Prof. Sharad Shah's
SHAH EDUCATIONAL ACADEMY
SEA
SCORE EXCEL ACHIEVE

SHAH'S ADVANTAGE PRELIM SERIES

A DIVISION OF "SHAH EDUCATIONAL ACADEMY"

B.COM, BAF, BMS, BBI, BFM, MCOM, CS, CPT, IPCC, Final CA.

H.O.: 5,6 & 7, Hill Crest Society, 16th Road, Bandra (W), Mum: 50. Ph: 26051635/ 9820422953
Branch: Avon Arcade, Shop No. A/121; 1st Floor, Vile Parle (W), Mum: 56. Ph: 9820418533

CLASS: S.Y.J.C *PRELIM SERIES NO. 1* Date: 18th Jan 2020	MATHS I & II (2019 -20)	Time : 3 Hrs. Marks: 80 Day:- Saturday

Note: (1) All questions are compulsory; (2) Figures to the right indicate full marks; (3) Answer to every question, must be started on a new page.; (4) Use separate answers books for each section. (5) Graph paper is not necessary for LLP.

Section – I

Q. 1. Attempt any SIX of the following : [12 marks]

(i) Write down the negation of the following statements :

(a) If it rains then match will be cancelled.

(b) Kiran is rich if and only if he is honest.

(ii) If $A = \begin{bmatrix} 6 & 3 \\ -4 & a \end{bmatrix}$ is a singular matrix, find a.

(iii) Find $\frac{dy}{dx}$ if $y = \cos^{-1}\left(1-2x^2\right)$.

(iv) Differentiate $x^{x\sin x}$ w.r.t. x.

(v) Divide 100 into two parts such that the sum of their squares is minimum.

(vi) Evaluate $\int \log x \, dx$

(vii) Evaluate : $\int_0^1 \frac{x}{x+1} dx$

(viii) Examine the continuity of the function

$$f(x) = \frac{\sin 5x}{3x}, \quad x \neq 0,$$
$$= \frac{3}{5}, \quad x = 0, \quad \text{at } x = 0.$$

Q. 2(A). Attempt any TWO of the following :[6 marks]

(i) Examine whether the following statement is a tautology or contradictions or neither.

$(\sim p \vee q) \rightarrow [p \wedge (q \vee \sim q)]$

(ii) Find the inverse of the matrix $\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ using row operations.

(iii) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$, find AB and $(AB)^{-1}$, using adj method.

Q. 2(B). Attempt any TWO of the following: [8 marks]

(i) A function $f(x)$ is defined by

$$f(x) = \frac{e^x + e^{-x} - 2}{x \sin x}, \quad \text{for } x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right];$$

$$= 0, \quad \text{otherwise.}$$

find $f(0)$ so that f is continuous at $x = 0$.

(ii) Evaluate $\int \frac{7x+3}{\sqrt{3+2x-x^2}} dx$

(iii) If $x^y = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

Q. 3(A). Attempt any TWO of the following: [6 marks]

(i) If p and q are true and r and s are false statements, find the truth values of the following statements :

(a) $(p \wedge q) \vee r$ (b) $p \wedge (r \rightarrow s)$ (c) $(p \vee s) \leftrightarrow (q \wedge r)$

(ii) Show that the following function is always increasing if $f(x) = x^3 - 9x^2 + 30x + 7$.

(iii) Evaluate : $\int e^x \cdot \frac{x+3}{(x+4)^2} dx$

Q. 3(B). Attempt any TWO of the following: [8 marks]

(i) Evaluate : $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$

(ii) Evaluate $\int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}} dx$

(iii) Find the value of k, if the function $f(x)$ is continuous at $x = \frac{\pi}{4}$; where

$$f(x) = \frac{1-\tan x}{1-\sqrt{2}\sin x}, \quad \text{for } x \neq \frac{\pi}{4};$$

$$= \frac{k}{2}, \quad \text{for } x = \frac{\pi}{4}.$$

Section – II

Q.4. Attempt any SIX of the following : [12 marks]

(i) Mr. Paralikar spends 30% of his income on food items and 15% on conveyance. If for the month of October 2012, he spent ₹ 1200 on conveyance, find his expenditure on food items during the same month.

(ii) A merchant pays his agent 8% ordinary commission and 3% delcredere. If the agent sells goods worth ₹ 5,300, find the amount which the merchant receives from the agent.

(iii) A flat is sold through Mr. Patil who charged 1.5% commission for the seller and got 2.5% commission from the buyer. Thus he earned a total amount of ₹ 15,000. Find the sales price of the flat.

(iv) A book seller paid ₹ 765 for a parcel of 50 books on which he had been given a discount of 15%. Find the list price of book.

(v) A person aged 45 insures his life for a certain amount and pays ₹ 313.50 as premium every six months. If the premium payable by him per annum is at the rate of ₹ 78.375 per thousand, find the value of his policy.

(vi) Calculate crude death rate for the population in a town as shown below :

Age Group (years)	Population in the town	
	No. of persons (in '000)	No. of deaths
Below 10	14	180
10 – 40	21	120
40 – 70	67	570
70 and above	11	460

(vii) In a bivariate data, $n = 20$, $\sum x = 300$, $\sum y = 400$, $\sum xy = 395$, find Cov(X, Y).

(viii) The following is the p.d.f. of a continuous random variable X is given by

$$f(x) = \frac{x}{3}, \quad 0 < x < 3$$
$$= 0, \quad \text{otherwise.}$$

Find the cumulative density function of X.

Q. 5(A). Attempt any TWO of the following: [6 marks]

(i) For the following table find the terms marked by '?'

x	l_x	d_x	q_x	p_x	L_x	T_x	e_x^0
5	90	?	0.11	?	?	290	
6	80					?	?

(ii) Find Karl Pearson's correlation coefficient between sales and expenses of the following 7 firms :

Firm No.	1	2	3	4	5	6	7
Sales	11	7	9	5	8	6	10
Expenses	10	8	6	5	9	7	11

(iii) A person fires 5 shots at a target. The probability that any shot will hit the target is $\frac{4}{5}$. Find the probability that the target is hit (i) exactly 3 times (ii) at least 3 times.

Q. 5(B). Attempt any TWO of the following: [8 marks]

(i) Find the equation of the line of regression of demand on price from the following coded data.

Price	2	4	6	10	16
Demand	72	54	36	18	12

Also find the estimated demand when the price is 7.5.

(ii) A team of 4 horses and 4 riders has entered the jumping show contest. The number of penalty points to be expected when each rider ride search horse is shown below. How should the horses be assigned to the riders so as to minimize the expected loss ? Also find the minimum expected loss.

Horses / Riders	H_1	H_2	H_3	H_4
R_1	12	3	3	2
R_2	1	11	4	13
R_3	11	10	6	11
R_4	5	8	1	7

(iii) An aeroplane can carry a maximum of 200 passengers. Baggage allowed for a first class passenger is 30 kgs and for an economy class passenger is 20 kgs. The maximum capacity of the aeroplane for the baggage is 4500 kgs. The profit on each first class ticket is ₹ 500 and that on an economy ticket is ₹ 300. It is required to determine how many tickets of each type be sold so as to get maximum profit. Formulate this problem as L.P.P. and solve it graphically. Also find feasible solution graphically and write the vertices of feasible region.

Q. 6(A). Attempt any TWO of the following :[6 marks]

(i) For the given data write the entries of d_x, q_x, p_x and L_x of the life table, whose notations have their usual meanings.

x	0	1	2	3	4	5
l_x	4000	3000	1000	200	40	0

(ii) Obtain the rank correlation coefficient between the variables X and Y from the following pairs of observed values.

X	50	55	65	50	55	60	50	65	70	75
Y	110	110	115	125	140	115	130	120	115	160

(iii) If a random variable X follows a Poisson distribution such that P(X = 3) = P(X = 4). Find the mean and standard deviation of the distribution. Also find P(X = 2). (Keep Ans. in e Form only)

Q. 6(B). Attempt any TWO of the following :[6 marks]

(i) Following equations represent two regression lines $5x + y = 21$ and $4x + 5y = 42$.

Find :

(i) mean values of x and y

(ii) correlation coefficient γ

(iii) standard deviation of y when Var(X) = 9.

(iv) most probable value of y when $x = 2$.

(ii) Find the optimal sequence that minimize total time required to complete five jobs, which have to be processed on each of three machines M_1, M_2, M_3 in the order M_1, M_2, M_3. If the processing time for each job on each machine is given below. Also find the total elapsed time and idle time for M_2.

Machine	Jobs				
	P	Q	R	S	T
M_1	6	7	5	11	5
M_2	4	3	2	5	1
M_3	3	8	7	4	9

(iii) The owner of a small machine shop has four machinists available to assign jobs for the day. The expected profits for each job done by each machinists are given below. Find the assignment of jobs to the machinists that will result in maximum profit. Also find the maximum profit. [One machine can be assigned only on job.]

Machinist		Jobs				
		A	B	C	D	E
	M_1	62	78	50	101	82
	M_2	71	84	61	73	59
	M_3	87	92	111	71	81
	M_4	48	61	87	77	80

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CLASS: S.Y.J.C
PRELIM SERIES NO. 1
Date: 18th Jan 2020

MATHS I & II
(2019 -20) (Solution)

Time : 3 Hrs.
Marks: 80
Day:- Saturday

Section-I

Q.1. Attempt any SIX of the following : [12]

(i) Write down the negation of the following statements :

(a) If it rains then match will be cancelled.

(b) Kiran is rich if and only if he is honest. (2)

Sol.: (i) (a) It rains but the match will not be cancelled. *(1 mark)*

(b) Kiran is rich and he is not honest. Or
Kiran is ~~not rich~~ not rich and he is ~~honest~~ honest. *(1 mark)*

(ii) If $A = \begin{bmatrix} 6 & 3 \\ -4 & a \end{bmatrix}$ is a singular matrix, find a. (2)

Sol.: Since A is a singular matrix, $|A| = 0$. *(1 mark)*

$\therefore \begin{vmatrix} 6 & 3 \\ -4 & a \end{vmatrix} = 0 \quad \therefore 6a + 12 = 0$

$\therefore 6a = -12 \quad \therefore a = -2$ *(1 mark)*

(iii) Find $\frac{dy}{dx}$ if $y = \cos^{-1}(1-2x^2)$. (2)

Sol.: $y = \cos^{-1}(1-2x^2)$

Put $x = \sin\theta \Rightarrow \theta = \sin^{-1} x$

$\therefore y = \cos^{-1}(1-2\sin^2\theta) = \cos^{-1}(\cos 2\theta)$

$= 2\theta = 2\sin^{-1} x$ *(1 mark)*

$\therefore \frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$ *(1 mark)*

(iv) Differentiate $x^{x\sin x}$ w.r.t. x. (2)

Sol.: Let $y = x^{x\sin x}$

Taking log on both sides,

$\log y = x\sin x \log x$

Differentiating w.r.t. x,

$\frac{1}{y}\frac{dy}{dx} = x\sin x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x\sin x)$

$= x\sin x\left(\frac{1}{x}\right) + \log x \cdot [x\cos x + \sin x]$ *(1 mark)*

$\therefore \frac{dy}{dx} = y[\sin x + x\cdot\cos x\cdot\log x + \log x\cdot\sin x]$

$= x^{x\sin x}[\sin x + x\cdot\cos x\cdot\log x + \log x\cdot\sin x]$ *(1 mark)*

(v) Divide 100 into two parts such that the sum of their squares is minimum. (2)

Sol.: Let x and y be the two parts.

$\therefore x + y = 100 \Rightarrow y = 100 - x$

$\therefore x$ and $100 - x$ be the required parts.

$\therefore f(x) = x^2 + (100-x)^2$

$\therefore f'(x) = 2x + 2(100-x)(-1) = 4x - 200$

$f''(x) = 4 > 0$

$\therefore f(x)$ has minimum.

For minimum $f'(x) = 0 \Rightarrow 4x - 200 = 0$ *(1 mark)*

$\Rightarrow 4x = 200 \quad \Rightarrow x = 50$

$\therefore f''(50) = 4 > 0.$

$\therefore f(x)$ has minimum at $x = 50$.

$\therefore$ Required parts are 50 and 50. *(1 mark)*

(vi) Evaluate $\int \log x\,dx$ (2)

Sol.: Let $I = \int \log x\,dx = \int \log x \cdot 1\,dx$

Integrating by parts,

$I = \log x\cdot(x) - \int \frac{1}{x}(x)\,dx$ *(1 mark)*

$= x\cdot\log x - \int 1\,dx$

$= x\cdot\log x - x + c$

$= x(\log x - 1) + c$ *(1 mark)*

(vii) Evaluate : $\int_0^1 \frac{x}{x+1}dx$ (2)

Sol.: Let $I = \int_0^1 \frac{x}{x+1}dx = \int_0^1 \frac{[(x+1)-1]}{x+1}dx$

$= \int_0^1\left[\frac{x+1}{x+1} - \frac{1}{x+1}\right]dx = \int_0^1 1\,dx - \int_0^1 \frac{1}{x+1}dx$

$= [x]_0^1 - [\log(x+1)]_0^1$ *(1 mark)*

$= [1-0] - [\log 2 - \log 1]$

$= 1 - \log 2 + 0$

$= 1 - \log 2$ *(1 mark)*

(viii) Examine the continuity of the function

$f(x) = \frac{\sin 5x}{3x}, \; x \neq 0,$

$= \frac{3}{5}, \quad x = 0, \quad$ **at $x = 0$. (2)**

Sol.: Given : $f(0) = \frac{3}{5}$ ....(i)

$\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{\sin(5x)}{3x}$

$= \frac{1}{3}\lim_{x\to 0}\frac{\sin(5x)}{5x} \times 5$

$= \frac{1}{3} \times 1 \times 5 = \frac{5}{3}$ ....(ii) *(1 mark)*

From (i) and (ii), $\lim_{x\to 0} f(x) \neq f(0)$

$\therefore f(x)$ is a not continuos at $x = 0$. *(1 mark)*

Q.2.(A) Attempt any TWO of the following : [6]

(i) Examine whether the following statement is a tautology or contradictions or neither.

$(\sim p \vee q) \rightarrow [p \wedge (q \vee \sim q)]$ **(3)**

Sol.: Let X = $(\sim p \vee q) \rightarrow [p \wedge (q \vee \sim q)]$

p	q	$\sim p$	$\sim p \vee q$	$\sim q$	$q \vee \sim p$	$p \wedge (q \vee \sim q)$	X
1	2	3	4	5	6	7	8
T	T	F	T	F	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	F	T	F	F
F	F	T	T	T	T	F	F

From the last column X, we can see that the given statement is neither tautology nor contradiction.

[1 mark each for column 4 and column 8 and one mark for the conclusion.]

(ii) Find the inverse of the matrix $\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ **(3)**

Sol.: Let $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

By the definition of inverse,

$AA^{-1} = I$, where I is the identity matrix.

$$\therefore \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ -4 & -3 & -3 \\ 4 & 4 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 + R_3$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 4 & 4 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 4R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 4 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -4 & 1 \end{bmatrix}$$ *(1 mark)*

$R_3 \rightarrow R_3 - 4R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ -4 & -4 & -3 \end{bmatrix}$$

$R_1 \rightarrow R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ -4 & -4 & -3 \end{bmatrix}$$ *(1 mark)*

$R_3 \to (-1)R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$ *(1 mark)*

Q 2)(A) (iii)

$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$= \begin{pmatrix} 2+9 & 0+3 \\ 1+6 & 0+2 \end{pmatrix} = \begin{pmatrix} 11 & 3 \\ 7 & 2 \end{pmatrix}$

$|AB| = \begin{vmatrix} 11 & 3 \\ 7 & 2 \end{vmatrix} = 22 - 21 = 1 \neq 0$

$\Rightarrow A$ is non singular matrix

$\Rightarrow A^{-1}$ exists

Minor of AB

$M_{11} = 2$

Cofactor $A_{11} = (-1)^{1+1}(2) = +2$

$M_{12} = 7$

$A_{12} = (-1)^{1+2}(7) = -7$

$M_{21} = 3$

$A_{21} = (-1)^{2+1}(3) = -3$

$M_{22} = 11$

$A_{22} = (-1)^{2+2}(11) = 11$

Matrix of Cofactor $= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

$= \begin{pmatrix} 2 & -7 \\ -3 & 11 \end{pmatrix}$

Adj(A) = Transpose

$= \begin{pmatrix} 2 & -3 \\ -7 & 11 \end{pmatrix}$

$(AB)^{-1} = \frac{1}{|A|} \cdot Adj(A)$

$= \frac{1}{1}\begin{pmatrix} 2 & -3 \\ -7 & 11 \end{pmatrix}$

$(AB)^{-1} = \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$

Q.2.(B) Attempt any two of the following : [8]

(i) A function $f(x)$ is defined by

$$f(x) = \frac{e^x + e^{-x} - 2}{x \sin x}, \quad \text{for } x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right];$$

$$= 0, \qquad \text{otherwise.}$$

find $f(0)$ so that f is continuous at $x = 0$. (4)

Sol.: Since f is continuous at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$\therefore$ We have to find

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left[\frac{e^x + e^{-x} - 2}{x \sin x}\right]$$

$$= \lim_{x \to 0} \left[\frac{e^x + \frac{1}{e^x} - 2}{x \sin x}\right] \qquad \textit{(1 mark)}$$

$$= \lim_{x \to 0} \left[\frac{e^x + 1 - 2e^x}{e^x \cdot x \sin x}\right]$$

$$= \lim_{x \to 0} \left[\frac{(e^x - 1)^2}{x \sin x \cdot e^x}\right] \qquad \textit{(1 mark)}$$

$$= \lim_{x \to 0} \left[\frac{\frac{(e^x - 1)^2}{x^2}}{\frac{x \sin x}{x^2}}\right] \cdot \lim_{x \to 0} \left(\frac{1}{e^x}\right)$$

$$= \frac{\lim_{x \to 0} \left(\frac{e^x - 1}{x}\right)^2}{\lim_{x \to 0} \left(\frac{\sin x}{x}\right)} \cdot \left(\frac{1}{e^0}\right) \qquad \textit{(1 mark)}$$

$$= \frac{(\log e)^2}{1} \cdot (1) = (\log e)^2$$

$$= (1)^2 = 1 \qquad \textit{(1 mark)}$$

$\therefore f(0) = 1.$

(ii) Evaluate $\int \frac{7x+3}{\sqrt{3+2x-x^2}}\,dx$ **(4)**

Sol.: Let $I = \int \frac{7x+3}{\sqrt{3+2x-x^2}}\,dx$

Put $7x+3 = A\frac{d}{dx}\left(3+2x-x^2\right)+B$

$7x+3 = A(2-2x)+B$ ....(i)

$7x+3 = 2A+B-x(2A)$

$\therefore\ 7x+3 = (-2A)x+(2A+B)$

Comparing coefficients on both sides,

we get, $-2A \Rightarrow 7 \quad \Rightarrow A = \frac{-7}{2}$

and $2A+B = 3 \quad \Rightarrow 2\left(\frac{-7}{2}\right)+B=3$

$\therefore\ B = 3+7 = 10$

Substituting values of A and B in eqn.(i),

$$7x+3 = \frac{-7}{2}(2-2x)+10$$

$$\therefore\ I = \int \frac{\frac{-7}{2}(2-2x)+10}{\sqrt{3+2x-x^2}} \qquad \textit{(1 mark)}$$

$$= -\frac{7}{2}\int \frac{2-2x}{\sqrt{3+2x-x^2}}\,dx + 10\int \frac{1}{\sqrt{3+2x-x^2}}\,dx$$

$$= \frac{-7}{2}\times 2\sqrt{3+2x-x^2}+10I_1, \quad \text{....(ii)} \ \textit{(1 mark)}$$

where

$$I_1 = \int \frac{1}{\sqrt{3+2x-x^2}}\,dx = \int \frac{1}{\sqrt{3+1-(x-1)^2}}\,dx$$

$$= \int \frac{1}{\sqrt{4-(x-1)^2}}\,dx = \int \frac{1}{\sqrt{(2)^2-(x-1)^2}}\,dx$$

$$= \sin^{-1}\left(\frac{x-1}{2}\right) \qquad \text{....(iii)} \qquad \textit{(1 mark)}$$

Substituting I_1 in eqn.(ii), we get

$$I = -7\sqrt{3+2x-x^2}+10\sin^{-1}\left(\frac{x-1}{2}\right)+c \qquad \textit{(1 mark)}$$

(iii) If $x^y = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$. **(4)**

Sol.: Given : $x^y = e^{x-y}$

Taking log on both sides,

$$y \log x = (x-y)\log e, \quad \text{but } \log e = 1$$

$$\therefore \quad y \log x = x - y \quad \text{....(i)} \quad \textit{(1 mark)}$$

Differentiate (i) w.r.t. x.

$$y\left(\frac{1}{x}\right) + \log x \cdot \frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{y}{x} + \log x \cdot \frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}[1 + \log x] = 1 - \frac{y}{x}$$

$$= \frac{x-y}{x}$$

$$\therefore \quad \frac{dy}{dx}(1 + \log x) = \frac{y \log x}{x} \quad \text{....[by eqn.(i)]}$$

$$\therefore \quad \frac{dy}{dx} = \frac{y \log x}{x(1+\log x)} \quad \text{....(ii)} \quad \textit{(1 mark)}$$

From eqn.(i),

$$x - y = y \log x \quad \Rightarrow \quad x = y \log x + y$$

$$\therefore \quad x = y(1 + \log x) \quad \text{....(iii)} \quad \textit{(1 mark)}$$

Substitute equation (iii) in equation (ii)

$$\therefore \quad \frac{dy}{dx} = \frac{y \log x}{y(1+\log x)(1+\log x)}$$

$$\therefore \quad \frac{dy}{dx} = \frac{\log x}{(1+\log x)^2} \quad \textit{(1 mark)}$$

Q.3.(A) Attempt any TWO of the following : [6]

(i) If p and q are true and r and s are false statements, find the truth values of the following statements :

(a) $(p \wedge q) \vee r$ (b) $p \wedge (r \rightarrow s)$

(c) $(p \vee s) \leftrightarrow (q \wedge r)$ (3)

Sol.: (a) $(p \wedge q) \vee r$.

As $p \wedge q$ is true, then $T \vee F = T$. *(1 mark)*

Hence, truth value of the given statement is true.

(b) $p \wedge (r \rightarrow s) = T \wedge (F \rightarrow F) = T \wedge T = T$ *(1 mark)*

Hence, the truth value of the given statement is true.

(c) $(p \vee s) \leftrightarrow (q \wedge r) = (T \vee T) \leftrightarrow (F \wedge F)$

$= T \leftrightarrow F = F$ *(1 mark)*

Hence, the truth value of the given statement is false.

(ii) Show that the following function is always increasing if $f(x) = x^3 - 9x^2 + 30x + 7$.

Sol.: Given : $f(x) = x^3 - 9x^2 + 30x + 7$

Differentiate w.r.t. x

$$f'(x) = 3x^2 - 18x + 30 \quad \textit{(1 mark)}$$

$$= 3\left[x^2 - 6x + 10\right]$$

$$= 3\left[\left(x^2 - 6x + 9\right) + 1\right]$$

$$= 3\left[(x-3)^2 + 1\right] > 0 \quad \textit{(1 mark)}$$

$\therefore f'(x) > 0$ for every real values of x.

Since $f'(x)$ is greater than zero,

the given $f(x)$ is increasing. *(1 mark)*

iii) **Evaluate :** $\int e^x \cdot \frac{x+3}{(x+4)^2} dx$ **(3)**

Sol.: Let $I = \int e^x \cdot \frac{x+3}{(x+4)^2} dx$

$$= \int e^x \cdot \left[\frac{(x+4)-1}{(x+4)^2}\right] dx \quad \textit{(1 mark)}$$

$$= \int e^x \cdot \left[\frac{x+4}{(x+4)^2} - \frac{1}{(x+4)^2}\right] dx$$

$$= \int e^x \cdot \left[\frac{1}{x+4} - \frac{1}{(x+4)^2}\right] dx$$

$$= \int e^x \cdot \left[\frac{1}{x+4} + \frac{-1}{(x+4)^2}\right] dx \quad \textit{(1 mark)}$$

$$= e^x\left(\frac{1}{x+4}\right) + c$$

[Using the result $\int e^x [f(x) + f'(x)] dx] = e^x \cdot f(x) + c$]

$$\therefore \quad I = \frac{e^x}{x+4} + c \quad \textit{(1 mark)}$$

Q.3.(B) Attempt any TWO of the following : [8]

(i) **Evaluate :** $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ **(4)**

Sol.: Let $I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$

$$I = \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x} dx + \int_{-\pi}^{\pi} \frac{2x \sin x}{1+\cos^2 x} dx \quad \textit{(1 mark)}$$

$$= I_1 + I_2 \quad \text{....(i)}$$

where, $I_1 = \int_{-\pi}^{\pi} \frac{2x(\sin x)}{1+\cos^2 x} dx$

Let $f_1(x) = \frac{2x \sin x}{1+\cos^2 x}$

$$\therefore f_1(-x) = \frac{2(-x)\sin(-x)}{1+\cos^2(-x)} = \frac{-2x\sin(-x)}{1+\cos^2 x}$$

$$= \frac{2x \sin x}{1+\cos^2 x} = f_1(x)$$

$\therefore f_1(x)$ is an even function.

$$\therefore I_2 = \int_{-\pi}^{\pi} \frac{2x \sin x}{1+\cos^2 x} dx = 2\int_0^{\pi} \frac{2x \sin x}{1+\cos^2 x} dx \quad \text{...(ii)}$$

Let $f_2(x) = \frac{2x}{1+\cos^2 x}$

$$\therefore f_2(-x) = \frac{2(-x)}{1+\cos^2(-x)} = \frac{-2x}{1+\cos^2 x} = -f_2(x)$$

$\therefore f_2(x)$ is an odd function.

$$\therefore I_1 = \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x}\,dx = 0 \quad \text{....(iii)} \qquad \textit{(1 mark)}$$

Substituting (ii) and (iii) in eqn. (i)

$$\therefore I = 0 + 2\cdot\int_0^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx$$

$$= 4\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx \quad \text{....(iv)}$$

By property of definite integral,

$$I = 4\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}\,dx$$

$$= 4\int_0^{\pi} \frac{(\pi-x)\sin x}{1+\cos^2 x}\,dx$$

$$= 4\int_0^{\pi} \frac{\pi\sin x}{1+\cos^2 x}\,dx - 4\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$$

$$= 4\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx - I \quad \text{....[by(iv)]}$$

$$\therefore 2I = 4\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx \qquad \textit{(1 mark)}$$

Put $\cos x = t \qquad \therefore \sin x\,dx = -dt$

When $x = 0,\ t = 0. \qquad$ When $x = \pi,\ t = -1$

$$\therefore 2I = -4\pi\int_1^{-1} \frac{dt}{1+t^2} = 4\pi\int_{-1}^{1} \frac{1}{1+t^2}\,dt$$

$$= 4\pi\left[\tan^{-1}(t)\right]_{-1}^{1} = 4\pi\left[\frac{\pi}{4}+\frac{\pi}{4}\right]$$

$$= 2\pi^2$$

$$\therefore I = \pi^2 \qquad \textit{(1 mark)}$$

(ii) Evaluate $\int^{\pi/2} \log\cos x\,dx$ **(4)**

(B)(2)

$$I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}}\,dx$$

$$= \int_0^{\pi/2} \frac{1}{1+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}\,dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad \text{— (I)}$$

using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

replace x by $\frac{\pi}{2} - x$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2 - x)}}{\sqrt{\cos(\frac{\pi}{2} - x)} + \sqrt{\sin(\frac{\pi}{2} - x)}} dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad - II$$

Add I and II:

$$I + I = \int_0^{\pi/2} __ \, dx + \int_0^{\pi/2} __ \, dx$$

$$2I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} + \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$2I = \int_0^{\pi/2} \cancel{\frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}} dx$$

$$2I = \int_0^{\pi/2} 1 \, dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} - 0$$

$$\boxed{I = \frac{\pi}{4}}$$ Ans.

(iii) Find the value of k, if the function $f(x)$ is continuous at $x = \frac{\pi}{4}$; where

$$f(x) = \frac{1 - \tan x}{1 - \sqrt{2}\sin x}, \quad \text{for } x \neq \frac{\pi}{4};$$

$$= \frac{k}{2}, \quad \text{for } x = \frac{\pi}{4}. \qquad \textbf{(4)}$$

Sol.: Since $f(x)$ is continuous at $x = \frac{\pi}{4}$

$$\lim_{x \to \frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right) \text{ Given : } f\left(\frac{\pi}{4}\right) = \frac{k}{2}$$

$$\therefore \frac{k}{2} = \lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2}\sin x} = \lim_{x \to \frac{\pi}{4}} \left[\frac{1 - \frac{\sin x}{\cos x}}{1 - \sqrt{2}\sin x}\right]$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos x\left(1-\sqrt{2}\sin x\right)} \quad \textit{(1 mark)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos x\left(1-\sqrt{2}\sin x\right)} \times \frac{1+\sqrt{2}\sin x}{1+\sqrt{2}\sin x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(\cos x - \sin x)\left(1+\sqrt{2}\sin x\right)}{\cos x\left(1-2\sin^2 x\right)}$$

$$= \lim_{x \to \frac{\pi}{4}} \left[\frac{\cos x - \sin x}{1-2\sin^2 x}\right] \cdot \lim_{x \to \frac{\pi}{4}} \left[\frac{1+\sqrt{2}\sin x}{\cos x}\right]$$

(1 mark)

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos^2 x + \sin^2 x - 2\sin^2 x} \times \frac{1+\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)}{\frac{1}{\sqrt{2}}}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos^2 x - \sin^2 x}\left[\frac{1+1}{1/\sqrt{2}}\right]$$

$$= 2\sqrt{2} \lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{(\cos x - \sin x)(\cos x + \sin x)}$$

(1 mark)

$$= 2\sqrt{2} \lim_{x \to \frac{\pi}{4}} \left[\frac{1}{\cos x + \sin x}\right]$$

$$= 2\sqrt{2}\left[\frac{1}{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}\right] = 2\sqrt{2}\left[\frac{\sqrt{2}}{2}\right]$$

$$\therefore \frac{k}{2} = 2 \quad \Rightarrow \quad k = 4$$

$$\therefore k = 4 \quad \textit{(1 mark)}$$

Section-II

Q.4. Attempt any SIX of the following : [12]

(i) Mr. Paralikar spends 30% of his income on food items and 15% on conveyance. If for the month of October 2012, he spent ₹ 1200 on conveyance, find his expenditure on food items during the same month. (2)

Sol.: We assume that Mr. Paralikar spent ₹ x during October 2012 on food items.

$\therefore$ We can equate the ratio.

$$30 : x = 15 : 1200 \quad \textit{(1 mark)}$$

$$\frac{30}{x} = \frac{15}{1200}$$

$$\therefore x = \frac{1200 \times 30}{15} = 2400. \quad \textit{(1 mark)}$$

$\therefore$ Expenditure on food items during October 2012 = ₹ 2400.

(ii) A merchant pays his agent 8% ordinary commission and 3% delcredere. If the agent sells goods worth ₹ 5,300, find the amount which the merchant receives from the agent. (2)

Sol.: Ordinary commission = 8% of ₹ 5,300

$$= 5300 \times \frac{8}{100} = 424$$

Delcredere = 3 % of ₹ 5,300

$$= 5300 \times \frac{3}{100} = 159 \quad \textit{(1 mark)}$$

Total commission = 424 + 159 = 583

$\therefore$ Amount which merchant receives

= 5300 − 583 = ₹ 4,717 *(1 mark)*

(iii) A flat is sold through Mr. Patil who charged 1.5% commission for the seller and got 2.5% commission from the buyer. Thus he earned a total amount of ₹ 15,000. Find the sales price of the flat. (2)

Sol.: Let x be the sales price.

Commission from buyer

= 2.5% of x = $0.025x$

Commission from the seller

= 1.5% of x = $0.015x$

Total commission = 15,000

$\Rightarrow 0.025x + 0.015x = 15{,}000$ *(1 mark)*

$\Rightarrow 0.04x = 15{,}000$

$\Rightarrow x = \frac{15{,}000}{0.04} = 3{,}75{,}000$ *(1 mark)*

$\therefore$ Sales price = x = ₹ 3,75,000

(iv) A book seller paid ₹ 765 for a parcel of 50 books on which he had been given a discount of 15%. Find the list price of book.

Sol.: Net price per book paid by the book seller

$$= \frac{765}{50} = 15.3 \quad \textit{(1 mark)}$$

Since the book seller has been given 15% discount on list price,

List price of a book

$$= \frac{15.3}{85} \times 100 = ₹\ 18 \quad \textit{(1 mark)}$$

(v) A person aged 45 insures his life for a certain amount and pays ₹ 313.50 as premium every six months. If the premium payable by him per annum is at the rate of ₹ 78.375 per thousand, find the value of his policy. (2)

Sol.: Yearly premium = 313.50 × 2 = ₹ 627 *(1 mark)*

Rate of annual premium = ₹ 78.375 per ₹ 1000.

$\therefore$ For ₹ 627 premium, the value of the policy is

$$= \frac{627}{78.375} \times 1000 = ₹\ 8000 \quad \textit{(1 mark)}$$

(vi) Calculate crude death rate for the population in a town as shown below :

Age Group (years)	Population in the town	
	No. of persons (in '000)	No. of deaths
Below 10	14	180
10 – 40	21	120
40 – 70	67	570
70 and above	11	460

Sol.: $CDR = \frac{D}{P} \times 1000$ **(2)**

$D = 180 + 120 + 570 + 460 = 1330$

$P = 14 + 21 + 67 + 11 = 113$ (in thousands)

(1 mark)

$\therefore CDR = \frac{1330}{113{,}000} \times 1000$

$= 11.77$ per thousand *(1 mark)*

(vii) In a bivariate data, $n = 20$, $\sum x = 300$, $\sum y = 400$, $\sum xy = 395$, find Cov(X, Y). (2)

Sol.: $\bar{x} = \frac{\sum x}{n} = \frac{300}{20} = 15,$

$\bar{y} = \frac{\sum y}{n} = \frac{400}{20} = 20$

$Cov(X, Y) = \frac{\sum xy}{n} - \bar{x} \cdot \bar{y}$ *(1 mark)*

$= \frac{395}{20} - 15 \times 20$

$= 19.75 - 300$

$= -280.25$ *(1 mark)*

(viii) The following is the p.d.f. of a continuous random variable X is given by

$f(x) = \frac{x}{3}, \quad 0 < x < 3$

$= 0, \quad$ otherwise.

Find the cumulative density function of X. **(2)**

Sol.: c.d.f. of continuous random variable is given by

$F(x) = \int_{-\infty}^{x} f(y)\,dy$, where $f(y) = \frac{y}{3}$.

$\therefore F(x) = \int_{0}^{x} f(y)\,dy = \int_{0}^{x} \frac{y}{3}\,dy$ *(1 mark)*

$= \frac{1}{3}\left(\frac{y^2}{2}\right)_0^x = \frac{1}{6}\left(x^2 - 0\right)$

$= \frac{x^2}{6}$ *(1 mark)*

Q.5.(A) Attempt any two of the following : [6]

(i) For the following table find the terms marked by '?' (3)

x	l_x	d_x	q_x	p_x	L_x	T_x	e_x^0
5	90	?	0.11	?	?	290	
6	80					?	?

Sol.: $d_5 = l_5 - l_6 = 90 - 80 = 10$

$p_5 = 1 - q_5 = 1 - 0.11 = 0.89$

$L_5 = L_x = \frac{l_x + l_{x+1}}{2} = \frac{90 + 80}{25} = 85$ *(1 mark)*

$T_x = L_x + T_{x+1}$

$T_6 = T_5 - L_5 = 290 - 85 = 205$ *(1 mark)*

$e_x^0 = \frac{T_6}{l_6} = \frac{205}{80} = 2.565$ *(1 mark)*

(ii) Find Karl Pearson's correlation coefficient between sales and expenses of the following 7 firms : (3)

Firm No.	1	2	3	4	5	6	7
Sales	11	7	9	5	8	6	10
Expenses	10	8	6	5	9	7	11

Sol.: Let x denote sales and y denote expense.

x	y	x^2	y^2	xy
11	10	121	100	110
7	8	49	64	56
9	6	81	36	54
5	5	25	25	25
8	9	64	81	72
6	7	36	49	42
10	11	100	121	110
56	56	476	476	469

$n = 7, \quad \sum x = 56, \quad \sum y = 56,$

$\sum x^2 = 476, \quad \sum y^2 = 476, \quad \sum xy = 469$

$\bar{x} = \frac{\sum x}{n} = \frac{56}{7} = 8$

$\bar{y} = \frac{\sum y}{n} = \frac{56}{7} = 8$

$Cov(X, Y) = \frac{\sum xy}{n} - \bar{x} \cdot \bar{y}$

$= \frac{469}{7} - 8 \times 8 = 3$ *(1 mark)*

$Var(X) = \frac{\sum X^2}{n} - (\bar{X})^2 = \frac{476}{7} - (8)^2$

$= 68 - 64 = 4$

$Var(Y) = \frac{\sum Y^2}{n} - (\bar{Y})^2 = \frac{476}{7} - (8)^2$

$= 68 - 64 \quad = 4$ *(1 mark)*

$$\gamma = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \cdot \text{Var}(Y)}}$$

$$= \frac{3}{\sqrt{4 \times 4}} = \frac{3}{4} = 0.75$$ *(1 mark)*

(iii) A person fires 5 shots at a target. The probability that any shot will hit the target is $\frac{4}{5}$. Find the probability that the target is hit (i) exactly 3 times (ii) at least 3 times. (3)

Sol.: Let X denote the number of shots out of 5 that hit the target.

$X \sim B(n, p)$, where $n = 5$, $p = \frac{4}{5}$.

$$q = 1 - p = 1 - \frac{4}{5} = \frac{1}{5}$$

$$\therefore P(X = x) = nC_x \cdot p^x q^{n-x}$$ *(1 mark)*

$$= 5C_x \left(\frac{4}{5}\right)^x \left(\frac{1}{5}\right)^{5-x}$$

(i) Required probability $= P(X = 3)$

$$= 5C_3 \left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right)^{5-3} = 5C_2 \left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right)^2$$

$$= \frac{5 \times 4}{1 \times 2} \times \frac{64}{3125} = \frac{128}{625}$$ *(1 mark)*

(ii) $P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$

$$= \frac{128}{625} + 5C_4 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^5$$

$$= \frac{128}{625} + \frac{256}{625} + \frac{1024}{3125}$$

$$= \frac{1408}{3125}$$ $\frac{2944}{3125}$ Check it *(1 mark)*

Q.5.(B) Attempt any TWO of the following : [8]

(i) Find the equation of the line of regression of demand on price from the following coded data.

Price	2	4	6	10	16
Demand	72	54	36	18	12

Also find the estimated demand when the price is 7.5. (4)

Sol.: Let the price and demand be denoted by x and y respectively.

We want the regression equation of y on x.

x	y	$u = x - 6$	$v = y - 36$	u^2	uv
2	72	– 4	36	16	– 144
4	54	– 2	18	4	– 36
6	36	0	0	0	0
10	18	4	– 18	16	– 72
16	12	10	– 24	100	– 240
	Total	8	12	136	– 492

(1 mark)

$n = 5$, $\sum u = 8$, $\sum v = 12$,

$\sum u^2 = 136$, $\sum uv = -492$

$$\bar{u} = \frac{\sum u}{n} = \frac{8}{5} = 1.6, \quad \bar{v} = \frac{\sum v}{n} = \frac{12}{5} = 2.4$$

$$\bar{X} = 6 + \bar{u} = 6 + 1.6 = 7.6$$

$$\bar{Y} = 36 + \bar{v} = 36 + 2.4 = 38.4$$

$$b_{yx} = b_{uv} = \frac{n\sum uv - (\sum u)(\sum v)}{n\sum u^2 - (\sum u)^2}$$

$$= \frac{5(-492) - (8)(12)}{5(136) - (8)^2} = -\frac{2556}{616}$$

$$= -4.149$$ *(1 mark)*

Regression equation of demand (y) on price (x) is given by

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

i.e. $y - 38.4 = -4.149(x - 7.6)$

i.e. $y = -4.149x + 69.93$ *(1 mark)*

$\therefore$ Estimate of demand (y) when price $x = 7.5$ is

$$y = -4.149(7.5) + 69.93$$

$$= 38.81$$ *(1 mark)*

(ii) A team of 4 horses and 4 riders has entered the jumping show contest. The number of penalty points to be expected when each rider ride search horse is shown below. How should the horses be assigned to the riders so as to minimize the expected loss ? Also find the minimum expected loss. (4)

Riders \ Horses	H_1	H_2	H_3	H_4
R_1	12	3	3	2
R_2	1	11	4	13
R_3	11	10	6	11
R_4	5	8	1	7

Sol.: Subtracting minimum of each row.

	H_1	H_2	H_3	H_4
R_1	12	3	3	2
R_2	1	11	4	13
R_3	11	10	6	11
R_4	5	8	1	7

	H_1	H_2	H_3	H_4
R_1	10	1	1	0
R_2	0	10	3	12
R_3	5	4	0	5
R_4	4	7	0	6

Only column 2 has non-zero elements. Subtracting minimum elements of column 2 from its elements, we get,

	H_1	H_2	H_3	H_4
R_1	10	0	1	0
R_2	0	9	3	12
R_3	5	3	0	5
R_4	4	6	0	6

	H_1	H_2	H_3	H_4
R_1	~~10~~	~~0~~	~~1~~	~~0~~
R_2	~~0~~	~~9~~	~~3~~	~~12~~
R_3	5	2	0	5
R_4	4	6	0	6

$$\begin{pmatrix}\text{Minimum number of lines} \\ \text{required to cover all zeros}\end{pmatrix} = 3 < \begin{pmatrix}\text{Order of the} \\ \text{matrix}\end{pmatrix}$$

∴ Optimal solution is not obtained. *(1 mark)*

∴ Subtracting minimum of the uncovered elements from the uncovered elements and adding it to the intersecting elements, we get

	H_1	H_2	H_3	H_4
R_1	10	0	4	0
R_2	0	9	6	12
R_3	2	0	0	2
R_4	1	3	0	3

	H_1	H_2	H_3	H_4
R_1	~~10~~	~~0~~	~~4~~	~~0~~
R_2	~~0~~	~~9~~	~~6~~	~~12~~
R_3	2	0	0	2
R_4	1	3	0	3

The minimum number of lines required to cover all zeros = 4 = order of the matrix.

∴ Optimal solution is reached. *(1 mark)*

∴ We can not make an assignment in the first row and the third row as they have more than one zero.

	H_1	H_2	H_3	H_4
R_1	10	0	4	0
R_2	$\boxed{0}$	9	6	12
R_3	2	0	~~0~~	2
R_4	1	3	$\boxed{0}$	3

	H_1	H_2	H_3	H_4
R_1	10	~~0~~	4	0
R_2	0	9	6	12
R_3	2	$\boxed{0}$	~~0~~	2
R_4	1	3	$\boxed{0}$	3

Horses / Riders	H_1	H_2	H_3	H_4
R_1	10	~~0~~	4	$\boxed{0}$
R_2	$\boxed{0}$	9	6	12
R_3	2	$\boxed{0}$	~~0~~	2
R_4	1	3	$\boxed{0}$	3

∴ The required optimal assignment is

$H_1 \to R_2$, $H_2 \to R_3$, $H_3 \to R_4$ and $H_4 \to R_1$ *(1 mark)*

∴ Minimum expected loss = 1 + 10 + 1 + 2

= 14 *(1 mark)*

(iii) An aeroplane can carry a maximum of 200 passengers. Baggage allowed for a first class passenger is 30 kgs and for an economy class passenger is 20 kgs. The maximum capacity of the aeroplane for the baggage is 4500 kgs. The profit on each first class ticket is ₹ 500 and that on an economy ticket is ₹ 300. It is required to determine how many tickets of each type be sold so as to get maximum profit. Formulate this problem as L.P.P. and solve it graphically. Also find feasible solution graphically and write the vertices of feasible region. (4)

Sol.: Let x tickets of 1st class and y tickets of economy class be sold.

∴ $x \geq 0$, $y \geq 0$ ....(i)

As the aeroplane can carry maximum 200 passengers

$x + y \leq 200$ ....(ii)

Now, we can tabulate information.

	1st class ticket	Economy class ticket	Maximum Capacity
Baggage in kgs.	30	20	4500
Profit in ₹	500	300	—

∴ $30x + 20y \leq 4500$ ....(iii)

∴ Total profit Z = $500x + 300y$ ....(objective function)

∴ LPP is maximized Z = $500x + 300y$

subject to $x + y \leq 200$

$30x + 20y \leq 4500$

$x \geq 0$, $y \geq 0$ *(1 mark)*

We plot the lines $x + y = 200$ ....(iv)

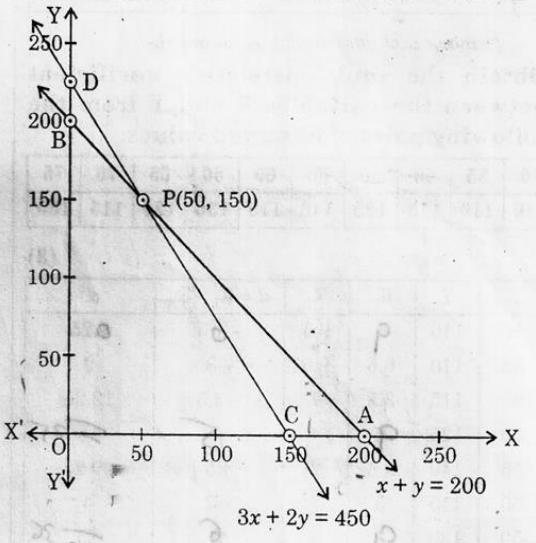
and $30x + 20y = 4500$

i.e. $3x + 2y = 450$ ....(v)

Consider $x + y = 200$ and consider $3x + 2y = 450$.

x	0	200
y	200	0

x	0	150
y	225	0



(1 mark)

The feasible region is the quadrilateral OCPB, where P is the point of intersection of the lines $x + y = 200$ and $3x + 2y = 450$.

Solving equations (iv) and (v), we get

$x = 50$ and $y = 150$.

∴ P ≡ (50, 150)

∴ Vertices of the feasible region O(0, 0), C(150, 0), P(50, 150) and B(0, 200).

Objective function Z = $500x + 300y$.

Hence, we can tabulate the vertices.

Vertex	Z = $500x + 300y$		
O(0, 0)	$Z_O = 500 \times 0 + 300 \times 0$	=	0
C(150, 0)	$Z_C = 500 \times 150 + 300 \times 0$	=	75,000
P(50, 150)	$Z_P = 500 \times 50 + 300 \times 150$	=	60,000
B(0, 200)	$Z_B = 500 \times 0 + 300 \times 200$	=	60,000

(1 mark)

Z is maximum at P and B

$\therefore$ Maximum profit = ₹ 60,000. *(1 mark)*

Q.6.(A) Attempt any TWO of the following : [6]

(i) For the given data write the entries of d_x, q_x, p_x and L_x of the life table, whose notations have their usual meanings.

x	0	1	2	3	4	5
l_x	4000	3000	1000	200	40	0

Sol.: $T_x = L_x + L_{x+1} + L_{x+2} + ...$ **(3)**

i.e. $T_x = L_x + T_{x+1}$

x	l_x	$d_x = l_x - l_{x+1}$	$q_x = \frac{d_x}{l_x}$	$p_x = 1 - q_x$	$L_x = \frac{l_x + l_{x+1}}{2}$
0	4000	1000	0.25	0.75	3500
1	3000	2000	0.66	0.34	2000
2	1000	800	0.80	0.20	600
3	200	160	0.80	0.20	120
4	40	40	1.00	0	20
5	0	—	—	—	—

[1 mark each for column d_x, p_x and L_x]

(ii) Obtain the rank correlation coefficient between the variables X and Y from the following pairs of observed values.

X	50	55	65	50	55	60	50	65	70	75
Y	110	110	115	125	140	115	130	120	115	160

(3)

Sol.:

X	Y	R_1	R_2	$d = R_1 - R_2$	d^2
50	110	9	9.5	– 0.5	0.25
55	110	6.5	9.5	– 3	9
65	115	3.5	7	– 3.5	12.25
50	125	9	4	5	~~16~~ 25
55	140	6.5	2	4.5	20.25
60	115	5	7	– 2	4
50	130	9	3	6	~~25~~ 36
65	120	3.5	5	– 1.5	2.25
70	115	2	7	– 5	25
75	160	1	1	0	0
				Total	~~116.00~~ 154

$n = 10$, $\sum d^2 = 116$. *(1 mark)*

1. Correction factor for 50, $CF_1 = \frac{m(m^2-1)}{12}$

   where $m = 3 \Rightarrow CF_1 = \frac{3(3^2-1)}{12} = 2$

2. Correction factor for 55, $CF_2 = \frac{m(m^2-1)}{12}$

   where $m = 2 \Rightarrow CF_2 = \frac{2(2^2-1)}{12} = 0.5$

3. Correction factor for 65, $CF_3 = \frac{m(m^2-1)}{12}$

   where $m = 2 \Rightarrow CF_3 = \frac{2(2^2-1)}{12} = 0.5$

4. Correction factor for 110, $CF_4 = \frac{m(m^2-1)}{12}$

   where $m = 2 \Rightarrow CF_4 = \frac{2(2^2-1)}{12} = 0.5$

5. Correction factor for 115, $CF_5 = \frac{m(m^2-1)}{12}$

   where $m = 3 \Rightarrow CF_5 = \frac{3(3^2-1)}{12} = 2$

$\therefore CF = CF_1 + CF_2 + CF_3 + CF_4 + CF_5$

$= 2 + 0.5 + 0.5 + 0.5 + +2 = 5.5$ *(1 mark)*

$$R = 1 - \frac{6\left[\sum d^2 + CF\right]}{n(n^2-1)} = 1 - \frac{6[116+5.5]}{10(10^2-1)}$$

$$= 1 - \frac{6 \times 121.5}{10 \times 99} = 1 - \frac{729}{990}$$

$$= \frac{261}{990} = 0.26$$ Ans 0.15 *(1 mark)*

(iii) If a random variable X follows a Poisson distribution such that P(X = 3) = P(X = 4). Find the mean and standard deviation of the distribution. Also find P(X = 2). (3)

Sol.: Given : $X \sim P(m)$

$$\therefore P(X = x) = \frac{e^{-m} \cdot m^x}{x!}$$

Given, $P(X = 3) = P(X = 4)$

$$\Rightarrow \frac{e^{-m} \cdot m^3}{3!} = \frac{e^{-m} \cdot m^4}{4!}$$

$$\Rightarrow \frac{1}{3!} = \frac{m}{4!}$$

$$\Rightarrow m = \frac{4!}{3!} = \frac{4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 4$$

$\therefore m = 4$

Mean $= m = 4$ *(1 mark)*

S.D. $= \sqrt{m} = \sqrt{4} = 2$ *(1 mark)*

$$\therefore P(X = x) = \frac{e^{-4} \cdot 4^x}{x!}$$

Put $x = 2$

$$P(X = 2) = \frac{e^{-4} \times 4^2}{2!} = \frac{e^{-4} \times 16}{2} = 8e^{-4}$$

$$\therefore P(X = 2) = \frac{8}{e^4}$$ *(1 mark)*

Q.6.(B) Attempt any TWO of the following : [8]

(i) Following equations represent two regression lines $5x + y = 21$ and $4x + 5y = 42$. Find :

(a) mean values of x and y

(b) correlation coefficient γ

(c) standard deviation of y when Var(X) = 9.

(d) most probable value of y when $x = 2$. (4)

Sol.: Solving the given equation of regression lines, we will get mean values of x and y.

$$5x + y = 21 \quad ....(i)$$
$$4x + 5y = 42 \quad ....(ii)$$

(i) × 5 $\Rightarrow$ $25x + 5y = 105$

(ii) $\Rightarrow$ $4x + 5y = 42$

Subtracting : $21x = 63$

$$\Rightarrow x = \frac{63}{21} = 3$$

Substituting $x = 3$ in equation (i), we will get

$5(3) + y = 21 \Rightarrow y = 21 - 15$

$\therefore y = 6$

(a) $\therefore$ Mean value of $x = \bar{x} = 3$

Mean value of $y = \bar{y} = 6$ *(1 mark)*

Let us assume that $5x + y = 21$ be the regression equation of y on x.

$\Rightarrow y = -5x + 21$

Now, b_{yx} = coefficient of $x = -5$

Take $4x + 5y = 42$ as regression equation of x on y.

$\Rightarrow 4x = -5y + 42$

$$\Rightarrow x = \frac{-5y + 42}{4}$$

$$\therefore b_{xy} = \text{coefficient of } y = -\frac{5}{4}.$$

Now, $\gamma^2 = b_{yx} \times b_{xy} = -5 \times -\frac{5}{4} = \frac{25}{4} > 1$

which is not possible as $-1 \le r \le +1$.

$\therefore$ Our assumption is wrong

i.e. $5x + y = 21$ is not regression equation of y on x.

$\therefore 5x + y = 21$ is the regression equation of x on y.

$\Rightarrow 5x = -y + 21$

$$\Rightarrow x = \frac{-y + 21}{5}$$

$$\therefore b_{xy} = \text{coefficient of } y = -\frac{1}{5}.$$

Now, $4x + 5y = 42$ is regression equation of y on x.

$\Rightarrow 5y = -4x + 42$

$$\Rightarrow y = \frac{-4x + 42}{5}$$

$$\therefore b_{yx} = \text{coefficient of } x = -\frac{4}{5}.$$

$$\therefore \gamma^2 = b_{yx} \times b_{xy} = \frac{-4}{5} \times \frac{-1}{5} = \frac{4}{25} < 1$$

$-1 \le r \le +1$ satisfied

$$\therefore r = \sqrt{\frac{4}{25}} = \pm\frac{2}{5}$$

Since b_{yx} and b_{xy} are negative.

(b) $\therefore r = -\frac{2}{5} = -0.4$ *(1 mark)*

Given : Var(X) = 9

$\Rightarrow$ S.D. of $x = \sigma_x = \sqrt{9} = 3$

We have $b_{yx} = \gamma \cdot \frac{\sigma_y}{\sigma_x}$

$$\Rightarrow \frac{-4}{5} = \frac{-2}{5} \cdot \frac{\sigma_y}{3}$$

(c) $\Rightarrow \sigma_y = \frac{-4 \times 3 \times 5}{-2 \times 5} = 6$ *(1 mark)*

(d) Regression equation of Y on X is $4x + 5y = 42$

Put $x = 2 \Rightarrow 4(2) + 5y = 42$

$\Rightarrow 5y = 42 - 8$

$$\Rightarrow y = \frac{34}{5} = 6.8$$

$\therefore$ Most probable value of y when $x = 2$ is 6.8

(1 mark)

(ii) Find the optimal sequence that minimize total time required to complete five jobs, which have to be processed on each of three machines M_1, M_2, M_3 in the order M_1, M_2, M_3. If the processing time for each job on each machine is given below. Also find the total elapsed time and idle time for M_2.

Machine	Jobs				
	P	Q	R	S	T
M_1	6	7	5	11	5
M_2	4	3	2	5	1
M_3	3	8	7	4	9

(4)

Sol.: For M_1 smallest time = 5

For M_2 smallest time = 5

$$\therefore \begin{pmatrix}\text{greatest time on}\\ \text{2nd machine}\end{pmatrix} \le \begin{pmatrix}\text{smallest time on}\\ \text{1st machine}\end{pmatrix}$$

is satisfied

Let the machines G and H be considered as $M_1 + M_2$ and $M_2 + M_3$ respectively.

Machine \ Jobs	P	Q	R	S	T
G	10	10	7	16	6
H	7	11	9	9	10

(1 mark)

Minimum time is 6 for the job T on the machine G.

$\therefore$ Place T to the left.

Next minimum time is 7 for job R on machine G and for job P on machine H.

$\therefore$ Place R to the left and P to the right.

T	R			P

Next minimum time is 9 for job S on machine H. So we place it on the right.

∴ Optimal sequence is

T	R	Q	S	P

(1 mark)

Job	Machines			M_1		M_2		M_3	
	M_1	M_2	M_3	In	Out	In	Out	In	Out
T	5	1	9	0	5	5	6	6	15
R	5	2	7	5	10	10	12	15	22
Q	7	3	8	10	17	17	20	22	30
S	11	5	4	17	28	28	33	33	37
P	6	4	3	28	34	34	38	38	41

Total elapsed time = 41 *(1 mark)*

Idle time for M_2 = 5 + 4 + 5 + 8 + 1 + 3

= 23 + 3 = 26 *(1 mark)*

(iii) The owner of a small machine shop has four machinists available to assign jobs for the day. The expected profits for each job done by each machinists are given below. Find the assignment of jobs to the machinists that will result in maximum profit. Also find the maximum profit. [One machine can be assigned only on job.]

Machinist		Jobs				
		A	B	C	D	E
	M_1	62	78	50	101	82
	M_2	71	84	61	73	59
	M_3	87	92	111	71	81
	M_4	48	61	87	77	80

(4)

Sol.: We first convert the maximisation problem to minimisation problem by subtracting each element of the matrix from the maximum demand i.e. 111.

	A	B	C	D	E
M_1	49	33	61	10	29
M_2	40	27	50	38	52
M_3	24	19	0	40	30
M_4	63	50	24	34	31

As (no. of jobs = 5) ≠ (Number of machinists = 4)

We introduce a dummy machinist M_5 with 0 cost for each job.

	A	B	C	D	E
M_1	49	33	61	10	29
M_2	40	27	50	38	52
M_3	24	19	0	40	30
M_4	63	50	24	34	31
M_5	0	0	0	0	0

Subtracting minimum element of each row from all its elements, we get

	A	B	C	D	E
M_1	39	23	51	0	19
M_2	13	0	23	11	25
M_3	24	10	0	40	30
M_4	39	26	0	10	7
M_5	0	0	0	0	0

Drawing minimum lines to cover all zeros

	A	B	C	D	E
M_1	~~39~~	~~23~~	~~51~~	~~0~~	~~19~~
M_2	~~13~~	~~0~~	~~23~~	~~11~~	~~25~~
M_3	24	10	0	40	30
M_4	39	26	0	10	7
M_5	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~

(1 mark)

(Min. no. of lines = 4) <(Order of the matrix = 5)

∴ Optimal solution is not reached.

Minimum uncovered elements = 7.

Subtracting 7 from uncovered and adding it to intersecting elements, we get

	A	B	C	D	E
M_1	39	23	58	0	19
M_2	13	0	30	11	25
M_3	17	12	0	33	23
M_4	32	19	0	3	0
M_5	0	0	0	0	0

Drawing minimum lines to cover all zeros

	A	B	C	D	E
M_1	~~39~~	~~23~~	~~58~~	~~0~~	~~19~~
M_2	~~13~~	~~0~~	~~30~~	~~11~~	~~25~~
M_3	17	12	0	33	23
M_4	~~32~~	~~19~~	~~0~~	~~3~~	~~0~~
M_5	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~

(Min. no. of lines = 5) =(Order of the matrix = 5)

∴ Optimal solution is reached. *(1 mark)*

We start with rows and make an assignment in the rows which have exactly one zero and cross the zeros of the column in which assignment is made.

	A	B	C	D	E
M_1	39	23	58	$\boxed{0}$	19
M_2	13	$\boxed{0}$	30	11	25
M_3	17	12	$\boxed{0}$	33	23
M_4	32	19	~~0~~	3	$\boxed{0}$
M_5	0	~~0~~	~~0~~	~~0~~	~~0~~

∴ The optimal assignment is

$B \to M_2$, $C \to M_3$, $D \to M_1$, $E \to M_4$ *(1 mark)*

∴ Maximum profit = 84 + 111 + 101 + 80

= ₹ 376 *(1 mark)*

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