Overview of Aberrations

Lens Design OPTI 517

Prof. Jose Sasian

College of Optical Sciences
The University of Arizona®

Aberration

From the **Latin**, aberrare, to wander from; **Latin**, ab, away, errare, to wander.

Symmetry properties

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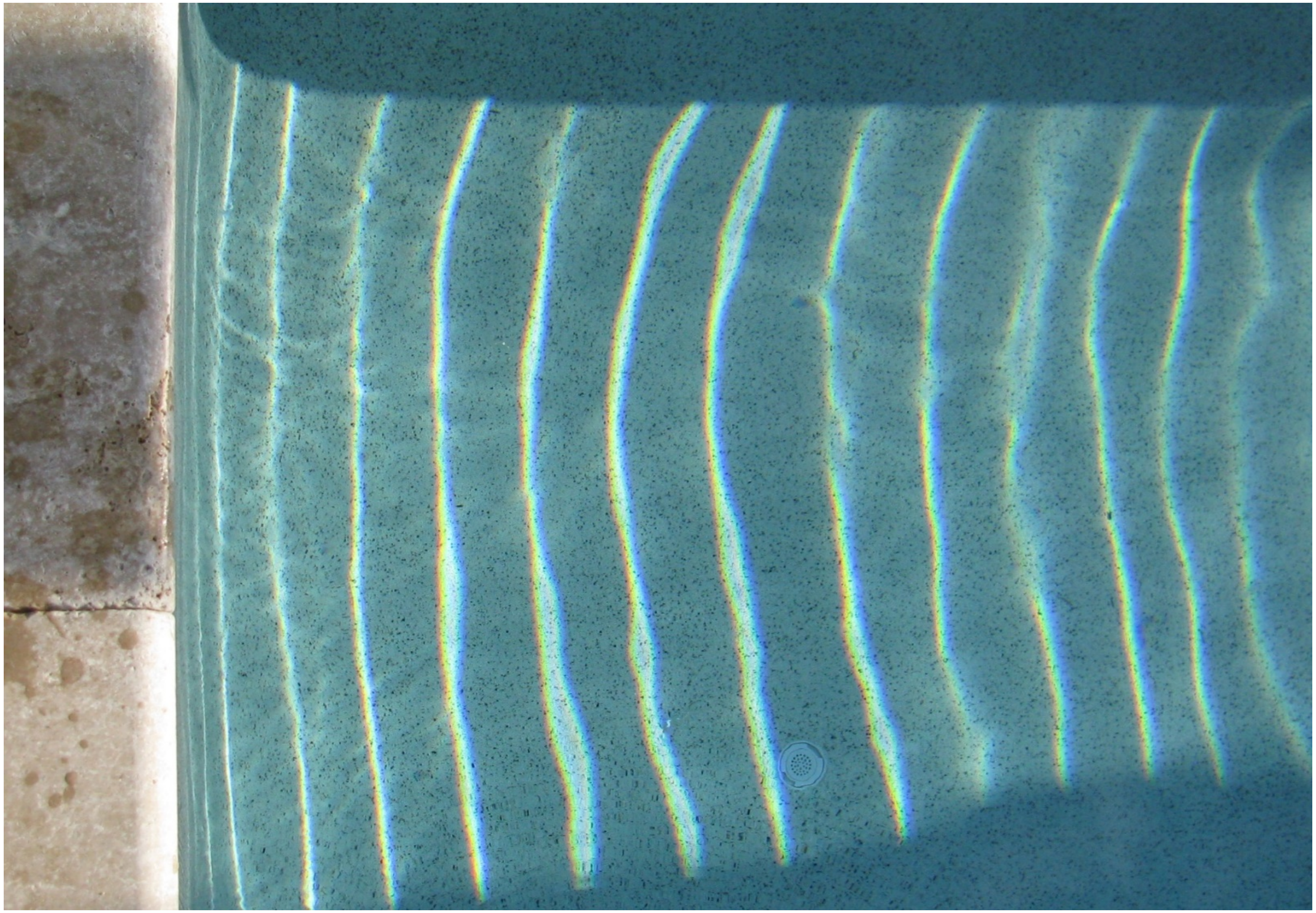
Overview of Aberrations
(Departures from ideal behavior)

- Basic reasoning
- Wave aberration function
- Aberration coefficients
- Aspheric contributions
- Stop shifting
- Structural aberration coefficients

Wavefront



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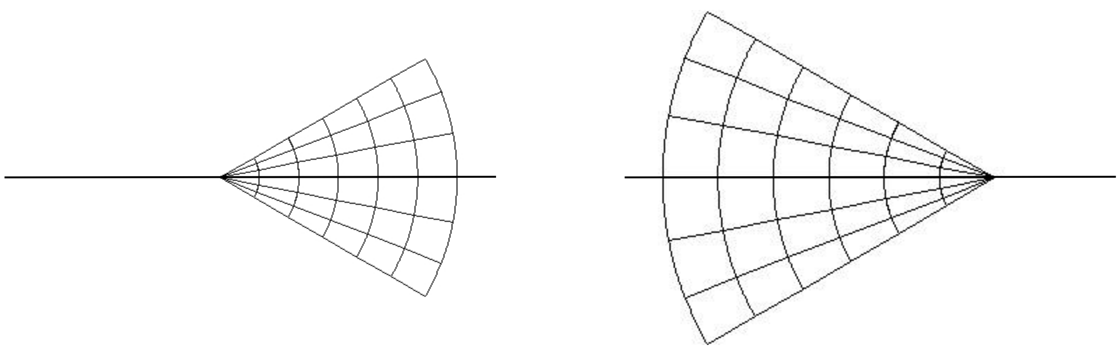
Basic reasoning

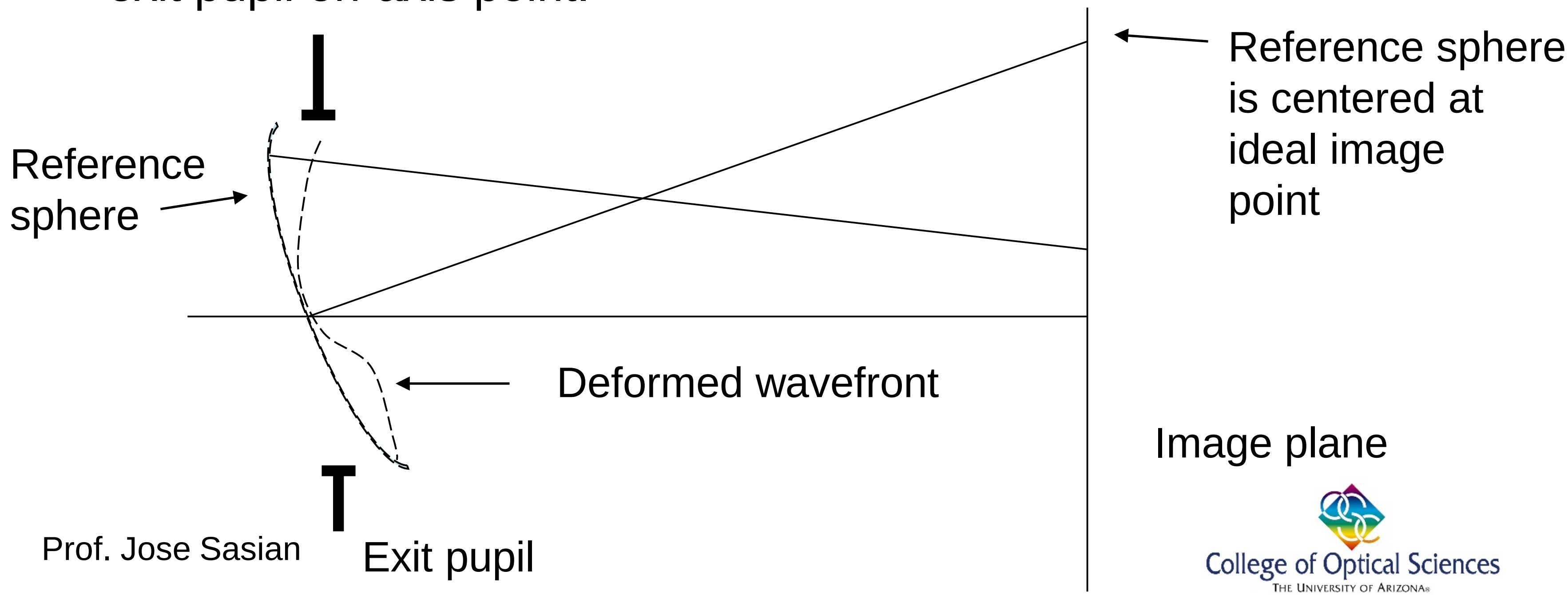
- Ideally wavefronts and rays converge to Gaussian image points. This implies that ideally wavefronts must be spherical and rays must be homocentric.



Basic reasoning

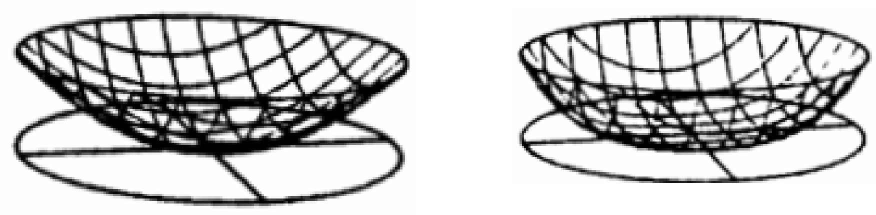
- Actual image degradation by an optical system implies that the collinear transformation can not model accurately imaging. In the wave picture for light propagation we notice that wavefronts must be deformed from the ideal spherical shape.

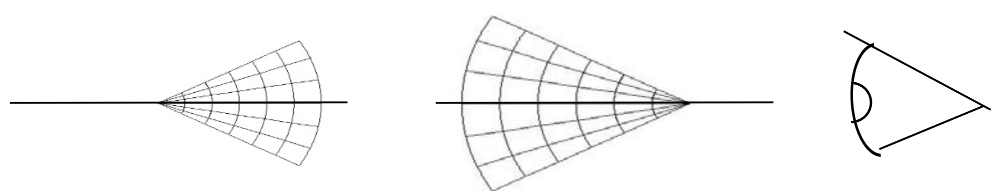
- Wavefront deformation is determined by the use of a <u>reference sphere</u> with center at the Gaussian image point and passing by the exit pupil on-axis point.



Basic reasoning

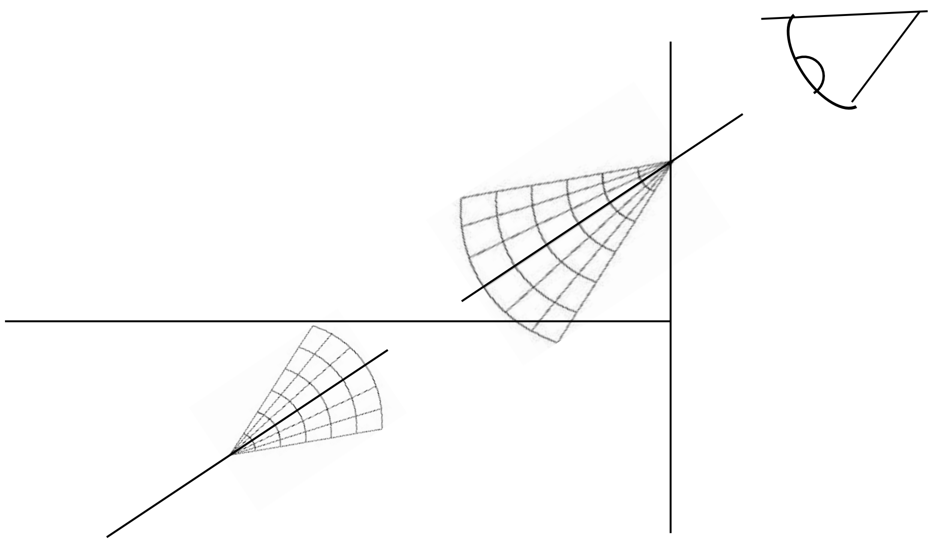
- An axially symmetric system can only have an axially symmetric wavefront deformation for an object point on-axis. In its simplest form this deformation can be quadratic or quartic with respect to the aperture. If the reference sphere is centered in the Gaussian image point then the quadratic deformation can not be present for the design wavelength.

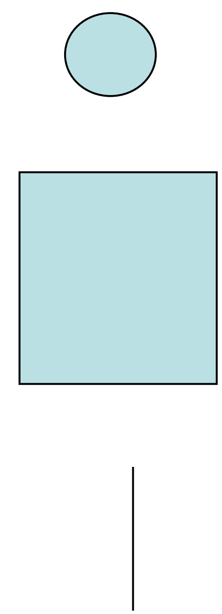




Basic reasoning

- For an object point that is off-axis the axial symmetry of the beam is lost and is reduced to plane symmetry. Therefore for that off-axis beam the wavefront deformation can have axial, plane, or double plane symmetry.



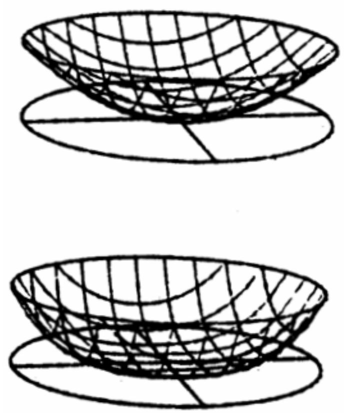


Basic reasoning

- The simplest plane symmetric wavefront deformation shapes represent the primary aberrations. These are:

• Spherical aberration	Axially symmetric
• Coma	Plane symmetric
• Astigmatism	Double plane symmetric
• Field curvature	Axially symmetric
• Distortion	Plane symmetric
• Longitudinal chromatic	Axially symmetric
• Lateral chromatic	Plane symmetric

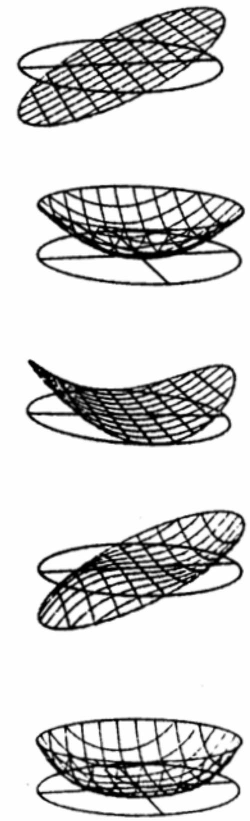
Aberration forms: symmetry considerations



Focus

Spherical aberration

On-axis



Distortion

Field curvature
Focus

Astigmatism

Coma

Spherical aberration

Off-axis

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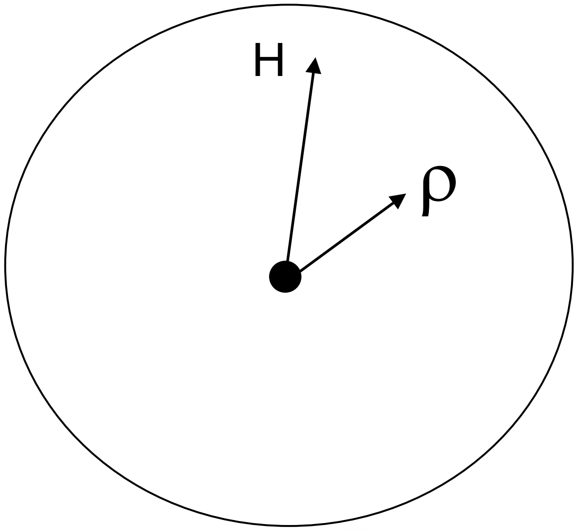
Wave aberration function

- The wave aberration function is a function of the field H and aperture ρ vectors. Because this function represents a scalar, which is the wavefront deformation at the exit pupil, it depends on the dot product of the field and aperture vectors. The assumed axial symmetry leads to a select set of terms.

$$W\{H,\rho\}=\sum_{j,m,n}W_{k,l,m}H^{k}\rho^{l}\cos^{m}\theta$$



$$\begin{aligned}W(H,\rho,\theta)&=W_{000}+W_{200}H^{2}+W_{020}\rho^{2}+W_{111}H\rho\cos\theta+\\&+W_{040}\rho^{4}+W_{131}H\rho^{3}\cos\theta+W_{222}H^{2}\rho^{2}\cos^{2}\theta+\\&+W_{220}H^{2}\rho^{2}+W_{311}H^{3}\rho\cos\theta+W_{400}H^{4}+\\&+\ldots\end{aligned}$$

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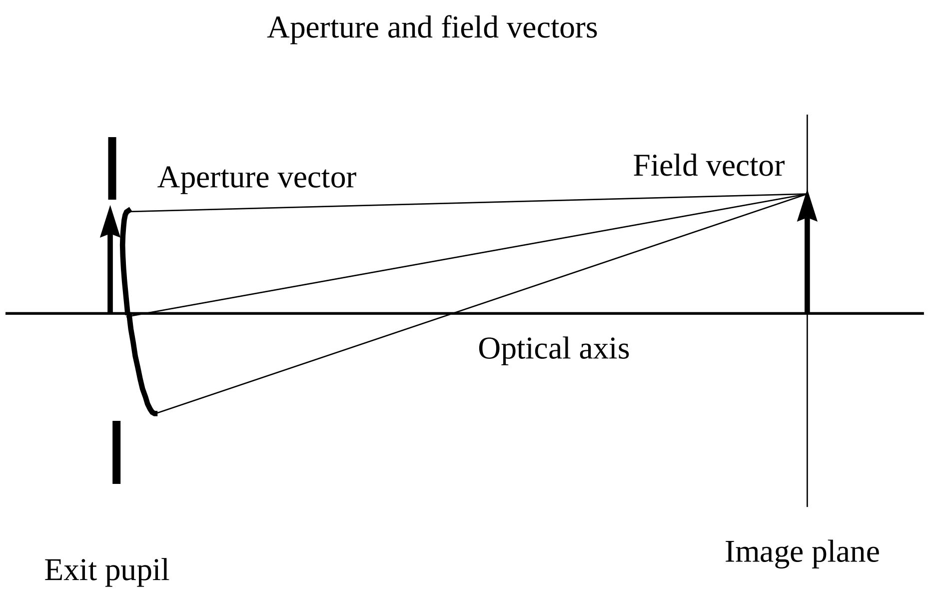
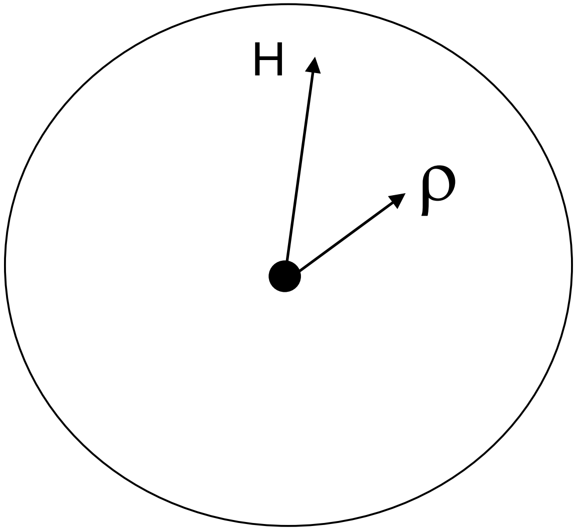
Wave aberration function

- The field vector has its foot at the center of the object plane and the aperture vector has its foot at the center of the exit pupil plane. Both are normalized and so their maximum magnitude is unity. For convenience we draw the first order image of the field vector.

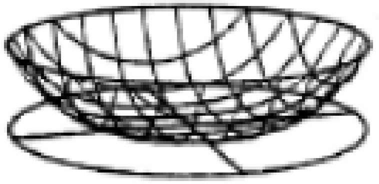
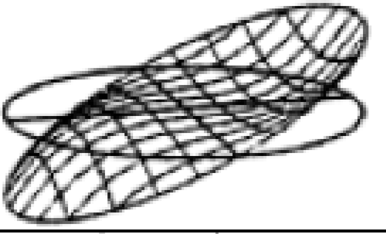
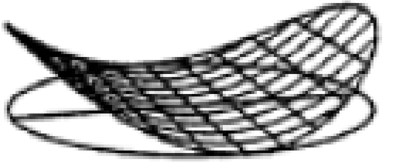
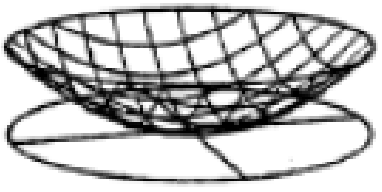
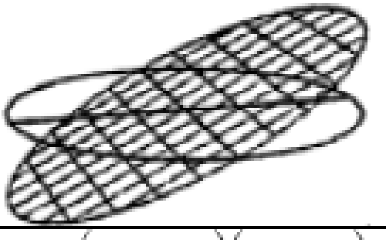
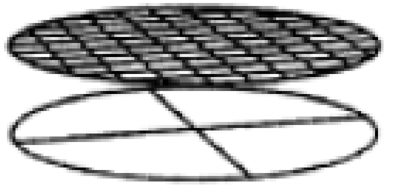




Wave aberration function

- Note that defocus W020 and the change of scale W111 terms are not needed because Gaussian optics accurately predict the location and size of the image. The piston terms W000, W200 and W400 represent a constant phase change that does not degrade the image. These piston terms do not depend on the aperture vector and so they do not produce transverse ray errors.
- Piston terms represent and advance or delay on the propagation of a wavefront.

Summary of primary aberrations

Fourth-order wavefront aberration shapes		
		
$W_{040}(\vec{\rho}\cdot\vec{\rho})^2$	$W_{131}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})$	$W_{222}(\vec{H}\cdot\vec{\rho})^2$
		
$W_{220}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho})$	$W_{311}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})$	$W_{400}(\vec{H}\cdot\vec{H})^2$

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Aberration coefficients for a system of q surfaces

$$W_{040} = \frac{1}{8} S_I$$

$$W_{131} = \frac{1}{2} S_{II}$$

$$W_{222} = \frac{1}{2} S_{III}$$

$$W_{220} = \frac{1}{4}\left(S_{IV} + S_{III} \right)$$

$$W_{311} = \frac{1}{2} S_V$$

$$S_I = -\sum_{j=1}^{q} A^2 y \Delta\left(\frac{u}{n}\right)$$

$$S_{II} = -\sum_{j=1}^{q} A\bar{A} y \Delta\left(\frac{u}{n}\right)$$

$$S_{III} = -\sum_{j=1}^{q} \bar{A}^2 y \Delta\left(\frac{u}{n}\right)$$

$$S_{IV} = -\sum_{j=1}^{q} Ж^2 P$$

$$S_V = -\sum_{j=1}^{q} \frac{\bar{A}}{A}\left[Ж^2 P + \bar{A}^2 y \Delta\left(\frac{u}{n}\right)\right]$$

$$S_V = -\sum_{j=1}^{q} \bar{A}\left[\bar{A}^2 \Delta\left(\frac{1}{n^2}\right) y - \left(Ж + \bar{A} y\right)\bar{y} P \right]$$

Aberration coefficients for chromatic aberrations

$$\delta\lambda W_{020} = \frac{1}{2} C_L \qquad \delta\lambda W_{111} = C_T$$

$$C_L = \sum_{j=1}^{q} A y \Delta\left(\frac{\delta n}{n}\right) \qquad C_T = \sum_{j=1}^{q} \bar{A} y \Delta\left(\frac{\delta n}{n}\right)$$

Aberration coefficient parameters

$$A = nu + nyc = ni$$

$$\bar{A} = n\bar{u} + n\bar{y}c = n\bar{i}$$

$$Ж = n\bar{u}y - nu\bar{y} \qquad c = \frac{1}{r} \qquad P = c\Delta(1/n)$$

$$\Delta\left(\frac{\delta n}{n}\right) = \frac{n-1}{n\nu} \qquad \nu = \frac{n_d - 1}{n_F - n_C}$$

Aberration coefficients

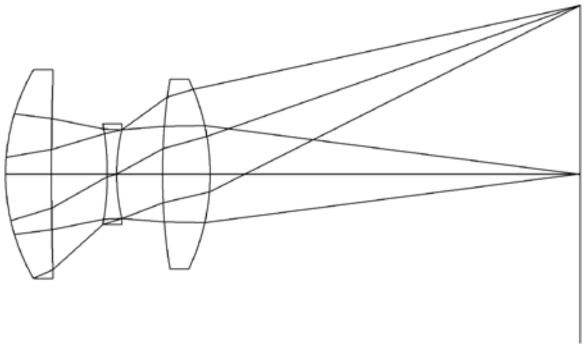
$\mathcal{K}$ is the Lagrange invariant.

- c is the surface curvature, ν is the ν number or reciprocal dispersive power.
- All with the marginal and chief ray first-order ray traces !!!
- Lens optimization started!!!

Comments on aberrations

- Third-order or fourth-order ?
- A well corrected system has its third-order aberrations almost zero
- Aberration cancellation is the main mechanism for image correction
- Presenting to the optimization routine a system with its third-order aberrations corrected is a good starting point
- Some simple systems are designed by formulas that relate third-order aberrations
- Note symmetry in third-order aberration coefficients
- Wave aberrations seem to be simpler to understand than transverse ray aberrations

Example



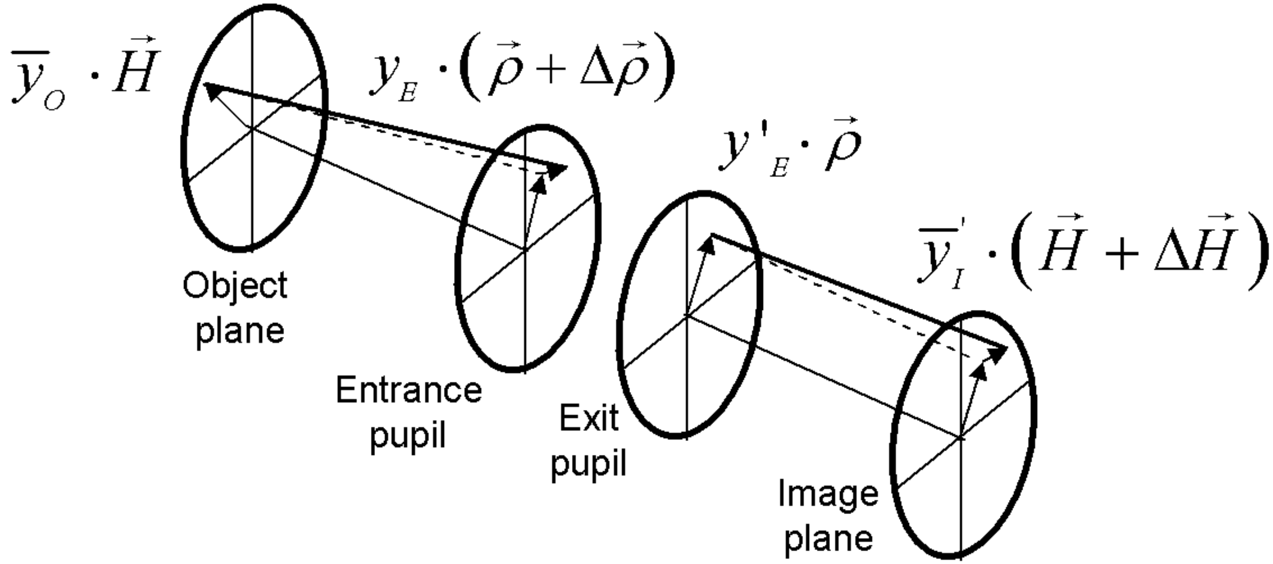
Constructional data of the Cooke triplet lens		
Radius	Thickness	Glass type
22.05	4.83	LAK9
371.58	5.86	
-30.10	0.98	SF5
20.01	4.82	
64.47	5.00	LAK9
-23.48		

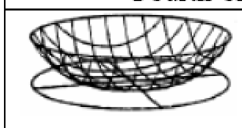
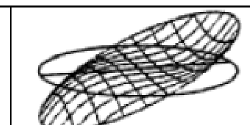
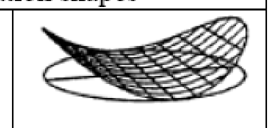
Example

Cooke triplet first-order ray trace						
Surface	y (mm)	u	A	$\bar{y}$ (mm)	$\bar{u}$	$\bar{A}$
1	6.13	-0.11	0.28	-4.37	0.30	0.17
2	5.58	-0.18	-0.17	-2.94	0.50	0.49
3	4.52	-0.05	-0.33	-0.03	0.30	0.50
4	4.47	0.07	0.29	0.25	0.50	0.52
5	4.80	0.01	-0.33	2.68	0.28	0.54
6	4.86	-0.12	-0.12	4.08	0.35	0.18

Wave aberration coefficients of a Cooke triplet (Waves at 587 nm)								
Surface	W_{040}	W_{131}	W_{222}	W_{220}	W_{331}	W_{400}	$\partial_{\lambda} W_{020}$	$\delta_{\lambda} W_{111}$
1	6.77	16.16	9.64	44.06	52.59	-4.83	-10.83	-12.93
2	3.78	-44.19	129.24	62.29	-364.36	47.54	-5.91	34.58
3	-16.16	96.72	-144.77	-100.68	301.39	-0.57	15.92	-47.64
4	-8.01	-56.45	-99.48	-92.30	-325.33	-4.7	13.9	48.99
5	1.34	20.24	76.6	51.72	391.53	57.08	-4.39	-33.26
6	14.94	-32.46	17.64	45.68	-49.63	-5.32	-10.24	11.13
Sum	2.66	0.02	-11.13	10.78	6.19	89.21	-1.57	0.87

Summary of aberrations



Fourth-order wavefront aberration shapes		
		
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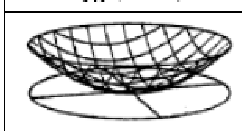	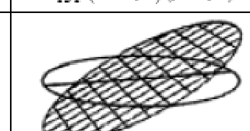	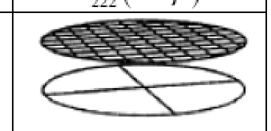
$W_{220}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho})$	$W_{311}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})$	$W_{400}(\vec{H}\cdot\vec{H})^2$

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Aspheric surfaces
(non-spherical)

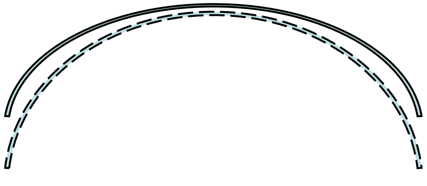
- Conic (or conicoids)
- Cartesian ovals
- Polynomials on x/y, r-theta, Zernikes
- Bernstein polynomials, Bezier curves
- Splines
- NURBS
- Freeform surfaces
- User defined

Conic plus polynomial
(much used in lens design)

$$Z(S) = \frac{cS^2}{1+\sqrt{\left[1-(K+1)c^2S^2\right]}} + A_4S^4 + A_6S^6 + A_8S^8 + A_{10}S^{10} + \ldots$$

$$S^2 = x^2 + y^2$$

C is 1/r where r is the radius of curvature; K is the conic constant; A’s are aspheric coefficients



Aspheric contribution can be thought of as a cap to the spherical part

Sag= sphere + aspheric cap

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Aspheric contributions to the Seidel sums

$$\delta S_I = a$$

$$\delta S_{II} = \frac{\bar{y}}{y} a$$

$$\delta S_{III} = \left(\frac{\bar{y}}{y} \right)^2 a$$

$$\delta S_{IV} = 0$$

$$\delta S_V = \left(\frac{\bar{y}}{y} \right)^3 a$$

$$\delta C_L = 0$$

$$\delta C_T = 0$$

$$a = -\varepsilon^2 c^3 y^4 \Delta n$$

$$a = 8 A_4 y^4 \Delta n$$

$$W_{040} = \frac{1}{8} S_I$$

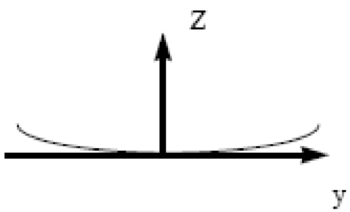
$$W_{131} = \frac{1}{2} S_{II}$$

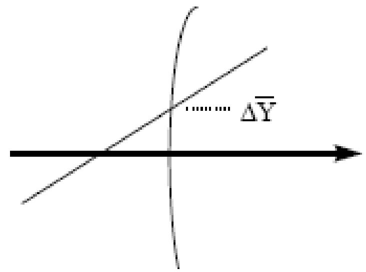
$$W_{222} = \frac{1}{2} S_{III}$$

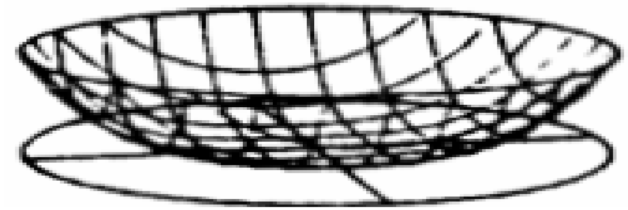
$$W_{220} = \frac{1}{4} \left(S_{III} + S_{IV} \right)$$

$$W_{311} = \frac{1}{2} S_V$$

Aspheric contributions explanation

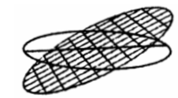




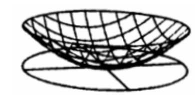


The surface sag is: $Z = A_4\, y^4$ and the wavefront deformation is:

$$W_{040} = (n'-n)\cdot A_4 y^4$$

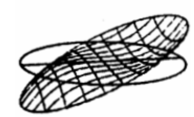


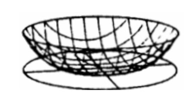
For off-axis beams y becomes $y+\Delta\bar{y}$ or $y+\bar{y}$ and therefore:



y^4 becomes $(y+\bar{y})^4 = \left(y^2+2y\bar{y}+\bar{y}^2\right)\cdot\left(y^2+2y\bar{y}+\bar{y}^2\right) =$

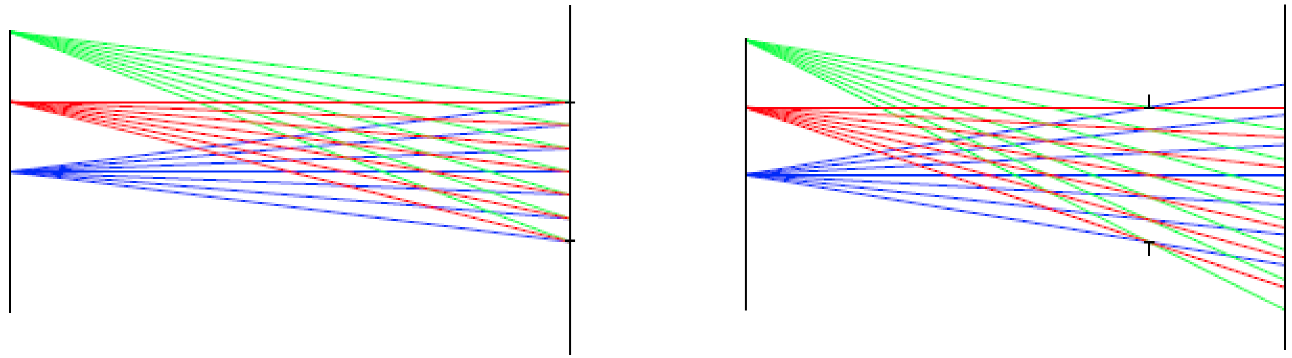
$$= y^4\left\{1+4\frac{\bar{y}}{y}+4\left(\frac{\bar{y}}{y}\right)^2+2\left(\frac{\bar{y}}{y}\right)^2+4\left(\frac{\bar{y}}{y}\right)^3+\left(\frac{\bar{y}}{y}\right)^4\right\}$$





The terms in the brackets represent spherical, coma, astigmatism, field curvature, distortion, and piston.

Aspheric contributions depend on chief ray height at the surface



•When the stop is at the aspheric surface only spherical aberration is contributed given that all the beams see the same portion of the surface

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•When the stop is away from the surface, different field beams pass through different parts of the aspheric surface and other aberrations are contributed

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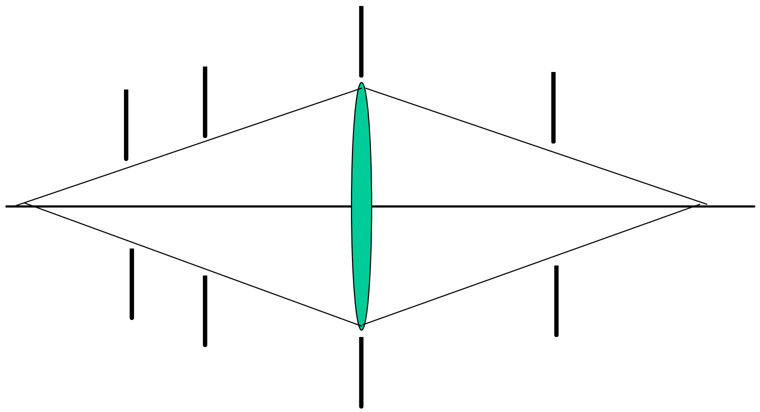
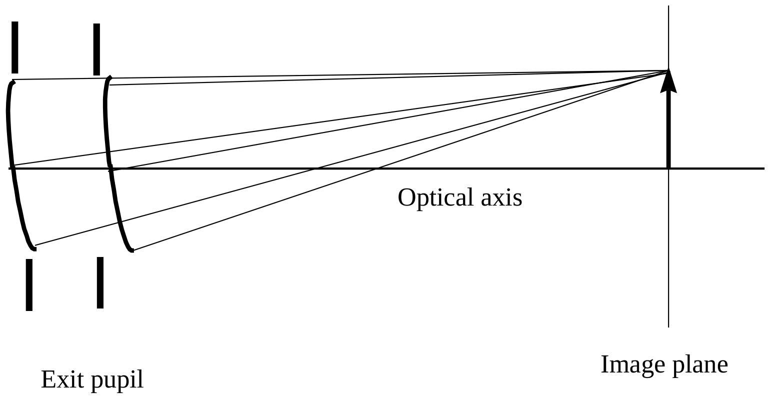
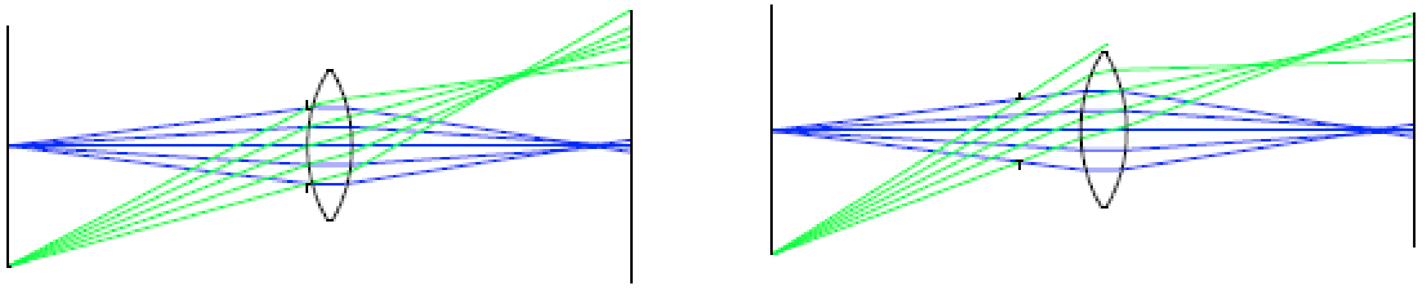
Stop Shifting





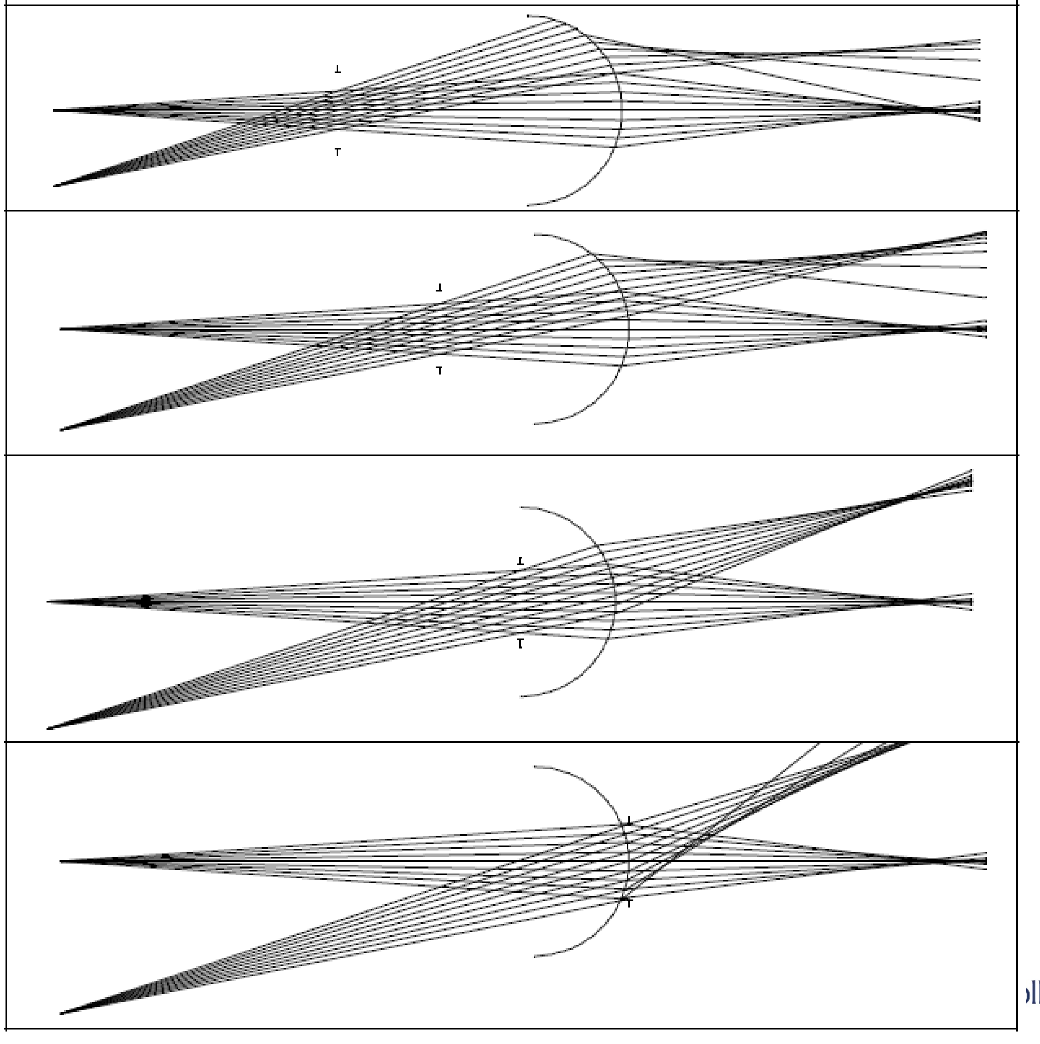
- Stop shifting is a change in the location of the aperture stop along the optical axis
- Stop shifting does not change the f/#
- Stop shifting does not change the optical throughput
- Stop shifting selects a different portion of the wavefront for off-axis beams

Stop shifting may produce light vignetting



Stop shifting

As the stop shifts different off-axis rays are selected and the on-axis rays remain the same. The stop diameter changes to maintain the F/#.



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Change of Seidel sums with stop shifting

$$\delta S_{I} = 0$$

$$\delta S_{II} = \frac{\delta \bar{y}}{y} S_{I}$$

$$\delta S_{III} = 2\frac{\partial \bar{y}}{y} S_{II} + \left(\frac{\delta \bar{y}}{y}\right)^{2} S_{I}$$

$$\delta S_{IV} = 0$$

$$\delta S_{V} = \frac{\delta \bar{y}}{y}\left\{S_{IV} + 3S_{III}\right\} + 3\left(\frac{\delta \bar{y}}{y}\right)^{2} S_{II} + \left(\frac{\delta \bar{y}}{y}\right)^{3} S_{I}$$

$$\delta C_{L} = 0$$

$$\delta C_{T} = \frac{\partial \bar{y}}{y} C_{L}$$

$$W_{040} = \frac{1}{8} S_{I}$$

$$W_{131} = \frac{1}{2} S_{II}$$

$$W_{222} = \frac{1}{2} S_{III}$$

$$W_{220} = \frac{1}{4}\left(S_{III} + S_{IV}\right)$$

$$W_{311} = \frac{1}{2} S_{V}$$

Wave coefficients in terms of Seidel sums

$$W_{040} = \frac{1}{8} S_I$$

$$W_{131} = \frac{1}{2} S_{II}$$

$$W_{222} = \frac{1}{2} S_{III}$$

$$W_{220} = \frac{1}{4} \left(S_{III} + S_{IV} \right)$$

$$W_{311} = \frac{1}{2} S_V$$

The ratio $\frac{\delta \bar{y}}{y}$

Can be calculated at any plane in the optical system

$$\bar{S} = \frac{\bar{u}_{new} - \bar{u}_{old}}{u} = \frac{\bar{y}_{new} - \bar{y}_{old}}{y} = \frac{\bar{A}_{new} - \bar{A}_{old}}{A}$$

$\bar{S}$ is the stop shifting parameter

Structural coefficients σ

$$S_I = \frac{1}{4} y_p{}^4 \phi^3 \sigma_I$$

$$S_{II} = \frac{1}{2} \text{Ж} y_p{}^2 \phi^2 \sigma_{II}$$

$$S_{III} = \text{Ж}^2 \phi \sigma_{III}$$

$$S_{IV} = \text{Ж}^2 \phi \sigma_{IV}$$

$$S_V = \frac{2\ \text{Ж}^3}{y_p{}^2} \sigma_V$$

$$C_L = y_p{}^2 \phi \sigma_L$$

$$C_T = 2\ \text{Ж} \sigma_T$$

$$y_p$$

Marginal ray height at
the principal planes

College of Optical Sciences
The University of Arizona

Structural coefficients: Thin lens (stop at lens)

$$S_I = \frac{1}{4} y^4 \phi^3 \left[AX^2 - BXY + CY^2 + D \right]$$

$$S_{II} = \frac{1}{2} \text{Ж} y^2 \phi^2 \left[EX - FY \right]$$

$$S_{III} = \text{Ж}^2 \phi$$

$$S_{IV} = \text{Ж}^2 \phi \frac{1}{n}$$

$$S_V = 0$$

$$C_L = y^2 \phi \frac{1}{\nu}$$

$$C_T = 0$$

$$X = \frac{c_1 + c_2}{c_1 - c_2} = \frac{r_2 + r_1}{r_2 - r_1}$$

$$Y = \frac{1+m}{1-m} = \frac{u' + u}{u' - u}$$

$$\phi = \Delta n \Delta c = (n-1)(c_1 - c_x)$$

$$A = \frac{n+2}{n(n-1)^2}$$

$$B = \frac{4(n+1)}{n(n-1)}$$

$$C = \frac{3n+2}{n}$$

$$D = \frac{n^2}{(n-1)^2}$$

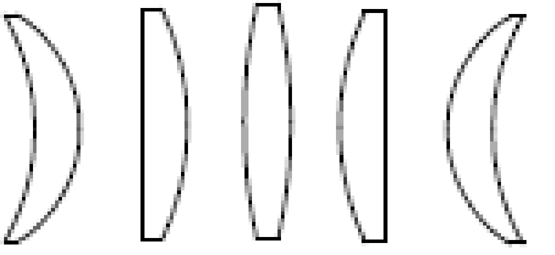
$$E = \frac{n+1}{n(n-1)}$$

$$F = \frac{2n+1}{n}$$

Surface optical power ϕ

Bending of a lens

- Maintains the optical power
- Changes the optical shape
- Meniscus, plano-convex, double-convex, etc.
- Shape factor



$$X = \frac{c_1 + c_2}{c_1 - c_2} = \frac{r_2 + r_1}{r_2 - r_1}$$

Using a lens design program

- Must be able to interpret correctly the information displayed by the program
- In some instances the program is right but we think it is wrong. So we must carefully review our assumptions
- When there is disagreement between you an the program, there is an opportunity to learn
- Verify that the program is modeling what you want
- Check and double check
- You must feel comfortable when using a program.
- Read the manual
- Play with the program to verify that it does what you think it does
- Must reach the point when it is actually fun to use the program

Summary

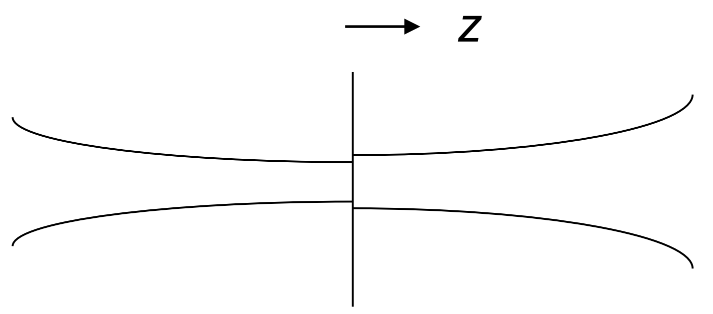
- Review of aberrations
- Aspheric surfaces
- Stop shifting
- Aberration coefficients
- Structural aberration coefficients

- Next class derivation of coefficients

Mode matching concept

- Same mode diameter
- Same amplitude distribution
- Same phase distribution
- Same polarization
- Same x,y,z position
- Same angular position

Fiber coupling efficiency



"overlap integral"

$$\int_{-\infty}^{+\infty} \Psi_m(x)\Psi_m^*(x)\,dx = 1 \qquad \int_{-\infty}^{+\infty} \bar{\Psi}_{\bar{m}}(x)\Psi_m^*(x)\,dx$$

$$\varepsilon = \frac{4}{\left(\frac{\omega_0}{\bar{\omega}_0} + \frac{\bar{\omega}_0}{\omega_0}\right)^2 + \left(\frac{\lambda}{\pi\bar{\omega}_0\omega_0}\right)^2 (\bar{z} + z)^2}$$

Herwig Kogelnik

Prof. Jose Sasian

College of Optical Sciences
THE UNIVERSITY OF ARIZONA