Higher-order aberrations

Lens Design OPTI 517

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Higher order Aberrations

- Aberration function
- Six-order terms (fifth-order transverse)
- Wavefront shapes and field dependence
- Coefficients
- Pupil matching
- References.

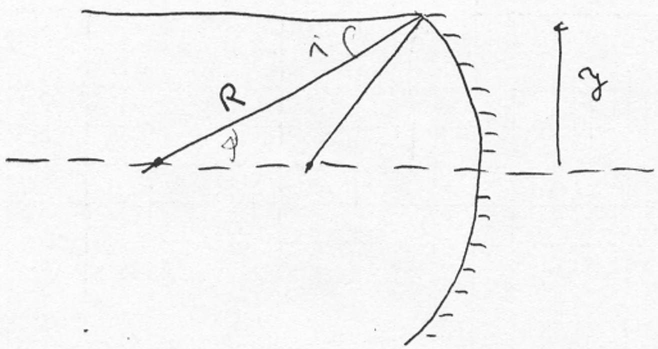
References

- 1) J. Sasian, “Introduction to Aberrations in optical imaging systems.”
- 2) O. N. Stavroudis, “Modular Optical Design”
- 3) Buchdahl, “Optical Aberration Coefficients”
- 4) M. Herzberger, “Modern Geometrical Optics”

“Back of the envelope”

- SPHERICAL ABERRATION

$$W_{040} = -\frac{1}{8} A^2 y \Delta\left\{\frac{u}{n}\right\}$$



$$i = \frac{y}{R}$$

$$u = 0 \;;\; u' = -\frac{2y}{R}$$

$$n = 1 \;;\; n' = -1$$

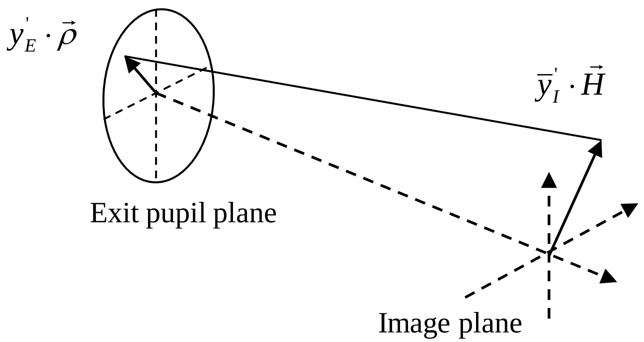
$$W_{040} = -\frac{1}{8} \frac{y^2}{R^2} y \left\{\frac{2y}{R}\right\} = -\frac{1}{4} \frac{y^4}{R^3}$$

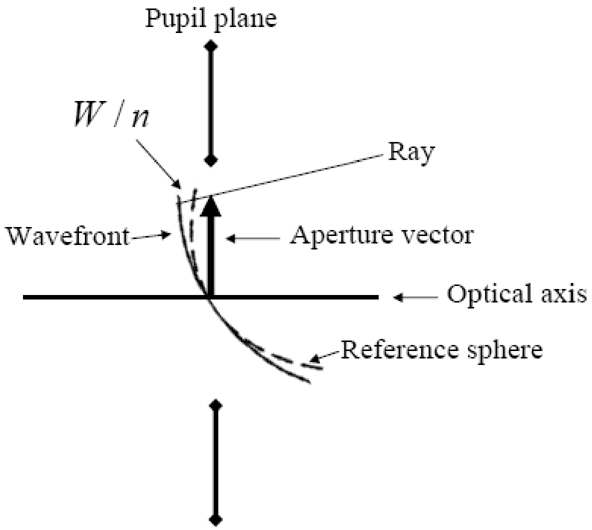
$Y = 2500$ mm $R = -18000$ F/1.8

$W_{040} = +1.6745 = 3044.5\ \lambda$ (@ $\lambda = 0.55\ \mu m$)

$$SAG_{sphere} = \frac{y^2}{2R} + \frac{1}{8} \frac{y^4}{R^3}$$

Coordinate system and reference sphere





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Aberration function

$$W(\vec{H},\vec{\rho}) = \sum_{j,m,n} W_{k,l,m} (\vec{H}\cdot\vec{H})^j \cdot (\vec{H}\cdot\vec{\rho})^m \cdot (\vec{\rho}\cdot\vec{\rho})^n$$

$$= W_{000} + W_{200}(\vec{H}\cdot\vec{H}) + W_{111}(\vec{H}\cdot\vec{\rho}) + W_{020}(\vec{\rho}\cdot\vec{\rho})$$

$$+ W_{040}(\vec{\rho}\cdot\vec{\rho})^2 + W_{131}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho}) + W_{222}(\vec{H}\cdot\vec{\rho})^2$$

$$+ W_{220}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho}) + W_{311}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho}) + W_{400}(\vec{H}\cdot\vec{H})^2$$

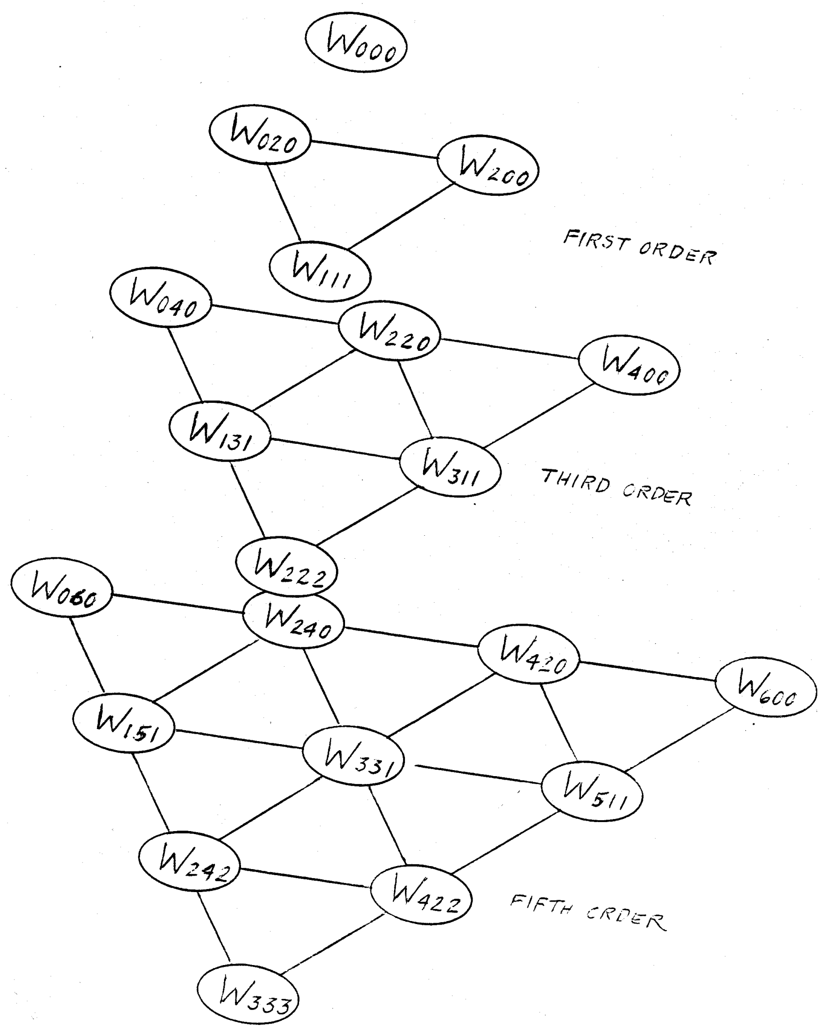
$$+ W_{240}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho})^2 + W_{331}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho}) + W_{422}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})^2$$

$$+ W_{420}(\vec{H}\cdot\vec{H})^2(\vec{\rho}\cdot\vec{\rho}) + W_{511}(\vec{H}\cdot\vec{H})^2(\vec{H}\cdot\vec{\rho}) + W_{600}(\vec{H}\cdot\vec{H})^3$$

$$+ W_{060}(\vec{\rho}\cdot\vec{\rho})^3 + W_{151}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})^2 + W_{242}(\vec{H}\cdot\vec{\rho})^2(\vec{\rho}\cdot\vec{\rho}) + W_{333}(\vec{H}\cdot\vec{\rho})^3$$

Wavefront aberrations					
Aberration name/order	Vector form	Algebraic form	j	m	n
Zero-order					
Uniform piston	W_{000}	W_{000}	0	0	0
Second-order,					
Quadratic piston	$W_{200}(\vec{H}\cdot\vec{H})$	$W_{200}H^2$	1	0	0
Magnification	$W_{111}(\vec{H}\cdot\vec{\rho})$	$W_{111}H\rho\cos(\phi)$	0	1	0
Focus	$W_{020}(\vec{\rho}\cdot\vec{\rho})$	$W_{020}\rho^2$	0	0	1
Fourth-order,					
Spherical aberration	$W_{040}(\vec{\rho}\cdot\vec{\rho})^2$	$W_{040}\rho^4$	0	0	2
Coma	$W_{131}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})$	$W_{131}H\rho^3\cos(\phi)$	0	1	1
Astigmatism	$W_{222}(\vec{H}\cdot\vec{\rho})^2$	$W_{222}H^2\rho^2\cos^2(\phi)$	0	2	0
Field curvature	$W_{220}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho})$	$W_{220}H^2\rho^2$	1	0	1
Distortion	$W_{311}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})$	$W_{311}H^3\rho\cos(\phi)$	1	1	0
Quartic piston	$W_{400}(\vec{H}\cdot\vec{H})^2$	$W_{400}H^4$	2	0	0
Sixth-order					
Oblique spherical aberration	$W_{240}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho})^2$	$W_{240}H^2\rho^4$	1	0	2
Coma	$W_{331}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})$	$W_{331}H^3\rho^3\cos(\phi)$	1	1	1
Astigmatism	$W_{422}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})^2$	$W_{422}H^4\rho^2\cos^2(\phi)$	1	2	0
Field curvature	$W_{420}(\vec{H}\cdot\vec{H})^2(\vec{\rho}\cdot\vec{\rho})$	$W_{420}H^4\rho^2$	2	0	1
Distortion	$W_{511}(\vec{H}\cdot\vec{H})^2(\vec{H}\cdot\vec{\rho})$	$W_{511}H^5\rho\cos(\phi)$	2	1	0
Piston	$W_{600}(\vec{H}\cdot\vec{H})^3$	$W_{600}H^6$	3	0	0
Spherical aberration	$W_{060}(\vec{\rho}\cdot\vec{\rho})^3$	$W_{060}\rho^6$	0	0	3
Un-named	$W_{151}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})^2$	$W_{151}H\rho^5\cos(\phi)$	0	1	2
Un-named	$W_{242}(\vec{H}\cdot\vec{\rho})^2(\vec{\rho}\cdot\vec{\rho})$	$W_{242}H^2\rho^4\cos^2(\phi)$	0	2	1
Un-named	$W_{333}(\vec{H}\cdot\vec{\rho})^3$	$W_{333}H^3\rho^3\cos^3(\phi)$	0	3	0

Aberration orders



Roland Shack's aberration diagram

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Terminology

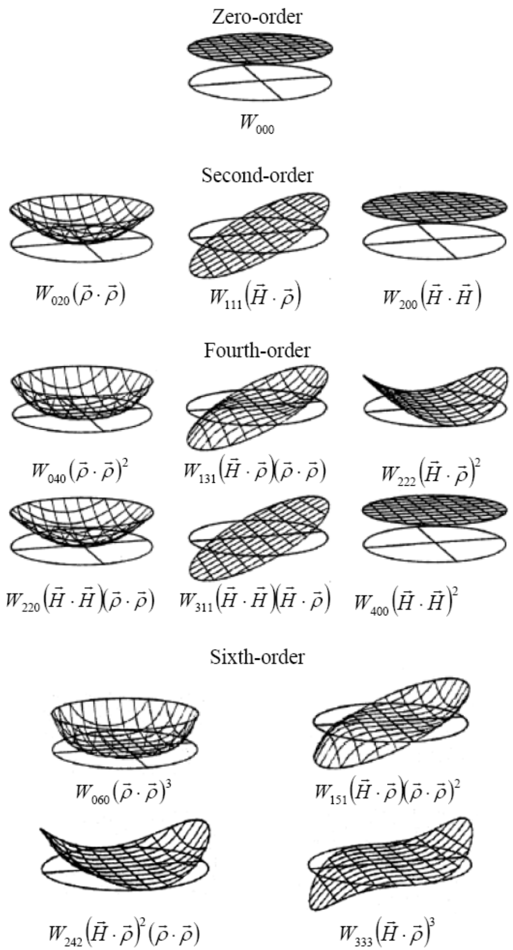
- W_{240} Oblique spherical aberration
- W_{331} Cubic coma
- W_{422} Quartic astigmatism
- W_{420} Six order field curvature
- W_{511} Six order distortion
- W_{600} Six order piston

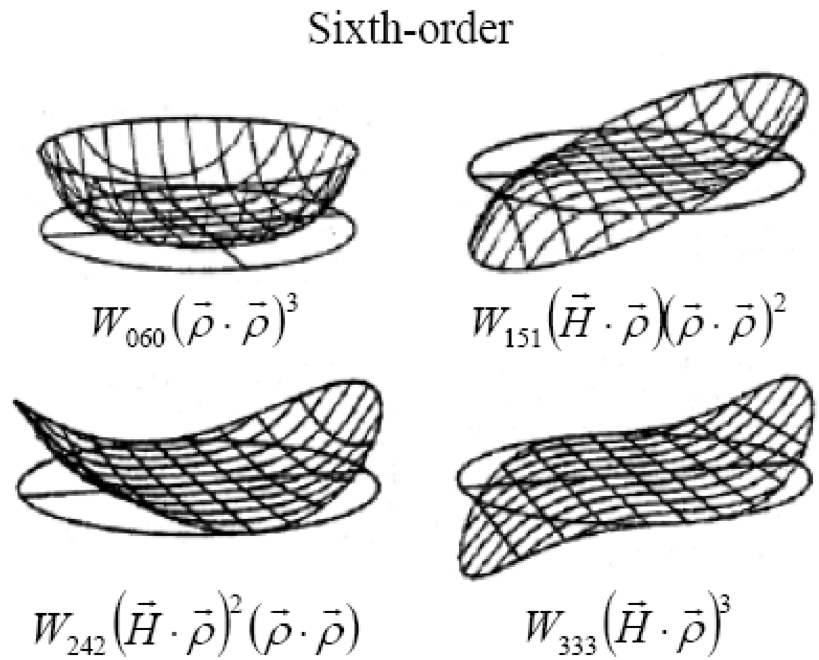
- W_{060} Six order spherical aberration
- W_{151}
- W_{242}
- W_{333}

Some earlier terminology

- Oblique spherical aberration
- Elliptical coma
- Line coma
- Secondary spherical aberration
- Secondary coma
- Lateral coma
- Lateral image curvature/astigmatism
- Trefoil

Wavefront deformation shapes





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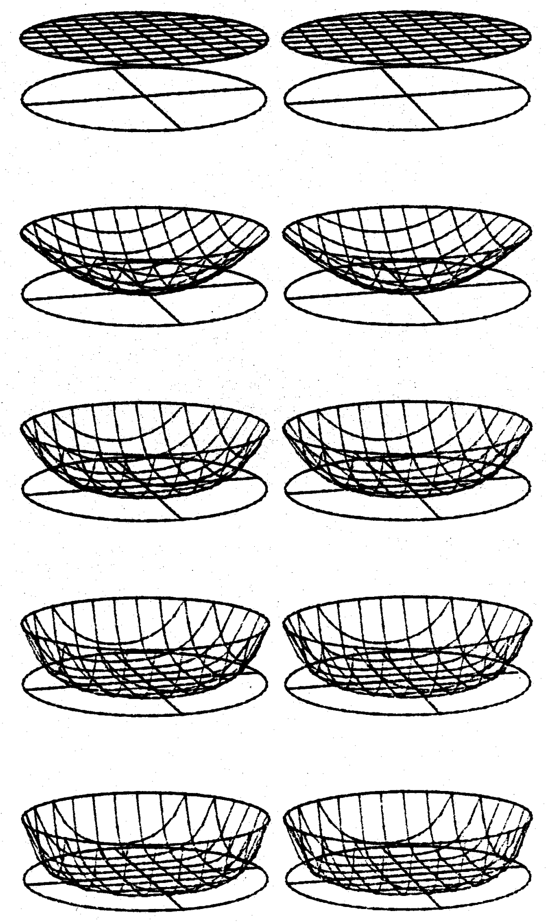
Spherical aberration: W_{060}



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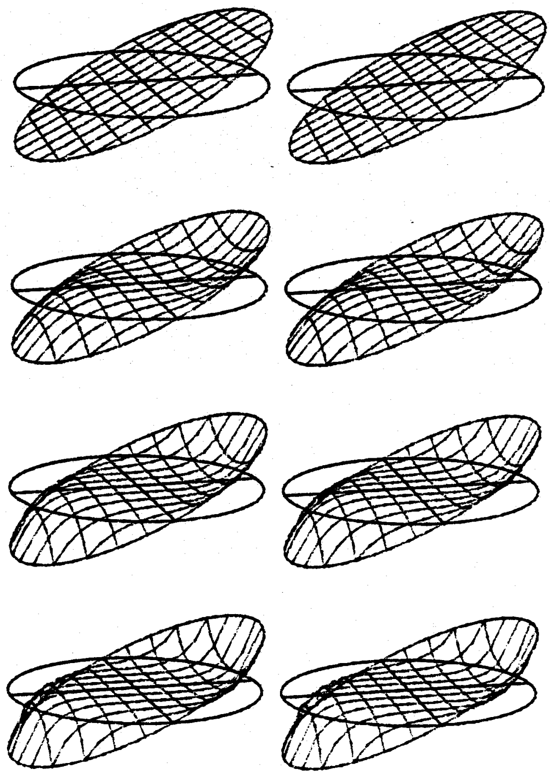
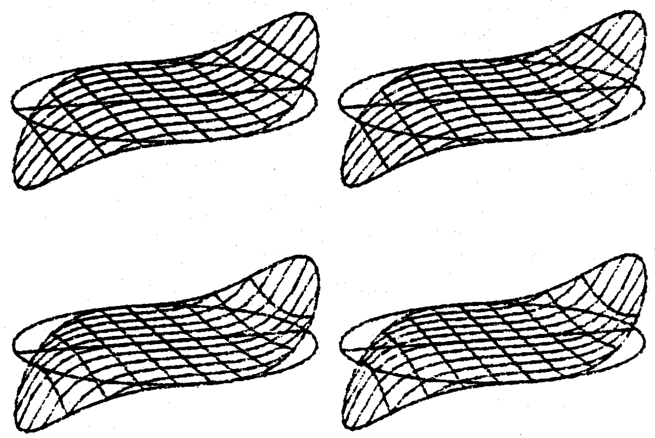
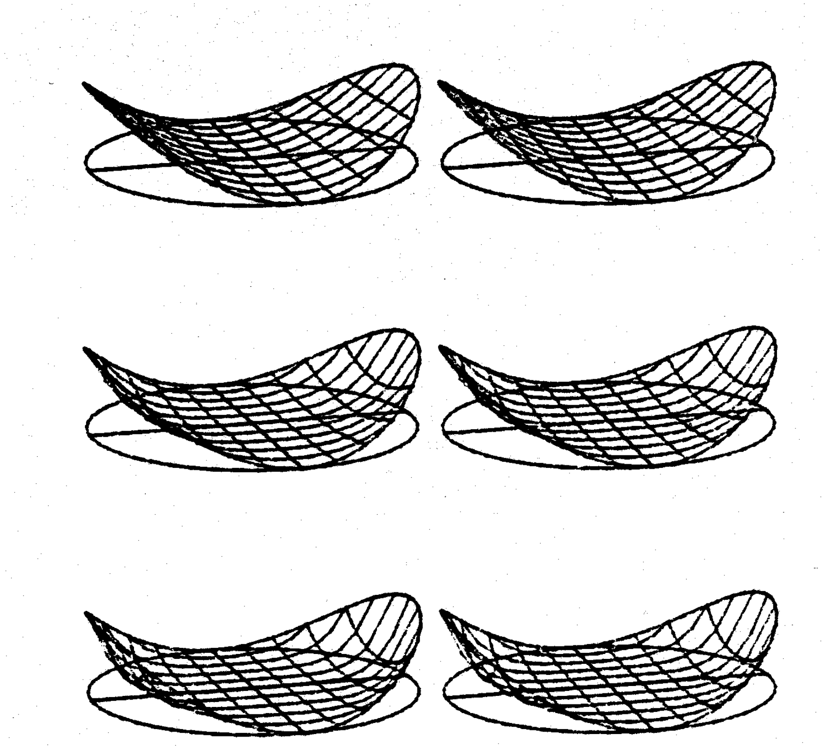
W_{151} & W_{333}





W_{242}



Higher-order aberration coefficients

- Harder to derive/calculate than fourth-order
- Intrinsic coefficients
- Extrinsic coefficients
- Depend highly on coordinate system

Intrinsic spherical aberration

$$W_{040} = -\frac{1}{8} A^2 y \Delta\left(\frac{u}{n}\right)$$

$$W_{060I}^{-} = W_{040}\left[\frac{1}{2}\frac{y^2}{r^2} - \frac{1}{2} A\left(\frac{u'}{n'} + \frac{u}{n}\right) + 2\frac{y}{r}u\right] + \frac{8}{Ж} W_{040} \cdot W_{040} \frac{\bar{y}}{y}$$

Aperture vector at entrance pupil

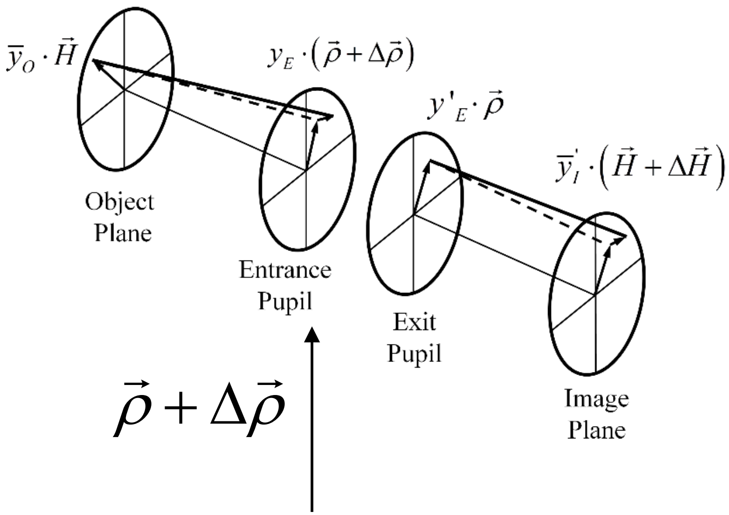
$$W_{060I}^{+} = W_{040}\left[\frac{1}{2}\frac{y^2}{r^2} - \frac{1}{2} A\left(\frac{u'}{n'} + \frac{u}{n}\right) + 2\frac{y}{r}u'\right] - \frac{8}{Ж} W_{040} \cdot W_{040} \frac{\bar{y}}{y}$$

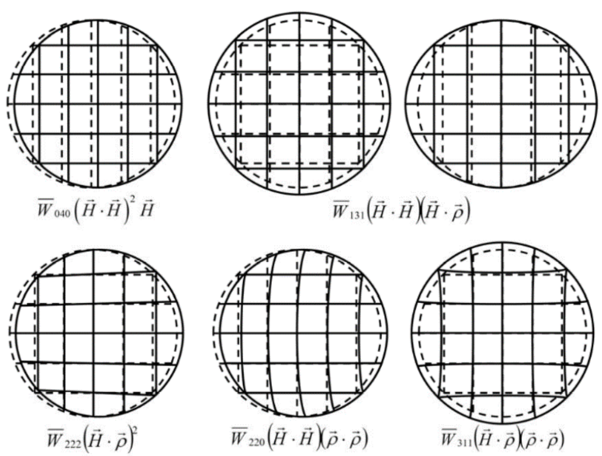
Aperture vector at exit pupil

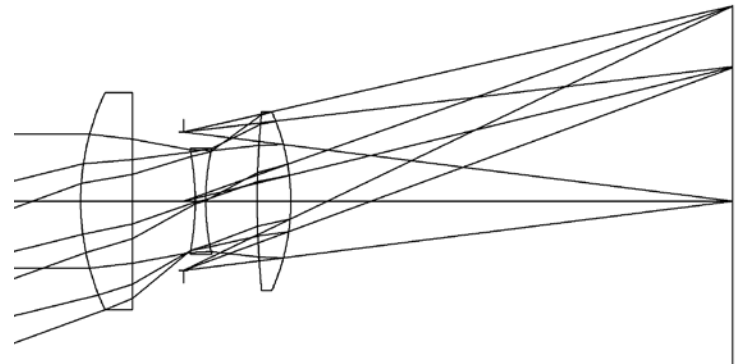
Pupil aberrations

$$\bar{W}\left(\vec{H},\vec{\rho}\right)=\bar{W}_{000}+\bar{W}_{200}\left(\vec{\rho}\cdot\vec{\rho}\right)+\bar{W}_{111}\left(\vec{H}\cdot\vec{\rho}\right)+\bar{W}_{020}\left(\vec{H}\cdot\vec{H}\right)$$
$$+\bar{W}_{040}\left(\vec{H}\cdot\vec{H}\right)^2+\bar{W}_{131}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+\bar{W}_{222}\left(\vec{H}\cdot\vec{\rho}\right)^2$$
$$+\bar{W}_{220}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+\bar{W}_{311}\left(\vec{\rho}\cdot\vec{\rho}\right)\left(\vec{H}\cdot\vec{\rho}\right)+\bar{W}_{400}\left(\vec{\rho}\cdot\vec{\rho}\right)^2$$

Distortion at entrance pupil represents a cross-section deformation







$$\Delta\vec{\rho} = -\frac{1}{Ж}\nabla_H \bar{W}\left(\vec{H},\vec{\rho}\right)$$

$$= -\frac{1}{Ж}\cdot\left\{\begin{matrix} 4\cdot\bar{W}_{040}\left(\vec{H}\cdot\vec{H}\right)\vec{H} + \bar{W}_{131}\left\{\left(\vec{H}\cdot\vec{H}\right)\vec{\rho} + 2\cdot\left(\vec{H}\cdot\vec{\rho}\right)\vec{H}\right\} + \\ 2\cdot\bar{W}_{222}\left(\vec{H}\cdot\vec{\rho}\right)\vec{\rho} + 2\cdot\bar{W}_{220}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{H} + \bar{W}_{311}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho} \end{matrix}\right\}$$

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Pupil grid mapping effects due to pupil aberrations in relation to the Gaussian pupil (dotted line grid). There is no effect from pupil piston.

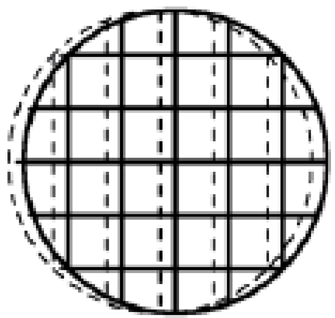
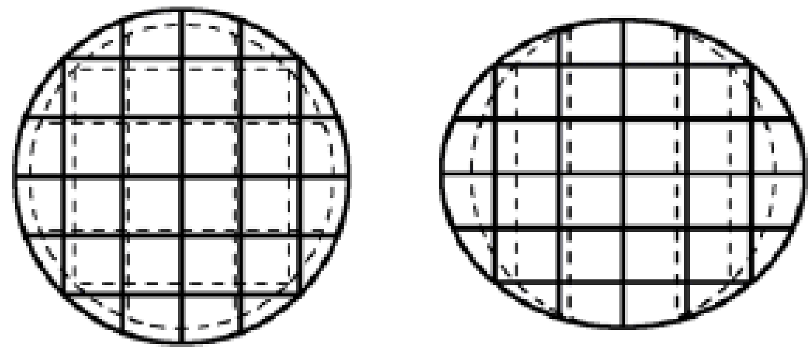
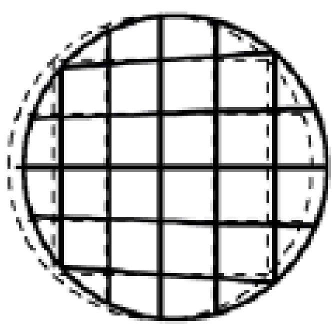
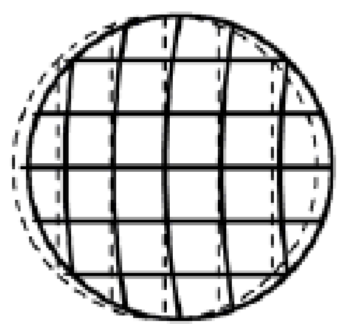
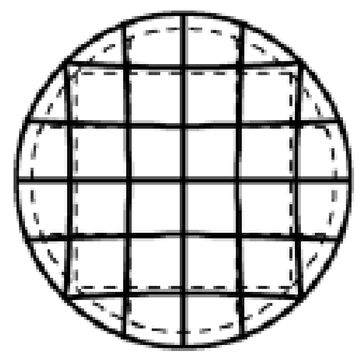
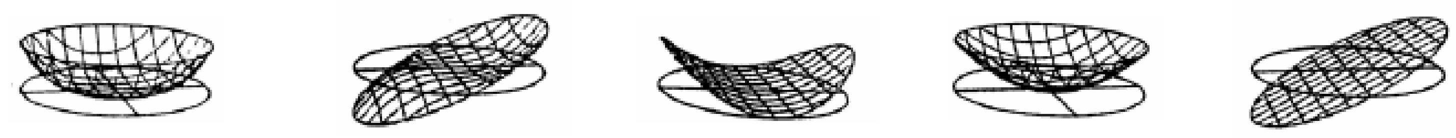
		
$\overline{W}_{040}(\vec{H}\cdot\vec{H})^2$	$\overline{W}_{131}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})$	
		
$\overline{W}_{222}(\vec{H}\cdot\vec{\rho})^2$	$\overline{W}_{220}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho})$	$\overline{W}_{311}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})$

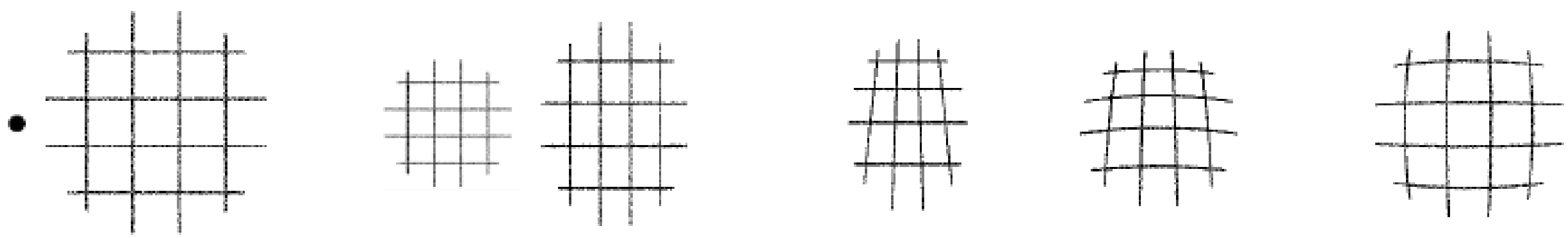
Image vs. Pupil aberrations

Basic wavefront deformation shapes



W_{040} W_{131} W_{222} W_{220} W_{311}

Basic cross-section deformation shapes

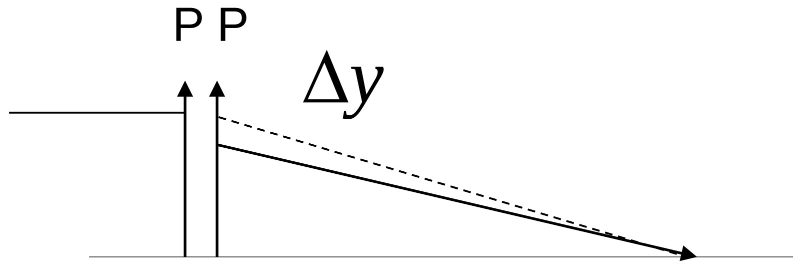


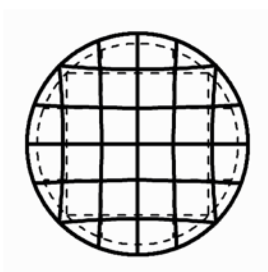
$\overline{W}_{040}$ $\overline{W}_{131}$ $\overline{W}_{222}$ $\overline{W}_{220}$ $\overline{W}_{311}$

Concept of pupil matching

- Not traditionally discussed.
- Pupil matching concept is important.
- Optical system connect: exit pupil of one connects with the entrance pupil of the next.
- Any pupil mismatch produces an effect.
- In general we have pupil mismatch

Example: f/#



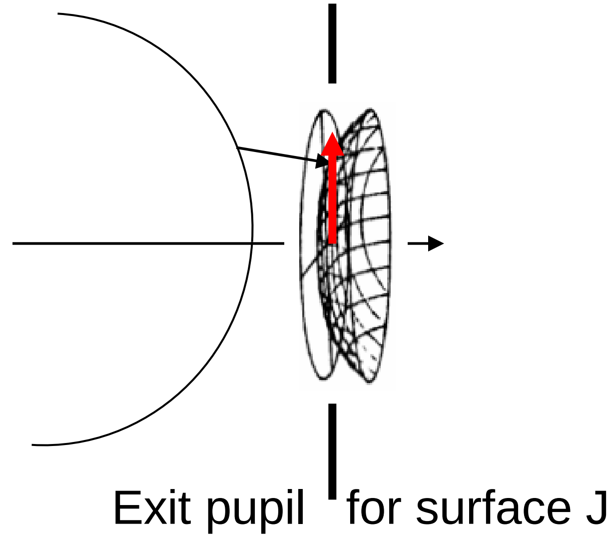


$$\Delta y = \frac{1}{\bar{u}} \overline{W}_{311}$$

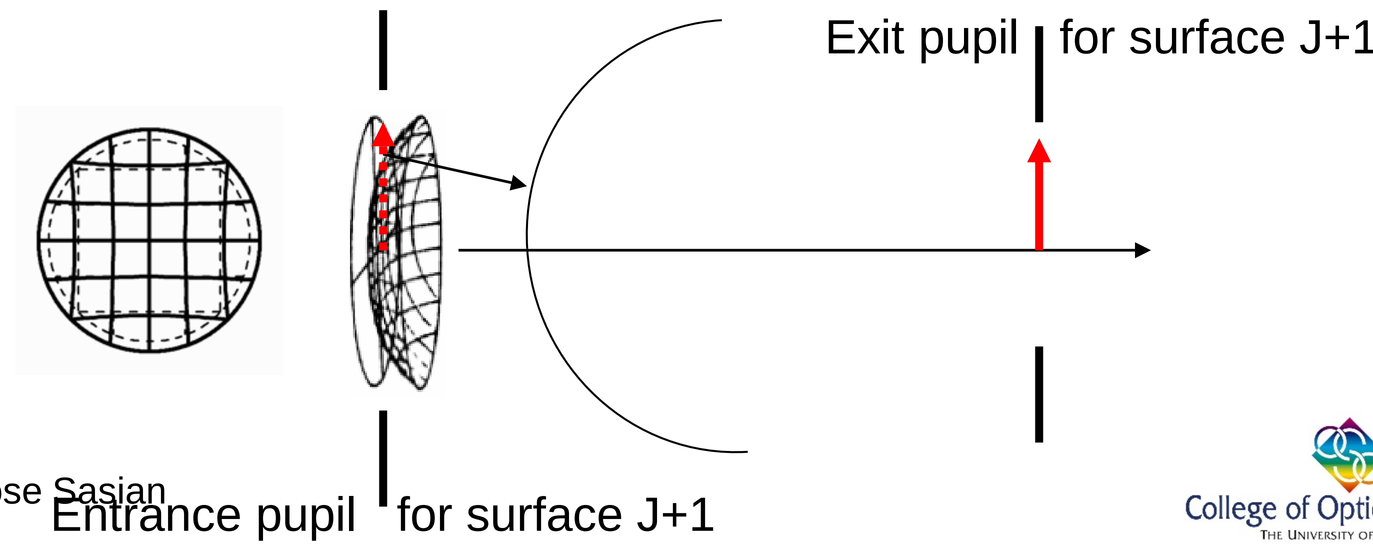
$$f/\# = \frac{f}{d - 2\Delta y}$$

(fourth-order contribution)

Exit pupil becomes entrance pupil for next surface



$$Y = y + c_3 y^3 + c_5 y^5 + c_7 y^7 + \ldots$$
$$= y + \Delta y = y\left(1 + \frac{\Delta y}{y}\right)$$



Extrinsic aberrations

$$W_A\left(\vec{H},\vec{\rho}\right)=W_A^4\left(\vec{H},\vec{\rho}\right)+W_A^6\left(\vec{H},\vec{\rho}\right)$$

$$W_B\left(\vec{H},\vec{\rho}\right)=W_B^4\left(\vec{H},\vec{\rho}\right)+W_B^6\left(\vec{H},\vec{\rho}\right)$$

$$W_A^4\left(\vec{H},\vec{\rho}+\Delta\vec{\rho}_B\right)=W_A^4\left(\vec{H},\vec{\rho}+\Delta\vec{\rho}_B\right)-W_A^4\left(\vec{H},\vec{\rho}\right)+W_A^4\left(\vec{H},\vec{\rho}\right)$$

$$=\nabla W_A^4\left(\vec{H},\vec{\rho}\right)\cdot\Delta\vec{\rho}_B+W_A^4\left(\vec{H},\vec{\rho}\right)$$

$$\boxed{W_E\left(\vec{H},\vec{\rho}\right)=-\frac{1}{Ж}\vec{\nabla}_\rho W_A\left(\vec{H},\vec{\rho}\right)\cdot\vec{\nabla}_H\bar{W}_B\left(\vec{H},\vec{\rho}\right)}$$

Extrinsic coefficients from the combination Of system A and system B	
$W_{060E}=-\frac{1}{\text{Ж}}\left(4W_{040}^{A}\bar{W}_{311}^{B}\right)$	$W_{331E}=-\frac{1}{\text{Ж}}\left(5W_{131}^{A}\bar{W}_{131}^{B}+4W_{220}^{A}\bar{W}_{220}^{B}+4W_{220}^{A}\bar{W}_{222}^{B}+4W_{222}^{A}\bar{W}_{220}^{B}+W_{311}^{A}\bar{W}_{311}^{B}+16W_{040}^{A}\bar{W}_{040}^{B}\right)$
$W_{151E}=-\frac{1}{\text{Ж}}\left(3W_{131}^{A}\bar{W}_{311}^{B}+8W_{040}^{A}\bar{W}_{220}^{B}+8W_{040}^{A}\bar{W}_{222}^{B}\right)$	$W_{422E}=-\frac{1}{\text{Ж}}\left(2W_{311}^{A}\bar{W}_{222}^{B}+4W_{220}^{A}\bar{W}_{131}^{B}+6W_{222}^{A}\bar{W}_{131}^{B}+8W_{131}^{A}\bar{W}_{040}^{B}\right)$
$W_{242E}=-\frac{1}{\text{Ж}}\left(2W_{222}^{A}\bar{W}_{311}^{B}+4W_{131}^{A}\bar{W}_{220}^{B}+6W_{131}^{A}\bar{W}_{222}^{B}+8W_{040}^{A}\bar{W}_{131}^{B}\right)$	$W_{420E}=-\frac{1}{\text{Ж}}\left(2W_{220}^{A}\bar{W}_{131}^{B}+2W_{311}^{A}\bar{W}_{220}^{B}+4W_{131}^{A}\bar{W}_{040}^{B}\right)$
$W_{333E}=-\frac{1}{\text{Ж}}\left(4W_{131}^{A}\bar{W}_{131}^{B}+4W_{222}^{A}\bar{W}_{222}^{B}\right)$	$W_{511E}=-\frac{1}{\text{Ж}}\left(3W_{311}^{A}\bar{W}_{131}^{B}+8W_{220}^{A}\bar{W}_{040}^{B}+8W_{222}^{A}\bar{W}_{040}^{B}\right)$
$W_{240E}=-\frac{1}{\text{Ж}}\left(2W_{131}^{A}\bar{W}_{220}^{B}+2W_{220}^{A}\bar{W}_{311}^{B}+4W_{040}^{A}\bar{W}_{131}^{B}\right)$	$W_{600E}=-\frac{1}{\text{Ж}}\left(4W_{311}^{A}\bar{W}_{040}^{B}\right)$

Buchdahl-Rimmer fifth-order aberrations

$$\varepsilon_y = B\cos(\phi)\rho^3 + F\left(2+\cos(2\phi)\right)\rho^2 H + (3C+\pi)\cos(\phi)\rho H^2 + EH^3$$
$$+B_5\cos(\phi)\rho^5 + \left(F_1 + F_2\cos(2\phi)\right)\rho^4 H + \left(M_1 + M_2 + M_3\cos^2(\phi)\right)\cos(\phi)\rho^3 H^2$$
$$+\left(N_1 + N_2\cos^2(\phi)\right)\rho^2 H^3 + (5C_5 + \pi_5)\cos(\phi)\rho H^4 + E_5 H^5$$

$$\varepsilon_x = B\sin(\phi)\rho^3 + F\sin(2\phi)\rho^2 H + (C+\pi)\sin(\phi)\rho H^2$$
$$+B_5\sin(\phi)\rho^5 + F_2\sin(2\phi)\rho^4 H + \left(M_2 + M_3\cos^2(\phi)\right)\sin(\phi)\rho^3 H^2$$
$$+N_3\sin(2\phi)\rho^2 H^3 + (C_5 + \pi_5)\sin(\phi)\rho H^4$$

12 fifth-order terms

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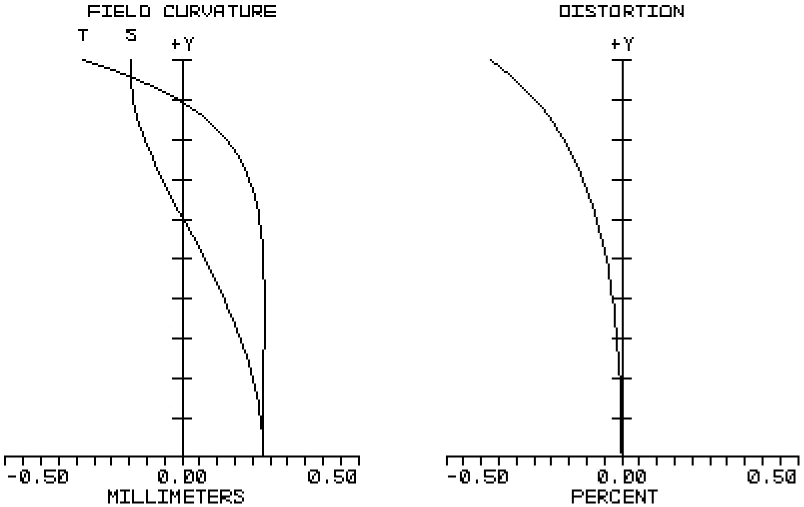
Aberration correction concepts

- Destroy an aberration (early days)
- Aberration correction (compensation): Add the opposite amount to have a net zero residual
- Aberration balancing: Add a different aberration and minimize or trade-off performance; fourth vs. higher order.
- Minimize an aberration.
- Do not generate an aberration
- Main mechanism for aberration correction is compensation and balancing

Aberration balancing: 4th order vs. higher order



Surface	W_{040}	W_{131}	W_{222}	W_{220}	W_{311}	W_{400}	$\partial_\lambda W_{020}$	$\delta_\lambda W_{111}$
1	6.77	16.16	9.64	39.24	52.59	-4.83	-10.83	-12.93
2	3.78	-44.19	129.24	-2.33	-364.36	47.54	-5.91	34.58
3	-16.16	96.72	-144.77	-28.29	301.39	-0.57	15.92	-47.64
4	-8.01	-56.45	-99.48	-42.55	-325.33	-4.7	13.9	48.99
5	1.34	20.24	76.6	13.42	391.53	57.08	-4.39	-33.26
6	14.94	-32.46	17.64	36.86	-49.63	-5.32	-10.24	11.13
Sum	2.66	0.02	-11.13	16.35	6.19	89.21	-1.57	0.87

Summary

- Higher-order aberrations
- Pupil aberrations
- Aberration correction and balancing