Prof. Sharad Shah's
SHAH EDUCATIONAL ACADEMY
SEA
SCORE EXCEL ACHIEVE

SHAH'S ADVANTAGE PRELIM SERIES

A DIVISION OF "SHAH EDUCATIONAL ACADEMY"

B.COM, BAF, BMS, BBI, BFM, MCOM, CS, CPT, IPCC, Final CA.

H.O.: 5,6 & 7, Hill Crest Society, 16th Road, Bandra (W), Mum: 50. Ph: 26051635/ 9820422953
Branch: Avon Arcade, Shop No. A/121; 1st Floor, Vile Parle (W), Mum: 56. Ph: 9820418533

CLASS: S.Y.J.C *PRELIM SERIES NO. 3* Date: 31st Jan 2020	MATHS I & II (2019 -20)	Time : 3 Hrs. Marks: 80 Day:- Friday

Note: (1) All questions are compulsory; (2) Figures to the right indicate full marks; (3) Answer to every question, must be started on a new page.; (4) Use separate answers books for each section. (5) Graph paper is not necessary for LLP.

Section – I

Q. 1. Attempt any SIX of the following : [12 marks]

(i) Write the negation of the following statements :

(a) Economic growth and per capita income is more in America.

(b) All students have completed their homework and the teacher is present.

(ii) If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -2 & 5 \end{bmatrix}$ use $R_1 \leftrightarrow R_2$ and $C_1 \to C_1 + 2C_2$ to give new matrix.

(iii) Find $\frac{dy}{dx}$ if $y = (2+x)^x$.

(iv) Evaluate $\int \frac{\tan\sqrt{x}}{\sqrt{x}}\,dx$.

(v) If the average revenue R_A is 50 and elasticity of demand η is 5, find marginal revenue R_m.

(vi) Evaluate $\int_0^{\pi/2} \cos^2 x\,dx$.

(vii) If $\int_1^a \left(3x^2 + 2x + 1\right) dx = 11$, find a.

(viii) The revenue function is given by $R = D^2 - 40D$, where D is demand of the commodity. For what values of D, the revenue is increasing ?

Q. 2(A). Attempt any TWO of the following :[6 marks]

(i) If the function f is continuous at $x = 1$, then find $f(1)$, where

$$f(x) = \frac{1+\cos\pi x}{(1-x)^2} \text{ for } x \neq 1.$$

(ii) Evaluate $\int \frac{1}{\sqrt{3+4x-4x^2}}\,dx$.

(iii) Find maximum & minimum of
$f(x) = 2x^3 - 3x^2 - 12x + 12$

Q.2 (B). Attempt any TWO of the following :- [8 marks]

(i)

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & x \\ 4 & y \end{bmatrix}$$

And $(A+B)^2 = A^2 + B^2$ find x, y

(ii) If $\operatorname{cosec}^{-1}\left(\frac{x^3 - y^3}{x^3 + y^3}\right) = \tan k$, Show $\frac{dy}{dx} = \frac{y}{x}$

(iii) Find the volume of the solid obtained by the complete revolution of the ellipse $\frac{x^2}{36} + \frac{y^2}{25} = 1$, about the X-axis.

Q. 3(A). Attempt any TWO of the following: [6 marks]

(i) Show that $(p \to q) \vee (q \to p)$ is a tautology.

(ii) Using rules of negation of compound statements write the negation of the following symbolic statements :

(1) $\sim p \wedge \sim q$ (2) $p \leftrightarrow q$ (3) $\sim p \vee \sim q$

(iii) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ Find the adjoint of the matrix A and hence find A^{-1}.

Q. 3(B). Attempt any TWO of the following: [8 marks]

(i) In a firm the cost function for output x is given as $C = \frac{x^3}{3} - 20x^2 + 70x$. Find the output for which

(i) Marginal Cost (MC) is minimum,

(ii) Average Cost (AC) is minimum.

(ii) If $f(x) = \frac{1 - \sin x}{(\pi - 2x)^2}$, for $x \neq \frac{\pi}{2}$, is continuous at $x = \frac{\pi}{2}$, then find $f\left(\frac{\pi}{2}\right)$

(iii) $\int_{\circ}^{\pi/2} \frac{1}{1+co+x} \, dx$

Section – II

Q.4. Attempt any SIX of the following : [12 marks]

(i) If the numerator of a function is increased by 20% and its denominator be diminished by 10%, the value of the fraction is $\frac{16}{21}$. Find the original fraction.

(ii) An agent charges 8% commission plus a 2% delcredere. If he sells goods worth ₹ 12,400/-, find his total earnings.

(iii) Obtain crude death rate (CDR) for the population given below:

Age group	Population	Deaths
Below 10	900	12
10 – 20	1500	8
20 – 60	4500	36
Above 60	600	30

(iv) Calculate mean and variance of Poisson variable X, if
$P(X = 4) = P(X = 5)$

(v) Calculate Karl Pearson's Coefficient of correlation if variance of x is 50, variance of y is 800 and covariance between x and y is 15.

(vi) Suppose X is a random variable with p.d.f.

$$f(x) = \frac{k}{\sqrt{x}}, \quad 0 < x < 4$$
$$= 0, \quad \text{elsewhere}$$

find the value of k.

(vii) Find the solution set represented by
$5x + 4y \geq 20, \quad x \leq 6, \quad y \leq 4, \quad x, y \geq 0.$

(viii) If the present worth of a certain amount due 6 months at 4% p.a. is ₹ 625, find the sum due and the true discount.

Q. 5(A). Attempt any TWO of the following: [6 marks]

(i) For a population the following information is available :

Number of persons who attain the exact age of 71 years = 100

Number of persons who attain the exact age of 72 years = 80.

Number of persons living from age 71 years until all of them die = 250.

Find :

(a) The probability that a person of age 71 years will survive till his next birthday.

(b) Number of persons living between the age 71 years and 72 years.

(c) The average number of years a person of age 72 years is expected to live under the prevailing conditions of mortality.

(ii) Calculate Karl Pearson's coefficient of correlation for the following data.

X	10	12	14	18	20	16
Y	20	25	30	35	26	20

(iii) It is observed that 3 out of 5 persons are swimmer. Find prob. That at least one person out of 3 is swimmer.

Q. 5(B). Attempt any TWO of the following: [8 marks]

(i) If $5x+y=21$ and $4x+5y=42$ are two regression equations, find the mean values of $\bar{x}$ and $\bar{y}$. Also find the coefficient of correlation r.

(ii) A chartered accountant's firm has accepted five new cases. The estimated number of days required by each of their five employees for each cases are given below, where '____' means that the particular employee cannot be assigned the particular case. Determine the optimal assignment of cases to the employees so that the total number of days required complete these five cases will be minimum. Also find the minimum number of days.

Employees	Cases				
	I	II	III	IV	V
E_1	5	2	4	2	6
E_2	3	4	-----	5	7
E_3	6	3	4	1	2
E_4	4	2	2	3	5
E_5	3	6	4	7	3

(iii) A firm manufactures two products A & B on which the profit per unit is ₹ 30 & ₹ 40 respectively. Each product is processed on two machines M_1 & M_2, which are available for use for at most 450 minutes and 600 minutes during any working day, respectively.

The product A requires 1 minute processing time on M_1 and 2 minutes time on M_2, per unit. The product B requires 3 minutes processing time M_1 and 1 minute processing time on M_2, per unit. It is required to find the number of units of products A & B to be produced per day so as to get maximum profit.

Formulate this problem as L.P.P. and show feasible region graphically and state the vertices of feasible region.

Q.6 (A). Attempt any TWO of the following: [6marks]

(i) Find the two regression lines for the following data and hence if a student obtained 18 marks in Mathematics estimate his marks in Statistics.

	Mathematics	Statistics
Mean	**20**	**25**
S.D.	**2**	**3**
Correlation coefficient r = 0.75		

(ii) If 2 % of electric bulbs are defective, find the probability that in a sample of 200 bulbs more than two are defective. ($e^{-4} = 0.0183$)

(iii) A bill of ₹ 7,500 was discounted for ₹ 7,290 at bank on 28th Oct 2006. If rate of interest is 14% p.a. Find legal due date.

Q.6.(B) Attempt any Two of the following: [6 marks]

(i) A salesman receives 8% commission on the total sales. If his sale exceeds ₹ 15,000 he receives an additional commission at 1% on the sales in excess of ₹ 15,000. If he receives ₹ 6,600 as commission, find his total sales.

(ii) For the given data write the entries in the columns of d_x; q_x; P_x, L_x of the life table where notations have their usual meanings.

Age (x)	0	1	2	3	4	5
l_x	100	90	80	60	30	0

(iii) Find the mean and the standard deviation of the continuous random variable. X whose p.d.f. is given by

$$f(x) = 6x(1-x), \quad 0 < x < 1$$
$$= 0, \quad \text{otherwise}$$

Prof. Sharad Shah's
SHAH EDUCATIONAL ACADEMY SEA SCORE EXCEL ACHIEVE

SHAH'S ADVANTAGE PRELIM SERIES

A DIVISION OF "SHAH EDUCATIONAL ACADEMY"

B.COM, BAF, BMS, BBI, BFM, MCOM, CS, CPT, IPCC, Final CA.

H.O.: 5,6 & 7, Hill Crest Society, 16th Road, Bandra (W), Mum: 50. Ph: 26051635/ 9820422953
Branch: Avon Arcade, Shop No. A/121; 1st Floor, Vile Parle (W), Mum: 56. Ph: 9820418533

CLASS: S.Y.J.C
PRELIM SERIES NO. 3
Date: 31st Jan 2020

MATHS I & II
(2019 -20) (Solution)

Time : 3 Hrs.
Marks: 80
Day:- Friday

Section-I

Q.1. Attempt any SIX of the following : [12]

(i) Write the negation of the following statements :

(a) Economic growth and per capita income is more in America.

(b) All students have completed their homework and the teacher is present. (2)

Sol.: (a) Economic growth is not more or per capita income is not more in America. *(1 mark)*

(b) Some of the students have not completed their homework or teacher is (not present) absent. *(1 mark)*

(ii) If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -2 & 5 \end{bmatrix}$ use $R_1 \leftrightarrow R_2$ and $C_1 \to C_1 + 2C_2$ to give new matrix. (2)

Sol.: As $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -2 & 5 \end{bmatrix}$

$\therefore R_1 \leftrightarrow R_2$ gives

$$A \sim \begin{bmatrix} 3 & -2 & 5 \\ 1 & 2 & -1 \end{bmatrix}$$ *(1 mark)*

Now, $C_1 \to C_1 + 2C_2$ gives

$$A \sim \begin{bmatrix} 3+2(-2) & -2 & 5 \\ 1+2(2) & 2 & -1 \end{bmatrix}$$

$$\therefore \quad A \sim \begin{bmatrix} -1 & -2 & 5 \\ 5 & 2 & -1 \end{bmatrix}$$ *(1 mark)*

(iii) Find $\frac{dy}{dx}$ if $y = (2+x)^x$ (2)

Sol.: $y = (2+x)^x$

Taking log on both sides

$\log y = \log\left[(2+x)^x\right] = x\log(2+x)$ *(1 mark)*

Differentiating w.r.t. x

$$\frac{1}{y}\frac{dy}{dx} = x\cdot\left(\frac{1}{2+x}\right) + \log(2+x)\cdot 1$$

$$\frac{dy}{dx} = y\left[\frac{x}{2+x} + \log(2+x)\right]$$

$$= (2+x)^x\left[\frac{x}{2+x} + \log(2+x)\right]$$ *(1 mark)*

(iv) Evaluate $\int \frac{\tan\sqrt{x}}{\sqrt{x}}dx$ (2)

Sol. $I = \int \frac{\tan\sqrt{x}}{\sqrt{x}}dx$

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt \Rightarrow \frac{1}{\sqrt{x}}dx = 2dt$

$\therefore I = \int \tan t\cdot(2dt)$ *(1 mark)*

$= 2\int \tan t\, dt$

$= 2\log|\sec t| + c$

$= 2\log\left|\sec\sqrt{x}\right| + c$ *(1 mark)*

(v) If the average revenue R_A is 50 and elasticity of demand η is 5, find marginal revenue R_m. (2)

Sol.: Given : $R_A = 50$ and $\eta = 5$.

We have, $R_m = R_A\left(1 - \frac{1}{\eta}\right)$ *(1 mark)*

$= 50\left(1 - \frac{1}{5}\right) = 50\left(\frac{4}{5}\right)$

$\therefore R_m = 40$ *(1 mark)*

(vi) Evaluate $\int_0^{\pi/2} \cos^2 x\,dx$ (2)

Sol.: Let $I = \int_0^{\pi/2} \cos^2 x\,dx$

$$= \int_0^{\pi/2} \frac{1+\cos 2x}{2}dx$$

$$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2}$$ *(1 mark)*

$$= \frac{1}{2}\left[x - \frac{2\sin x\cos x}{2}\right]_0^{\pi/2}$$

$$= \frac{1}{2}\left[x - \sin x\cos x\right]_0^{\pi/2}$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{4}$$ *(1 mark)*

(vii) If $\int_1^a (3x^2 + 2x + 1)\,dx = 11$, find a. (2)

Sol.: Let $I = \int_1^a (3x^2 + 2x + 1)\,dx = 11$ (given)

$\Rightarrow \left[\frac{3x^3}{3} + \frac{2x^2}{2} + x\right]_1^a = 11$ *(1 mark)*

$\Rightarrow \left[x^3 + x^2 + x\right]_1^a = 11$

$\Rightarrow (a^3 + a^2 + a) - (1+1+1) = 11$

$\Rightarrow a^3 + a^2 + a = 11 + 3$

$\Rightarrow a(a^2 + a + 1) = 14$

$\Rightarrow a = 2$ *(1 mark)*

(viii) The revenue function is given by $R = D^2 - 40D$, where D is demand of the commodity. For what values of D, the revenue is increasing ? (2)

Sol.: Given : $R = D^2 - 40D$

Differentiating w.r.t. x

$$\frac{dR}{dD} = 2D - 40 \qquad \textit{(1 mark)}$$

As revenue is increasing, $\frac{dR}{dD} > 0$.

$\Rightarrow 2D - 40 > 0$

$\Rightarrow 2D > 40$

$\therefore D > 20$ *(1 mark)*

$\therefore$ Revenue is increasing for $D > 20$.

Q.2.(A) Attempt any TWO of the following : [6]

(i) If the function f is continuous at $x = 1$, then find $f(1)$, where

$$f(x) = \frac{1+\cos\pi x}{(1-x)^2} \quad \text{for } x \neq 1. \qquad (3)$$

Sol.: Given : f is continuous at $x = 1$.

$$\therefore f(1) = \lim_{x\to 1} f(x) = \lim_{x\to 1}\frac{1+\cos\pi x}{(1-x)^2}$$

Put $1 - x = t \qquad \therefore x = 1 - t$

When $x \to 1, \quad t \to 0$

$$\therefore f(1) = \lim_{t\to 0}\frac{1+\cos[\pi(1-t)]}{t^2} \qquad \textit{(1 mark)}$$

$$= \lim_{t\to 0}\frac{1-\cos(\pi-\pi t)}{t^2} = \lim_{t\to 0}\frac{1-\cos\pi t}{t^2}$$

$$= \lim_{t\to 0}\frac{2\sin^2\left(\frac{\pi t}{2}\right)}{t^2} \qquad \textit{(1 mark)}$$

$$= 2\cdot\lim_{t\to 0}\left[\frac{\sin\left(\frac{\pi t}{2}\right)}{t}\right]^2 = 2\lim_{t\to 0}\left[\frac{\sin\left(\frac{\pi t}{2}\right)}{\frac{\pi t}{2}}\times\frac{\pi}{2}\right]^2$$

$$= 2\times 1\times\frac{\pi^2}{4} = \frac{\pi^2}{2} \qquad \textit{(1 mark)}$$

$$\therefore f(1) = \frac{\pi^2}{2}$$

(ii) Evaluate $\int \frac{1}{\sqrt{3+4x-4x^2}}\,dx$ (3)

Sol.: Let $I = \int \frac{1}{\sqrt{3+4x-4x^2}}\,dx$

$$= \frac{1}{2}\int\frac{1}{\sqrt{\frac{3}{4}+x-x^2}}\,dx \qquad \textit{(1 mark)}$$

$$= \frac{1}{2}\int\frac{dx}{\sqrt{\frac{3}{4}+\frac{1}{4}-\left(x-\frac{1}{2}\right)^2}}$$

$$= \frac{1}{2}\int\frac{dx}{\sqrt{1-\left(x-\frac{1}{2}\right)^2}}$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{x-\frac{1}{2}}{1}\right)+c \qquad \textit{(1 mark)}$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{2x-1}{2}\right)+c \qquad \textit{(1 mark)}$$

(iii) Find maximum & minimum of $f(x) = 2x^3 - 3x^2 - 12x + 12$

Sol.: $f(x) = 2x^3 - 3x^2 - 12x + 12$

differentiating w.r.t.x

$f^1(x) = 6x^2 - 6x - 12 + 0$

$= 6x^2 - 6x - 12$

Diff. w.r.t.x

$f^{11}(x) = 12^x - 6$

For extreme value sale $f^1(x)$

$f^1(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x - 2 = 0$ OR $x + 1 = 0$

$x = 2$ $\quad x = -1$

when $x = 2$

$f^{11}(x) = 12x - 6$

$= 12(2) - 6$

$= 18 > 0$

= f is minimum at x = 2

(2nd desivative test)& min value of f is

$2(2)^3 - 3(2)^2 - 12(2) + 12$

$16 - \cancel{12} - 24 + \cancel{12} - 8$

When x = 1

$f^{11}(x) = 12(-1) - 6$

$= -18 < 0$

= f is max of x = -1

(2nd D. Test) & max value of f is

$2(-1)^3 - 3(-11^2 - 12(-1) + 12$

$-2 - 3 + 12 + 12$

$-5 + 24 = 19$

Q.2.(B) Attempt any TWO of the following : [8]

(i)

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & x \\ 4 & y \end{bmatrix}$$

And $(A+B)^2 = A^2 + B^2$ find x, y.

$(A+B)^2 = A^2 + B^2$ (given)

i.e. $(A+B)(A+B) = A^2 + B^2$

i.e. $A^2 + AB + BA + B^2 = A^2 + B^2$

i.e. $AB + BA = 0$

i.e. $AB = -BA$

$$\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & x \\ 4 & y \end{bmatrix} = -\begin{bmatrix} 1 & x \\ 4 & y \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 4 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1-4 & x-y \\ 2-4 & 2x-y \end{bmatrix} = -\begin{bmatrix} 1+2x & -1-x \\ 4+2y & -4-y \end{bmatrix}$$

$$\begin{bmatrix} -3 & x-y \\ -2 & 2x-y \end{bmatrix} = \begin{bmatrix} -1-2x & 1+x \\ -4-2y & 4+y \end{bmatrix}$$

By equality of matrices compare corr. Element

$-3 = -1 - 2x$	$-2 = -4 - 2y$
$2x = -1 + 3$	$2y = -4 + 2$
$2x = 2$	$2y = -2$
$x = 1$	$y = -1$

(ii) If $\operatorname{cosec}^{-1}\left(\dfrac{x^3 - y^3}{x^3 + y^3}\right) = \tan k$,

then show that $\dfrac{dy}{dx} = \dfrac{y}{x}$. **(4)**

Sol.: Given : $\operatorname{cosec}^{-1}\left(\dfrac{x^3 - y^3}{x^3 + y^3}\right) = \tan k$

$$\Rightarrow \quad \sin^{-1}\left(\frac{x^3 + y^3}{x^3 - y^3}\right) = \tan k$$

$$\Rightarrow \quad \left(\frac{x^3 + y^3}{x^3 - y^3}\right) = \sin(\tan k) = a, \text{ (say)}$$

i.e. $\dfrac{x^3 + y^3}{x^3 - y^3} = a$ *(1 mark)*

$\Rightarrow \quad x^3 + y^3 = a(x^3 - y^3)$

$= ax^3 - ay^3$

$\Rightarrow \quad y^3 + ay^3 = ax^3 - x^3$

$\Rightarrow \quad y^3(a+1) = x^3(a-1)$

$\Rightarrow \quad \dfrac{y^3}{x^3} = \dfrac{a-1}{a+1} = b$, (say)

i.e. $\dfrac{y^3}{x^3} = b$ ....(i)

$\Rightarrow \quad y^3 = bx^3$ ....(ii) *(1 mark)*

Differentiate w.r.t. x

$$3y^2\frac{dy}{dx} = b(3x^2)$$

i.e. $y^2\dfrac{dy}{dx} = b(x^2)$

where $b = \dfrac{y^3}{x^3}$ from eqn (i)

$\Rightarrow \quad y^2\dfrac{dy}{dx} = \dfrac{y^3}{x^3}(x^2)$ *(1 mark)*

$\Rightarrow \quad y^2\dfrac{dy}{dx} = \dfrac{y^3}{x}$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{y}{x}$ *(1 mark)*

12

Q3(B) iii) $I = \int_0^{\frac{\pi}{2}} \frac{1}{1+\cot x}\, dx$

$= \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{\cos x}{\sin x}}\, dx$

$\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx$ ——— (I)

using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

replace x by $(\frac{\pi}{2} - x)$ } must write

$I = \int_0^{\pi/2} \frac{\sin(\frac{\pi}{2} - x)}{\sin(\frac{\pi}{2} - x) + \cos(\frac{\pi}{2} - x)}\, dx$

$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\, dx$ ——— $\overline{(I)}$

Adding I & $\overline{I}$

$I + I = \int_0^{\pi/2} \text{———}\, dx + \int_0^{\pi/2} \text{———}\, dx$

$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\cos x + \sin x}\, dx$

$2I = \int_0^{\pi/2} (1)\, dx$

$2I = [x]_0^{\pi/2}$

$2I = \frac{\pi}{2} - 0$

$\boxed{I = \frac{\pi}{4}}$ Ans.

Q.3.(A) Attempt any TWO of the following : [6]

(i) Show that $(p \to q) \vee (q \to p)$ is a tautology. (3)

Sol.:

p	q	$p \to q$	$q \to p$	$(p \to q) \vee (q \to p)$
1	2	3	4	5
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

From the above table, the truth value of the given statement is T.

$\therefore (p \to q) \vee (q \to p)$ is a tautology.

[Steps 3, 4, 5 one mark each.]

(ii) Using rules of negation of compound statements write the negation of the following symbolic statements : (3)

(1) $\sim p \wedge \sim q$ (2) $p \leftrightarrow q$ (3) $\sim p \vee \sim q$

Sol.: (1) $p \vee q$ *(1 mark)*

(2) $(p \wedge \sim q) \vee (q \wedge \sim p)$ *(1 mark)*

(3) $p \wedge q$ *(1 mark)*

(iii) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ Find the adjoint of the matrix A and hence find A^{-1}. (3)

Sol.: $|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$

$= 1(1+3) + 1(2+3) + 1(2-1)$

$= 4 + 5 + 1$

$= 10 \neq 0.$

$\therefore A^{-1}$ exists. *(1 mark)*

$M_{11} = \begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} = 1 + 3 = 4$

$M_{12} = \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = 2 + 3 = 5$

$$M_{13} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 2 - 1 = 1$$

$$M_{21} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = (-1 + -1) = -2$$

$$M_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 - 1 = 0$$

$$M_{23} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1 + 1 = 2$$

$$M_{31} = \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = 3 - 1 = 2$$

$$M_{32} = \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -3 - 2 = -5$$

$$M_{33} = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1 + 2 = 3$$

∴ Co-factor Matrix of A

$$= \begin{bmatrix} M_{11} & -M_{12} & M_{13} \\ -M_{21} & M_{22} & -M_{23} \\ M_{31} & -M_{32} & M_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$$ *(1 mark)*

∴ Adjoint of the matrix is transpose of the co-factor matrix.

$$\therefore \text{Adj.A} = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj.A}}{|A|} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 2/5 & 1/5 & 1/5 \\ -1/2 & 0 & 1/2 \\ -1/10 & -1/5 & 3/10 \end{bmatrix}$$ *(1 mark)*

Q.3.(B) Attempt any TWO of the following : [8]

(i) In a firm the cost function for output x is given as $C = \frac{x^3}{3} - 20x^2 + 70x$. Find the output for which

(a) Marginal Cost (MC) is minimum,

(b) Average Cost (AC) is minimum. (4)

Sol.: Given : $C = \frac{x^3}{3} - 20x^2 + 70x$ ....(1)

(a) Marginal Cost (MC) $= \frac{dC}{dx}$

Differentiating equation(1) with respect to x,

$$\therefore \text{MC} = \frac{3x^2}{3} - 40x + 70$$

$$= x^2 - 40x + 70 \quad ....(2)$$

Now, we have to find the output x, for which MC is minimum.

Let $f(x) = x^2 - 40x + 70$

$f'(x) = 2x - 40$ ....(3) *(1 mark)*

For the minimum Marginal Cost (MC), $f'(x) = 0$.

$\Rightarrow 2x - 40 = 0 \quad \Rightarrow 2x = 40 \quad \Rightarrow x = \frac{40}{2} = 20$

$\therefore x = 20.$

Now, for the second derivative test, differentiating equation (3) with respect to x,

$f''(x) = 2$

$\therefore f''(20) = 2 > 0$

$\therefore f(x)$ is minimum for $x = 20$.

$\therefore$ M.C. is minimum for $x = 20$. *(1 mark)*

(b) Average Cost (AC) $= \frac{C}{x}$

$\therefore AC = \left(\frac{x^3}{3} - 20x^2 + 70x\right) \cdot \frac{1}{x}$

$\therefore AC = \frac{x^2}{3} - 20x + 70$ ....(4)

Now, we have to find the output x, for which AC is minimum.

$\therefore$ Differentiate equation (4) with respect to x,

$\frac{d(AC)}{dx} = \frac{2x}{3} - 20$ ....(5) *(1 mark)*

For minimum Average Cost, $\frac{d(AC)}{dx} = 0$.

$\Rightarrow \frac{2x}{3} - 20 = 0 \quad \Rightarrow \frac{2x}{3} = 20$

$\Rightarrow 2x = 60 \quad \Rightarrow x = \frac{60}{2} = 30$

$\therefore x = 30$

Now by second derivative test,

$\frac{d^2(AC)}{dx^2} = \frac{d}{dx}\left(\frac{2x}{3} - 20\right) = \frac{2}{3}$

When $x = 30$,

$\frac{d^2(AC)}{dx} = \frac{2}{3} > 0$

$\therefore$ Average cost is minimum for the output $x = 30$. *(1 mark)*

(ii) If $f(x) = \frac{1 - \sin x}{(\pi - 2x)^2}$, for $x \neq \frac{\pi}{2}$, is continuous at $x = \frac{\pi}{2}$, then find $f\left(\frac{\pi}{2}\right)$. **(4)**

Sol.: Given : $f(x)$ is continuous at $x = \frac{\pi}{2}$.

$\therefore f\left(\frac{\pi}{2}\right) = \lim_{x \to \frac{\pi}{2}} \left[\frac{1 - \sin x}{(\pi - 2x)^2}\right]$

Put $\pi - 2x = t \quad \Rightarrow 2x = \pi - t \quad \Rightarrow x = \frac{\pi - t}{2}$

$\therefore x = \frac{\pi}{2} - \frac{t}{2}$ when $\Rightarrow \frac{\pi}{2}$, $t \to 0$ *(1 mark)*

$\therefore f\left(\frac{\pi}{2}\right) = \lim_{t \to 0} \left[\frac{1 - \sin\left(\frac{\pi}{2} - \frac{t}{2}\right)}{t^2}\right]$

$= \lim_{t \to 0} \left[\frac{1 - \cos\left(\frac{t}{2}\right)}{t^2}\right]$

$= \lim_{t \to 0} \left[\frac{1}{t^2} \cdot 1 - \cos\left(\frac{t}{2}\right)\right]$

$= \lim_{t \to 0} \left[\frac{1}{t^2} \cdot 2\sin^2\left(\frac{t}{4}\right)\right]$ *(1 mark)*

$= 2 \lim_{t \to 0} \left[\frac{\sin\left(\frac{t}{4}\right)}{t}\right]^2$

$= 2 \lim_{t \to 0} \left[\frac{\sin\left(\frac{t}{4}\right)}{\left(\frac{t}{4}\right)} \times \left(\frac{1}{4}\right)\right]^2$ *(1 mark)*

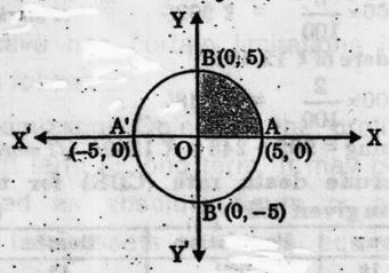
$= 2 \times 1 \times \frac{1}{16}$

$\therefore f\left(\frac{\pi}{2}\right) = \frac{1}{8}$ *(1 mark)*

(iii) By using integration, find the area of circle $x^2 + y^2 = 25$. **(4)**

Sol.: Given : $x^2 + y^2 = 25 \quad \Rightarrow y^2 = 25 - x^2$

$\Rightarrow y = \sqrt{25 - x^2}$ Put $y = 0$, $x = \pm 5$



Area of circle *(1 mark)*

$= 4 \times$ [Area of the circle in first quadrant]. *(1 mark)*

$= 4\int_0^5 \sqrt{25 - x^2}\,dx = 4\int_0^5 \sqrt{(5)^2 - x^2}\,dx$

$$= 4\left[\frac{x}{2}\sqrt{25-x^2}+\frac{25}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^5 \quad \textit{(1 mark)}$$

$$= 4\left[0+\frac{25}{2}\sin^{-1}\left(\frac{5}{5}\right)\right]$$

$$= 4\times\frac{25}{2}\times\frac{\pi}{2} = 25\pi$$

$\therefore$ Area of circle $= 25\pi$ sq.units. *(1 mark)*

Section-II

Q.4. Attempt any SIX of the following : [12]

(i) If the numerator of a function is increased by 20% and its denominator be diminished by 10%, the value of the fraction is $\frac{16}{21}$. Find the original fraction. (2)

Sol.: Let $\frac{p}{q}$ be the original fraction.

If the numerator is increased by 20% then it becomes $p\times\frac{120}{100}$.

Similarly if the denominator is diminished by 10% then it becomes $q\times\frac{90}{100}$.

Given the final fraction is $\frac{16}{21}$.

$$\therefore \frac{p\times\frac{120}{100}}{q\times\frac{90}{100}} = \frac{16}{21} \quad \textit{(1 mark)}$$

$$\Rightarrow \frac{p}{q} = \frac{16\times\frac{90}{100}}{21\times\frac{120}{100}} = \frac{16\times 9}{21\times 12} = \frac{4}{7} \quad \textit{(1 mark)}$$

(ii) An agent charges 8% commission plus a 2% delcredere. If he sells goods worth ₹ 12,400/-, find his total earnings. (2)

Sol.: 8% commission of ₹ 12,400

$$= 12{,}400\times\frac{8}{100} = ₹\ 992/- \quad \textit{(1 mark)}$$

2% delcredere of ₹ 12,400

$$= 12{,}400\times\frac{2}{100} = ₹\ 248/-$$

Total earning = 992 + 248 = ₹ 1240/- *(1 mark)*

(iii) Obtain crude death rate (CDR) for the population given below: (2)

Age group	Population	Deaths
Below 10	900	12
10 – 20	1500	8
20 – 60	4500	36
Above 60	600	30

Sol.: We have crude death rate,

$$\text{CDR} = \frac{\text{Total death}}{\text{Total population}}\times 1000$$

$$= \frac{12+8+36+30}{900+1500+4500+600}\times 1000 \quad \textit{(1 mark)}$$

$$= \frac{86}{7500}\times 1000 = \frac{860}{75} = 11.467 \quad \textit{(1 mark)}$$

(iv) Calculate mean and variance of Poisson variable X, if P(X = 4) = P(X = 5). (2)

Sol.: $X \sim P(m)$

$$\therefore P(X=x) = \frac{e^{-m}\cdot m^x}{x!}$$

Given : $P(X=4) = P(X=5)$

i.e. $\frac{e^{-m}\cdot m^4}{4!} = \frac{e^{-m}\cdot m^5}{5!}$ *(1 mark)*

$\Rightarrow \frac{1}{4!} = \frac{m}{5\times 4!}$

$\Rightarrow m = 5$

$\therefore$ Mean = Variance $= m = 5$ *(1 mark)*

(v) Calculate Karl Pearson's Coefficient of correlation if variance of x is 50, variance of y is 800 and covariance between x and y is 15. (2)

Sol.: Given: Var(X) = 50, Var(Y) = 800 and Cov(X, Y) = 15.

$$r = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\cdot\text{Var}(Y)}}$$

$$= \frac{15}{\sqrt{50\times 800}} = \frac{15}{\sqrt{40000}} \quad \textit{(1 mark)}$$

$$= \frac{15}{200} = \frac{3}{40} = 0.075 \quad \textit{(1 mark)}$$

(vi) Suppose X is a random variable with p.d.f.

$$f(x) = \frac{k}{\sqrt{x}}, \quad 0<x<4$$

$$= 0, \quad \text{elsewhere}$$

find the value of k. (2)

Sol.: Since X is a random variable, $\int_0^4 f(x)\,dx = 1$.

$$\Rightarrow \int_0^4 \frac{k}{\sqrt{x}}\,dx = 1 \quad \textit{(1 mark)}$$

$$\Rightarrow k\int_0^4 x^{-1/2}\,dx = 1 \quad \Rightarrow k\left[\frac{x^{1/2}}{1/2}\right]_0^4 = 1$$

$$\Rightarrow 2k\left[\sqrt{x}\right]_0^4 = 1 \quad \Rightarrow 2k\left[\sqrt{4}-0\right] = 1$$

$$\Rightarrow 2k[2] = 1 \quad \Rightarrow 4k = 1$$

$$\Rightarrow k = \frac{1}{4} \quad \textit{(1 mark)}$$

(vii) Find the solution set represented by $5x+4y\ge 20,\ x\le 6,\ y\le 4,\ x, y\ge 0$. (2)

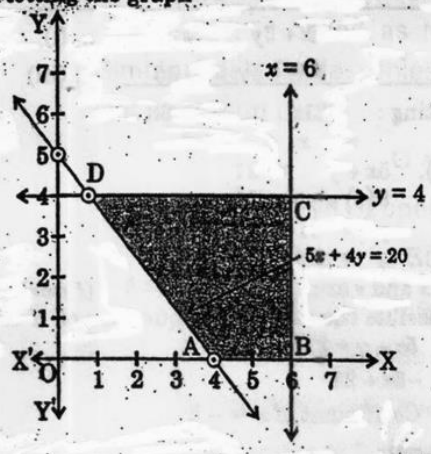
Sol.: Consider $5x+4y = 20$

x	0	4
y	5	0

$x = 6$ is a line parallel to Y-axis.

$y = 4$ is a line parallel to X-axis.

Plotting the graph



(1 mark)

ABCD is the required solution set. *(1 mark)*

(viii) If the present worth of a certain amount due 6 months at 4% p.a. is ₹ 625, find the sum due and the true discount. (2)

Sol.: True discount

= Interest on present worth

= Interest on ₹ 625 for 6 months at 4% p.a.

$= 625 \times \frac{6}{12} \times \frac{4}{100} = 12.5$ *(1 mark)*

∴ Sum due = Present worth + True discount

$= 625 + 12.5 = 637.50$

= ₹ 637.5 *(1 mark)*

Q.5.(A) Attempt any TWO of the following : [6]

(i) For a population the following information is available :

Number of persons who attain the exact age of 71 years = 100

Number of persons who attain the exact age of 72 years = 80.

Number of persons living from age 71 years until all of them die = 250.

Find :

(a) The probability that a person of age 71 years will survive till his next birthday.

(b) Number of persons living between the age 71 years and 72 years.

(c) The average number of years a person of age 72 years is expected to live under the prevailing conditions of mortality. (3)

Sol.: Using the given informations and with the usual notations, we get $l_{71} = 100$, $l_{72} = 80$ and $T_{71} = 250$.

As $P_x = \frac{l_{x+1}}{l_x}$

$P_{71} = \frac{l_{72}}{l_{71}} = \frac{80}{100} = \frac{4}{5}$ *(1 mark)*

∴ The probability that a person of age 71 will survive till his next birthday is $\frac{4}{5}$

Number of persons living between the age 71 years and 72 years

$= L_{71} = \frac{l_{71} + l_{72}}{2} = 90$

As $T_{72} = T_{71} - L_{71}$

$= 250 - 90 = 160$ *(1 mark)*

∴ The average number of years a person of age 72 years is expected to live

$= e^0_{72} = \frac{T_{72}}{l_{72}} = \frac{160}{80} = 2$ *(1 mark)*

(ii) Calculate Karl Pearson's coefficient of correlation for the following data. (3)

X	10	12	14	18	20	16
Y	20	25	30	35	26	20

Sol.:

X	Y	X^2	Y^2	XY
10	20	100	400	200
12	25	144	625	300
14	30	196	900	420
18	35	324	1225	630
20	26	400	676	520
16	20	256	400	320
90	156	1420	4226	2390

(1 mark)

$n = 6$, $\sum x = 90$, $\sum y = 156$,

$\sum x^2 = 1420$, $\sum y^2 = 4226$, $\sum xy = 2390$

$\bar{X} = \frac{\sum x}{n} = \frac{90}{6} = 15$,

$\bar{Y} = \frac{\sum y}{n} = \frac{156}{6} = 26$

$Cov(X, Y) = \frac{\sum xy}{n} - \bar{x} \cdot \bar{y}$

$= \frac{2390}{6} - 15 \times 26$

$= 398.33 - 390$

$= 8.33$

$Var(X) = \frac{\sum X^2}{n} - (\bar{X})^2 = \frac{1420}{6} - (15)^2$

$= 236.67 - 225 = 11.67$

$Var(Y) = \frac{\sum Y^2}{n} - (\bar{Y})^2 = \frac{4226}{6} - (26)^2$

$= 704.33 - 676 = 28.33$ *(1 mark)*

$r = \frac{Cov(X, Y)}{\sqrt{Var(X) \cdot Var(Y)}}$

$= \frac{8.33}{\sqrt{11.67 \times 28.33}}$

$= 0.4583$ *(1 mark)*

(iii) It is observed that 3 out of 5 persons are swimmer. Find prob. that at least one person out of 3 is swimmer. (3)

Sol.:

Let x denote no. of persons who swim

3 out of 5 is swimmer

$\therefore p = 3/5$

$q = 1 - p$

$= 1 - 3/5$

$= 2/5$ (who do not swim)

$n = 3$

At least one person is swimmer mean 1 or more.

$p(n - 8) = 1 - p(\text{none})$

also binomial dist

$p(\text{attempt } 1) = 1 - p(x = 0)$

$= 1 - n_{cr}(p)^r (q)^{n-r}$

$= 1 = 3_{co} \left[\frac{3}{5}\right]^0 \left[\frac{2}{5}\right]^{3-0}$

$= 1 - 1 \times 1 \times \left[\frac{2}{5}\right]^3$

$= 1 - \frac{8}{125}$

$= \frac{125 - 8}{125}$

Ans:- $= \frac{117}{125}$

Q.5.(B) Attempt any TWO of the following : [8]

(i) If $5x + y = 21$ and $4x + 5y = 42$ are two regression equations, find the mean values of $\bar{x}$ and $\bar{y}$. Also find the coefficient of correlation r. (4)

Sol.: The mean values of x and y will be obtained by solving the simultaneous equations

$5x + y = 21$ ...(i) and $4x + 5y = 42$ ....(ii)

(ii) − (i) × 5 gives

$$4x + 5y = 42 \quad ....(i)$$
$$25x + 5y = 105 \quad ....(iii)$$

Subtracting : $-21x = -63$

$\Rightarrow x = 3$

From (1), $5x + y = 21$

$5(3) + y = 21$

$y = 21 - 15 = 6$

$\therefore y = 6$

$\therefore$ Mean of x and y are : $\bar{x} = 3$ and $\bar{y} = 6$ *(1 mark)*

Let if possible the regression equations of Y on X be $5x + y = 21$

$\Rightarrow y = -5x + 21$

$\therefore b_{yx}$ = **Coefficient of $x = -5$**

i.e. $b_{yx} = -5$

Then, $4x + 5y = 42$ **will be the regression equation of X on Y.**

$\Rightarrow 4x = -5y + 42$

$\Rightarrow x = \frac{-5y + 42}{4}$

$\therefore b_{xy}$ = **Coefficient of $y = \frac{-5}{4}$**

i.e. $b_{xy} = \frac{-5}{4}$

$\therefore b_{yx} \times b_{xy} = -5 \times \frac{-5}{4} = \frac{25}{4}$ *(1 mark)*

$\therefore b_{yx} \times b_{xy} = \frac{25}{4} > 1$

which is not possible .... $\left(b_{yx} \cdot b_{xy} = r^2 \le 1 \right)$

$\therefore$ Our assumption is wrong.

$\therefore$ $5x + y = 21$ is not the regression equation of Y on X.

$\therefore$ $5x + y = 21$ is the regression equation of X on Y and $4x + 5y = 42$ is that of of Y on X.

$\therefore b_{xy} = \frac{-1}{5}$ **and** $b_{yx} = \frac{-4}{5}$ *(1 mark)*

$\therefore r^2 = b_{yx} \times b_{xy} = \frac{-1}{5} \times \frac{-4}{5} = \frac{4}{25}$

$\therefore r = \sqrt{\frac{4}{25}} = \pm\frac{2}{5}$

Since b_{yx} and b_{xy} are negative

$\therefore r = -\frac{2}{5}$ *(1 mark)*

(ii) A chartered accountant's firm has accepted five new cases. The estimated number of days required by each of their five employees for each cases are given below, where '__' means that the particular employee cannot be assigned the particular case. Determine the optimal assignment of cases to the employees so that the total number of days required to complete these five cases will be minimum Also find the minimum number of days.

Employees	Cases				
	I	II	III	IV	V
E_1	5	2	4	2	6
E_2	3	4	—	5	7
E_3	6	3	4	1	2
E_4	4	2	2	3	5
E_5	3	6	4	7	3

(4)

Sol.: We replace '—' by a very large number M so that adding or subtracting a small number does not affect M.

Subtracting Row minimum,

$$\begin{bmatrix} 3 & 0 & 2 & 0 & 4 \\ 0 & 1 & M & 2 & 4 \\ 5 & 2 & 3 & 0 & 1 \\ 2 & 0 & 0 & 1 & 3 \\ 0 & 3 & 1 & 4 & 0 \end{bmatrix}$$

Subtracting Column minimum,

$$\begin{bmatrix} 3 & 0 & 2 & 2 & 3 \\ 0 & 1 & M & 0 & 3 \\ 5 & 2 & 3 & 0 & 0 \\ 2 & 0 & 0 & 1 & 2 \\ 0 & 3 & 1 & 4 & 0 \end{bmatrix}$$ *(1 mark)*

Drawing lines covering maximum zeros.

Total number of lines = 5 = order of matrix.

$\therefore$ Optimal solution is reached.

$\therefore$ We start assigning,

$E_2 \to I$ $\quad$ $E_5 \not\to I$

$E_5 \to V$ $\quad$ $E_3 \not\to V$

Then $E_4 \to III$ $\quad$ $\therefore$ $E_4 \not\to II$ *(1 mark)*

	I	II	III	IV	V
E_1	3	0	2	~~0~~	3
E_2	0	1	M	2	3
E_3	5	2	3	0	~~0~~
E_4	2	~~0~~	0	1	2
E_5	~~0~~	3	1	4	0

We go on assigning the zeros and cross the other zeros in that particular rows and columns are cancelled.

$\therefore$ The optimal assignment is

$I \to E_2$, $II \to E_1$, $III \to E_4$, $IV \to E_3$, $V \to E_5$. *(1 mark)*

The minimum number of days

$= 3 + 2 + 2 + 1 + 3 = 11$ days *(1 mark)*

(iii) A firm manufactures two products A and B on which the profit per unit is ₹ 30 and ₹ 40 respectively. Each product is processed on two machines M_1 and M_2, which are available for use for at most 450 minutes and 600 minutes during any working day, respectively.

The product A requires 1 minute processing time on M_1 and 2 minutes time on M_2, per unit. The product B requires 3 minutes processing time on M_1 and 1 minute processing time on M_2, per unit. It is required to find the number of units of products A and B to be produced per day so as to get maximum profit.

Formulate this problem as L.P.P. and show feasible region graphically and state the vertices of feasible region. **(4)**

Sol.: Let x units of A and y units of B produced.

$\therefore$ $x \geq 0$, $\quad y \geq 0$.

It can be tabulated as :

Machines	Product A (x)	Product B (y)	Maximum available time in minutes
M_1	1	3	450
M_2	2	1	600
Profit in ₹	30	40	—

$x + 3y \leq 450$, $\quad 2x + y \leq 600$ *(1 mark)*

The total profit, when x units of A and y units of B are sold is given by $Z = 30x + 40y$.

$\therefore$ L.P.P. is

Maximize $Z = 30x + 40y$ subject to

$x + 3y \leq 450$, $2x + y \leq 600$, $x \geq 0$, $y \geq 0$ *(1 mark)*

We plot the lines $\quad x + 3y = 450 \quad$ ....(i)

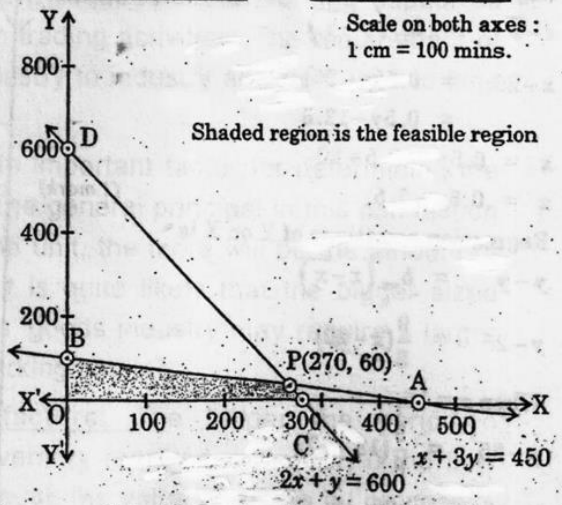
and $\quad 2x + y = 600 \quad$ ....(ii)

For the line (i)

Point	x	y
A	450	0
B	0	150

For the line (ii)

Point	x	y
C	300	0
D	0	600



(1 mark)

The quadrilateral OCPB is the feasible region, where P is the point of intersection of the lines.

$$x + 3y = 450 \quad ....(i)$$
$$2x + y = 600 \quad ....(ii)$$
$$\Rightarrow \quad 2x + 6y = 900$$
$$2x + y = 600$$

Subtracting : $5y = 300$

$$y = 60$$
$$\therefore \; x = 270$$

$\therefore$ **P = (270, 60)**

$\therefore$ **Vertices of the feasible region are:**
O(0,0), C(300,0), P(270, 60) and B(0, 150)

(1 mark)

Q.6.(A) Attempt any TWO of the following : [6]

(i) Find the two regression lines for the following data and hence if a student obtained 18 marks in Mathematics estimate his marks in Statistics.

	Mathematics	Statistics
Mean	20	25
S.D.	2	3
Correlation coefficient $r = 0.75$		

(3)

Sol.: Let x be the marks in Mathematics

y be the marks in Statistics.

$\bar{x} = 20, \; \bar{y} = 25, \; \sigma_x = 2, \; \sigma_y = 3, \; r = 0.75$

$$b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y} = 0.75 \times \frac{2}{3} = 0.5$$

$$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} = 0.75 \times \frac{3}{2} = \frac{9}{8} = 1.12 \quad \textit{(1 mark)}$$

Regression equations of X on Y is

$$x - \bar{x} = b_{xy}\left(y - \bar{y}\right)$$
$$\Rightarrow \; x - 20 = 0.5(y - 25)$$
$$= 0.5y - 12.5$$
$$\therefore \; x = 0.5y - 12.5 + 20$$
$$\therefore \; x = 0.5y + 7.5 \quad \textit{(1 mark)}$$

Regression equations of Y on X is

$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$$
$$\Rightarrow \; y - 25 = \frac{9}{8}(x - 20)$$

When $x = 18$,

$$y - 25 = \frac{9}{8}(18 - 20)$$
$$\therefore \; y = 25 - 2.25 = 22.75$$
$$\therefore \; y = 22.75 \quad \textit{(1 mark)}$$

(ii) If 2% of electric bulbs are defective, find the probability that in a sample of 200 bulbs more than two are defective. ($e^{-4} = 0.0183$) **(3)**

Sol.: $p = 2\% = \frac{2}{100} = 0.02$

$n = 200$

$\therefore m = np = 200 \times 0.02 = 4$

Let X denote the number of defective bulbs

$X \sim P(m)$

$\therefore P(X = x) = p(x) = \frac{e^{-m} \cdot m^x}{x!}, x = 0, 1, 2, 3, \ldots$

Put $m = 4$

$P(X = x) = P(X) = \frac{e^{-4} \cdot 4^x}{x!}$ *(1 mark)*

Required Probability

$= P(X > 2)$

$= 1 - P(X \le 2)$

$= 1 - \{P(X = 2) + P(X = 1) + P(X = 0)\}$

$= 1 - \left\{\frac{e^{-4} \times 4^2}{2!} + \frac{e^{-4} \times 4^1}{1!} + \frac{e^{-4} \times 4^0}{0!}\right\}$ *(1 mark)*

$= 1 - e^{-4}\left\{\frac{16}{2} + \frac{4}{1} + \frac{1}{1}\right\}$

$= 1 - e^{-4}\left\{\frac{16 + 8 + 2}{2}\right\}$

$= 1 - e^{-4}\{13\}$, but given $e^{-4} = 0.0183$

$= 1 - 0.0183 \times 13$

$= 1 - 0.2379$

$= 0.7621$ *(1 mark)*

Q 6A)(iii) FV = Rs 2500

CV = Rs 2290

Discounted on = 28th oct 2006

r = 14% pa

BD = FV − CV

= 2500 − 2290

= Rs 210

$$BD = \frac{FVNR}{100}$$

$$210 = \frac{2500 \times N \times 14}{100 \times 365}$$

$$\frac{210 \times 365 \times 100}{2500 \times 14} = N$$

N = 73 days

These are days from discounted on to Legal due date

oct	3 (31−28)	73
Nov	− 30	−64
Dec	− 31	9
Jan	$\boxed{9}$	

Legal Due Date = 9th Jan 2007

Q.6.(B) Attempt any TWO of the following : [8]

(i) A salesman receives 8% commission on the total sales. If his sale exceeds ₹ 15,000 he receives an additional commission at 1% on the sales in excess of ₹ 15,000. If he receives ₹ 6,600 as commission, find his total sales.

Solution: Let the total sales be ₹ x

∴ Commission at 8% on total sales

$$= x \times \frac{8}{100} = \frac{8x}{100}$$

Sale in excess of ₹ 15,000 $= x - 15{,}000$

∴ Commission at 1% on excess sales

$$= (x - 15{,}000) \times \frac{1}{100}$$

But total commission earned = ₹ 6,600

$$\therefore \quad \frac{8x}{100} + \frac{(x - 15{,}000)}{100} = 6{,}600$$

$$\therefore \quad \frac{8x + x - 15{,}000}{100} = 6{,}600$$

$$\therefore \quad 9x - 15000 = 6{,}60{,}000$$

$$9x = 6{,}60{,}000 + 15{,}000$$

$$= 6{,}75{,}000$$

$$\therefore \quad x = 75{,}000$$

∴ His total sales are ₹ 75,000.

(ii) For the given data write the entries in the columns of d_x, q_x, P_x, L_x of the life table where notations have their usual meanings.

Age (x)	0	1	2	3	4	5
l_x	100	90	80	60	30	0

(4)

Sol.: Life table : $T_x = L_x + L_{x+1} + L_{x+2} + \ldots$

x	l_x	$d_x = l_x - l_{x+1}$	$q_x = \frac{d_x}{l_x}$	$p_x = 1 - q_x$	$L_x = \frac{l_x + l_{x+1}}{2}$
0	100	10	0.10	0.90	95
1	90	10	0.11	0.89	85
2	80	20	0.25	0.75	70
3	60	30	0.50	0.50	45
4	30	30	1.00	0	15
5	—	—	—	—	—

[1 mark each for columns d_x, q_x, P_x and L_x]

(iii) Find the mean and the standard deviation of the continuous random variable. X whose p.d.f. is given by

$$f(x) = 6x(1-x), \quad 0 < x < 1$$
$$= 0, \quad \text{otherwise}$$

Solution :

$$\text{Mean} = E(X)$$

$$= \int_0^1 x(6x - 6x^2)\,dx$$

$$= 6\int_0^1 x^2\,dx - 6\int_0^1 x^3\,dx$$

$$= 6\left[\frac{x^3}{3}\right]_0^1 - 6\left[\frac{x^4}{5}\right]_0^1$$

$$= 6\left[\frac{1}{3}\right] - 6\left[\frac{1}{4}\right]$$

$$= 2 - \frac{3}{2} = \frac{1}{2} = 0.5$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$\text{Now } E(X)^2 = \int_0^1 x^2 f(x)\,dx$$

$$= \int_0^1 6x^3(1-x)\,dx$$

$$= 6\int_0^1 x^3\,dx - 6\int_0^1 x^4\,dx$$

$$= 6\left[\frac{x^4}{4}\right]_0^1 - 6\left[\frac{x^5}{5}\right]_0^1$$

$$= \frac{6}{4} - \frac{6}{5} = \frac{6}{20} = \frac{3}{10} = 0.3$$

$$\therefore \quad \text{Var}(X) = 0.3 - (0.5)^2$$

$$= 0.3 - 0.25$$

$$= 0.05$$

$$\therefore \quad \text{S.D.}(X) = \sqrt{\text{Var}(X)}$$

$$= \sqrt{0.05}$$

****