Prof. Sharad Shah's
SHAH EDUCATIONAL ACADEMY SEA SCORE EXCEL ACHIEVE

SHAH'S ADVANTAGE PRELIM SERIES

A DIVISION OF "SHAH EDUCATIONAL ACADEMY"

B.COM, BAF, BMS, BBI, BFM, MCOM, CS, CPT, IPCC, Final CA.

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CLASS: S.Y.J.C ***PRELIM SERIES NO. 2*** **Date: 25th Jan 2020**	**MATHS I & II** **(2019 -20)**	**Time : 3 Hrs.** **Marks: 80** **Day:- Saturday**

Note: (1) All questions are compulsory; (2) Figures to the right indicate full marks; (3) Answer to every question, must be started on a new page.; (4) Use separate answers books for each section. (5) Graph paper is not necessary for LLP.

Section – I

Q.1 Attempt any Six questions: **(12)**

(i) $\int \frac{1}{\sin^2 x.\cos^2 x}dx$

(ii) Find derivative of $\tan^{-1}\left[\frac{16x}{1-64x^2}\right]$

(iii) If $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ find $(A + I).(A - I)$

(iv) Write the negation of the following statement.
$2 + 3 = 0$ if and only if $2i$ is a real number.

(v) Find k it the following function is continuous at $x = 2$
$f(x) = 2 + 3k$ for $x \le 2$
$= 1 - 7x$ for $x > 2$

(vi) Evaluate $\int_1^2 \left(x - \frac{1}{x}\right)^2 dx$

(vii) The demand function is $D = \frac{4}{p}$. Find elasticity of demand w.r.t. price when price is 2 units.

(viii) Find $\int \sin^2 4x dx$

Q.2 (a) Attempt any two of the questions: **(6)**

(i) Differentiate $\log(1 + x^2)$ w.r.x. $\tan^{-1} 4x$

(ii) $\int e^x.\frac{(1-x)^2}{(x^2+1)^2}dx$

(iii) Solve the equations $2x + 3y = 9$ and $y - x = -2$ using the method of reduction.

(b) Attempt any two of the following : **(8)**

(i) $\int_0^1 \frac{x+4}{x^2+5}dx$

(ii) The function given below is continuous on domain (0,5) then find the value of $\frac{a+b}{2}$

$$f(x) = \begin{cases} x^2 + ax + b & x < 2 \\ 3x + 2 & 2 \le x \le 4 \\ 2ax + 5b & x > 4 \end{cases}$$

(iii) If $x^2.y^k = (x+y)^{2+k}$ then find $\frac{dy}{dx}$

Q.3 (a) Attempt any two questions: **(6)**

(i) $\int_0^{\pi} \frac{1}{1+\sin x} dx$

(ii) Find $\frac{du}{dv}$ if $u = \sin^3\theta$, $v = \cos^3\theta$

(iii) Verify that $\sim(p \to q) \equiv p \wedge \sim q$

(b) Attempt any two questions: **(8)**

(i) Evaluate $\int_0^2 \frac{dx}{x+\sqrt{4-x^2}}$

(ii) Find $\int \frac{x^2+1}{(x^2+4)(x^2+25)} dx$

(iii) Divide 60 in two parts such that the product of square of one part and the other is maximum.

Section – II

Q.1 Attempt any Six questions: **(12)**

(i) An article is marked at ₹ 1,500. A trader allows a discount at 3% and still gains 20% on the cost. Find the cost price of the article.

(ii) What must be subtracted from each of the numbers 5, 7 and 10, so that the resulting numbers are in continued proportion?

(iii) For a bivariate date, the regression coefficient of Y on X is 0.4 and the regression coefficient of X on Y is 0.9. Find the value of variance of Y if variance of X is 9.

(iv) Coefficient of rank correlation between X and Y is 0.5 and $\sum d_1^2 = 42$. Assuming that no ranks are repeated, find the number of pairs of observations.

(v) In the following table, c.d.f. of a.r.v. X is given

x_i	-2	-1	0	1	2
$F(x_i)$	0.2	0.5	0.65	0.9	1

(i) Find p.m.f. of X. (ii) Compute $P(0 < X < 2)$, $P(X > 1)$

(vi) The probability of defective bolt in an workshop is 40%. Find mean and variance of the defective bolts out of 10 defective bolts.

(vii) Determine the optimal sequence of jobs that minimizes the total elapsed time for the data given below (processing time on machines is given in hours and passing is not allowed). Also find minimum total elapsed time T & the idle time for three machines.

Job	**A**	**B**	**C**	**D**	**E**	**F**	**G**
Machine M_1	3	8	7	4	9	8	7
Machine M_2	4	3	2	5	1	4	3
Machine M_3	6	7	5	11	5	6	12

(viii) The number of complaints received per day is passion distribution with parameter 4. Find prob. That the manager will receive at most 2 complains on any day ($e^{-4} = 0.0183$)

Q.2 (a) Attempt any two of the questions: (6)

(i) The equation of the two lines of regression are $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$ (i) Estimate Y for X = 2.

(ii) The p.d.f. of continuous random variable X is given by

$$f(x) = \frac{x}{8} \qquad 0 < x < 4$$
$$= 0 \qquad \text{otherwise}$$

Find (i) $P(X \le 2)$ (ii) $P(2 < X \le 3)$

(iii) Find common region for the following system of linear inequations. $8x+5y \le 60$, $4x + 5y \le 40$, $x \ge 0$, $y \ge 0$

(b) Attempt any two of the following : (8)

(i) A pharmaceutical company has four branches, one each at city A, B, C, D. A branch manager is to be appointed one at each city, out of four candidates P, Q, R and S. The monthly business depends upon the city and the effectiveness of the branch manager in hat city is given below:

Branch Manager	**City**			
	A	**B**	**C**	**D**
	Monthly business (₹ lakh)			
P	10	10	8	8
Q	12	15	10	9
R	11	16	12	7
S	15	13	15	11

Which manager should be appointed at which city so as to get the maximum total monthly business?

(ii) 5000 copies of a book of price ₹ 80 each were insured for $\left(\frac{3}{5}\right)^{th}$ of their value. During the transit some copies were damaged, and reduced to 60% of the value. If the amount recovered against the damage was ₹ 12,000 find the number of copies that were damaged.

(iii) The difference between true discount and banker's discount on a bill due 6 months hence at 4% is ₹ 80. Calculate true discount, banker's discount and amount of bill.

Q.3 (a) Attempt any two questions: **(6)**

(i) An alloy of zinc and tin contains 37% zinc by weight. Find the weight of zinc which must be added to 400 kg of this alloy if the final percentage of zinc is to be 70.

(ii)

Age group	Population	No. of Deaths
Below 10	900	12
10 – 20	1500	x
20 – 60	4500	38
60 and above	600	30

From the above information, find x, if the CDR = 12.0 per thousand.

(iii) There are give jobs, each of which is to be processed through three machines, A, B and C in the order ABC. Processing times in hours are as shown in the following table.

Job	1	2	3	4	5
A	3	8	7	5	4
B	4	5	1	2	3
C	7	9	5	6	10

Determine the optimum sequence for the five jobs and the minimum elapsed time. Also, find the idle time for three machines.

(b) Attempt any two questions: **(8)**

(i) Mrs. Menon plans to save for her daughter's marriage. She wants to accumulate a sum of ₹4,00,000 at the end of 4 years. How much should she invest at the end of each year from now, if she can get interest compounded at 10% p.a.? (Given $(1.1)^4 = 1.4641$)

(ii) Find the number of pairs of observations from the following data.
$r = 0.15, \sum(x-\bar{x})(y-\bar{y}) = 12, \sigma_y = 4$ and $\sum(x-\bar{x})^2 = 40$

(iii) The p.d.f. of a r.v. X is given by

$$f_x(x) = \begin{cases} \frac{k}{\sqrt{x}} & 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

Determine k. Determine c.d.f. of X and hence find $P(X \le 2)$, $P(2 < x < 3)$, $P(X \ge 1)$ and find E(x) and V(x)

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CLASS: S.Y.J.C
PRELIM SERIES NO. 2
Date: 25th Jan 2020

MATHS I & II
(2019 -20) (Solution)

Time : 3 Hrs.
Marks: 80
Day:- Saturday

Paper I

Q.1 Attempt any Six questions: **(12)**

(i) $\int \frac{1}{\sin^2 x.\cos^2 x}dx$

Ans. Let $I = \int \frac{1}{\sin^2 x.\cos^2 x}dx$

$\therefore I = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x.\cos^2 x}dx$

$\therefore I = \left(\int \frac{\sin^2 x}{\sin^2 x.\cos^2 x} + \frac{\cos^2 x}{\sin^2 x.\cos^2 x}\right)dx$

$I = \int\left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right)dx$

$\therefore I = \int(\sec^2 x dx + \int \operatorname{cosec}^2 x)dx$

$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$

$= \tan x - \cot x + c$

$= \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} + c$

$= \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} + c$

$= \frac{-(\cos^2 x - \sin^2 x)}{\sin x.\cos x} + c$

$= -\frac{\cos 2x}{\sin x \cos x} + c$

$= -\frac{2\cos 2x}{2\sin x.\cos x} + c$

$= -\frac{2\cos 2x}{\sin 2x} + c$

$\therefore I = -2\cot 2x + c$

(ii) Find derivative of $\tan^{-1}\left[\frac{16x}{1-64x^2}\right]$

Ans.

Let $y = \tan^{-1}\left(\frac{16x}{1-64x^2}\right)$

$\therefore y = \tan^{-1}\left(\frac{8x+8x}{1-(8x)(8x)}\right)$

$\therefore y = \tan^{-1}(8x) + \tan^{-1}(8x)$

as $\tan^{-1}\frac{a+b}{1-a.b} = \tan^{-1}a + \tan^{-1}b$

diff. w.r.t. x

$\frac{dy}{dx} = \frac{1}{1+(8x)^2}.\frac{d8x}{dx} + \frac{1}{1+(8x)^2}.\frac{d8x}{dx}$

$\therefore \frac{dy}{dx} = \frac{1}{1+64x^2}.8 + \frac{1}{1+64x^2}.8$

$\therefore \frac{dy}{dx} = \frac{8}{1+64x^2} + \frac{8}{1+64x^2} = \frac{16}{1+64x^2}$

(iii) If $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ find $(A + I).(A - I)$

Ans. $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore A + I = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore A + I = \begin{bmatrix} 1+1 & -2+0 \\ 0+0 & 1+1 \end{bmatrix}$

$\therefore A + I = \begin{bmatrix} 2 & -2 \\ 0 & 2 \end{bmatrix}$

Similarly $A - I = \begin{bmatrix} 0 & -2 \\ 0 & 0 \end{bmatrix}$

$\therefore (A + I).(A - I) = \begin{bmatrix} 2 & -2 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & -4+0 \\ 0+0 & 0+0 \end{bmatrix}$

$\therefore (A + I).(A - I) = \begin{bmatrix} 0 & -4 \\ 0 & 0 \end{bmatrix}$

(iv) Write the negation of the following statement.
2 + 3 = 0 if and only if 2i is a real number.

Ans. 2 + 3 = 0 if and only if 2i is a real number,
Let P : 2 + 3 = 0 and q : 2i is a real number
$\therefore$ Given statement becomes $\underline{2}$ $p \leftrightarrow q$
Formula : $\sim (p \rightarrow q) \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$
$\therefore$ Negation of given statement is 2 + 3 = 0 and 2i is not a real number or 2 + 3 ≠ 0 and 2i is a real number.

(v) Find k it the following function is continuous at x = 2
f(x) = 2 + 3k for x ≤ 2
= 1 − 7x for x > 2

Ans. f(x) = 2 + 3k for x ≤ 2
= 1 − 7x for x > 2
If f(x) is continuous at x = 2

then $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x) = f(2)$ .....(1)

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2} (2 + 3k) = 2 + 3k$ ..... (2)

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2} (1 - 7x) = 1 - 7(2) = -13$ .....(3)

$\therefore$ from (1), (2) and (3)

$\therefore \lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x)$

$\therefore 2 + 3k = -13$

$\therefore 3k = -15$

$\therefore k = -5$

(vi) Evaluate $\int_1^2 \left(x - \frac{1}{x}\right)^2 dx$

Ans. Let $I = \int_1^2 \left(x - \frac{1}{x}\right)^2 dx$

$$= \int_1^2 \left(x^2 - 2 + \frac{1}{x^2}\right) dx$$

$$= \int_1^2 x^2 dx - \int_1^2 2dx + \int_1^2 \frac{1}{x^2} dx$$

$$= \left[\frac{x^3}{3}\right]_1^2 - [2x]_1^2 + \int_1^2 x^{-2} dx$$

$$= \left[\frac{(2)^3}{3} - \frac{(1)^3}{3}\right] - [2(2) - 2(1)] + \left[\frac{x^{-1}}{-1}\right]_1^2$$

$$= \left(\frac{8}{3} - \frac{1}{3}\right) - (4 - 2) + \left[\frac{-1}{x}\right]_1^2$$

$$= \frac{7}{3} - 2 + \left[\left(\frac{-1}{2}\right) - \left(\frac{-1}{1}\right)\right]$$

$$= \frac{7-6}{3} + \left(-\frac{1}{2} + 1\right)$$

$$= \frac{1}{3} + \frac{1}{2}$$

$$= \frac{2+3}{6}$$

$$\therefore I = \frac{5}{6}$$

(vii) The demand function is $D = \frac{4}{p}$. Find elasticity of demand w.r.t. price when price is 2 units.

Ans. The demand function is $D = \frac{4}{p}$

$$D = \frac{4}{p} \quad .....(i)$$

$$\therefore \frac{dD}{dp} = \frac{d}{dp}\left(\frac{4}{p}\right)$$

$$\therefore \frac{dD}{dp} = \frac{4d}{dp}\left(\frac{1}{p}\right)$$

$$\therefore \frac{dD}{dp} = 4\left(\frac{-1}{p^2}\right)$$

$$\therefore \frac{dD}{dp} - \frac{-4}{p^2} \quad .....(ii)$$

Elasticity of demand w.r.t. price is given by

$$\frac{ED}{Ep} = \frac{-p}{D} \cdot \frac{dD}{dp}$$

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$$\therefore \frac{ED}{Ep} = \frac{-p}{\left(\frac{4}{p}\right)} \cdot \frac{-4}{p^2}$$ from (i) and (ii)

$$\therefore \eta = \frac{-p \times p}{4} \cdot \frac{-4}{p^2}$$

$$\therefore \eta = \frac{+p^2}{p^2}$$

$$\therefore \eta = 1$$

$$\therefore (\eta)_{\text{when } p=2} = 1$$

(viii) Find $\int \sin^2 4x dx$

Ans. Let $I = \int \sin^2 4x dx$

Now $1 - \cos 2x = 2\sin^2 x$

$$\therefore \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\therefore \sin^2 4x = \frac{1 - \cos 8x}{2}$$

$$\therefore I = \int \frac{1 - \cos 8x}{2} dx$$

$$\therefore I = \frac{1}{2} \int (1 - \cos 8x) dx$$

$$\therefore I = \frac{1}{2} \left(\int 1 dx - \int \cos 8x dx \right)$$

$$\therefore I = \frac{1}{2} \left(x - \frac{\sin 8x}{8} \right) + c$$

$$\therefore I = \frac{x}{2} - \frac{\sin 8x}{16} + c$$

Q.2 (a) Attempt any two of the questions: Teach. **(6)**

(i) Differentiate $\log(1+x^2)$ w.r.x. $\tan^{-1} 4x$.

Ans.

(i) Let $u = \log(1 + x^2)$ and $v = \tan^{-1} 4x$

$\therefore$ we have to find du/dv

$u = \log(1 + x^2)$

diff wrt x

$$\frac{dy}{dx} = \frac{1}{1 + x^2} \frac{d}{dx} (1 + x^2)$$

$$= \frac{1}{1 + x^2} (0+2x)$$

$$= \frac{2x}{1 + x^2}$$

$v = \tan^{-1} 4x$

$\therefore \dfrac{dv}{dx} = \dfrac{d}{dx}\tan^{-1} 4x$

$\therefore \dfrac{dv}{dx} = \dfrac{1}{1+(4x)^2} \cdot \dfrac{d}{dx} 4x$

$\therefore \dfrac{dv}{dx} = \dfrac{1}{1+16x^2} \cdot 4$

$\therefore \dfrac{dv}{dx} = \dfrac{4}{1+16x^2}$ .....(ii)

Now $\dfrac{du}{dv} = \dfrac{\left(\dfrac{du}{dx}\right)}{\left(\dfrac{dv}{dx}\right)}$

$= \dfrac{\dfrac{2x}{1+x^2}}{\dfrac{4}{1+16x^2}} = \dfrac{2x}{1+x^2} \times \dfrac{1+16x^2}{4}$

$= \dfrac{x(1+16x^2)}{2(1+x^2)}$

(ii) $\int e^x . \dfrac{(1-x)^2}{(x^2+1)^2} dx$

Ans. Let $I = \int e^x . \dfrac{(1-x)^2}{(x^2+1)^2} dx$

$\therefore I = \int e^x . \dfrac{(1-2x+x^2)}{(x^2+1)^2} dx$

$\therefore I = \int e^x \left[\dfrac{1+x^2-2x}{(x^2+1)^2}\right] dx$

$\therefore I = \int e^x \left[\dfrac{1+x^2}{(x^2+1)^2} - \dfrac{2x}{(x^2+1)^2}\right] dx$

$\therefore I = \int e^x \left(\dfrac{1}{x^2+1} - \dfrac{2x}{(x^2+1)^2}\right) dx$

Now, $\dfrac{d}{dx}\left(\dfrac{1}{x^2+1}\right) = \dfrac{-1}{(x^2+1)^2} \cdot \dfrac{d(x^2+1)}{dx}$

$= \dfrac{-1}{(x^2+1)^2} . (2x+0) = \dfrac{-2x}{(x^2+1)^2}$

Also $\int e^x [f(x) + f'(x)] dx = e^x . f(x) + c$

$\therefore I = e^x . \dfrac{1}{x^2+1} + c$

$\therefore I = \dfrac{e^x}{(x^2+1)} + c$

(iii) Solve the equations $2x + 3y = 9$ and $y - x = -2$ using the method of reduction.

Ans. $2x + 3y = 9$ and $y - x = -2$ i.e. $-x + y = -2$

Let $\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 9 \\ -2 \end{bmatrix}$

$\therefore A.X = B$

$\therefore \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}, X = \begin{bmatrix} 9 \\ -2 \end{bmatrix}$

$R_2 \to 2R_2 + R_1$

$\therefore \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}, X \begin{bmatrix} 9 \\ 5 \end{bmatrix}$

$\therefore \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$

$\therefore \begin{bmatrix} 2x + 3y \\ 5y \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$

$\therefore 2x + 3y = 9$ .....(i)

and $5y = 5$

$\therefore y = 1$ sub-in equation .....(i) $x = 3$

(b) **Attempt any two of the following :** 4 each **(8)**

(i) $\int_0^1 \frac{x+4}{x^2+5} dx$

Ans. Let $I = \int_0^1 \frac{x+4}{x^2+5} dx$

$\therefore I = \int_0^1 \left(\frac{x}{x^2+5} + \frac{4}{x^2+5} \right) dx$

$\therefore I = \int_0^1 \frac{x}{x^2+5} dx + \int_0^1 \frac{4}{x^2+5} dx$

$I = \frac{1}{2} \int_0^1 \frac{2x}{x^2+5} dx + 4 \int_0^1 \frac{1}{(x)^2 + (\sqrt{5})^2} dx$

$\therefore I = \frac{1}{2} \int_0^1 \frac{\frac{d}{dx}(x^2+5)}{x^2+5} dx + 4. \left[\frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x}{\sqrt{5}} \right) \right]_0^1$

$\therefore \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c$

$\therefore I = \frac{1}{2} \left[\log|x^2+5| \right]_0^1 + 4 \left[\frac{1}{\sqrt{5}} . \tan^{-1}\left(\frac{1}{\sqrt{5}} \right) - \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{0}{5} \right) \right]$

$\therefore I = \frac{1}{2} \left[\log|(1)^2+5| - \log|(0)^2+5| \right] + 4 \left[\frac{1}{\sqrt{5}} . \tan^{-1}\left(\frac{1}{\sqrt{5}} \right) - \frac{1}{\sqrt{5}} \tan^{-1}(0) \right]$

$\therefore I = \frac{1}{2} [\log 6 - \log 5] + 4 \left[\frac{1}{\sqrt{5}} . \tan^{-1}\left(\frac{1}{\sqrt{5}} \right) - \frac{1}{\sqrt{5}} (0) \right]$

$\therefore I = \frac{1}{2} \log\left(\frac{6}{5} \right) + \frac{4}{\sqrt{5}} . \tan^{-1}\left(\frac{1}{\sqrt{5}} \right)$

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(ii) The function given below is continuous on domain (0,5) then find the value of $\frac{a+b}{2}$

$f(x) = x^2 + ax + b \qquad x < 2$
$\quad = 3x + 2 \qquad 2 \le x \le 4$
$\quad = 2ax + 5b \qquad x > 4$

Ans. $f(x) = x^2 + ax + b \qquad x < 2$
$\quad = 3x + 2 \qquad 2 \le x \le 4$
$\quad = 2ax + 5b \qquad x > 4$

f(x) is continuous on (0,5)

$\therefore$ It is continuous at x = 2 and at x = 4.

Part (i) f(x) is continuous at x = 2

$\therefore \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$ .....(i)

Now, $\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (x^2 + ax + b)$

$= (2)^2 + a(2) + b$

$= 4 + 2a + b$ .....(ii)

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2} (3x + 2) = 3(2) + 2 = 8$ .....(iii)

$\therefore$ from (i), (ii) and (iii)

$4 + 2a + b = 8 \qquad 2a + b = 4$ .....(A)

Part (ii) f(x) is cont. at x = 4

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = f(4)$ .....(iv)

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4} (3x + 2) = 3(4) + 2 = 14$ .....(v)

$\lim_{x \to 4^+} f(x) = \lim_{x \to 4} (2ax + 5b) = 2a(4) + 5b = 8a + 5b$ ....(vi)

From (iv), (v) and (vi)

$8a + 5b = 14$ .....(B)

Solving eqn (A) and (B)

$2a + b = 4$ multiplying by 5

$8a + 5b = 14$

$10a + 5b = 20$

$8a + 5b = 14$

$- \quad - \quad -$

$2a = 6$

$\boxed{\therefore a = 3 \quad \therefore b = -2}$ $\quad 2(3) + b = 4$

$\frac{a+b}{2} = \frac{3-2}{2} = \frac{1}{2}$

(iii) If $x^2.y^k = (x + y)^{2+k}$ then find $\frac{dy}{dx}$

$x^2.y^k = (x + y)^{2+k}$

Ans. $\therefore \log(x^2.y^k) = \log(x + y)^{2+k}$

$\therefore \log x^2 + \log y^k = (2 + k)\log(x + y)$

$\therefore 2\log x + k\log y = (2 + k).\log(x + y)$

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$$\therefore \frac{d}{dx}2\log x + \frac{d}{dx}k\log y = \frac{d}{dx}(2+k).\log(x+y)$$

$$\therefore 2.\frac{1}{x} + k.\frac{1}{y}\frac{dy}{dx} = (2+k).\frac{1}{x+y}.\frac{d}{dx}(x+y)$$

$$\therefore \frac{2}{x} + \frac{k}{y}\frac{dy}{dx} = \frac{2+k}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\therefore \frac{2}{x} + \frac{k}{y}\frac{dy}{dx} = \frac{2+k}{x+y} + \frac{2+k}{x+y}.\frac{dy}{dx}$$

$$\therefore \frac{k}{x}\frac{dy}{dx} - \frac{2+k}{x+y}\frac{dy}{dx} = \frac{2+k}{x+y} - \frac{2}{x}$$

$$\therefore \frac{dy}{dx}\left(\frac{k}{y} - \frac{2+k}{x+y}\right) = \frac{2+k}{x+y} - \frac{2}{x}$$

$$\therefore \frac{dy}{dx}\left[\frac{kx+ky-2y-ky}{y(x+y)}\right] = \frac{2x+kx-2x-2y}{(x+y).x}$$

$$\therefore \frac{dy}{dx}\left[\frac{kx-2y}{y(x+y)}\right] = \frac{kx-2y}{(x+y).x}$$

$$\therefore \frac{dy}{dx} = \frac{kx-2y}{(x+y).x} \times \frac{y.(x+y)}{kx-2y}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}$$

Q.3 (a) Attempt any two questions: 3 each **(6)**

(i) $\int_0^{\pi} \frac{1}{1+\sin x}dx$

Ans. Let $I = \int_0^{\pi} \frac{1}{1+\sin x}dx$

$$\therefore I = \int_0^{\pi}\left(\frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}\right)dx$$

$$\therefore I = \int_0^{\pi}\left(\frac{1-\sin x}{1+\sin^2 x}\right)dx$$

$$\therefore I = \int_0^{\pi}\left(\frac{1-\sin x}{\cos^2 x}\right)dx$$

$$\therefore I = \int_0^{\pi}\left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\right)dx$$

$$\therefore I = \int_0^{\pi}\left(\sec^2 x - \sec x.\tan x\right)dx$$

$$\therefore I = [\tan x - \sec x]_0^{\pi}$$

$$\therefore I = [\tan\pi - \sec\pi] - [\tan 0 - \sec 0]$$

$$\therefore I = (0 - (-1) - (0 - 1)$$

$$\therefore I = 1 + 1$$

$$\therefore I = 2$$

S.Y.B.A./S.Y.J.C

(ii) Find $\frac{du}{dv}$ if $u = \sin^3\theta$, $v = \cos^3\theta$

Ans.

$u = \sin^3\theta$

$u = (\sin\theta)^3$

diff wrt 4

$\frac{du}{d\theta} = 3(\sin\theta)^2 \frac{d}{d\theta} \sin\theta$

$= 3\sin^2\theta \cos\theta$ ..... (1)

$V = \cos^3\theta = (\cos\theta)^3$

Diff wrt v

$\frac{dv}{d\theta} = 3\cos^2\theta \frac{d}{d\theta} \cos\theta$

$= 3\cos^2\theta(-\sin\theta)$

$= -3\cos^2\theta \sin\theta$ .....(2)

Using parametric rule

$\frac{du}{dv} = \frac{\frac{du}{d\theta}}{\frac{dv}{d\theta}} = \frac{3\sin^2\theta \cos\theta}{-3\cos^2\theta\sin\theta}$

$= -\tan\theta$

(iii) Verify that $\sim(p \to q) \equiv p \wedge \sim q$

Ans. To verity that $\sim(p \to q) \equiv p \wedge \sim q$

Truth table

(1)	(2)	(3)	(4)	(5)	(6)
p	q	$p \to q$	$\sim(p \to q)$	$\sim q$	$p \wedge \sim q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

Since coloumn (4) and coloumn (6) are identical $\sim(p \to q) \equiv p \wedge \sim q$

(b) **Attempt any two questions:** 4 each **(8)**

(i) Evaluate $\int_0^2 \frac{dx}{x + \sqrt{4 - x^2}}$

Ans. Let $I = \int_0^2 \frac{1}{x + \sqrt{4 - x^2}} dx$

Put $x = 2\sin\theta$

Diff w.r.t. x

$\therefore 1 = 2\cos\theta \frac{d\theta}{dx}$

$\therefore dx = 2\cos\theta . d\theta$

When $x = 0.2\sin\theta = 0$ $\therefore \sin\theta = 0$ $\therefore \theta = 0$

When $x = 2$, $2\sin\theta = 2$ $\therefore \sin\theta = 1$, $\therefore \theta = \frac{\pi}{2}$

$$\therefore \quad I = \int_0^{\frac{\pi}{2}} \frac{1}{2\sin\theta + \sqrt{4 - (2\sin\theta)^2}} 2\cos\theta d\theta$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{2\cos\theta}{2\sin\theta + \sqrt{4 - 4\sin^2\theta}} d\theta$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{2\cos\theta}{2\sin\theta + \sqrt{4(1 - \sin^2\theta)}} d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{2\cos\theta}{2\sin\theta + \sqrt{4\cos^2\theta}} d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{2\cos\theta}{2\sin\theta + 2\cos\theta} d\theta$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{2\cos\theta}{2(\sin\theta + \cos\theta)} d\theta$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta + \cos\theta} d\theta \quad \text{.....(i)}$$

Now $\int_0^a f(x)dx = \int_0^a f(a - x)dx)$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)} d\theta$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\cos\theta + \sin\theta} d\theta \quad \text{.....(ii)}$$

Adding (i) & (ii)

$$\therefore 2I = \int_0^{\frac{\pi}{2}} \left(\frac{\cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta}{\sin\theta + \cos\theta} \right) d\theta$$

$$\therefore 2I = \int_0^{\frac{\pi}{2}} \left(\frac{\sin\theta + \cos\theta}{\sin\theta + \cos\theta} \right) d\theta \quad \therefore 2I = \int_0^{\frac{\pi}{2}} 1\, d\theta \quad \therefore 2I = [0]_0^{\frac{\pi}{2}}$$

$$\therefore 2I = \frac{\pi}{2} - 0 \quad \therefore I = \frac{\pi}{4}$$

(ii) Find $\int \frac{x^2 + 1}{(x^2 + 4)(x^2 + 25)} dx$

Ans. Let $I = \int \frac{x^2 + 1}{(x^2 + 4)(x^2 + 25)} dx$ .....(i)

Put $x^2 = t$ for partial fraction method

$$\therefore \frac{x^2 + 1}{(x^2 + 4).(x^2 + 25)} = \frac{t + 1}{(t + 4)(t + 25)} \quad \text{....(ii)}$$

S.Y.J.C.

Let $\frac{t+1}{(t+4).(t+5)} = \frac{A}{t+4} + \frac{B}{t+5}$ .....(iii)

$\therefore \frac{t+1}{(t+4)(t+5)} = \frac{A(t+5)+B(t+4)}{(t+4)(t+5)}$

$\therefore t + 1 = A(t+5) + B(t+4)$ .....(iv)

Sub. t = -4 in (iv)

$\therefore -4 + 1 = A(-4+5) + B(-4+4)$

$\therefore -3 = A$

Sub. t = -5 in (iv)

$\therefore -5 + 1 = A(-5+5) + B(-5+4)$

$\therefore -4 = -A(-20) + B(-21)$

$\therefore -4 = -B \quad \therefore B = 4$

$A = \frac{-1}{7}, B = \frac{8}{7}$ correct values

From (i), (ii) and (iii) and resubstituting

$\therefore I = \int\left(\frac{-2}{x^2+4} + \frac{4}{x^2+25}\right)dx$

$\therefore I = \int\frac{-3}{x^2+4}dx + \int\frac{4}{x^2+25}dx$

$\therefore I = -3\int\frac{1}{(x)^2+(2)^2}dx + 4\int\frac{1}{(x)^2+(5)^2}dx$

$\therefore I = -3.\frac{1}{2}.\tan^{-1}\left(\frac{x}{2}\right) + 4.\frac{1}{5}.\tan^{-1}\left(\frac{x}{5}\right) + c$

$\therefore I = .\frac{-3}{2}.\tan^{-1}\left(\frac{x}{2}\right) + \frac{4}{5}.\tan^{-1}\left(\frac{x}{5}\right) + c$

(iii) Divide 60 in two parts such that the product of square of one part and the other is maximum.

Ans. Let the two parts of 60 be x and 60 – x

$\therefore$ The product of square of one part and the other = $x^2.(60 - x)$

Let $p = x^2 . (60 - x)$

$\therefore p = 60x^2 - x^3$

$\therefore \frac{dp}{dx} = 120x - 3x^2$

For maximum product $\frac{dp}{dx} = 0$ and $\frac{d^2p}{dx^2} < 0$

Let $\frac{dp}{dx} = 0$

$\therefore 120x - 3x^2 = 0 \qquad \therefore 3x(40 - x) = 0$

$3x = 0$ or $40 - x = 0 \qquad \therefore x = 0$ or $x = 40$

Also $\frac{d^2p}{dx^2} = \frac{d}{dx}\left(\frac{dp}{dx}\right)$

$\therefore \frac{d^2p}{dx^2} = \frac{d}{dx}(120x - 3x^2) \qquad \therefore \frac{d^2p}{dx^2} = 120 - 6x$

Sub. x = 40

$\therefore \frac{d^2p}{dx^2} = 120 - 6(40) \qquad \therefore \frac{d^2p}{dx^2} = 120 - 240 = -120 < 0$

$\therefore$ When x = 40 product is maximum Also when x = 40 other part is 60 – 40 = 20

$\therefore$ The two parts of 60 are 40 and 20.

Paper II

Q.1 Attempt any Six questions: **(12)**

(i) An article is marked at ₹ 1,500. A trader allows a discount at 3% and still gains 20% on the cost. Find the cost price of the article.

Ans. List price of the article = ₹ 1500

Discount at 3% on ₹ 1500 $= 1500 \times \frac{3}{100} =$ ₹ 45

∴ Selling price = 1500 − 45 = ₹ 1455

If the cost price is ₹100, then the selling price is ₹ 120

∴ For selling price = 1455

The cost price $\frac{100 \times 1455}{120} =$ ₹ 1212.50

∴ Cost price of the article is = 1212.50.

(ii) What must be subtracted from each of the numbers 5, 7 and 10, so that the resulting numbers are in continued proportion?

Ans. Let x be subtracted from each of the numbers 5, 7 and 10 so that resulting numbers are in continued proportion.

∴ by definition $\frac{5-x}{7-x} = \frac{7-x}{10-x}$

$\therefore (5-x)(10-x) = (7-x)^2$

$\therefore 50 - 5x - 10x + x^2 = 49 - 14x + x^2$

$\therefore 50 - 15x = 49 - 14x$

$\therefore -15x + 14x = 49 - 50$

$\therefore -x = -1$

$\therefore x = 1$

(iii) For a bivariate date, the regression coefficient of Y on X is 0.4 and the regression coefficient of X on Y is 0.9. Find the value of variance of Y if variance of X is 9.

Ans. Regression coeff. of Y on X I,e, bYX = 0.4

Regression coeffi. of X on Y i.e. $b_{XY} = 0.9$

$r = \pm\sqrt{b_{xy}.b_{yx}}$

$\therefore r = \sqrt{0.4 \times 0.9}$

$\therefore r = \sqrt{0.36}$

$\therefore r = 0.6$

Variance of x = 9 $\sigma_x = 3$

$b_{YX} = r.\frac{\sigma Y}{\sigma X}$

$\therefore 0.4 = 0.6 \times \frac{\sigma Y}{3}$

$\therefore \frac{0.4 \times 3}{0.6} = \sigma Y$

$\therefore \sigma Y = \frac{1.2}{0.6} = 2$

V(y) = 4

S.T.A./S.Y.J.C.

(iv) Coefficient of rank correlation between X and Y is 0.5 and $\sum d_1^2 = 42$. Assuming that no ranks are repeated, find the number of pairs of observations.

Ans. Given R = 0.5, $\sum d_1^2 = 42$, n = ?

$$R = \frac{1 - 6\sum d^2}{n(n^2 - 1)} \quad \text{as no Rank is repeated}$$

$$\therefore 0.5 = 1 - \frac{6(42)}{n(n^2 - 1)}$$

$$\therefore 0.5 - 1 = \frac{-252}{n(n^2 - 1)}$$

$$\therefore -0.5 = \frac{-252}{n(n^2 - 1)}$$

$$\therefore n(n^2 - 1) = \frac{-252}{-0.5}$$

$$\therefore n(n-1)(n+1) = \frac{252}{0.5}$$

$\therefore$ (n -1) n.(n + 1) = 504

$\therefore$ (n – 1). n. (n + 1) = 504

$\therefore$ (n – 1). n. (n + 1) = 7 x 8 x 9

$\therefore$ n = 8

(v) In the following table, c.d.f. of a.r.v. X is given

x_i	-2	-1	0	1	2
$F(x_i)$	0.2	0.5	0.65	0.9	1

(i) Find p.m.f. of X. (ii) Compute P(0 < X <2), P(X > 1), P(X ≤ 2 / X > 0)

Ans. c.d.f. of a r.v.of x is

x_i	$F(x_i)$
-2	0.2
-1	0.5
0	0.65
1	0.9
2	1

$\therefore$ p.d.f. of X is

x_i	$F(x_i)$
-2	0.2
-1	0.5 – 0.2 = 0.3
0	0.65 – 0.5 = 0.15
1	0.9 – 0.65 = 0.25
2	1 – 0.9 = 0.1

(i) P(0 < x < 2)

= p(x = 1)

= 0.25

(ii) P(X > 1)

= P(X = 2)

= 0.1

S.T.M./S.Y.J.C.

(vi) The probability of defective bolt in an workshop is 40%. Find mean and variance of the defective bolts out of 10 defective bolts.

Ans. The probability of defective both = 40%

$\therefore P = \frac{40}{100} = \frac{2}{5}$

$\therefore q = 1 - p \quad \therefore q = 1 - \frac{1}{5} = \frac{3}{5}$

No. of defective both = 10 $\quad \therefore n = 10$

Let r.v.x be no. of defective bulbs

$\therefore$ X follows a binomial distribution

$\therefore$ Mean – $n.p = 10 \times \frac{2}{5} = 4$

Variance = npq $4 \times \frac{3}{5} = \frac{12}{5} = 2.4$

(vii) Determine the optimal sequence of jobs that minimizes the total elapsed time for the data given below (processing time on machines is given in hours and passing is not allowed). Also find minimum total elapsed time T & the idle time for three machines.

Job	A	B	C	D	E	F	G
Machine M_1	3	8	7	4	9	8	7
Machine M_2	4	3	2	5	1	4	3
Machine M_3	6	7	5	11	5	6	12

Ans. A D G B F C E

Let x denote no. of complains received by bant on a given day

X is RV which follows passion distribution with parameter m = 4

x takes the values 0,1,2,3.....

PMF of x is given by

$P(x) = \frac{m^x c^{-m}}{x!}$ ..... (1)

Prob that manger receiver at most 2

Complaints means 1 = 0,1,2

$P(0,1,2) = P(0) + P(1) + P(2)$

$= \frac{4^0 e^{-4}}{0!} + \frac{4^1 e^{-4}}{1!} + \frac{4^2 e^{-4}}{2!}$

$= e^{-4}\left(\frac{1}{1} + \frac{4}{1} + \frac{16}{2}\right)$

$= e^{-4} \times 13$

$= 0.0183 \times 13$

$= 0.2381$

0.2379

(viii) A building worth ₹ 5,00,000 is insured for $\left(\frac{4}{5}\right)^{th}$ of its value at a premium of 5%. What is the amount of premium? Also find commission of the agent if the rate of commission is 3%.

Ans. Property value = ₹ 5,00,000

Policy value $= \frac{4}{5} \times 5{,}00{,}000$ = ₹ 4,00,000

Rate of Premium = 5%

$\therefore$ Amount of premium $= 4{,}00{,}000 \times \frac{5}{100}$

$\therefore$ Premium amount is ₹ 20,000

$\therefore$ Commission at 3% is $20{,}{,}000 \times \frac{3}{100}$ = ₹ 600.

Q 2 **(a)** **Attempt any two of the questions:** 3 each. **(6)**

(i) The equation of the two lines of regression are $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$ (i) Estimate Y for X = 2.

Ans. The two regression lines equations are $3x + 2y - 26 = 0$ (i) and $6x + y - 31 = 0$.(ii)

To estimate y when x = 2 we have to Identify the regression equation of y on x line. we assume that the equation (i) is of X on Y line. And equation (ii) is of Y on X line.

Consider equation $3x + 2y - 26 = 0$

$\therefore 3x = -2y + 26$

Dividing by 3

$\therefore x = \frac{-2y}{3} + \frac{26}{3}$

Comparing with the standard form $x - my + c$

$m = \frac{-2}{3}$

$\therefore b_{xy} = \frac{-2}{3}$.(iii)

Consider equation $6x + y - 31 = 0$

$\therefore y = -6x + 31$

Comparing with the standard form $y = mx + c$

$m = -6 \quad \therefore b_{xy} = -6$

$\therefore r = \pm\sqrt{b_{xy}.b_{yx}}$

$\therefore r = -\sqrt{\left(-\frac{2}{3}\right).(-6)^2}$

$\therefore r = -\sqrt{4}$

$\therefore r = -2$

which is not true as $-1 \le r \le 1$ our assumption is wrong

1st line is y on x

put x = 2 in (1)

$3x + 2(2) - 26 = 0$

$3x = 22$

$x = \frac{22}{3} = 7.3$

$3(2) + 2y - 26 = 0$

$2y = 20$

$y = 10$

(ii) The p.d.f. of continuous random variable X is given by

$f(x) = \frac{x}{8} \qquad 0 < x < 4$

$= 0 \qquad$ otherwise

Find (i) $P(X \le 2)$ (ii) $P(2 < X \le 3)$

Ans. (i) $P(X \le 2) = \int_0^2 f(x)dx$

$$= \int_0^2 \frac{x}{8} dx$$

$$= \left[\frac{x^2}{16}\right]_0^2$$

$$= \frac{4}{16} - 0 = \frac{1}{4}$$

(ii) $P[2 < X \le 3] = \int_2^3 f(x)dx$

$$= \int_2^3 \frac{x}{8} dx$$

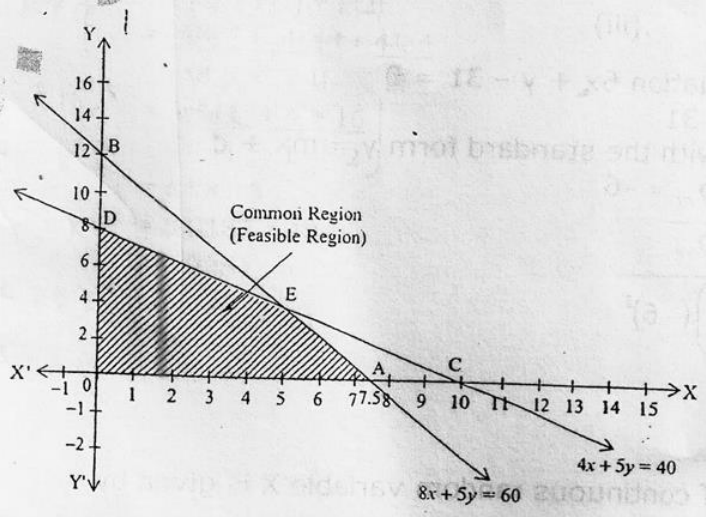
$$= \left[\frac{x^2}{16}\right]_2^3$$

$$= \frac{9}{16} - \frac{4}{16} = \frac{5}{16}$$

(iii) Find common region for the following system of linear inequations. $8x + 5y \le 60$, $4x + 5y \le 40$, $x \ge 0$, $y \ge 0$

Ans.

Inequation	Equation	Double intercept form	Points (x,y)	Region
$8x+5y \le 60$	$8x+5y=60$	$\frac{x}{7.5} + \frac{y}{12} = 1$	A(7.5,0), B(0,12)	8(0)+5(0)=0<60 ∴ Origin side
$4x+5y \le 40$	$4x+5=40$	$\frac{x}{10} + \frac{y}{8} = 1$	C(10,0) D(0,8)	4(0)+5(0)=0<40 ∴ origin side
$x \ge 0$	$x = 0$	-	-	RHS of y-axis
$y \ge 0$	$y = 0$	-	-	Above x-axis



The shaded portion OAED represents the common region.

(16)

(b) **Attempt any two of the following :** **(8)**

(i) A pharmaceutical company has four branches, one each at city A, B, C, D. A branch manager is to be appointed one at each city, out of four candidates P, Q, R and S. The monthly business depends upon the city and the effectiveness of the branch manager in that city is given below:

Branch Manager	City			
	A	B	C	D
	Monthly business (₹ lakh)			
P	10	10	8	8
Q	12	15	10	9
R	11	16	12	7
S	15	13	15	11

Which manager should be appointed at which city so as to get the maximum total monthly business?

Ans. Since it is a maximization problem subtract each of the elements in the table from the largest element (16 here). This gives rise to data as shown in Table

Branch Manager	City			
	A	B	C	D
P	6	6	8	8
Q	4	1	6	7
R	5	0	4	9
S	1	3	1	5

Since the data in the table represents opportunity losses, minimization assignment technique can be applied. Step – by – step computation is shown in table

"Minimum element of each row is subtracted from every element in that row"

Branch Manager	City			
	A	B	C	D
P	0	0	2	2
Q	3	0	5	6
R	5	0	4	9
S	0	2	0	4

"Minimum element in each column is subtracted from every element in that column"

Branch Manager	City			
	A	B	C	D
P	0	0	2	0
Q	3	0	5	4
R	5	0	4	7
S	0	2	0	2

"Zero elements are covered with minimum number of straight lines"

Branch Manager	City			
	A	B	C	D
P	0	0	2	0
Q	3	0	5	4
R	5	0	4	7
S	0	2	0	2

Since number of lines covering all zeros is less than number of rows / columns the optimal solution has not reached.

"Smallest uncovered element is subtracted from every uncovered element and added to the element which lies on the intersection of lines"

Branch Manager	City			
	A	B	C	D
P	0	3	2	0
Q	0	0	2	1
R	2	0	1	4
S	0	5	0	2

"Zero elements are covered by minimum number of straight lines:

Branch Manager	City			
	A	B	C	D
P	0	3	2	0
Q	0	0	2	1
R	2	0	1	4
S	0	5	0	2

"Since number of straight lines covering zeros equal 4, optimal solution has reached. Optimal assignment can be made in the table

Branch Manager	City			
	A	B	C	D
P	~~0~~	3	2	$\boxed{0}$
Q	$\boxed{0}$	~~0~~	2	1
R	2	$\boxed{0}$	1	4
S	~~0~~	5	$\boxed{0}$	2

Optimum allocation is shown as follows:

Branch Manager	Assigned to City	Business (₹ in lakhs)
P	D	8
Q	A	12
R	B	16
S	C	15

With total (Maximum) business

= 8 + 12 + 16 + 15 = ₹ 51 lakhs.

(ii) 5000 copies of a book of price ₹ 80 each were insured for $\left(\frac{3}{5}\right)^{th}$ of their value. During the transit some copies were damaged, and reduced to 60% of the value. If the amount recovered against the damage was ₹ 12,000, find the number of copies that were damaged.

Ans. Total Number of copies = 5000

Cost of one book = ₹ 80

Insured value = $\left(\frac{3}{5}\right)^{th}$ of total value

Insured claim = ₹ 12,000.

$$\text{Now claim} = \frac{\text{Insured value}}{\text{Property value}} \times \text{loss}$$

$\therefore 12{,}000 = \frac{3}{5} \times \text{loss}$

$\therefore$ Loss $= 12{,}000 \times \frac{5}{3}$

= ₹ 20,000

This amount was equal to 40% of the damage.

$\therefore$ Total damage $= 20{,}000 \times \frac{100}{40} =$ ₹ 50,000

Since cost of one book was ₹ 80, the number of books damaged $= \frac{50{,}000}{80} = 625$

$\therefore$ 625 books were damaged.

(iii) The difference between true discount and banker's discount on a bill due 6 months hence at 4% is ₹ 80. Calculate true discount, banker's discount and amount of bill.

Ans. Let T.D. = ₹ x

B.G. = B.D. – TD = interest on T.D. for 6 months at 4% p.a.

$\therefore 80 = x \times \frac{1}{2} \times \frac{4}{100} = \frac{x}{50}$

$\therefore x = 80 \times 50 = 4000$

$\therefore$ True discount is ₹ 4,000

$\therefore$ B.D. = B.G. + T.D.

= 80 + 4000 = ₹ 4,080

$\therefore$ Banker's discount is ₹ 4,080.

B.D. = interest on F.V. for 6 months at 4% p.a.

Let the face value (E.V.) be y

$\therefore$ B.D. $= y \times \frac{1}{2} \times \frac{4}{100}$

$\therefore 4080 = \frac{y}{50}$

$\therefore y = 2{,}04{,}000$

$\therefore$ Amount of the bill is ₹ 2,04,000

Q.3 (a) Attempt any two questions: **(6)**

(i) An alloy of zinc and tin contains 37% zinc by weight. Find the weight of zinc which must be added to 400 kg of this alloy if the final percentage of zinc is to be 70.

Ans. Let x kg of zinc be added.

$\therefore$ From the given condition we get

$$\frac{\frac{37}{100}(400) + x}{400 + x} = \frac{70}{100}$$

$$\therefore \frac{148 + x}{400 + x} = \frac{7}{10}$$

$\therefore 10(148 + x) = 7(400 + x)$

$\therefore 1480 + 10x = 2800 + 7x$

$\therefore 3x = 2800 - 1480 \quad \therefore 3x = 1320$

$x = 440$

440 kg of zinc is added.

S.T.A. / S.Y.J.C.

(ii)

Age group	Population	No. of Deaths
Below 10	900	12
10 – 20	1500	X
20 – 60	4500	38
60 and above	600	30

From the above information, find x, if the CDR = 12.0 per thousand.

Ans. Since age-group wise information is given, we have

$$CDR = \frac{\sum D_i}{\sum P_i} \times 1000$$

Hence, $12.0 = \frac{\sum D_i}{\sum P_i} \times 1000$

Thus, $12.0 = \frac{80 + x}{7500} \times 1000$

i.e. $10(80 + x) = 900$

i.e. $800 + 10x = 900$

$\Rightarrow 10x = 100$

$\therefore x = 10$

(iii) There are five jobs, each of which is to be processed through three machines, A, B and C in the order ABC. Processing times in hours are as shown in the following table.

Job	1	2	3	4	5
A	3	8	7	5	4
B	4	5	1	2	3
C	7	9	5	6	10

Determine the optimum sequence for the five jobs and the minimum elapsed time. Also, find the idle time for three machines.

Ans. Here, min MA = 3, max MB = 5 and min MC = 5.

Since min MC ≥ max MB is satisfied, the problem can be converted into 5 jobs and 2 machine problem.

Consider two fictitious machines, such that G = A + B , H = B + C

The problem is:

Job	1	2	3	4	5
G	7	13	8	7	7
H	11	14	6	8	13

Using optimal sequence algorithm, we have following optimal sequence

4	1	5	2	3

For the total elapsed time, we have

Job	Machine A		Machine B		Machine C	
	In	Out	In	Out	In	Out
4	0	5	5	7	7	13
1	5	8	8	12	13	20
5	8	12	12	1	20	30
2	12	20	20	25	30	39
3	20	27	27	28	39	44

Minimum elapsed time = 44 hrs

Idle time

for A = 44 – 27 = 17 hrs

for B = 44 – 15 = 29 hrs

for C = 44 – 37 = 7 hrs

Q3 **(b)** **Attempt any two questions:** **(8)**

(i) Mrs. Menon plans to save for her daughter's marriage. She wants to accumulate a sum of ₹4,00,000 at the end of 4 years. How much should she invest at the end of each year from now, if she can get interest compounded at 10% p.a.?
(Given $(1.1)^4 = 1.4641$)

Ans. For annuity due
Accumulated amount i.e. A = ₹ 4,00,000
Period is 4 years
$\therefore$ n = 4 as annuity is yearly
Rate of interest 10% P.A.

$$\therefore i = \frac{10}{100} = 0.1$$

C = ? $(1.1)^4 = 1.4641$ .....given

$$A = \frac{c}{i}\left[(1+i)^n - 1\right]$$

$$\therefore 4,00,000 = \frac{c}{i}\left[(1+0.1)^4 - 1\right]$$

$$4,00,000 \times 0.1 = c\left[(1+1)^4 - 1\right]$$

$\therefore 40,000 = c(1.4641 - 1)$
$\therefore 40,000 = c(0.4641)$

$$\therefore \frac{40,000}{0.4641} = c$$

$\therefore c = 86,188.32$

(ii) Find the number of pairs of observations from the following data.
$r = 0.15, \sum(x-\bar{x})(y-\bar{y}) = 12, \sigma_x = 4$ and $\sum(x-\bar{x})^2 = 40$

Ans. $r = 0.15, \sum(x-\bar{x})(y-\bar{y}) = 12, \sigma_x = 4, \sum(x-\bar{x})^2 = 40$, n = ?

Formula $r = \dfrac{cov(x,y)}{\sigma_x \sigma_y}$ .....(i)

$$cov(x,y) = \frac{\sum(x-\bar{x})(y-\bar{y})}{n} = \frac{12}{n}$$

$$\sigma_x = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{40}{n}} = \frac{2\sqrt{10}}{\sqrt{n}}$$

$$\therefore \text{from (i)} \quad 0.15 = \frac{\frac{12}{n}}{\frac{2\sqrt{10}}{\sqrt{n}} \times 4}$$

$$0.15 = \frac{12}{n} \rightarrow \frac{8\sqrt{10}}{\sqrt{n}}$$

$$\therefore 0.15 = \frac{12}{\frac{n}{\sqrt{n}}} \rightarrow \frac{\sqrt{10}}{8\sqrt{10}}$$

S.T.M./S.Y.J.C.

Note
If n = 0.1 give
Full marks since some
thing wrong with question

$$\therefore 0.15 = \frac{3}{2\sqrt{10}.\sqrt{n}}$$

$$\therefore \sqrt{n} = \frac{3}{2\sqrt{10}.0.15}$$

Squaring both the side,

$$\therefore n = \frac{9}{40 \times 0.0225}$$

$$\therefore n = \frac{9}{0.9}$$

$$\therefore n = 10$$

n = 0.1

(iii) The p.d.f. of a r.v. X is given by

$$f_x(x) = \begin{cases} \frac{k}{\sqrt{x}} & 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

Determine k. and find E(x) and V(x)

Ans. The p.d.f. is

$$f(x) = \frac{k}{\sqrt{x}} \qquad 0 < x < 4$$
$$= 0 \qquad \text{otherwise}$$

For a p.d.f. $\int_{-\infty}^{\infty} f(x)dx = 1$

$$\therefore \int_0^4 \frac{k}{\sqrt{x}} dx = 1$$

$$\therefore k\int_0^4 \frac{1}{\sqrt{x}} dx = 1$$

$$\therefore k\left[2\sqrt{x}\right]_1^4 = 1$$

$$\therefore k\left[2\sqrt{4} - 2\sqrt{10}\right] = 1 \rightarrow k(4-0) = 1$$

$$\therefore k(4) = 1$$

$$\therefore k = \frac{1}{4}$$

$$\therefore f(x) = \frac{1}{4\sqrt{x}} \qquad 0 < x < 4$$
$$= 0 \qquad \text{otherwise}$$

Now $E(x) = \int_{-\infty}^{\infty} x.f(x)dx$

$$E(x) = \int_0^4 x.f(x)dx$$

$$\therefore E(x) = \int_0^4 x^{\sqrt{x}}.\frac{1}{4\sqrt{x}} dx$$

$$\therefore E(x) = \frac{1}{4}\int_0^4 \sqrt{x}dx$$

$$\therefore E(x) = \frac{1}{4}\int_0^4 x^{\frac{1}{2}}dx$$

$$\frac{1}{4}\left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^4$$

$$\frac{1}{4}\left[\frac{x^{3/2}}{3/2}\right]_0^4$$

$$\frac{1}{4} \times \frac{2}{3}\left[4^{3/2} - 0\right]$$

$$\frac{1}{6}\left[(2^2)^{3/2}\right] = \frac{8}{6} = \frac{4}{3} = 1.3$$

now

$$x^2 f(x) = x^2 \cdot \frac{1}{4\sqrt{x}}$$

$$= \frac{1}{4}\frac{x^2}{x^{1/2}}$$

$$= \frac{1}{4}x^{2-\frac{1}{2}} = \frac{1}{4}x^{3/2}$$

$$V(x) = \int_a^b x^2 f(x)\,dx - (E(x))^2$$

$$= \int_0^4 \frac{1}{4}x^{3/2}dx - (1.3)^2$$

$$\frac{1}{4}\int_0^4 x^{3/2}dx - 1.69$$

$$\frac{1}{4}\left(\frac{x^{5/2}}{5/2}\right)_0^4 - 1.69$$

$$\frac{1}{4} \times \frac{2}{5}\left[4^{5/2} - 0\right] - 1.69$$

$$\frac{1}{10}\left[(2^2)^{5/2}\right.$$

$$\frac{32}{10} - 1.69$$

$$3.2 - 1.69$$

$$1.51$$

S.T.A./S.T.C.

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