Prof. Sharad Shah's
SHAH
EDUCATIONAL ACADEMY
Bandra West | Vile Parle West

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S.Y.J.C (2019-20)	MATHS PAPER - I	19/12/19 Thursday	Time:- 1 ½ hrs. Marks: 40
Unit Test 2	*Topic: Logic, Matrices; Application of Derivative & Applications of Definite Integrals*		08.00 am to 09.30 am

Q.1. **(8)**

(i) Express the following statement in symbolic form and write its truth value.
"If 4 is an odd number, then 6 is divisible by 3."

(ii) Find the values of x and y, if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

(iii) Prove that the following statement pattern is equivalent
$(p \vee q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$

(iv) The price P for demand D is given as $P = 183 + 120D - 3D^2$.
Find D for which the price is increasing.

Q.2. **(20)**

(i) Find the inverse of the following matrix by elementary row transformations if it exists.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{bmatrix}$$

(ii) If $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ show that $A^2 - 3A + I = 0$ Hence Find A^{-1}

(iii) The expenditure E_c of a person with income I is given by $E_c = (0 \cdot 000035)\,I^2 + (0 \cdot 045)\,I$. Find marginal propensity to consume (MPC) and marginal propensity to save (MPS) when I = 5000. Also find A (average) PC and A (average) PS.

(iv) Express the truth of each of the following statements by Venn diagram :

(a) Some hardworking students are obedient.

(b) No circles are polygons.

(c) All teachers are scholars and scholars are teachers.

(d) If a person is sincere then he/she is teacher

(V) Discuss extreme values of the function $f(x) = x \log x$

(12)

Q.3. (i) If p : It is a day time.

q : It is warm.

Give the verbal statements for the following symbolic statements :

(a) $p \wedge \sim q$ (b) $p \vee q$ (c) $p \leftrightarrow q$

(ii) Using the truth table, examine whether the statement pattern $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$ is a tautology, a contradiction or a contingency.

(iii) The cost C of producing x articles is given as

$C = x^3 - 16x^2 + 47x.$

For what values of x will the average cost be decreasing ?

(iv) Find the volume of a solid obtained by the complete revolution of the ellipse

$\frac{x^2}{36} + \frac{y^2}{25} = 1$ about X – axis.

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Q1) (i) p : 4 is odd No
q : 6 is divisible by 3
Given statement can be written in symbol as
$p \to q$

p is False, q is True
$F \to T = T$
Given statement has Truth value T

(ii) $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\begin{bmatrix} 2+y & 6+0 \\ 0+1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

By Equality of Matrices

$2+y=5$ | $2x+2=8$
$\boxed{y=3}$ | $2x=6$
 | $\boxed{x=3}$

(3) $(p \vee q) \to r$ and $(p \to r) \wedge (q \to r)$

(PT C)

p	q	r	$p \vee q$	$(p \vee q) \to r$	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T
1	2	3	4	5	6	7	8

Entries in col 5 & col 8 are equal

Hence

$(p \vee q) \to r$ and $(p \to r) \wedge (q \to r)$ are equivalent

(iv) $P = 183 + 120D - 3D^2$

diff w.r.t. D

$\frac{dP}{dD} = 0 + 120 - 6D$

$= 120 - 6D$ — I

For increasing price

$\frac{dP}{dD} > 0$

$120 - 6D > 0$

$120 > 6D$

$\frac{120}{6} > D$

$20 > D$

i.e. $D < 20$

but D cannot be −ve

$0 < D < 20$

Q 2). $A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{vmatrix}$

$= 1(0-3) - 2(0+1) - 2(0-2)$

$= -3 - 2 + 4 = -1 \neq 0$

$\Rightarrow$ A is non singular

$\Rightarrow A^{-1}$ exists

$A A^{-1} = I$

$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_3 \to R_3 + R_1$

$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ 0 & 5 & -2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$R_3 \to R_3 + 2R_2$

$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

$R_2 \leftrightarrow R_3$

$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$R_1 + 2R_2$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$$

$R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{bmatrix}$$

$R_1 + 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{pmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 0 & 2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix} \text{ Ay}$$

ii) $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

$$A^2 - 3A + I = AA - 3A + I$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} - 3\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} 4+1 & 2+1 \\ 2+1 & 1+1 \end{bmatrix} + \begin{bmatrix} -6 & -3 \\ -3 & -3 \end{bmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} + \begin{pmatrix} -6 & -3 \\ -3 & -3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} 5-6 & 3-3 \\ 3-3 & 2-3 \end{bmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} -1+1 & 0+0 \\ 0+0 & -1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

PTO

$A^2 - 3A + I = 0$

pre multiply both sides by A^{-1}

$A^{-1}A^2 - 3A^{-1}A + 3A^{-1}I = 0$

$A - 3I + 3A^{-1} = 0$

$3A^{-1} = 3I - A$

$3A^{-1} = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} + \begin{pmatrix} -2 & -1 \\ -1 & -1 \end{pmatrix}$

$3A^{-1} = \begin{bmatrix} 3-2 & 0-1 \\ 0-1 & 3-1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

or $\begin{bmatrix} 1/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$ A

iii) $E_C = 0.000035I^2 + 0.045I$

$MPC = \frac{d}{dI}(E_C)$

$= \frac{d}{dI}(0.000035I^2 + 0.045I)$

$= 2(0.000035I) + 0.045$

$= \cancel{0.000070I + 0.045}$

$= 0.00007I + 0.045$

$I = 5000$

$MPC = 0.00007 \times 5000 + 0.045$

$= 0.35000 + 0.045$

$= 0.35 + 0.045$

$= \boxed{0.395}$

$APC = \dfrac{EC}{I} = \dfrac{0.000035 I^2 + 0.045 I}{I}$

$= \dfrac{\cancel{I}(0.000035 I + 0.045)}{\cancel{I}}$

$= 0.000035 I + 0.045$

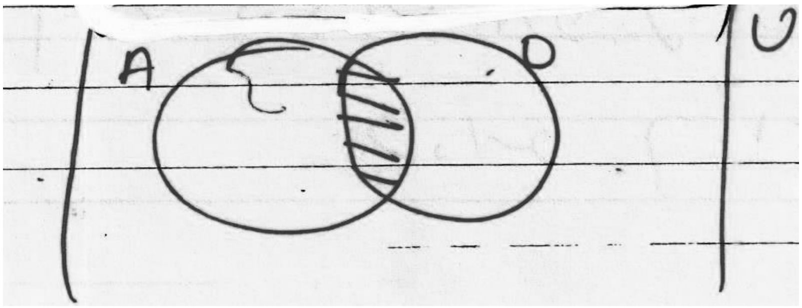
$I = 5000$

$APC = 0.000035 \times 5000 + 0.045$

$= 0.175000 + 0.045$

$= 0.175 + 0.045 = 0.220$

$\boxed{0.22}$.

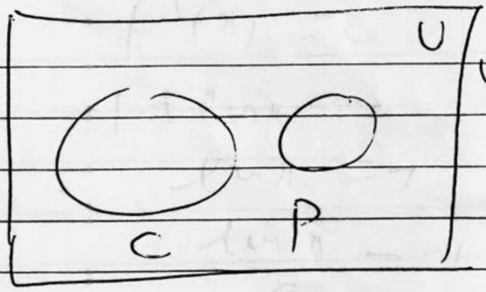


U = universal set of ~~[illegible]~~ in students

H = hardworking students

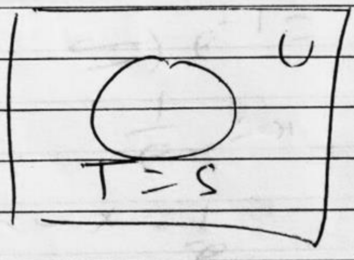
O = obedient students

(b)



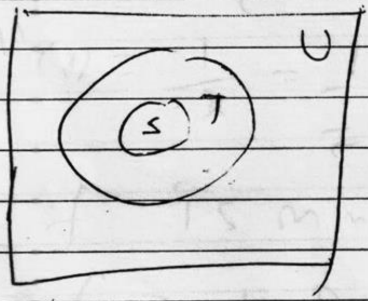
U = Universal set of 2 Dimension figure or closed figure
C = circle
P = polygon

(c)



U = Universal set of human being
T = Teachers
S = Scholars

(d)



U = universal set of human being
S = Sincere person
T = Teachers

(v) $f(x) = x \ln x$

diff. w.r.t. x, UV rule

$f'(x) = x \frac{d}{dx} \ln x + \ln x \frac{d}{dx} x$

$= x \cdot \frac{1}{x} + \ln x (1)$

$= 1 + \ln x$

diff. again

$f''(x) = 0 + \frac{1}{x} = \frac{1}{x}$

For extreme values

Since $f'(x)$

$$f'(x) = 0$$
$$1 + \ln x = 0$$
$$\ln x = -1$$
$$e^{\ln x} = e^{-1}$$
$$\Rightarrow e^{-1} = x$$
$$\frac{1}{e} = x$$

When $x = \frac{1}{e}$

$$f''(x) = \frac{1}{x} = \frac{1}{\frac{1}{e}} = e > 0$$

$\Rightarrow f$ is minimum at $x = \frac{1}{e}$

(2nd Derivative Test)

& its min value is

$$f(x) = x \ln x$$
$$= \frac{1}{e} \ln\left(\frac{1}{e}\right)$$
$$= \frac{1}{e}(\ln(1) - \ln e)$$
$$= \frac{1}{e}(0 - 1)$$
$$= \frac{-1}{e}$$

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