Group Decision and Negotiation (2023) 32:807–833
https://doi.org/10.1007/s10726-023-09825-1

Uncertainty Analysis in Group Decisions through Interval Ordinal Priority Approach

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Accepted: 8 March 2023 / Published online: 17 April 2023

Abstract
In multiple criteria decision-making (MCDM) problems, ranking alternatives is usually the end. However, in real life, it is rarely an end in itself and serves as a means to an end. Further, in real-life problems, the selection of alternatives is usually made with the aid of unique experts, and thus minimizing the influence of the uncertainty of their subjective judgements on the optimality of the decisions is a critical issue. The current study proposes a novel Interval Ordinal Priority Approach to objectively solve these and other issues by allowing uncertainty analysis and quantification. The study argues that when the model's input contains uncertainty (even if represented by crisp numbers), expecting the output to be free from uncertainty is an unrealistic conjecture. Therefore, unlike the conventional MCDM models producing crisp weights, the proposed approach yields interval weights with the length of the interval representing the uncertainty (inconsistency among the experts' judgements). Also, instead of resorting to the subjective measurement of thresholds to qualify or disqualify a set of inputs based on the degree of uncertainty, a novel objective measure of threshold is put forward. The validity of the proposed method is demonstrated through illustrative examples and comparative analysis. Later, the study is concluded with the implications for real-world decision-making.

Keywords Group decisions · Multiple criteria decision analysis · Ordinal priority approach · Uncertainty analysis · Uncertainty quantification

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1 Introduction

In physics and engineering, uncertainty in observations is usually represented by tolerance level (Taguchi and Tsai 1993), and in statistics and probability, confidence limits/intervals are used (Gnedenko and Ushakov 1997; Wentzel 1986). Both measures aim to manifest uncertainty in observations and give us confidence that the values of interest lie within specific intervals. However, their use is uncommon in multiple criteria [group] decision-making (MCDM) problems. Since, in general, repeated measurements are inconvenient and unrealistic, historical data are unavailable (as most MCDM problems are unique), and objective and precise measurements (as one may obtain from machines and scientific equipment) are next to impossible, tolerance levels are of less use in the MCDM problems. Also, confidence intervals, which are widely used in social science research, require an assumption that the sample follows a distribution (usually, normal distribution). In contrast, in MCDM problems, such assumptions' validity is unproven; thus, they are rarely reported in the MCDM literature.

In the MCDM literature, there are abundant studies where the uncertainty in input is modelled as fuzzy/linguistic variables (Qi et al. 2022), linguistic/grey numbers (Rajesh and Ravi 2015; Hu et al. 2019), rough sets (Shojaei and Bolvardizadeh 2020), interval numbers (Ahn 2017), among others. These methodologies capture the uncertainty in the *intension* (e.g., fuzzy numbers-based methods) or *extension* (e.g., grey number or interval-based methods) of the "true" value. The output of most of these methodologies is crisp weights or scores, thus revealing no measure of uncertainty. This is more a matter of convenience than of necessity. Though few studies did attempt to present the optimal weights as intervals (uncertainty), the lower and upper limits of the interval weights either simply came from two distinct weight estimation methods (Esangbedo et al. 2021) or the uncertainty resulting from the pairwise comparisons provided by the expert and not from the uncertain preferences of the group of experts (Rezaei 2016). Thus, it can be argued that the research concerning the quantification of uncertainty in the input data available for a MCDM problem is still in its infancy. Meanwhile, when the criteria are qualitative, the only viable way is to represent their performance values through ordinal relations (Yang et al. 2021), and thus quantifying uncertainty is not easy.

While guiding us toward best practices, Briggs et al. (2012) suggested that all modelling studies should include uncertainty assessment, and the decision-makers' role should be considered when presenting uncertainty analyses. Bickel and Bratvold (2008) surveyed 494 oil and gas professionals and found that though most respondents agreed that uncertainty quantification in the industry had improved over the years, this improvement was not necessarily translated into improved decision-making. They presented a strong case for decision-focused uncertainty quantification. Literature is full of uncertainty analysis and quantification methodologies; many are so complicated that real-life decision-makers may rarely use them in practice. Thus, in real life, why many organizations and managers avoid employing decision-making methods and instead decide intuitively should not surprise us (Asadabadi et al. 2019). Especially, a trend toward 'hybridization', where the solution to every problem is looked for in merging two (or more) methodologies without evaluating the computational cost, practical utility, and robustness of the resultant methodology, is a solution that may give rise to new problems. No matter, one may argue, how good a methodology is on a piece of paper, as long as the end-user (decision-makers) is reluctant to use it, its practical utility is hard to validate.

Keeping in view the gaps in the literature and the issues mentioned above, the current study proposes a novel Interval Ordinal Priority Approach (Interval OPA), which quantifies the uncertainty in the subjective opinion of the experts by extending the classical ordinal priority approach with the aid of the mathematical theory of uncertainty analysis. In the proposed approach, firstly, the experts should be ranked, later experts should rank the criteria, and finally, alternatives should be ranked by the experts. The current study aims to solve the ordinal multiple criteria group decision-making problem in an environment where uncertainty is not readily visible from the input data and at a reasonable computational cost. The key contributions of the study will be: (a) Unlike contemporary studies where one method is for estimating criteria weights, the other one for prioritizing experts, and another one for ranking alternatives, the proposed approach would do all in one go using one method; (b) Unlike contemporary literature on interval MCDM, where uncertainty usually implies input data in the form of interval/range or fuzzy/linguistic variables, the current study would elicit the uncertainty hidden in the crisp data; (c) It would present a highly original attempt, and possibly one of the first attempts, to quantify uncertainty in the crisp ordinal input available for the ordinal group MCDM problems without any 'hybridization' or unnecessary complexities; (d) To automate the decision support system, the proposed methodology aims to intelligently guide, the study also introduces a novel Hypothesis Testing mechanism, and the Consistency Control Charts. Grey Relational Analysis (GRA) is a well-known MCDM method with unique features (Kuo et al. 2008). To check the reliability of the results of the proposed approach, it has been compared with the Dynamic GRA in the current study.

The rest of the study is organized as follows: the second section reviews the literature and debates on uncertainty analysis in multiple criteria group decision-making process. The third section introduces some preliminaries, followed by the proposed methodology. The fourth section shows the application of the proposed methodology to a real-life case. The comparative analyses with a benchmark model and a discussion of results are also presented. The fifth section concludes the study with significant implications.

2 Literature Review

2.1 Uncertainty Analysis in Group Decision Making

In group decision-making, one gets input from multiple experts, each with unique experience and knowledge of the problem under study. Such data is frequently found in the form of crisp numbers. On the one hand, if the crisp numbers provide convenience, on the other hand, they can behold uncertainty within them and may give a false impression to the decision-maker (or the data analyst) that there is no uncertainty in the data because data is not expressed linguistically or through intervals (Sugihara et al. 2004). Thus, analysis of uncertainty is an important part of a decision-making process, especially in group decision-making, where the presence of multiple experts with different opinions further complicates the decision-making process. Uncertainty analysis is not decision-making but directly aids decision-makers in improving decisions that would otherwise be made differently (Bickel and Bratvold 2008). By guiding decision-making, it directly influences effective risk analysis (Abdo and Flaus 2016).

In literature, uncertainty has been defined and classified in various ways. Abdo and Flaus (2016) classified uncertainty in the risk assessment process into two types; stochastic (or aleatory) and epistemic. They argued that aleatory uncertainty could be handled using probability theory while several approaches (e.g., fuzzy numbers) can deal with epistemic uncertainty. Briggs et al. (2012) identified four types of uncertainty: stochastic, parameter, structural, and heterogeneity. Their classification is more comprehensive, objective, and guided by evidence-based principles of medical sciences. This classification guided the classification of uncertainty in the current study. Hanea et al. (2021) identified two ways in which uncertainty is acknowledged and modelled through data collected from a group of experts: (1) consider the variability among the experts' opinions as a measure of uncertainty, or (2) ask experts about their subjective distribution associated with the measurable variable. Later, uncertainty is quantified through probabilities or percentiles (Hanea et al. 2021). Since probability can be interpreted in different ways (e.g., it can be defined as a subjective degree of belief, relative frequency, or a theoretical distribution, among others) and in either way is a subjective measure in group decision-making, how to objectively elicit it is a challenge no small. Also, in the case of unique events such as real-life MCDM problems, the use of theoretical distributions can be questioned as an 'ideal' distribution is usually unknown. From a policy or practical perspective, as Briggs et al. (2012) argued, the importance of model-based evaluation lies not only in its ability to produce a precise point estimate for a particular outcome but also in the systematic analysis and responsible reporting of the uncertainty surrounding that outcome, and the final decision being made. Thus, uncertainty analysis can serve two main purposes (Briggs et al. 2012): (a) to assess confidence in a chosen alternative and (b) to ascertain the value of collecting additional information to inform the decision better.

All decision models include parameters, and parameter uncertainty may be represented via probabilistic or deterministic sensitivity analysis (Briggs et al. 2012). The uncertainty representation depends on the nature of uncertainty analysis. For probabilistic sensitivity analysis, a probability distribution is defined through the parameters. For deterministic sensitivity analysis, as is the case in the current study, an interval estimate representing a degree of confidence or belief about the parameter's likely range is required (Briggs et al. 2012). Further, the interval can also be used to express parameter uncertainty (second-order uncertainty) (Hanea et al. 2021). However, the proposed approach neither operationalized the interval as percentiles of the experts' subjective distribution nor by specifying the probability of a one-off event. Instead, it uses the estimate of the weight(s) obtained through the Ordinal Priority Approach as a probability that a given alternative (or expert or criterion) has priority over the other(s), and analysed the uncertainty via interval-based deterministic sensitivity analysis of the estimates (weights), which are probabilistic in nature.

2.2 Multiple Criteria Decision Analysis with Order Preferences

Aristotle, in his work *Nicomachean Ethics,* defined 'preferences' as 'rational desires' and, thus, possibly made one of the first attempts at associating rational decision-making with human desires (preferences) (Köksalan et al. 2016). Preferences play an important role in real-life decision-making, and it's not easy to define them through conceptually measurable and observable utility functions. Today, the multiple criteria group decision-making problems are generally grouped into two categories; cardinal and ordinal. Unlike cardinal problems, where the experts (humans) are supposed to express the degree of preference between two alternatives through precise numbers, ordinal problems only rely on the preferential relation between them, and thus demand for information from the experts is lower (Contreras 2010). The use of ordinal data brings convenience to objective reporting of subjective observations and experiences (Wang et al. 2021) because most humans lack the required granulation capacity, which is necessary to produce numerically precise information (Danielson and Ekenberg 2017).

Even though most of the MCDM methods rely on cardinal data (collected through interval or ratio scales) (Dhurkari 2022), the use of ordinal data is not rare (though not as popular as in the social science research involving the Likert scale). In group decision-making, it is almost impossible for decision-makers to agree on the exact weights of various criteria; however, agreement on their relative priority (represented through ranks) is more likely to ensue (Sureeyatanapas 2016). Thus, the widespread use of ordinal data (or data collected through an ordinal scale) in decision-making under uncertainty literature should not come to us as a surprise (Yager 1999; Whalen 2001; Gilbert 2015). Some of the popular rank-based methods that have seen application in the MCDM context are discussed below.

Stillwell et al. (1981) discussed three methods in their influential study. These were rank reciprocal (RR), rank sum (RS), and rank exponent (RE) weights. The rank order centroid (ROC), proposed by Solymosi and Dombi (1986) and Barron (1992), is another important method. Barron and Barrett (1996) showed that the ROC outperformed the RR and the RS. On the contrary, Roberts and Goodwin (2002) showed the RS outperforming the ROC, followed by the RR. While noting the models RS and RR to be almost opposites and arguing the preferences to lie somewhere between the two extremes, Danielson and Ekenberg (2014) proposed the Sum and Reciprocal (SR) weight function, an additive combination of the RS and the RR functions. Meanwhile, Olson and Dorai (1992) compared the ROC weights with the Analytic Hierarchy Process (AHP) weights and reported the ROC achieved comparable accuracy using much lower computational cost and requiring much less effort from the decision-makers. It should be noted that in the literature, most of these rank-based methods are known for estimating the weights of the criteria against which multiple alternatives are to be evaluated. There are some methods like Deng Julong's Grey Relational Analysis (Kuo et al., 2008) and the QUALIFLEX method (Paelinck 1978) that are not used for weighing criteria but can handle qualitative criteria and accept ordinal input data. They are used primarily for ranking alternatives. However, like most other MCDM methods, none of them can estimate criteria weights by themselves. Since the QUALIFLEX method selects the best alternative from the total number of permutations, which depends on the total number of alternatives, checking each permutation becomes cumbersome as the problem becomes bigger. Most decision analysis methods, including the ordinal ones discussed above, at most, estimate the weights (or scores) of criteria and alternatives. Accurate and reliable experts are vital in group decision-making (Klee 1972), yet their performance is rarely weighed. Like the criteria weight estimation methods, some standalone techniques are available for estimating the weights of the experts (see, e.g., Yue 2012); however, if one looks at the enormous MCDM literature, one rarely sees the experts being weighed. While solving an MCDM problem, the individual judgements are usually aggregated into a collective judgement using arithmetic, geometric, or some other mean (or weighted mean, if they are not equally important) (Javed et al. 2020; Saaty and Vargas 2007), and the weights of the experts are rarely calculated. The Ordinal Priority Approach was proposed to estimate the weights of experts, criteria, and alternatives simultaneously using an easy-to-use linear programming model (Ataei et al. 2020). One can see that there are several rank-based methods to solve ordinal decision-making problems; however, none of them can simultaneously estimate the weights of experts, criteria, and alternatives. In fact, two of the rank-based methods—rank reciprocal (Stillwell et al. 1981) and rank order centroid (Barron and Barrett 1996)—are special cases of the Ordinal Priority Approach on the criterion weighting and alternative weighting dimensions, respectively. Unlike other studies, where these methods are usually presented as competitors, in the OPA, their role is complementary. The OPA represents a significant development in multiple criteria decision analysis with order preferences and would be the central theme of the succeeding section.

3 Interval Ordinal Priority Approach

In literature, "interval" MCDM methods are found in different forms. Dymova et al. (2013) proposed a direct interval extension of the TOPSIS method. They proposed interval extension of TOPSIS and argued that in many cases, it is hard to present precisely the exact ratings of alternatives with respect to local criteria, and as a result, these ratings are considered intervals (Dymova et al. 2013). Meanwhile, Jahanshahloo et al. (2009) extended TOPSIS with interval data, and Hafezalkotob et al. (2019) proposed Interval MULTIMOORA Method Integrating Interval Borda Rule and Interval Best–Worst-Method-Based Weighting Model. In both methods, uncertainty in input data is represented through intervals. Ahn (2017) proposed the analytic hierarchy process (AHP) with interval preference statements. Their method involved uncertainty analysis while the input was an interval ratio, whereas the alternative weights'

Table 1 Classification of literature on "interval" MCDM

Classes	Input	Output	Representative method	Literature
Class I	Interval	Crisp	Interval preference-based AHP	Arbel (1989)
			Direct interval extension of TOPSIS	Dymova et al. (2013)
			Interval type-2 fuzzy sets-based QUALIFLEX	Chen et al. (2013)
			Interval GRA	Wei (2011)
			Interval BWM for criteria weight estimation	Hafezalkotob et al. (2019)
			Interval MULTIMOORA-Borda for alternative ranking	
			Interval-valued BWM with normal distribution for criteria weight estimation	Qu et al. (2021)
			Interval 2-tuple linguistic BWM for criterion weight estimation	Qi et al. (2022)
			Interval 2-tuple linguistic TODIM for ranking alternatives	
Class II	Interval	Interval	Interval AHP models for interval data	Sugihara et al. (2004)
			TOPSIS with interval data	Jahanshahloo et al. (2009)
			AHP with interval preference statements	Ahn (2017)
Class III	Crisp	Interval	Possibilistic AHP for Crisp Data	Sugihara et al. (2004)
			Interval OPA	The current study

*AHP: Analytic Hierarchy Process; BWM: Best–Worst Method; TOPSIS: Technique for Order Preference by Similarity to Ideal Solution; MULTIMOORA: Multiple Objective Optimization on the basis of Ratio Analysis plus Full Multiplicative Form; GRA: Grey Relational Analysis

**It is possible that in some cases, the direct output of the method was interval/non-crispy and was aggregated into a crisp number through some operations. In cases where such operators were not directly listed in the algorithm/computational steps (e.g., Sugihara et al. 2004), the output is considered 'interval' otherwise not (e.g., Wei 2011)

absolute bounds were reported as an interval. Arbel (1989) proposed an approach for AHP where instead of defining preferences through crisp values taken from the 1–9 preference scale, the decision-maker can express the strength of preference as a range of scale values. Jahanshahloo et al. (2009) extended TOPSIS with interval data. The input was interval, and so was the output (the TOPSIS distance), and when data was crisp, their model was allegedly comparable to the classical TOPSIS method. Later, to compare the interval numbers, they used approaches by Sengupta and Pal (2000) and Delgado et al. (1998). This and other interval extensions of TOPSIS have some serious limitations that prompted Dymova et al. (2013) to propose a direct interval extension of TOPSIS. Their method's input was interval, and so was the output, which needed to be compared using the methods for interval comparison. On the other hand, Lo and Liou (2018) used intuitionistic grey value assessment to manage information uncertainty in the failure mode and effect analysis. Qu et al. (2021) extended the best worst method (BWM) using normally distributed interval fuzzy numbers, and Qi et al. (2022) proposed BMW and TODIM-based fuzzy approach where the input is defined through interval 2-tuple linguistic variables.

In short, the existing literature on "interval" MCDM methods can be classified into three categories (see Table 1). One, where input is interval while output is crisp (e.g., Qi et al. 2022; Qu et al. 2021; Dymova et al. 2013). Second, where both input and output are in ranges/intervals (e.g., Ahn 2017), and third, where input is crisp, and output

Table 2 Uncertainty, its analysis, and quantification in the Interval Ordinal Priority Approach

	Name	Concept
Uncertainty type	Parameter uncertainty (second-order uncertainty)	The uncertainty in the estimates of the weights
	Heterogeneity	The variability between experts can be attributed to their characteristics (such as experience, age, speed, ability to work under time pressure, focus, information processing capability, granulation capacity, domain knowledge, etc.)
Uncertainty analysis	Deterministic sensitivity analysis	The representation of uncertainty in an interval (plausible range of the estimates of the weights)
Uncertainty quantification	Standard Deviation (SD) and percentage SD	Absolute and percentage measures of dispersion

is interval (e.g., Sugihara et al. 2004). If one looks at the literature, the significance of the third approach is usually overlooked. For example, Sugihara et al. (2004) proposed an interval extension of AHP and wisely argued that "even if pairwise comparison values with respect to preference are given as crisp values, the priority weights should be estimated as intervals because of a decision maker's uncertainty of judgments". Even though most studies used intervals as input data, it should be noted that "interval representations and similar ways of expressing uncertainty seem also to be unnecessarily restrictive" (Danielson and Ekenberg 2007). The Interval OPA proposed in the current study falls in the third category, where the input (preference information) is not taken at its face value, and the decision-makers' *uncertainty of judgments* is duly considered.

Therefore, generally, the advantages of the Interval OPA lie not only in the crystallization of the past achievements in the rank-based methodologies but also specifically in the fact that it performs sensitivity analysis in a way that the aspects of both probabilistic and deterministic sensitivity analyses can be envisaged in the parameter estimates (i.e., weights) it produces (Mahmoudi et al. 2022). In the case of ties, objects (experts, criteria, or alternatives) can have the same rank, and in the case of limited information, the decision matrix is allowed to have empty blanks representing incomplete information. The absolute value of these weights reveals the probabilistic aspect, while the range gives the deterministic aspect. The definition and representation of uncertainty in the Interval OPA are presented in Table 2.

3.1 The Method

In this section, the steps of the Interval OPA are explained. First, the variables and parameters of the proposed method are defined as follows, as understanding these notations would facilitate us in comprehending the Interval OPA and other definitions that would follow this method (Figs. 1, 2, 3, 4 and 5).

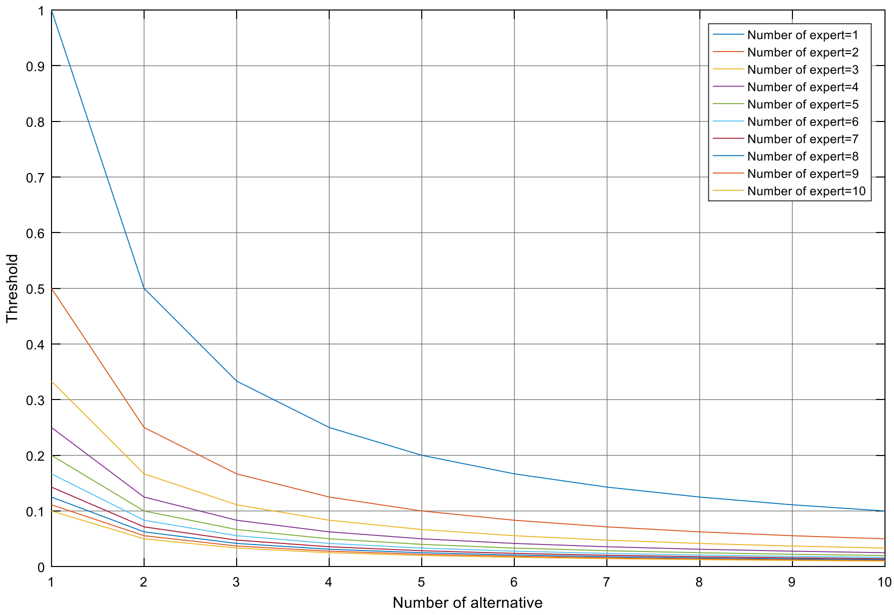


Fig. 1 The nonlinear relationship between the alternatives and its thereshold

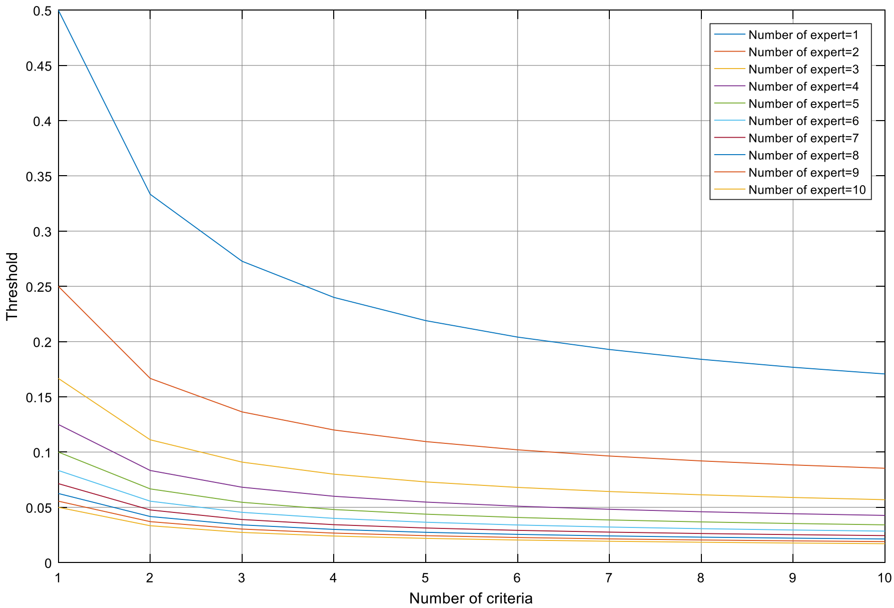


Fig. 2 The nonlinear relationship between the criteria and its thereshold

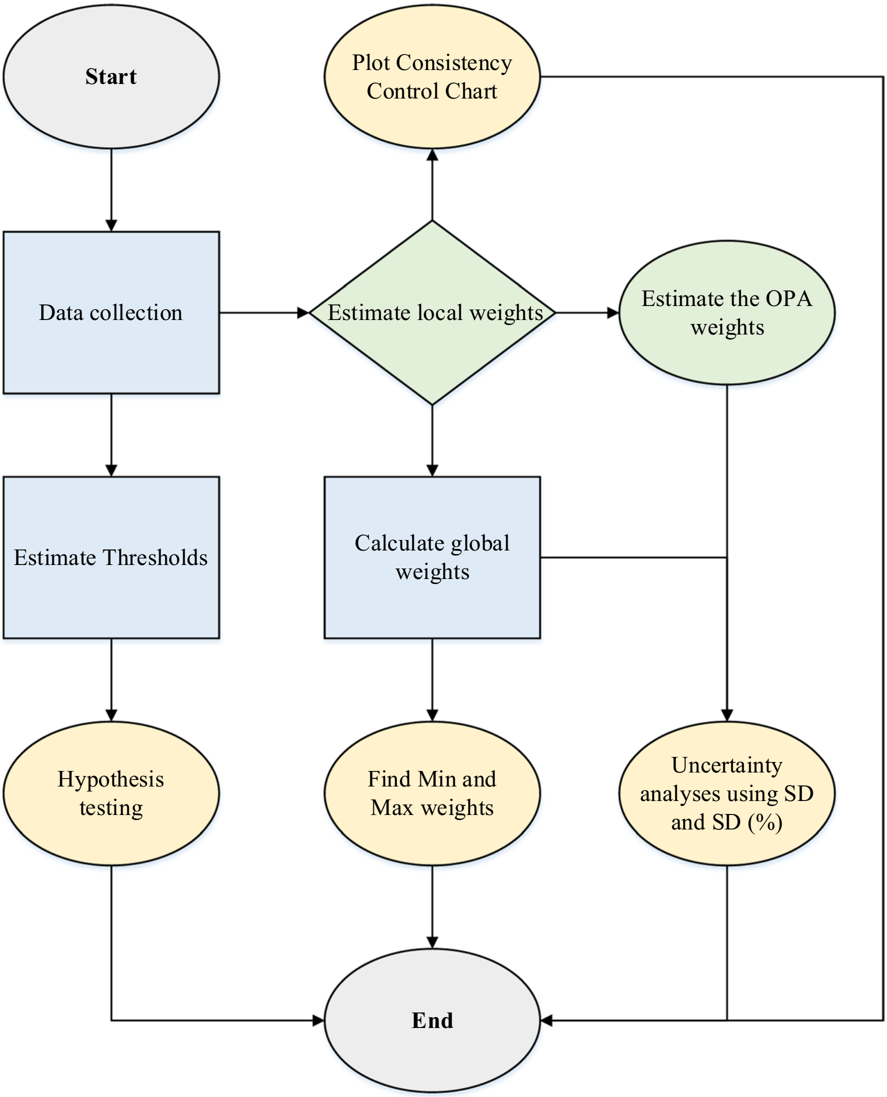
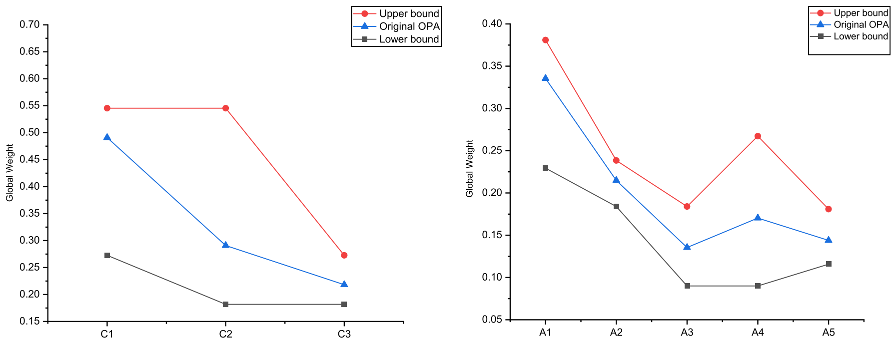


Fig. 3 The proposed framework



(a) The criterion weights

(b) The alternative weights

Fig. 4 The OPA weights, and the lower and upper bounds of the Interval OPA weights

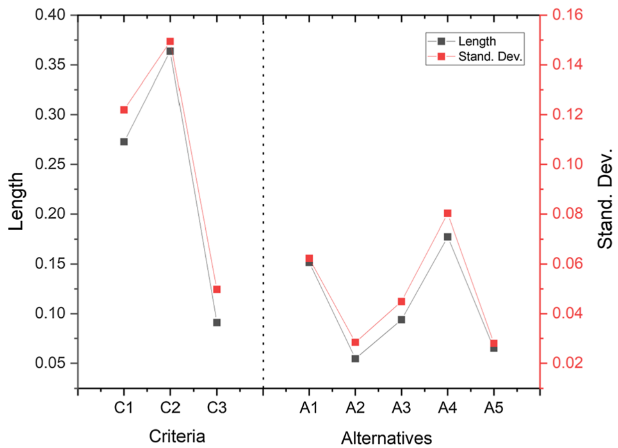


Fig. 5 The ordinal priority standard deviations and the length of the interval weights

Sets	
Set of experts $\forall i \in$ I	I
Set of criteria $\forall j \in$ J	J
Set of alternatives $\forall k \in$ K	K
Indexes/subscripts	
Index of the experts $(1, \dots, p)$	i
Index of the criteria $(1, \dots, n)$	j
Index of the alternatives $(1, \dots, m)$	k
Variables	
Objective function	Z
Weight (importance) of k th alternative based on j th criterion by ith expert at r_k^{th} rank	$W_{ijk}^{r_k}$
Parameters	
The rank of expert i	r_i

The rank of criterion j	r_j
The rank of alternative k	r_k
The difference between two consecutive ranks (generally, $r=1$)	r

Considering the variables and parameters defined at the start of Sect. 3.1, the computing steps of the Interval OPA, which is a generalization of the classical OPA (Mahmoudi and Javed 2022), are listed below. The Interval OPA-based framework of the current study is also illustrated in Fig. 3.

Step 1 The important criteria should be identified carefully while ensuring no important criterion has been missed in light of the decision-making problem under study.

Step 2 Experts should be identified and ranked based on their knowledge and/or experience in the related field (see *Heterogeneity* in Table 2). Experts can be ranked based on one or more distinguishing characteristics.

Step 3 In this stage, the experts should rank the criteria based on their viewpoints.

Step 4 In this stage, alternatives should be ranked by the expert(s) in each criterion.

Step 5 Using the information from steps 1 to 4, Model (1) should be formed and solved.

$$
\begin{aligned}
& Max\ Z \\
& \text{S.t.}: \\
& Z \le r_i\left(r_j\left(r_k\left(W_{ijk}^{r_k} - W_{ijk}^{r_k+1}\right)\right)\right) \quad \forall i, j \text{ and } r_k \\
& Z \le r_i r_j r_m W_{ijk}^{r_m} \quad \forall\ i,\ j \text{ and } r_k = r_m \\
& \sum_{i=1}^{p}\sum_{j=1}^{n}\sum_{k=1}^{m} W_{ijk} = 1 \\
& W_{ijk} \ge\ 0 \quad \forall\ i,\ j \text{ and } k
\end{aligned}
\tag{1}
$$

where Z is Unrestricted in sign.

After solving Model (1), the local alternatives' weight can be calculated using Eq. (2).

$$
W_k = \left[\min_i \left\{W_{ik}\right\}, \max_i \left\{W_{ik}\right\}\right] \forall\ k \tag{2}
$$

where $W_{ik} = \sum_{j=1}^{n} W_{ijk}\ \forall\ k\ and\ i.$

To calculate the local weights of the criteria, Eq. (3) can be utilized.

$$W_j = \left[\min_i \left\{W_{ij}\right\}, \max_i \left\{W_{ij}\right\}\right] \forall j \tag{3}$$

where $W_{ij} = \sum_{k=1}^{m} W_{ijk} \forall j \; and \; i$

Meanwhile, the experts' weights, if need, can be determined employing Eq. (4).

$$W_i = \sum_{j=1}^{n} \sum_{k=1}^{m} W_{ijk} \forall i \tag{4}$$

3.2 Preliminaries

In this section, two propositions and some definitions associated with the OPA and the Interval OPA are presented.

Proposition 1 *The weighing method of rank order centroid is a special case of the Ordinal Priority Approach to estimate the weights of alternatives in a multiple criteria decision-making problem.*

Proposition 2 *The weighing method of rank reciprocal is a special case of the Ordinal Priority Approach to estimate the weights of criteria in a multiple criteria decision-making problem.*

Definition 1 Threshold of the alternatives

Let m is the number of alternatives and p is the number of experts, then the threshold of m alternatives is given by

$$T_A = \frac{1}{mp} \tag{5}$$

Equation (5) is used to check alternatives' local weights for equally-important experts. Otherwise, p=1 is used against global weights. The relationship between the threshold of the alternatives and the number of alternatives is nonlinear and is illustrated in Fig. 1. As seen from the figure, in group decision-making, as the number of experts increases, the relationship becomes re linear, resulting in a declining threshold. Thus, instead of arbitrarily or subjectively defining the threshold, a larger sample size is suggested if the objective is to lower the threshold, and if it is not affordable, then the threshold should be raised.

Proof Let k be the index of alternatives, r_k is the rank of kth alternative, and m is the total number of alternatives. Consider Proposition 1, and the formula of the rank order centroid (Barron and Barrett 1996) is as follows:

$$W_k = \frac{1}{m}\sum_{k=1}^{m}\frac{1}{r_k} \tag{6}$$

Therefore,

$$W_k^{r_k} - W_{k'}^{r_k+1} = \frac{1}{m}\left(\left(\frac{1}{r_k} + \frac{1}{r_k+1}\right) - \left(\frac{1}{r_k+1}\right)\right) = \frac{1}{mr_k} \tag{7}$$

If W_k represents the weight of alternative k, then $W_k^{r_k}$ and $W_{k'}^{r_k+1}$ represent the weights of alternative k with rank r_k and the weights of alternative k' with rank $r_k + 1$ respectively. Since we want to estimate the maximum difference (soft threshold) between the two sequential ranks, we should consider the top ranks. Therefore, $r_k = 1, r_k + 1 = 2$, and consequently, we have the following results:

$$W_k^{r_k} - W_{k'}^{r_k+1} = \frac{1}{m} \tag{8}$$

Thus, the threshold between the top two ranks is

$$T_A = \frac{1}{m} \tag{9}$$

In the case of group decision-making involving p experts, Eq. (9) becomes:

$$T_A = \frac{1}{mp} \tag{10}$$

Hence, the theorem reported in Definition 1 is proved.

Definition 2 Threshold of the criteria

Let $j = 1, \ldots, n$ where n is the total number of criteria, and p is the number of experts, then the threshold of n criteria is given by

$$T_C = \frac{1}{2p\left(\sum_{j=1}^{n}\frac{1}{j}\right)} \tag{11}$$

Equation (11) is used to check criteria's local weights for equally-important experts. Otherwise, p=1 is used against global weights. The relationship between the threshold of the criteria and the number of alternatives is nonlinear and is illustrated in Fig. 2. As can be seen from the figure, in group decision-making, as the number of experts increases, the relationship becomes more linear, resulting in a declining threshold. Thus, instead of arbitrarily or subjectively defining threshold, a larger sample size is suggested if the objective is to lower the threshold, and if it is not affordable, then the threshold should be raised.

Proof Let j is the index of criteria, r_j is the rank of jth criterion and n. is the total number of criteria. Consider Proposition 2 and the formula for the rank reciprocal weights (Stillwell et al. 1981):

$$W_j = \frac{1/r_j}{\sum_{j=1}^{n} 1/j} \tag{12}$$

Therefore we can have:

$$W_j^{r_j} - W_{j'}^{r_j+1} = \frac{1/r_j}{\sum_{j=1}^{n} 1/j} - \frac{1/(r_j+1)}{\sum_{j=1}^{n} 1/j} = \frac{1/r_j - 1/(r_j+1)}{\sum_{j=1}^{n} 1/j} \tag{13}$$

If W_j represents the weight of criterion j then $W_j^{r_j}$ and $W_{j'}^{r_j+1}$ represent the weights of criterion j with rank r_j and the weights of alternative j' with rank $r_j + 1$, respectively. Since we want to calculate the maximum difference (soft threshold) between two sequential ranks, we should consider top ranks. Therefore, $r_j = 1$, $r_j + 1 = 2$, and consequently, we have the following results:

$$W_j^{r_j} - W_{j'}^{r_j+1} = \frac{1/1 - 1/2}{\sum_{j=1}^{n} 1/j} = \frac{1}{2\sum_{j=1}^{n} 1/j} \tag{14}$$

Thus, the threshold between the top two ranks is

$$T_C = \frac{1}{2\sum_{j=1}^{n} 1/j} \tag{15}$$

In the case of group decision-making involng p experts, Eq. (15) becomes:

$$T_C = \frac{1}{2p\left(\sum_{j=1}^{n} \frac{1}{j}\right)} \tag{16}$$

Hence, the theorem reported in Definition 2 is proved.

Definition 3 Maximum and minimum thresholds

Let $\underline{W}_k$ and $\overline{W}_k$ represents the minimum and maximum local weights of the criteria W_{ik}, then the Maximum Threshold $T_{A,max}$ and Minimum Threshold $T_{A,min}$ will be respectively given by,

$$T_{A,max} = \underline{W}_k + T_A,\ \underline{W}_k = \min\left(W_{ik}\right) \tag{17}$$

and

$$T_{A,min} = \overline{W}_k - T_A,\ \overline{W}_k = \max\left(W_{ik}\right) \tag{18}$$

Likewise, if $\underline{W}_j$ and $\overline{W}_j$ represents the minimum and maximum local weights of the alternatives W_{ij}, then the Maximum Threshold $T_{C,max}$ and Minimum Threshold $T_{C,min}$ will be respectively given by,

$$T_{C,max} = \underline{W}_j + T_C, \underline{W}_j = \min\left(W_{ij}\right) \tag{19}$$

and

$$T_{C,min} = \overline{W}_j - T_C, \overline{W}_j = \max\left(W_{ij}\right) \tag{20}$$

The local weights of the criteria W_{ij} and alternatives W_{ik} will be given by the proposed Interval OPA, which was defined in Sect. 3.1.

Definition 4 Length of the interval

The interval length (or, simply length) is the gap between the upper and lower bounds of the weights. A lower gap shows a lower uncertainty. The lengths of the *local* weights, for criteria and alternatives, are respectively given by

$$\Delta_j = \overline{W}_j - \underline{W}_j \tag{21}$$

and

$$\Delta_k = \overline{W}_k - \underline{W}_k \tag{22}$$

where, $\underline{W}_* = \min\left(W_{1*}, W_{2*}, \ldots, W_{p*}\right)$, $\overline{W}_* = \max\left(W_{1*}, W_{2*}, \ldots, W_{p*}\right)$ and W_{i*} is estimated using the Interval OPA model, i.e., from Eqs. (2) and (3).

In general, for criteria and alternatives, the lengths of the *global* weights are respectively given by

$$\Delta_j = \overline{\omega}_j - \underline{\omega}_j \tag{23}$$

and

$$\Delta_k = \overline{\omega}_k - \underline{\omega}_k \tag{24}$$

where, $\underline{\omega}_* = \min\left(\omega_{1*}, \omega_{2*}, \ldots, \omega_{p*}\right)$, $\overline{\omega}_* = \max\left(\omega_{1*}, \omega_{2*}, \ldots, \omega_{p*}\right)$, and $\omega_{i*} = \frac{\omega_{i*}}{\sum_* \omega_{i*}}, i = 1, 2, \ldots, p; * = \{j, k\}$. Here, it should be noted that ω is reserved for global weight, and W is reserved for the local weight, which is a pre-normalized form of global weight.

After *normalizing*[1] the local weights, the global weights (ω_{i*}) are obtained. The minimum ($\underline{\omega}_*$) and maximum values ($\overline{\omega}_*$) of the global weights will give the lower and upper bounds of the interval weights, the interval revealing the likely range of the weights of the criteria (or alternatives). Here, $*$ is a symbol denoting k (in case of criteria) or j (in case of alternatives). If the global weights reveal the relative

[1] Normalization is not performed to normalize/rescale the data between 0 and 1 (as the data is already between 0 and 1) but to obtain global weights against criteria (or alternatives).

weights of criteria (or alternatives) against each expert, then the interval weights can be viewed as the global weights that provide [overall] relative weights of criteria (or alternatives) in the group decision-making problem.

Definition 5 Standard Deviation, $\boldsymbol{\sigma}$

Let ω_j be the global criterion weight obtained through the OPA, and ω_{ij} is the vector of the global weights of the jth criterion against th experts. The Ordinal Priority Standard Deviation (SD), σ, is given by

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{p} \left(\omega_j - \omega_{ij}\right)^2}{p-1}} \tag{25}$$

Let ω_k is the global alternative weight obtained through the OPA, and ω_{ik} is the vector of the global weights of the kth alternative against ith experts. The Ordinal Priority Standard Deviation (SD), σ, is given by

$$\sigma_k = \sqrt{\frac{\sum_{i=1}^{p} \left(\omega_k - \omega_{ik}\right)^2}{p-1}} \tag{26}$$

The ordinal priority standard deviation has a cost-type characteristic (lower the better). A high value of σ implies a problem (high level of inconsistency) in the MCDM problem.

Definition 6 Percentage Standard Deviation, $\boldsymbol{\sigma}(\%\,)$

If the Standard Deviation of the criteria is defined as in Eq. (25), the Percentage Ordinal Priority Standard Deviation, $\sigma(\%)$, is given by

$$\sigma_j(\%) = \frac{\sigma_j}{\omega_j} \times 100 \tag{27}$$

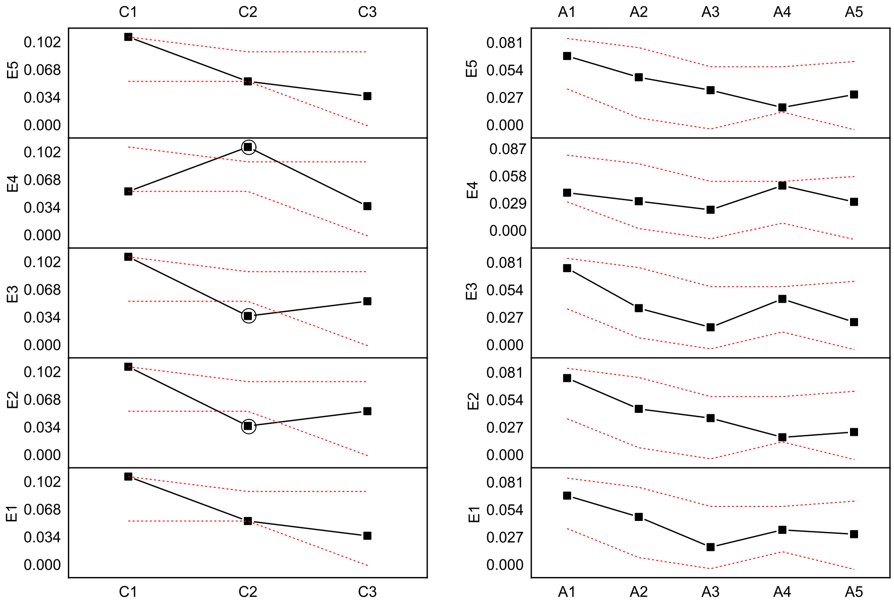
If the ordinal priority standard deviation of the alternatives is defined as in Eq. (26), the Percentage Ordinal Priority Standard Deviation, $\sigma(\%)$, is given by

$$\sigma_k(\%) = \frac{\sigma_k}{\omega_k} \times 100 \tag{28}$$

$\sigma(\%)$ is a valuable metric to know the percentage deviation of the interval weights with respect to the OPA weight.

Definition 7 The Consistency Control Chart

The Consistency Control (CC) Chart is the plotting of three values (over the y-axis)—Maximum Threshold, Minimum Threshold, and global weights—against each criterion (over the x-axis) for each expert. The Maximum and Minimum Thresholds are represented by dotted/dash lines, while global weights are



(a) The criteria (b) The alternatives

Fig. 6 The Consistency Control Charts for (**a**) criteria and, (**b**) alternatives

represented by the solid line(s). For the sake of visual convenience, the lines representing Maximum and Minimum Thresholds can be colored red, symbolizing the thresholds or limits that should not be crossed to stay *consistent*. To save space, we have omitted a representative illustration of the consistency control chart, however, some real-life examples of the CC charts will be shown in Fig. 6.

Definition 8 Hypothesis Testing

Let the interval length (Δ) and the threshold (T) be defined as in Definition 4 and Definition 1 (or Definition 2), respectively. In the Interval OPA, there are two hypotheses; null and alternative. If the interval length is smaller than the threshold, the null hypothesis is accepted, and if not, the alternative hypothesis is accepted. Both hypotheses cannot be true, and thus acceptable, simultaneously. These hypotheses are written as:

Null hypothesis: $H_0 : \Delta \leq T$

Alternative hypothesis: $H_1 : \Delta > T$.

To avoid confusion, it should be noted that the proposed hypothesis testing framework is not the one we are accustomed to in statistics. It is a novel construct. Here, what we are really testing is: 'whether the weight of the object (expert, criterion or alternative) is within the threshold or not?' If it is within the threshold, the null hypothesis is true and if not, the alternative hypothesis (the opposite of the null hypothesis) is true.

4 Application

4.1 Results and Discussion

In this section, to demonstrate the feasibility and validity of the proposed model, an example has been created. The results obtained through the Interval OPA, and the classical OPA will be presented along with the standard deviations and the Consistency Control Charts.

Let's assume we have five alternatives A_1, A_2, A_3, A_4 and A_5 to be evaluated on three criteria C_1, C_2 and C_3 by five equally-important experts E_1, E_2, E_3, E_4 and E_5. The relative importance of criteria for each criterion is shown in Table 3, and the relative importance of the alternatives in each criterion for all experts is shown in Table 4.

In the Interval OPA, uncertainty is defined through the interval weights, while uncertainty analysis is conducted through a system of hypothesis testing involving the length of the interval weight (Definition 3) and the threshold (Definitions 1 and 2). Meanwhile, uncertainty quantification is done through measures of

Table 3 The relative importance of the criteria against each expert (input)

Criteria	E1	E2	E3	E4	E5
C1	1	1	1	2	1
C2	2	3	3	1	2
C3	3	2	2	3	3

Table 4 The relative importance of the alternatives in each criterion against each expert (input)

	E1	E2	E3	E4	E5
(C1)					
A1	1	1	1	1	1
A2	2	2	3	2	2
A3	4	3	4	3	3
A4	3	4	2	5	4
A5	5	5	5	4	5
(C2)					
A1	5	5	5	5	5
A2	2	2	3	3	2
A3	4	3	4	4	3
A4	3	4	2	1	4
A5	1	1	1	2	1
(C3)					
A1	1	1	1	1	1
A2	3	3	2	3	3
A3	4	2	4	2	2
A4	2	4	3	5	4
A5	5	5	5	4	5

dispersions; Standard Deviation (Definition 5) and Percentage Standard Deviation (Definition 6).

The Interval OPA's hypothesis testing process begins with hypothesizing two hypotheses for each case:

Null hypothesis: $H_0 : \Delta \leq T$
Alternative hypothesis: $H_1 : \Delta > T$.

Afterward, we tend to want to reject the null hypothesis. Table 5 shows that we failed to reject the null hypothesis H_0 for the second criterion C_2 and thus the alternative hypothesis H_1 has been rejected. Consequently, C_2 is very likely to be an outlier. However, we succeed in rejecting the null hypotheses for all other cases; thus, the alternative hypotheses are accepted. In case of unequally-important experts one could use p=1 and compare with the global weights (see Table 6).

According to the percentage standard deviation (Definition 6), the second criterion (51.3%) and fourth alternative (47.2%) involve 'high uncertainty', whereas the uncertainty in the criterion is greatest. The results are shown in Table 6. In the third last column of the table, the expected weight (ω), which is obtained through the classical OPA, is also shown. One can see that, $\underline{\omega} \leq \omega \leq \overline{\omega}$. This can be illustrated in Fig. 4 as well. When an object is 'highly uncertain', there are two counter-strategies. If $\Delta \leq T$, it is acceptable (as is the case of our fourth alternative) and if $\Delta > T$, it is unacceptable and must be rejected (as is the case of our second criterion). In case of rejection, either the object should be removed altogether (Note: the current study does not recommends this move as it can be controversial in real-life decision-making), or a new expert should be added to dilute the effect of the inconsistent expert(s). The current study suggests the latter move.

Few important inferences can be drawn from this exercise. One can see from the uncertainty analysis, in general, and hypothesis testing, in particular, that C_2 is very likely to be an outlier. Also, the uncertainty quantification through the percentage standard deviation reveals that C_2 is 'highly uncertain'. However, the uncertainty quantification provides another piece of information that among all

Table 5 Local weights, interval length, and hypotheses testing

	E1	E2	E3	E4	E5	$\underline{W}$	$\overline{W}$	Length	Threshold	$H_1 : \Delta > T$
C1	0.109	0.109	0.109	0.055	0.109	0.055	0.109	0.055	0.055	Accept
C2	0.055	0.036	0.036	0.109	0.055	0.036	0.109	0.073	0.055	Reject
C3	0.036	0.055	0.055	0.036	0.036	0.036	0.055	0.018	0.055	Accept
A1	0.069	0.076	0.076	0.046	0.069	0.046	0.076	0.030	0.040	Accept
A2	0.048	0.046	0.037	0.037	0.048	0.037	0.048	0.011	0.040	Accept
A3	0.018	0.037	0.018	0.028	0.035	0.018	0.037	0.019	0.040	Accept
A4	0.035	0.018	0.046	0.053	0.018	0.018	0.053	0.035	0.040	Accept
A5	0.031	0.023	0.023	0.036	0.031	0.023	0.036	0.013	0.040	Accept

Table 6 Global weights, interval weights, and dispersion measurement

	E1	E2	E3	E4	E5	$[\underline{\omega}, \overline{\omega}]$	OPA ω	Length	σ	$\sigma(\%)$
C1	0.545	0.545	0.545	0.273	0.545	[0.273, 0.545]	0.491	0.273	0.122	24.8
C2	0.273	0.182	0.182	0.545	0.273	[0.182, 0.545]	0.291	0.364	0.149	51.3
C3	0.182	0.273	0.273	0.182	0.182	[0.182, 0.273]	0.218	0.091	0.050	22.8
A1	0.343	0.381	0.381	0.229	0.343	[0.229, 0.381]	0.335	0.152	0.062	18.6
A2	0.238	0.229	0.184	0.184	0.238	[0.184, 0.238]	0.215	0.055	0.028	13.2
A3	0.090	0.184	0.090	0.138	0.175	[0.090, 0.184]	0.135	0.094	0.045	33.1
A4	0.175	0.090	0.229	0.267	0.090	[0.090, 0.267]	0.170	0.177	0.080	47.2
A5	0.154	0.116	0.116	0.181	0.154	[0.116, 0.181]	0.144	0.065	0.028	19.5

alternatives, the fourth alternative A_4 is 'highly uncertain'; however, ousting it from the pool of alternatives at the given threshold level is not recommended. If one considers the OPA weights, the third alternative A_3 is the worst option despite having 'moderate uncertainty.' If one considers the Interval OPA, the percentage standard deviation suggests that A_2 is the most reliable (at the lowest value of percentage standard deviation; 'low uncertainty') alternative, and among the criteria C_3 is the most reliable one. The same conclusion can be drawn if one considers the distance between the interval length and the threshold, i.e., the distance is maximum for them. Also, even though both A2 (0.0280) and A5 (0.0285) reported almost similar standard deviation (see the second last column of Table 6), their mutual difference is clearer by looking at the percentage standard deviation: A2 (13.2%) is better than A5 (19.5%). Thus, to conclude the discussion, it is stated that the ranking of the alternatives should be produced based on the OPA weights, but the decision to disqualify an alternative should be made after performing an uncertainty analysis and quantification using the Interval OPA and the percentage standard deviation. For instance, in real-life supplier selection problems, it is possible a supplier receives higher weight (and rank) but involves 'high uncertainty'; thus, the procurement manager must select it cautiously. In some situations, when supplier selection is critical for business success, the procurement manager may likely prefer a lower-weight supplier with 'low uncertainty' over a higher-weight supplier with 'highly uncertain'. Thus, the proposed system can effectively serve as an early-warning tool for decision-makers making critical decisions from subjective opinions. Also, one can see that when the interval length is greater, the ordinal priority standard deviation is also higher and vice versa (see, Fig. 5). Thus, this directly proportional relationship between the deviation and the interval length confirms that the proposed interval weights can appropriately represent uncertainty.

Another contribution of the current study is the Consistency Control Chart (Fig. 6), which shows the global weights of the criteria (or alternatives) against each expert and the gap between these weights and the thresholds. The point(s) where the global weight(s) cross the minimum or maximum threshold (upper or lower bounds of the Interval OPA weights) is the point(s) of attention. These points (which are encircled in Fig. 6a) represent inconsistencies and thus can be a source of concern for the decision-makers. These figures provide a quick glimpse of the problem of the MCDM process. Thus, the Consistency Control Charts can help the decision-makers monitor and control inconsistency/consistency in the multiple criteria group decision-making processes.

The proposed system also guarantees flexibility for its users. In real-life, decision-makers may choose to define the threshold subjectively. However, the current study suggests that in order to avoid conflict resulting from the subjective threshold, one may simply increase the value of rank r (see, Eqs. (7) and (13)) (or increase the number of experts as suggested before) and thus lower the threshold value to the desired level. By doing so, the inherent relationship between the number of criteria and the objective threshold (as defined through Eq. (11)) is preserved. This relation was shown in Fig. 2. From the figure, one can notice that, in general,[2] when we have one criterion the threshold is always 1, which is a very realistic case. In other words, when we have only one criterion the threshold is maximum, and therefore the interval length cannot be greater than it, and thus it's impossible to have an outlier. Thus, in the Interval OPA, the relationship between the number of criteria and the threshold is not merely a mathematical equation representing nonlinear relationship between the threshold and the number of criteria, it is a mathematical representation of common sense.

4.2 Comparative Analysis

Looking at the literature, one may notice that not many methods can be directly compared with the proposed method. For instance, if one method can rank the alternatives (e.g., TOPSIS), it cannot produce the criteria weights, and if one method can produce the criteria weights (e.g., the ROC) it cannot be used to rank the alternatives. Even if one finds a method that can do both tasks (e.g., AHP), one may need some mandatory procedures (e.g., pairwise comparisons, normalization of input data) that have no equivalent and need in the OPA. Different normalization methods can produce different rankings (Palczewski and Sałabun 2019), so which normalization technique should be used? One that produces ranks most similar to that of the OPA or the one that produces the most dissimilar ranks? Also, since the input for the OPA is ordinal, one is left with very few options (as seen from Sect. 2.2). Meanwhile, not many methods have focused on eliciting uncertainty hidden in the crisp data (see, Table 1). Thus, the

[2] At $r=1$, which is the case when one defines priorities through equally-distanced ordinal numbers. r implies the gap between two consecutive ranks, e.g., $r=1$ implies the top rank is 1, and the second top rank is 2; $2-1=1$.

Table 7 Comparative analysis of the Interval OPA ranks and the Dynamic GRA ranks

Alternatives	E1	E2	E3	E4	E5	Overall*
Interval OPA						
A1	1	1	1	2	1	1
A2	2	2	3	3	2	2
A3	5	3	5	5	3	5
A4	3	5	2	1	5	3
A5	4	4	4	4	4	4
Dynamic GRA						
A1	1	1	1	2	1	1
A2	2	2	3	3	2	2
A3	5	3	5	5	3	5
A4	3	4	2	1	5	3
A5	4	5	4	4	4	4

*Overall rank vector of the Interval OPA is from the OPA, while that for the Dynamic GRA is an ensembled ranking

interval methods that rely on linguistic or interval input cannot be compared with the Interval OPA that relies on the crip input (Table 7).

In the current section, the current study chooses the Dynamic Grey Relational Analysis (Dynamic GRA) for the comparative analysis because it can accept crisp ordinal information as its input while avoiding normalization of the input data. The introduction, steps and equations of the method can be found in Javed et al. (2022). The weights of the criteria were obtained through the OPA. The Dynamic GRA estimates a correlational measure called the Grey Relational Grade (GRG) that is used to rank the alternatives (see Table A.1 in Appendix). A higher value of GRG implies a better rank. The summary of the results is shown in Table 7. The Dynamic GRA was applied on each of the five decision matrices against the five experts independently. Later, by following the majority voting rule, the ensemble rank was obtained.

5 Conclusion

From a set of experiments done in a natural science laboratory, it is not difficult to objectively choose the 'best' one based on the criteria against which objective measurements are available. However, in social sciences, 'measurements' (observations) come directly from 'subjective' experts, where human intuition plays a definite role. This subjectivity in real-life multiple criteria decision-making (MCDM) problems is one of the reasons there is not a single 'best' MCDM technique and each technique attempts to define and analyze uncertainty differently. In most of the literature, explicit uncertainty as obvious from linguistic or interval numbers has been studied and relatively insignificant attention was paid to the analysis of uncertainty hidden in the crisp ordinal data.

The current study extends the Ordinal Priority Approach to the Interval Ordinal Priority Approach (Interval OPA) and enables it to elicit uncertainty hidden in the crisp data by identifying the conflict (inconsistency) among the expert opinions. The proposed system produces ‘interval weights’, a range of plausible weights for the criteria and alternatives. The validity of these weights is confirmed through the standard deviation. To minimize the subjectivity surrounding the real-life MCDM problems, an objective measure of the threshold is furnished that provides an intelligent early-warning tool to decision-makers to gauge whether a given criterion (or alternative) is reliable (less uncertain) or not (more uncertain). The concept of the Consistency Control Chart has been introduced for detailed analyses of a problem and to find the root cause of the problem. Thus, the benefits of the proposed system are multi-fold and can help decision-makers make sound decisions objectively while minimizing the influence of subjectivity on the results. The proposed system is flexible, and thus, if necessary, the decision-makers are allowed to make certain changes without disturbing the inherent relationships between parameters, such as between the number of criteria and the threshold. Moreover, the proposed approach was compared with the Dynamic GRA to check the feasibility of the results. The obtained overall ranks through both methods are exactly the same, which confirms the performance of the proposed approach.

In the current study, the standard deviation was used to confirm the reliability of the interval weights, in the future other measures can also be used. Also, new ways to define the threshold and comparison of the interval weights can be proposed. In the future, the proposed approach can be extended to help deal with the interval, grey or fuzzy/linguistic inputs. With the application of the Interval OPA on sensitive problems where uncertainty analysis is of crucial importance new insights would be drawn and the methodology can be further enriched with new empirical evidences.

Appendix

See Table A.1.

Table A.1 The Dynamic GRA’s grey relational grades

	E1	E2	E3	E4	E5
A1	0.864	0.909	0.909	0.727	0.864
A2	0.776	0.764	0.703	0.703	0.776
A3	0.571	0.641	0.571	0.639	0.691
A4	0.691	0.634	0.764	0.773	0.571
A5	0.636	0.591	0.591	0.696	0.636

Acknowledgements The authors thank the editor and the anonymous reviewers for their important suggestions. More information regarding the Ordinal Priority Approach (OPA) and interval OPA software can be found at www.ordinalpriorityapproach.com.

Declarations

Conflict of interest No potential conflict of interest was reported by the authors.

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