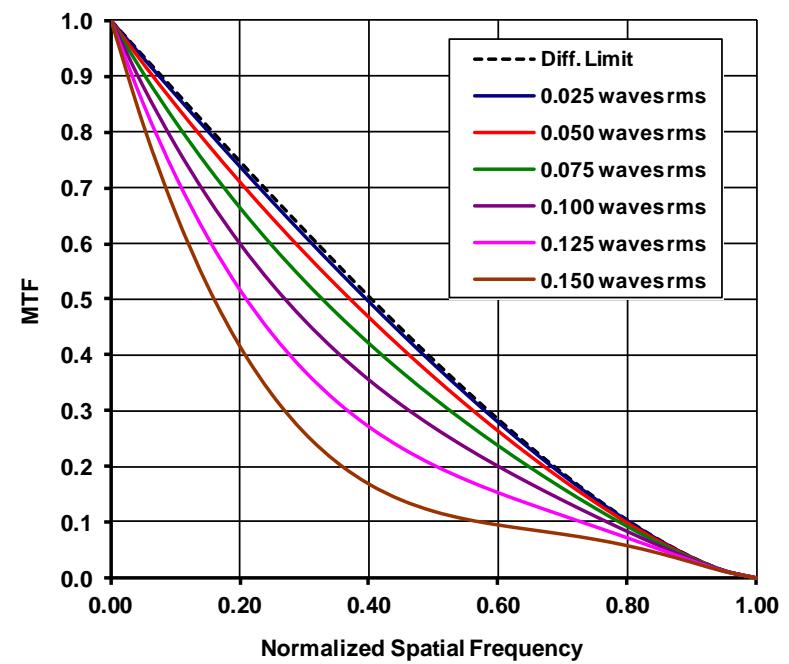
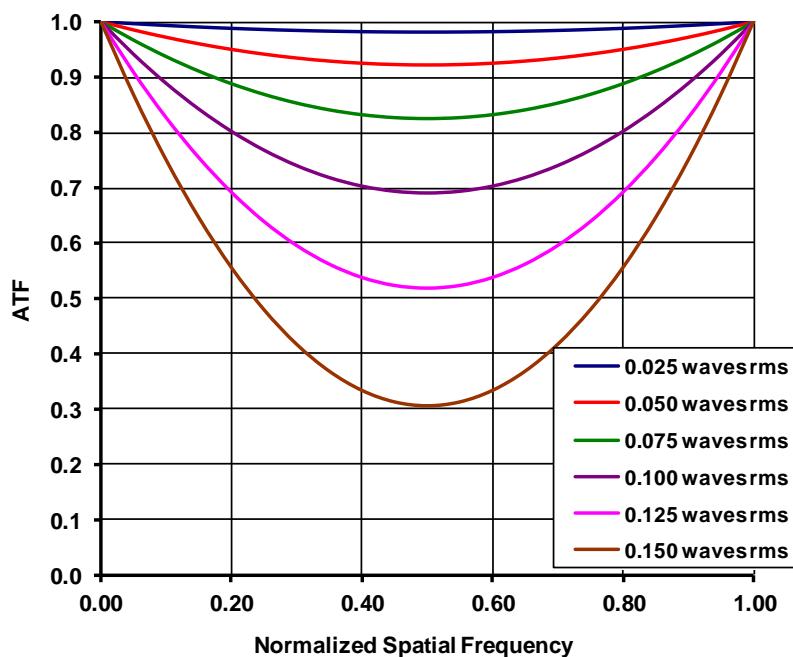
$$DTF(v) = \frac{2}{\pi} \left[\cos^{-1} v - v \sqrt{1-v^2} \right]$$

$$v = f / f_{co}$$

$$ATF(v) = 1 - \left(\frac{W_{rms}}{0.18} \right)^2 \left(1 - 4(v - 0.5)^2 \right)$$

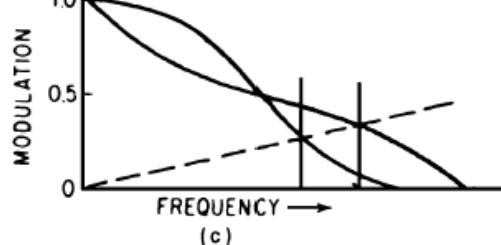
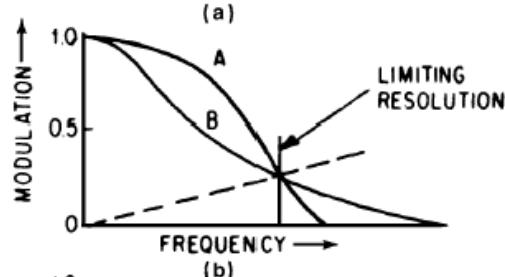
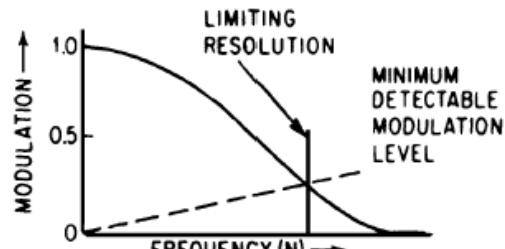


Bob
Shannon



Demand Contrast Function

- The eye requires more modulation for smaller objects to be able to resolve them
 - The amount of modulation required to resolve an object is called the **demand contrast function**
 - This and the MTF limits the highest spatial frequency that can be resolved

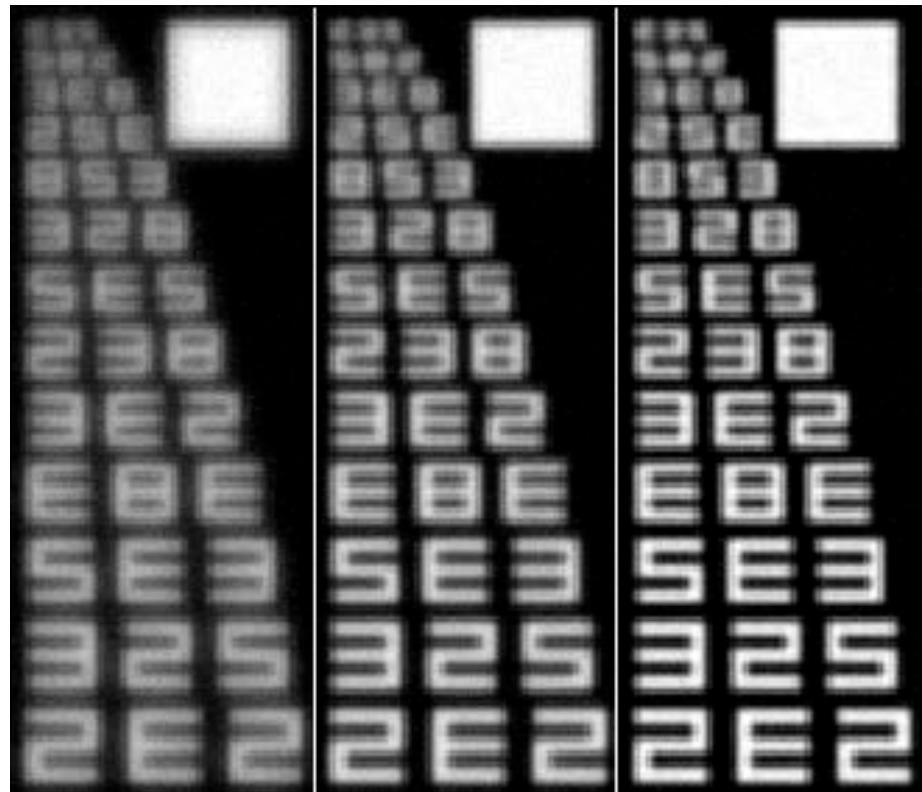
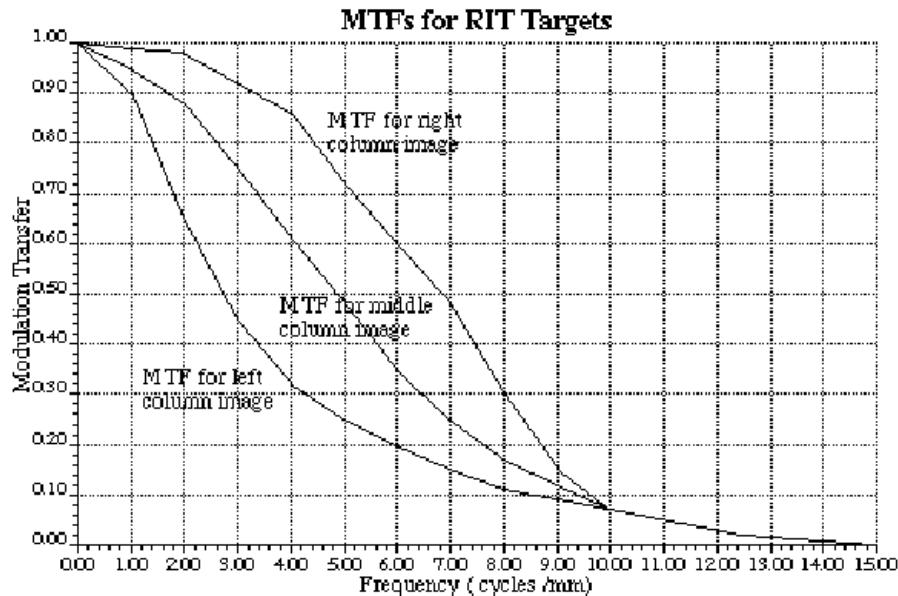


The limiting resolution is where the Demand Contrast Function intersects the MTF

System A will produce a superior image although it has the same limiting resolution as System B

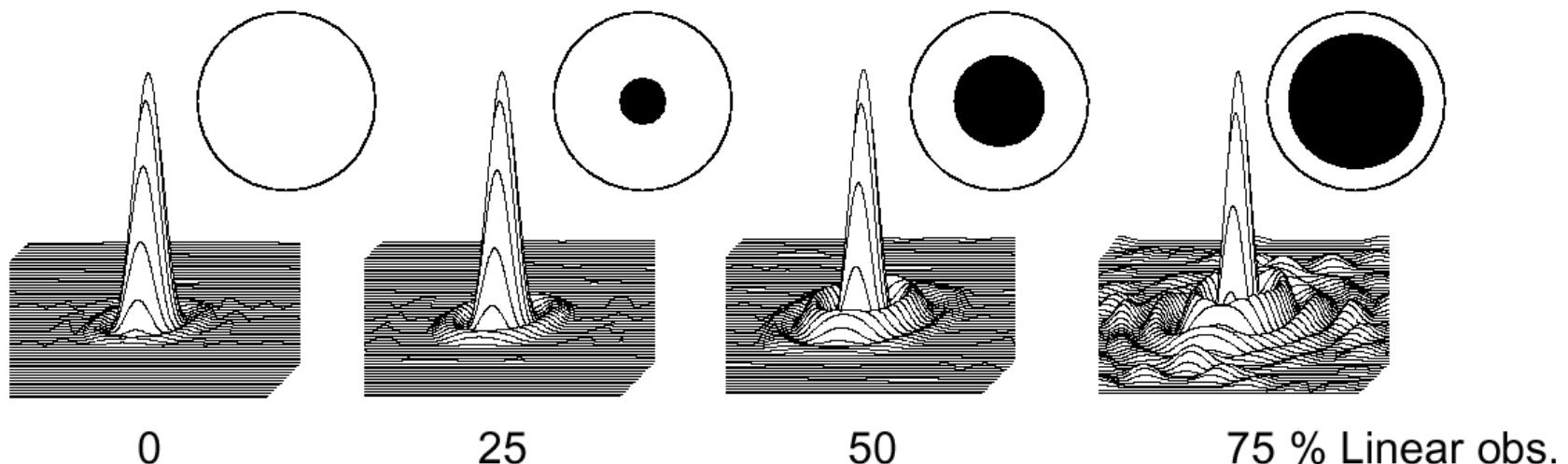
System A has a lower limiting resolution than System B even though it has higher MTF at lower frequencies

Example of Different MTFs on RIT Target



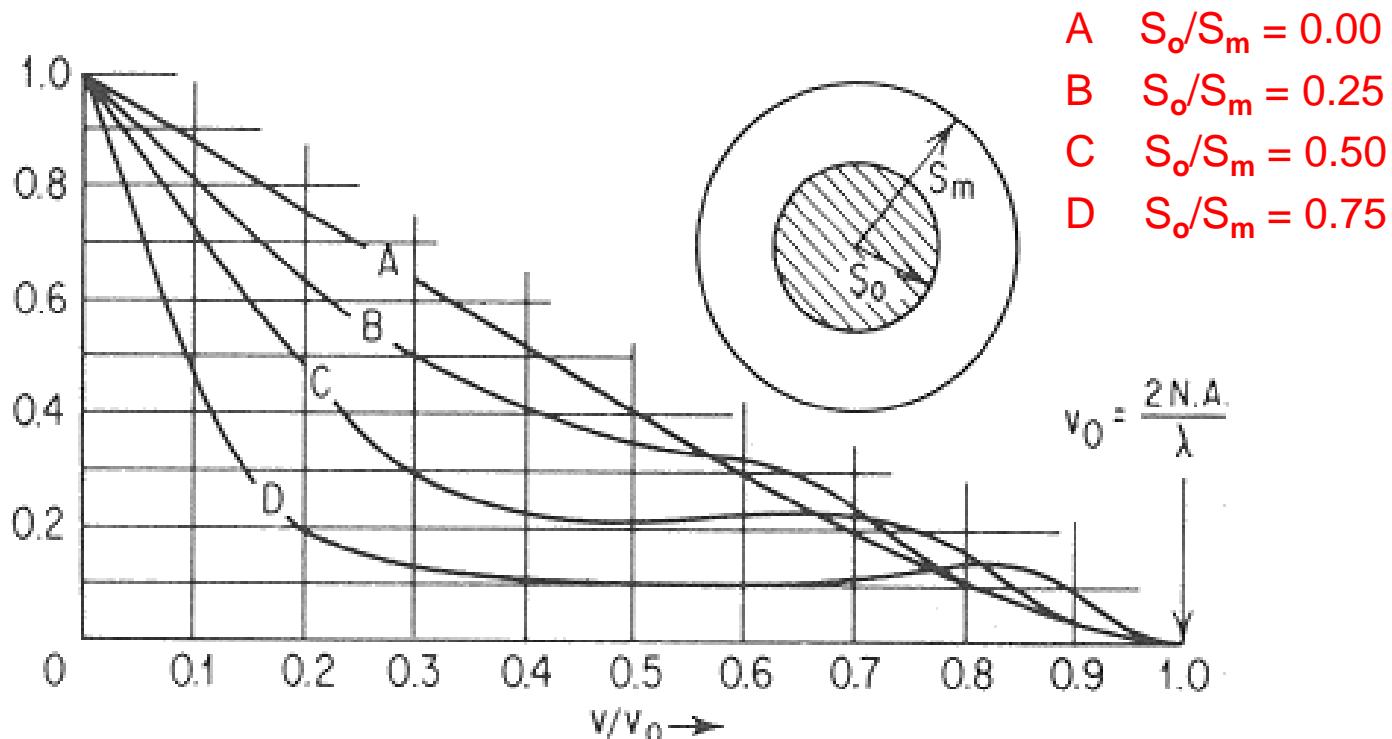
Central Obscurations

- In on-axis telescope designs, the obscuration caused by the secondary mirror is typically 30-50% of the diameter
 - Any obscuration above 30% will have a noticeable effect on the Airy disk, both in terms of dark ring location and in percent energy in a given ring (energy shifts out of the central disk and into the rings)
- Contrary perhaps to expectations, as the obscuration increases the diameter of the first Airy ring decreases (the peak is the same, and the loss of energy to the outer rings has to come from somewhere)



Central Obscurations

- Central obscurations, such as in a Cassegrain telescope, have two deleterious effects on an optical system
 - The obscuration causes a loss in energy collected (loss of area)
 - The obscuration causes a loss of MTF



Coherent Illumination

- Incoherent illumination fills the whole entrance pupil
- Partially coherent illumination fills only part of the entrance pupil
 - Coherent illumination essentially only fills a point in the entrance pupil

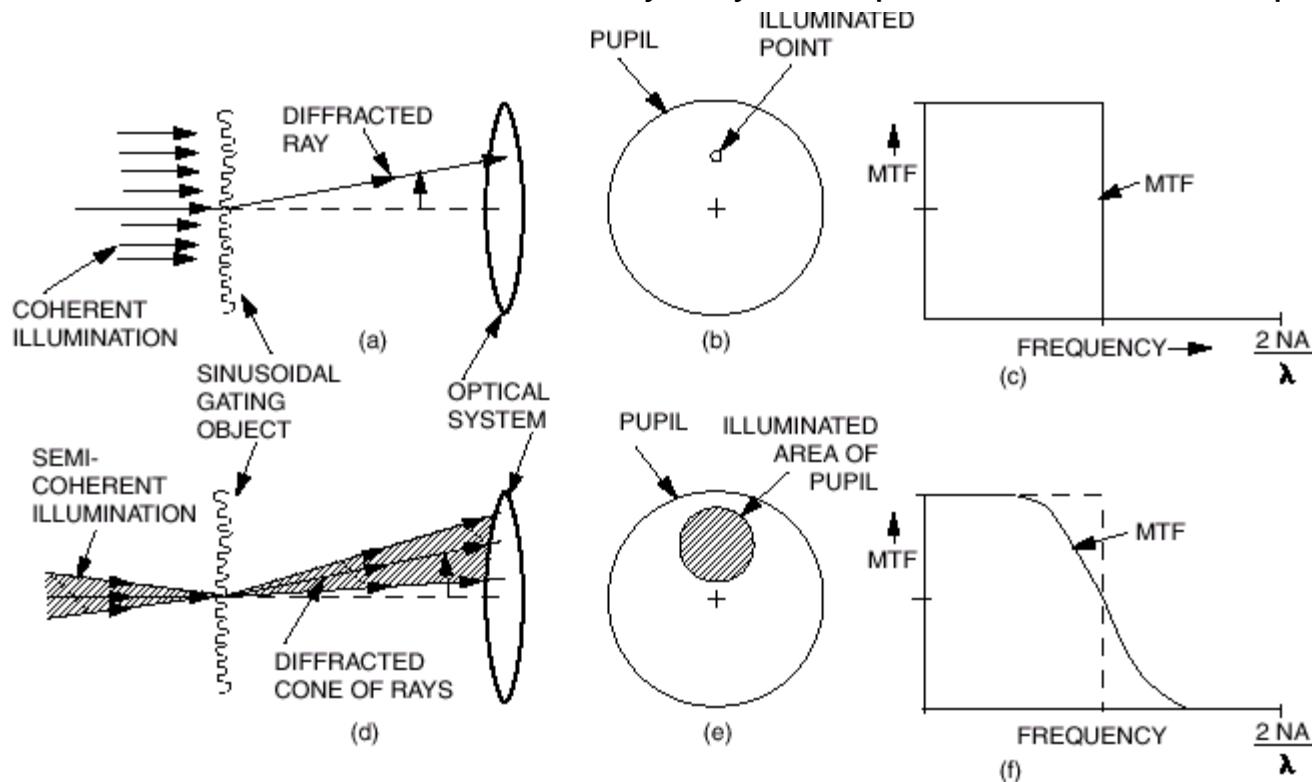


Figure 11.20 (a–c) The MTF with coherent illumination. (d–f) The MTF with semicoherent illumination (which partially fills the pupil).

MTF of Partially Coherent Illumination

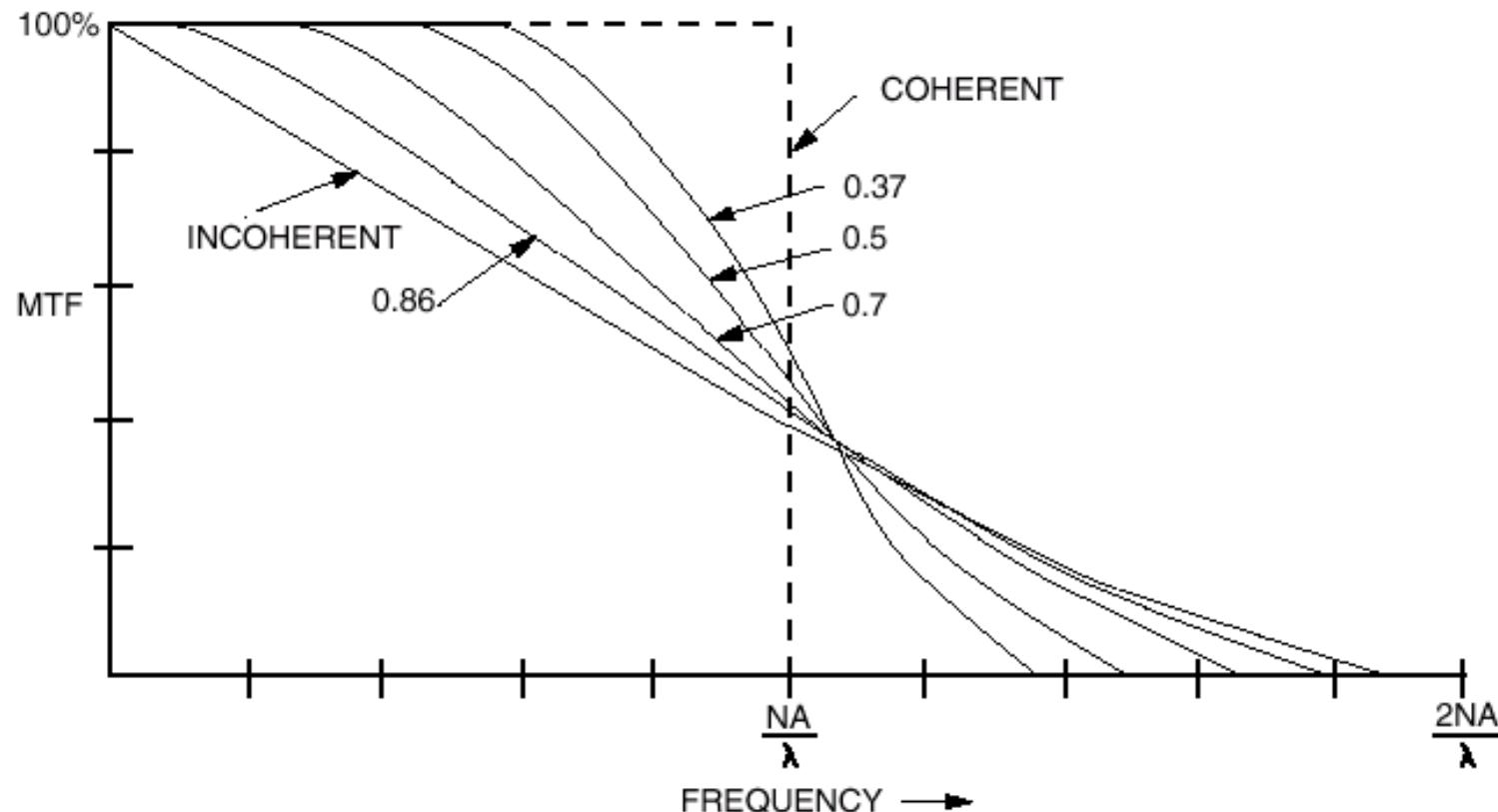
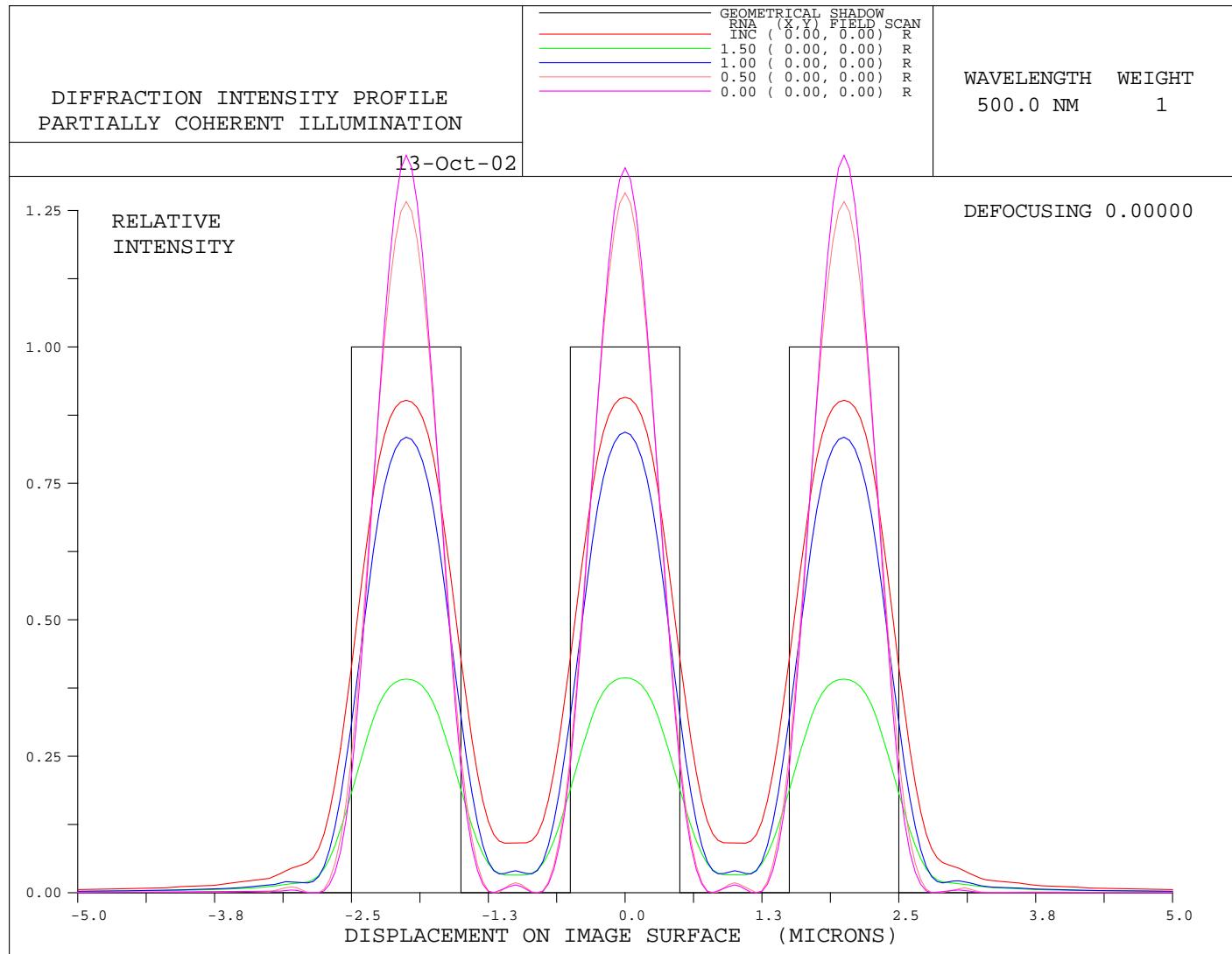
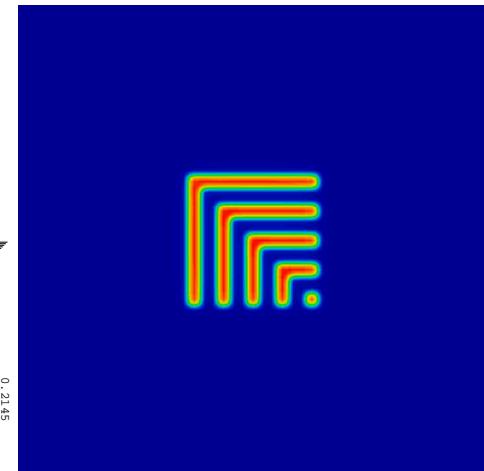
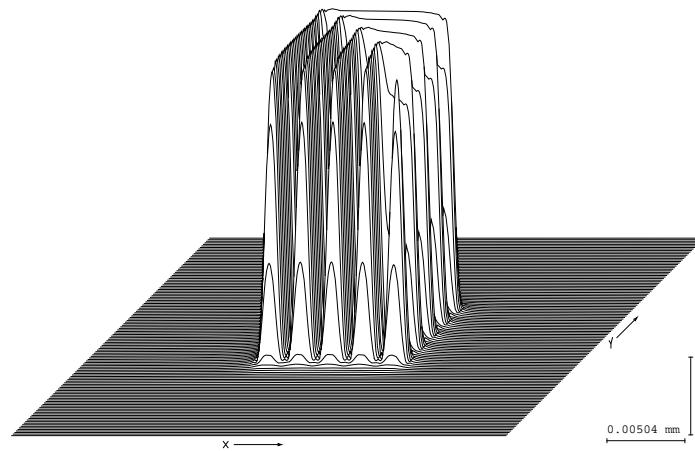
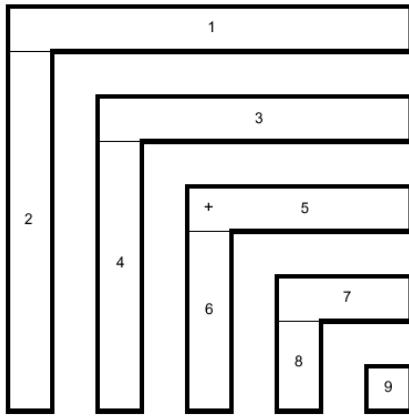


Figure 11.21 MTF vs. frequency for a partially filled pupil (semicoherent illumination). Numbers are the ratio of illuminating system NA to optical system NA.

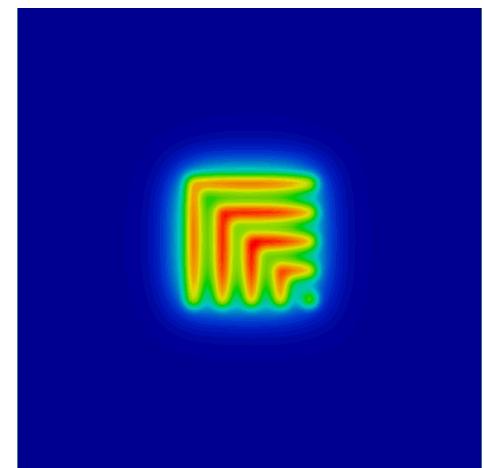
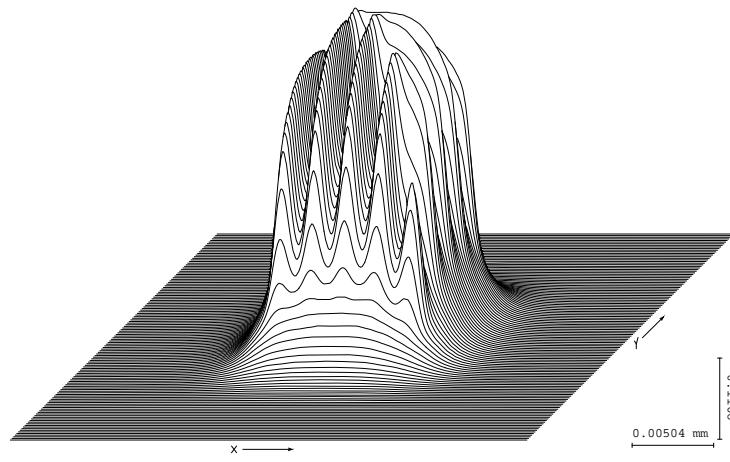
Partial Coherent Image of a 3-Bar Target



Example of Elbows Imaged in Partially Coherent Light



With 1 wave of
spherical aberration

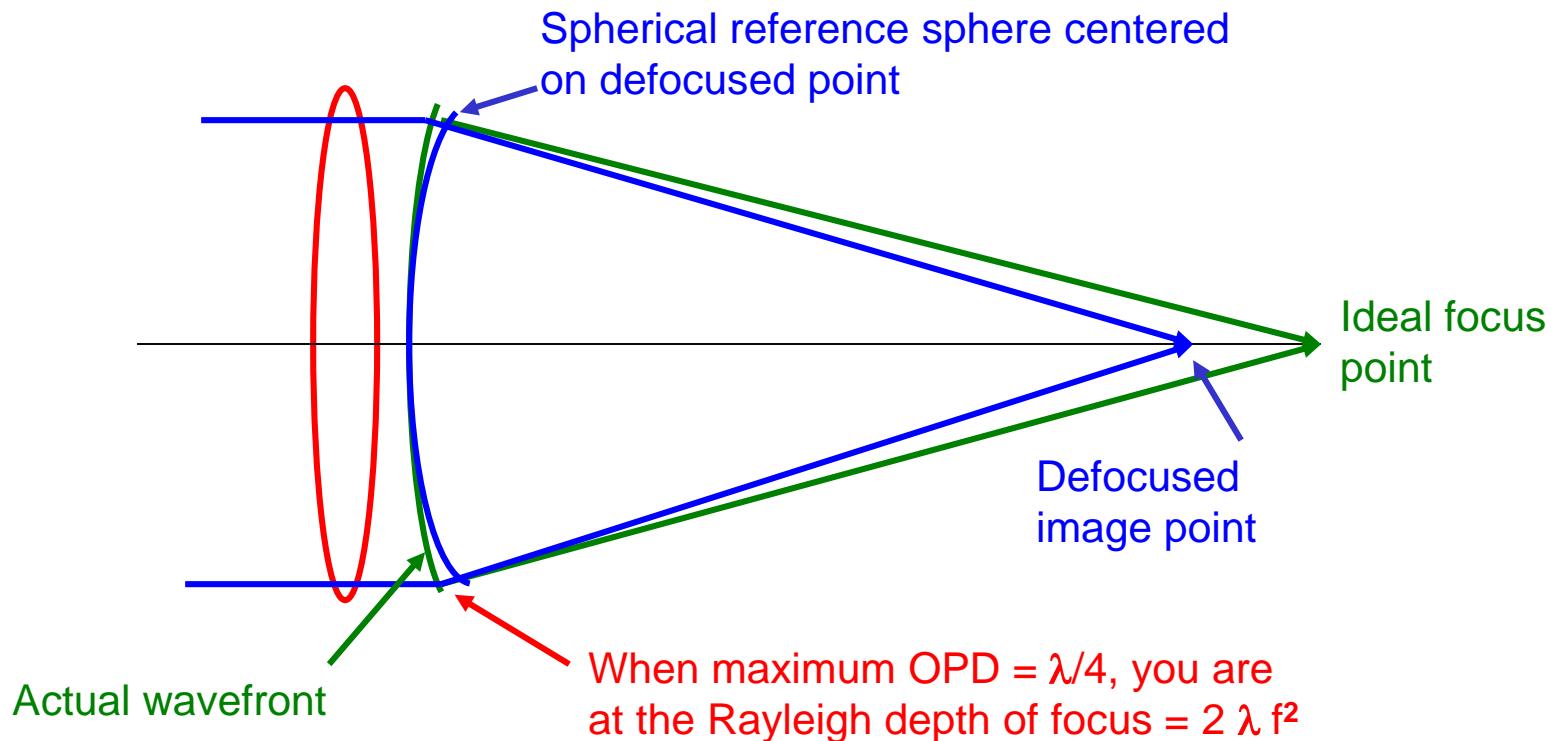


The Main Aberrations in an Optical System

- Defocus – the focal plane is not located exactly at the best focus position
- Chromatic aberration – the axial and lateral shift of focus with wavelength
- The Seidel aberrations
 - Spherical Aberration
 - Coma
 - Astigmatism
 - Distortion
 - Curvature of field

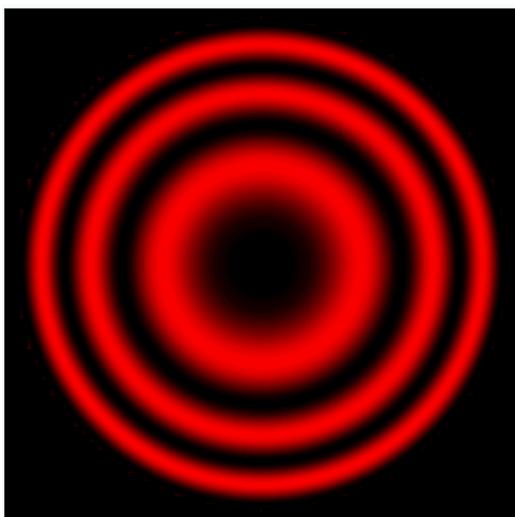
Defocus

- Technically, defocus is not an aberration in that it can be corrected by simply refocusing the lens
- However, defocus is an important effect in many optical systems

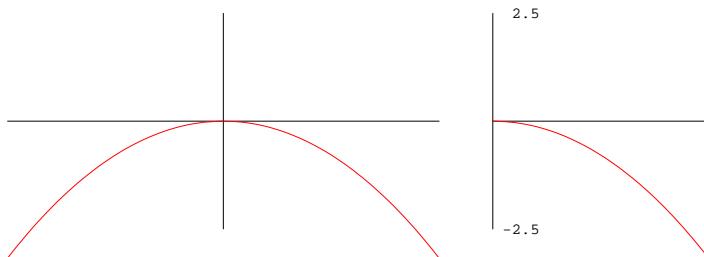
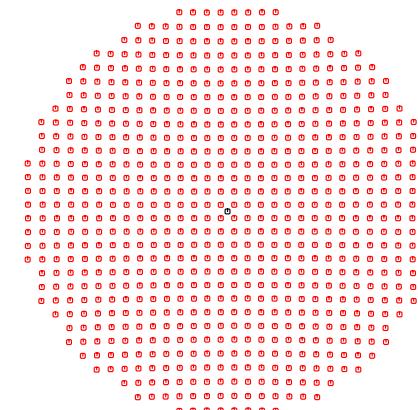


Defocus Ray Aberration Curves

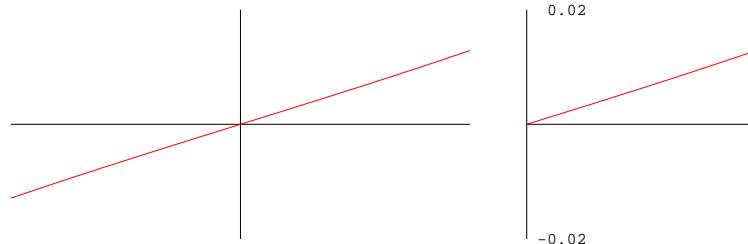
Wavefront map



Spot diagram



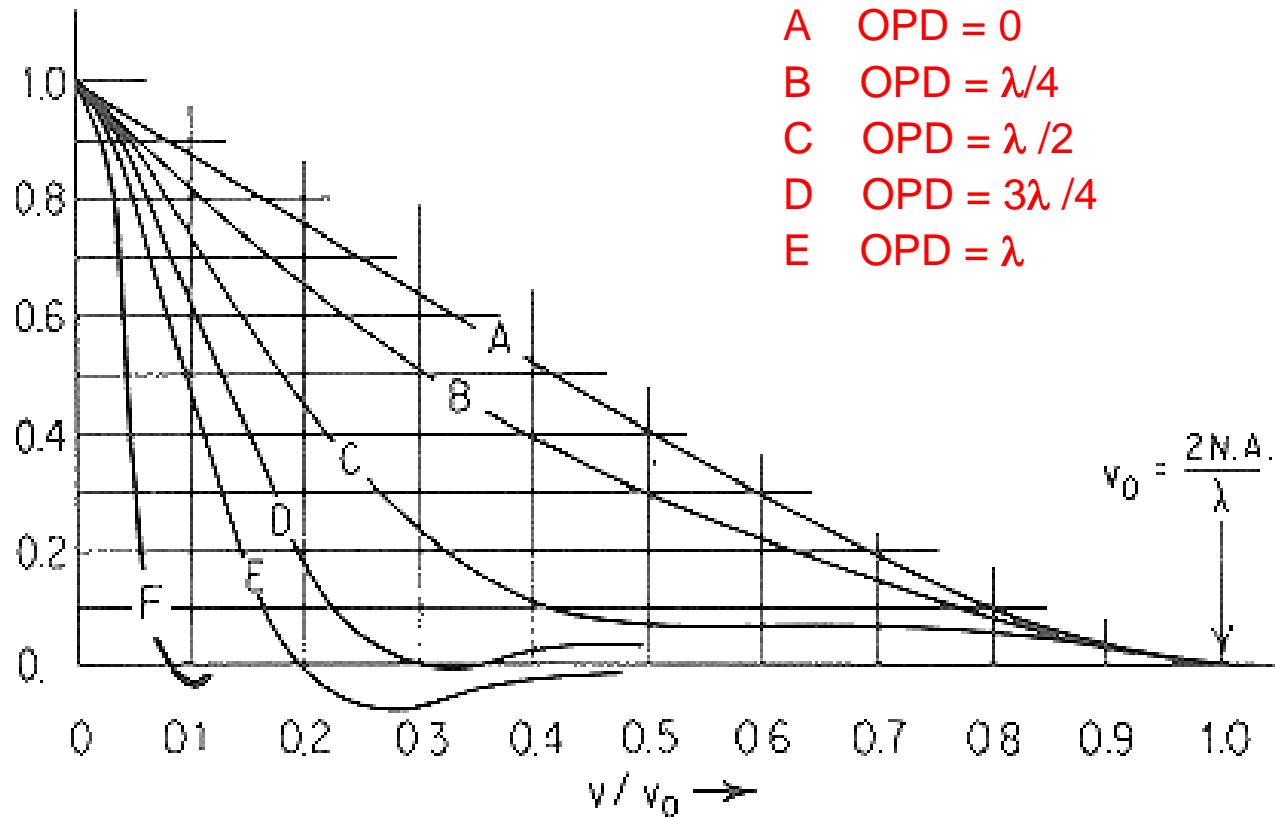
Wavefront error



Transverse ray aberration

MTF of a Defocused Image

- As the amount of defocus increases, the MTF drops accordingly

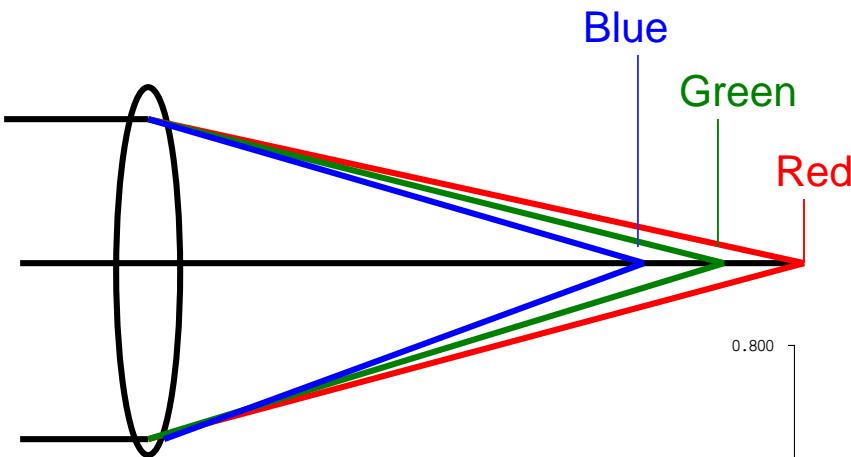


Sources of Defocus

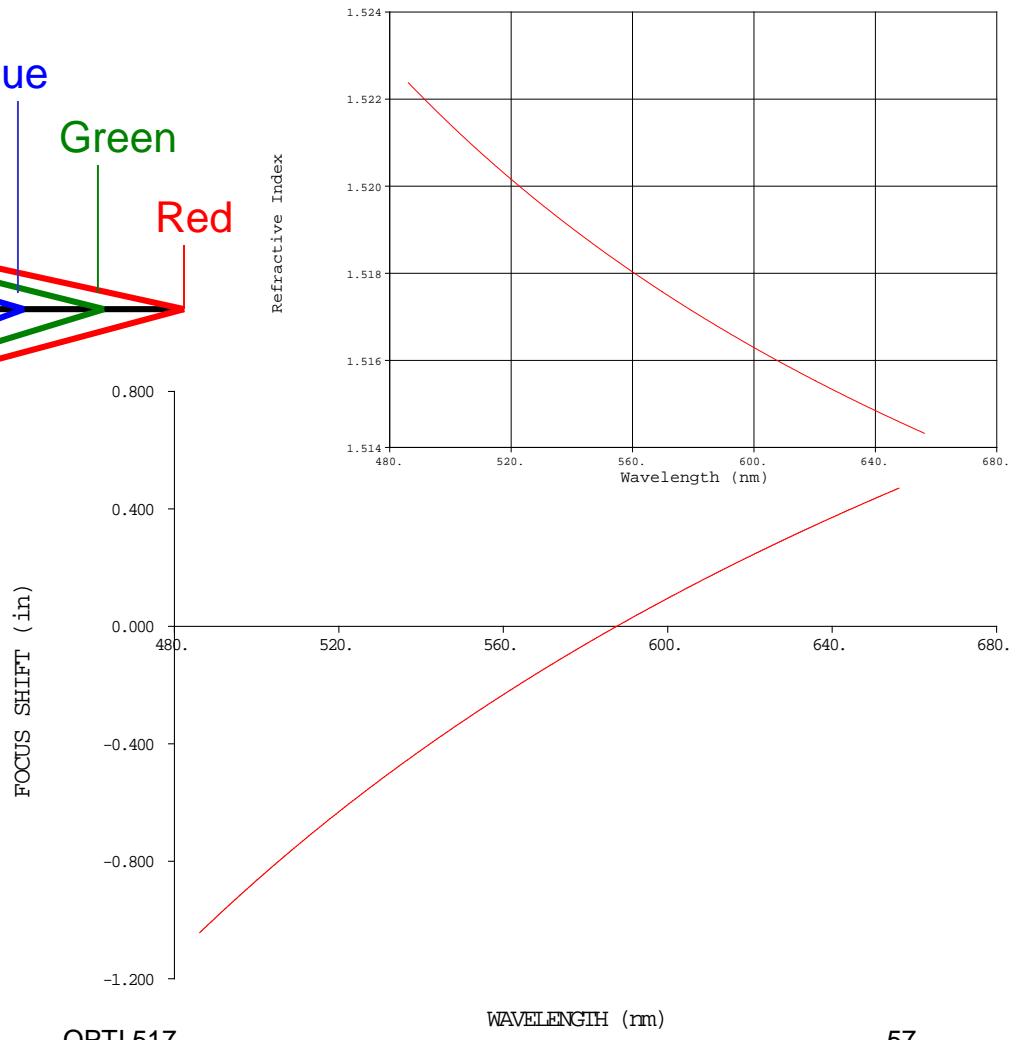
- One obvious source of defocus is the [location of the object](#)
 - For lenses focused at infinity, objects closer than infinity have defocused images
 - There's nothing we can do about this (unless we have a focus knob)
- [Changes in temperature](#)
 - As the temperature changes, the elements and mounts change dimensions and the refractive indices change
 - This can cause the lens to go out of focus
 - This can be reduced by design (material selection)
- Another source is the [focus procedure](#)
 - There are two possible sources of error here
 - Inaccuracy in the measurement of the desired focus position
 - Resolution in the positioning of the focus (e.g., shims in 0.001 inch increments)
 - The focus measurement procedure and focus position resolution must be designed to not cause focus errors which can degrade the image quality beyond the IQ specification

Chromatic Aberration

- Chromatic aberration is caused by the lens's refractive index changing with wavelength



The shorter wavelengths focus closer to the lens because the refractive index is higher for the shorter wavelengths

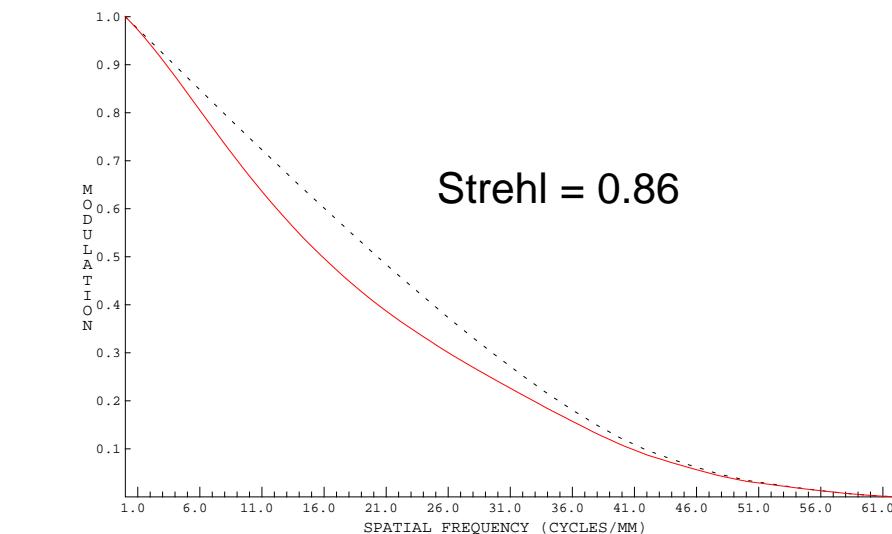
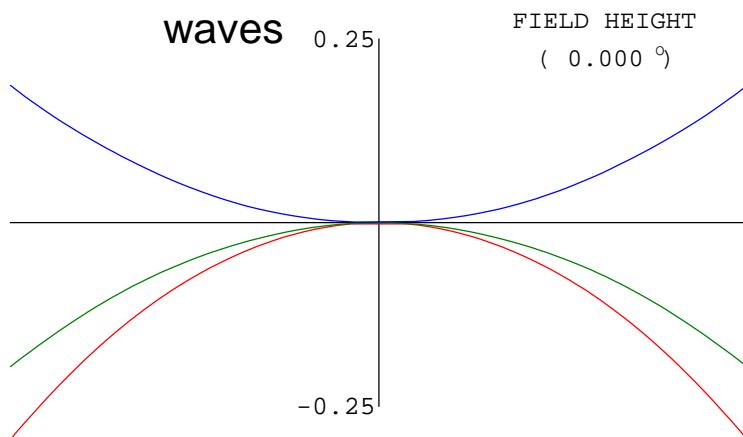


Computing Chromatic Aberration

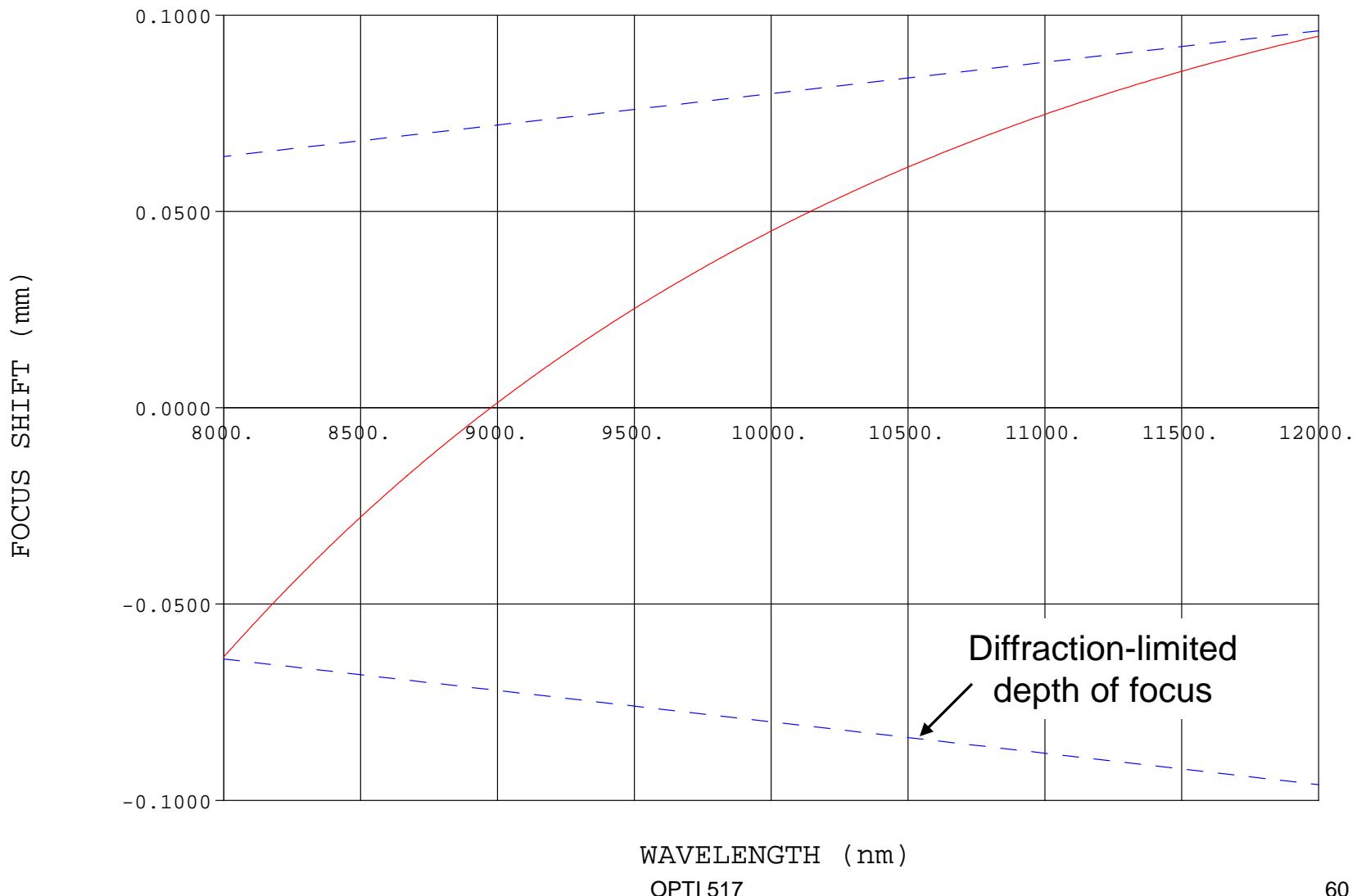
- The chromatic aberration of a lens is a function of the dispersion of the glass
 - Dispersion is a measure of the change in index with wavelength
- It is commonly designated by the Abbe V-number for three wavelengths
 - For visible glasses, these are F (486.13), d (587.56), C (656.27)
 - For infrared glasses they are typically 3, 4, 5 or 8, 10, 12 microns
 - $V = (n_{\text{middle}} - 1) / (n_{\text{short}} - n_{\text{long}})$
- For optical glasses, V is typically in the range 35-80
- For infrared glasses they vary from 50 to 1000
- The axial (longitudinal) spread of the short wavelength focus to the long wavelength focus is F/V
 - Example 1: N-BK7 glass has a V-value of 64.4. What is the axial chromatic spread of an N-BK7 lens of 100 mm focal length?
 - Answer: $100/64.4 = 1.56 \text{ mm}$
 - Note that if the lens were f/2, the diffraction DOF = $\pm 2\lambda f^2 = \pm 0.004 \text{ mm}$
 - Example 2: Germanium has a V-value of 942 (for 8 – 12 μ). What is the axial chromatic spread of a germanium lens of 100 mm focal length?
 - Answer: $100/942 = 0.11 \text{ mm}$ Note: DOF(f/2) = $\pm 2\lambda f^2 = \pm 0.08 \text{ mm}$

Chromatic Aberration Example - Germanium Singlet

- We want to use an f/2 germanium singlet over the 8 to 12 micron band
- Question - What is the longest focal length we can have and not need to color correct? (assume an asphere to correct any spherical aberration)
- Answer
 - Over the 8-12 micron band, for germanium $V = 942$
 - The longitudinal defocus = $F / V = F / 942$
 - The 1/4 wave depth of focus is $\pm 2\lambda f^2$
 - Equating these and solving gives $F = 4 * 942 * \lambda * f^2 = 150 \text{ mm}$

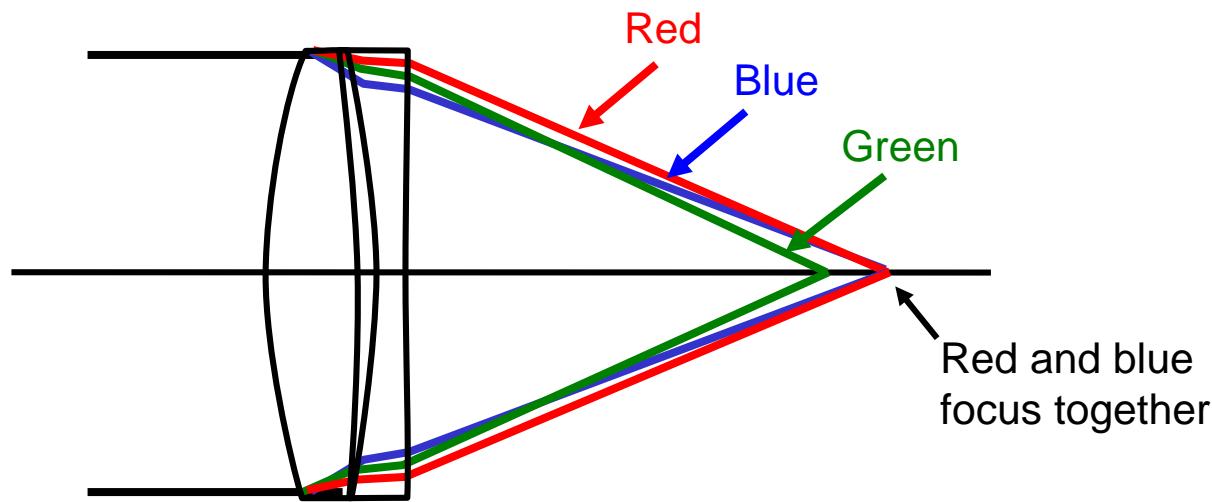


Focus Shift vs. Wavelength (Germanium singlet)



Correcting Chromatic Aberration

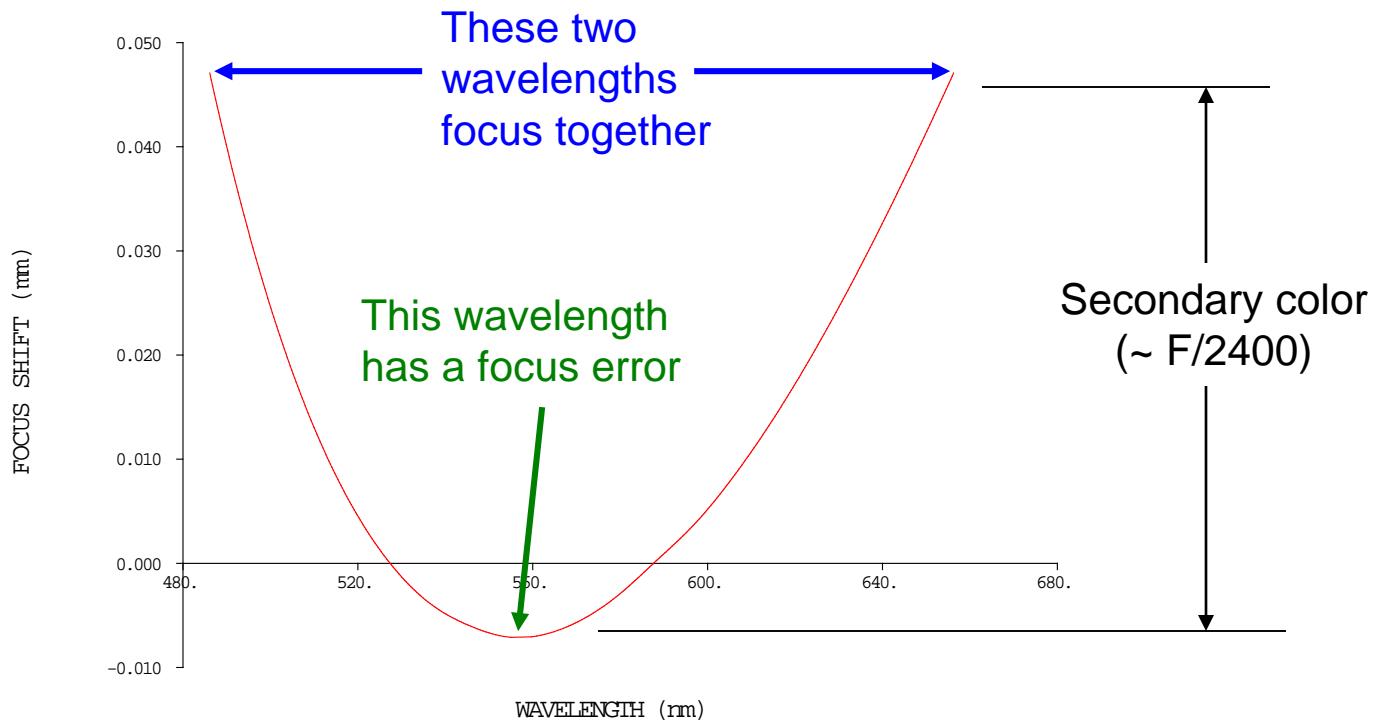
- Chromatic aberration is corrected by a combination of two glasses
 - The positive lens has low dispersion (high V number) and the negative lens has high dispersion (low V number)



- This will correct primary chromatic aberration
 - The red and blue wavelengths focus together
 - The green (or middle) wavelength still has a focus error
 - This residual chromatic spread is called secondary color

Secondary Color

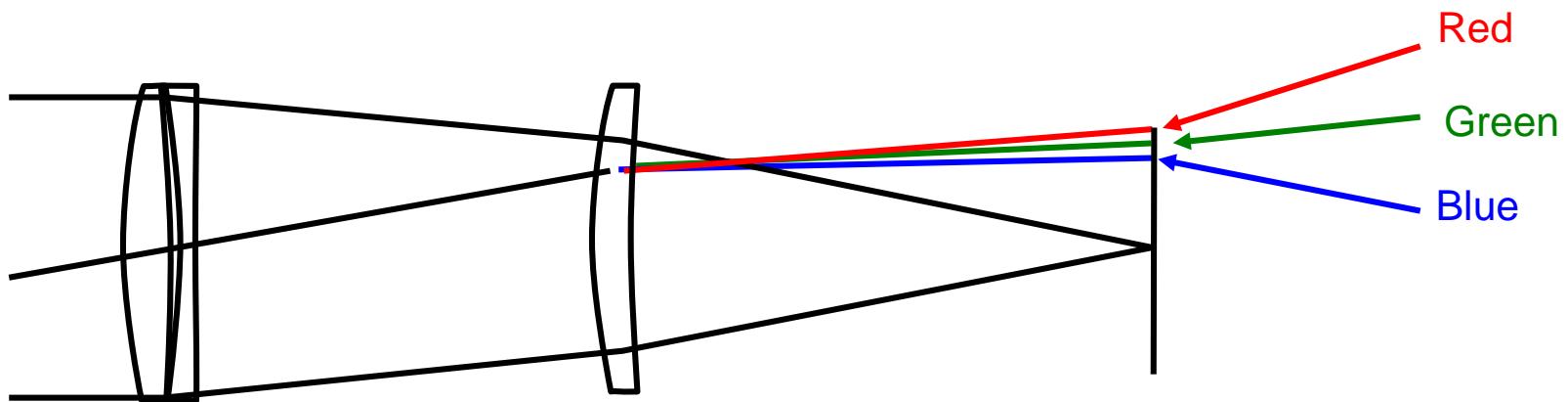
- Secondary color is the residual chromatic aberration left when the primary chromatic aberration is corrected



- Secondary color can be reduced by selecting special glasses
 - These glasses cost more (naturally)

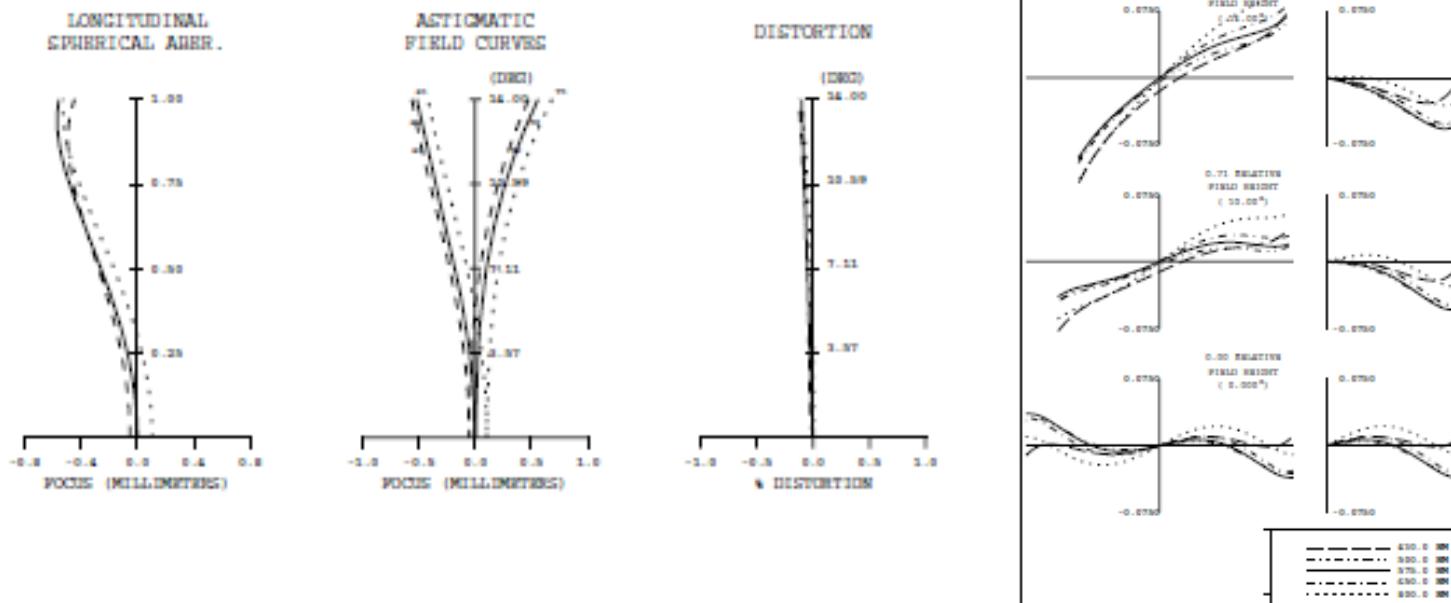
Lateral Color

- Lateral color is a change in focal length (or magnification) with wavelength
 - This results in a different image size with wavelength
 - The effect is often seen as color fringes at the edge of the FOV
 - This reduces the MTF for off-axis images



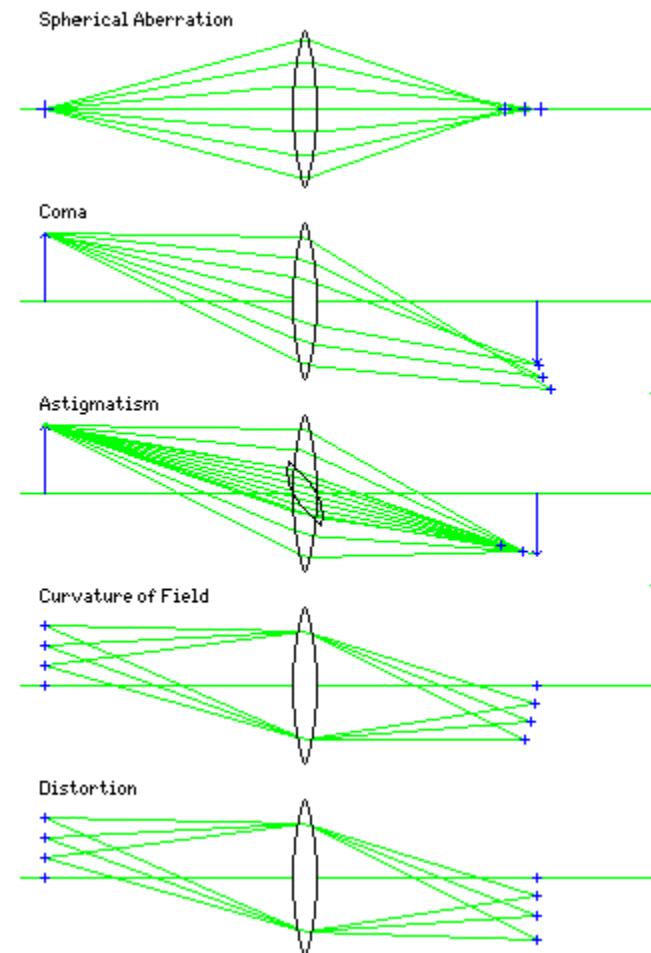
Higher-order Chromatic Aberrations

- For broadband systems, the chromatic variation in the third-order aberrations are often the most challenging aberrations to correct (e.g., spherochromatism, chromatic variation of coma, chromatic variation of astigmatism)
 - These are best studied with ray aberration curves and field plots



The Seidel Aberrations

- These are the classical aberrations in optical design
 - Spherical aberration
 - Coma
 - Astigmatism
 - Distortion
 - Curvature of field
- These aberrations, along with defocus and chromatic aberrations, are the main aberrations in an optical system



The Importance of Third-order Aberrations

- The ultimate performance of any unconstrained optical design is almost always limited by a specific aberration that is an intrinsic characteristic of the design form
- A familiarity with aberrations and lens forms is an important ingredient in a successful optimization that makes optimal use of the time available to accomplish the design
- A knowledge of the aberrations
 - Allows "spotting" lenses that are at the end of the road with respect to optimization
 - Gives guidance in what direction to "kick" a lens that has strayed from the optimal solution

Aberration Dependence on EPD and Field Angle

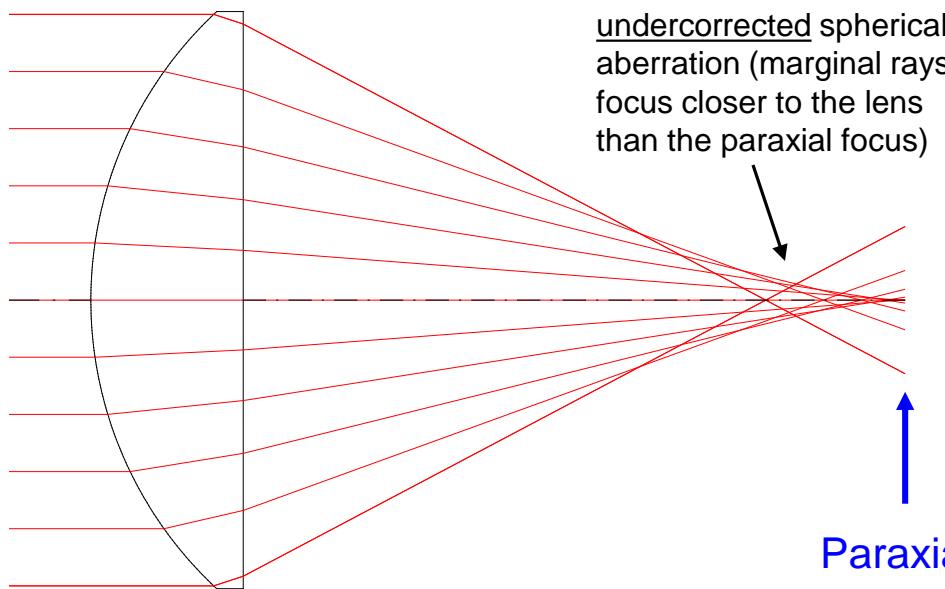
- The various Seidel aberrations have different dependencies on the aperture (EPD) radius y and the field angle θ = field height/focal length
- For the third-order aberrations, the variation with y and θ are as follows:

	Aperture	Angle
Longitudinal spherical aberration	y^2	-
Transverse spherical aberration	y^3	-
Coma	y^2	θ
Astigmatism	-	θ^2
Field curvature	-	θ^2
Linear distortion	-	θ^3
Percent distortion	-	θ^2
Axial chromatic aberration	-	-
Lateral chromatic aberration	-	θ

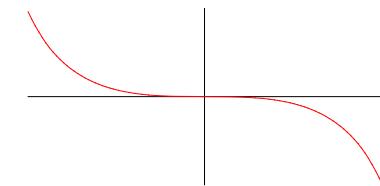
- Knowing the functional dependence of an aberration will allow you to estimate the change in a given aberration for a change in f/number or field angle

Spherical Aberration

- Spherical aberration is an on-axis aberration
- Rays at the outer parts of the pupil focus closer to or further from the lens than the paraxial focus

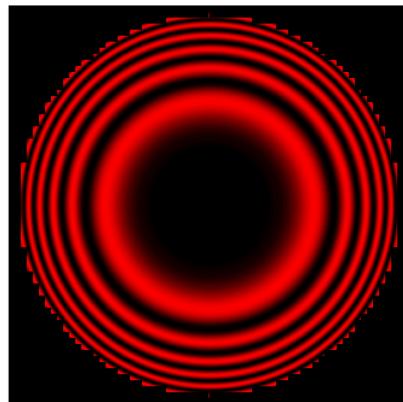
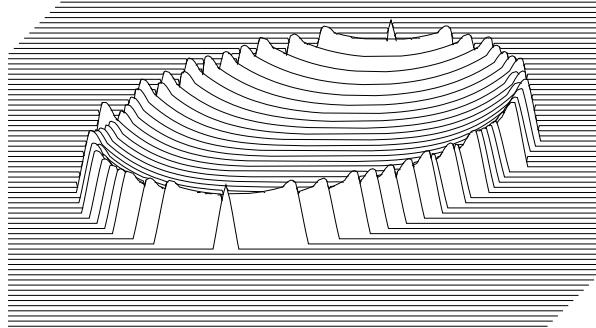


Ray aberration curve



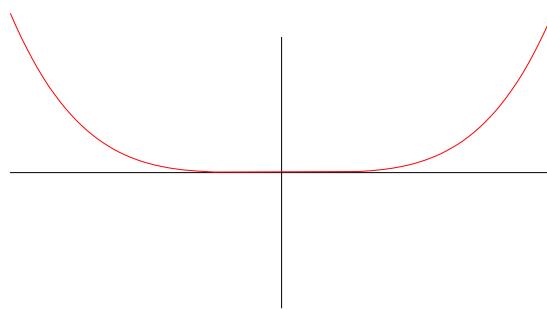
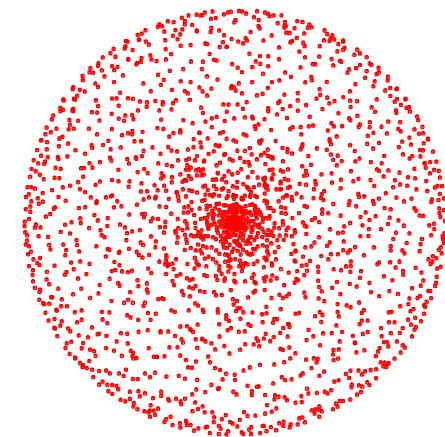
- The magnitude of the (third-order) spherical aberration goes as the cube of the aperture (going from $f/2$ to $f/1$ increases the SA by a factor of 8)

Third-order SA Ray Aberration Curves

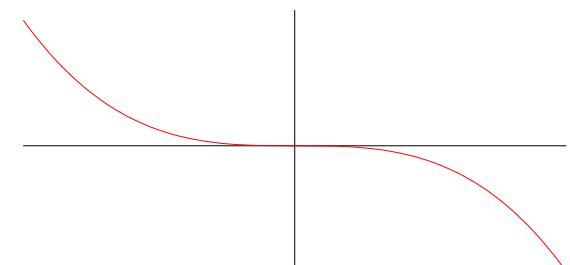


Wavefront map

Spot diagram

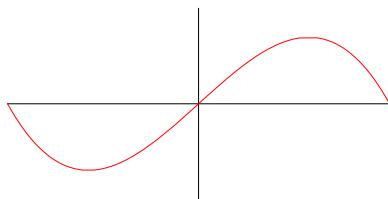
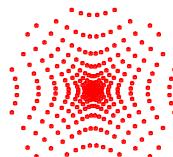
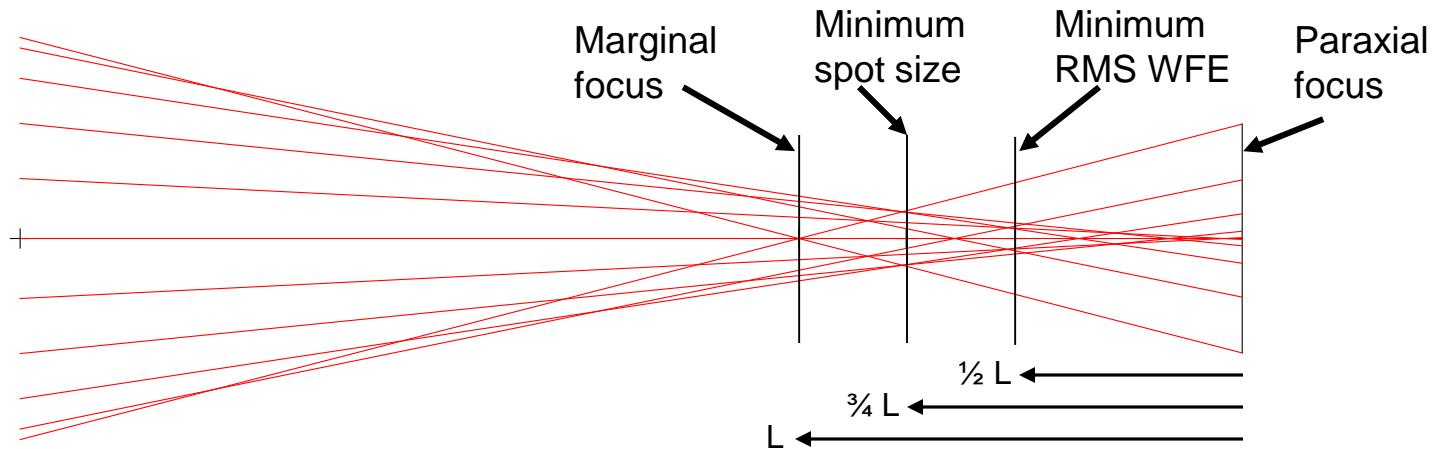


Wavefront error

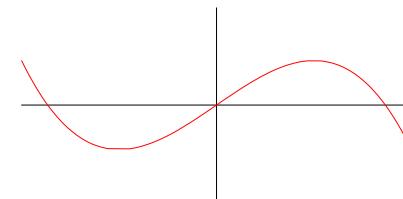


Transverse ray
aberration curve

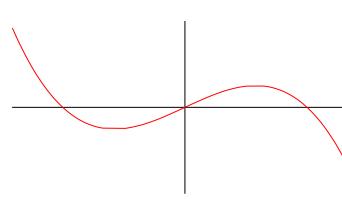
Spherical Aberration



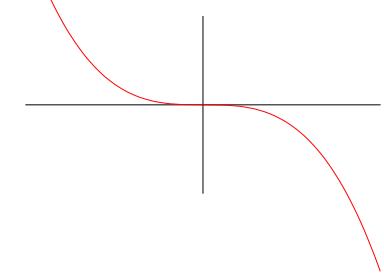
Marginal focus



Minimum spot size



Minimum RMS WFE

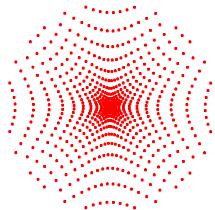


Paraxial focus

Scaling Laws for Spherical Aberration

$$\propto (f/\#)^{-3}$$

f/3



f/4



f/5



Spot size goes as the
cube of the EPD (or
inverse cube of the f/#)

$$\propto \theta^0$$

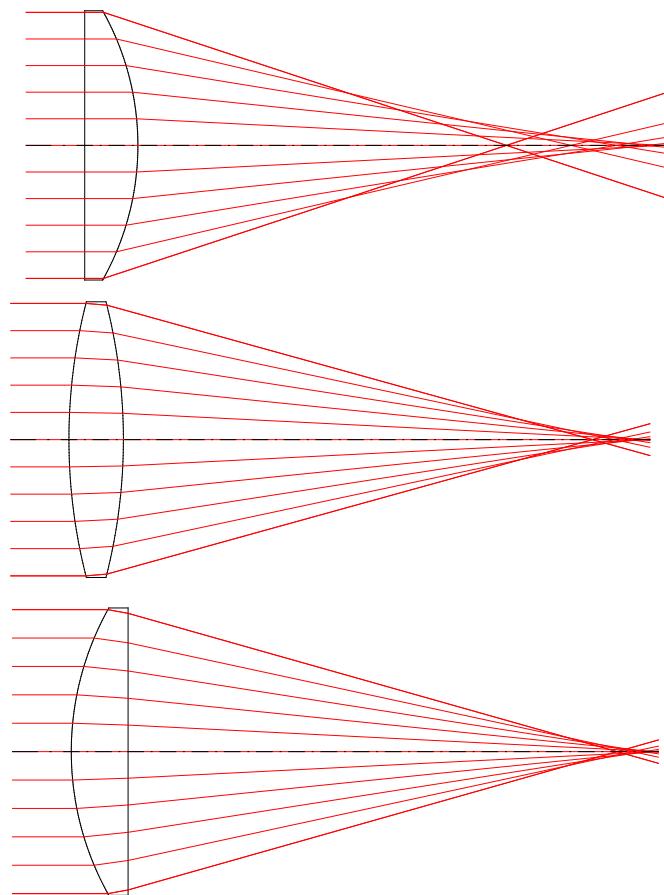
Field Angle (deg)
5.0
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0

-5.0 -4.0 -3.0 -2.0 -1.0 0.0 1.0 2.0 3.0 4.0 5.0
Field Angle (deg)

Spot size not dependent
on field position

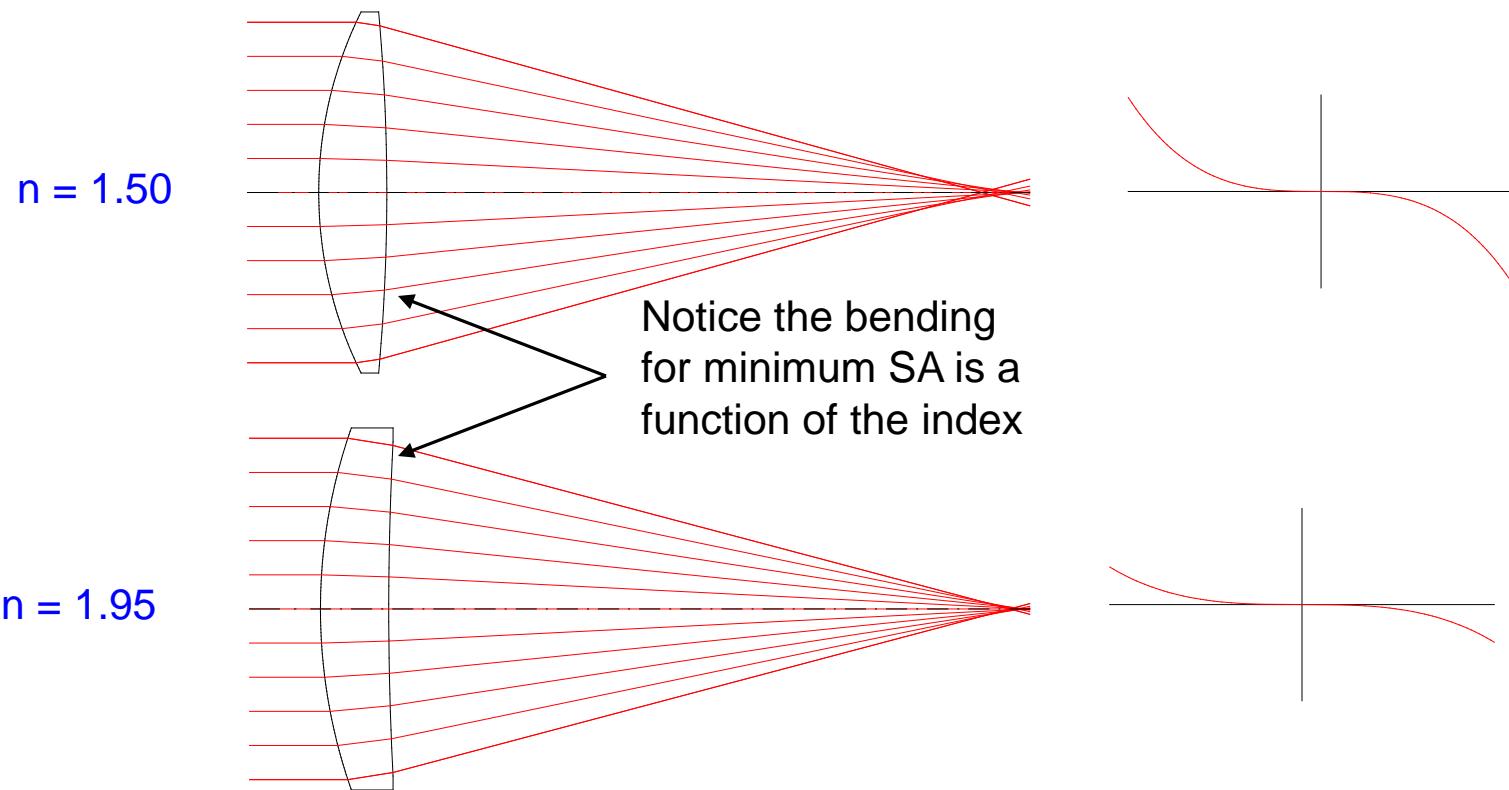
Spherical Aberration vs. Lens Shape

- The spherical aberration is a function of the lens bending, or shape of the lens

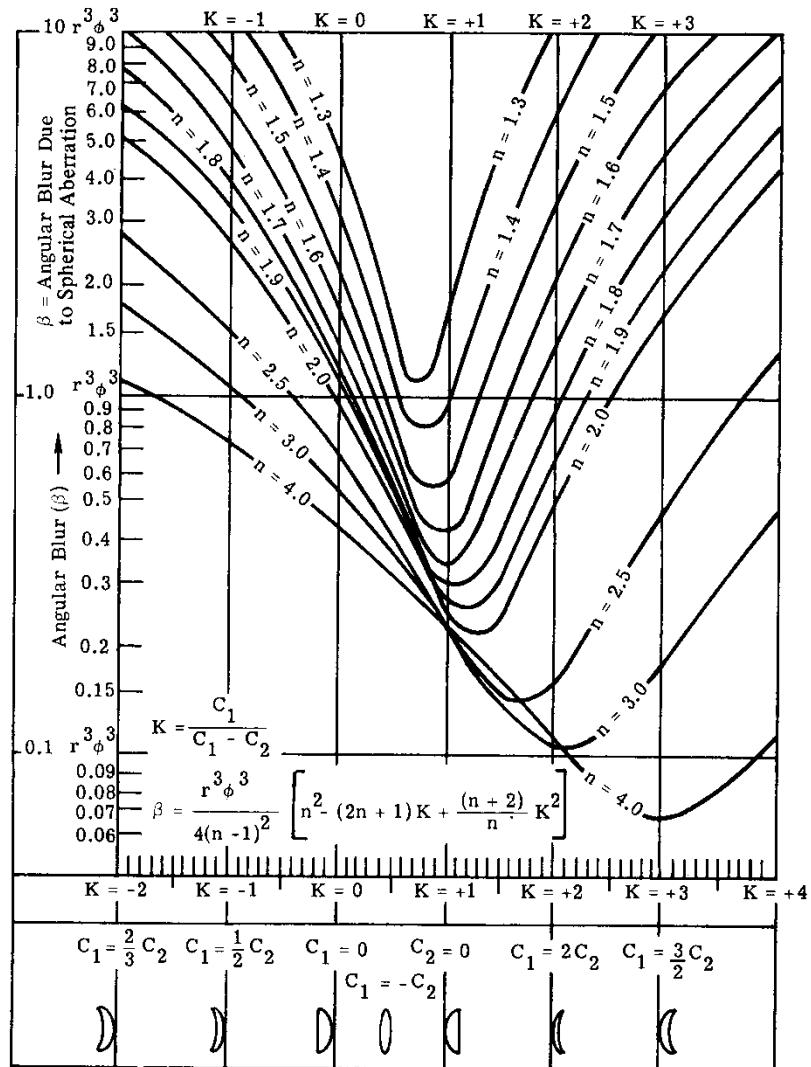


Spherical Aberration vs. Refractive Index

- Spherical aberration is reduced with higher index materials
 - Higher indices allow shallower radii, allowing less variation in incidence angle across the lens

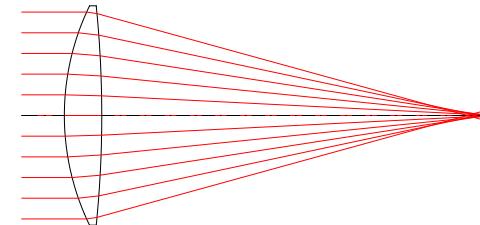


Spherical Aberration vs. Index and Bending

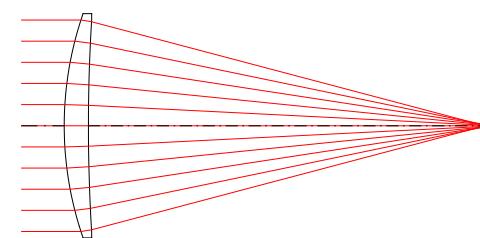


$$\beta \text{ at } K_{\min} = r^3 \phi^3 \frac{4n^2 - n}{16(n-1)^2(n+2)}$$

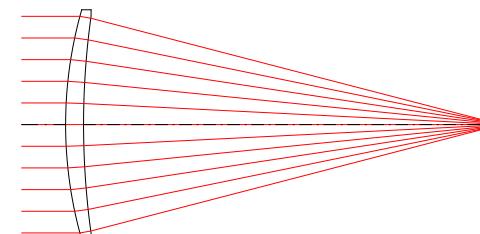
$n = 1.5$



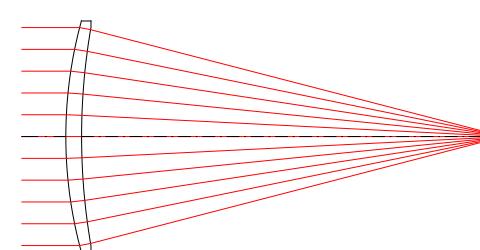
$n = 2.0$



$n = 3.0$

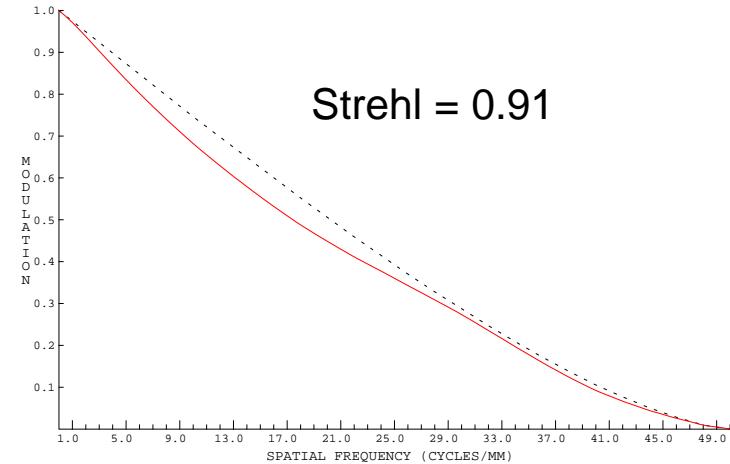
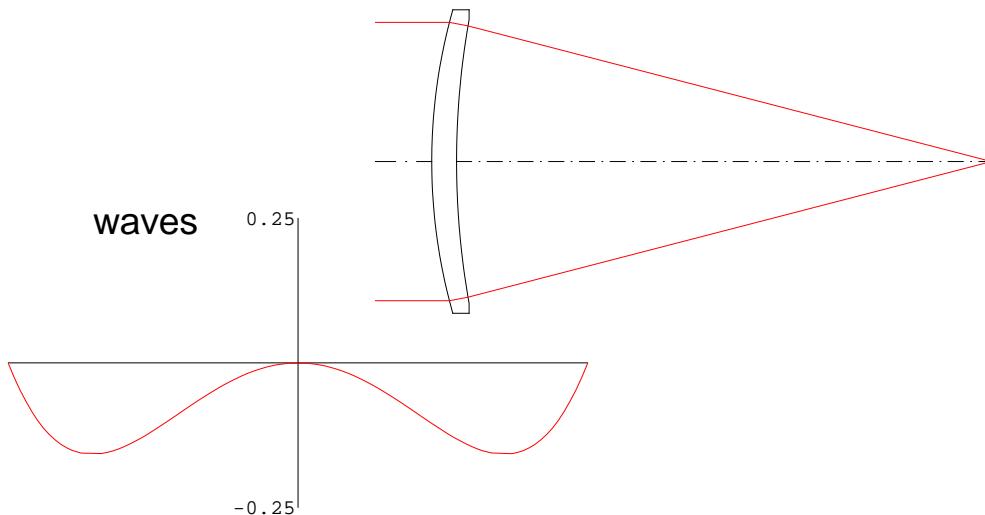


$n = 4.0$



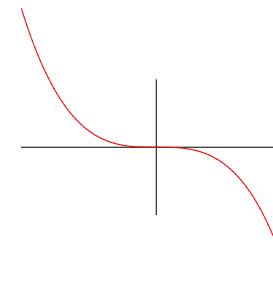
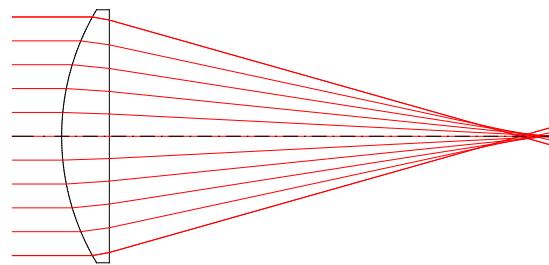
Example - Germanium Singlet

- We want an f/2 germanium singlet to be used at 10 microns (0.01 mm)
- Question - What is the longest focal length we can have and not need aspherics to correct spherical aberration?
- Answer
 - Diffraction Airy disk angular size is $\beta_{\text{diff}} = 2.44 \lambda/D$
 - Spherical aberration angular blur is $\beta_{\text{sa}} = 0.00867 / f^3$
 - Equating these gives $D = 2.44 \lambda f^3 / 0.00867 = 22.5 \text{ mm}$
 - For f/2, this gives $F = 45 \text{ mm}$

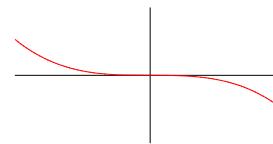
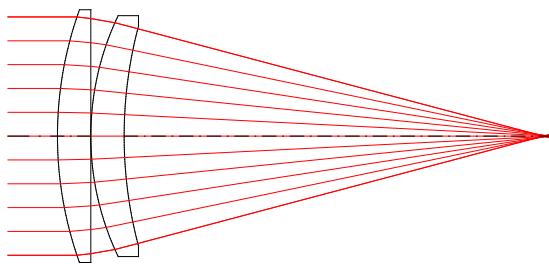


Spherical Aberration vs. Number of Lenses

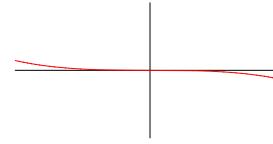
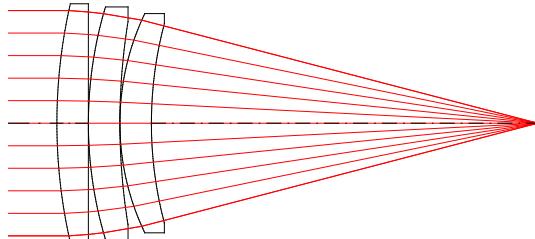
- Spherical aberration can be reduced by splitting the lens into more than one lens



SA = 1
(arbitrary units)



SA = 1/4
(arbitrary units)



SA = 1/9
(arbitrary units)

Spherical Aberration vs. Number of Lenses

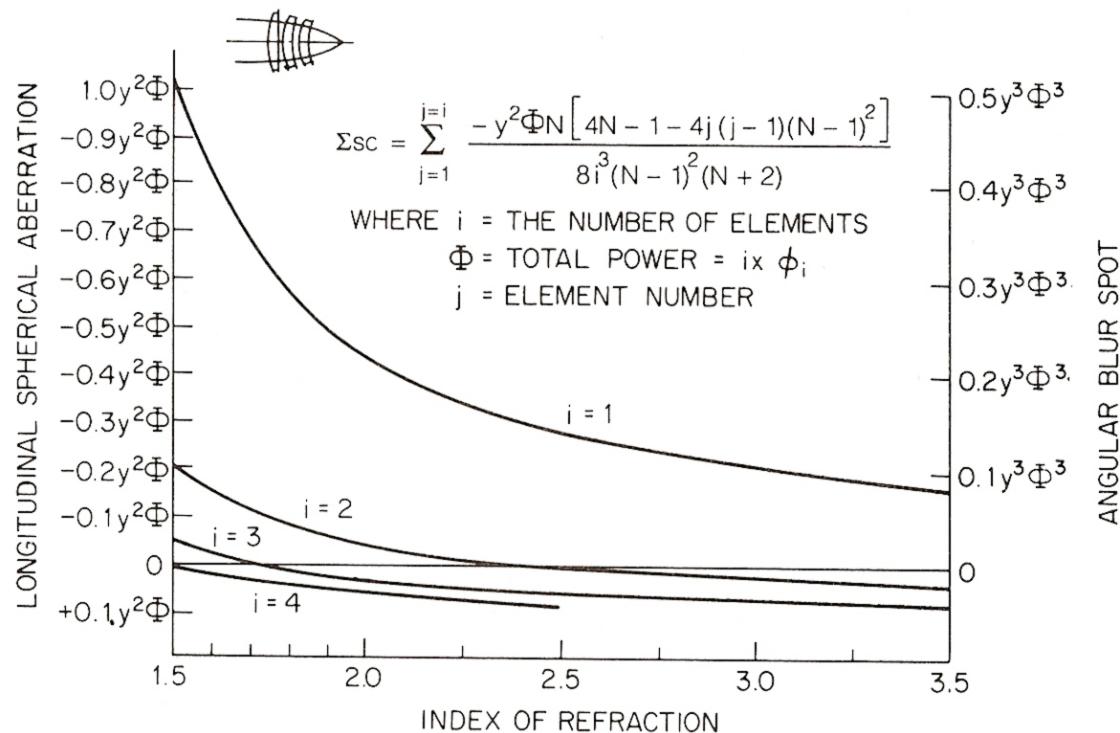
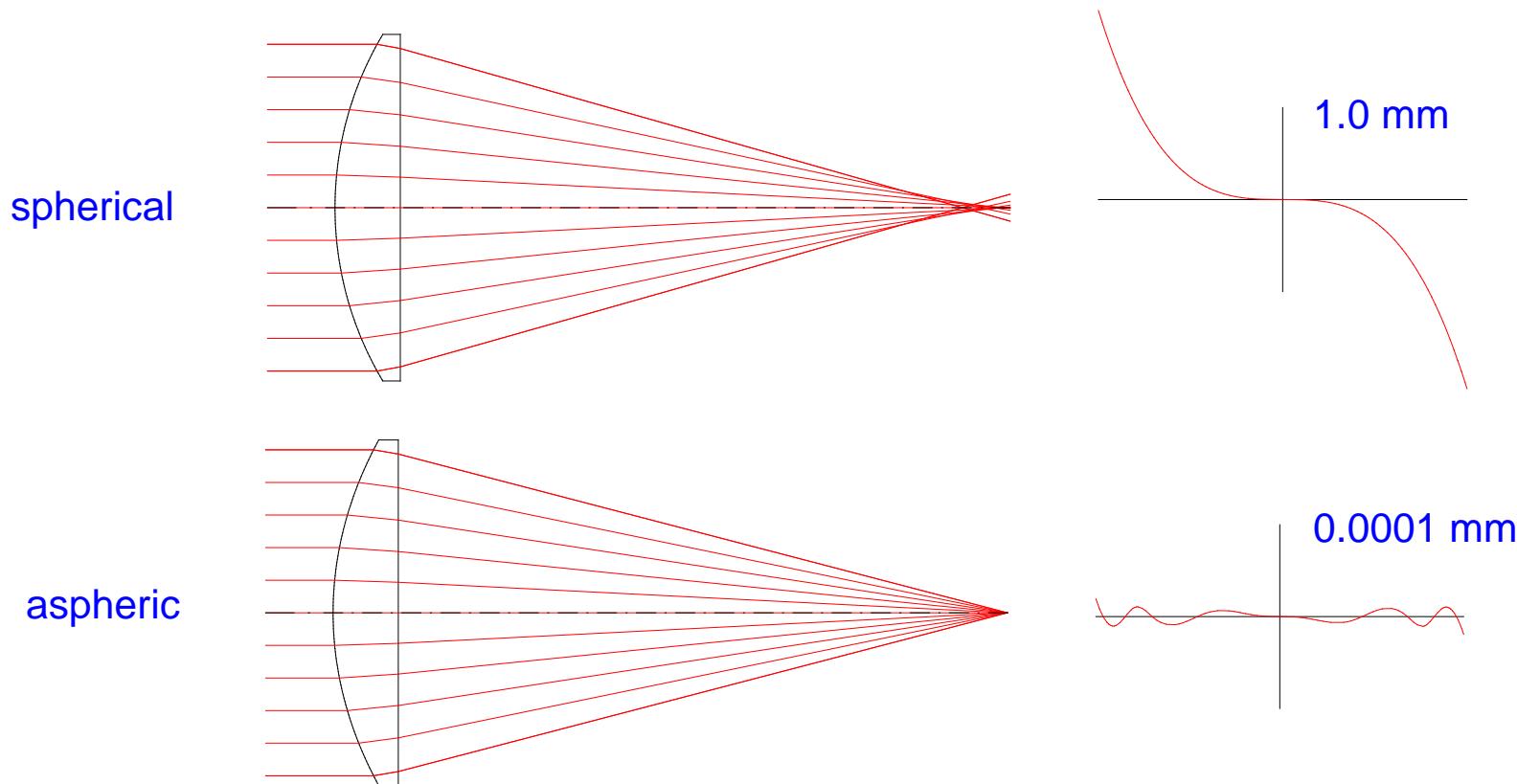


Figure 3.2 The spherical aberration of one, two, three, and four thin positive elements, each bent for minimum spherical aberration, plotted as a function of the index of refraction, and showing the reduction in the amount of aberration produced by splitting a single element into two or more elements (of the same total power). Each plot is labeled with i , the number of elements in the set. (The object is at infinity.)

Spherical Aberration and Aspherics

- The spherical aberration can be reduced, or even effectively eliminated, by making one of the surfaces aspheric



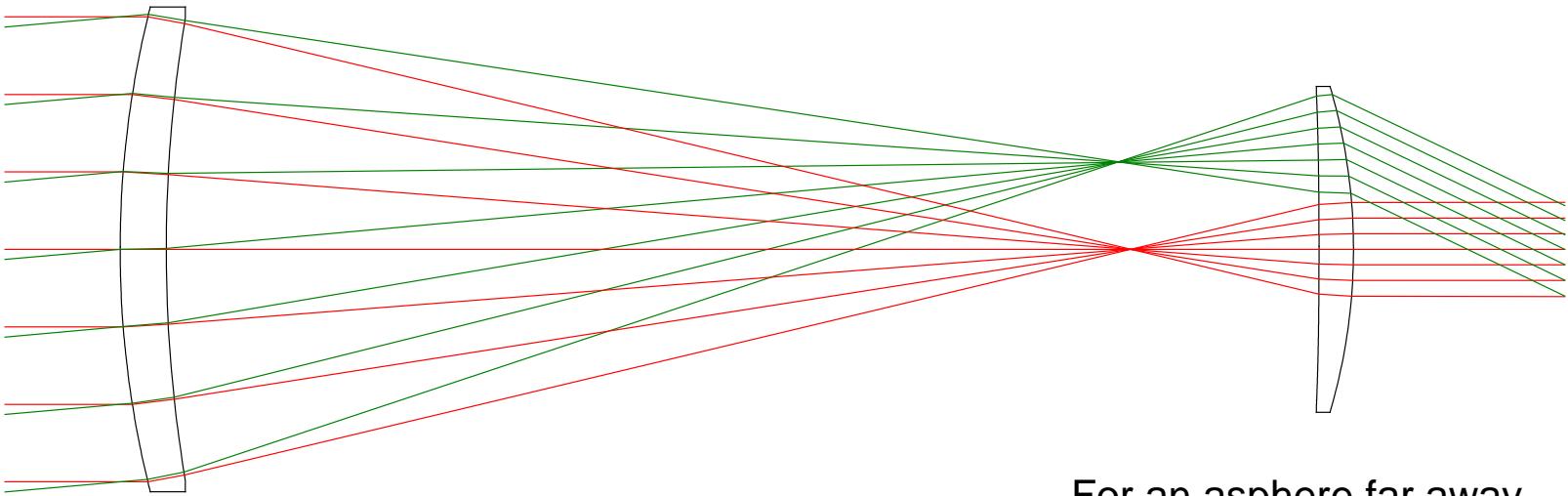
Aspheric Surfaces

- Aspheric surfaces technically are any surfaces which are not spherical, but usually refer to a polynomial deformation to a conic

$$z(r) = \frac{r^2/R}{1 + \sqrt{1 - (k+1)(r/R)^2}} + Ar^4 + Br^6 + Cr^8 + Dr^{10} + \dots$$

- The aspheric coefficients (A, B, C, D, ...) can correct 3rd, 5th, 7th, 9th, ... order spherical aberration
- When used near a pupil, aspherics are used primarily to correct spherical aberration
- When used far away (optically) from a pupil, they are primarily used to correct astigmatism by flattening the field
- Before using aspherics, be sure that they are necessary and the increased performance justifies the increased cost
 - Never use a higher-order asphere than justified by the ray aberration curves

Optimizing Aspherics

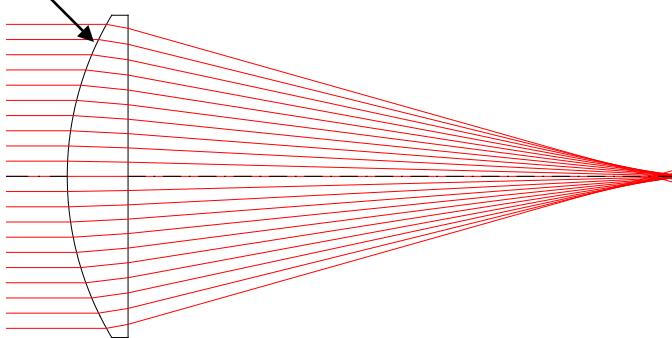


For an asphere at (or near) a pupil, there need to be enough **rays** to sample the pupil sufficiently. This asphere primarily corrects spherical aberration.

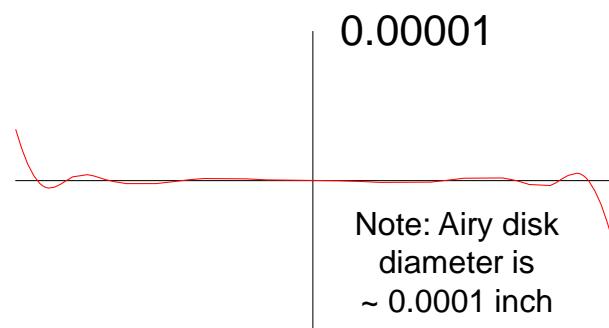
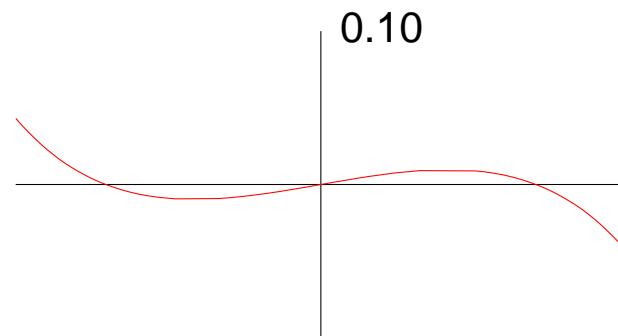
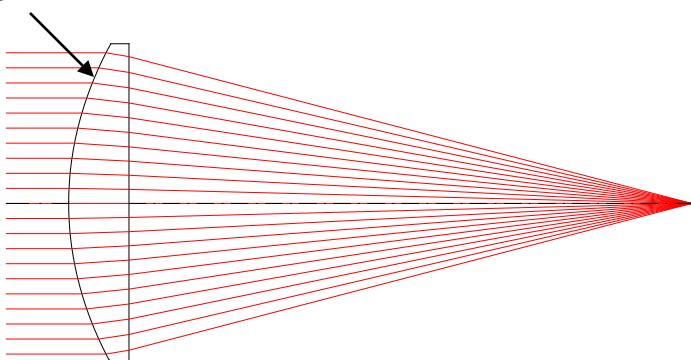
For an asphere far away (optically) from a pupil, the ray density need not be high, but there must be a sufficient number of overlapping **fields** to sample the surface accurately. This asphere primarily corrects field aberrations (e.g., astigmatism).

Asphere Example

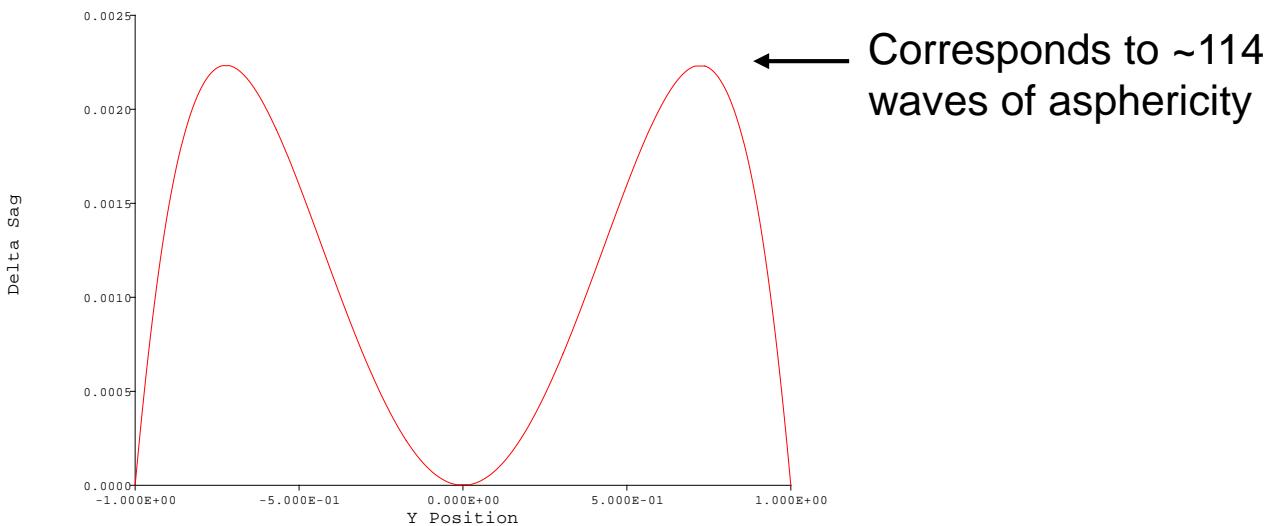
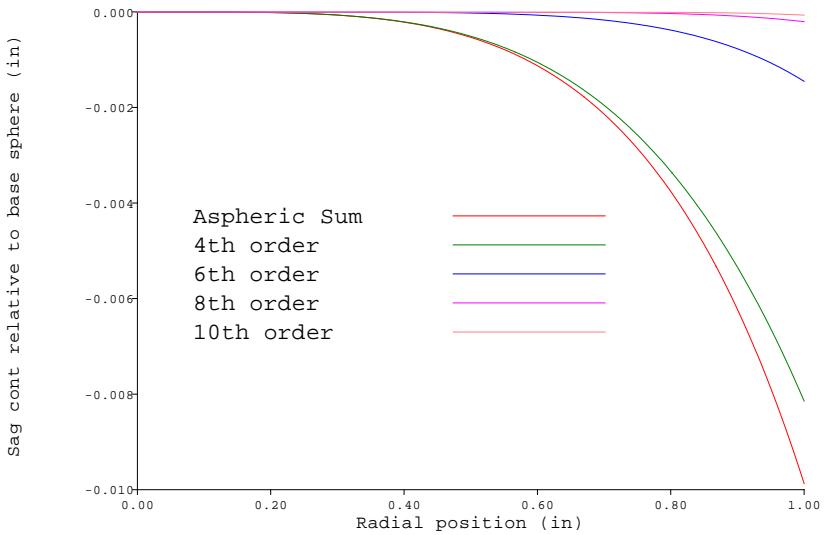
- 2 inch diameter, f/2 plano-convex lens
sphere



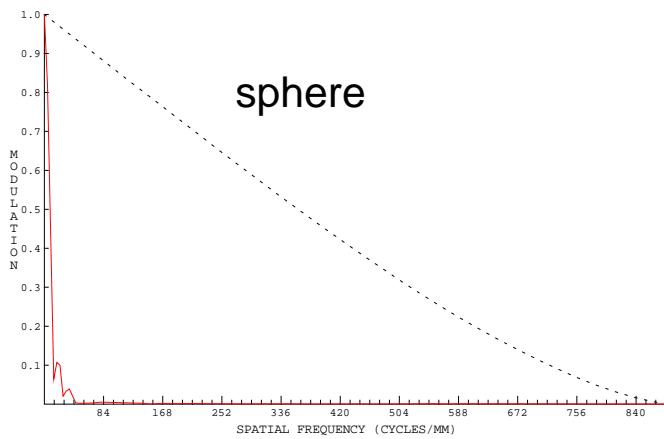
asphere



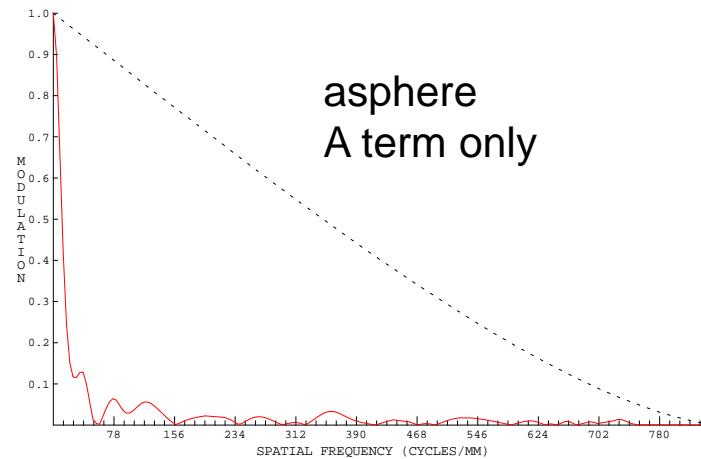
Aspheric Orders



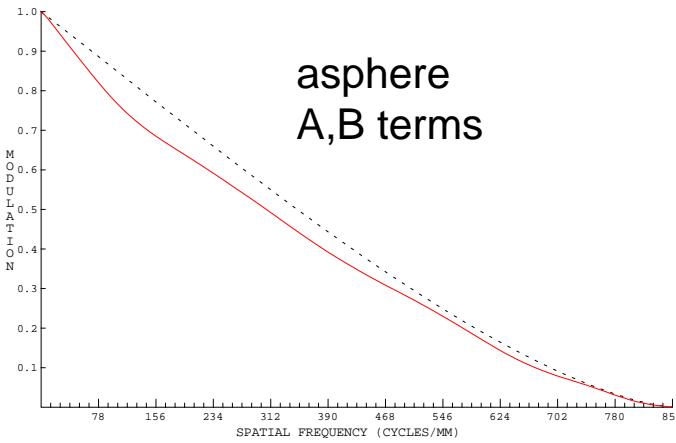
MTF vs. Aspheric Order



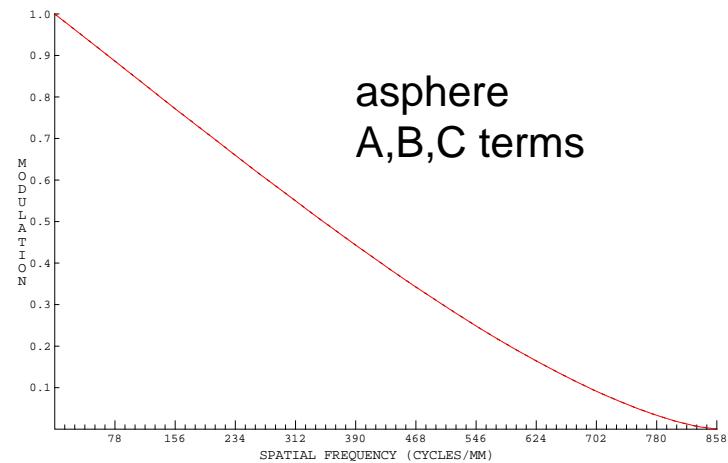
sphere



asphere
A term only



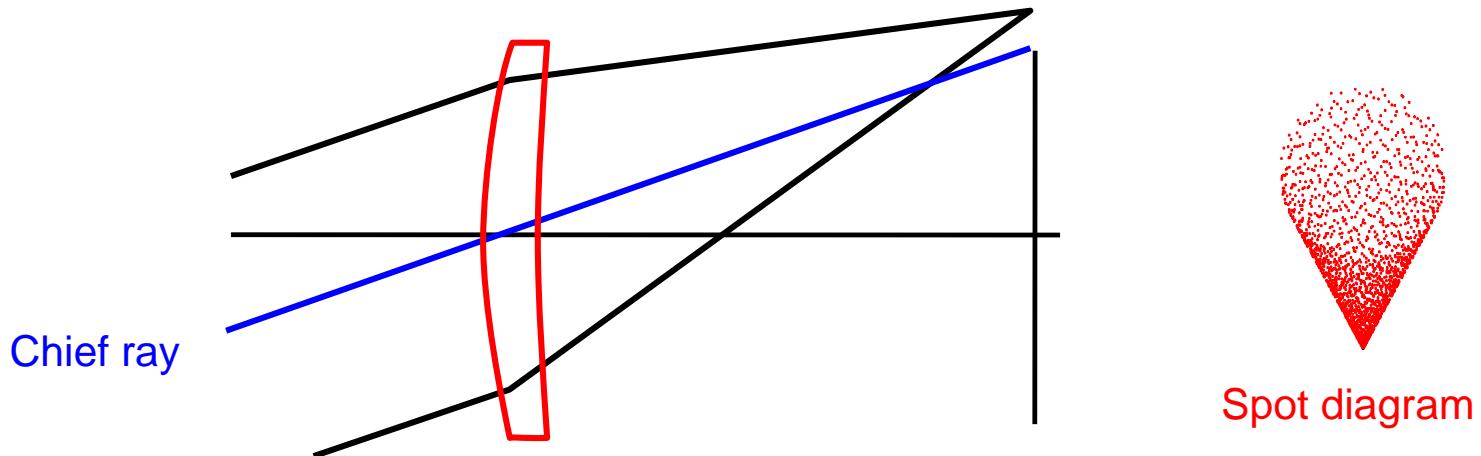
asphere
A,B terms



asphere
A,B,C terms

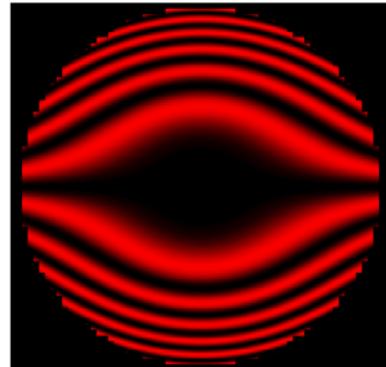
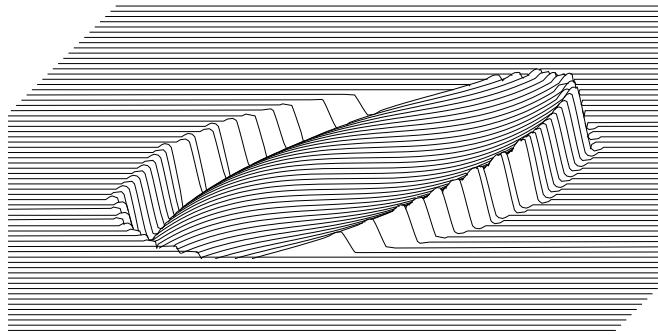
Coma

- Coma is an off-axis aberration
- It gets its name from the spot diagram which looks like a comet (coma is Latin for comet)
- A comatic image results when the periphery of the lens has a higher or lower magnification than the portion of the lens containing the chief ray



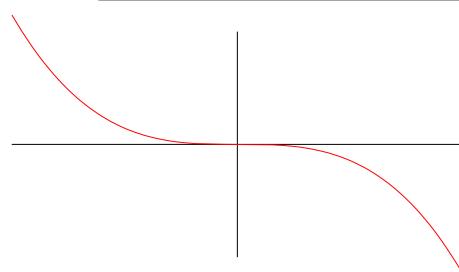
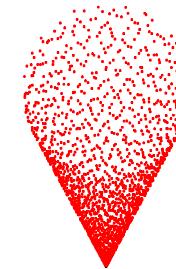
- The magnitude of the (third-order) coma is proportional to the square of the aperture and the first power of the field angle

Transverse vs. Wavefront 3rd-order Coma

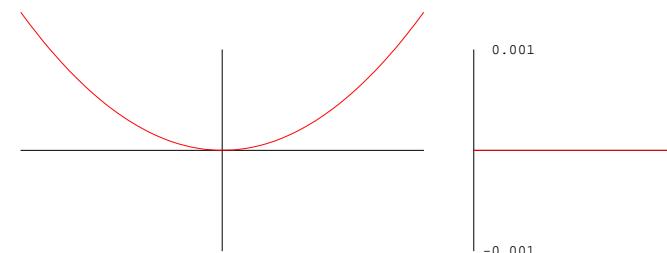
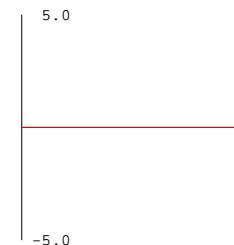


Wavefront map

Spot diagram

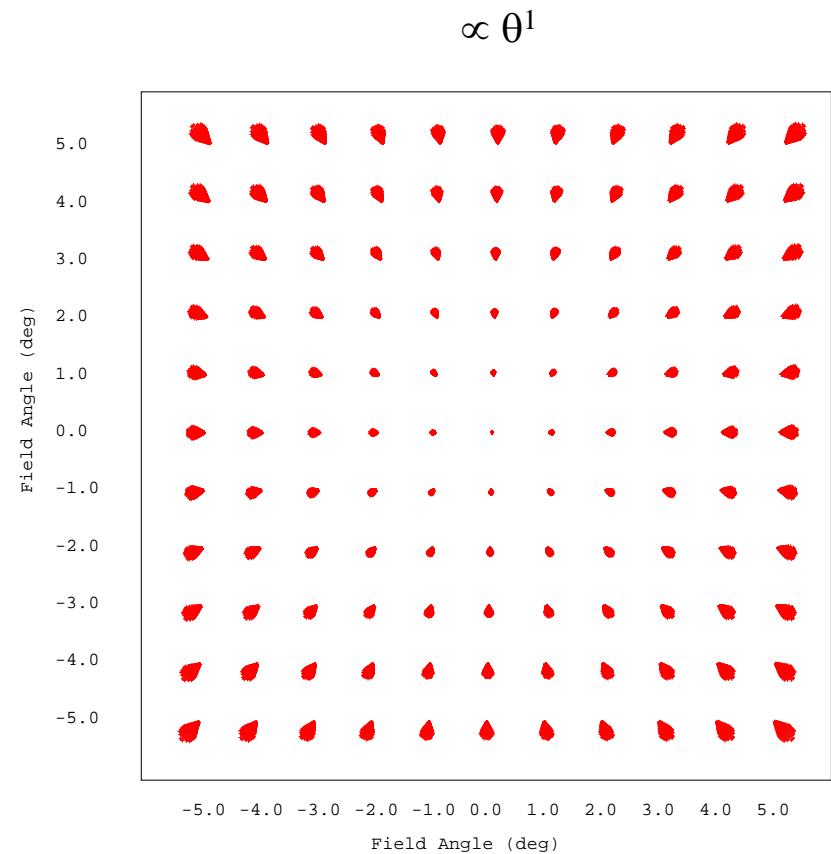
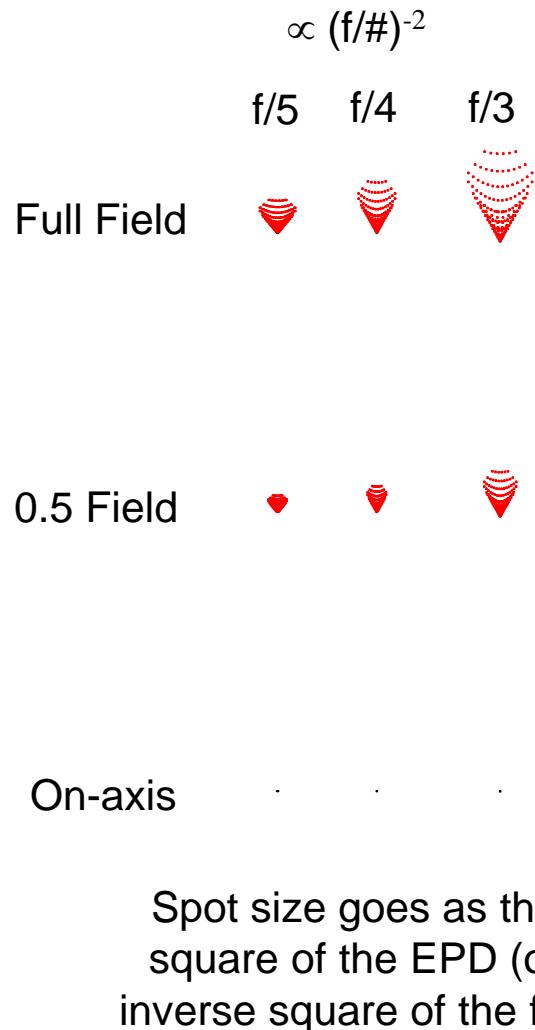


Wavefront error



Transverse ray aberration

Scaling Laws for Coma

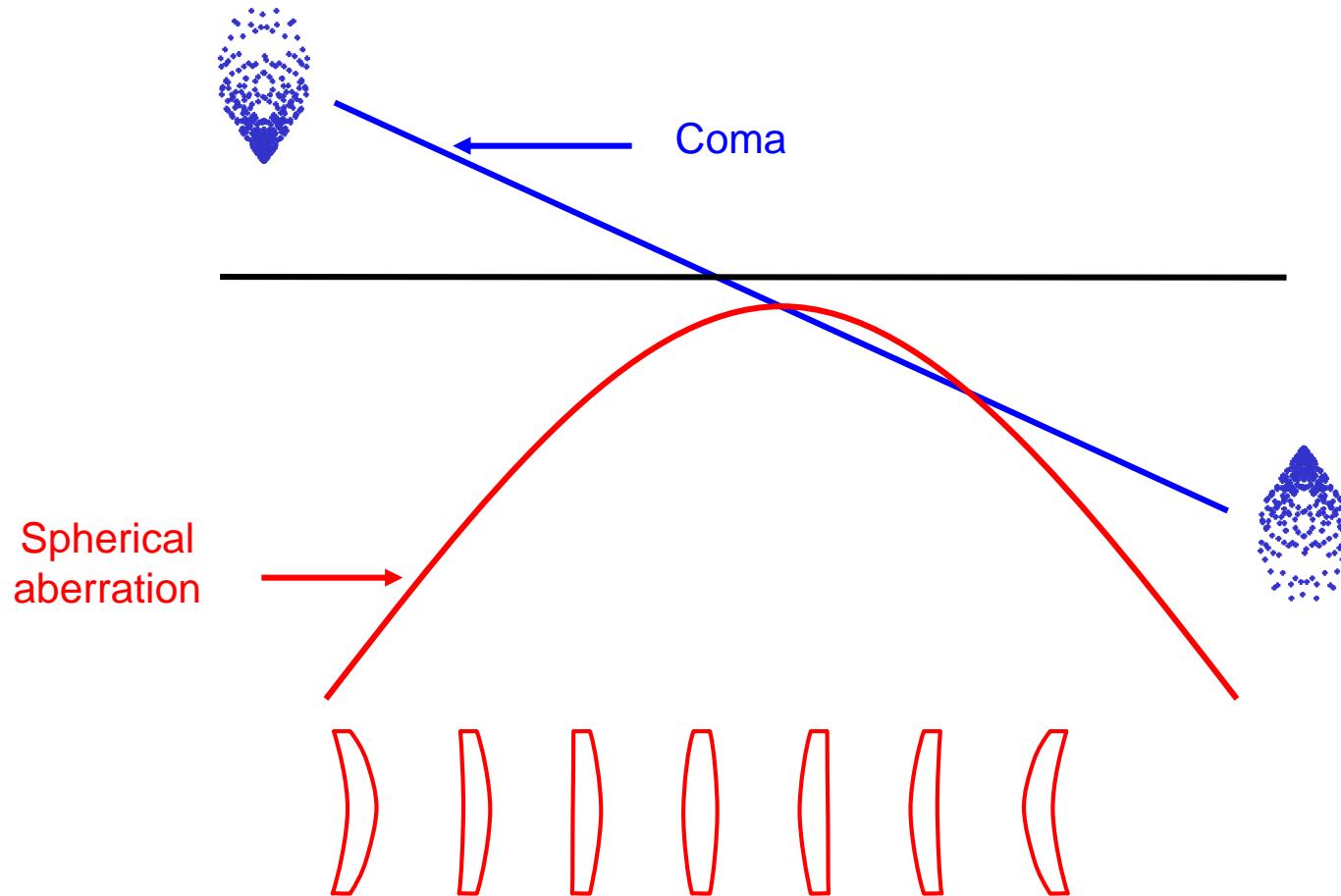


Spot size goes as the square of the EPD (or inverse square of the f/#)

Spot size is linearly dependent on field height

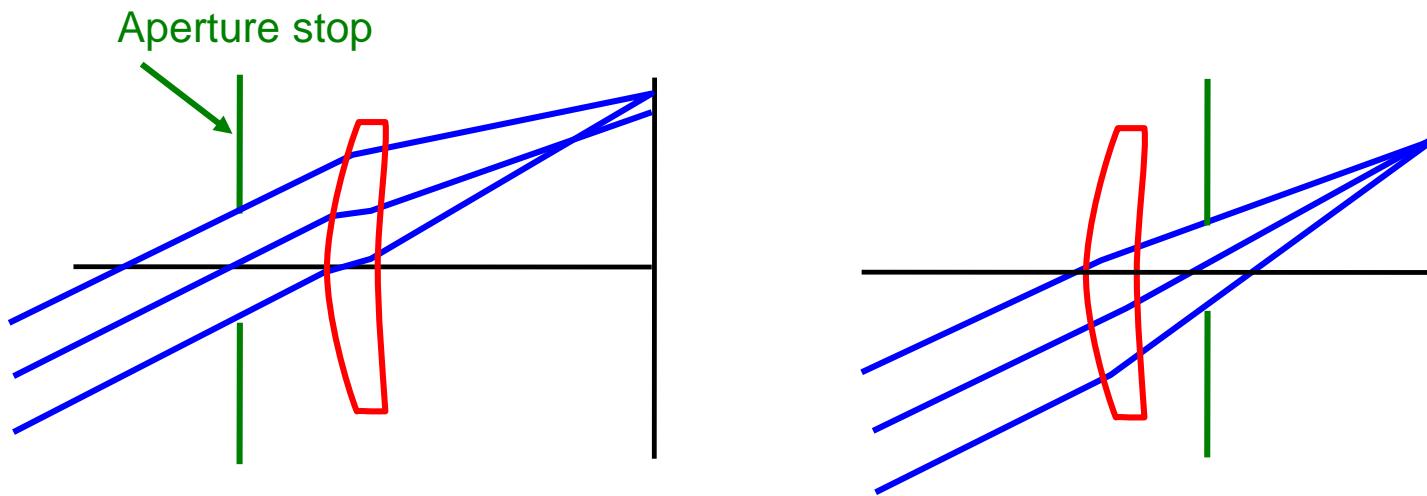
Coma vs. Lens Bending

- Both spherical aberration and coma are a function of the lens bending



Coma vs. Stop Position

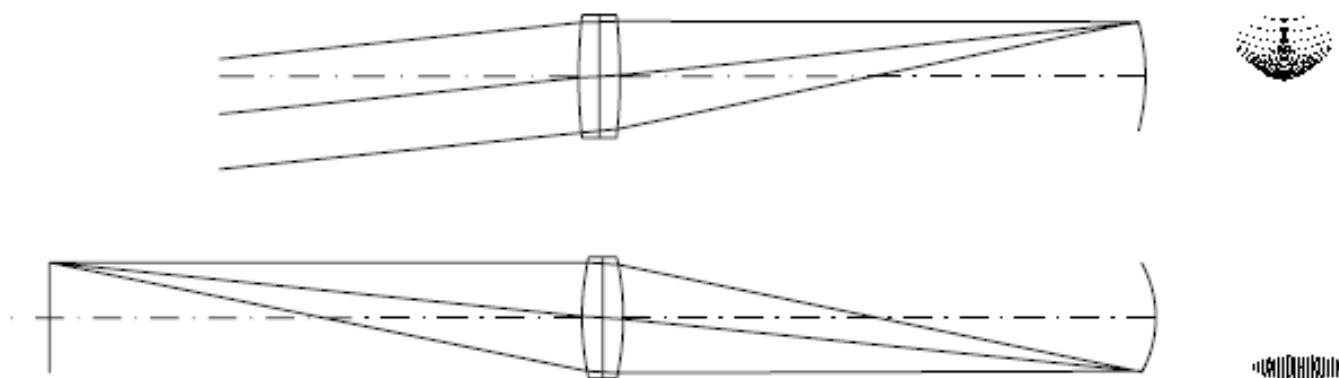
- The size of the coma is also a function of the stop location relative to the lens



Coma is reduced due to increased lens symmetry around the stop

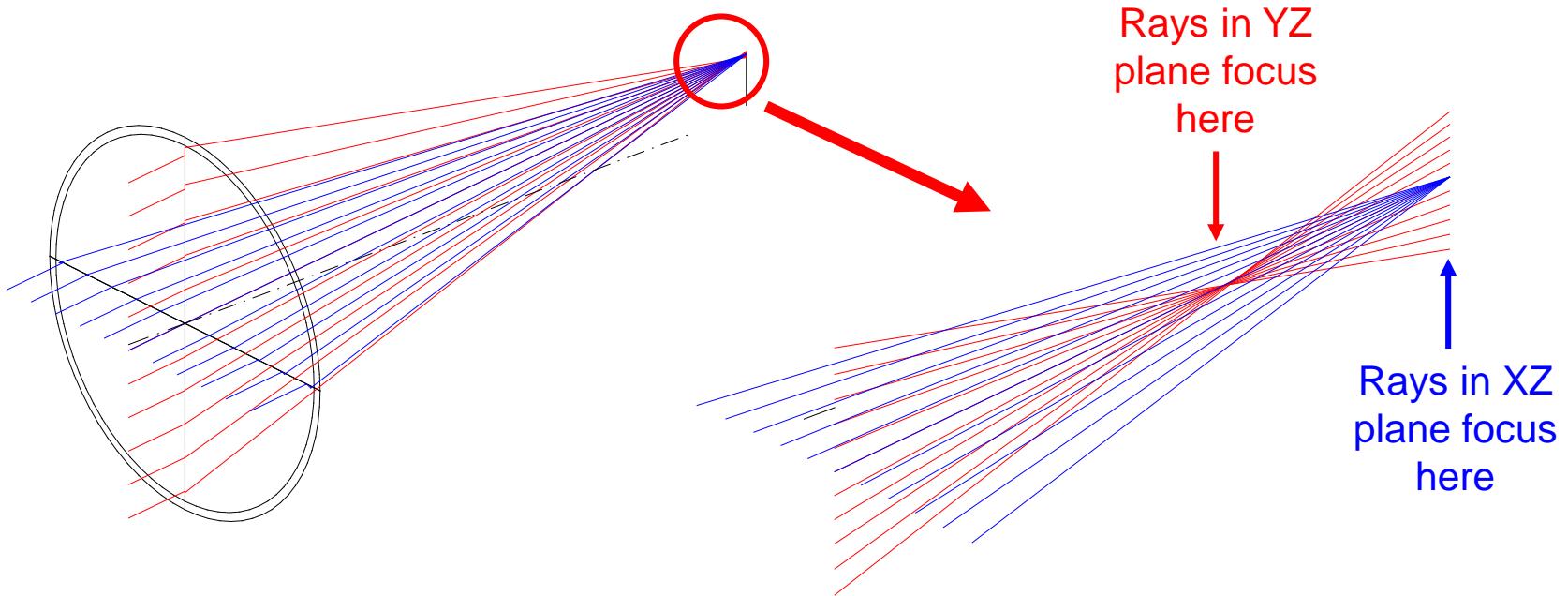
Coma is an Odd Aberration

- Any completely symmetric optical system (including the stop location) is free of all orders of odd field symmetry aberrations (coma and distortion)



Astigmatism

- Astigmatism is caused when the wavefront has a cylindrical component
 - The wavefront has different spherical power in one plane (e.g., tangential) vs. the other plane (e.g., sagittal)
- The result is different focal positions for tangential and sagittal rays



- The magnitude of the (third-order) astigmatism goes as the first power of the aperture and the square of the field angle

Cause of Astigmatism

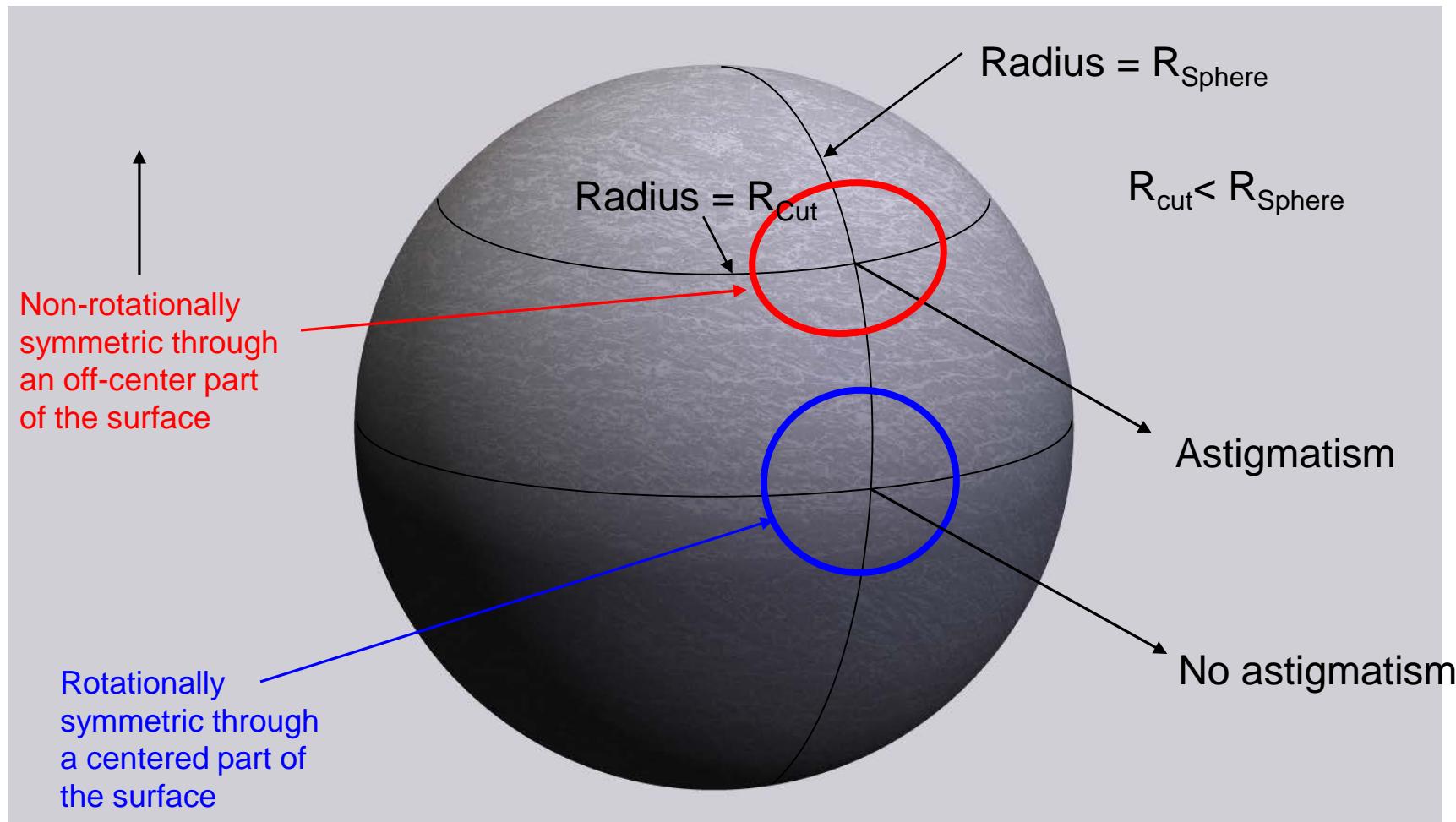
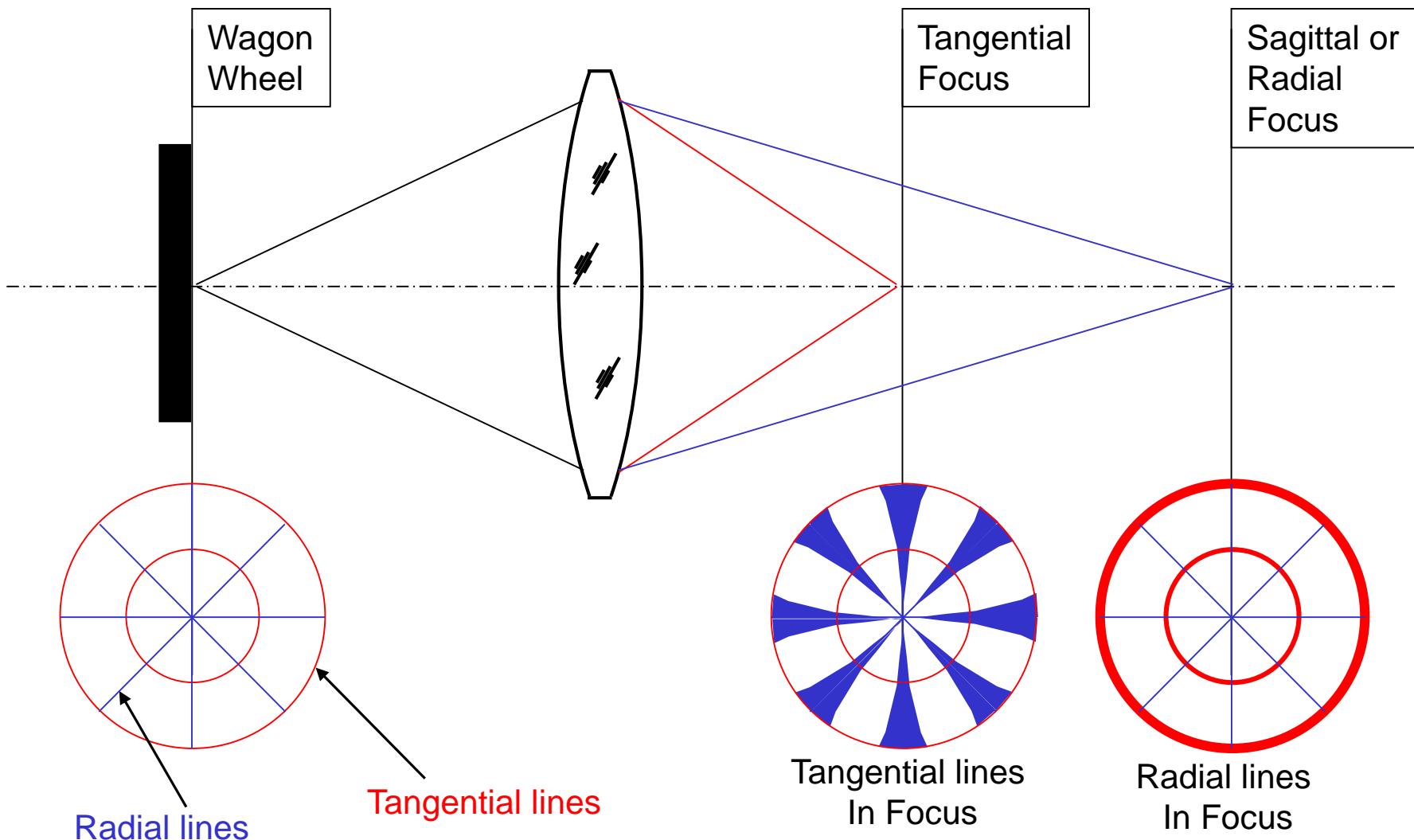
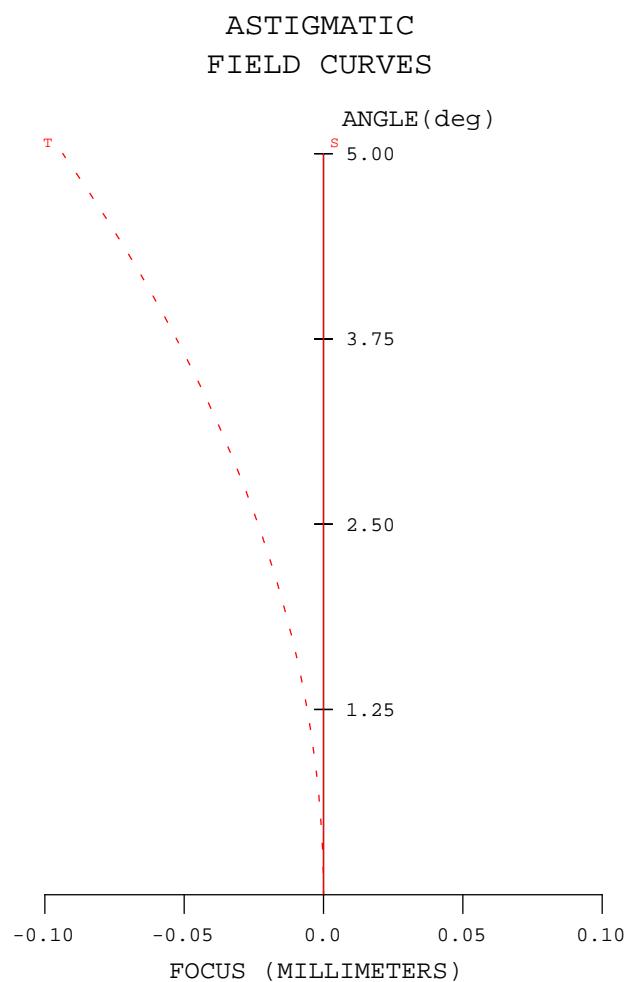
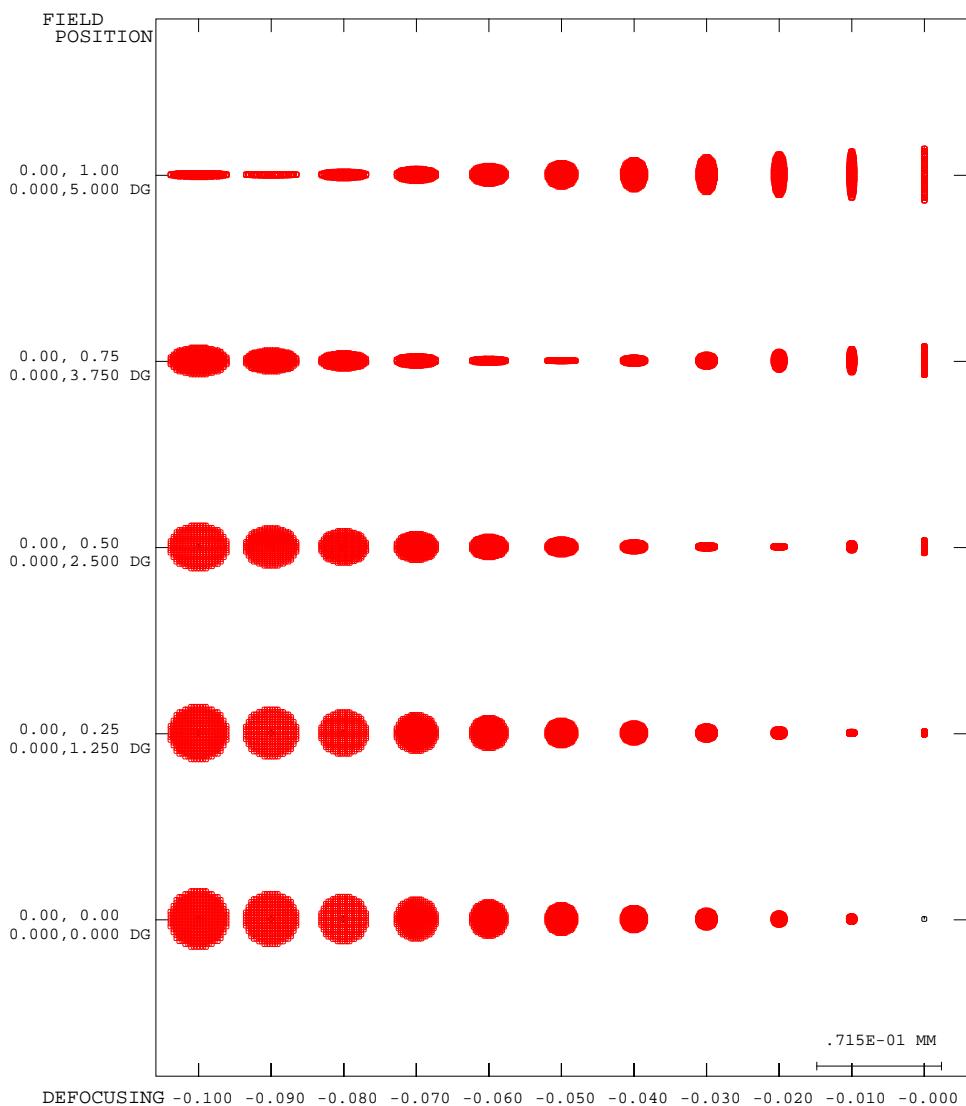


Image of a Wagon Wheel With Astigmatism

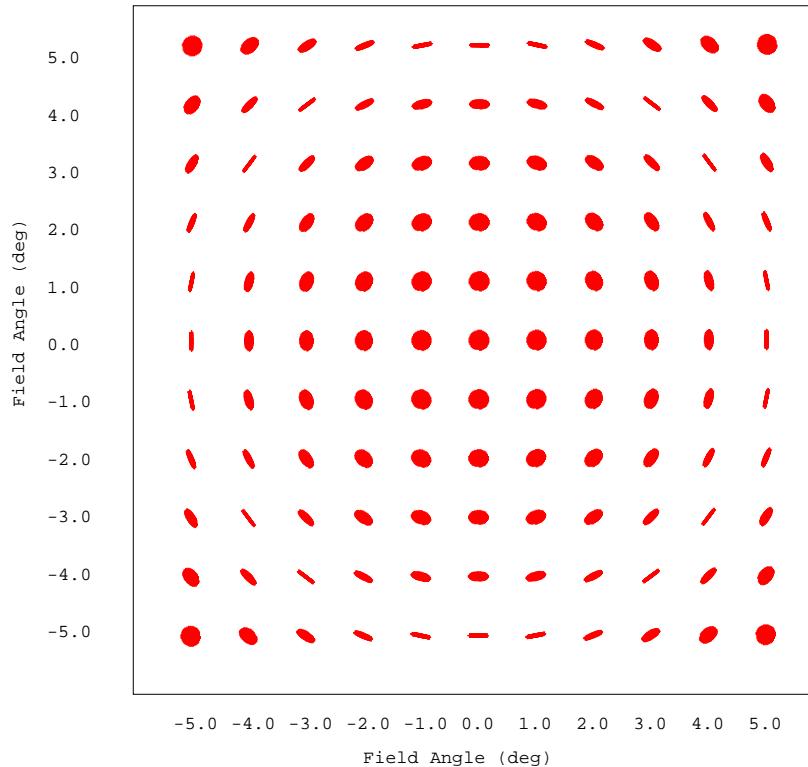


Astigmatism vs. Field



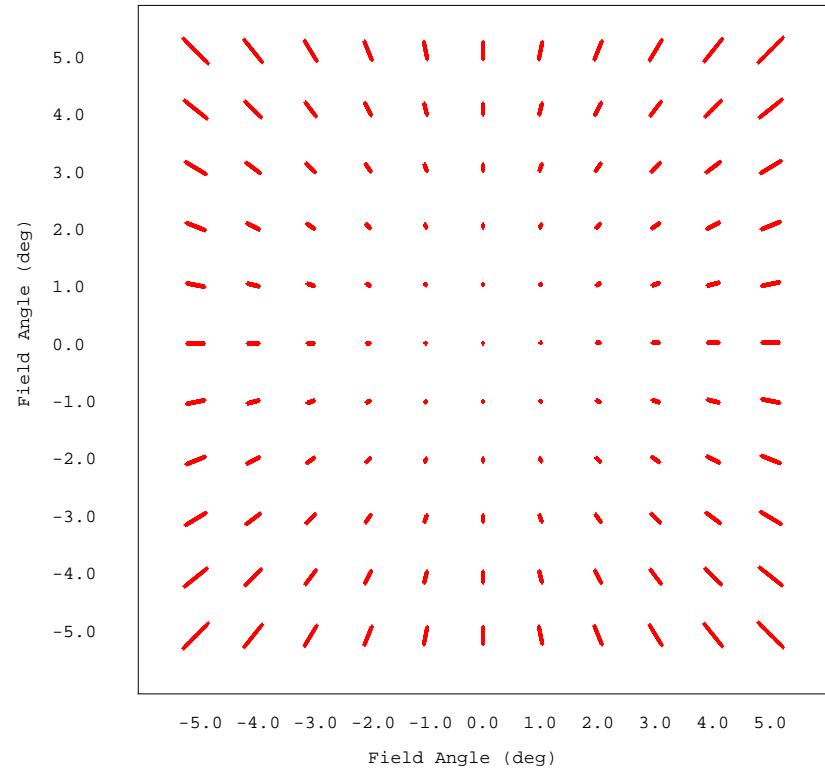
Scaling Laws for Astigmatism

$$\propto (f/\#)^{-1} \theta^2$$



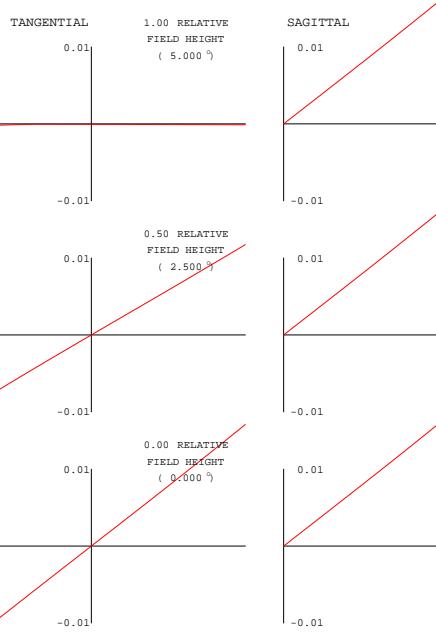
Tangential focus

$$\propto (f/\#)^{-1} \theta^2$$

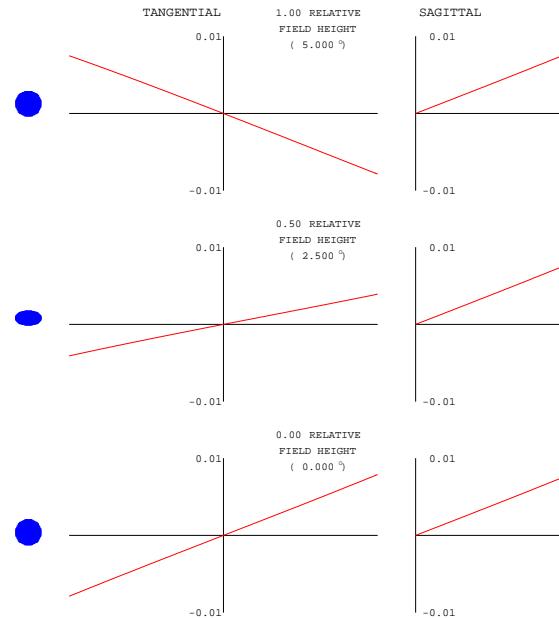


Sagittal focus

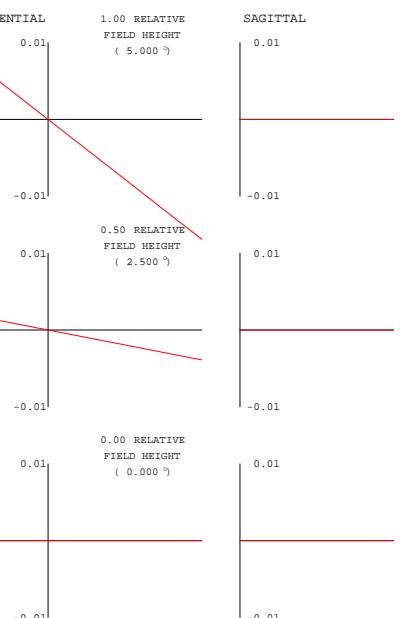
Astigmatism Ray Aberration Plots



Tangential focus



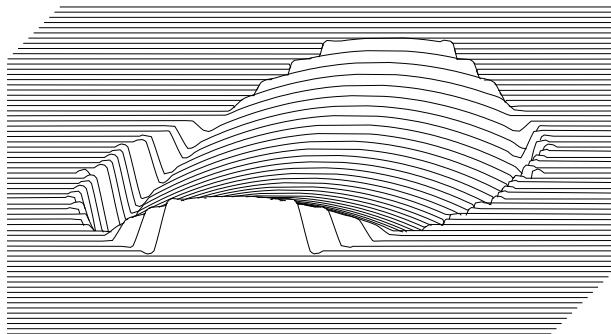
Medial focus
(best diffraction focus)
Occurs halfway
between sagittal and
tangential foci



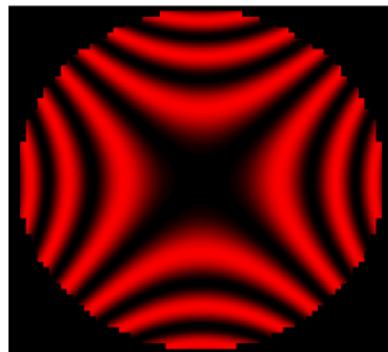
Sagittal focus

Note: the sagittal focus does
not always occur at the
paraxial focus

Transverse vs. Wavefront Astigmatism

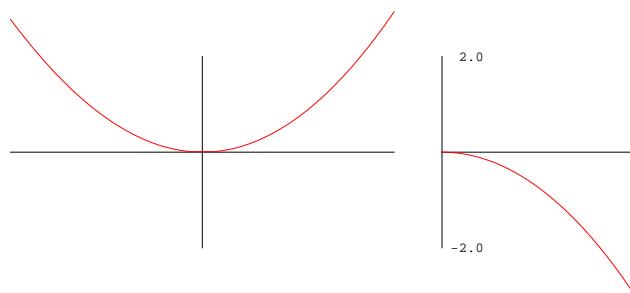
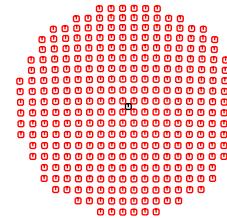


At medial focus

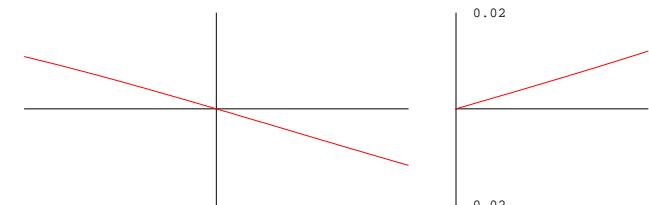


Wavefront map

Spot diagram

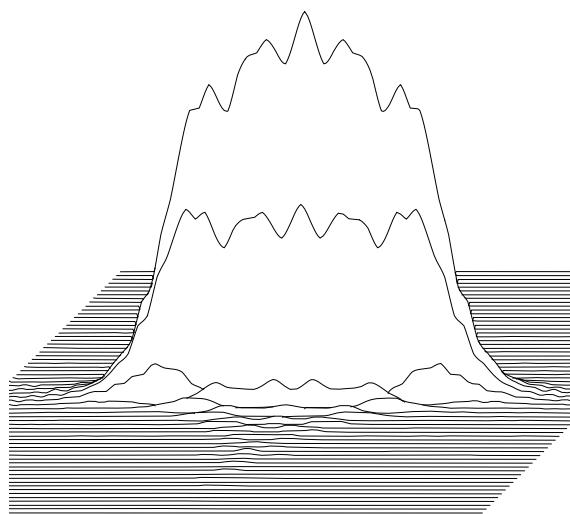


Wavefront error

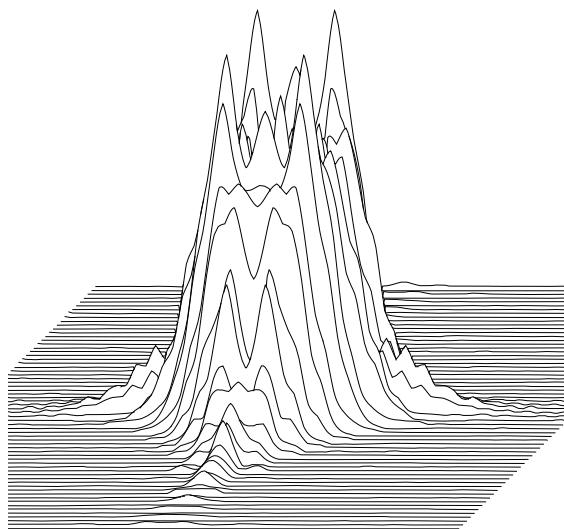


Transverse ray aberration

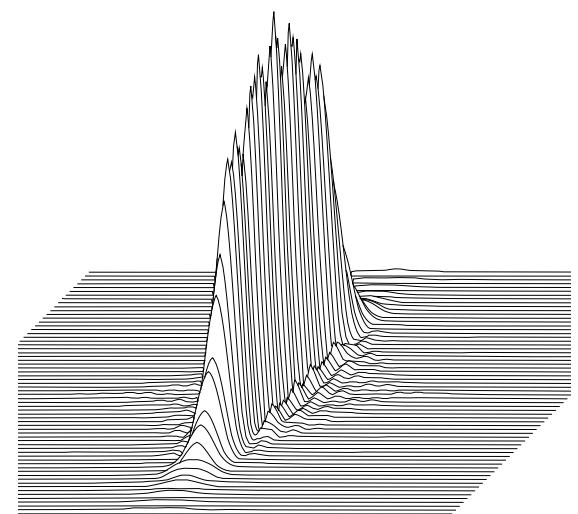
PSF of Astigmatism vs. Focus Position



Tangential focus



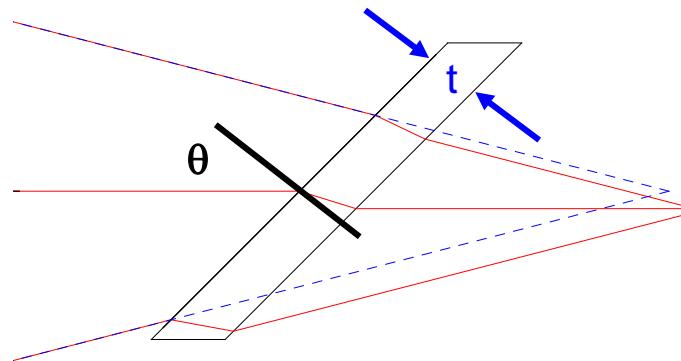
Medial focus
(best diffraction focus)



Sagittal focus

Astigmatism of a Tilted Flat Plate

- Placing a tilted plane parallel plate into a diverging or converging beam will introduce astigmatism



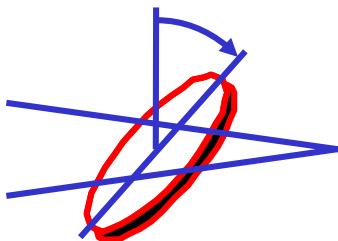
- The amount of the longitudinal astigmatism (focus shift between the tangential and sagittal foci) is given by

$$Ast = \frac{t}{\sqrt{n^2 - \sin^2\theta}} \left[\frac{n^2 \cos^2\theta}{n^2 - \sin^2\theta} - 1 \right] \quad \text{Exact}$$

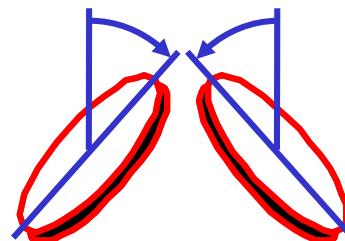
$$Ast = \frac{-t \theta^2 (n^2 - 1)}{n^3} \quad \text{Third-order}$$

Correcting the Astigmatism of a Tilted Flat Plate

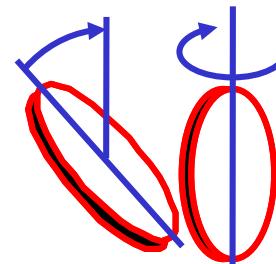
- The astigmatism introduced by a tilted flat plate can be corrected by
 - Adding cylindrical lenses
 - Adding tilted spherical lenses
 - Adding another plate tilted in the orthogonal plane



To correct
for this



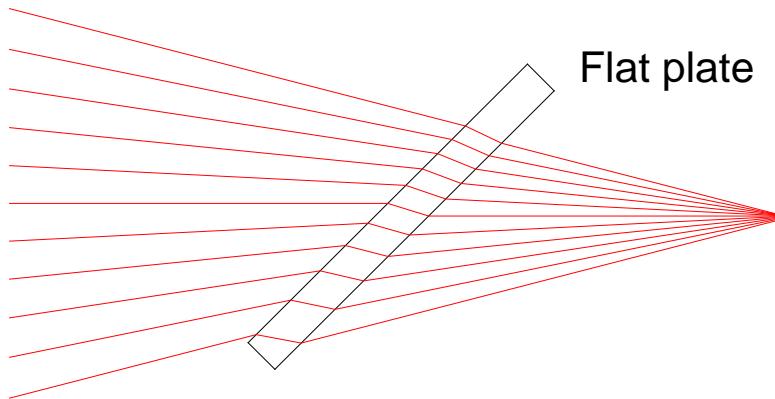
Do not do this
(it will double the
astigmatism)



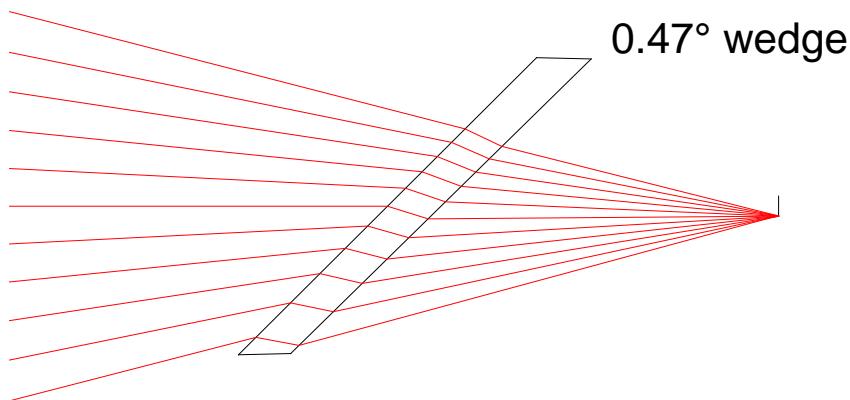
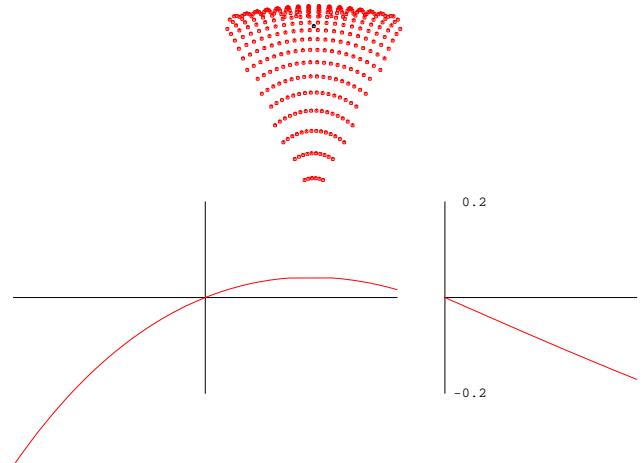
Do this

Reducing the Astigmatism of a Tilted Flat Plate

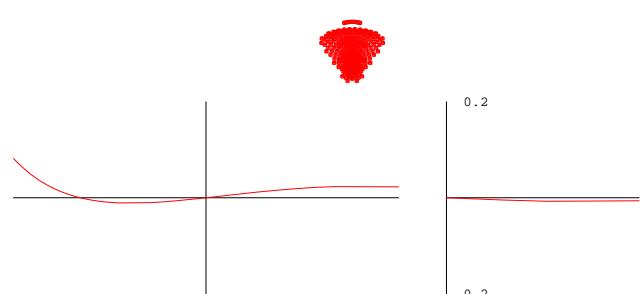
- Astigmatism of a flat plate can be reduced by adding a slight wedge to the plate



Flat plate



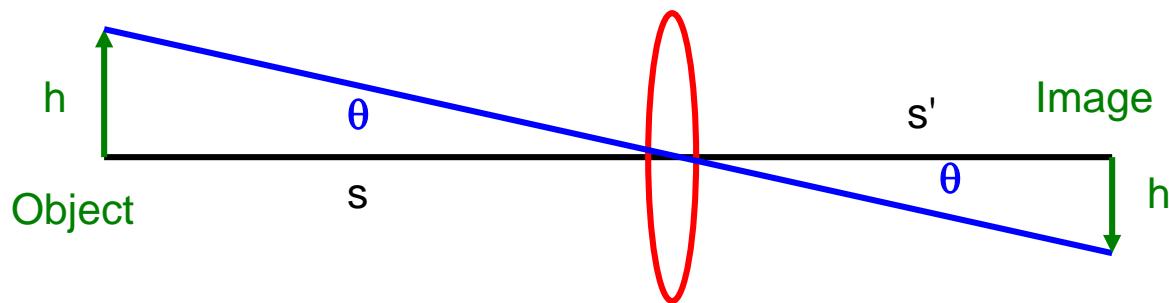
0.47° wedge



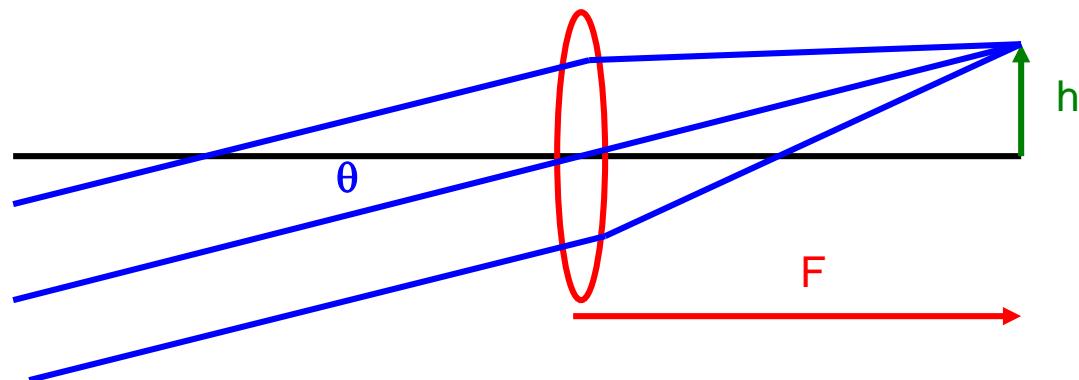
Transverse ray aberration

Rectilinear Imaging

- Most optical systems want to image rectilinear objects into rectilinear images

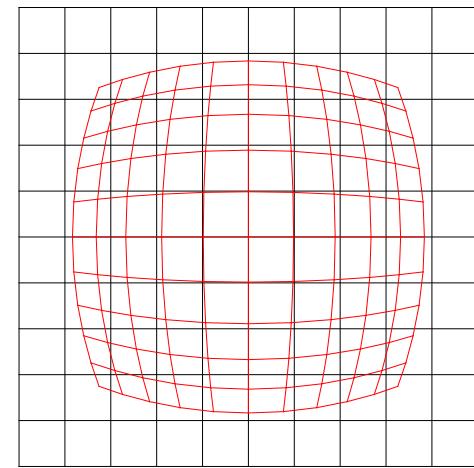
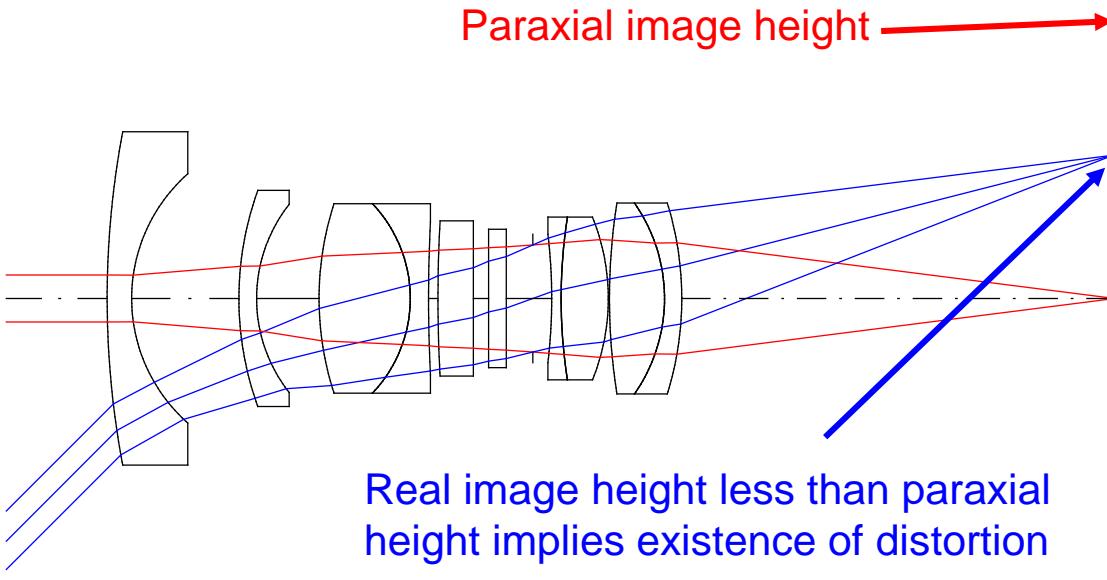


- This requires that $m = -s'/s = -h'/h = \text{constant}$ for the entire FOV
- For infinite conjugate lenses, this requires that $h' = F \tan\theta$ for all field angles



Distortion

- If rectilinear imaging is not met, then there is distortion in the lens
- Effectively, distortion is a change in magnification or focal length over the field of view

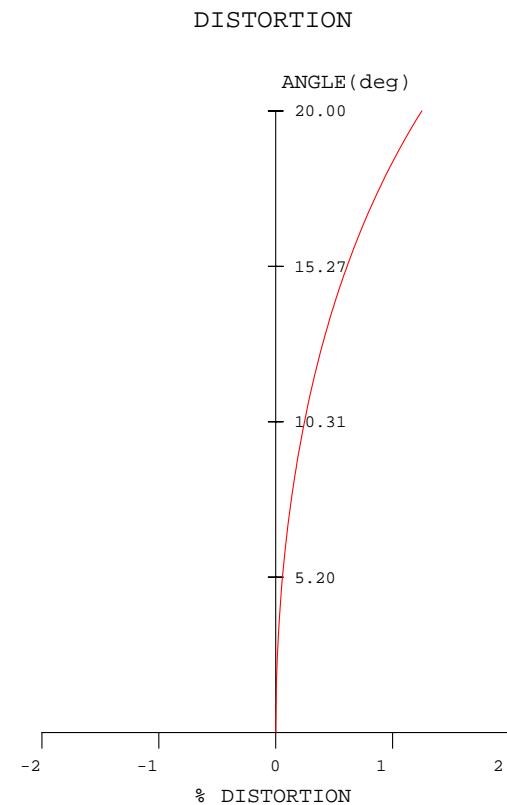


Plot of distorted FOV

- Negative distortion (shown) is often called barrel distortion
- Positive distortion (not shown) is often called pincushion distortion

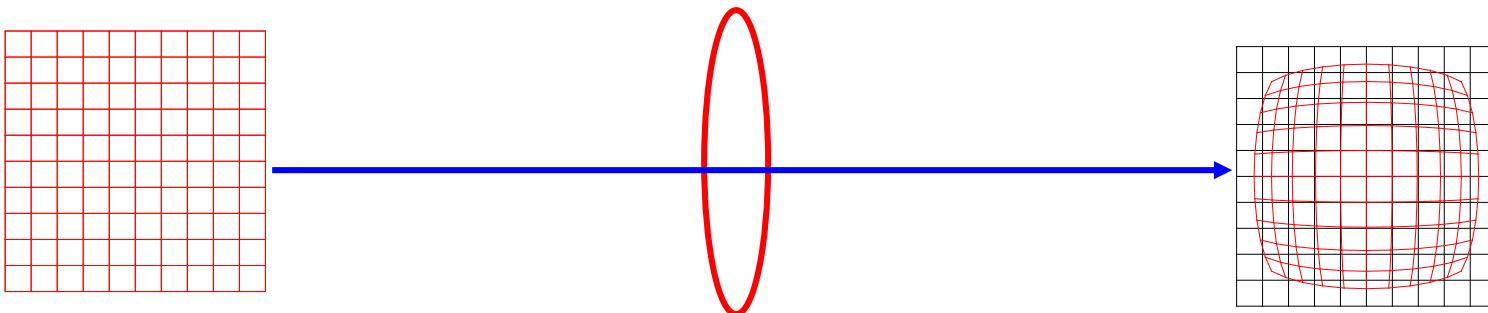
More on Distortion

- Distortion does not result in a blurred image and does not cause a reduction in any measure of image quality such as MTF
- Distortion is a measure of the displacement of the image from its corresponding paraxial reference point
- Distortion is independent of f/number
- Linear distortion is proportional to the cube of the field angle
- Percent distortion is proportional to the square of the field angle

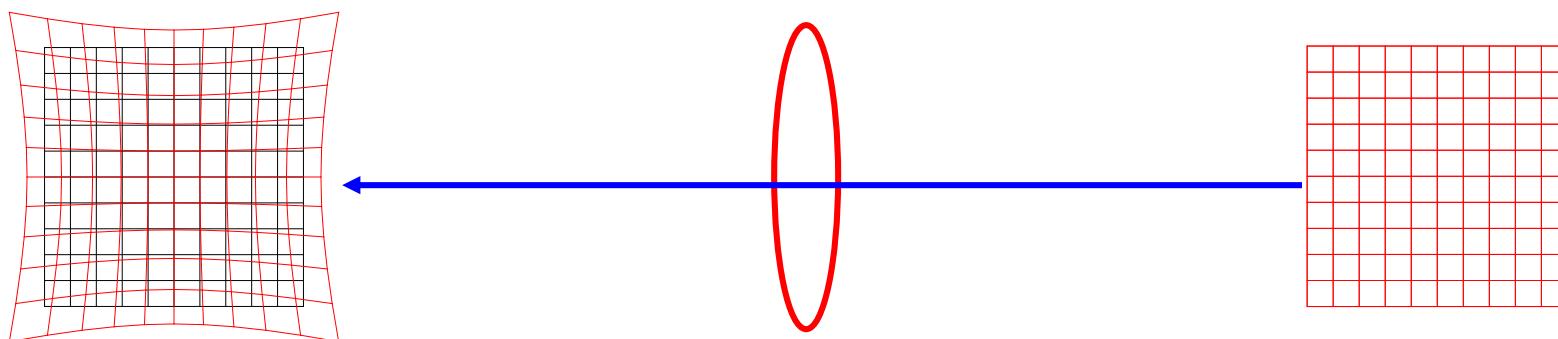


Implications of Distortion

- Consider negative distortion
 - A rectilinear object is imaged inside the detector

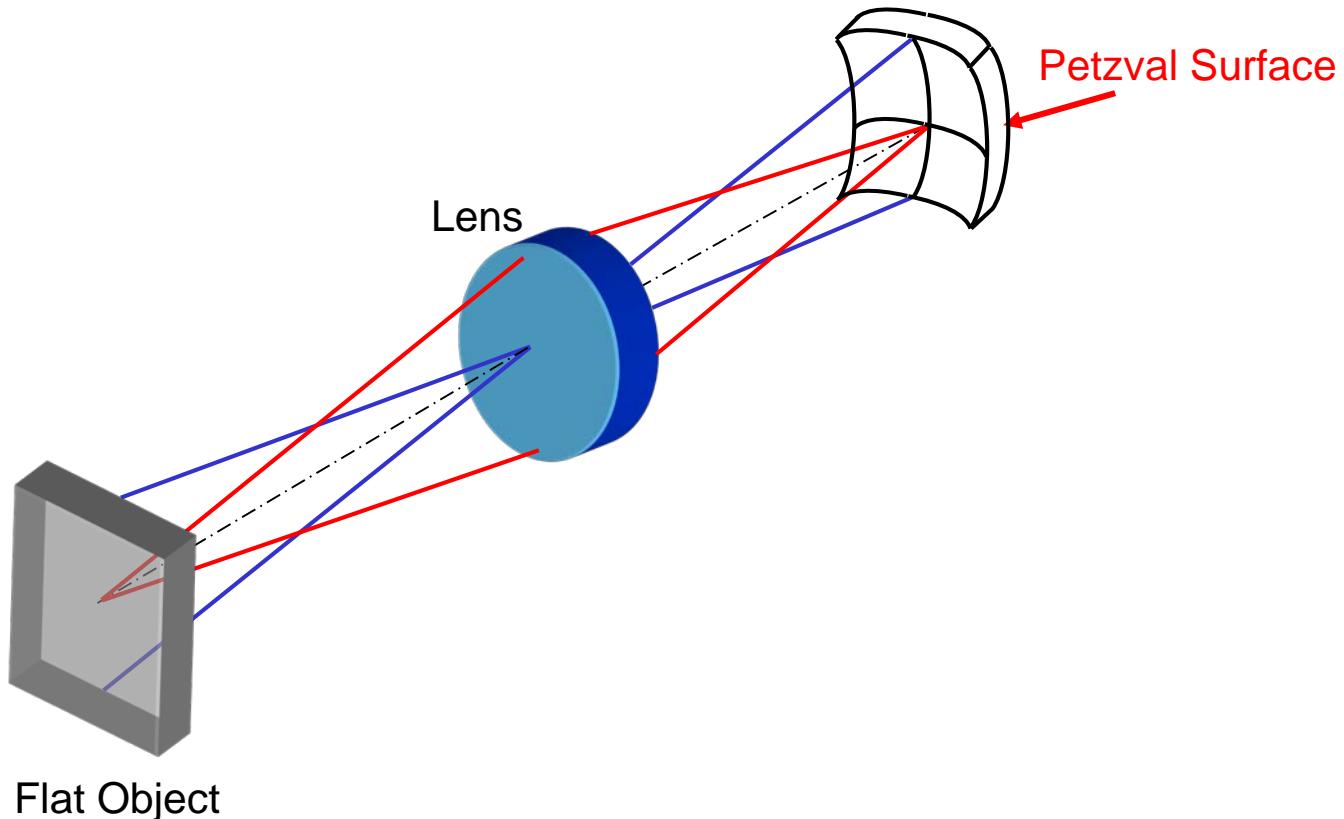


- This means a rectilinear detector sees a larger-than-rectilinear area in object space

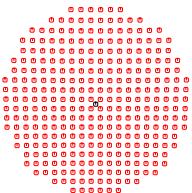
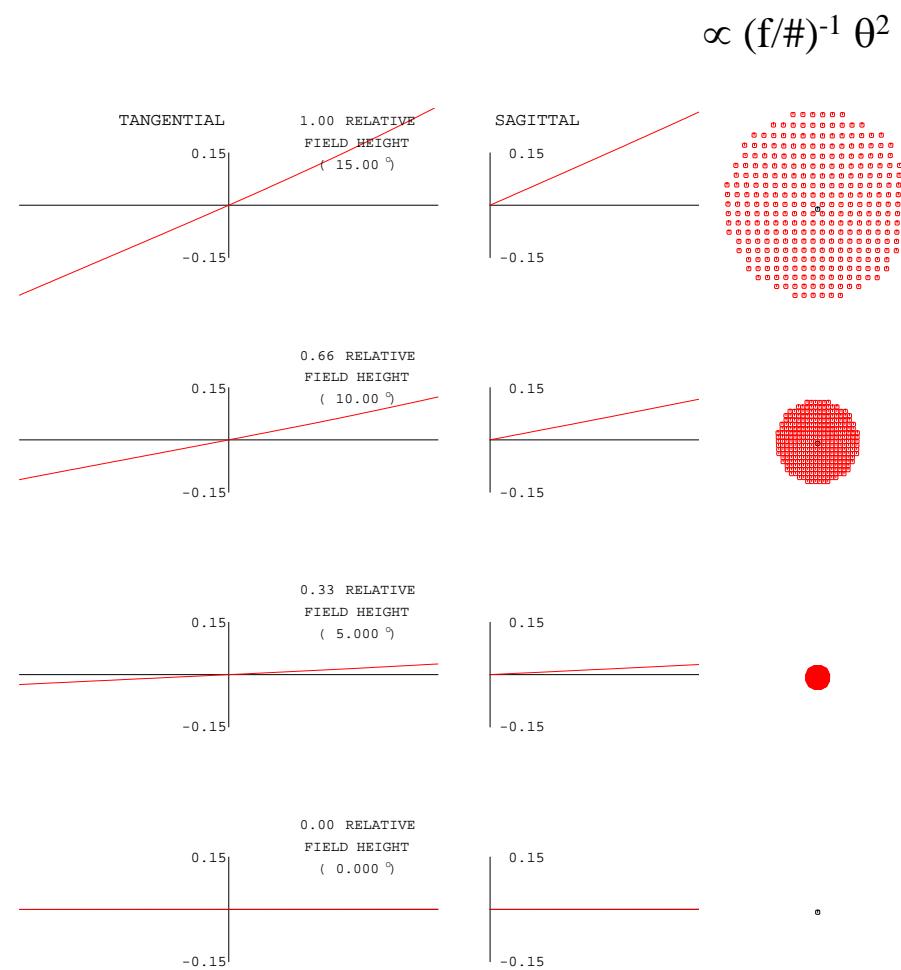
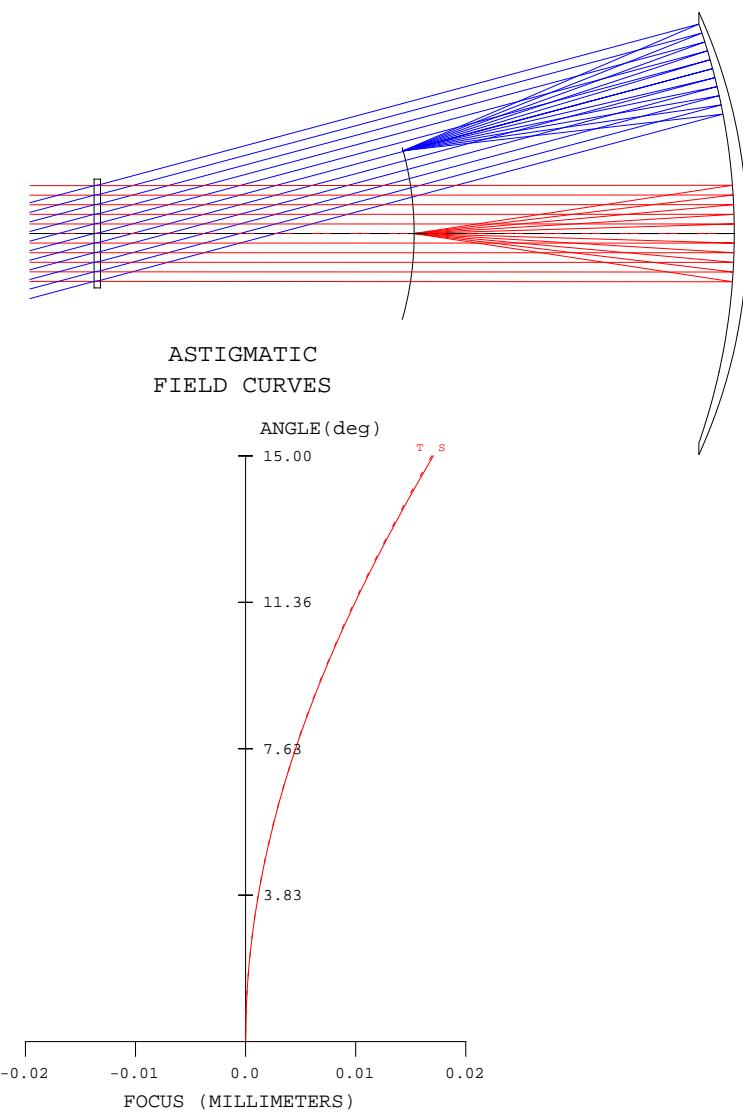


Curvature of Field

- In the absence of astigmatism, the focal surface is a curved surface called the Petzval surface



Third-order Field Curvature



Aberrations relative to a flat image surface

The Petzval Surface

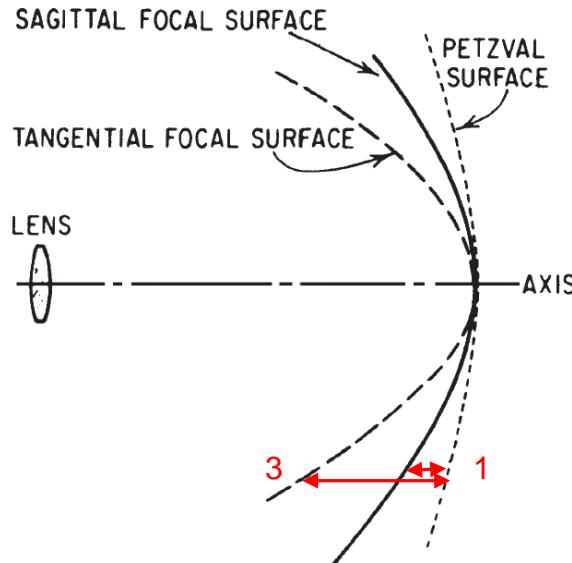
- The radius of the Petzval surface is given by

$$\frac{1}{R_{\text{Petzval}}} = \sum_i \left(\frac{1}{n_i F_i} \right)$$

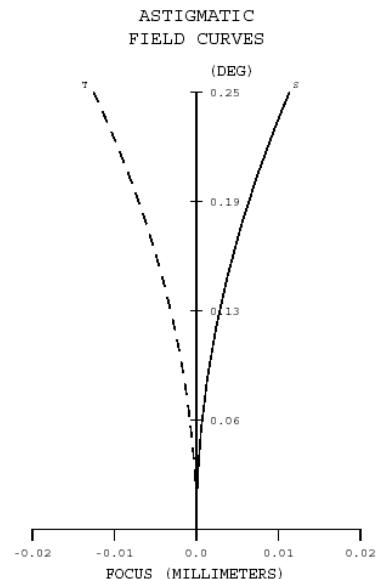
- For a singlet lens, the Petzval radius = $n F$
- Obviously, if we have only positive lenses in an optical system, the Petzval radius will become very short
 - We need some negative lenses in the system to help make the Petzval radius longer (i.e., flatten the field)
- This, and chromatic aberration correction, is why optical systems need some negative lenses in addition to all the positive lenses

Field Curvature and Astigmatism

- As an aberration, field curvature is not very interesting
- As a design obstacle, it is the basic reason that optical design is still a challenge
- The astigmatic contribution starts from the Petzval surface
 - If the axial distance from the Petzval surface to the sagittal surface is 1 (arbitrary units), then the distance from the Petzval surface to the tangential surface is 3



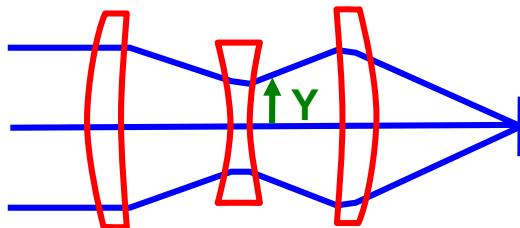
Field curvature and astigmatism can be used together to help flatten the image plane and improve the image quality



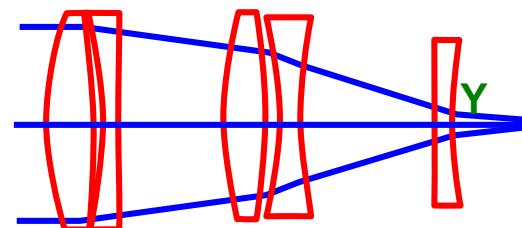
Flat Medial Field

Flattening the Field

- The contribution of a lens to the focal length is proportional to yF where F is lens power ($1/F$)
- The contribution of a lens to the Petzval sum is proportional to F/n
- Thus, if we include negative lenses in the system where y is small we can reduce the Petzval sum and flatten the field while holding the focal length

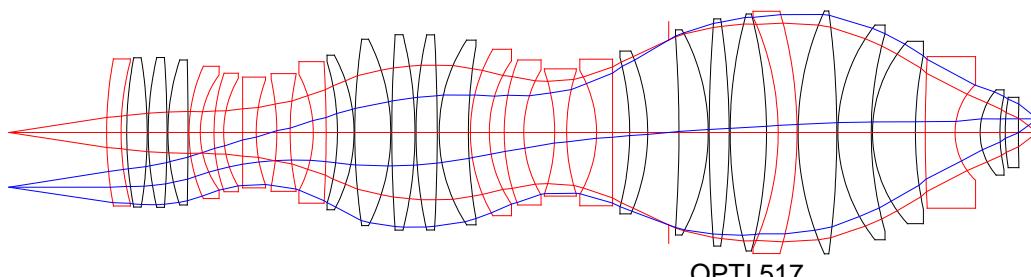


Cooke Triplet



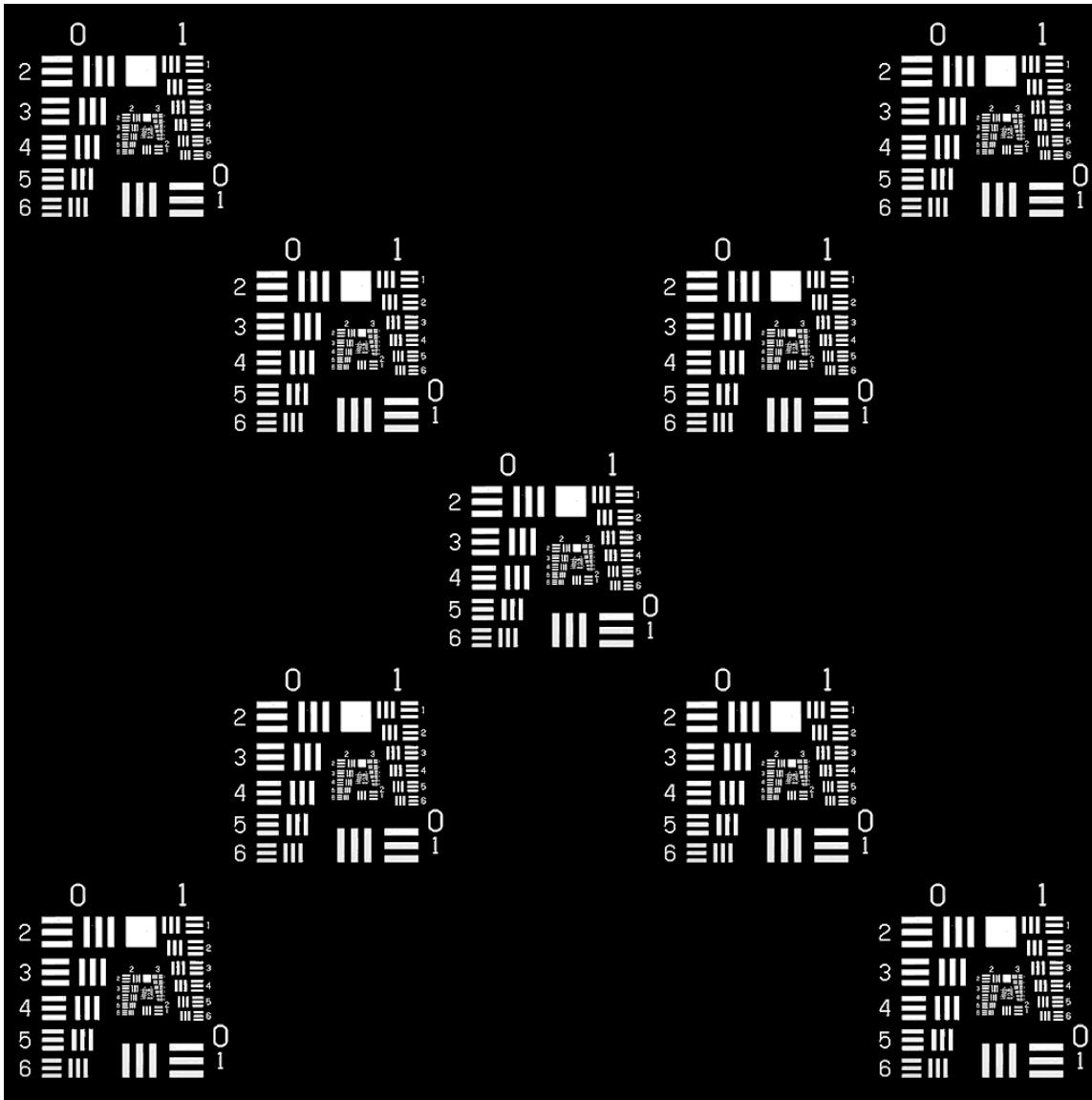
Lens With Field Flattener
(Petzval Lens)

- Yet another reason why optical systems have so darn many lenses



Flat-field lithographic lens
Negative lenses in RED

Original Object



Spherical Aberration

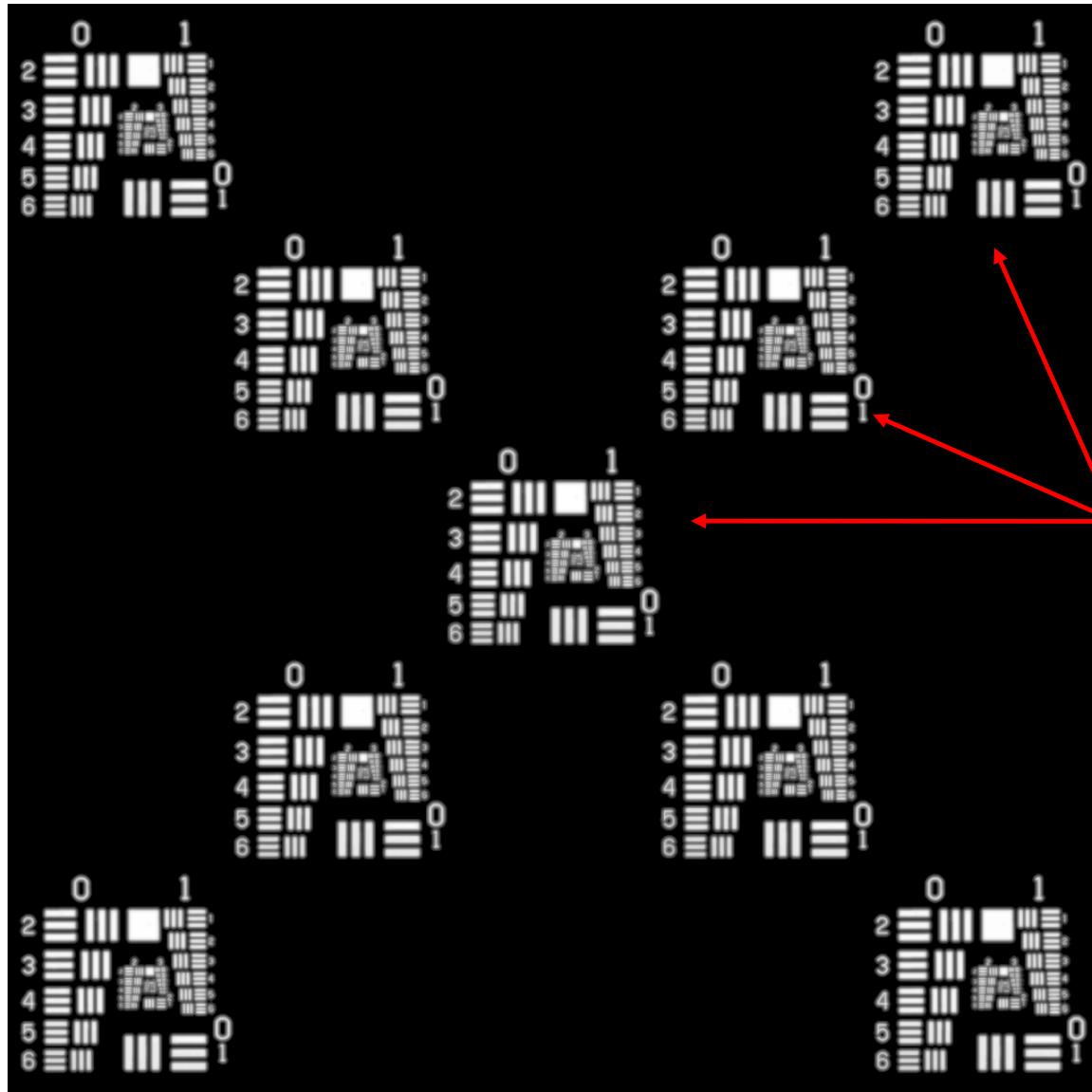


Image blur is
constant over
the field

Coma

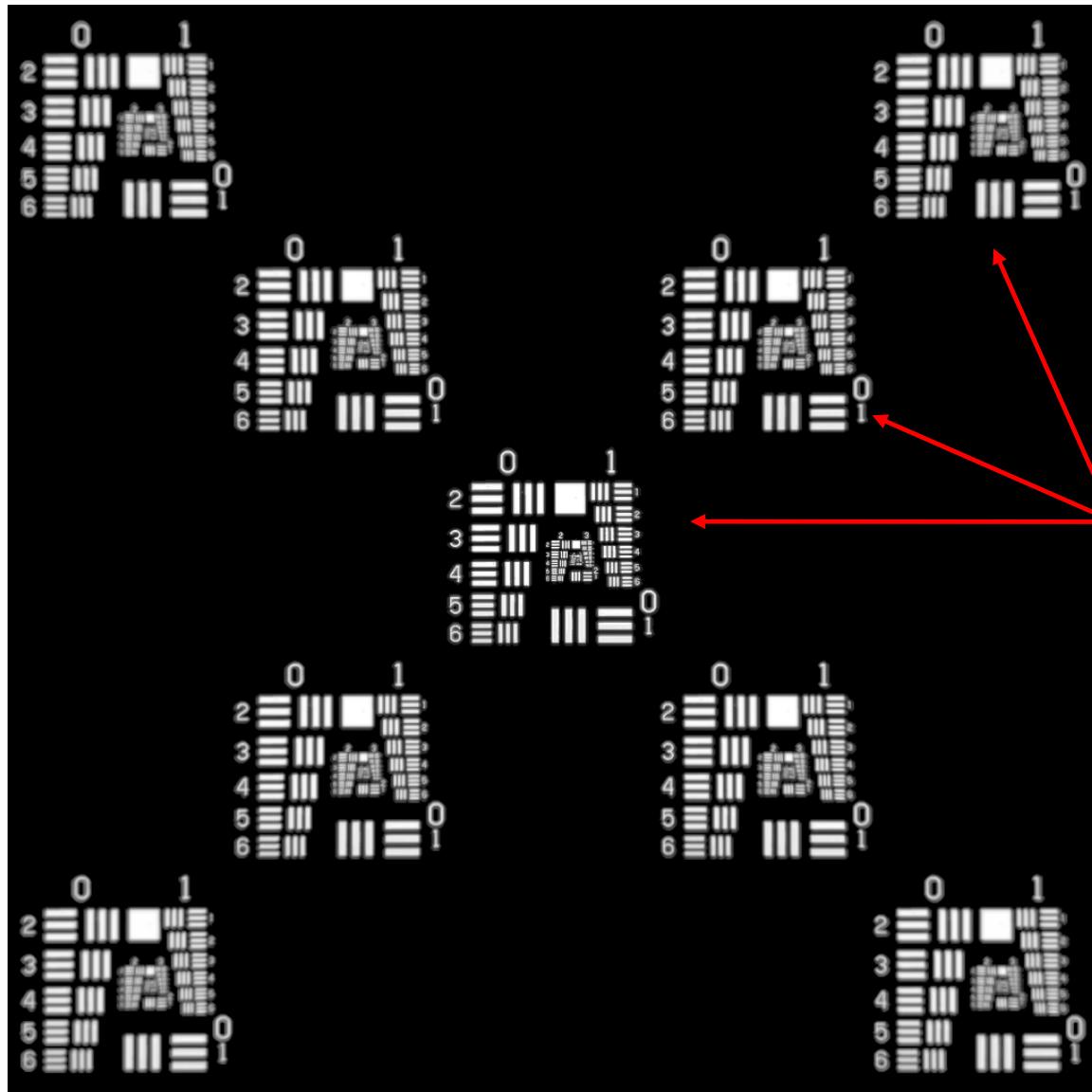


Image blur
grows linearly
over the field

Astigmatism

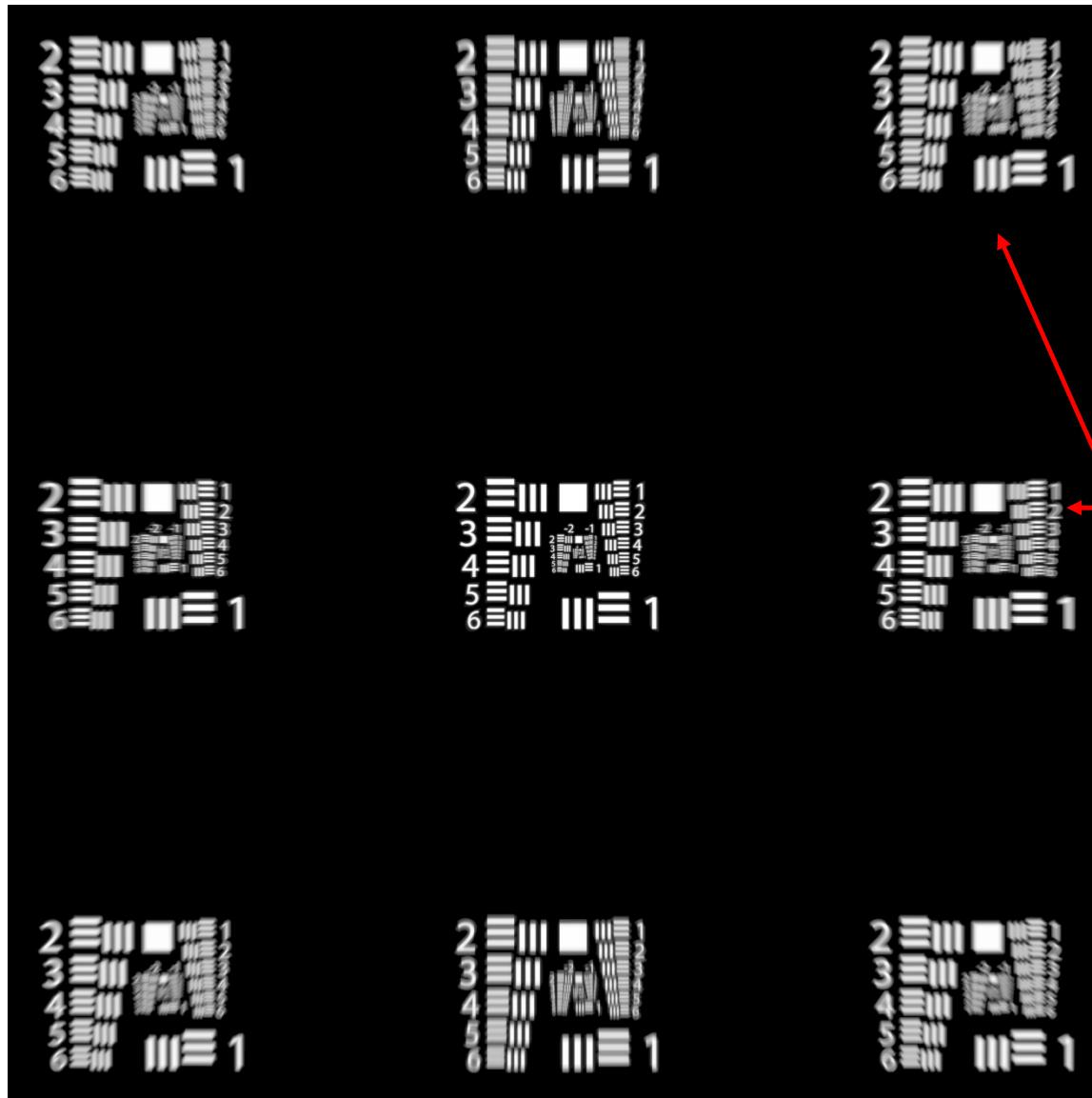
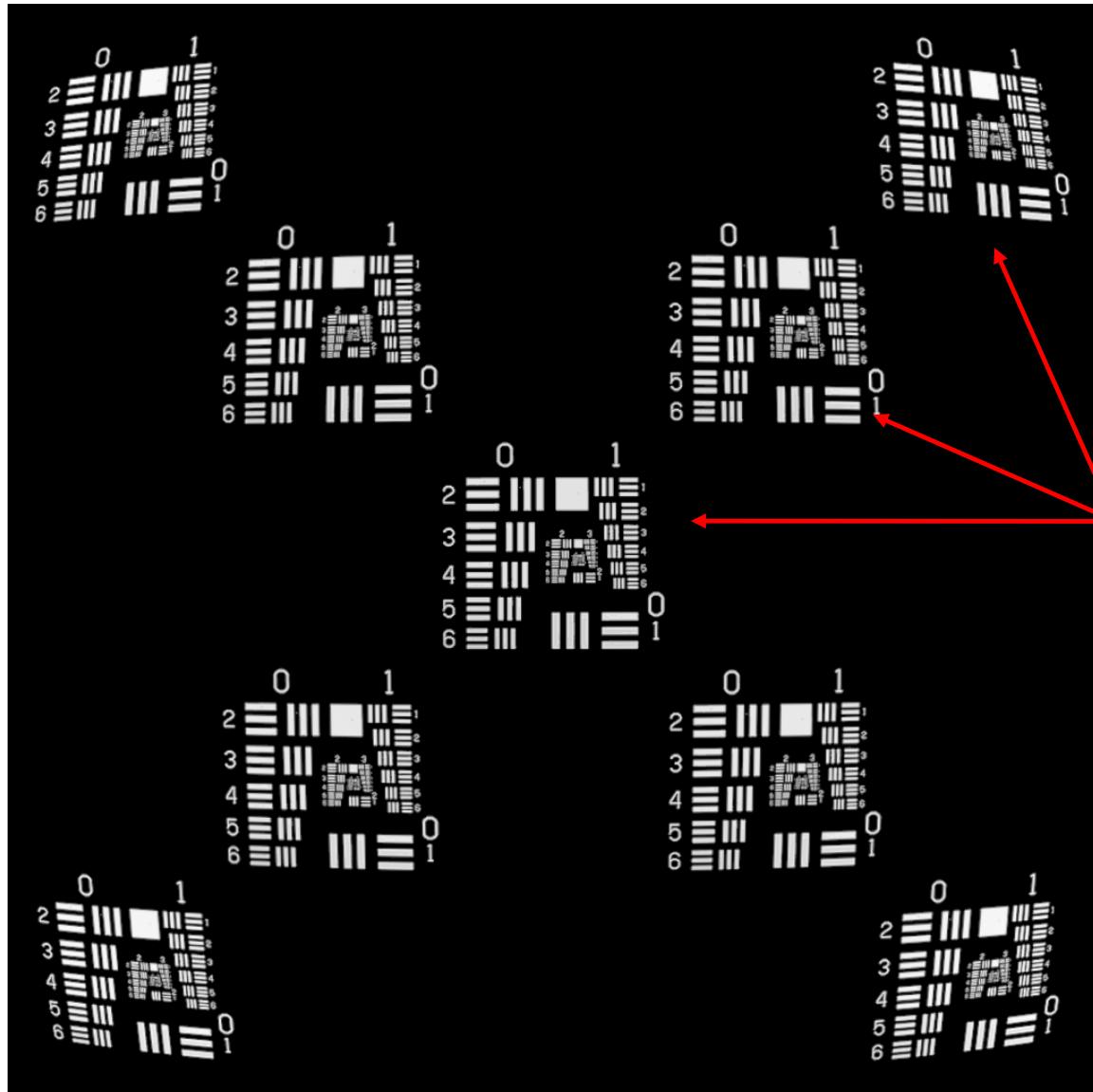


Image blurs more in one direction over the field

Distortion



No image degradation
but image locations are shifted

Curvature of Field

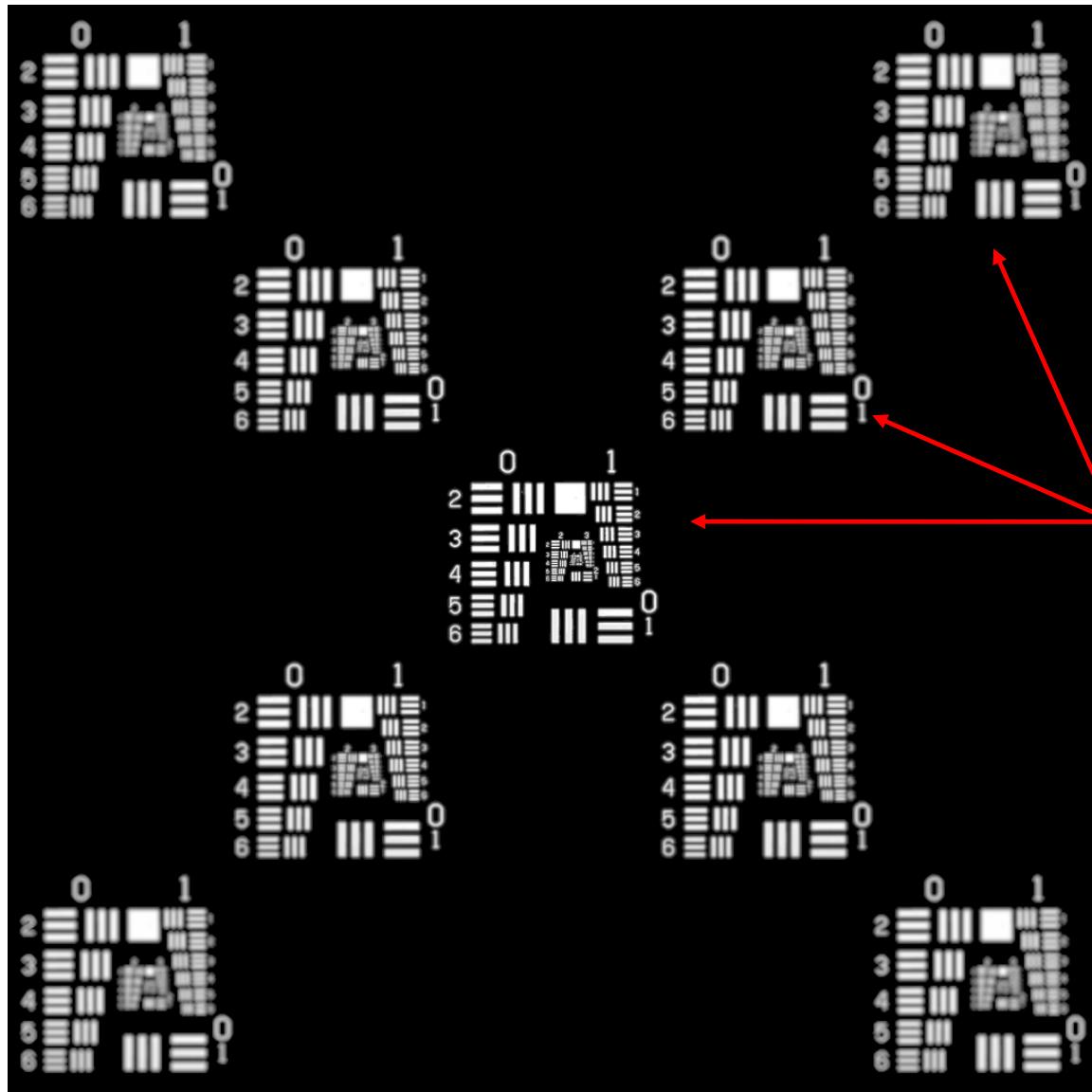
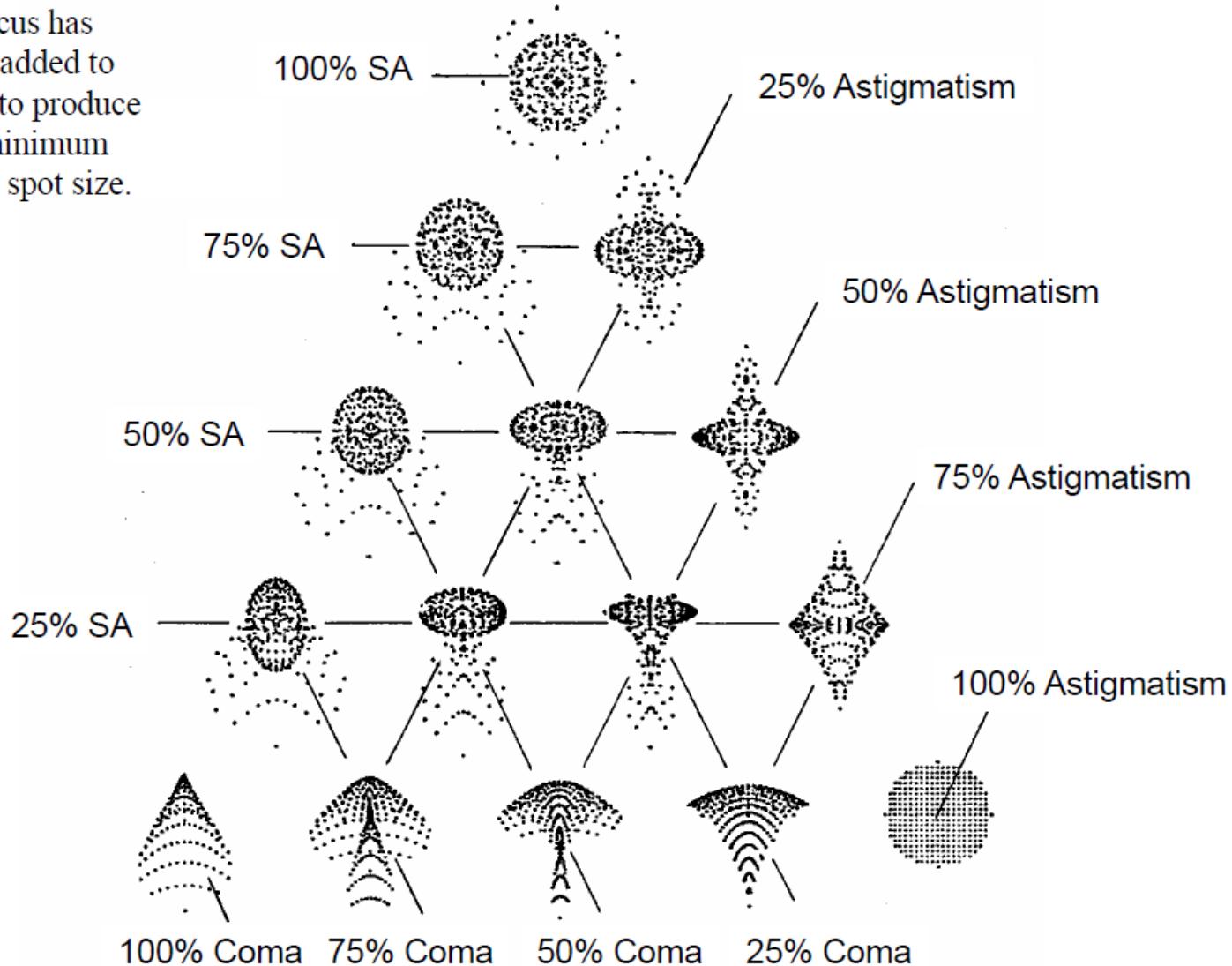


Image blur
grows
quadratically
over the field

Combined Aberrations – Spot Diagrams

Defocus has been added to each to produce the minimum RMS spot size.

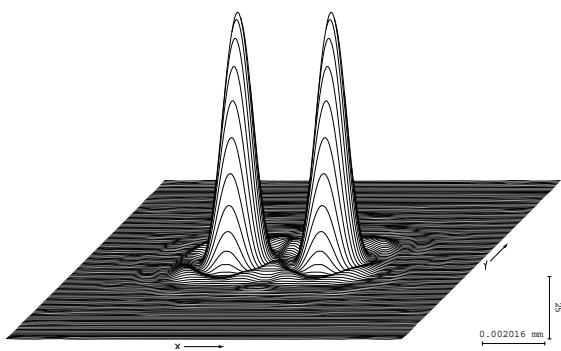


Balancing of Aberrations

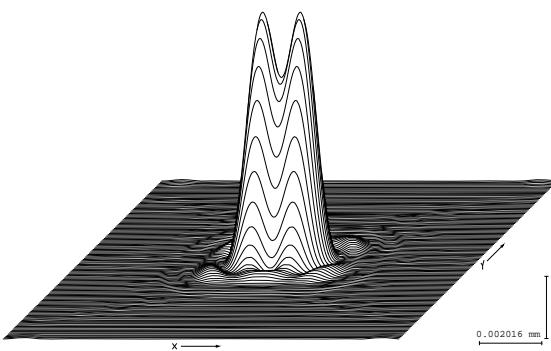
- Different aberrations can be combined to improve the overall image quality
 - Spherical aberration and defocus
 - Astigmatism and field curvature
 - Third-order and fifth-order spherical aberration
 - Longitudinal color and spherochromatism
 - Etc.
- Lens design is the art (or science) of putting together a system so that the resulting image quality is acceptable over the field of view and range of wavelengths

Resolution

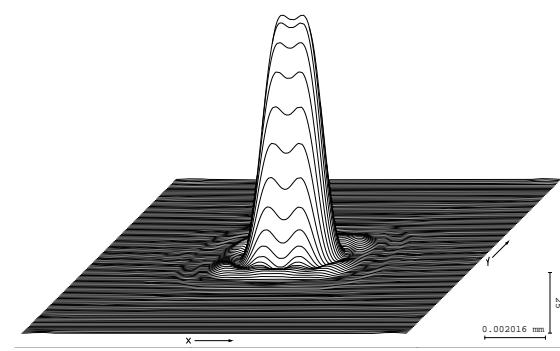
- Resolution is an important aspect of image quality
- Every image has some resolution associated with it, even if it is the Airy disk
 - In this case, the resolution is dependent on the aberrations of the system
- Resolution is the smallest detail you can resolve in the image
 - It determines whether you can resolve two closely spaced objects



Well resolved



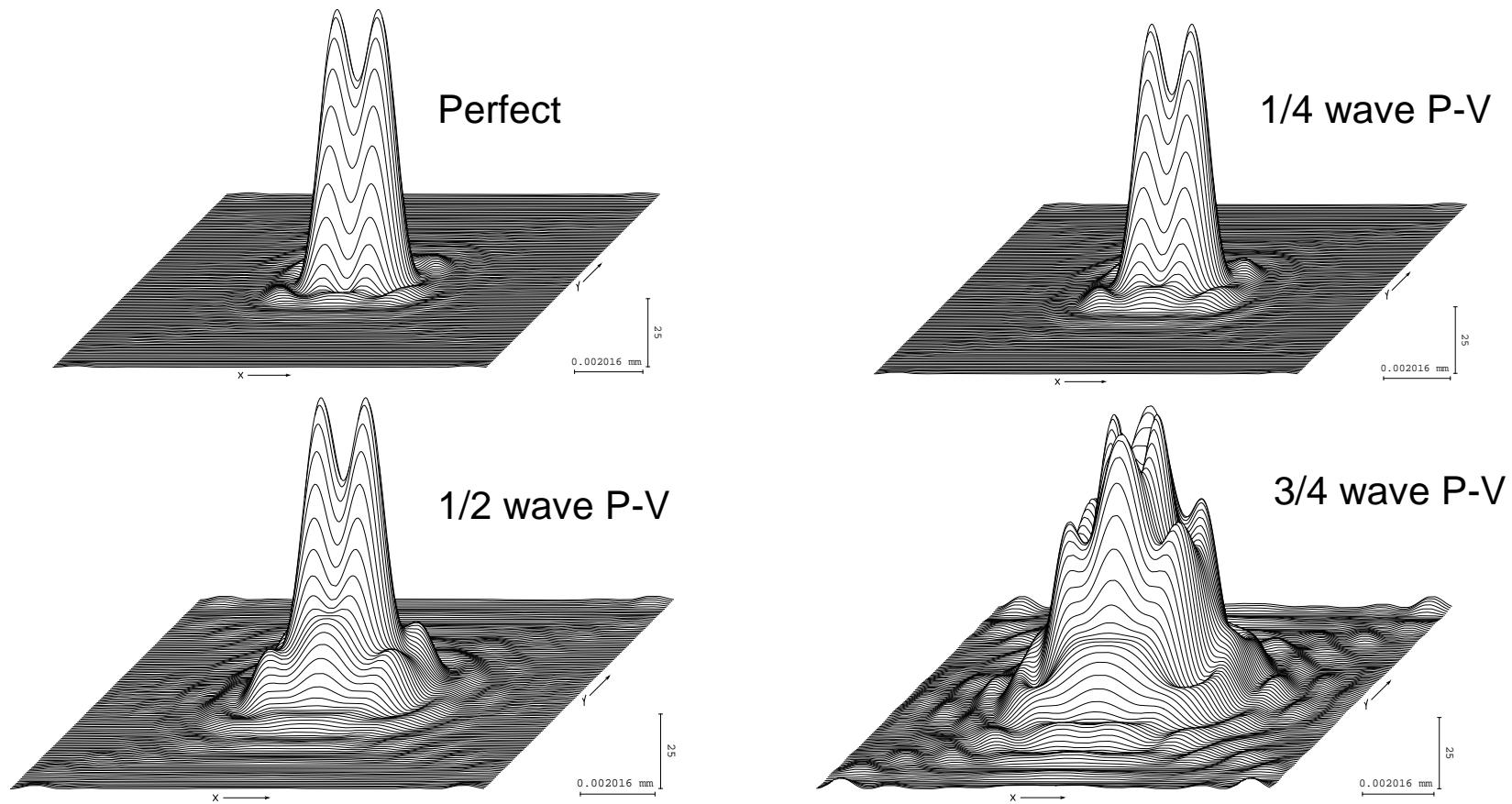
Rayleigh criterion
peak of 2nd at
1st zero of first



Sparrow criterion
overlap at FWHM

Resolution vs. P-V Wavefront Error

- The 1/4 wave rule was empirically developed by astronomers as the greatest amount of P-V wavefront error that a telescope could have and still resolve two stars separated by the Rayleigh spacing (peak of one at 1st zero of the other)



Resolution Examples

- Angular resolution is given by $\beta \approx 2.44 \lambda/D$
 - Limited only by the diameter, not by the focal length or f/number
- U of A is building 8.4 meter diameter primary mirrors for astronomical telescopes
- For visible light ($\sim 0.5 \mu\text{m}$), the Rayleigh spacing corresponds to an angular separation of $(2.44 * 0.5 \times 10^{-6} / 8.4)/2 = 0.073 \times 10^{-6}$ radian (~ 0.015 arc second)
- Assume a binary star at a distance of 200 light years ($\sim 1.2 \times 10^{15}$ miles)
 - This would have a resolution of 90 million miles
 - Perhaps enough resolution to "split the binary"
- A typical cell phone camera has an aperture of about 0.070 inch
 - This gives a Rayleigh spacing of about 0.34 mrad (for reference, the human eye has an angular resolution of about 0.3 mrad)
 - For an object 10 feet away, this is an object resolution of about 1 mm

Film Resolution

- Due to the grain size of film, there is an MTF associated with films

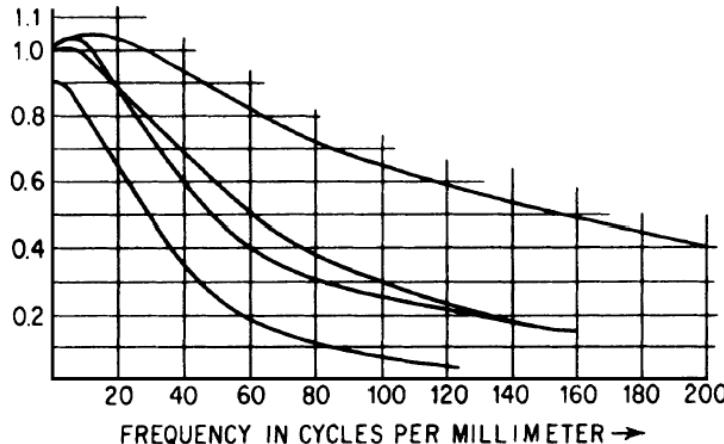
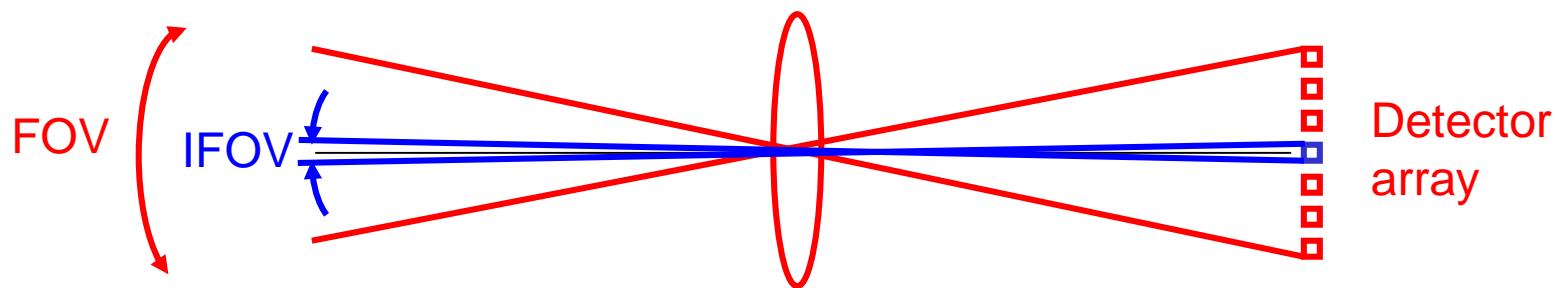


Figure 11.22 Modulation transfer functions of several photographic emulsions.

- A reasonable guide for MTF of a camera lens is the 30-50 rule: 50% at 30 lp/mm and 30% at 50 lp/mm
- For excellent performance of a camera lens, use 50% at 50 lp/mm
- Another criterion for 35 mm camera lenses is 20% at 30 lp/mm over 90% of the field (at full aperture)
- As a rough guide for the resolution required in a negative, use 200 lines divided by the square root of the long dimension in mm

Detectors

- All optical systems have some sort of detector
 - The most common is the human eye
 - Many optical systems use a 2D detector array (e.g., CCD)
- No matter what the detector is, there is always some small element of the detector which defines the detector resolution
 - This is referred to as a **picture element (pixel)**
- The size of the pixel divided by the focal length is called the **Instantaneous FOV (IFOV)**
 - The IFOV defines the angular limit of resolution in object space
 - IFOV is always expressed as a full angle



Implications of IFOV

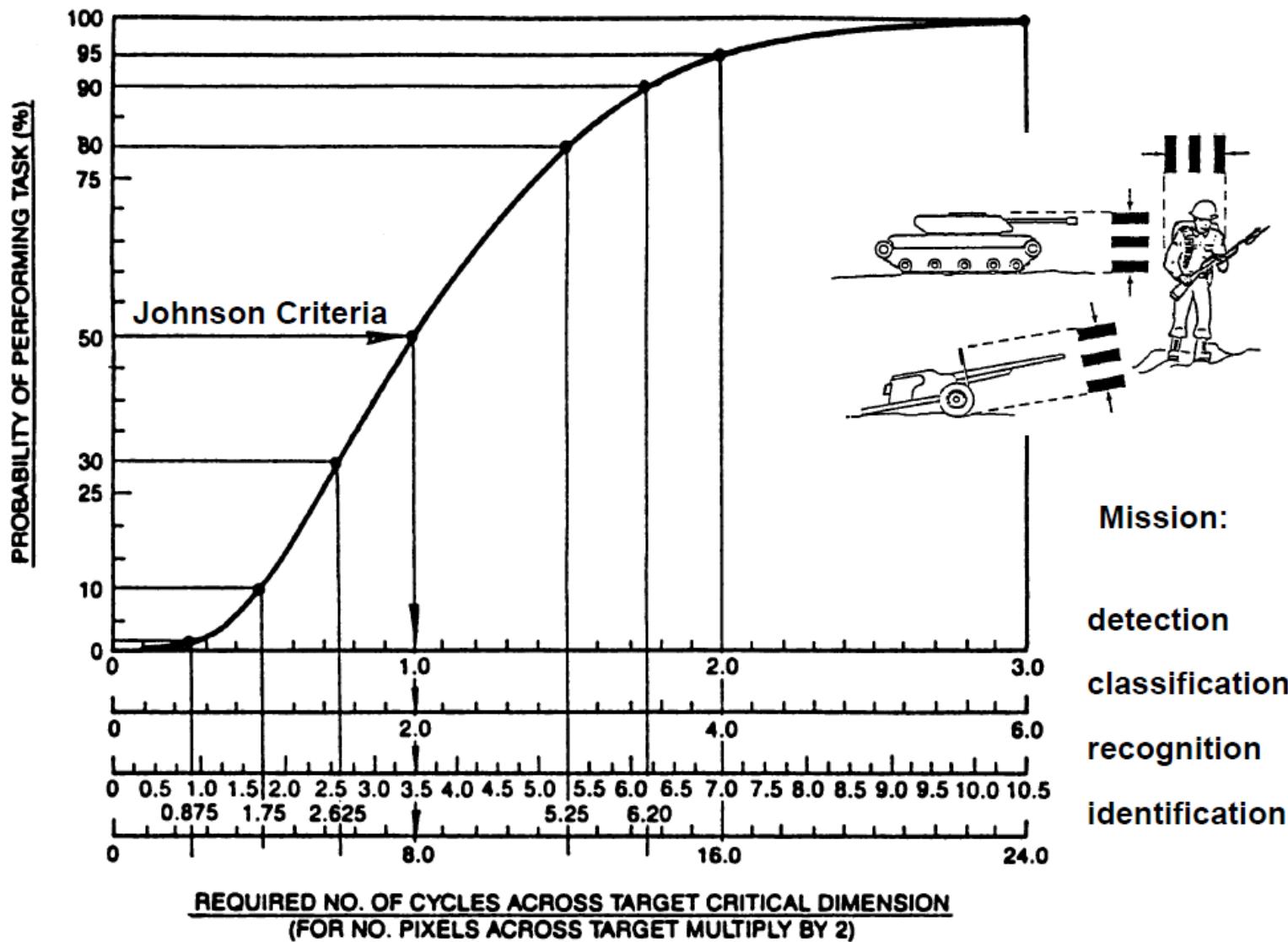
- If the target angular size is smaller than an IFOV, it is **not resolved**
 - It is essentially a point target
 - Example is a star
- If the target annular size is larger than an IFOV it may be resolved
 - This does not mean that you can always tell what the object is

Practical Resolution Considerations

- Resolution required to photograph written or printed copy:
 - Excellent reproduction (serifs, etc.) requires 8 line pairs per lower case e
 - Legible reproduction requires 5 line pairs per letter height
 - Decipherable (e, c, o partially closed) requires 3 line pairs per height
- The correlation between resolution in cycles/minimum dimension and certain functions (often referred to as the Johnson Criteria) is

– Detect	1.0 line pairs per dimension
– Orient	1.4 line pairs per dimension
– Aim	2.5 line pairs per dimension
– Recognize	4.0 line pairs per dimension
– Identify	6-8 line pairs per dimension
– Recognize with 50% accuracy	7.5 line pairs per height
– Recognize with 90% accuracy	12 line pairs per height

Johnson Resolution Criteria



Examples of the Johnson Criteria

Detect
1 bar pair



Maybe something
of military interest

Recognize
4 bar pairs



Tank

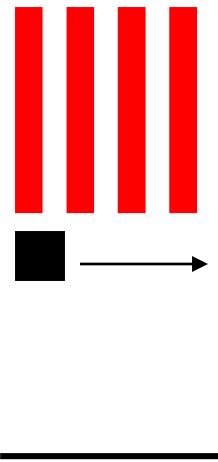
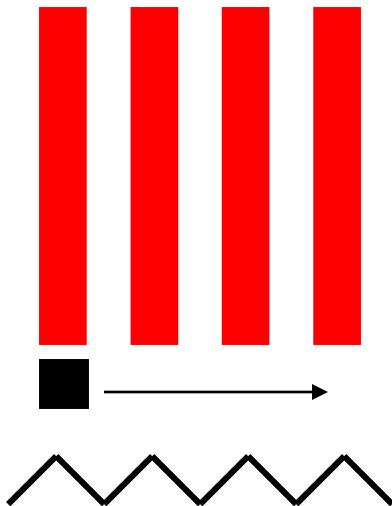
Identify
7 bar pairs



Abrams Tank

MTF of a Pixel

- Consider a fixed size pixel scanning across different sized bar targets



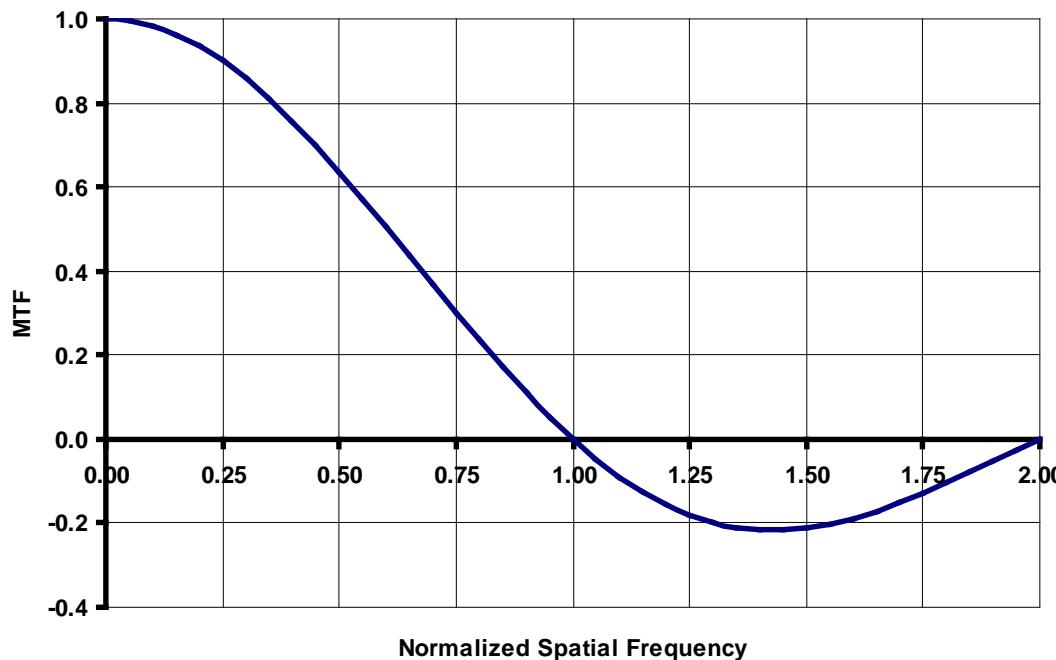
When the pixel size equals the width of a bar pair (light and dark) there is no more modulation

MTF of a Pixel

- If the pixel is of linear width Δ , the MTF of the pixel is given by

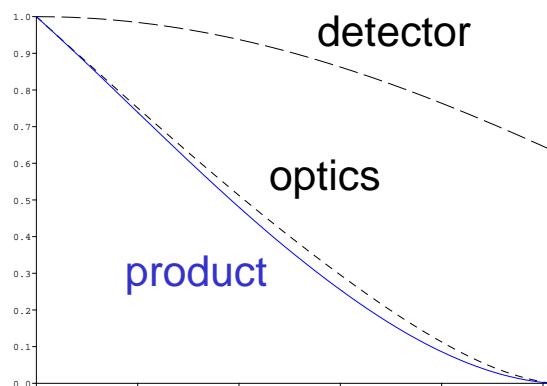
$$\text{MTF}(f) = \frac{\sin(\pi f \Delta)}{\pi f \Delta}$$

- The cutoff frequency (where the MTF goes to zero) is at a spatial frequency $1/\Delta$

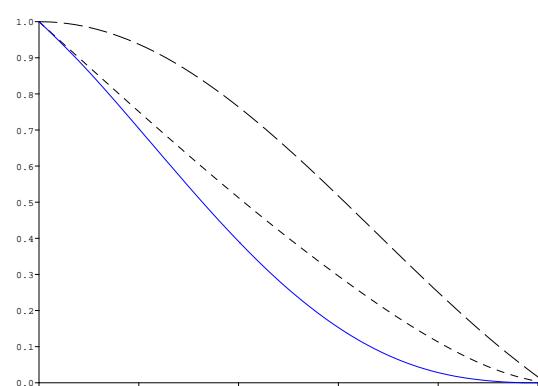


Optical MTF and Pixel MTF

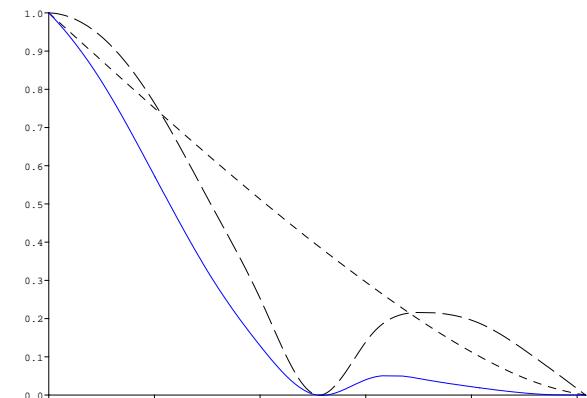
- The total MTF is the product of the optical MTF and the pixel MTF



Case 1 - Optics limited
Best for high resolution over-sampling



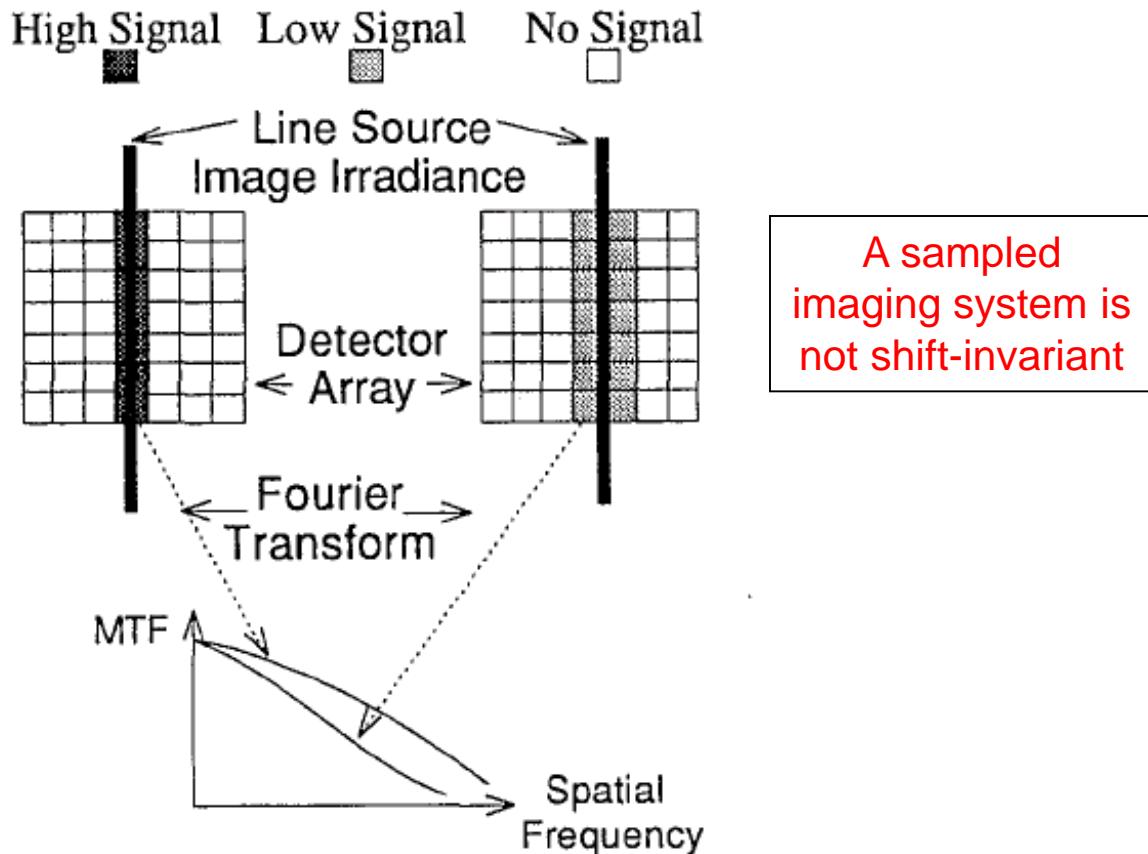
Case 2 - Optics and detector are matched
Best for most FLIR-like mapping systems



Case 3 - Detector limited
Best for detecting dim point targets

- Of course, there are other MTF contributors to total system MTF
 - Electronics, display, jitter, smear, eye, turbulence, etc.

Effects of Signal/CCD Alignment on MTF

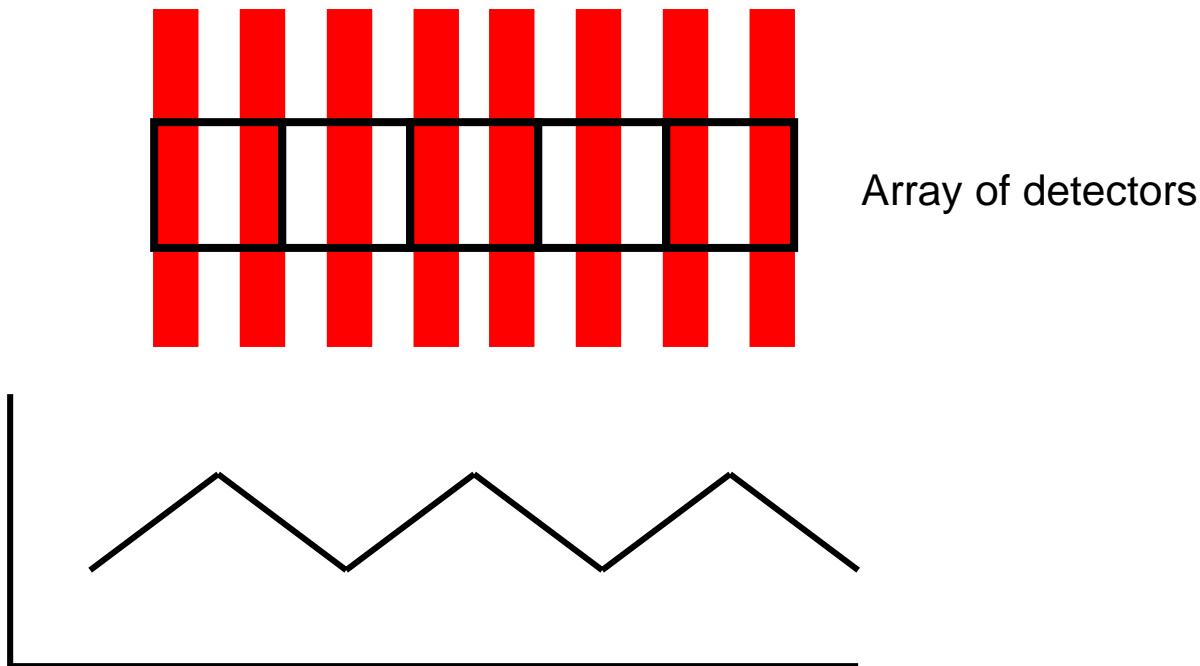


MTF of Alignment

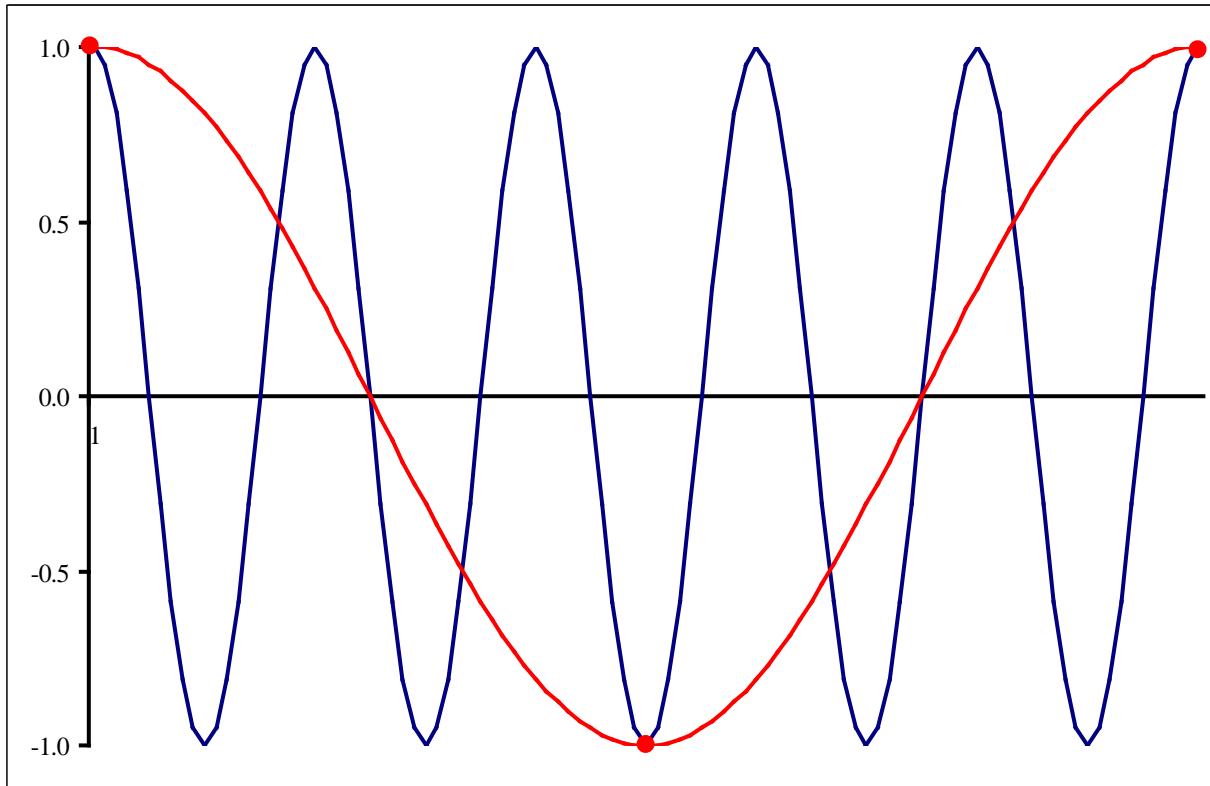
- When performing MTF testing, the user can align the image with respect to the imager to produce the best image
 - In this case, a sampling MTF might not apply
- A natural scene, however, has no net alignment with respect to the sampling sites
- To account for the average alignment of unaligned objects a sampling MTF must be added
 - $MTF_{sampling} = \sin(\pi f \Delta x) / (\pi f \Delta x)$ where Δx is the sampling interval
 - This MTF is an ensemble average of individual alignments and hence is statistical in nature

Aliasing

- Aliasing is a very common effect but is not well understood by most people
- Aliasing is an image artifact that occurs when we insufficiently sample a waveform
 - It is evidenced as the imaging of high frequency objects as low frequency objects



Sampling of a Sine Wave

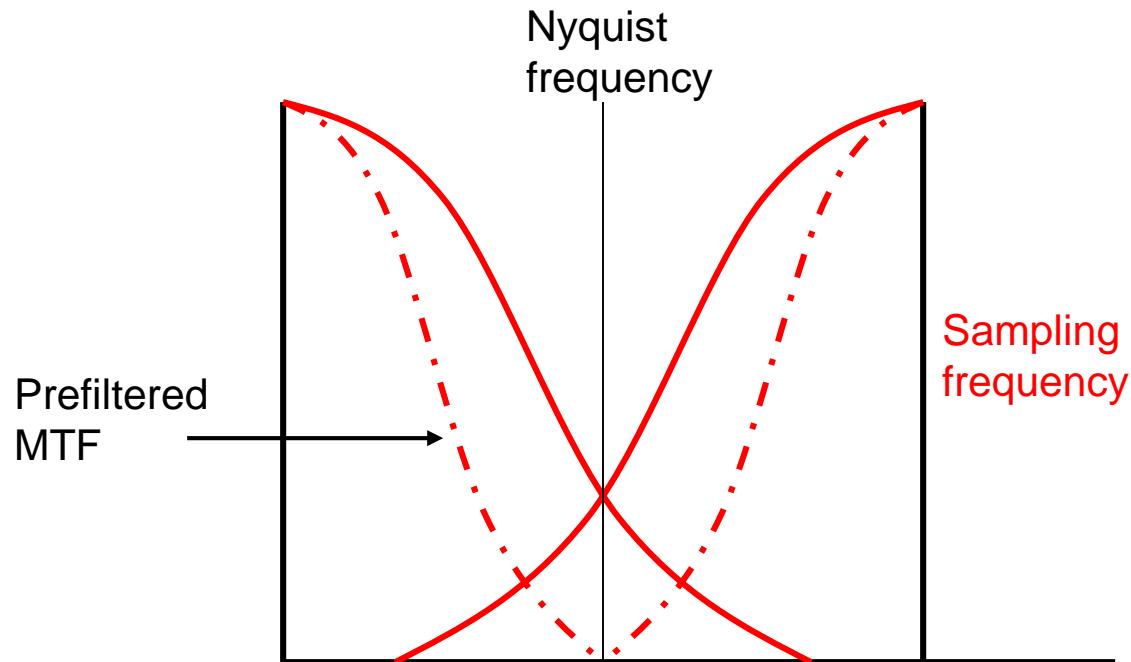


Nyquist Condition

- If we choose a sampling interval sufficiently fine to locate the peaks and valleys of a sine wave, then we can reconstruct that frequency from its sample values
- The Nyquist condition says we need at least two samples per cycle to reproduce a sine wave
 - For a sine wave period ξ , we need a sampling interval $\Delta x < \xi/2$

MTF Fold Over

- The effect of sampling is to replicate the MTF back from the sampling frequency
 - This will cause higher frequencies to appear as lower frequencies



- The solution to this is to prefilter the MTF so it goes to zero at the Nyquist frequency
 - This is often done by blurring

Final Comments

- Image quality is essentially a measure of how well an optical system is suited for the expected application of the system
- Different image quality metrics are needed for different systems
- The better the needed image quality, the more complex the optical system will be (and the harder it will be to design and the higher the cost will be to make it)
- The measures of image quality used by the optical designer during the design process are not necessarily the same as the final performance metrics
 - It's up to the optical designer to convert the needed system performance into appropriate image quality metrics as needed