Computer Vision 1: Homework 8

Important: Mark the homeworks you solved in the homework sheet and bring your solutions with you to the exercise class. For each homework problem, one student will be chosen at random to present their solution.

Programming tasks.

A second order polynomial follows the functional form

$$f(x) = a_3 x^2 + a_2 x + a_1,$$

where the three parameters $a_i \in \mathbb{R}$.

- Load the two files `x.npy` and `y.npy` from Moodle using `numpy.load`. These depict the x and y values of the dataset shown in Figure 1. The values contain inliers that are close to the polynomial, and other outliers that are at random locations.
- Implement RANSAC for fitting a second order polynomial to the data. Use an inlier threshold of 0.05 for the Euclidean distance, and run 1000 iterations. Draw a plot similar to Figure 1 using the best parameters you found. You can check your work by comparing the values you find to the true values $a_3 = 2.0, a_2 = -4.2, a_1 = 1.0$.

Hints: Given the three randomly sampled points (x_1, y_1), (x_2, y_2), (x_3, y_3), create a coefficient matrix

$$X = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix}.$$

The estimated model coefficients $\hat{a}_i$ are then obtained by solving the linear group of equations

$$X \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}.$$

You may use `numpy.linalg.lstsq` or another similar tool to solve the least squares problems for finding the model parameters.

Other tasks.

1. Suppose $\epsilon \in (0, 1)$ is the probability that any single randomly drawn point is an outlier. We need to draw a sample of $m \in \mathbb{N}$ points to fit our model. Find the minimum number of iterations $k \in \mathbb{N}$ required for RANSAC such that with probability p at least one sample is good[1].
2. We have applied the Hough transform for lines in the Hesse normal form, and found a line with parameters $\rho = 2, \theta = 90°$. Draw the detected line in the image coordinate system.
3. The corresponding filters for computing D_{xx} and D_{yy} are $\begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, respectively. These filters are obtained by convolving the 1st-order derivative filters with themselves with proper zero-paddings: $\begin{bmatrix} 1 & -1 \end{bmatrix} \star \begin{bmatrix} 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \star \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$.

[1] In a "good" sample, all of the m randomly drawn points are inliers.

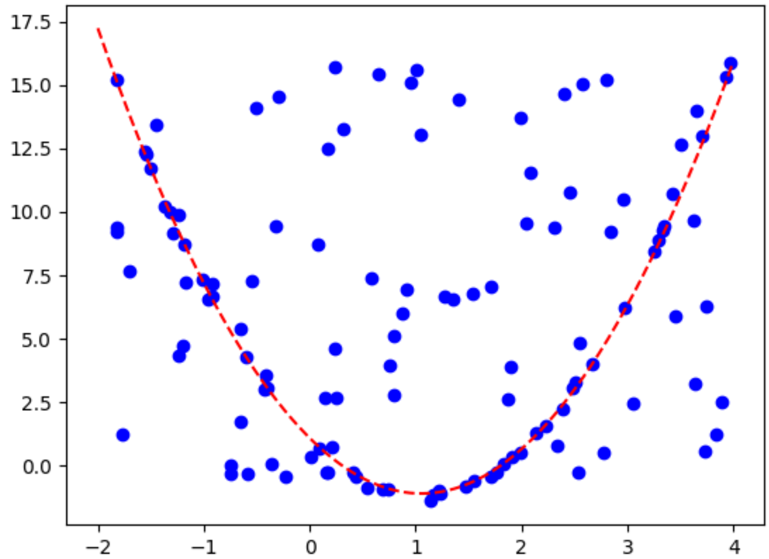

Figure 1: Dataset shown by blue dots. Second order polynomial fit found by RANSAC shown in red.

Using similar approach, compute $D_{xy} = D_{yx} = \begin{bmatrix} 1 & -1 \end{bmatrix} \star \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. The result should be a 2×2 matrix.

4. Explain why $r = \frac{\alpha}{\beta} > 0$, with α, β being the eigenvalues of the Hessian matrix at a keypoint. Using this result, prove that when $r > 0$, the function $f(r) = \frac{(r+1)^2}{r}$ has a minimum at $r = 1$.