Diffractive Optical Elements

Lens Design OPTI 517





Diffractive Lenses



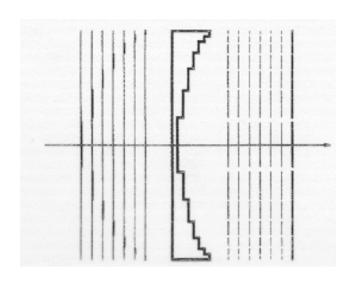
- What they are
- How they work
- Zone spacing and blaze profile roles
- First order properties
- Dispersion
- Two point construction model
- Phase model
- Sweatt model
- Efficiency
- Diffractive landscape lens

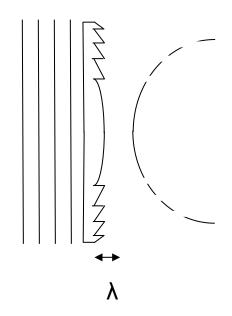


Terminology

- Diffractive optical element: generic term
- Fresnel lens: Scale of zones and lack of organized phasing
- Kinoform: Phased Fresnel lens. Phase modulation from surface relief
- Holographic optical element: Produced by interfering two or more beams
- Binary optics: Made by staircases that approximate the ideal surface relief
- Fresnel zone plate: A particular pattern that produces amplitude modulation.
- Hybrid lens: combined refractive and diffractive power
- Computer generated hologram: A hologram produced by calculations in a computer

The work of a diffractive optical element

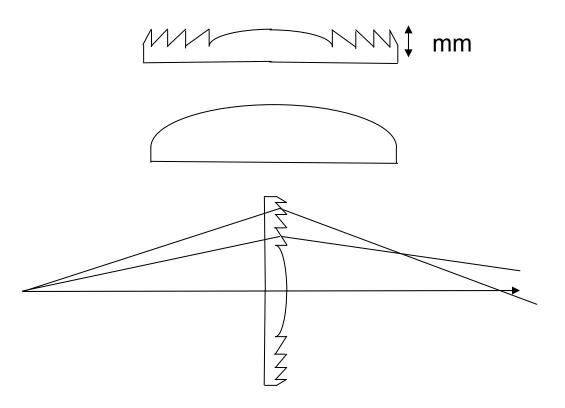




Organized rearrangement of the wavefront



Fresnel Lens



A Fresnel lens reduces the amount of bulk glass.

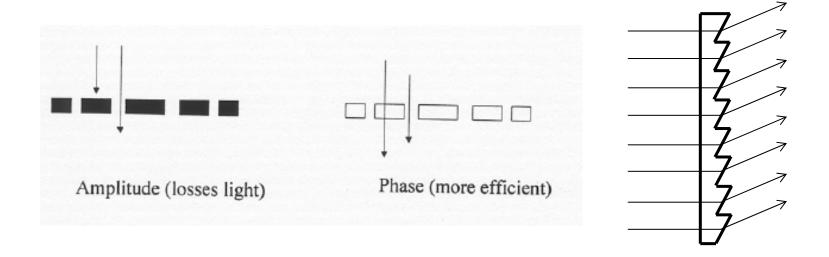
Scale of zones is large and the wavefront segments are not rearranged to re-create a spherical wavefront.

The ring-zone segments is not properly organized.

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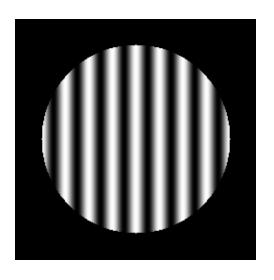
Prof. Jose Sasian

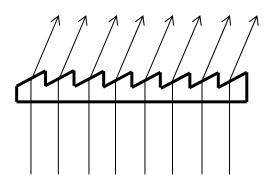
Two contexts for DOE: amplitude and phase



- •Blaze determines amplitude of diffracted orders
- Geometry of zone boundary determines wavefront shape (phase)
- •The wavefront deformation introduced by a DOE is equal to the wavefront deformation represented by the DOE when it is thought of as an interferogram

Example

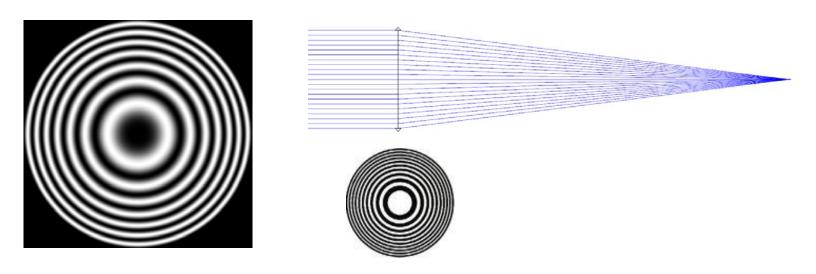




 Straight fringes represent tilt and so the beam is deviated



Example



- Circular fringes represent defocus and so a DOE with these zone boundaries will introduce optical power
- Depending on the spacing, spherical aberration can also be introduced



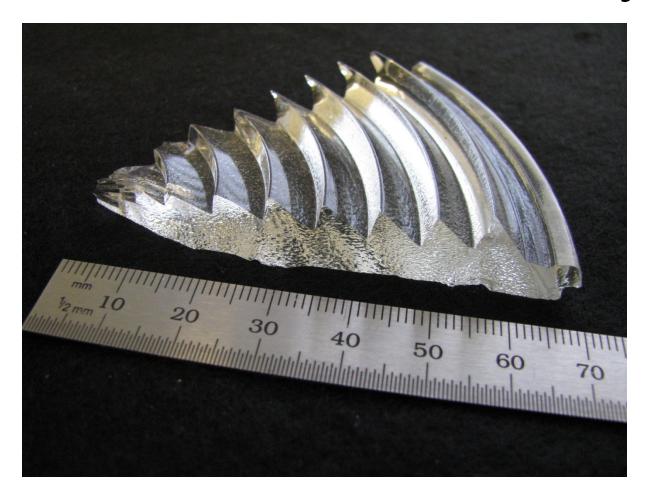
An infrared DOE



From Michael Morris

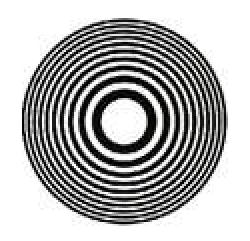


A Fresnel lens cut-away





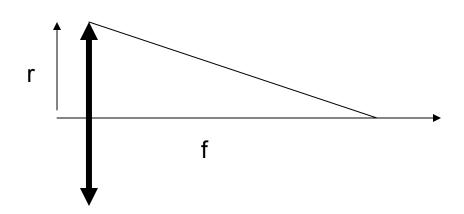
First-order properties



$$\sqrt{f^2 + r_n^2} = f + n\lambda$$

$$f^2 + r_n^2 = f^2 + 2nf\lambda + n^2\lambda^2$$

$$r_n \cong \sqrt{2nf\lambda}$$



Given a focal length the zone boundaries are defined. The optical path difference Between zones is one wavelength



Paraxial diffractive lens definition

$$r_n = \sqrt{2nf\lambda}$$

Design of a wide field diffractive landscape lens

Dale A. Buralli and G. Michael Morris



Zone Spacing

$$r_n^2 \cong 2nf\lambda$$

$$r_n^2 - r_{n-1}^2 = (r_n + r_{n-1})(r_n - r_{n-1}) \cong 2r_n dr = 2f\lambda$$

$$Spacing = dr \cong \frac{f}{2r_n} 2\lambda \cong F / \#_{micrometers}$$





Focal length for a given spacing

$$f = \frac{r_n \cdot dr}{\lambda_{construction}} \times \frac{\lambda_{construction}}{\lambda_{reconstruction}} = f_0 \times \frac{\lambda_{construction}}{\lambda_{reconstruction}}$$

Designed for

$$\lambda_{construction}$$

Used at



Abbe's number for a refractive lens

$$\phi_{refractive} = \frac{(n-1)}{R}$$

$$\frac{\partial \phi}{\partial \lambda} = \frac{1}{R} \frac{\partial n}{\partial \lambda}$$

$$\partial \phi = \frac{1}{R} (n_d - 1) \frac{n_f - n_c}{n_d - 1} = \phi_d \frac{n_f - n_c}{n_d - 1} = \frac{\phi_d}{\nu}$$

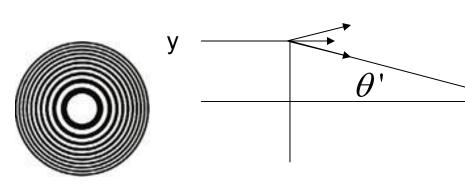
$$\nu_{refractive} = \frac{\phi}{\partial \phi}$$



Diffractive V-number

$$\frac{\Delta \varphi}{\varphi} = \frac{r}{n_d - 1} \frac{n_f - n_c}{r} = \frac{n_f - n_c}{n_d - 1} = \frac{1}{v_{refractive}}$$

$$n'\sin(\theta') - n\sin(\theta) = \frac{m\lambda}{d}$$



$$\theta'$$
 $f = \frac{1}{\varphi} \approx \frac{y}{\sin(\theta')} = \frac{y}{m\lambda/d}$

$$\frac{\Delta \varphi}{\varphi} = \frac{y}{m\lambda_d / d} \frac{m(\lambda_f - \lambda_c) / d}{y} = \frac{\lambda_f - \lambda_c}{\lambda_d} = \frac{1}{v_{diffractive}} \approx \frac{1}{-3.5}$$



Diffractive focal length from grating perspective

$$f = \frac{1}{\varphi} \cong \frac{y}{\sin(\theta')} = \frac{y}{m\lambda/d}$$

$$= \frac{y}{m\lambda_{construction}} \times \frac{\lambda_{construction}}{\lambda_{reconstruction}}$$

$$= f_0 \times \frac{\lambda_{construction}}{\lambda_{construction}}$$

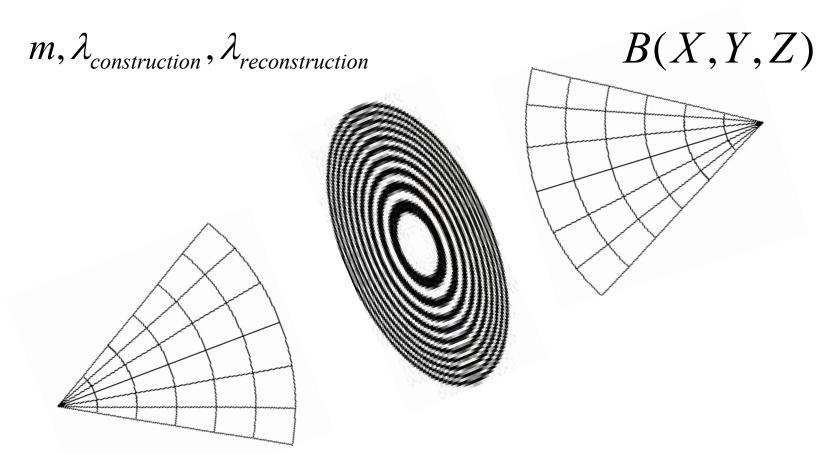


Modeling Diffractive Optics

- Two point construction model
- Phase function
- Sweatt model



Two point construction model



A(X,Y,Z)

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Nonparaxial Imaging, Magnification, and Aberration Properties in Holography*

EDWIN B. CHAMPAGNE

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THE UNIVERSITY OF ARIZONA®

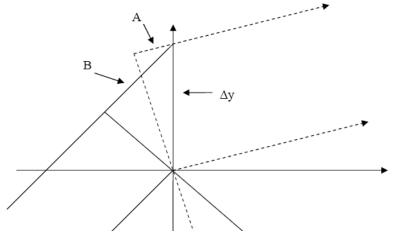
Phase model

$$\phi(\rho) = 2\pi \cdot (a\rho^2 + b\rho^4 + c\rho^6 + d\rho^8 +)$$

$$\rho = \sqrt{x^2 + y^2}$$



Phase model



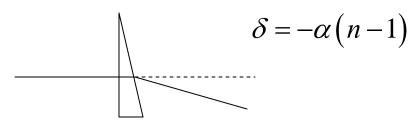
$$n'\sin(I') \cdot \Delta y = n\sin(I) \cdot \Delta y$$

$$n'\sin(I') \cdot \Delta y - n\sin(I) \cdot \Delta y = \Delta \varphi(y)$$

$$n'\sin(I') - n\sin(I) = \frac{\Delta\varphi(y)}{\Delta y} \to \frac{\partial\phi(y)}{\partial y}$$
$$\frac{\partial\phi(y)}{\partial y} = n'\sin(I') - n\sin(I)$$

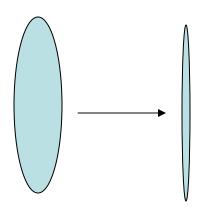


Sweatt's model



 α

For n~10,000 alpha must be very small to maintain The same deviation



$$\varphi = \frac{n-1}{r}$$

For a plano convex lens with n~10,000 The radius must be very long to maintain The same optical power.



Sweatt Model justification

Start with the diffraction grating equation

$$n'\sin(I') - n\sin(I) = \left[n'\cos(I') - n\cos(I)\right] \cdot \frac{m\lambda (1/d)}{n'\cos(I') - n\cos(I)}$$
$$n'\sin(I') - n\sin(I) = \left[n'\cos(I') - n\cos(I)\right] \cdot \tan(\alpha)$$

$$n'\{\sin(I') - \cos(I')\tan(\alpha)\} = n\{\sin(I) - \cos(I)\tan(\alpha)\}$$

$$n'\{\cos(\alpha)\sin(I')-\cos(I')\sin(\alpha)\}=n\{\cos(\alpha)\sin(I)-\cos(I)\sin(\alpha)\}$$

$$n'\{\sin(I'-\alpha)\} = n\{\sin(I-\alpha)\}$$



Sweatt's Model

$$n'\{\sin(I'-\alpha)\} = n\{\sin(I-\alpha)\}$$

$$\tan(\alpha) = \frac{m\lambda \ (1/d)}{n'\cos(I') - n\cos(I)}$$

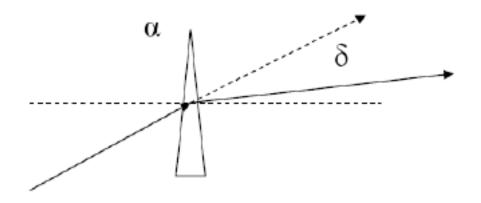
For large n's then α is negligible and we have:

$$n'\sin(I) = n\sin(I)$$

Thus for high index diffraction becomes like refraction!



Dispersion in Sweatt's model



$$\begin{split} & \delta = -\alpha \left(n - 1 \right) \\ & Sin\left(I' \right) \cong Sin\left(I \right) + \left(n_d - 1 \right) \alpha \\ & \Delta \cong Sin\left(I'_F \right) - Sin\left(I'_C \right) \cong \left(n_F - n_C \right) \alpha \\ & \frac{\delta}{\Delta} = \nu_{refractive} = \frac{\left(n_d - 1 \right) \alpha}{\left(n_F - n_C \right) \alpha} = \frac{\lambda_d \left(10,000 \right)}{\lambda_F \left(10,000 \right) - \lambda_C \left(10,000 \right)} = \frac{\lambda_d}{\lambda_F - \lambda_C} \cong -3.5 \end{split}$$



Dispersion in Sweatt's model

Consistent with diffraction case

$$Sin(I'_{d}) - Sin(I_{d}) = \frac{m\lambda_{d}}{d} \cong \delta$$

$$\Delta \cong Sin(I'_{F}) - Sin(I'_{C}) = m\frac{\lambda_{F} - \lambda_{C}}{d}$$

$$\frac{\delta}{\Delta} = \nu_{refractive} \cong \frac{m\frac{\lambda_{d}}{d}}{m\frac{\lambda_{F} - \lambda_{C}}{d}} = \frac{\lambda_{d}}{\lambda_{F} - \lambda_{C}}$$

In conclusion:

To include dispersion in the Sweatt model make the index of refraction equal to the wavelength times 10,000

Schott: $n(\lambda)^2 = A + B\lambda^2 + ...$



Structural coefficients: Thin lens (stop at lens)

 $X = \frac{c_1 + c_2}{c_1 - c_2} = \frac{r_2 + r_1}{r_2 - r_1}$

 $\phi = \Delta n \Delta c = (n-1)(c_1 - c_x)$

 $Y = \frac{1+m}{1-m} = \frac{u'+u}{u'-u}$

$$S_I = \frac{1}{4} y^4 \phi^3 \left[AX^2 - BXY + CY^2 + D \right]$$

$$S_{II} = \frac{1}{2} \mathcal{K} y^2 \varphi^2 \left[EX - FY \right]$$

$$S_{III} = \mathcal{K}^2 \varphi$$

$$S_{IV} = \mathcal{K}^2 \varphi \frac{1}{n}$$

$$S_V = 0$$

$$C_L = y^2 \phi \frac{1}{v}$$

$$C_T = 0$$

$$A = \frac{n+2}{n(n-1)^2}$$

$$B = \frac{4(n+1)}{n(n-1)}$$

$$C = \frac{3n+2}{n}$$

$$D = \frac{n^2}{\left(n-1\right)^2}$$

$$E = \frac{n+1}{n(n-1)}$$

$$F = \frac{2n+1}{n}$$



Diffractive lens (n very large @ X=0)

Structural aberration coefficients of a thin lens (Stop at lens)	
Paraxial identities	
$\phi = (n'-n) \cdot (c_1 - c_2) = (n'-n) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$	
$X = \frac{c_1 + c_2}{c_1 - c_2} = -\frac{R_1 + R_2}{R_1 - R_2}$	$Y = \frac{w' + w}{w' - w} = \frac{1 + m}{1 - m}$
$c_1 = \frac{1}{2} \frac{\phi}{n-1} (X+1)$	$c_2 = \frac{1}{2} \frac{\phi}{n-1} (X - 1)$
$w = u = -\frac{1}{2}(Y - 1)(\phi \cdot y)$	$w' = u' = -\frac{1}{2}(Y+1)(\phi \cdot y)$
Structural aberration coefficients	
$\sigma_I = AX^2 - BXY + CY^2 + D$	$A = \frac{n+2}{n(n-1)^2}$
$\sigma_{II} = EX - FY$	$B = \frac{4(n+1)}{n(n-1)}$
$\sigma_{I\!I}=1$	$C = \frac{3n+2}{n}$
$\sigma_{IV} = \frac{1}{n}$	$D = \frac{n}{(n-1)^2}$
$\sigma_{\nu} = 0$	$E = \frac{n+1}{n(n-1)}$
$\sigma_L = \frac{1}{\nu}$ $\sigma_T = 0$	$F = \frac{2n+1}{n}$
$\sigma_T = 0$	

$$\sigma_{I} = 3Y^{2} + 1$$
 $\sigma_{II} = -2Y$
 $\sigma_{III} = 1$
 $\sigma_{IV} = 0$
 $\sigma_{V} = 0$
 $\sigma_{L} = \frac{1}{v_{diffractive}}$
 $\sigma_{T} = 0$
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C=3; D=1; F=2

Aberration coefficients for Y=1; X=0

$$S_{II} = \frac{y^4}{f^3} \left(\frac{\lambda}{\lambda_0}\right)^3 \qquad S_{III} = \frac{\mathcal{K}^2}{f} \left(\frac{\lambda}{\lambda_0}\right)$$

$$S_{II} = \frac{-y^2}{f^2} \mathcal{K} \left(\frac{\lambda}{\lambda_0}\right)^2 \qquad S_{IV} = 0$$

For general case one needs to be careful as the shape depends on the index for a given power.



Structural coefficients for diffractive lens

Ctt1-1t	
Structural aberration coefficients of a thin lens (Stop at lens)	
Paraxial identities	
$\phi = (n'-n) \cdot (c_1 - c_2) = (n'-n) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$	
$X = \frac{c_1 + c_2}{c_1 - c_2} = -\frac{R_1 + R_2}{R_1 - R_2}$	$Y = \frac{w' + w}{w' - w} = \frac{1 + m}{1 - m}$
$c_1 = \frac{1}{2} \frac{\phi}{n-1} (X+1)$	$c_2 = \frac{1}{2} \frac{\phi}{n-1} (X - 1)$
$w = u = -\frac{1}{2}(Y - 1)(\phi \cdot y)$	$w' = u' = -\frac{1}{2}(Y+1)(\phi \cdot y)$
Structural aberration coefficients	
$\sigma_I = AX^2 - BXY + CY^2 + D$	$A = \frac{n+2}{n(n-1)^2}$
$\sigma_{II} = EX - FY$	$B = \frac{4(n+1)}{n(n-1)}$
$\sigma_{III}=1$	$C = \frac{3n+2}{n}$
$\sigma_{IV} = \frac{1}{n}$	$D = \frac{n^2}{(n-1)^2}$
$\sigma_{v}=0$	$E = \frac{n+1}{n(n-1)}$
$\sigma_L = \frac{1}{\nu}$	$F = \frac{2n+1}{n}$
$\sigma_T = 0$	

$$\sigma_{I} = \frac{4}{(\phi R_{2})^{2}} - \frac{8Y}{\phi R_{2}} + 3Y^{2} + 1$$

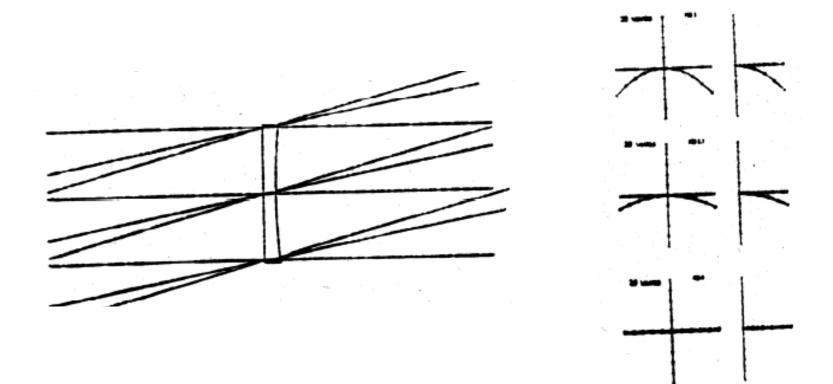
$$\sigma_{II} = \frac{2}{\phi R_2} - 2Y$$

$$\sigma_{IV} = 0$$
 $\sigma_{V} = 0$

$$\sigma_L = \frac{1}{\text{Prof. Jose Sas}}$$

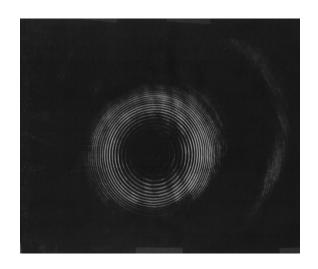
$$\sigma_T = 0$$
 $\sigma_{III} = 1$ College of Optical Sciences

Field curvature correction hybrid lens





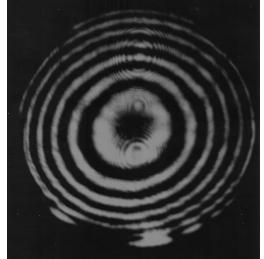
Verification

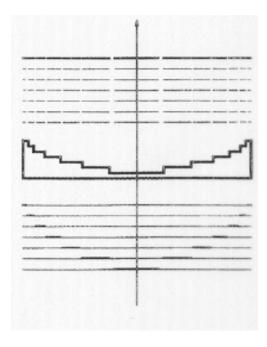




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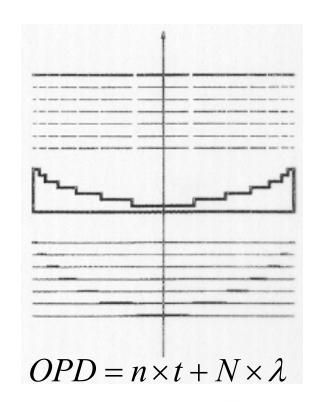




OPD Alternate view

 OPD has two parts. One is due to material dispersion, the other to due to diffraction

$$\begin{aligned} OPD_F &= \frac{y^2}{2R} \Biggl((n_F - 1) + (n_d - 1) \frac{\lambda_F}{\lambda_d} \Biggr) \\ OPD_F - OPD_C &= \frac{y^2}{2R} \Biggl((n_F - 1) + (n_d - 1) \frac{\lambda_F}{\lambda_d} \Biggr) \\ &- \frac{y^2}{2R} \Biggl((n_C - 1) + (n_d - 1) \frac{\lambda_C}{\lambda_d} \Biggr) \\ &= \frac{y^2}{2R} \Biggl((n_F - n_C) + (n_d - 1) \frac{\lambda_F - \lambda_C}{\lambda_d} \Biggr) \\ &= \frac{y^2}{2} \phi \Biggl(\frac{1}{v_{ref}} + \frac{1}{v_{diff}} \Biggr) \end{aligned}$$





Spherical aberration

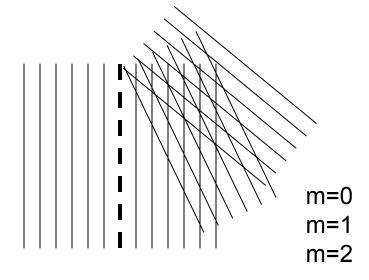
 Depending on the zone boundary distribution DOE axially symmetric DOE can introduce different orders of spherical aberration

$$\phi(\rho) = 2\pi \cdot (a\rho^2 + b\rho^4 + c\rho^6 + d\rho^8 +)$$



Calculating order efficiency

- Simple case of an amplitude device with a square wave profile
- Duty cycle



$$\psi(x,y) = A_p Comb(x - nx_0) **rect\left(\frac{x}{d}\right)$$



Square wave

$$F(v) \cong \frac{A}{2}SINC\left(\frac{v}{2v_0}\right) \sum_{-\infty}^{\infty} \partial(v - nv_0) = \frac{A}{2} \sum_{-\infty}^{\infty} SINC\left(\frac{n}{2}\right) \partial(v - nv_0)$$

$$f(t) = square \ wave = \frac{A}{2} \sum_{-\infty}^{\infty} SINC\left(\frac{n}{2}\right) e^{i2\pi nv_0 t}$$

$$= \frac{A}{2} + \frac{A}{\pi} \left[e^{i2\pi n v_0 t} + e^{-i2\pi n v_0 t} \right] + \frac{A}{3\pi} \left[e^{i2\pi n 3 v_0 t} + e^{-i2\pi n 3 v_0 t} \right]$$

$$+\frac{A}{5\pi} \left[e^{i2\pi n5\nu_0 t} + e^{-i2\pi n5\nu_0 t} \right] + \frac{A}{7\pi} \left[e^{i2\pi n7\nu_0 t} + e^{-i2\pi n7\nu_0 t} \right] + \dots$$

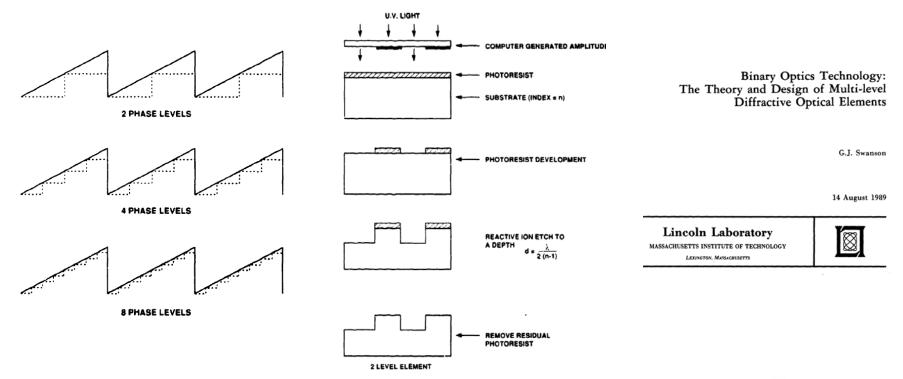
$$\nu_0 = T^{-1}$$



50% duty cycle
$$\left(\frac{1}{\pi}\right)^2 \approx 0.1$$



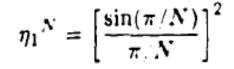
Binary optics technology

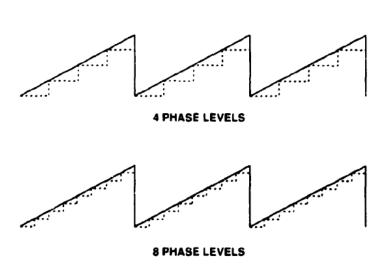




Efficiency for binary optics

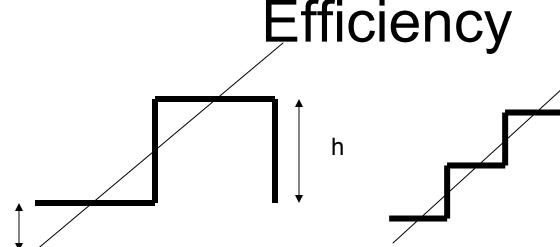






Number of Levels N	First-Order Efficiency $\eta_1{}^N$
2	0 41
3	0.68
. 4	0.81
5	0.87
6	0.91
8	0.95
12	0.98
16	0.99
<u>. </u>	





$$\sigma^{2} = (n-1)^{2} \frac{1}{2} \int_{-1}^{1} \left(\frac{hx}{N}\right)^{2} dx = (n-1)^{2} \left(\frac{h}{N}\right)^{2} \frac{1}{2} x^{3} \frac{1}{3} \Big|_{-1}^{1} = \frac{1}{3} (n-1)^{2} \left(\frac{h}{N}\right)^{2}$$

$$h = 1$$

$$(n-1)2h = \lambda$$

$$\sigma^{2} = \frac{1}{3} \frac{4}{4} (n-1)^{2} \left(\frac{h}{N}\right)^{2} = \frac{1}{12} \lambda^{2} \left(\frac{1}{N}\right)^{2}$$

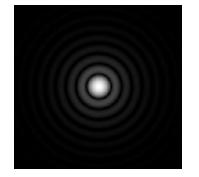
$$S \approx 1 - \frac{\pi^2}{3} \left(\frac{1}{N}\right)^2$$

$$N = 2$$
; $S = 0.17$

$$N = 4$$
; $S = 0.794$

$$N = 8 : S = 0.948$$

$$N = 16 : S = 0.987$$



$$S \approx 1 - \left(\frac{2\pi}{\lambda}\sigma\right)^2$$

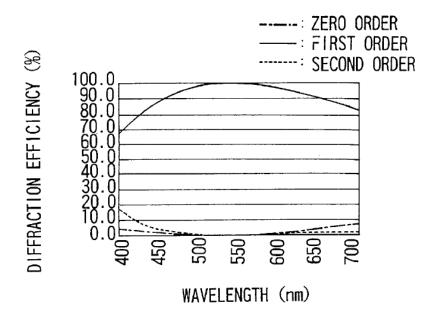


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h/N

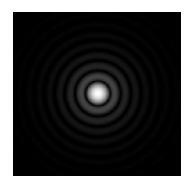
Efficiency

$$\varepsilon = \sin c^{2} \left(\pi \left[\frac{\lambda_{construction}}{\lambda_{reconstruction}} \frac{n(\lambda_{reconstruction}) - 1}{n(\lambda_{construction}) - 1} - m \right] \right)$$



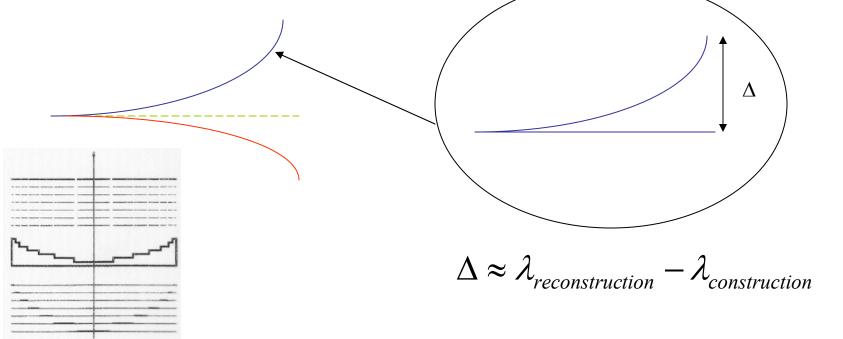


Efficiency



$$\varepsilon \approx 1 - \left(\frac{2\pi}{\lambda}\sigma\right)^{2} \approx 1 - \left(\frac{2\pi}{\lambda_{reconstruction}} \frac{\lambda_{reconstruction}}{\lambda_{reconstruction}} \frac{\lambda_{reconstruction}}{3}\right)^{2}$$

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Comparison

Standard lens, Fresnel lens and DOE lens







Refracting lens
Prof. Jose Sasian



Fresnel lens



DOE lens



Images of extended objects

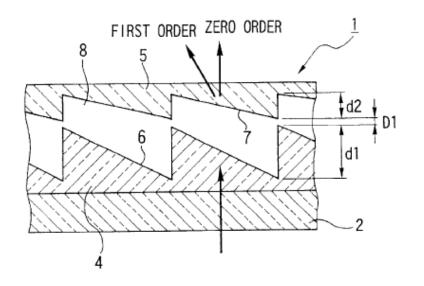


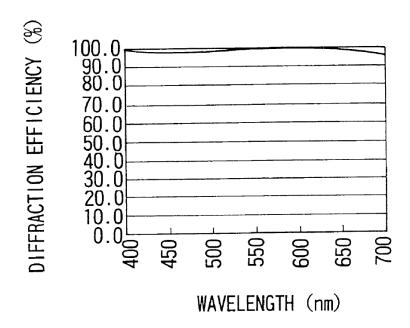
Acrylic powerless lens

Other orders produce images at different magnifications
Like ghost images



Canon's multilayer DOE's







How does it work?

US006507437B1

(12) United States Patent Nakai

(10) Patent No.: US 6,507,437 B1 (45) Date of Patent: *Jan. 14, 2003

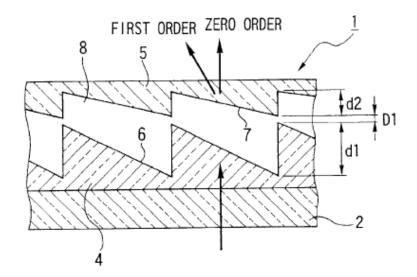
(54)	DIFFRACTIVE OPTICAL ELEMENT AND
	PHOTOGRAPHIC OPTICAL SYSTEM
	HAVING THE SAME

(75) Inventor: Takehiko Nakai, Kawasaki (JP)

(73) Assignee: Canon Kabushiki Kaisha, Tokyo (JP)

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JP	10-133149	5/1998





How does it work?

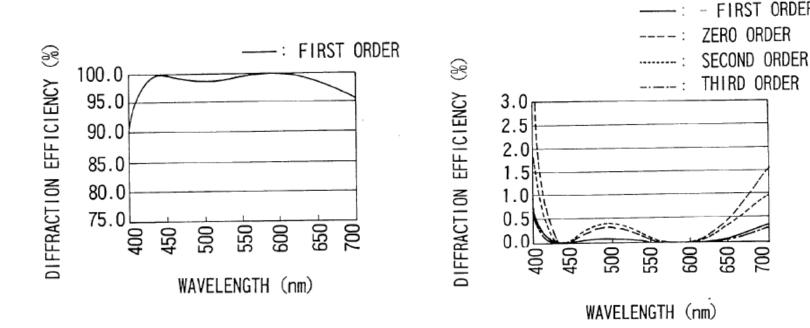
$$\varepsilon(\lambda) = \sin c^{2} \left(\pi \left[\frac{\lambda_{construction}}{\lambda_{reconstruction}} \frac{n(\lambda_{reconstruction}) - 1}{n(\lambda_{construction}) - 1} - m \right] \right)$$

$$\varepsilon(\lambda) = \sin c^{2} \left(\pi \left[d \frac{n(\lambda_{reconstruction}) - 1}{\lambda_{reconstruction}} - m \right] \right) = \sin c^{2} \left(\pi \left[\frac{d_{construction}}{d_{reconstruction}} - m \right] \right)$$

$$\varepsilon(\lambda) = \sin c^{2} \left(\pi \left[d_{2} \frac{n_{2} (\lambda_{reconstruction}) - 1}{\lambda_{reconstruction}} \pm d_{1} \frac{n_{1} (\lambda_{reconstruction}) - 1}{\lambda_{reconstruction}} - m \right] \right)$$
or $d_{2} \left(n_{2} (\lambda_{reconstruction}) - 1 \right) \pm d_{1} \left(n_{1} (\lambda_{reconstruction}) - 1 \right) = \lambda_{reconstruction}$



100% at two wavelengths



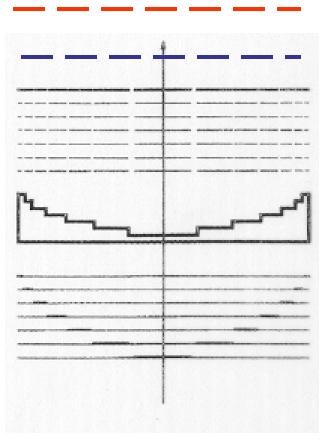


650

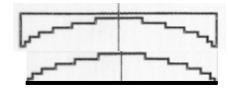
200

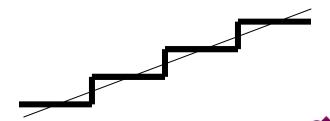
FIRST ORDER

Alternate view 100% efficiency at 2λ (no ripple)



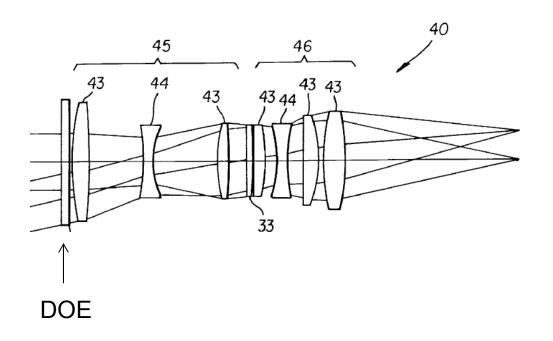
$$\lambda_2 = 2\lambda_1 = 2(450nm)$$







An actual lens application for controlling chromatic change of magnification



(12) United States Patent Harrigan

- (54) MOVIE PROJECTION LENS
- (75) Inventor: Michael Harrigan, Webster, NY (US)
- (73) Assignee: Eastman Kodak Company, Rochester, NY (US)

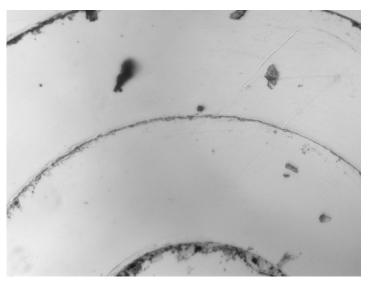
(10) Patent No.: US 6,317,268 B1

(45) **Date of Patent:** Nov. 13, 2001

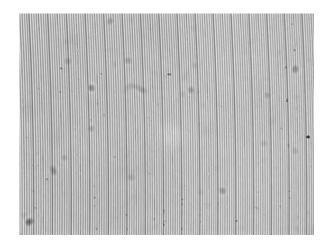
Note lack of lens symmetry about the stop



Some Fresnel lens and DOE photographs



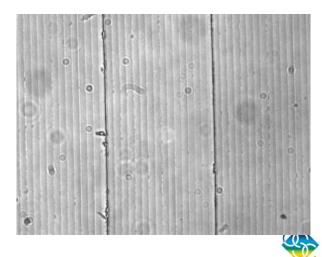
Plastic Fresnel lens; Diamond turned and replicated



Prof. J. Bingrya levels



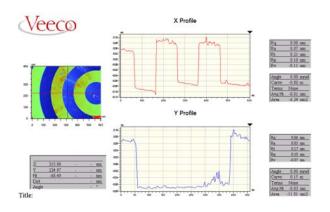
Gray scale; note binary edge



College of Optical Sciences

Binary 16 levels

Measurement of a DOE

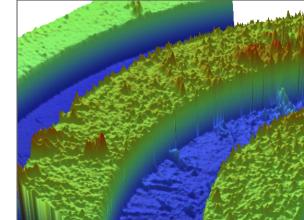


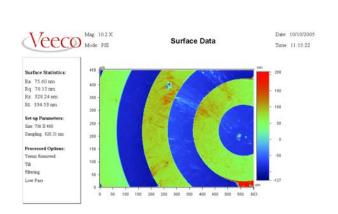
3-Dimensional Interactive Display

Date: 10/10/2005

Time: 11:15:22

College of Optical Sciences





Surface Stats:

Ra: 75.60 nm Rq: 78.15 nm

Rt: 334.53 nm

Measurement Info:

Magnification: 10.24 Measurement Mode: PSI Sampling: 820.31 nm

Array Size: 736 X 480

Beware

- Modeling assumes DOEs having no physical structure
- Real modeling faces sampling issues
- Scalar treatment
- Zones are about ~7λ or more
- Light scattered at boundaries and zone shadowing effects
- Fabrication: Diamond turning, microlithography printing techniques, Grey scale techniques.



Examples

- Diffractive landscape lens
- Correction of chromatic change in the landscape lens, eyepieces, fish-eye lenses, unsymmetrical lenses
- Null-corrector Certifier
- Modeling a few zones

