Exam Final

IFT6561

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Part 1

a

Experimental Results

Beta vector with CV
(0.00628, 0.000347, 0.00138, 0.00501, 0.00458, 0.00271, 0.00276, 0.00703)
Average without CV : 0.11448
Average with CV : 0.11291
Variance without CV : 0.025091
Variance with CV : 0.010342
Variance reduction factor : 2.4261
Confidence interval student 95 without CV : 0.11448 $\pm$ 0.0031046
Confidence interval student 95 with CV : 0.11291 $\pm$ 0.0019932

As shown in the data, the confidence interval with control variates (CV) is about 1.5 times tighter than without CV. The average of the estimator without CV is also contained within the 95% student confidence interval of the estimator with CV. Overall, the strategy using CV is pretty good, decreasing the variance by a factor of 2.4. This was expected, as stated in example 6.26.

b

The whole concept of stochastic derivatives is explained at p.101 of the book, I'm not rewriting it here, we'll just state the core of the matter.
The first necessary observation is that both θ_2 and θ_4 are parameters for the mean of exponential distributions. Using the same proof as given in example 1.49 of the book, we can explain why our estimator is unbiased.
Suppose we want to estimate $\frac{\delta \mathbb{E}[T]}{\delta \theta_i}$, we can write T as a function of θ_i.
Say f is the function. $f^{'}(\theta_i, U) = V_i^{'}(\theta_i)$ if the link i is in the longest path and 0 otherwise. Since, in this case, both θ_i are the means of exponential random variables, we have $Y_i = Y_i(\theta_i) = -\theta_i ln(1 - U_i)$ and its derivative is $-ln(1 - U_i)$. Using the corollary 6.6 to prove that we can interchange the expectation and derivative operators to obtain that our estimator of $f^{'}(\theta_i)$ is unbiased.

Basically, this is the rest of the example 1.49.

c

As stated in example 1.50 of the book, if the function $f_j(\theta_j, U) = \mathbb{1}[T > x]$, the estimator can only take two values, 0 and 1. Its derivative is either undefined, when the function jumps exactly at θ_j or is 0. It cannot be an unbiased estimator of the derivative because the original function is discontinuous at some point as a function of θ_j.

d

For one thing, the CMC estimator is continuous everywhere in each θ_i so it can't jump from zero to one and yield a value of $1/\delta$ at some point, which ruins the dominated convergence theorem for the naive indicator function.

e

f

g

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