SIMULATING ULTRACOLD ATOMS

CHRISTOPHER JON WATKINS

From classical to quantum gases

December 2013 – version 0.0.1

Ohana means family.
Family means nobody gets left behind, or forgotten.

— Lilo & Stitch

Dedicated to the loving memory of Rudolf Miede.

1939 – 2005

ABSTRACT

Short summary of the contents...

PUBLICATIONS

Some ideas and figures have appeared previously in the following publications:

Put your publications from the thesis here. The packages `multibib` or `bibtopic` etc. can be used to handle multiple different bibliographies in your document.

We have seen that computer programming is an art,
because it applies accumulated knowledge to the world,
because it requires skill and ingenuity, and especially
because it produces objects of beauty.

— **?** [**?**]

ACKNOWLEDGEMENTS

Put your acknowledgements here.

Many thanks to everybody who already sent me a postcard!

Regarding the typography and other help, many thanks go to Marco Kuhlmann, Philipp Lehman, Lothar Schlesier, Jim Young, Lorenzo Pantieri and Enrico Gregorio[1], Jörg Sommer, Joachim Köstler, Daniel Gottschlag, Denis Aydin, Paride Legovini, Steffen Prochnow, Nicolas Repp, Hinrich Harms, Roland Winkler, and the whole LaTeX-community for support, ideas and some great software.

Regarding LyX: The LyX port was intially done by Nicholas Mariette in March 2009 and continued by Ivo Pletikosić in 2011. Thank you very much for your work and the contributions to the original style.

1 Members of GuIT (Gruppo Italiano Utilizzatori di TeX e LaTeX)

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LIST OF FIGURES

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LISTINGS

ACRONYMS

DRY Don't Repeat Yourself

API Application Programming Interface

UML Unified Modeling Language

Part I

INTRO MATERIAL

You can put some informational part preamble text here. Illo principalmente su nos. Non message occidental angloromanic da. Debitas effortio simplificate sia se, auxiliar summarios da que, se avantiate publicationes via. Pan in terra summarios, capital interlingua se que. Al via multo esser specimen, campo responder que da. Le usate medical addresses pro, europa origine sanctificate nos se.

1 INTRODUCTION

This template for LaTeX has two goals:

1. Provide students with an easy-to-use template for their Master's or PhD thesis (though it might also be used by other types of authors for reports, books, etc.).

2. Provide a classic, high-quality typographic style that is inspired by **?** 's "The Elements of Typographic Style" [**?**].

The bundle is configured to run with a full MiKTeX or TeXLive installation right away and, therefore, it uses only freely available fonts.

People interested only in the nice style and not the whole bundle can now use the style stand-alone via the file `classicthesis.sty`. This works now also with "plain" LaTeX.

As of version 3.0, `classicthesis` can also be easily used with LyX[1] thanks to Nicholas Mariette and Ivo Pletikosić. The LyX version of this manual will contain more information on the details.

This should enable anyone with a basic knowledge of LaTeX 2_ε or LyX to produce beautiful documents without too much effort. In the end, this is my overall goal: more beautiful documents, especially theses, as I am tired of seeing so many ugly ones.

The whole template and the used style is released under the GNU General Public License.

If you like the style then I would appreciate a postcard:

Andre Miede
Detmolder Strasse 32
31737 Rinteln
Germany

The postcards I received so far are available at:

`http://postcards.miede.de`

IMPORTANT NOTE: Some things of this style might look unusual at first glance, many people feel so in the beginning. However, all things are intentionally designed to be as they are, especially these:

- No bold fonts are used. Italics or spaced small caps do the job quite well.
- The size of the text body is intentionally shaped like it is. It supports both legibility and allows a reasonable amount of information to be on a page. And, no: the lines are not too short.

1 `http://www.lyx.org`

- The tables intentionally do not use vertical or double rules. See the documentation for the `booktabs` package for a nice discussion of this topic.[2]
- And last but not least, to provide the reader with a way easier access to page numbers in the table of contents, the page numbers are right behind the titles. Yes, they are not neatly aligned at the right side and they are not connected with dots that help the eye to bridge a distance that is not necessary. If you are still not convinced: is your reader interested in the page number or does she want to sum the numbers up?

Therefore, please do not break the beauty of the style by changing these things unless you really know what you are doing! Please.

1.1 ORGANIZATION

You can use these margins for summaries of the text body...

A very important factor for successful thesis writing is the organization of the material. This template suggests a structure as the following:

- `Chapters/` is where all the "real" content goes in separate files such as `Chapter01.tex` etc.
- `FrontBackMatter/` is where all the stuff goes that surrounds the "real" content, such as the acknowledgments, dedication, etc.
- `gfx/` is where you put all the graphics you use in the thesis. Maybe they should be organized into subfolders depending on the chapter they are used in, if you have a lot of graphics.
- `Bibliography.bib`: the BibTEX database to organize all the references you might want to cite.
- `classicthesis.sty`: the style definition to get this awesome look and feel. Bonus: works with both LATEX and PDFLATEX... and LYX.
- `ClassicThesis.tcp` a TEXnicCenter project file. Great tool and it's free!
- `ClassicThesis.tex`: the main file of your thesis where all the content gets bundled together.
- `classicthesis-config.tex`: a central place to load all nifty packages that are used. In there, you can also activate backrefs in order to have information in the bibliography about where a source was cited in the text (i. e., the page number).

  Make your changes and adjustments here. This means that you specify here the options you want to load `classicthesis.sty` with. You

2 To be found online at http://www.ctan.org/tex-archive/macros/latex/contrib/booktabs/.

also adjust the title of your thesis, your name, and all similar information here. Refer to Section 1.3 for more information.

This had to change as of version 3.0 in order to enable an easy transition from the "basic" style to LyX.

In total, this should get you started in no time.

1.2 STYLE OPTIONS

There are a couple of options for `classicthesis.sty` that allow for a bit of freedom concerning the layout:

. . . or your supervisor might use the margins for some comments of her own while reading.

- General:
  - `drafting`: prints the date and time at the bottom of each page, so you always know which version you are dealing with. Might come in handy not to give your Prof. that old draft.
- Parts and Chapters:
  - `parts`: if you use Part divisions for your document, you should choose this option. (Cannot be used together with `nochapters`.)
  - `nochapters`: allows to use the look-and-feel with classes that do not use chapters, e. g. , for articles. Automatically turns off a couple of other options: `eulerchapternumbers`, `linedheaders`, `listsseparated`, and `parts`.
  - `linedheaders`: changes the look of the chapter headings a bit by adding a horizontal line above the chapter title. The chapter number will also be moved to the top of the page, above the chapter title.
- Typography:
  - `eulerchapternumbers`: use figures from Hermann Zapf's Euler math font for the chapter numbers. By default, old style figures from the Palatino font are used.
  - `beramono`: loads Bera Mono as typewriter font. (Default setting is using the standard CM typewriter font.)
  - `eulermath`: loads the awesome Euler fonts for math. (Palatino is used as default font.)
  - `pdfspacing`: makes use of pdftex' letter spacing capabilities via the `microtype` package.[3] This fixes some serious issues regarding math formulæ etc. (e. g. , "ß") in headers.
  - `minionprospacing`: uses the internal `textssc` command of the `MinionPro` package for letter spacing. This automatically enables the `minionpro` option and overrides the `pdfspacing` option.

3 Use `microtype`'s `DVIoutput` option to generate DVI with pdftex.

- Table of Contents:
  - `tocaligned`: aligns the whole table of contents on the left side. Some people like that, some don't.
  - `dottedtoc`: sets pagenumbers flushed right in the table of contents.
  - `manychapters`: if you need more than nine chapters for your document, you might not be happy with the spacing between the chapter number and the chapter title in the Table of Contents. This option allows for additional space in this context. However, it does not look as "perfect" if you use `\parts` for structuring your document.
- Floats:
  - `listings`: loads the `listings` package (if not already done) and configures the List of Listings accordingly.
  - `floatperchapter`: activates numbering per chapter for all floats such as figures, tables, and listings (if used).
  - `subfig(ure)`: is passed to the `tocloft` package to enable compatibility with the `subfig(ure)` package. Use this option if you want use classicthesis with the `subfig` package.

The best way to figure these options out is to try the different possibilities and see, what you and your supervisor like best.

In order to make things easier in general, `classicthesis-config.tex` contains some useful commands that might help you.

1.3 CUSTOMIZATION

This section will give you some hints about how to adapt `classicthesis` to your needs.

The file `classicthesis.sty` contains the core functionality of the style and in most cases will be left intact, whereas the file `classicthesis-config.tex` is used for some common user customizations.

The first customization you are about to make is to alter the document title, author name, and other thesis details. In order to do this, replace the data in the following lines of `classicthesis-config.tex`:

Modifications in `classicthesis-config.tex`

```
1 \newcommand{\myTitle}{A Classic Thesis Style\xspace}
  \newcommand{\mySubtitle}{An Homage to ...\xspace}
  \newcommand{\myDegree}{Doktor-Ingenieur (Dr.-Ing.)\xspace}
```

Further customization can be made in `classicthesis-config.tex` by choosing the options to `classicthesis.sty` (see Section 1.2) in a line that looks like this:

```
\PassOptionsToPackage{eulerchapternumbers,listings,drafting,
    pdfspacing, subfig,beramono,eulermath,parts}{classicthesis}
```

If you want to use backreferences from your citations to the pages they were cited on, change the following line from:

```
\setboolean{enable-backrefs}{false}
```

to

```
\setboolean{enable-backrefs}{true}
```

Many other customizations in `classicthesis-config.tex` are possible, but you should be careful making changes there, since some changes could cause errors.

Modifications in `classicthesis.sty`

Finally, changes can be made in the file `classicthesis.sty`, although this is mostly not designed for user customization. The main change that might be made here is the text-block size, for example, to get longer lines of text.

1.4 ISSUES

This section will list some information about problems using `classicthesis` in general or using it with other packages.

Beta versions of `classicthesis` can be found at the following Google code repository:

http://code.google.com/p/classicthesis/

There, you can also post serious bugs and problems you encounter.

Compatibility with the `glossaries` Package

If you want to use the `glossaries` package, take care of loading it with the following options:

```
\usepackage[style=long,nolist]{glossaries}
```

Thanks to Sven Staehs for this information.

Compatibility with the (Spanish) `babel` Package

Spanish languages need an extra option in order to work with this template:

```
\usepackage[spanish,es-lcroman]{babel}
```

Thanks to an unknown person for this information (via Google Code issue reporting).

Compatibility with the `pdfsync` Package

Using the `pdfsync` package leads to linebreaking problems with the `graffito` command. Thanks to Henrik Schumacher for this information.

1.5 FUTURE WORK

So far, this is a quite stable version that served a couple of people well during their thesis time. However, some things are still not as they should be. Proper documentation in the standard format is still missing. In the long run, the style should probably be published separately, with the template bundle being only an application of the style. Alas, there is no time for that at the moment... it could be a nice task for a small group of LaTeXnicians.

Please do not send me email with questions concerning LaTeX or the template, as I do not have time for an answer. But if you have comments, suggestions, or improvements for the style or the template in general, do not hesitate to write them on that postcard of yours.

1.6 LICENSE

2 EXAMPLES

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2.1 A NEW SECTION

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Examples: *Italics*, ALL CAPS, Small Caps, low small caps[1].

2.1.1 *Test for a Subsection*

Note: The content of this chapter is just some dummy text.

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1 Footnote example.

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2.2 ANOTHER SECTION IN THIS CHAPTER

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2 De web nostre historia angloromanic.

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This statement requires citation **?** [**?**].

2.2.1 *Personas Initialmente*

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2.2.1.1 *A Subsubsection*

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A PARAGRAPH EXAMPLE Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium

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Table 2.1: Autem timeam deleniti usu id. **?**

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A. Enumeration with small caps

B. Second item

Another statement requiring citation **?** [**?**] but this time with text after the citation.

2.2.2 *Figure Citations*

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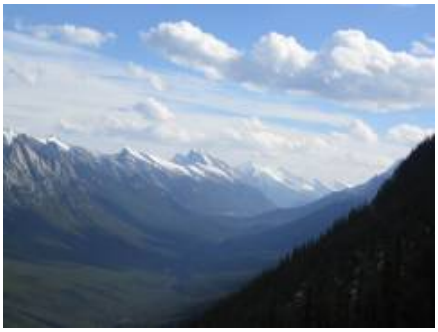
(a) Asia personas duo.

(b) Pan ma signo.



(c) Methodicamente o uno.

(d) Titulo debitas.

Figure 2.1: Tu duo titulo debitas latente.

3 MATH TEST CHAPTER

Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl.

3.1 SOME FORMULAS

Due to the statistical nature of ionisation energy loss, large fluctuations can occur in the amount of energy deposited by a particle traversing an absorber element[1]. Continuous processes such as multiple scattering and energy loss play a relevant role in the longitudinal and lateral development of electromagnetic and hadronic showers, and in the case of sampling calorimeters the measured resolution can be significantly affected by such fluctuations in their active layers. The description of ionisation fluctuations is characterised by the significance parameter κ, which is proportional to the ratio of mean energy loss to the maximum allowed energy transfer in a single collision with an atomic electron:

You might get unexpected results using math in chapter or section heads. Consider the `pdfspacing` option.

$$\kappa = \frac{\xi}{E_{max}} \tag{3.1}$$

E_{max} is the maximum transferable energy in a single collision with an atomic electron.

$$E_{max} = \frac{2m_e\beta^2\gamma^2}{1 + 2\gamma m_e/m_x + (m_e/m_x)^2},$$

where $\gamma = E/m_x$, E is energy and m_x the mass of the incident particle, $\beta^2 = 1 - 1/\gamma^2$ and m_e is the electron mass. ξ comes from the Rutherford scattering cross section and is defined as:

$$\xi = \frac{2\pi z^2 e^4 N_{Av} Z\rho\delta x}{m_e\beta^2 c^2 A} = 153.4\frac{z^2}{\beta^2}\frac{Z}{A}\rho\delta x \quad \text{keV},$$

where

1 Examples taken from Walter Schmidt's great gallery: http://home.vrweb.de/~was/mathfonts.html

z	charge of the incident particle
N_{Av}	Avogadro's number
Z	atomic number of the material
A	atomic weight of the material
ρ	density
δx	thickness of the material

κ measures the contribution of the collisions with energy transfer close to E_{max}. For a given absorber, κ tends towards large values if δx is large and/or if β is small. Likewise, κ tends towards zero if δx is small and/or if β approaches 1.

The value of κ distinguishes two regimes which occur in the description of ionisation fluctuations:

1. A large number of collisions involving the loss of all or most of the incident particle energy during the traversal of an absorber.

   As the total energy transfer is composed of a multitude of small energy losses, we can apply the central limit theorem and describe the fluctuations by a Gaussian distribution. This case is applicable to non-relativistic particles and is described by the inequality $\kappa > 10$ (i. e. , when the mean energy loss in the absorber is greater than the maximum energy transfer in a single collision).

2. Particles traversing thin counters and incident electrons under any conditions.

   The relevant inequalities and distributions are $0.01 < \kappa < 10$, Vavilov distribution, and $\kappa < 0.01$, Landau distribution.

3.2 VARIOUS MATHEMATICAL EXAMPLES

If $n > 2$, the identity

$$t[u_1, \ldots, u_n] = t\big[t[u_1, \ldots, u_{n_1}], t[u_2, \ldots, u_n]\big]$$

defines $t[u_1, \ldots, u_n]$ recursively, and it can be shown that the alternative definition

$$t[u_1, \ldots, u_n] = t\big[t[u_1, u_2], \ldots, t[u_{n-1}, u_n]\big]$$

gives the same result.

4 INTRODUCTION

I am going to need to introduce the different trapping potentials in here some where I think.

4.1 MAGNETIC TRAPPING

Ioffe Pritchard trap

$$\mathbf{B}_{IP}(x,y,z) = B_0 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + B' \begin{bmatrix} x \\ -y \\ 0 \end{bmatrix} + \frac{1}{2} B'' \begin{bmatrix} -xz \\ -yz \\ z^2 - \frac{1}{2}\left(x^2+y^2\right) \end{bmatrix} \tag{4.1}$$

the magnitude of this field can be approximated as the following for small position

$$|\mathbf{B}_{IP}| = \frac{1}{2}\left(B''_\rho\left(x^2+y^2\right) + B''z^2\right), \tag{4.2}$$

where

$$B''_\rho = \frac{B'^2}{B_0} - \frac{B''}{2}.$$

Quadrupole trap

$$\mathbf{B}_Q(x,y,z) = \frac{1}{2} B_z \begin{bmatrix} x \\ y \\ -2z \end{bmatrix} \tag{4.3}$$

Part II

DSMC

You can put some informational part preamble text here. Illo principalmente su nos. Non message occidental angloromanic da. Debitas effortio simplificate sia se, auxiliar summarios da que, se avantiate publicationes via. Pan in terra summarios, capital interlingua se que. Al via multo esser specimen, campo responder que da. Le usate medical addresses pro, europa origine sanctificate nos se.

5 SIMULATING COLLISIONS IN THERMAL GASES

Need to introduce the usefulness of the method here. References to many kinds of applications and such things. I also need to perform a literature review of all DSMC used in cold atoms physics.

Compare to molecular dynamic approaches, when is DSMC appropriate / good? When does it fail?

Find out the knudsen number for typical cold atom conditions. Wade has numbers for stamper kern and shvatchuck.

DSMC - Birds Book [1]

Evaporative Cooling and Expansion Dynamics: [2–4]

Bosonic Collective-Mode dynamics: [5–9], Can't find Jackson Zaremba 2002 Laser Physics, 12, 93

Fermion Dynamics: [10–13] (see also [14–17])

Sympathetic Cooling: [18, 19]

Applications - Rayleigh Bernard Flow: [20]

Spacecraft aerodynamics: [21]

Chemical reactions: [22] Goldsworthy?

Microfluidics: [23]

Acoustics on Earth, Mars and Titan: [24]

Volcanic plumes on Jupiter: [25]

Read: ~~[26]~~, [45]

Refer to cuda section 10 Should also discuss the development of this parallel implementation of the code. Compare to CPU implementations. Goldsworthy has a few references for other CUDA codes.

The heart of the DSMC method is to simplify the simulation of inter-particle interactions in the form of two body collisions. In this sense there are two aspects we need to ensure are modelled accurately; the number and frequency of collisions (collision rate) and the collisions themselves, wether or not they are correctly distributing the kinetic energy. In the following sections I carefully analyse these two aspects to ensure the utmost accuracy in our simulations.

5.1 COLLISION RATES IN THERMAL GASES

Overall collision rate, spatial collision rate, talk about number of cells, the occupancy of cells and the effect of inhomogeniety.

One of the most basic tests of the application of the DSMC method to cold atom physics is to investigate the collision rate for a thermal gas. Using the Boltzmann equation we can derive [27] the thermally averaged

Maybe say something about the Boltzmann equation here and give some references.

TRAPPING POTENTIAL	TRAP POWER	EFFECTIVE VOLUME, V_e	COLLISION RATE, ${\tau_c}^{-1}$
Homogeneous Gas	∞?	V	$\frac{1}{2^{1/2}} n_0 \bar{v} \sigma$
Spherical Quadrupole	1	$256\pi \left(\frac{k_B T}{g_s \mu_B B_z}\right)^3$	$\frac{1}{2^{7/2}} n_0 \bar{v} \sigma$
Ioffe Pritchard	2	$\frac{8}{\sqrt{B''} B_\rho''} \left(\frac{\pi k_B T}{g_s \mu_B}\right)^{3/2}$	$\frac{1}{2^2} n_0 \bar{v} \sigma$
Isotropic Power Law	$3/\gamma$	$\frac{4}{3}\pi {r_e}^3 \Gamma[\gamma+1] \left(\frac{k_B T}{U_0}\right)^\gamma$	$\frac{1}{2^{\gamma+0.5}} n_0 \bar{v} \sigma$

Table 5.1: Collision rates for different trapping potentials.

collision rate per unit density for a single species atomic gas bound by the potential $U(\mathbf{r})$,

$$ {\tau_c}^{-1} = \frac{1}{2} n_0 \langle v\sigma \rangle \frac{V_{2e}}{V_e}, \tag{5.1} $$

where $n_0 = N/V_e$ is the central density of the gas, $\langle v\sigma \rangle$ is the thermally averaged product of the atomic velocity and collision cross section, $V_e = \int \exp[-U(\mathbf{r})/kBT] \, d\mathbf{r}$ is the effective volume of the gas, and $V_{2e} = \int \exp[-2U(\mathbf{r})/kBT] \, d\mathbf{r}$ the effective volume corresponding to the distribution of pairs. For the bulk of this work we will consider collisions in three unique trapping potentials: no trapping potential, i. e. a homogeneous gas, an Ioffe Pritchard trap(cite) and a spherical quadrupole trap(cite - is it really spherical?)[1]. In table 5.1 we have derived the expressions for the effective volume and average collision rates for each of these traps. We have also included the results for a general isotropic power law trap, the potential of which is described by

The effective volume, V_e, of an inhomogeneous gas equals the volume of a homogeneous gas with the same number of atoms and density.

$$ U_{PL}(\mathbf{r}) = U_0 \left(\frac{\mathbf{r}}{r_e}\right)^{3/\gamma}, \tag{5.2} $$

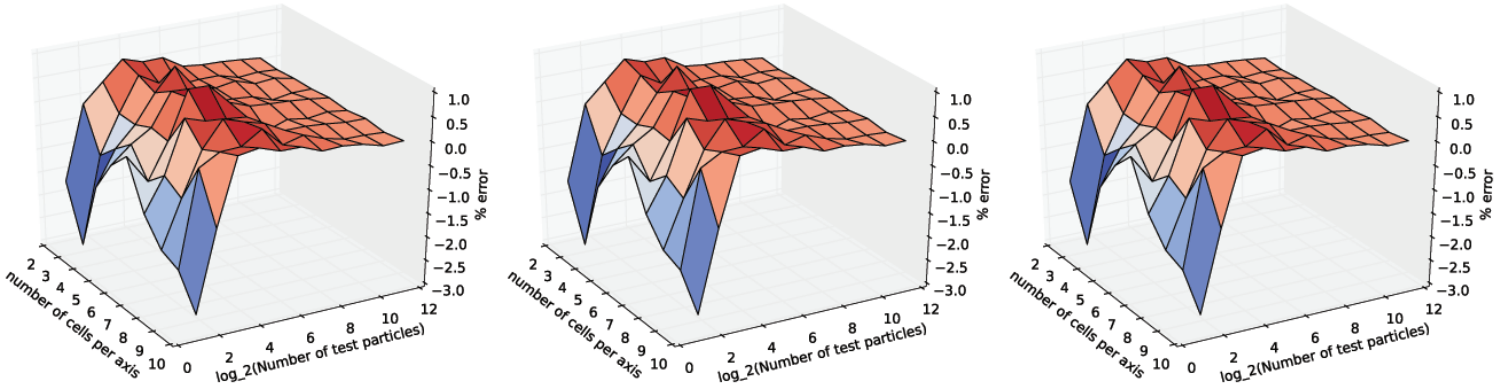
where the trap has a characteristic trap size r_e and a trap strength of U_0. The parameter, $\bar{v}$, in table 5.1 is the thermally averaged atomic speed and is given by

$$ \bar{v} = \sqrt{\frac{8 k_B T}{\pi m}}. $$

We can make a few interesting observations from these calculations. The most interesting is that the collision rate in the trapped gases increases as the temperature decreases, which is the converse to the homogeneous gas[2]. This is because for a trapped gas the central density increases at a rate greater than the decrease of the average thermal velocity, $\bar{v}$. For the homogeneous gas the density remains constant as the gas cools, thus the

1 We have a more in depth discussion of magnetic trapping in appendix B.

2 This relationship between collision rate and temperature for trapped gases is what gives rise to the "runaway" evaporation observed during atom cooling experiments.



(a) Homo err (b) IP err (c) Quad err

Figure 5.1: Error of DSMC method as a function of test particle number and cell number.

decrease in the velocity of the atoms results in an overall decrease in the collision rate. We can also see that if we can hold the trap strength and effective trap size constant the central density of the trap will increase as the power of the trap decrease (or as γ increases). This is often spoken about in terms of "tightness" and is a strong motivator behind the use of the quadrupole trap. In section 5.4 we will show that it is the tightness of a trap that determines it's efficiency in evaporative cooling.

5.2 DSMC SIMULATIONS OF COLLISIONS

The DSMC method offers some free parameters which we can optimise to balance the accuracy and efficiency of the algorithm, namely the number of cells, n_c, and the number of test particles, N_p. We wish to see the effect of varying these parameters on the results of the simulation.

**Include a surface plot of the collision rate as a function of cell number and test particle number. Should make the z axis percentage error. See fig 14. in wade. **

Figure 5.5a shows how well the DSMC method performs over a wide range of test particle and cell numbers in a homogeneous gas. We can see in the corner the method beginning to fail. This is the region where, on average, we have less than two test particles per cell. This means that there is not enough atoms to perform a collision within a cell. One way to avoid this (which has not been implemented here) is to search neighbouring cells for collision pairs when a partner can not be found in the current cell. The main thing to observe here is the increase in the error as the number of test particles is reduced.

Smaller cells -> larger fluctuations.

Not adaptive since slowly changing cloud.

5.3 THERMALISATION

Once we are convinced that we have the correct number of collisions we must confirm that they are performing as they should. The perfect way to test this is through the investigation of thermal relaxation or the process of thermalisation through elastic collisions. There are many ways that a system can be out of equilibrium, but here we will investigate three scenarios from the literature that have a concrete theoretical backing to which we can compare our simulations.

*Thermalisation is the generic name for all kinds of processes giving rise to relaxation towards thermal equilibrium starting from a non-equilibrium situation. **REWORD, NOT MY WORDS***

5.3.1 *Walraven Thermalisation*

The simplest example for rethermalisation is somewhat reminiscent of evaporative cooling, that is the case is simply perturbing the equilibrium by adding a small number of atoms whose average energy is different to that of the bulk gas. In [27] (the analysis of which we step through in detail in appendix A.2.1) we are shown that the rethermalisation time, $\tau_{th}{}^{-1}$, is given by

$$\tau_{th}^{-1} = \frac{1}{2\,(\gamma + 3/2)}\tau_c^{-1}. \tag{5.3}$$

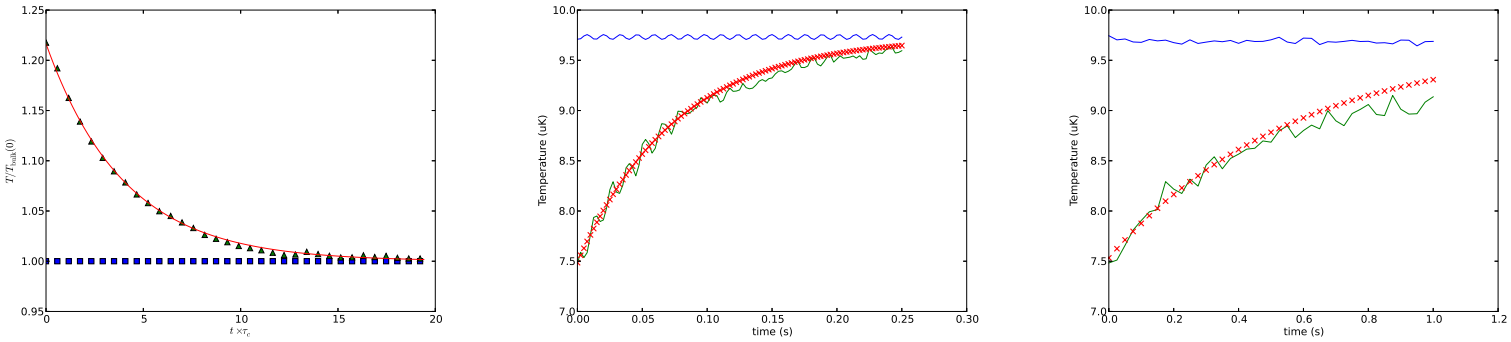
If we use the values for the trapping parameter, γ, given in table 5.1 we would expect the thermalisation time in a homogeneous, IP and quadrupole trap to be 3, 6 and 9 times the collision time in each trap respectively. Anderlini and Guéry-Odelin [28] have taken this analysis one step further, considering the effect including all partial waves in the collision integrals has on a homogeneous box trap and a harmonic potential. Interesingly in the limit of constant cross section the results for the homogeneous trap reduces to that given in equation (5.3) with $\gamma = 0$. In fact they go on to show that for the harmonic trap the thermalisation time is longer by a factor of 2, which again, agrees with equation (5.3) with $\gamma = 3/2$.

Before we consider these simulations numerically the reader might be a little confused by the above result. Looking at equation 5.3 it is quite clear that as the trapping parameter, γ, increases the number of collision required for thermalisation increases. This might be a little counter intuitive. We just stated in section 5.1 that evaporative cooling, a process driven by rethermalisation is more efficient in tighter trapping potentials i. e. larger trapping parameters. Aderlini et al. explain this best when they say

> ... the fact that the space of configuration is larger for a non-homogeneous gas, and that the thermalization affects both the space and velocity degrees of freedom.

We can understand this better if we consider an effective volume in configuration space, V_{cs}, (analogous to the effective volume in real space, V_e)

$$V_{cs} = \int \exp\left[-\frac{H(\mathbf{r},\mathbf{v})}{k_B T}\right] d\mathbf{r}d\mathbf{v} = V_e \left(\frac{3\pi m}{k_B T}\right)^{3/2}. \tag{5.4}$$



(a) Walraven homegeneous gas thermalisation. $\tau_c^{-1}/\tau^{-1} = 1.2$ should equal 3?

(b) Walraven ip gas thermalisation. $\tau_c^{-1}/\tau^{-1} = 1.6$ should equal 6?

(c) Walraven quad gas thermalisation. $\tau_c^{-1}/\tau^{-1} = 10.45$ should equal 9?

Figure 5.2: Walraven rethermalisation. I want to do this simulation for some different perturbation temperatures. One more higher and one lower too.

So we can see that for a given temperature the region occupied in configuration space increases with the effective volume, which table 5.1 shows increases with γ. Again this might make the reader uncomfortable, we seem to have convincigly shown that tighter trapping potentials take longer to thermalise. However, even though the number of collisions required to thermalise the gas increases with the trapping parameter, γ, the average time between collisions, τ_c, decreases. Thus the absolute time required for a gas to thermalise is lower for tighter trapping potentials i. e. larger γ.

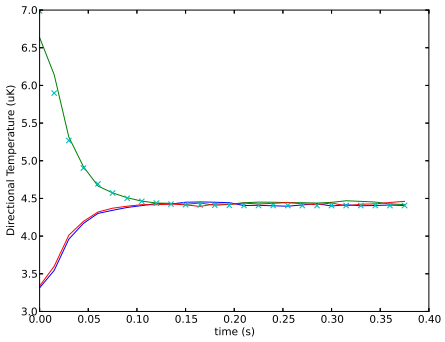
In figure 5.2 we have simulated a million physical atoms, $N_P = 10^6$, at 10 μK in three different trapping potentials: homogeneous, IP and quadrupole. Each gas began in an initially thermal distribution with 10% of the particles at set to have an average temperature of 12.5 μK. For each simulation we have chosen the trapping parameters such that the collision rate was approximately 25 collisions per atom per second, as this number is similar to atoms of this temperature in an evaporation experiment. We used one million test particle, $N_T = 10^6$, in each simulation so that the ratio of physical atoms to test particles was one, $\alpha = 1$.

When choosing the initial perturbed distribution we must keep in mind the virial theorem []. The key result here is that in a thermal distribution we have $\langle E_p \rangle = \frac{2\gamma}{3}\langle E_k \rangle = \gamma k_B T$. So when choosing the width for the spatial distribution we must keep in mind the effective power of the trapping potential.

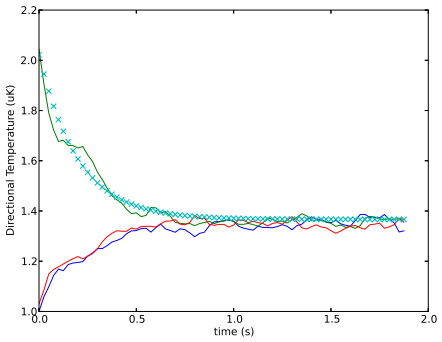
For the homogeneous gas simulation shown in figure 5.2a I set the width of the box containing the atoms to be 100 μm in each direction. This corresponds to a collision rate of 24.64 s^{-1}. I found the thermalisation occurred in 3.14 ± 0.02 collision times, closely resembling the result in equation (5.3).

For the Ioffe Pritchard trap simulation shown in figure 5.2b we set $B_0 = 0.01$ T, $B' = 33.54\,Tm^{-1}$ and $B'' = 75,000\,Tm^{-2}$ which equates to a ${B_\rho}'' = 75,000\,Tm^{-2}$. Choosing these trapping parameters gives a collision rate of 24.72 s. I found the thermalisation occurred in 6 ± 0.1 collision times, again, in perfect agreement with equation (5.3).

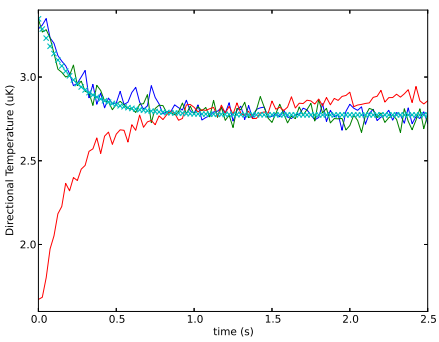
Finally in the quadrupole trap simulation shown in figure 5.2c we set $B_z = 2.8\,Tm^{-1}$ resulting in a collision rate of 25.48 s. I found the thermalisation occurred in 9 ± 0.1 collision times, again, in perfect agreement with equation (5.3).



(a) Monroe homegeneous gas thermalisation. $\tau_c^{-1}/\tau^{-1} = 0.77$



(b) Walraven homegeneous gas thermalisation. $\tau_c^{-1}/\tau^{-1} = 2.21$



(c) Walraven quad gas thermalisati $\tau_c^{-1}/\tau^{-1} = 6.09$

Figure 5.3: Walraven rethermalisation. I want to do this simulation for some different perturbation temperatures. One more higher and one lower too.

We can note from all simulations shown in figure 5.2 that the thermalisation time appears to be independent of the size of the perturbation.

5.3.2 *Monroe Thermalisation*

Another interesting experiment that has been investigated in some depth is the rethermalisation of a directional anisotropy. In these experiments the kinetic energy is changed in one cartesian direction only (the spatial distribution is reshaped accordingly), creating a directional anisotropy in the distribution. This squeezed distribution is then allowed to rethermalise through elastic collisions. The original theoretical development of Myatt [29] predicts that in an harmonic trap the thermalisation time for a directional anisotropy is approximately 2.7. This result has been used to experimentally determine the collision cross section, σ, of atoms in ultra-cold gas experiments [30, 31]. This simulation has also been repeated by Wu and Foot [2] using the DSMC method. Here I have extended this investigation to consider the thermalisation times in our three favourite trapping potentials: homogeneous, Ioffe Pritchard and quadrupole. As we saw in section 5.3.1 we can't expect the thermalisation time to be the same in different trapping potentials. In fact, if the perturbation is kept the same, it was purely a function of the trapping parameter, γ.

Also do it for different temperatures or trap numbers.

*** MUST REDO ALL OF THESE SIMULATIONS WITH COLLISIONS WORKING CORRECTLY (I.E. WITH THE NEW SORTING FIX IMPLEMENTED). ***

Comment on directional temperatures being a useful tool for checking that collisions are working effectively.

5.4 EVAPORATION

Compare some results to those predicted by the theory of walraven [27] and the other guy [32] .

As demonstrated by Wu et al [2, 3] we can use the DSMC method to simulate evaporative cooling. There has been a lot of theoretical investigation into the evolution of cold gases under forced evaporative cooling [31–33], giving us a good basis for analysis. We will use the results of Luiten et al [32] to validate to our simulations. In their work Luiten et al find that the rate of change of total energy due to evaporation is given by

$$\dot{E} = \left(\eta + \frac{W_{ev}}{V_{ev}}\right) \dot{N} k_B T,$$

where the effective volume for elastic collisions leading to evaporation, V_{ev} is given by

$$V_{ev} = \frac{\Lambda}{k_B T} \int_0^{\epsilon_t} \rho(\epsilon) \left[(\epsilon_t - \epsilon - k_B T)\, e^{-\epsilon/k_B T} + k_B T e^{-\eta} \right] d\epsilon$$

and the volume $W_{ev} = V_{ev} - X_{ev}$, with

$$X_{ev} = \frac{\Lambda}{k_B T} \int_0^{\epsilon_t} \rho(\epsilon) \left[k_B T e^{-\epsilon/k_B T} - (\epsilon_t - \epsilon + k_B T)\, e^{-\eta} \right] d\epsilon.$$

If we use the expression for an isotropic power law potential (5.2) from section 5.1 we can find a (rather complicated) expression for this ratio of volumes

$$\frac{X_{ev}}{V_{ev}} = \frac{2\left(2(2\gamma+2\eta+5)\eta^{\gamma+\frac{3}{2}} + e^{\eta}\left((2\gamma+3)(2\gamma+5)\Gamma\left(\gamma+\frac{3}{2},\eta\right) - 4\Gamma\left(\gamma+\frac{7}{2}\right)\right)\right)}{e^{\eta}(-2\gamma+2\eta-5)\left((2\gamma+3)(2\gamma+5)\Gamma\left(\gamma+\frac{3}{2},\eta\right) - 4\Gamma\left(\gamma+\frac{7}{2}\right)\right) - 2(2\gamma+5)^2\eta^{\gamma+\frac{3}{2}}},$$

where $\Gamma[a,z] = \int_z^\infty t^{a-1}e^{-t}\,dt$ is the incomplete gamma function and $\Gamma[a] = \int_0^\infty t^{a-1}e^{-t}\,dt$ is the Euler gamma function. It might be immediately obvious from the equation above, but for a given γ this ratio has a maximum of 1 at $\eta = 0$ and is a monotonically decreasing function of η. I have included this to illustrate a rather unintuitive result. That is that the rate of change of the total energy for an evaporatively cooled gas is not only a function of the evaporation parameter η, but also a function of the trapping parameter γ. Perhaps if we reflect on the thermalisation experiments we have done in the previous sections 5.3.1 and **??** it is not so surprising that this is the case.

If we now differentiate the equation for the total energy of the gas, $E = \left(\frac{3}{2}+\gamma\right) N k_B T$, with respect to time and combine it with equation (**??**) we can show

$$\frac{\dot{T}}{T} = \left(\frac{\eta + \frac{W_{ev}}{V_{ev}}}{\frac{3}{2}+\gamma} - 1\right) \frac{\dot{N}}{N}. \tag{5.5}$$

Keeping in mind the maximal value for W_{ev}/V_{ev} is 1 we can see from the above that we require $\eta > \gamma + 1/2$ for the temperature to decrease as the

number of atoms decreases. Further more we can note the larger η is the more efficient the evaporative cooling will be i. e. a smaller loss of atoms will result in a larger decrease in temperature.

We can also investigate how the density of the gas changes with the number of atoms. Recall in section 5.1 we claimed the density would increase as we removed atoms (how can this be?!). Using the relationship $N = n_0 V_e$ and the subsequent derivative $\dot{N} = \dot{n}_0 V_e + n_0 \dot{V}_e$, and combining this with the results from table 5.1 and equation (5.5) we have

$$\frac{\dot{n}_0}{n_0} = \left(1 - \gamma\left(\frac{\eta + \frac{W_{ev}}{V_{ev}}}{\frac{3}{2} + \gamma} - 1\right)\right)\frac{\dot{N}}{N}. \tag{5.6}$$

Thus we see that for the density of the gas to increase as the number of atoms decreases we require that $\eta > \gamma + 5/2 + 3/2\gamma$. It is clear from equation (5.6) that the larger γ is the greater increase in density we will have for a given loss of atoms.

Finally we can see how the degeneracy parameter, $D = n_0\Lambda^3$, changes with the loss of atoms

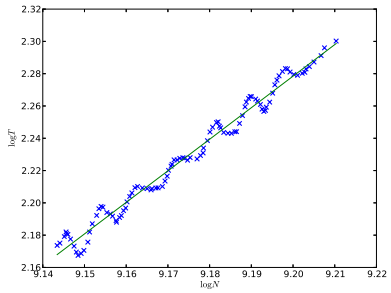
$$\frac{\dot{D}}{D} = \left(\gamma - \eta - \frac{W_{ev}}{V_{ev}} + \frac{5}{2}\right)\frac{\dot{N}}{N}. \tag{5.7}$$

So for $\eta > \gamma + 3/2$ we will have an increase in the degeneracy parameter as the number of atoms decreases. Here is the definitive result that drives the desire to evaporate in a quadrupole potential. We can see that the larger γ is the greater the increase in the degeneracy parameter will be for a given η. Thus if we can use a quadrupole trap, with $\gamma = 3$, we will reach the quantum limit with the minimum atom loss.

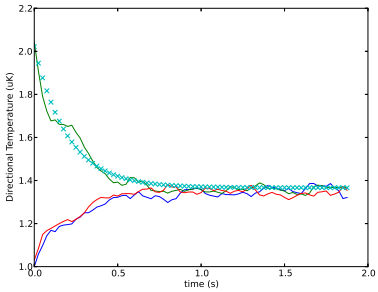
5.5 DSMC SIMULATIONS OF EVAPORATION

5.6 ADIABATICITY

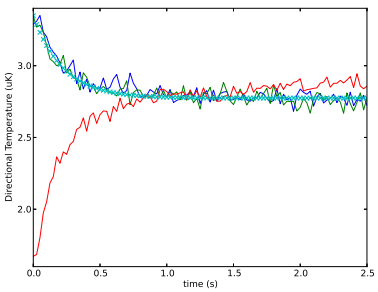
Have a look at squeezing the magnetic trap both diabaticaly and adiabatically.



(a) η should equal 7, here it is equal to 7.86

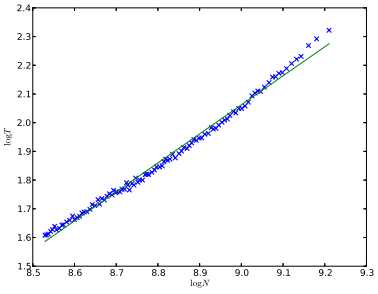


(b) I want this to be a plot of no vs N.

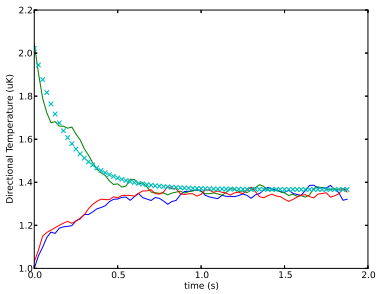


(c) Not sure what plot tp put here

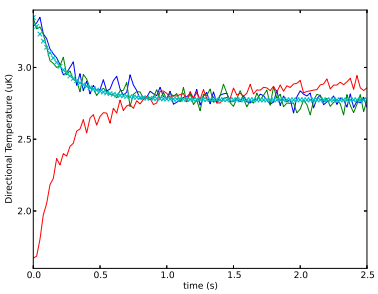
Figure 5.4: IP trap evaporation.



(a) η should equal ?(check the paper), here it is equal to 5



(b) I want this to be a plot of no vs N.



(c) Not sure what plot tp put here

Figure 5.5: Quadrupole trap evaporation.

6 MAJORANA INTERULDE

6.1 MAJORNAN SPIN FLIPS

Talk about Majorana problem, history, derive formula etc

6.2 LANDAU ZENER FORMULA

Content

7 DSMC WITH SPIN - EHRENFEST

7.1 SCHRÖDINGER SIMULATIONS

Content

7.1.1 *Simulating Schrödinger Equation*

Content

7.1.2 *Single Atom Spin Flips*

Content

7.2 FULL GAS SIMULATIONS

Content

7.2.1 *Ioffe Pritchard Trap*

Content

7.2.2 *Quadrupole Trap*

Content

8

DSMC WITH SPIN - MCWF

8.1 SCHRÖDINGER SIMULATIONS

Content

8.1.1 *Simulating Schrödinger Equation*

Content

8.1.2 *Single Atom Spin Flips*

Content

8.2 FULL GAS SIMULATIONS

Content

8.2.1 *Ioffe Pritchard Trap*

Content

8.2.2 *Quadrupole Trap*

Content

Part III

FEMDVR

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9 FEMDVR

9.1 1D FEMDVR

Content

9.1.1 *Scaling Comparison to CPU*

Content

9.1.2 *Real Simulation*

Content

9.2 3D FEMDVR

Content

9.2.1 *Knots*

Content

Part IV

CUDA

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10 CUDA DSMC

10.1 CUDA

Content

10.1.1 *Parallelisation*

Content

10.2 SPEED UP

Content

10.2.1 *Some simulations*

Content

Part V

APPENDIX

A

THERMAL PHYSICS

A.1 COLLISION RATES IN THERMAL GASES

A.2 THERMALISATION

A.2.1 *Walraven Thermalisation*

B

MAGNETIC TRAPPING

C

MOTION INTEGRATION

In this appendix we will explain in detail the different methods of integrating the Newtonian motion of particles and specifically how we have gone about it.

C.1 EULER METHOD

$$x_{n+1} = x_n + v_n \Delta t, \tag{C.1}$$
$$v_{n+1} = v_n + a_n \Delta t. \tag{C.2}$$

C.2 SEMI-IMPLICIT EULER METHOD

$$x_{n+1} = x_n + v_n \Delta t, \tag{C.3}$$
$$v_{n+1} = v_n + a_{n+1} \Delta t. \tag{C.4}$$

C.3 VERLET ALGORITHM

Sometimes referred to as the Störmer-Verlet method (see wikipedia page it's pretty good), it was made popular by Verlet in 1976 [34]. The Verlet algorithm can be derived from the Taylor series expansions for position as follows,

$$\mathbf{r}(t_{n+1}) = \mathbf{r}(t_n) + k\mathbf{v}(t_n) + \frac{1}{2}k^2\mathbf{a}(t_n) + \mathcal{O}(k^3), \tag{C.5a}$$
$$\mathbf{r}(t_{n-1}) = \mathbf{r}(t_n) - k\mathbf{v}(t_n) + \frac{1}{2}k^2\mathbf{a}(t_n) + \mathcal{O}(k^3), \tag{C.5b}$$

we can now add equations (C.5a) and (C.5b) together to get

$$\begin{aligned} \mathbf{r}(t_{n+1}) + \mathbf{r}(t_{n-1}) &= 2\mathbf{r}(t_n) + k^2\mathbf{a}(t_n) + \mathcal{O}(k^4), \\ \Rightarrow \mathbf{r}(t_{n+1}) &= 2\mathbf{r}(t_n) - \mathbf{r}(t_{n-1}) + k^2\mathbf{a}(t_n) + \mathcal{O}(k^4), . \end{aligned} \tag{C.6}$$

We can see from equation (C.6) that the leap frog method is fourth order in time. However, we can also note that the velocities do not explicitly appear in the method, this means we need to derive them from positions. A simple approximation would be to use the midpoint method

$$\mathbf{v}(t_n) = \frac{1}{2}k\left(\mathbf{r}(t_{n+1}) - \mathbf{r}(t_{n-1})\right) \tag{C.7}$$

C.4 LEAP FROG METHOD

[35]

$$x_n = x_{n-1} + v_{i-1/2}\Delta t, \tag{C.8}$$
$$v_{n+1/2} = v_{n-1/2} + a_n \Delta t. \tag{C.9}$$

C.5 VELOCITY VERLET

[36]

$$x_{n+1} = x_n + v_i \Delta t + \frac{1}{2} a_n \Delta t^2, \tag{C.10}$$
$$v_{n+1} = v_n + \frac{1}{2} (a_n + a_{n+1}) \Delta t. \tag{C.11}$$

C.6 BEEMAN'S ALGORITHM

[37]

$$x_{n+1} = x_n + v_n \Delta t + \frac{1}{6} (4a_n - a_{n-1}) \Delta t^2, \tag{C.12}$$
$$v_{n+1} = v_n + \frac{1}{6} (2a_{n+1} + 5a_n - a_{n-1}) \Delta t. \tag{C.13}$$

D

DIRECT SIMULATION MONTE CARLO

Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl.

Etiam ac leo a risus tristique nonummy. Donec dignissim tincidunt nulla. Vestibulum rhoncus molestie odio. Sed lobortis, justo et pretium lobortis, mauris turpis condimentum augue, nec ultricies nibh arcu pretium enim. Nunc purus neque, placerat id, imperdiet sed, pellentesque nec, nisl. Vestibulum imperdiet neque non sem accumsan laoreet. In hac habitasse platea dictumst. Etiam condimentum facilisis libero. Suspendisse in elit quis nisl aliquam dapibus. Pellentesque auctor sapien. Sed egestas sapien nec lectus. Pellentesque vel dui vel neque bibendum viverra. Aliquam porttitor nisl nec pede. Proin mattis libero vel turpis. Donec rutrum mauris et libero. Proin euismod porta felis. Nam lobortis, metus quis elementum commodo, nunc lectus elementum mauris, eget vulputate ligula tellus eu neque. Vivamus eu dolor.

D.1 APPENDIX SECTION TEST

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Table D.1: Autem usu id.

Listing D.1: A floating example

```
for i:=maxint to 0 do
begin
{ do nothing }
end;
```

D.2 ANOTHER APPENDIX SECTION TEST

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E

NON-DIMENSIONALISATION

E.1 QUASI - 1D GPE

Let's just start with writing out the full quasi-one-dimensional three component equation in a harmonic potential

$$\imath\hbar\frac{\partial}{\partial t}f_+ = \Big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \frac{1}{2}m{\omega_z}^2 z^2 + E_+ + E_\perp + c_0 N\eta\rho + c_2 N\eta\left(\rho_+ + \rho_0 - \rho_-\right)\Big) f_+ + c_2 N\eta {f_0}^2 {f_-}^*, \tag{E.1a}$$

$$\imath\hbar\frac{\partial}{\partial t}f_0 = \Big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \frac{1}{2}m{\omega_z}^2 z^2 + E_0 + E_\perp + c_0 N\eta\rho + c_2 N\eta\left(\rho_+ + \rho_-\right)\Big) f_0 + 2c_2 N\eta f_+ f_- {f_0}^*, \tag{E.1b}$$

$$\imath\hbar\frac{\partial}{\partial t}f_- = \Big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \frac{1}{2}m{\omega_z}^2 z^2 + E_- + E_\perp + c_0 N\eta\rho + c_2 N\eta\left(\rho_- + \rho_0 - \rho_+\right)\Big) f_- + c_2 N\eta {f_0}^2 {f_+}^*, \tag{E.1c}$$

$$\rho\frac{\partial E_\perp}{\partial\chi} + \left(\frac{c_0 N}{2}\rho^2 + \frac{c_2 N}{2}S_2\right)\frac{\partial\eta}{\partial\chi} = 0. \tag{E.1d}$$

If we make the Thomas Fermi ansatz then the transverse mode energy and the scaling factor are given by

$$E_\perp = \frac{\hbar\omega_\perp}{6}\frac{\chi^2}{{a_\perp}^2}, \tag{E.2}$$

$$\eta = \frac{4}{3\pi\chi^2}, \tag{E.3}$$

where $a_\perp = \sqrt{\hbar/m\omega_\perp}$. If we substitute (E.2) and (E.3) into (E.1d) then we find

$$\chi = \left(\frac{4c_0 N\rho^2 + c_2 N S_2}{m\pi\rho{\omega_\perp}^2}\right)^{\frac{1}{4}}. \tag{E.4}$$

Now substituting (E.4) back into (E.2) and (E.3) and simplifying we have

$$E_\perp = \sqrt{\frac{mN{\omega_\perp}^2\left(c_0\rho^2 + c_2 S_2\right)}{9\pi\rho}}, \tag{E.5}$$

$$\eta = \frac{2}{3}\sqrt{\frac{m\rho{\omega_\perp}^2}{\pi N\left(c_0\rho^2 + c_2 S_2\right)}}. \tag{E.6}$$

Now we can begin to non-dimensionalise the equations. Let us only consider the non-dimensionalisation of the positive component since the procedure will be exactly the same for all components. We begin by making the substitutions

$$\begin{aligned} t &\to t_c \tau, \\ z &\to z_c \zeta, \\ f_+ &\to f_c u_+. \end{aligned}$$

Equation (E.1a) now becomes

$$\begin{aligned} \imath\hbar \frac{f_c}{t_c}\frac{\partial}{\partial\tau}u_+ = \Bigg(&-\frac{\hbar^2}{2m{z_c}^2}\frac{\partial^2}{\partial\zeta^2} + \frac{1}{2}m\omega_z{}^2{z_c}^2\zeta^2 + f_c E_+ + f_c E_\perp + c_0 f_c N\eta\rho \\ &+ c_2 f_c N\eta\left(\rho_+ + \rho_0 - \rho_-\right)\Bigg) f_c u_+ + c_2 {f_c}^2 N\eta {u_0}^2 {u_-}^*, \\ \imath\frac{m{z_c}^2}{\hbar t_c}\frac{\partial}{\partial\tau}u_+ = \Bigg(&-\frac{1}{2}\frac{\partial^2}{\partial\zeta^2} + \frac{1}{2}\frac{m^2\omega_z{}^2{z_c}^4}{\hbar^2}\zeta^2 + \frac{m{z_c}^2}{\hbar^2} f_c \left[E_+ + E_\perp + c_0 N\eta\rho \right. \\ &\left. + c_2 N\eta\left(\rho_+ + \rho_0 - \rho_-\right)\right]\Bigg) u_+ + \frac{m{z_c}^2}{\hbar^2} c_2 f_c N\eta {u_0}^2 {u_-}^*. \end{aligned}$$

Looking at the coefficient of the ζ^2 term we can see that if we set it to $1/2$ we will be able to solve for z_c,

$$\begin{aligned} 1 &= \frac{m^2 w_z{}^2 {z_c}^4}{\hbar^2}, \\ \Rightarrow z_c &= \sqrt{\frac{\hbar}{m\omega_z}}, \end{aligned} \tag{E.7}$$

which is the harmonic oscillator length along the z axis, a natural length scale for the z dimension. Now we can turn our attention to the coefficient of the time derivative, and set it to $\imath$,

$$\begin{aligned} 1 &= \frac{m{z_c}^2}{\hbar t_c}, \\ &= \frac{m\hbar}{m\hbar\omega_z t_c}, \\ \Rightarrow t_c &= \frac{1}{w_z}, \end{aligned} \tag{E.8}$$

which is the angular period of the oscillator, again a natural length scale. Finally we can consider the energy terms. With these we need to choose f_c such that the dimension of the energy term is one. To cut a long story short, this makes a suitable choice for f_c to be $1/\sqrt{z_c}$. Giving us the final form of the non-dimensionalised equation

$$\begin{aligned} \imath\frac{\partial}{\partial\tau}u_+ = \Bigg(&-\frac{1}{2}\frac{\partial^2}{\partial\zeta^2} + \frac{1}{2}\zeta^2 + \frac{m}{\hbar^2}\left(\frac{\hbar}{m\omega_z}\right)^{\frac{3}{4}}\left[E_+ + E_\perp + c_0 N\eta\rho \right. \\ &\left. + c_2 N\eta\left(\rho_+ + \rho_0 - \rho_-\right)\right]\Bigg) u_+ + \frac{m}{\hbar^2}\left(\frac{\hbar}{m\omega_z}\right)^{\frac{3}{4}} c_2 N\eta {u_0}^2 {u_-}^*. \end{aligned} \tag{E.9}$$

E.2 MAJORANA PROBLEM SPIN HALF

The potential energy operator [**?**] for a magnetic dipole in a field is given by

$$\hat{V} = -\hat{\mu} \cdot \mathbf{B}, \tag{E.10}$$

where $\hat{\mu}$ is the magnetic dipole operator and $\mathbf{B}$ is the magnetic field. Which for a spin half particle is

$$\hat{V} = \frac{1}{2}\mu_B g_s \begin{bmatrix} B_z & B_x - \imath B_y \\ B_x + \imath B_y & -B_z \end{bmatrix},$$

where μ_B is the Bohr magneton [**?**] and g_s is the Landé g-factor of the spin-$\frac{1}{2}$ particle. Now we can write the time dependant Schrödinger equation for our system (need to introduce kinetic energy operator as well)

$$\imath\hbar\partial_t\psi_\uparrow = -\frac{\hbar^2}{2m}\partial_{zz}\psi_\uparrow + \frac{1}{2}\mu_B g_s B_z\psi_\uparrow + \frac{1}{2}\mu_B g_s \left(B_x - \imath By\right)\psi_\downarrow, \tag{E.11}$$

$$\imath\hbar\partial_t\psi_\downarrow = -\frac{\hbar^2}{2m}\partial_{zz}\psi_\downarrow - \frac{1}{2}\mu_B g_s B_z\psi_\downarrow + \frac{1}{2}\mu_B g_s \left(B_x + \imath By\right)\psi_\uparrow. \tag{E.12}$$

Maybe before we non-dimensionalise we will insert or actual values for the magnetic field, $\mathbf{B} = (B_x, 0, -dB_z z)$. Now to non-dimensionalise. We make the substitutions

$$\begin{aligned} z &\to z_c\zeta, \\ t &\to t_c\tau, \\ \psi_i &\to \psi_c\phi. \end{aligned}$$

From here we will only consider the equation for the spin up component as the two will have the same non-dimensionalisation. After making these substitutions the above equation becomes

$$\imath\hbar\frac{\psi_c}{t_c}\partial_\tau\phi_\uparrow = -\frac{\hbar^2}{2m}\frac{\psi_c}{{z_c}^2}\partial_{\zeta\zeta}\phi_\uparrow - \frac{1}{2}\mu_B g_s dB_z z_c\zeta\psi_c\phi_\uparrow + \frac{1}{2}\mu_B g_s\psi_c B_x\phi_\downarrow,$$

rearranging so that the coefficient of the highest derivative is dimensionless

$$\imath\frac{m}{\hbar}\frac{{z_c}^2}{t_c}\partial_\tau\phi_\uparrow = -\frac{1}{2}\partial_{\zeta\zeta}\phi_\uparrow - \frac{1}{2}\frac{\mu_B g_s m dB_z {z_c}^3}{\hbar^2}\zeta\phi_\uparrow + \frac{1}{2}\frac{\mu_B g_s m {z_c}^2}{\hbar^2}B_x\phi_\downarrow.$$

From here we can see

$$z_c = \frac{B_x}{dB_z}, \tag{E.13}$$

$$t_c = \frac{\hbar}{B_x g_s \mu_B}, \text{1 / Larmor frequency around Bx} \tag{E.14}$$

$$\psi_c = \frac{dB_z{}^2\hbar^2}{B_x{}^3 g_s m\mu_B}. \tag{E.15}$$

Leaving us with the non-dimensionalised equation

$$\partial_\tau\phi_\uparrow = \frac{\imath}{2}\partial_{\zeta\zeta}\phi_\uparrow + \frac{\imath}{2}\zeta\phi_\uparrow - \frac{\imath}{2}\phi_\downarrow.$$

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Table E.1: Autem usu id.

Listing E.1: A floating example

```
for i:=maxint to 0 do
begin
{ do nothing }
end;
```

E.3 ANOTHER APPENDIX SECTION TEST

Curabitur tellus magna, porttitor a, commodo a, commodo in, tortor. Donec interdum. Praesent scelerisque. Maecenas posuere sodales odio. Vivamus metus lacus, varius quis, imperdiet quis, rhoncus a, turpis. Etiam ligula arcu, elementum a, venenatis quis, sollicitudin sed, metus. Donec nunc pede, tincidunt in, venenatis vitae, faucibus vel, nibh. Pellentesque wisi. Nullam malesuada. Morbi ut tellus ut pede tincidunt porta. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam congue neque id dolor.

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F

FINITE ELEMENT METHOD

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Clayton, Victoria, December 2013

Christopher Jon Watkins,
November 25, 2014