Computer Vision 1: Homework 6

Important: Mark the homeworks you solved in the homework sheet and bring your solutionswith you to the exercise class. For each homework problem, one student will be chosen at random to present their solution.

Programming tasks.

The ideal lowpass filter studied in the exercise suffers from the problem of ringing. Using a Gaussian lowpass filter (GLPF) instead significantly reduces ringing. Using the same notation as in Exercise sheet 6, the GLPF kernel in the frequency domain is defined as

$$H(u, v) = \exp\left(-D^2(u, v)/(2D_0^2)\right). \tag{1}$$

where $D^2(u, v)$ is the squared distance from the center, and D_0 is the cutoff frequency parameter. The filter H is visualized in Figure 1.

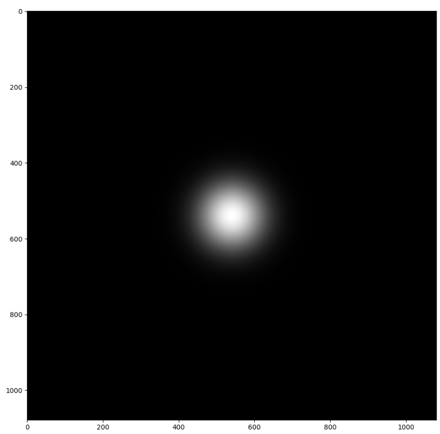

Figure 1: A Gaussian lowpass filter in frequency domain.

Use $D_0 = 60.0$ in the following tasks. Convert the image to grayscale before filtering.

1. Apply GLPF to `MaruTaro.jpg` and visualize the result side-by-side with the result of the ideal lowpass filter.
2. Given an ideal or Gaussian lowpass filter $H(u, v)$, a complementary highpass filter kernel is given by $H_{\text{HP}}(u, v) = 1 - H(u, v)$. 1) Create a filter kernel for the ideal highpass filter, and 2) create the filter kernel for the Gaussian highpass filter, and finally 3) apply both to the image and visualize the results side by side. The expected result is shown in Figure 2.

Other tasks.

In all of the exercises below, assume $f(x, y)$ represents a real-valued image of size M-by-N. Recall the definition of the 2D DFT

$$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \exp\left[-j2\pi(ux/M + vy/N)\right]$$

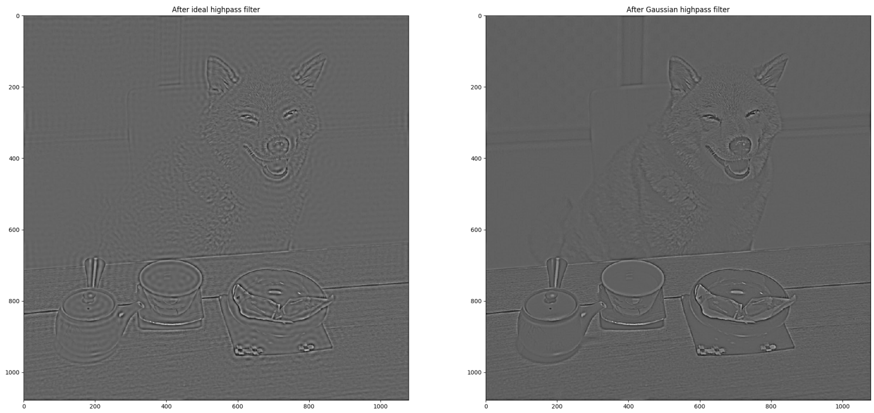

Figure 2: The image after applying the ideal highpass filter (left) or the Gaussian highpass filter (right).

and the inverse DFT

$$f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{M-1} F(u, v) \exp\left[j2\pi(ux/M + vy/N)\right].$$

1. Show that the 2D DFT of real-valued $f(x, y)$ is conjugate symmetric, i.e., that $F^*(u, v) = F(-u, -v)$, where the operator * denotes complex conjugate[1].

   Using the result, prove that the magnitude spectrum has even symmetry about the origin, i.e., that $|F(u, v)| = |F(-u, -v)|$. This result explains why the magnitude spectrum of an image is symmetric when visualized.

2. Prove the following shifting properties of 2D DFT by showing that the two expressions form a Fourier transform pair.

   (a) $f(x, y) \exp\left[j2\pi(u_0 x/M + v_0 y/N)\right] \Leftrightarrow F(u - u_0, v - v_0)$

   (b) $f(x - x_0, y - y_0) \Leftrightarrow F(u, v) \exp\left[-j2\pi(x_0 u/M + y_0 v/N)\right]$.

   Multiplying an image with the shown complex exponential shifts the spectrum in the frequency domain. Multiplying the spectrum with the corresponding negative complex exponential shifts the image in the spatial domain[2]. What effect does shifting have on the magnitude $|F(u, v)|$?

3. Consider the GLPF defined in Equation (1) above. Suppose we lowpass filter the image $f(x, y)$ by applying the GLPF K times. What is the resulting image as $K \to \infty$, and why?

[1] $F(-u, -v)$ is well defined due to periodicity of the 2D Fourier transform: $F(u, v) = F(u + k_1 M, v + k_2 N)$ for all $k_1, k_2 \in \mathbb{Z}$.

[2] Due to periodicity possibly negative arguments are not a problem; $\forall k_1, k_2 \in \mathbb{Z} : f(x, y) = f(x + k_1 M, y + k_2 M)$.