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S.Y.J.C **MATHS PAPER - I** **Saturday** **Time:- 1hr 15 min**
VCR **Marks: 30**

TEST NO. - 1 ***Topic: Matrices & Logic*** **Date:- 4/5/19**

Q.1. (A) Attempt any four: **(8)**

1) Prove that $(p\wedge q)\vee\sim q$ is logically equivalent to $p \vee \sim q$. Hence state its dual result.

2) Determine whether the statement $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology, contradiction or a contingency.

3) If p = Rita is lazy and
q = Rita fail in the examination.
Translate the following in the verbal form
(a) $p \to \sim q$ (b) $q \leftrightarrow p$

4) Represent the following statement by Venn diagram ''No quadrilateral is a triangle''

5) Find the negation of the following:
(a) $(p \to q)\vee(q \to r)$
(b) $p\wedge (q \vee r)$

(B) Write Converse, Contrapositive and Inverse of, If r is rational then r is a real **(3)**

Q.2.(A) Attempt any three. **(12)**

1) $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$ show $A^2 - 2A - I = 0$ *and hence find* A^{-1}

2) Solve by reduction method.
$x - y + z = 1$
$3x - y + 2z = 1$
$2x - 2y + 3z = 2$

3) $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ *Show A is non singular and hence find* A^{-1} *by row operations.*

4) $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 5 & 0 \\ 3 & -1 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 & 5 \\ 3 & -2 & 1 \\ 4 & 5 & 0 \end{bmatrix}$
Show $(AB)' = B'A'$

(B) If $A = \begin{bmatrix} 4 & 2 \\ -6 & 1 \\ 3 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 4 & -2 \end{bmatrix}$ *show that AB is singular matrix* **(2)**

(C) *Find the inverse of the matrix* $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ *by using adjoint method.* **(5)**

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S.Y.J.C | **MATHS PAPER - I** | **Saturday** | **Time:- 1hr 15 min**
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TEST NO. - 1 | ***Topic: Matrices & Logic (Solution)*** | **Date:- 4/5/19**

SYJC
Test 1 :- Matrices & logic (Solution)
(1)

Q1(A) (1)

p	q	$p \wedge q$	$\sim q$	$(p\wedge q)\vee\sim q$	$p \vee \sim q$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	F	F	F	F
F	F	F	T	T	T
(1)	(2)	(3)	(4)	(5)	(6)

all entries in 5th & 6th col are equal

$\therefore (p\wedge q)\vee \sim q \equiv p \vee \sim q$

The dual of $(p\wedge q)\vee\sim q$ is $(p \vee q)\wedge \sim q$

(2)

p	q	$p\to q$	$\sim p$	$\sim q$	$\sim q \to \sim p$	$(p\to q)\leftrightarrow(\sim q\to\sim p)$
T	T	T	F	F	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	F	T	T	T	T	T
(1)	(2)	(3)	(4)	(5)	(6)	(7)

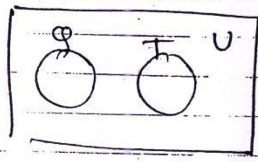
Since all entries in last col i.e. 7th col are all T

given statement is Tautology

(iii) i) If Rita is lazy then Rita doesnot fail in exam

ii) Rita fails in exam if and only if Rita is lazy

(iv)



U = universal set of polygon
Q = set of quadrilateral
T = set of triangles

(v) (i) $(p\to q)\vee(q\to r)$
$\sim[(p\to q)\vee(q\to r)]$
$\equiv \sim(p\to q)\wedge\sim(q\to r)$
$\equiv (p\wedge\sim q)\wedge(q\wedge\sim r)$

$p\wedge(q\vee r)$
$\sim[p\wedge(q\vee r)]$
$\equiv \sim p\vee\sim(q\vee r)$
$\equiv \sim p \vee(\sim q\wedge\sim r)$

Q1 (B) If r is rational then r is real

p : r is rational q : r is real

given statement is $p\to q$

(1) using $q\to p$ Converse : If r is real then r is rational

(ii) $\sim q\to\sim p$ Contrapositive If r is not real then r is not rational

(iii) $\sim p\to\sim q$ Inverse If r is not rational then r is not real

Q 2 (A)

(1) $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$

$= A^2 - 2A - I$

$= AA - 2A - I$

$= \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} - 2\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 4+1 & 2+0 \\ 2+0 & 1+0 \end{pmatrix} + \begin{bmatrix} -4 & -2 \\ -2 & 0 \end{bmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -4-1 & -2+0 \\ -2+0 & 0-1 \end{bmatrix}$

$\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -5 & 2 \\ -2 & -1 \end{bmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0 = Req$

$\therefore A^2 - 2A - I = 0$ (proved)

Pre multiply b.s by A^{-1}

$A^{-1}A^2 - 2A^{-1}A - A^{-1}I = 0$

$A - 2I - A^{-1} = 0$

$A - 2I = A^{-1}$

$\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A^{-1}$

$\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = A^{-1}$

$A^{-1} = \begin{pmatrix} 2-2 & 1+0 \\ 1+0 & 0-2 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$

(2) The Equations Can be written in Matrix Form

$\begin{bmatrix} 1 & -1 & 1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

$R_2 - 3R_1$
$R_3 - 2R_1$ $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$

By reduction method (compare 3rd row

$0 + 0 + z = 0$

$z = 0$

Compare 2nd eq

$0 + 2y + 0 = -2$

$\therefore y = -1$

Compare 1st eq

$x - y + z = 1$

$x - (-1) + 0 = 1$

$x + 1 = 0$

$x = 0$

$x = 0 \quad y = -1 \quad z = 0$

iii) $|A| = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix}$

$1(16 - 9) - 3(4 - 3) + 3(3 - 4)$

$7 - 3(1) - 3 = 1 \neq 0$

$\Rightarrow A$ is non singular matrix

$\therefore A^{-1}$ exists

$A A^{-1} = I$

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 - R_1$

$R_3 - R_1$

$$\begin{bmatrix} 1 & 3 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$R_1 - 3R_2$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 4 & -3 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$R_1 - 3R_3$

(4)

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}A^{-1} = \begin{bmatrix}7&-3&-3\\-1&1&0\\-1&0&1\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix}7&-3&-3\\-1&1&0\\-1&0&1\end{bmatrix}$$

(iv) $$AB = \begin{bmatrix}3&2&1\\4&5&0\\3&-1&-3\end{bmatrix}\begin{bmatrix}2&0&5\\3&-2&1\\4&5&0\end{bmatrix}$$

$$= \begin{bmatrix}6+6+4 & 0-4+5 & 15+2+0\\8+15+0 & 0-10+0 & 20+5+0\\6-3-12 & 0+2-15 & 15-1+0\end{bmatrix}$$

$$AB = \begin{bmatrix}16&1&17\\23&-10&25\\-9&-13&14\end{bmatrix}$$

$$(AB)' = \begin{bmatrix}16&23&-9\\1&-10&-13\\17&25&14\end{bmatrix} \text{ — I}$$

$$B'A' = \begin{bmatrix}2&3&4\\0&-2&5\\5&1&0\end{bmatrix}\begin{bmatrix}3&4&3\\2&5&-1\\1&0&-3\end{bmatrix}$$

$$= \begin{bmatrix}6+6+4 & 8+15+0 & 6-3-12\\0-4+5 & 0-10+0 & 0+2-15\\15+2+0 & 20+5+0 & 15-1+0\end{bmatrix}$$

$$\begin{bmatrix}16&23&-9\\1&-10&-13\\17&25&14\end{bmatrix} \text{ — II}$$

from I & II $(AB)' = B'A'$

Q2(B) $$AB = \begin{bmatrix}4&2\\-6&1\\3&5\end{bmatrix}\begin{bmatrix}2&1&0\\3&4&-2\end{bmatrix}$$

$$\begin{bmatrix}8+6 & 4+8 & 0-4\\-12+3 & -6+4 & 0-2\\6+15 & 3+20 & 0-10\end{bmatrix}$$

By reduction method (another 3rd m

(5)

$AB = \begin{bmatrix} 14 & 12 & -4 \\ -9 & -2 & -2 \\ 21 & 23 & -10 \end{bmatrix}$

$|AB| = 14(20+46) - 12(90+42) - 4(-207+42)$

$= 14(66) - 12(132) - 4(\cancel{249}) - 4(-165)$

$= 924 - 1584 - \cancel{996} + 660$

$\cancel{= -1056}$

$\cancel{\neq 0}$

$\cancel{\Rightarrow AB \text{ is non singular}}$

$1584 - 1584$

$= 0$

$\Rightarrow AB$ is singular matrix

Ex for
Q2(c) let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{vmatrix}$

$= 1(7-20) - 2(7-10) + 3(4-2)$

$= -13 - 2(-3) + 3(2)$

$= -13 + 6 + 6$

$= -1$

$M_{11} = \begin{vmatrix} 1 & 5 \\ 4 & 7 \end{vmatrix}$

$= 7 - 20 = -13$

$A_{11} = (-1)^{1+1}(-13) = -13$

$M_{12} = \begin{vmatrix} 1 & 5 \\ 2 & 7 \end{vmatrix} = 7 - 10$
$= -3$

$A_{12} (-1)^{1+2}(-3) = -1(-3) = 3$

$M_{13} = \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} = 4 - 2 = 2$

$A_{13} = (-1)^{1+3}(2) = 2$

$M_{21} = \begin{vmatrix} 2 & 3 \\ 4 & 7 \end{vmatrix}$

$= 14 - 12 = 2$

$A_{21} = (-1)^{2+1}(2) = -2$

$M_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix}$

$= 7 - 6 = 1$

$A_{22} = (-1)^{2+2}(1) = 1$

$M_{23} = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 4 - 4$
$= 0$

$A_{23} = 0$

$M_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix}$

$= 10 - 3 = 7$

$A_{31} = (-1)^{3+1}(7) = 7$

$M_{32} = \begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix}$

$= 5 - 3 = 2$

$A_{32} = (-1)^{3+2}(2)$
$= -2$

$M_{33} = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$

$= [redacted] 1 - 2$
$= -1$

$A_{33} = (-1)^{3+3}(-1)$

$= -1$

Matrix of Cofactors $= \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$

$$= \begin{bmatrix} -13 & 3 & 2 \\ -2 & 1 & 0 \\ 7 & -2 & -1 \end{bmatrix}$$

Adj(A) = Transpose of above matrix

$$= \begin{bmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \operatorname{Adj}(A)$$

$$= \frac{1}{-1} \begin{bmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{pmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & +1 \end{pmatrix}$$

(6)