Introduction to Machine Learning

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September 23, 2025

Outline

The problems of Machine Learning (1 week) Introduction

Estimation

Answering a scientific problem Pandas and dataframes Single variable models Two variable models

Statistics, validation and model selection

Course summary Course Contents

For next week Assigment Reading

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Machine Learning And Data Mining

The nuts and bolts

- Models
- ► Algorithms
- ► Theory
- Practice

■Workflow

- Scientific question
- Formalisation of the problem
- Data collection
- Analysis and model selection

Types of <u>I</u> statistics / ***** machine learning problems

- Classification
- Regression
- ▶ Density estimation
- ► Reinforcement learning



Machine learning

Data Collection

- Downloading a clean dataset from a repository
- Scraping data from the web
- Conducting a survey
- Performing experiments, and obtaining measurements.

Modelling

- Simple: the bias of a coin
- Complex: a language model.
- The model depends on the data and the problem

Algorithms and Decision Making

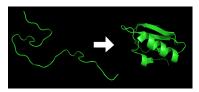
- ▶ We want to use models to make decisions.
- Decisions are made every step of the way.
- Both humans and algorithms can make decisions.



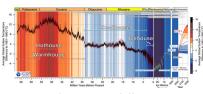
The main problems in machine learning and statistics



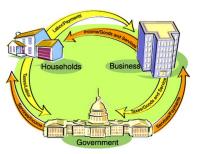
Matter



Protein Folding

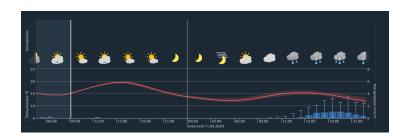


Climate Modelling



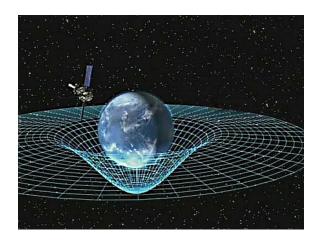
Economic Policy

Prediction



- ▶ Will it rain tomorrow?
- ► How much will bitcoin be worth next year?
- ▶ When is the next solar eclipse?

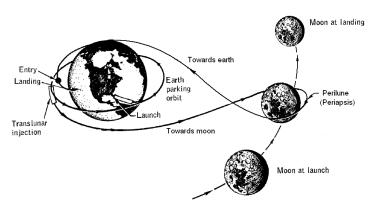
Inference



- ▶ What is the law of gravitation?
- ▶ Where is the spaceship now?
- ▶ Does my poker opponent have two aces?



Decision Making



- ► What data should I collect?
- ► Which model should I use?
- Should I fold, call, or raise in my poker game?
- How can I get a spaceship to the moon and back?
- ./fig/artemis.gif



The need to learn from data

Problem definition

- ▶ What problem do we need to solve?
- ► How can we formalise it?
- ▶ What properties of the problem can we learn from data?

Data collection

- ▶ Why do we need data?
- What data do we need?
- ► How much data do we want?
- ► How will we collect the data?

Modelling and decision making

- ► How will we compute something useful?
- ► How can we use the model to make decisions?

Course Material

Moodle

- Assignments and proejct
- Additional reading material
- Asking questions

Course Github

- .org files for notes, PDF for slides
- source code for examples

Course literature

- An Introduction to Statistical Learning with Python
- Book chapters will be mentioned in the course





Assignment, teaching and questions

Assignments and project

- ► Indidivual weekly assignments in the first half
- Group project in the second half
- Project presentation
- No exam.

Other questions

- Use Moodle for technical/administrative questions: That way everybody gets the same information.
- Use email for personal problems or extra help, if the moodle is not enough.
- Complicated questions can be answered at the next lecture

Office hours

- ▶ Tuesdays 13:15-14:00: book with an email to avoid clashes.
- ► Email me for an appointment outside those hours.



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Problem definition

Example: Health, weight and height

Example (Health questions regarding height and weight)

- ▶ What is a normal height and weight?
- ► How are they related to health?
- What variables affect height and weight?

Define a research question

Find a non-sensitive variable that we can easily measure via a survey, e.g. related to sleep, smoking, exercise, food, politics, sports, hobbies etc.

- Discuss in small groups and post suggestions
- We then vote for what to measure

Data collection

Think about which variables we need to collect to answer our research question.

Necessary variables

The variables we need to know about

- Weight
- Height
- Dependent: (health/vote/opinion/salary)

Auxiliary variables

Measurable factors related to the variables of interest

Possible confounders

Hidden factors that might affect variables

Class data and variables

▶ The class enters their data into the excel file.



Pay attention to the variables we wish to measure

Privacy

▶ Is the use of a pseudonym sufficient to hide your identity?

Variables

The class data looks like this

First Name	Gender	Height	Weight	Age	Nationality	Smoking
Lee	М	170	80	20	Chinese	10
Fatemeh	F	150	65	25	Turkey	0
Ali	Male	174	82	19	Turkish	0
Joan	N	5'11	180	21	American	4

► X: Everybody's data

 \triangleright x_t : The t-th person's data

 \triangleright $x_{t,k}$: The k-th feature of the t-th person.

 $ightharpoonup x_k$: Everybody's k-th feature

Raw versus neat data

▶ Neat data: $x_t \in \mathbb{R}^n$

▶ Raw data: web pages, handwritten text, graphs, data packets, with missing/incorrect values, etc

Types of learning problems

Unsupervised learning (unconditional estimation)

- ▶ Predict the gender of an unknown individual.
- Predict the height.
- Predict the height and weight?

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Supervised learning problems (conditional estimation)

- ► Classification: Can we predict gender from height/weight?
- ▶ Regression: Can we predict weight from height and gender?
- ► In both cases we <u>predict</u> <u>output</u> variables from <u>input</u> variables

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Variables

- ▶ Input variables: aka features, predictors, independent variables
- Output variables: aka response, dependent variables, labels, or targets.
- ▶ The input/output dichotomy only exists in some prediction problems.



Python pandas for data wrangling

Reading class data

```
import pandas as pd
X = pd.read_excel("data/class.xlsx")
X["First Name"]
```

- Array columns correspond to features
- Columns can be accessed through namesx

Summarising class data

```
X.hist()
import matplotlib.pyplot as plt
plt.show()
```

Pandas and DataFrames

- Data in pandas is stored in a DataFrame
- ▶ DataFrame is not the same as a numpy array.

Core libraries

```
import pandas as pd
import numpy as np
```

Series: A sequence of values

```
# From numpy array:
s = pd.Series(np.random.randn(3), index=["a", "b", "c"])
# From dict:
d = {"a": 1, "b": 0, "c": 2}
s = pd.Series(d)
# accessing elemets
s.iloc[2] #element 2
s.iloc[1:2] #elements 1,2
s.array # gets the array object
s.to_numpy() # gets the underlying numpy array
```

DataFrames

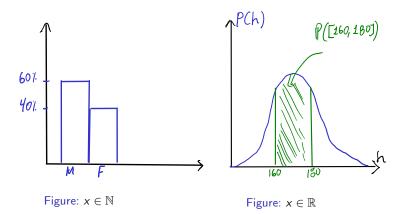
Constructing from a numpy array

Constructing from a dictionary

Access

```
X["First Name"] # get a column
X.loc[2] # get a row
X.at[2, "First Name"] # row 2, column 'first name'
X.loc[2].at["First Name"] # row 2, element 'first name' of the s
X.iat[2,0] # row 2, column 0
```

Modelling single variables



Means using python

Example (Calculating the mean of our class data)

```
X.mean() # gives the mean of all the variables through pandas.co
X["Height"].mean()
np.mean(X["Weight"])
```

- ▶ The mean here is fixed because we calculate it on the same data.
- ▶ If we were to collect new data then the answer would be different.

Example (Calculating the mean of a random variable)

```
import numpy as np
X = np.random.gamma(170, 1, size=20)
X.mean()
np.mean(X)
```

▶ The mean is random, so we get a different answer everytime.

Definition (The expected value)

 $ightharpoonup \Omega$: random outcome space

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- ▶ i.i.d. variables $x_1, \ldots, x_t, \ldots, x_T$: with $x_t = x(\omega_t), \omega_t \sim P$.
- ▶ The sample mean of $x_1, ..., x_T$ is

$$\frac{1}{T} \sum_{t=1}^{T} x_t$$

The sample mean is $O(1/\sqrt{T})$ -close to $\mathbb{E}_P[x_t]$ with high probability.



Reminder: expectations of random variables

A gambling game

What are the expected winnings if you play this game?

- ► [a] With probability 1%, you win 100 CHF
- ▶ [b] With probability 40%, you win 20 CHF.
- ► [c] Otherwise, you win nothing

Solution

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- Let x be the amount won, then x(a) = 100, x(b) = 20, x(c) = 0.
- ► We need to calculate

$$\mathbb{E}_{P}(x) = \sum_{\omega \in \{a,b,c\}} x(\omega)P(\omega) = x(a)P(a) + x(b)P(b) + x(c)P(c)$$

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▶ P(c) = 59%, as $P(\Omega) = 1$. Substituting,

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, as $P(\Omega)=1$. Substituting, $\mathbb{E}_P(x)=1+8+0=9$.

Models

Models as summaries

- ▶ They summarise what we can see in the data
- ► The ultimate model of the data is the data

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Example models

- A numerical mean
- A linear classifier
- A linear regressor
- A deep neural network
- ► A Gaussian process
- A large language model



Estimates and decisions

We always need to make decisions based on some estimates.

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Estimate the bias of a coin

- ▶ I give you a coin that, lands with some fixed probability on heads.
- ▶ You are allowed to experiment with the coin.
- ► I will pay you 1 CHF if you guess the throw correctly
- Otherwise you pay me x CHF.
- How much should I ask you to pay for the bet to be fair?
- What do you need to know to determine this?

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Example (If the coin is fair)

- ► If the coin is fair, then you only have 50% proability of guessing correctly.
- ▶ If you bet x CHF, your expected return is x

Definition (Bernoulli distribution)

We say that $x \in \{0,1\}$ has Bernoulli distribution with parameter θ and write

$$x \sim \text{Bernoulli}(\theta),$$

when

$$\mathbb{P}(x) = \begin{cases} \theta & x = 1 \\ 1 - \theta & x = 0. \end{cases}$$

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Example (Applications of the Bernoulli distribution)

- ► A biased coin flip.
- Classification errors.

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Example (Applications of the Bernoulli distribution)

- A biased coin flip.
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Exercise: The expected value

If x is Bernoulli with parameter θ , then what is the expected value of

- ▶ The variable f(x) = x 1?
- ► The variable $g(x) = (x-1)^2$?



Two-variable models

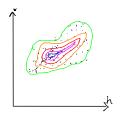


Figure: $x \in \mathbb{R}^2$

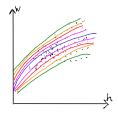


Figure: $x \in \mathbb{R} \to y \in \mathbb{R}$

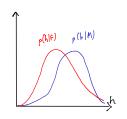


Figure: $x \in \mathbb{N} \to y \in \mathbb{R}$

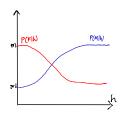


Figure: $x \in \mathbb{R} \to y \in \mathbb{N}$

Predicting y from x, discrete case.

Consider two variables, x, y. We can either care about

- $ightharpoonup \mathbb{E}[y|x]$ the expectation of y for all x.
- $ightharpoonup \mathbb{P}[y|x]$ the distribution of y for all x.

Probability table for P(x, y)

P(x,y)	y = 0	y = 1
x = 0	54%	6%
x = 1	16%	24%

- ► How can we graph this?
- ▶ What is P(x)?

Conditional probability table for P(y|x)

$P(y \mid x)$	y = 0	y = 1
$\times = 0$	90%	10%
x = 1	40%	60%

▶ What is $\mathbb{E}[y \mid x]$?



Distributions of two variables

In this setting, both x and y have a Bernoulli distribution. If we use a model whereby x is sampled first, and then y, then we can define two Bernoulli distributions. The first, for x is unconditional, while the second, for y, depends on the value of x:

$$x \sim \text{Bernoulli}(\theta)$$

 $y \mid x \sim \text{Bernoulli}(\phi_x).$

In our example, $\phi_0 = 0.1$ and $\phi_1 = 0.6$.

Homework

Probability table for P(x, y)

P(x,y)	y = -1	y = 0	y = 1
x = 0	10%	20%	10%
x = 1	30%	20%	10%

Given the above table, calculate

- ► *P*(*x*)
- ▶ The conditional probability table for P(y|x).
- $ightharpoonup \mathbb{E}_P[f]$ where f(x,y) = x + y.
- ▶ $\mathbb{E}_P[g]$ where g(x, y) = xy.
- $ightharpoonup \mathbb{E}[y|x]$ for all values of x.

Two variables: conditional expectation

The height of different genders

The conditional expected height

$$\mathbb{E}[\textit{h} \mid \textit{g} = 1] = \sum_{\omega \in \Omega} \textit{h}(\omega) \textit{P}[\omega \mid \textit{g}(\omega) = 1]$$

The empirical conditional expectation

$$\mathbb{E}[h \mid g = 1] \approx \frac{\sum_{t:g(\omega_t)=1} h(\omega_t)}{|\{t:g(\omega_t)=1\}|}$$

Python implementation

Two variables: conditional expectation

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Python implementation

```
h[g==1] / sum(g==1)
## alternative
import numpy as np
np.mean(h[g==1])
```

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Populations, samples, and distributions

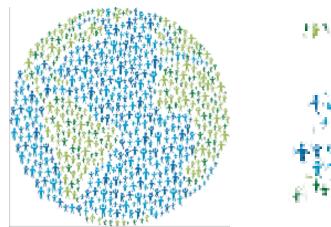


Figure: The world population



Figure: A sample

Statistical assumptions

Independent, Identically Distributed data

- $lackbox{}\omega_t \sim P$: individuals $\omega_t \in \Omega$ are drawn from some distribution P
- $ightharpoonup x_t riangleq x(\omega_t)$ are some features of the t-th individual
- ightharpoonup Here we are interested in properties of the unknown distribution P.

Representative sample from a fixed population

- ▶ Finite population $\Omega = \{\omega_1, \omega_2, \dots, \omega_N\}$
- A subset S ⊂ Ω of size T < N is selected with a uniform distribution, i.e. so that</p>

$$P(S) = T/N, \quad \forall S \subset \Omega.$$

- ▶ Here we are interested in statistics of the unknown population Ω .
- ▶ We assume an underlying distribution *P* for convenience.
- We can tried both cases essentially the same.



Unsupervised learning

- ▶ Given data $x_1, ..., x_T$.
- ► Learn about the data-generating process.
- Example: Estimation, compression, text/image generation

Unsupervised learning

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Supervised learning

- ightharpoonup Given data $(x_1, y_1), \ldots, (x_T, y_T)$
- ▶ Learn about the relationship between x_t and y_t .
- Example: Classification, Regression

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Online learning

- ▶ Sequence prediction: At each step t, predict x_{t+1} from x_1, \ldots, x_t .
- Conditional prediction: At each step t, predict y_{t+1} from $x_1, y_1, \dots, x_t, y_t, x_{t+1}$

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Reinforcement learning

Learn to act in an unknown world through interaction and rewards



Training data

- ► Calculations, optimisation
- ▶ Data exploration

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Validation data

- Fine-tuning
- ► Model selection

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Test data

▶ Performance comparison

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Simulation

- ► Interactive performance comparison
- ▶ White box testing

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Real-world testing

Actual performance measurement



Model selection

- ► Train/Test/Validate
- ► Cross-validation
- ► Simulation

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Two variable models

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For next week Assigment Reading

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Models

- k-Nearest Neighbours.
- Linear models and perceptrons.
- Multi-layer perceptrons (aka deep neural networks).
- ► Bayesian Networks

Algorithms

- ► (Stochastic) Gradient Descent.
- Bayesian inference.

Reproducibility

- Modelling assumptions
- Interactions and feedback

Fairness

- ► Implicit biases in training data
- ► Fair decision rules and meritocracy



The problems of Machine Learning (1 week) Introduction

Estimation

Answering a scientific problem Pandas and dataframes
Single variable models
Two variable models

Statistics, validation and model selection

Course Summary
Course Contents

For next week Assigment Reading

Assignment

► See Moodle

Reading for this week

ISLP Chapter 1