Computer Vision 1: Homework 5

Important: Mark the homeworks you solved in the homework sheet and bring your solutions with you to the exercise class. For each homework problem, one student will be chosen at random to present their solution.

Programming tasks.

Correlation filters respond most strongly to regions of images that look like the filter itself. This property can be used for *template matching*. In template matching, we cross correlate a template with an image. The template is another image that is a prototype for what we want to localize in an image. The cross-correlation is the strongest where the image looks most like the template.

- Download a clock template image and an image containing a clock from Moodle under the names `coco264316clock.jpg` and `coco264316.jpg`, respectively.
- Read the template and the image. Convert them to grayscale.
- Use the function `skimage.feature.match_template` to compute the normalized correlation between two images. Visualize the matching result. Verify that the brightest pixel in the result corresponds to the clock.
- Flip the template horizontally. Verify that the clock can no longer be found.

Other tasks.

1. Consider the image

$$I = \begin{bmatrix} 5 & 7 & 9 \\ 4 & 1 & 3 \\ 5 & 8 & 0 \end{bmatrix}.$$

   a) Using the central difference method, calculate the gradient $\nabla I = \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix}$ at the center pixel.

   b) Draw the gradient vector on a coordinate axis. Use the same coordinate system as in the lecture.

   c) Calculate the magnitude $||\nabla I||$ and direction (orientation) θ of the gradient. Indicate the magnitude and orientation in your drawing.

2. The Laplacian operator ∇^2 on a function g is defined as

$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}.$$

   Informally, the Laplacian $\nabla^2 g(x, y)$ may be interpreted as the average rate of change of g at the point (x, y).

   a) The Gaussian function is $g(x, y) = e^{\left(-\frac{x^2+y^2}{2\sigma^2}\right)}$. Calculate the Laplacian of a Gaussian (LoG) $\nabla^2 g(x, y)$.

b) Prove that the LoG is isotropic. That is, show that the rate of change of the Gaussian is equal in all directions by proving that

$$\nabla^2 g(x, y) = \nabla^2 g(x', y')$$

where

$$\begin{cases} x' = x\cos\theta - y\sin\theta \\ y' = x\sin\theta + y\cos\theta \end{cases}$$

are the coordinates rotated by an angle $\theta \in [0, 2\pi)$.

3. Consider a discrete signal of length $M = 4$:

$$f(x) = \begin{cases} 1 & x = 0, 2 \\ 0 & x = 1, 3 \end{cases}.$$

a) Calculate the M-point discrete Fourier transform F of f. Draw the values of $F(u)$ for $u = 0, 1, 2, 3$ on the complex plane.

b) What is the relation of the average value of f and $F(0)$?