Computer Vision 1: Homework 3

Important: Mark the homeworks you solved in the homework sheet and bring your solutions with you to the exercise class. For each homework problem, one student will be chosen at random to present their solution.

Programming tasks.

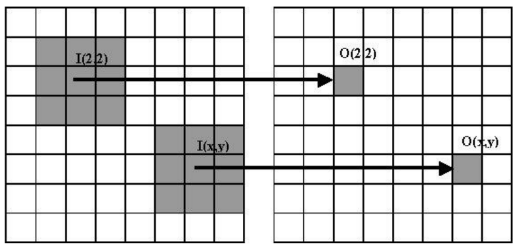
Use the Gaussian filter kernel $K_{gaussian}$ defined in the exercise. Download the image `woman.png` from Moodle and use it as I. Following the definitions of the Gaussian pyramid and Laplacian pyramid given on the lecture slides.

1. Compute the layers G_0, G_1, G_2, and G_3, of a Gaussian pyramid for I. Print out the shape of each layer. Draw images to display each layer.

2. Compute the layers L_0, L_1, L_2, and L_3 of a Laplacian pyramid for I. Print out the shape of each layer. Draw images to display each layer.

Other tasks.

1. Consider the image below. Recall how an image I (gridded area on the left) is cross-correlated with a kernel H (shaded area on the left) by sliding the kernel over the image and storing the output values (gridded area on the right) obtained by a dot product of the image content and the kernel values. The cross correlation is denoted as $H \otimes I$.



   Give an example of I and H, where $H \otimes I \neq I \otimes H$. Pad the signals with zeros as required. This shows that cross-correlation is not commutative.

2. Consider two 2-dimensional signals F and H. The convolution $H * F$ at index (i, j) is equal to

$$[H * F](i, j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H(u, v) F(i - u, j - v).$$

   a) Show that convolution is homogeneous, that is, for any $\alpha \in \mathbb{R}$, $\alpha(H * F) = \alpha H * F$.

   b) Show that convolution is distributive over addition, that is, $H * (F_1 + F_2) = H * F_1 + H * F_2$.

3. Prove that convolution is commutative, that is, $H * F = F * H$.

   Hint: Start from the definition given in Problem 2 and make two changes of variables: $u' = i - u$ and $v' = j - v$. Then, make use of the property that signals are infinite: the sum over any $(2k + 1)$ terms is equal to the sum from $-k$ to k.