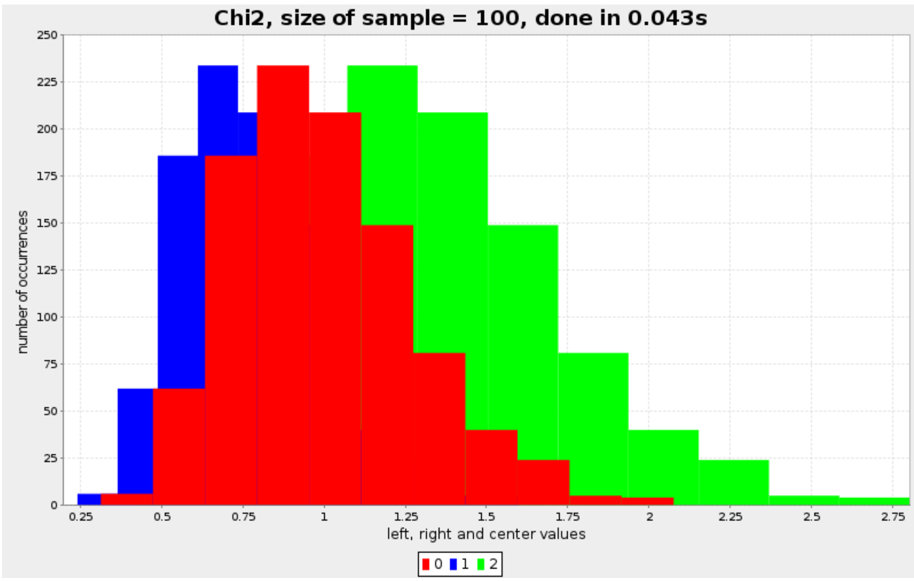
Exercise 3

a) and b)

Confidence intervals for the variance of 100 samples (chi-square).



Empirical values obtained

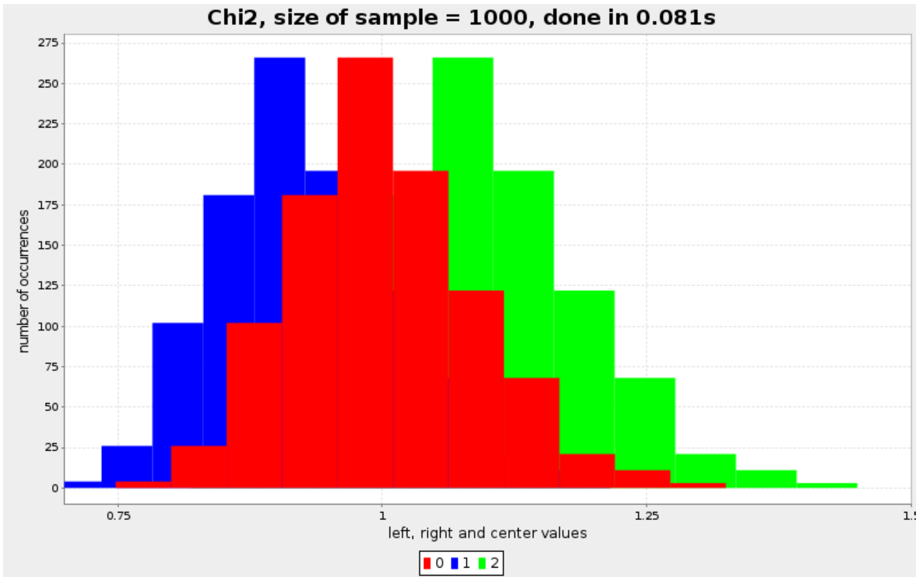
$\mathbb{E}[w] \;=\; 0.5762 \pm\; 0.0098$

$\hat{p} = 0.687 \pm 0.028$

The left sides, centers and right sides of the confidence intervals calculated are plotted in the same fashion for every figure.

note: the confidence interval for $\hat{p}$ is calculated using the student confidence interval

Confidence interval for the variance of 1000 samples



Empirical values obtained

$\mathbb{E}[w] \;=\; 0.176186 \pm\; 0.00094$

$\hat{p} = 0.697 \pm 0.028$

Comparison between 100 and 1000 samples

When using more data, the confidence interval on the variance estimated using the chi-squared distribution is narrower. Although the interval is narrower, the coverage percentage stays basically the same (about 69%).

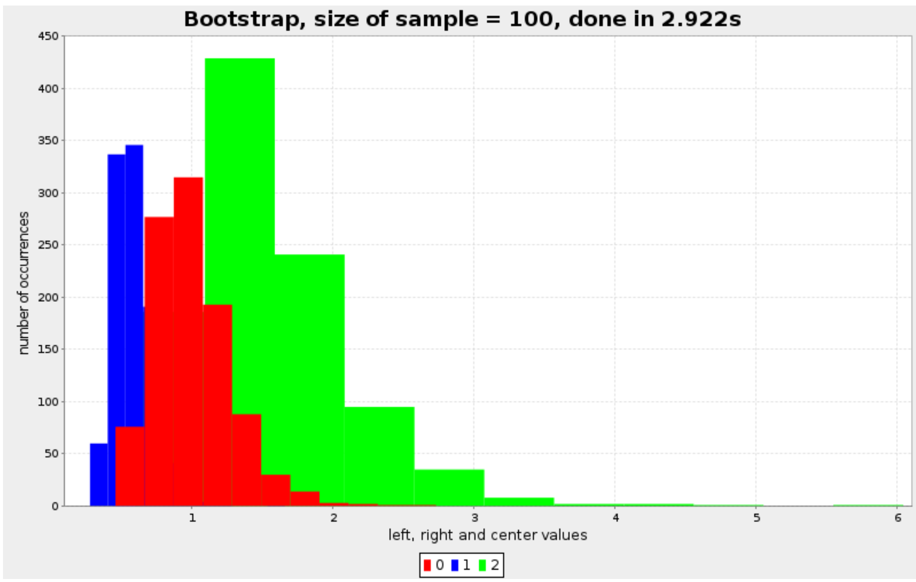
Basically, you get a better precision, but the values tend to the same and the coverage isn't great.

Let's see what happens when the bootstrap procedure is used to get the confidence interval.

c)

The same procedure as in a) and b) was done, but this time the variance confidence interval is estimated using the basic nonparametric bootstrap procedure.

Confidence interval for the variance of 100 samples

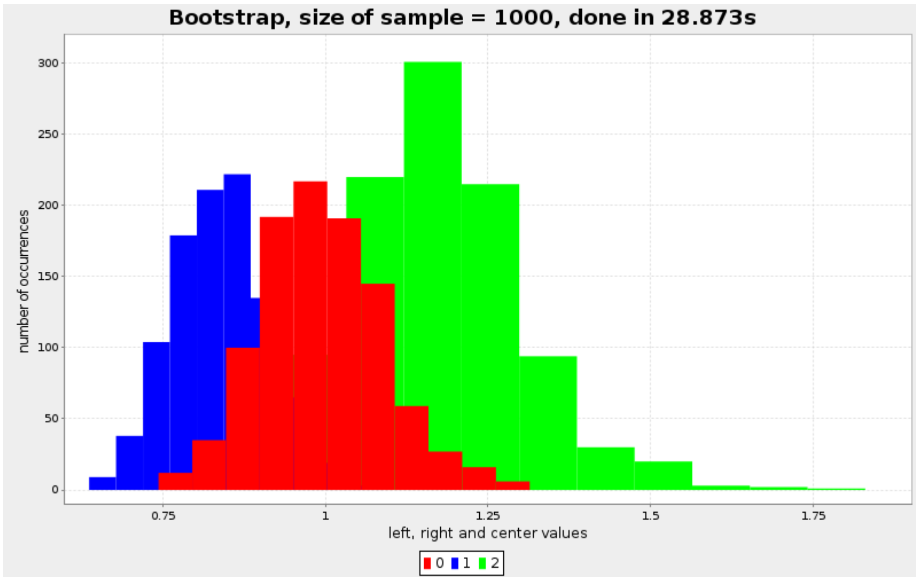


Empirical values obtained

$\mathbb{E}[w] \; = \; 0.960 \pm \; 0.029$

$\hat{p} = 0.867 \pm 0.021$

Confidence interval for the variance of 1000 samples



Empirical values obtained

$\mathbb{E}[w] \;=\; 0.3397 \pm\; 0.0045$

$\hat{p} = 0.904 \pm 0.018$

When using 100 as sample size, the interval boundaries is pretty unstable as can be seen on the figure. and it's width is very big (0.960 ± 0.029) but contains the true value much more often than both of the previous chi-squared versions.

When using 1000 as sample size, the interval boundaries get much smaller (0.3397 ± 0.0045) and the coverage gets even better (0.904 ± 0.018).

This improvement in performance comes at a price however, the performance. In the chi-squared estimations, the time necessary to compute the values was 0.043 and 0.081s whereas the necessary time for the bootstrap procedure was 2.922 and 28.873s.

To evaluate the performance of an estimator, there are multiple factors such as the variance, the bias and the cost. In this case, the chi-squared estimator is biased and the nonparametric bootstrap isn't. However the cost of the chi-squared estimator is much smaller than the one of the bootstrap.

Therefore, each estimator has its advantage, depending on what is important.