Computer Vision 1: Homework 4

Important: Mark the homeworks you solved in the homework sheet and bring your solutions with you to the exercise class. For each homework problem, one student will be chosen at random to present their solution.

Programming tasks.

Download the image `woman.png` from Moodle and use it as I. Use the Gaussian filter implementation from `scikit-image`: `skimage.filters.gaussian`.

1. Apply the Difference-of-Gaussians filter on I with $\sigma_1 = 2$ and $\sigma_2 = 5$. Display the result.
2. Create a Difference-of-Boxes filter where the larger box has size 43 and the smaller box has size 19. Display the filter kernel.
3. Apply the Difference-of-Boxes filter on I, and display the result.

Other tasks.

1. Consider the following two images:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 0 \\ 3 & 1 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 4 & 5 & 6 & 8 \\ 7 & 8 & 9 & 6 \end{bmatrix}.$$

   Calculate median$(A + B)$ and median(A) + median(B) with filter size 3-by-3. Pad the borders with the necessary number of zeros so the output is the same size as A and B. Are the results the same? Why/Why not?

2. Let

$$X = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

   a) Which two filters do X and Y correspond to?

   b) Calculate the convolutions $X \star Y$ and $X^T \star Y^T$ by hand, padding with zeros. The result should be 3-by-3. Which filter related to edge detection is the output equal to?

3. By the singular value decomposition (SVD), any n-by-n matrix K can be represented as $K = \sum_{i=1}^{n} \sigma_i \vec{u}_i \vec{v}_i^T$, where σ_i is the ith singular value, and $\vec{u}_i, \vec{v}_i \in \mathbb{R}^n$ are the ith left and right singular vectors, respectively.

   Using the SVD of a n-by-n filter matrix K, find the conditions when $K = \vec{k}_1 \vec{k}_2^T$ for some $\vec{k}_1, \vec{k}_2 \in \mathbb{R}^n$. Apply your answer to find $\vec{k}_1$ and $\vec{k}_2$ such that $\vec{k}_1 \vec{k}_2^T$ is the 3-by-3 box filter.

   Note: To apply your answer, you may use a software package to compute the SVD of the box filter instead of doing it by hand.