Alternative Ranking-Based Clustering and Reliability Index-Based Consensus Reaching Process for Hesitant Fuzzy Large Scale Group Decision Making

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***Abstract*—Recently, large scale group decision making (LSGDM) problems have become a hotspot. This paper focuses on hesitant fuzzy LSGDM problems where decision makers (DMs) use hesitant fuzzy preference relations (HFPRs) to express their assessment information. HFPRs can represent the fuzziness and hesitancy of DM assessment information well. To improve the efficiency of hesitant fuzzy LSGDM problems, we first propose a reliability index-based consensus reaching process (RI-CRP). By assessing the ordinal consistency of DM's assessment information and measuring the deviation from collective opinion, the DM's opinion reliability index is given. To avoid unreliable information, we propose an unreliable DM management method to be used in the RI-CRP, based on the computation of DM's opinion reliability index. Moreover, an alternative ranking-based clustering (ARC) method with HFPRs is proposed to improve the efficiency of the RI-CRP. The similarity index between two DMs' opinions is provided to ensure the ARC method can be effectively implemented. Compared with those clustering methods which need to preset several correlated parameters, the presented ARC method is more objective with a different approach based on the alternative ranking. A numerical example shows that the proposed ARC method and the RI-CRP are feasible and effective for hesitant fuzzy LSGDM problems.**

***Index Terms*—Alternative ranking-based clustering (ARC), consensus reaching process (CRP), large scale group decision making (LSGDM), reliability index (RI).**

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I. Introduction

With the rapid development and application of science and technology, such as e-democracy [1], social networks [2], [3], and public participation [6], more and more decision makers (DMs) are involved in decision-making problems. This has led to large scale group decision making (LSGDM) problems to become a hotspot [7]–[10], [42], [43]. As there is a huge number of DMs involved in LSGDM problems, it is of great importance for them to be managed effectively and for the efficiency of LSGDM problems to be improved.

In LSGDM problems, there are a large number of DMs involved. They may have different cultures, educational backgrounds, and personal interest preferences. Meanwhile, there are also fuzzy and hesitant natures in human judgment. Thus, when expressing their assessment information, DMs may have several possible numerical values and may hesitate before giving decisions [11]. We focus our research on the hesitant fuzzy set (HFS) [12]–[15].

In the decision-making process, preference relation is one of the most common preference structures used to express DM's assessment information. Hesitant fuzzy preference relation (HFPR) [16] is an effective tool used to express DM's hesitancy and fuzziness. Meanwhile, the HFPR is widely used in the decision making events [14], [17]–[19]. In HFPR, DM's assessment information consists of hesitant fuzzy elements (HFEs), which denote all possible preference values and can be utilized to effectively express DM's hesitant and fuzzy information in LSGDM problems.

As we all know, in LSGDM problems, it is really hard to ensure the acceptance of the final decision by each DM, as such a large number of DMs participate. Thus, the consensus reaching processes (CRPs) [20]–[22] were proposed to improve the efficiency of LSGDM problems [7]–[9], [23]–[26]. Additionally, to improve the efficiency of CRPs, clustering methods were proposed and widely used in the CRPs for LSGDM problems [7], [24]. All the existing CRP models play important roles in improving the efficiency of LSGDM problems. However, there are still some flaws LSGDM researches that need to be discussed.

1) All the existing CRP models are mostly based on the hypothesis that all the DMs' opinions provided for LSGDM problems are reliable. Their assessment information is used directly in the decision-making process without checking their reliability. Actually, it is very difficult

to ensure that each DM's assessment information is reliable in LSGDM problems. The reason is that there are a huge number of DMs that participate in LSGDM problems, some of which may give dishonest or contradictory opinions, which are given taking into account their own interests only. Once the unreliable opinions are utilized in the CRPs, the validity and reliability of the final decision will be greatly decreased for LSGDM problems.

2) Most of the existing clustering methods for LSGDM problems are almost the expansions of fuzzy-C means [7], [24], [27], [28], and interval fuzzy C-means clustering [29]. All these methods usually need to preset several subjective clustering coefficients, which may reduce the objectiveness of the clustering results. Additionally, some innovative clustering methods are provided with a fuzzy set [30], interval-valued intuitionistic fuzzy set [25], interval type-2 fuzzy [26], rather than HFS. Whether they are applicable to hesitant fuzzy LSGDM problems or not requires further verification.

3) In the CRPs for LSGDM problems, some clusters' opinions may be far from the collective opinion and the majority of DMs in them may do not compromise despite the guidance of the moderator. Those DMs prefer to stick with their own opinions, which are good for their own interests. We refer to the cluster's behavior that contains these DMs as "noncooperative behavior". To achieve a high level of consensus and improve the efficiency of the CRPs in LSGDM problems, these noncooperative clusters need to be managed reasonably.

In order to tackle these three gaps in LSGDM problems mentioned above, we propose an alternative ranking-based clustering (ARC) method with HFPRs, and a corresponding reliability index-based CRP (RI-CRP). The proposed ARC method and the RI-CRP for hesitant fuzzy LSGDM problems are mainly based on the following hypotheses.

1) As DMs always give the assessment information which is conducive to their own interests, the assessment information given by DMs may be unreliable in hesitant fuzzy LSGDM problems. The unreliability can be reflected in the following two aspects. One is the contradictory views in DMs' assessment information, and the other is the excessive deviation between individual and collective opinions. If the unreliable DMs' opinions are used in the decision-making process, the validity and efficiency of the CRPs for hesitant fuzzy LSGDM problems will be greatly reduced.

2) The aim of the clustering method is to classify those DMs which provide similar opinions into a group. Generally, the similarity between two DMs' opinions can be reflected by the DMs' HFPRs alternative ranking. According to the majority principle in decision making, those DMs who express the majority of similar opinions in HFPRs alternative ranking should be classified into a same group. We introduce the similarity index (SI) between two DMs' opinions. The implementation of the ARC method can greatly improve the efficiency of the CRPs for hesitant fuzzy LSGDM problems. This is a greatly different hypothesis in comparison with the literature, grouping experts by their preferences instead of alternative ranking.

3) Although the DMs, which provide the reliable assessment information, can participate in the further hesitant fuzzy LSGDM process, some of them may not compromise so as to protect their interests in the CRPs. This causes the clusters that contain those noncooperative DMs to contribute less to the consensus. Thus, in order to reach a high level of consensus for hesitant fuzzy LSGDM problems, the clusters that contain these DMs need to be managed reasonably.

The improvements of the ARC method and the RI-CRP for hesitant fuzzy LSGDM problems in this paper can be shown mainly in the following three points.

1) By assessing the ordinal consistency of DMs' opinions and measuring the deviation between individual and collective opinions, DM's opinion reliability index (ORI) is given. Meanwhile, an algorithm of DM's opinion reliability detection is provided. By checking the DM's ORI, the unreliable DMs' reasonable management processes are proposed. This allows us to guarantee that all the DMs involved in the CRPs can provide reliable assessment information, which ensures that the final decision is reasonable and reliable.

2) The ARC method is given with DMs' HFPRs alternative ranking. By comparing the number of alternatives with the same position (NASP) between two DMs, the SI between them is provided, which ensures the ARC method can be effectively implemented. Additionally, the algorithm of the ARC method for hesitant fuzzy LSGDM problems is proposed. Compared with those clustering methods that need to preset several correlated parameters, the ARC method is more objective with a different approach based on the alternative ranking.

3) In the RI-CRP, the group consensus index (GCI) is given to measure the consensus level. To achieve a high level of consensus, the management processes for noncooperative clusters in the RI-CRP are proposed. The weights of the noncooperative clusters which are unwilling to make any compromise will be punished. The implementation of the weight punishment makes the RI-CRP more efficient for hesitant fuzzy LSGDM problems.

The proposed ARC method and the RI-CRP for hesitant fuzzy LSGDM problems are examined using a numerical example. The example shows that the utilization of the ARC method and RI-CRP can effectively improve the efficiency of hesitant fuzzy LSGDM problems. From the example results, we can find that the unreliable DMs and the noncooperative clusters are effectively managed. Meanwhile, the consensus is reached up to the threshold in a limited three rounds of the RI-CRP, which shows the efficiency of the proposed ARC method and the RI-CRP for hesitant fuzzy LSGDM problems.

The rest of this paper is organized as follows. In Section II, we review some preliminaries related to fuzzy preference relation

(FPR), HFS, HFPR, the score function of HFPR, and the assessment method of ordinal consistency for FPR. In Section III, DM's opinion reliability detection process is proposed, and the corresponding reasonable management methods for unreliable DMs are given. In Section IV, the ARC method is given, detailing the steps for clustering process. In Section V, the RI-CRP for hesitant fuzzy LSGDM problems is proposed. A numerical example and analysis of the proposed ARC method, as well as some comparisons and discussions, are discussed in Section VI. Finally, some of the conclusions of this paper are summarized in Section VII.

II. Preliminaries

Before explaining the ARC method and the RI-CRP, some related preliminaries are presented in this section. In Section II-A, we first provide the preliminary knowledge regarding the FPR, HFS, and HFPR. Subsequently, we review the score function of HFEs and HFPR in Section II-B, which provide the basis for the ARC method of hesitant fuzzy LSGDM problems. Finally, the assessment method of ordinal consistency for FPR is provided, which is used in the RI-CRP to detect the DM's opinion reliability. For simplicity, we denote $M = \{1, 2, \ldots, m\}$ and $N = \{1, 2, \ldots, n\}$ as the number of DMs and alternatives, respectively.

A. Basic concepts of FPR, HFS, and HFPR

Definition 1 (see [31]): An additive FPR R on a finite set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ is a fuzzy relation on the product set $X \times X$ with membership function $\mu_R : X \times X \to [0, 1]$, $\mu_R(x_i, x_j) = r_{ij}$, verifying

$$r_{ij} + r_{ji} = 1, r_{ii} = 0.5, i, j \in N.$$

Generally, an FPR is represented by an $n \times n$ matrix $R = (r_{ij})_{n\times n}$, in which r_{ij} denotes the preference degree of x_i over x_j, where $r_{ij} = 0.5$ implies indifference between x_i and x_j ($x_i \sim x_j$); $r_{ij} = 1$ indicates that x_i is definitely preferred to x_j ($x_i \succ x_j$); $0.5 < r_{ij} < 1$ means that x_i is preferred to x_j ($x_i \succ x_j$); $0 \le r_{ij} < 0.5$ indicates that x_j is preferred to x_i, the smaller r_{ij} the stronger the preference of x_j over x_i.

Definition 2 (see [15]): Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set of alternatives. An HFS A on X is characterized by a membership function $h_A(x)$ that when applied to X returns a subset of $[0, 1]$, which can be represented by a mathematical expression

$$A = \{<x, h_A(x)> | x \in X\}$$

where $h_A(x)$ is a set of some different values in $[0, 1]$, denoting the possible hesitant membership degree of the elements $x \in X$ to A. For convenience, $h = h_A(x)$ is called an HFE.

A detailed review of HFS and further use are provided in [32] and [33]. Based on HFS and FPR, the concept of HFPR is defined by Xu *et al*. [14] as follows.

Definition 3 (see [14]): Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set of alternatives, then an HFPR H on X is represented by a matrix $H = (h_{ij})_{n\times n} \subset X \times X$, where $h_{ij} = \{h_{ij}^{(l)} | l = 1, \ldots, \#h_{ij}\}$ ($\#h_{ij}$ is the number of elements in h_{ij}) is an HFE, which indicates all the possible values of the preference degree of the alternative x_i over x_j. For all $i, j \in N$, h_{ij} should satisfy the following conditions:

$$h_{ij}^{(l)} + h_{ji}^{(l)} = 1,\ h_{ii} = \{0.5\},\ \#h_{ij} = \#h_{ji}$$

where $h_{ij}^{(l)}$ and $h_{ji}^{(l)}$ are the lth elements in h_{ij}, respectively.

Remark 1: The Definition 3 is different from definition of HFPR in [34], it does not have the constraint that the values in h_{ij} are supposed to be arranged in ascending order, i.e., $h_{ij}^{(l)} < h_{ij}^{(l+1)}$, $h_{ji}^{(l+1)} < h_{ji}^{(l)}$. The detailed explanation can be seen in Remark 1 of [14]. Generally, the number of values in different h_{ij} is different. In order to operate correctly, there is a normalization process in [14] and [35], which ensures that the different HFEs have the same number of values. In this paper, we assume the HFPRs offered by the DMs are normalized.

B. Basic Concepts of the Score Function of HFE and HFPR

To compare the HFEs, Xia and Xu [36] defined the following comparison laws.

Definition 4 (see [36]): For an HFE h, $s(h) = \frac{1}{\#h}\sum_{\gamma\in h}\gamma$ is called the score function of h, and $\#h$ is the number of the elements in h. For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then h_1 is superior to h_2, denoted by $h_1 \succ h_2$; if $s(h_1) = s(h_2)$, then h_1 is indifferent to h_2, denoted by $h_1 \sim h_2$.

According to Definitions 3 and 4, suppose an HFPR $H = (h_{ij})_{n\times n}$, where h_{ij} represents the preference degree between alternative x_i and x_j, $\#h_{ij}$ is the number of the elements in h_{ij}, and $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set of alternatives, then the score value of each alternative $s(x_i)$, $i \in N$, can be calculated as follows:

$$s(x_i) = \sum_{j=1}^{n}\left(\frac{1}{\#h_{ij}}\sum_{\gamma\in h_{ij}}\gamma\right). \tag{1}$$

Then, the alternative ranking with HFPRs can be obtained based on the overall score values, as well as the best alternative(s) can be selected.

C. Ordinal Consistency for an FPR

In [37], the definition of ordinal consistency for an FPR is introduced as follows.

Definition 5 (see [37]): Let $R = (r_{ij})_{n\times n}$ be an FPR, for all $i, j, k \in N$, $i \ne j \ne k$.

1) If $r_{ik} > 0.5$, $r_{kj} \ge 0.5$; or $r_{ik} \ge 0.5$, $r_{kj} > 0.5$, we have $r_{ij} > 0.5$.
2) If $r_{ik} = 0.5$, and $r_{kj} = 0.5$, we have $r_{ij} = 0.5$.

Then, an FPR R is said to be ordinally consistent.

Remark 2: Definition 5 is the minimum requirement that a consistent FPR should possess, and it is the usual transitivity condition that a logical and consistent DM should use if she/he does not want to provide a contradictory opinion.

Then, Xu *et al*. [37] discussed the ordinal consistency of FPR from the perspective of graph theory, and presented some basic theory of digraph as follows.

Definition 6 (see [37]): Let $R = (r_{ij})_{n\times n}$ be an FPR, the adjacency matrix $E = (e_{ij})_{n\times n}$ of R is defined by

$$e_{ij} = \begin{cases} 1, & r_{ij} \geq 0.5, (i \neq j); \\ 0, & \text{otherwise} \end{cases}. \tag{2}$$

Then, a digraph $G = (V, A)$ of R is constructed, where $V = \{v_1, v_2, \ldots, v_n\}$ denotes the node set, $A = \{(V_i, V_j)|i \neq j, r_{ij} \geq 0.5\}$ denotes the arc set. That is, if $i \neq j, r_{ij} > 0.5$, then there is a directed arc in G from v_i to v_j, denoted by (v_i, v_j) or $v_i \to v_j$. Therefore, if $r_{ij} = 0.5\ (i \neq j)$, then there are two arcs between v_i and v_j, one from v_i to v_j, and another from v_j to v_i.

According to Definition 5, if *R* is not ordinally consistent, then there are some unreasonable judgment elements in *R*, satisfying one of the following.

1) $r_{ik} \geq 0.5$, $r_{kj} > 0.5$, but $r_{ij} \leq 0.5$.
2) $r_{ik} > 0.5$, $r_{kj} \geq 0.5$, but $r_{ij} \leq 0.5$.
3) $r_{ik} = 0.5$, $r_{kj} = 0.5$, but $r_{ij} \neq 0.5$.

In each situation, there is a directed cycle of length 3 (simplified 3-cycle) $(v_i \to v_k \to v_j \to v_i)$ in the digraph *G* of *R*, That is, the inconsistent judgments could be represented by 3-cycle in *G*.

Theorem 1 (see [37]): Let $R = (r_{ij})_{n\times n}$ be an FPR, there is a directed 3-cycle $(v_i \to v_k \to v_j \to v_i)$ in the digraph *G* of *R*, if and only if the elements $r_{ik}, r_{kj}, r_{ji}\ (i \neq j \neq k)$ are present, satisfying one of the following.

1) $r_{ik} > 0.5$, $r_{kj} \geq 0.5$, or $r_{ik} \geq 0.5$, $r_{kj} > 0.5$, but $r_{ji} \geq 0.5$.
2) $r_{ik} = 0.5$, $r_{kj} = 0.5$, but $r_{ji} \neq 0.5$.
3) $r_{ik} = 0.5$, $r_{kj} = 0.5$, $r_{ji} = 0.5$.

Remark 3: Theorem 1 shows that a directed 3-cycle $(v_i \to v_k \to v_j \to v_i)$ would be determined from the above three cases. When r_{ik}, r_{kj}, and r_{ji} satisfy the third case of Theorem 1, there would be two 3-cycles $(v_i \to v_j \to v_k \to v_i, v_i \to v_k \to v_j \to v_i)$ in the digraph *G*. But these judgment elements would be considered reasonable, because x_i, x_k, and x_j are indifferent $(x_i \sim x_k \sim x_j)$. Thus, these two 3-cycles would not result in order inconsistency.

Based on the analysis of 3-cycle, Xu *et al*. [37] introduced the ordinal consistency index (OCI) of an FPR as follows.

Definition 7 (see [37]): Let $R = (r_{ij})_{n\times n}$ be an FPR, $E = (e_{ij})_{n\times n}$ is the adjacency matrix of *R*, and $B = (b_{ij})_{n\times n} = E^2 \circ E^T$, we call

$$\text{OCI} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}}{3} - \chi \tag{3}$$

is the OCI of *R*, where χ is the number of 3-cycles that satisfies the condition (c) in Theorem 1. Meanwhile, the $B = (b_{ij})_{n\times n}$ is the *Hadamard* product of E^2 and E^T. Suppose that $E^2 = (e^2_{ij})_{n\times n}$ and $E^T = (e^T_{ij})_{n\times n}$, then, $b_{ij} = (e^2_{ij}) \times (e^T_{ij})$.

Theorem 2 (see [37]): Let *R* be an FPR, *R* is ordinally consistent, if and only if $\text{OCI} = 0$.

Proof: The proof process of Theorem 2 can be seen in [37].

III. Opinion Reliability Detection and the Management for Unreliable DMs

Almost all the remaining methods for LSGDM problems are based on the hypothesis that the assessment information provided by DMs is reliable and can be utilized in the decision-making process directly. Actually, some DMs may give unreliable opinions in LSGDM problems, which may reduce the reliability of the final decision. Thus, it would be of great importance to detect the reliability of DMs' opinions, as well as to manage the unreliable DMs in LSGDM problems. In this section, the specific processes of DMs' opinions reliability detection are shown in Section III-A. For unreliable DMs, the corresponding management methods are presented in Section III-B.

A. Opinion Reliability Detection

Usually, if a DM provides an unreliable opinion, it may be reflected by the following two aspects.

1) DM's opinion is contradictory, that is, the preference relation given by the DM does not have ordinal consistency.
2) The deviation between individual and collective opinions is excessively large. Namely, the DM's contribution to the CRPs may be lower than those DMs that have a low deviation level from the collective opinion.

Compared with the second aspect, the contradiction remaining in DMs' opinions can reflect more objective of DMs' unreliability. Thus, in the DMs' opinions reliability detection processes, we first assess the contradictory degree of DMs' HFPRs based on the ordinal consistency.

1) DM's Opinion Contradictory Detection Based on the Ordinal Consistency: Let $D = \{d_1, d_2, \ldots, d_m\}$ denote the DMs set, the $H_\varphi = (h_{ij,\varphi})_{n\times n}\ (\varphi \in M)$ be an HFPR provided by DM d_φ. Then, based on Theorem 2, the contradictory degree of DM's HFPR is defined as follows.

Definition 8: Let $H_\varphi = (h_{ij,\varphi})_{n\times n}$ be an HFPR, and $R^{H_\varphi}_{v\in\{1,\ldots,\#h_{ij}\}} = (r_{v,ij})_{n\times n}$ denotes the FPRs transformed by $H_\varphi = (h_{ij,\varphi})_{n\times n}$, where $r_{v,ij} \in \{h_{ij,\varphi}\}$, and $\#h_{ij}$ is the number of the elements in $h_{ij,\varphi}$. Then the contradictory degree of DM d_φ's HFPR is defined as follows:

$$\tau_{(\varphi)} = \#h_{ij} - \#\left(\text{OCI}\left(R^{H_\varphi}_{v\in\{1,\ldots,\#h_{ij}\}}\right) = 0\right). \tag{4}$$

Remark 4: In Definition 8, the $\#(\text{OCI}(R^{H_\varphi}_{v\in\{1,\ldots,\#h_{ij}\}}) = 0)$ is the number of $\text{OCI}(R^{H_\varphi}_{v\in\{1,\ldots,\#h_{ij}\}}) = 0$ in the HFPR $H_\varphi = (h_{ij,\varphi})_{n\times n}$, and the $\text{OCI}(R^{H_\varphi}_{v\in\{1,\ldots,\#h_{ij}\}})$ can be calculated by (3).

Obviously, $\tau_{(\varphi)}$ has the following characteristics.

1) $0 \leq \tau_{(\varphi)} \leq \#h_{ij}$.
2) If $\tau_{(\varphi)} = 0$, all the FPRs obtained by an HFPR are ordinally consistent, then, the opinion provided by d_φ is completely a logical opinion without any contradictions.
3) If $0 < \tau_{(\varphi)} < \#h_{ij}$, some of FPRs transformed by HFPR are ordinally consistent. Then, the opinion provided by d_φ is considered partially contradictory.
4) If $\tau_{(\varphi)} = \#h_{ij}$, all the FPRs transformed by HFPR are ordinally inconsistent. Then, the opinion provided by d_φ

is a completely contradictory opinion, which is regarded as a completely unreliable opinion.

In real hesitant fuzzy LSGDM problems, DM's opinion is completely contradictory, or absolutely logical, belonging to two relatively extreme phenomena. Thus, we consider the acceptable ordinal consistency as a way to assess the contradictory degree of DM's opinion in this paper. We assume that if $0 \le \tau_{(\varphi)} < (\#h_{ij}) \times \alpha$, (where α is an acceptable ordinal consistency parameter, and $\alpha \in [0,1]$), then d_φ is considered to provide an acceptable ordinal consistency HFPR. For those DMs in which $(\#h_{ij}) \times \alpha \le \tau_{(\varphi)} \le \#h_{ij}$, their opinions are regarded as unreliable.

Based on the majority principle, we suppose that $\alpha = 0.5$ in this paper. That is, if $0 \le \tau_{(\varphi)} \le (\#h_{ij}) \times 0.5$, then d_φ's HFPR is considered to be acceptably ordinally consistent, and d_φ can participate in the next stage of DM's opinion reliability detection. If d_φ gives partly contradictory opinions, but not within the acceptable level, namely, $(\#h_{ij}) \times 0.5 \le \tau_{(\varphi)} < \#h_{ij}$. Then, d_φ will be involved in the management process for unreliable DMs. Moreover, if $\tau_{(\varphi)} = \#h_{ij}$, d_φ's opinion will be directly rejected. See Example 1 for the detailed calculation process.

Example 1: Assume that there are four alternatives $X = \{x_1, x_2, x_3, x_4\}$ for a hesitant fuzzy LSGDM problem, and DM d_1 provides his/her HFPR as follows:

$$H_1 = \begin{bmatrix} \{0.5\} & \{0.7, 0.1\} & \{0.9, 0.6\} & \{0.5, 0.7\} \\ \{0.3, 0.9\} & \{0.5\} & \{0.6, 0.8\} & \{0.7, 0.4\} \\ \{0.1, 0.4\} & \{0.4, 0.2\} & \{0.5\} & \{0.8, 0.9\} \\ \{0.5, 0.3\} & \{0.3, 0.6\} & \{0.2, 0.1\} & \{0.5\} \end{bmatrix}$$

First, this HFPR can be transformed into two FPRs as follows:

$$R_1^{H_1} = \begin{bmatrix} 0.5 & 0.7 & 0.9 & 0.5 \\ 0.3 & 0.5 & 0.6 & 0.7 \\ 0.1 & 0.4 & 0.5 & 0.8 \\ 0.5 & 0.3 & 0.2 & 0.5 \end{bmatrix}, R_2^{H_1} = \begin{bmatrix} 0.5 & 0.1 & 0.6 & 0.7 \\ 0.9 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.9 \\ 0.3 & 0.6 & 0.1 & 0.5 \end{bmatrix}.$$

By (3), we can calculate $\mathrm{OCI}(R_1^{H_1}) = 2$, $\mathrm{OCI}(R_2^{H_1}) = 2$. Obviously, we have $\#(\mathrm{OCI}(R_{v\in\{1,2\}}^{H_1}) = 0) = 0$. Then, utilizing (4), we have $\tau_{(1)} = 2 > (\#h_{ij}) \times \alpha = 1$ $(\alpha = 0.5)$, namely, all the FPRs $R_{v\in\{1,2\}}^{H_1}$ transformed from the HFPR H_1 are ordinally inconsistent. Thus, we conclude that the DM d_1 provides a completely contradictory opinion.

Remark 5: Actually, for Example 1, based on Definition 1, we can also clearly find the contradiction of d_1's opinion. Such as in $R_1^{H_1}$, d_1 provides $r_{1,12}^{H_1} = 0.7$, means $x_1 \succ x_2$; $r_{1,23}^{H_1} = 0.6$, indicates $x_2 \succ x_3$; and $r_{1,34}^{H_1} = 0.8$, denotes $x_3 \succ x_4$, then according to Definition 5, we should have $x_1 \succ x_4$. However, the d_1 gives $r_{1,14}^{H_1} = 0.5$, implying $x_1 \sim x_4$. It is obvious that d_1's opinion is illogical and contradictory. Similarly, we can easily find contradictions in $R_2^{H_1}$ between x_2 and x_4.

2) Deviation Measure Between Individual and Collective Opinions: To achieve a high level of consensus in the CRPs for hesitant fuzzy LSGDM problems, after the contradiction detection processes, we need to further detect the DM's opinion reliability by measuring the deviation level (DL) between the individual opinion and the collective opinion, which is the second aspect mentioned in Section III-A.

Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)^T$ be the weight vector of DMs, where $\sum_{\varphi=1}^{m} \lambda_\varphi = 1, \lambda_\varphi \in [0,1], \varphi \in M$. Considering the fairness, we suppose the weights of DMs are equal and $\lambda_1 = \lambda_2 = \cdots = \lambda_m = 1/m$. By using the weighted arithmetic average (WAA) operator, the collective HFPR $H = (h_{ij})_{n\times n}$ can be calculated as follows:

$$h_{ij} = \sum_{i,j=1}^{n} \sum_{\varphi=1}^{m} \lambda_\varphi h_{ij,\varphi}. \tag{5}$$

Then, the DL between individual and collective opinions is defined as follows.

Definition 9: Let $H_\varphi = (h_{ij,\varphi})_{n\times n}$, and $H = (h_{ij})_{n\times n}$ be the individual HFPR and the collective one, respectively. Then, the $\mathrm{DL}_{(\varphi)}$ can be calculated as follows:

$$\mathrm{DL}_{(\varphi)} = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} d(h_{ij,\varphi}, h_{ij}) \tag{6}$$

where $d(h_{ij,\varphi}, h_{ij}) = \frac{1}{\#h_{ij}} \sum_{h_{ij,\varphi}^{(l)} \in h_{ij,\varphi}; h_{ij}^{(l)} \in h_{ij}} |h_{ij,\varphi}^{(l)} - h_{ij}^{(l)}|$, $\#h_{ij}$ is the number of the elements in h_{ij}. $h_{ij,\varphi}^{(l)}$, $h_{ij}^{(l)}$ are the lth elements in $h_{ij,\varphi}$ and h_{ij}, respectively. Obviously, the $\mathrm{DL}_{(\varphi)} \in [0,1]$.

As with DM's opinion contradiction detection, we should consider an acceptable reliability level. Then, the ORI of DM d_φ is defined as follows.

Definition 10: Let $H_\varphi = (h_{ij,\varphi})_{n\times n}$, and $H = (h_{ij})_{n\times n}$ be the individual preferences and collective opinion, respectively, and $\mathrm{DL}_{(\varphi)}$ as calculated by (6). Then the DM's ORI is given by

$$\mathrm{ORI}_{(\varphi)} = \sigma - \mathrm{DL}_{(\varphi)} \tag{7}$$

where σ is the acceptable deviation threshold and $\sigma \in [0,1]$.

Additionally, we suppose the following condition.

1) If $\mathrm{ORI}_{(\varphi)} \ge 0$, then d_φ is considered to provide an acceptable reliability opinion.
2) If $\mathrm{ORI}_{(\varphi)} < 0$, then d_φ is considered to provide an unreliable opinion.

In addition, the detail processes of DM's opinion reliability detection are shown in Algorithm 1.

B. Unreliable DMs Management Process

To ensure fairness and democracy in hesitant fuzzy LSGDM problems, a moderator is introduced to persuade the DMs with unreliable opinions to make some modifications. Additionally, in order to guarantee the efficiency of the RI-CRP in hesitant fuzzy LSGDM problems, we need to preset the maximum modification rounds $T_{\max}$. By checking DM's opinion reliability, we can obtain an unreliable DMs set. The corresponding management methods for those unreliable DMs are provided in this section.

The unreliable DMs are obtained considering two aspects: one is the DMs with unacceptable contradictions in Step 3 of Algorithm 1, and the other is the DMs which are too biased against collective opinion obtained in Step 4 of Algorithm 1.

Correspondingly, the management of unreliable DMs is carried out in the following two parts.

Algorithm 1 DM's Opinion Reliability Detection.

Step 1: Transform $H_\varphi = (h_{ij,\varphi})_{n\times n}$ into FPRs $R^{H_\varphi}_{v\in\{1,\ldots,\#h_{ij}\}} = (r_{v,ij})_{n\times n}$, $r_{v,ij} \in \{h_{ij,\varphi}\}$,$\#h_{ij}$ is the number of the elements in $h_{ij,\varphi}$.

Step 2: Compute the $\mathrm{OCI}(R^{H_\varphi}_{v\in\{1,\ldots,\#h_{ij}\}})$ by (3).

Step 3: Compute the $\tau_{(\varphi)}$ of d_φ using (4).

- If $0 \le \tau_{(\varphi)} < (\#h_{ij}) \times \alpha$, $\alpha \in [0,1]$, then d_φ is considered to provide an acceptable ordinal consistency preference relation. Turn to Step 4.
- Otherwise, d_φ's opinion is contradictory and d_φ needs to be managed reasonably.

Step 4: By (6) and (7), we have the $\mathrm{ORI}_{(\varphi)}$ of d_φ.

- If $\mathrm{ORI}_{(\varphi)} \ge 0$, then d_φ provides an acceptable reliability opinion, and is allowed to participate in further LSGDM processes.
- Otherwise, d_φ is considered to provide unreliable opinions and needs to be managed reasonably.

Output: The reliable set and the unreliable set.

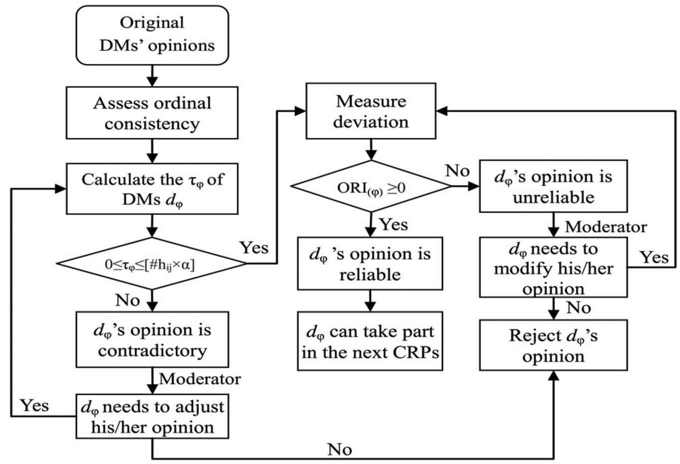

Fig 1. Processes of DM's opinion reliability detection and management of unreliable DMs.

1) Management for DMs Who Offer Contradictory Opinions: By (4), we can obtain the $\tau_{(\varphi)}$ of d_φ. For the DMs $(\#h_{ij}) \times \alpha \le \tau_{(\varphi)} \le \#h_{ij}$, $\alpha \in [0,1]$, their opinions are regarded as contradictory and they need the following management to modify their opinions.

1) If $\tau_{(\varphi)} = \#h_{ij}$, DM d_φ provides a completely contradictory opinion, then his/her opinion will be directly rejected to ensure the reliability of the final decision.
2) If $(\#h_{ij}) \times \alpha \le \tau_{(\varphi)} < \#h_{ij}$, $\alpha \in [0,1]$, DM d_φ gives partially contradictory opinions. A moderator will be introduced to persuade d_φ to make some modifications.
   a) If d_φ follows the persuasion, then d_φ's HFPR ordinal consistency degree will be reconsidered after he/she makes a modification within the maximum modification rounds. If d_φ's revised preference relation is of ordinal consistency, then d_φ's opinion reliability will be further redetected by measuring the deviation from collective opinion.
   b) If d_φ is unwilling to make any adjustments, or in the maximum permissible modification rounds, d_φ's revised opinion still does not have the property of ordinal consistency, then, d_φ's opinion will be rejected directly.

Additionally, in order to retain as much of the original preference information as possible, we allow the DMs to select how many FPRs they want to modify in the HFPR, but the minimum cannot be lower than the acceptable ordinal consistency level. For example, an HFPR can be transformed into 4 FPRs, that is, $\#h_{ij} = 4$, and by (4), we have $\tau_{(\varphi)} = 3$, ($\alpha = 0.5$). Thus, the DM d_φ needs to modify at least one of the FPRs to meet the acceptable ordinal consistency requirements.

2) Management for DMs Who Are Too Biased Against Collective Opinion: According to the deviation measure shown in Step 4 in Algorithm 1, if $\mathrm{ORI}_{(\varphi)} < 0$, then d_φ is considered to provide an unreliable opinion. The moderator will try to persuade d_φ to make some modifications to his/her opinion.

1) For those DMs with unreliable opinions that are willing to modify their opinions, we will redetect d_φ's opinion reliability after he/she has made a modification, and allow d_φ to participate in the next decision making when $\mathrm{ORI}_{(\varphi)} \ge 0$ within the limited modification rounds.
2) For the DMs who are unwilling to make a compromise, or in the case of maximum permissible modification rounds, the DM's revised opinion still does not meet the reliability requirement. Then, their opinions will be rejected directly to ensure the final decision is reliable.

After applying the unreliable DM management process, only the DMs who provide acceptable reliable opinions can enter the following decision-making process. To clarify, the specific processes of DM's opinion reliability detection and the management of unreliable DMs are depicted in Fig. 1.

IV. Alternative Ranking-Based Cluster Method for Hesitant Fuzzy LSGDM Problems

Clustering methods aim to classify the DMs who provide similar opinions into a group to improve the CRPs efficiency in LSGDM problems. A novel ARC method for hesitant fuzzy LSGDM problems, based on the DM's HFPR alternative ranking, is presented in this section.

Based on the score function of HFPRs mentioned in Section II, DM's alternative ranking can be obtained. Then, we can get the position order of alternatives for each DM. The detailed process is shown in Example 2.

Example 2: Suppose there are four alternatives $X = \{x_1, x_2, x_3, x_4\}$, and DM d_2 provides his/her normalized HFPR $H_2 = (h_{ij,2})_{4\times 4}$ as follows:

$$H_2 = \begin{bmatrix} \{0.5\} & \{0.2, 0.3\} & \{0.4, 0.3\} & \{0.1, 0.3\} \\ \{0.8, 0.7\} & \{0.5\} & \{0.6, 0.7\} & \{0.4, 0.2\} \\ \{0.6, 0.7\} & \{0.4, 0.3\} & \{0.5\} & \{0.3, 0.1\} \\ \{0.9, 0.7\} & \{0.6, 0.8\} & \{0.7, 0.9\} & \{0.5\} \end{bmatrix}.$$

By (1), we have the score value of each alternative $s(x_i)$ $(i = 1, 2, 3, 4)$

$$s(x_1) = 1.3, s(x_2) = 2.2, s(x_3) = 1.7, s(x_4) = 2.8.$$

Thus, the alternative ranking for d_2 is $x_4 \succ x_2 \succ x_3 \succ x_1$, and the alternatives position order of d_2 is $O(d_2) = (4, 2, 3, 1)$.

In addition, if there are equivalent alternatives given by DM, in order to obtain a reasonable clustering result, we propose that all the possible alternative position orders should be considered. For instance, the alternative ranking of d_2 is $x_4 \succ x_2 \sim x_3 \succ x_1$, then we have $O^1_{(d_2)} = (4, 2, 3, 1)$ and $O^2_{(d_2)} = (4, 3, 2, 1)$.

The alternative position order can be used to find the similarity degree of each pair of DMs, which can be further used in the clustering method based on alternative ranking.

Definition 11: Let $n \in N$ be the number of alternatives, for two DMs $d_\varphi, d_m \in D$, their opinions SI is defined as follows:

$$\mathrm{SI}(d_\varphi, d_m) = \mathrm{NASP}(d_\varphi, d_m), SI \in N^+ \tag{8}$$

where $\mathrm{NASP}(d_\varphi, d_m)$ is the number of alternatives that hold the same position between d_φ and d_m. If $\mathrm{SI}(d_\varphi, d_m) \geq \mu \times n$ (where μ denotes the SI parameter, and $\mu \in [1/n, (n-1)/n]$), then we consider that there are similar or consistent opinions between d_φ and d_m.

Remark 6: For (8), the $\mathrm{SI}(d_\varphi, d_m) = n - 1$ already means that the opinions between d_φ and d_m are completely consistent. Thus, we set $\mu \in [1/n, (n-1)/n]$, instead of $\mu \in [1/n, 1]$.

Considering the feasibility of the numerical example in Section VI-A, we assume that $\mu = 0.5$ in this paper. It means that if $\mathrm{SI}(d_\varphi, d_m) = \mathrm{NASP}(d_\varphi, d_m) \geq 0.5 \times n$, then, we consider that d_φ and d_m share the majority of the same opinions, which are then classified into one group. The detailed cluster analysis steps are depicted in Algorithm 2.

Remark 7: To reduce the complexity of alternative position order comparisons among DMs, in Algorithm 2, we first classify the DMs that have the completely consistent opinions. Furthermore, the $f(C_s)$ represents the average of the cluster C_s, which can be computed by the arithmetic mean method. The $d(f(C_s), f(C_{s'}))$ denotes the distance between $f(C_s)$ and $f(C_{s'})$, which can be obtained by (6). Other symbols that have the same formula as $d(f(C_s), f(C_{s'}))$ and $f(C_s)$ also share the similar meaning to them.

Algorithm 2 The ARC Method for Hesitant Fuzzy LSGDM Problems.

Step 1: According to (1), we can calculate the alternative score function values of DMs with HFPRs. Then, the alternatives position order of DMs can be obtained.
Step 2: Firstly, cluster the DMs with a completely consistent alternative position order, and so the remaining DMs are considered to be a unique cluster, then we have the initial clustering results C_s, $1 \leq s \leq m$.
Step 3: These different clusters are then compared with each other to obtain the SI amongst them. Suppose that C_s, C'_s and $C_{s''}$ are the different clusters.
1) If $\mathrm{SI}(C_s, C'_s) \geq \mu \times n$, $\mathrm{SI}(C'_s, C_{s''}) \geq \mu \times n$, and $\mathrm{SI}(C_s, C_{s''}) \geq \mu \times n$, then C_s, C'_s and $C_{s''}$ are divided into one group.
2) If $\mathrm{SI}(C_s, C_{s'}) \geq \mu \times n$, $\mathrm{SI}(C_{s'}, C_{s''}) \geq \mu \times n$, but $\mathrm{SI}(C_s, C_{s''}) < \mu \times n$, then.
   i) If $\mathrm{SI}(C_s, C_{s'}) > \mathrm{SI}(C_{s'}, C_{s''})$, then, C_s and $C_{s'}$ are divided into one group.
   ii) If $\mathrm{SI}(C_s, C_{s'}) < \mathrm{SI}(C_{s'}, C_{s''})$, then, $C_{s'}$ and $C_{s''}$ are divided into one group.
   iii) If $\mathrm{SI}(C_s, C_{s'}) = \mathrm{SI}(C_{s'}, C_{s''})$, then.
      a) If $d(f(C_s), f(C_{s'})) > d(f(C_{s'}), f(C_{s''}))$, $C_{s'}$ and $C_{s''}$ are divided into one group.
      b) If $d(f(C_s), f(C_{s'})) < d(f(C_{s'}), f(C_{s''}))$, C_s and $C_{s'}$ are classified into one group.

Step 4: End.

V. RI-CRP FOR HESITANT FUZZY LSGDM PROBLEMS

In this section, we first present a consensus measure for hesitant fuzzy LSGDM problems, as well as the GCI of hesitant fuzzy LSGDM problems being given in Section V-A. Subsequently, the management processes for noncooperative clusters in the RI-CRP are given in Section V-B. Finally, the flowchart of RI-CRP for hesitant fuzzy LSGDM problems is provided in Section V-C.

A. Consensus Measure for Hesitant Fuzzy LSGDM Problems

Using the ARC method which is provided in Section IV, the remaining DMs can be divided into S $(1 \leq S \leq m)$ clusters, denoted C_s $(s = 1, 2, \ldots, S)$. Based on two rules: (a) DMs in the same cluster can be assigned the same weight because they have similar opinions, and (b) clusters that have large number DMs should be assigned larger weights based on the majority principle. The weight of DM d_φ in different clusters is calculated as follows:

$$\hat{\lambda}_\varphi = 1/o_s$$

where $d_\varphi \in C_s$, $\varphi = 1, 2, \ldots o_s$, o_s is the number of DMs in cluster C_s. The weight of cluster C_s can be obtained

$$w_s = o_s \Big/ \sum\nolimits_{s=1}^{S} o_s. \tag{9}$$

It is obvious that $0 < w_s \leq 1$ and $\sum_{s=1}^{S} w_s = 1$. Meanwhile, the decision matrix of cluster C_s can be obtained as $P^s = (p^s_{ij})_{n \times n}$

$$p^s_{ij} = \hat{\lambda}_\varphi \times \sum_{\varphi=1}^{o_s} h_{ij,\varphi}. \tag{10}$$

Similarly, the group decision matrix can be represented as $G^c = (g^c_{ij})_{n \times n}$

$$g^c_{ij} = \sum_{s=1}^{S} w_s p^s_{ij}. \tag{11}$$

In order to obtain the GCI, we give the following definition based on distance measure.

Definition 12: Let $P^s = (p^s_{ij})_{n \times n}$ be the decision matrix of cluster C_s, and $G^c = (g^c_{ij})_{n \times n}$ be the group decision matrix obtained by (10) and (11), respectively. Then the deviation degree between the individual cluster matrix P^s and the group decision matrix G^c is defined as follows:

$$\vartheta^s = d(P^s, G^c) = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} d(p_{ij}^s, g_{ij}^c) \tag{12}$$

where the $d(p_{ij}^s, g_{ij}^c) = \frac{1}{Y}\sum_{p_{ij,l}^s \in p_{ij}^s, g_{ij,l}^c \in g_{ij}^c} |p_{ij,l}^s - g_{ij,l}^c|$ and Y is the number of the elements in p_{ij}^s and g_{ij}^c. Furthermore, $p_{ij,l}^s$ and $g_{ij,l}^c$ are the lth elements in p_{ij}^s and g_{ij}^c, respectively.

It is clear that, ϑ^s has the following properties.

1) $0 \le \vartheta^s \le 1$.
2) $\vartheta^s = 0$, if and only if $P^s = G^c$, namely, there is no deviation between P^s and G^c.
3) $\vartheta^s = 1$, if and only if P^s and G^c are completely dissimilar, that is, they are contrary.

Accordingly, the weighted sum of all the ϑ^s, then the GCI can be defined as follows.

Definition 13: Let w_s and ϑ^s be the weight and deviation degree of cluster C_s, respectively. By the WAA operator, the GCI can be calculated as follows:

$$\text{GCI} = \sum_{s=1}^{S} w_s \vartheta^s. \tag{13}$$

Obviously, if $\text{GCI} = 0$, there is no deviation between clusters' opinions. Generally, we suppose that if $\text{GCI} \le \delta$ (where δ is consensus threshold, and $\delta \in [0,1]$), then the acceptable consensus is reached in the RI-CRP for hesitant fuzzy LSGDM problems.

B. Management Processes for Noncooperative Clusters in the RI-CRP

In the RI-CRP for hesitant fuzzy LSGDM problems, some clusters' opinions may be far from the collective opinion, and the DMs in them may not compromise despite the guidance of the moderator. We refer to the clusters' behavior that contains these DMs as "noncooperative behavior". To achieve a high level of consensus and improve the efficiency of the RI-CRP, these noncooperative clusters need to be managed reasonably. The detailed RI-CRP for hesitant fuzzy LSGDM problems can be seen in Algorithm 3.

Remark 8: In order to ensure the proper management of noncooperative DMs, the RI-CRP for hesitant fuzzy LSGDM problems proposes flexible penalties for noncooperative DMs. That is, in each iteration process, when the DM changes his/her cooperative attitude, we will reconsider his/her opinion and implement reasonable management methods for them. For example, if a DM is unwilling to make a compromise in an iteration step t, then, his/her weight will be punished to achieve a high level of consensus, as well as to improve the efficiency of CRPs for hesitant fuzzy LSGDM problems. But, in the iteration step $t+1$, if he/she follows the advice of moderator, and makes some modifications to his/her opinion, then we allow them to revise their opinions and will not punish them. In this way, we can guarantee that the mechanisms for managing noncooperative DMs are not manipulated, but open to all the DMs.

Algorithm 3 The RI-CRP for Hesitant Fuzzy LSGDM Problems.

Input: The alternatives set $X = \{x_1, x_2, \ldots, x_n\}$, the individual HFPRs $H_\varphi = (h_{ij,\varphi})_{n\times n}$ $(\varphi \in M)$, the DMs set $D = \{d_1, d_2, \ldots, d_m\}$, the initial weights vector of DMs $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)^T$, the maximum modification rounds $T_{\max} \ge 1$ in the reliability detection process, the maximum number of iterations $t_{\max} \ge 1$ in the RI-CRP, the predefined deviation threshold σ, consensus threshold δ. Meanwhile, the acceptable ordinal consistency parameter α, and the SI parameter μ.

Output: The group consensus level $\text{GCI}^{(t)}$, the number of iteration t, and the best alternative(s) selection.

Step 1: Utilize Algorithm 1 to detect the DM's opinion reliability. Let ψ_1 denote the DM set that provides completely contradictory opinions. Let ψ_2 denote the DM set that provides partial contradictory opinions, and ψ mean the DM set that provides acceptable ordinal consistency opinions. DMs in ψ_1 are directly rejected.

Step 2: For DMs in ψ_2 a moderator is introduced to persuade them to make some adjustments to their opinions, and then go to Step 1. If the DM is unwilling to compromise, or in the case of $T_{\max} \ge 1$, the revised preference relations that remain ordinally inconsistent will be rejected directly.

Step 3: Let Ω denote the DM set that provides reliable opinions, and ψ_3 denote the unreliable DMs set. For the DM d_φ in ψ, we use (7) to calculate d_φ's $\text{ORI}_{(\varphi)}$. If $\text{ORI}_{(\varphi)} \ge 0$, then d_φ belongs to Ω; otherwise, d_φ belongs to ψ_3.

Step 4: Similar to Step 2, for the DMs which belongs to ψ_3, the moderator has to persuade them to make modifications to their opinions. If they follow the suggestion go to Step 3. Otherwise, reject their opinions directly.

Step 5: Suppose that there are remaining $q(q \in M)$ DMs after Step 4. Using Algorithm 2, the remaining DMs are divided into S $(1 \le S \le m)$ small clusters $C_s(s = 1, 2, \ldots, S)$. By (9) and (10), the weights w_s and the decision matrix $P^{s(t)} = (p_{ij}^{s(t)})_{n\times n}$ of C_s can be obtained, respectively.

Step 6: The group decision matrix $G^{c(t)} = (g_{ij}^{c(t)})_{n\times n}$ can be obtained using (11). The $\text{GCI}^{(t)}$ can be calculated using (13). If $\text{GCI}^{(t)} \le \delta$, then go to Step 8; otherwise go to the next step.

Step 7: Find the cluster C_s which has the largest deviation from the collective opinion, namely, the one that has the maximal value of $\vartheta_{\max}^s$. Let the moderator persuade the DMs in this cluster to modify their preferences. For the noncooperative clusters which are unwilling to make any compromise, their weights will be punished. The principle of punishment is as follows:

$$w_s^{(t+1)} = w_s^{(t)} \times \zeta \tag{14}$$

where the ζ is the punishment parameter, and $\zeta \in [0,1]$. Let Q be the number of clusters excluding noncooperative clusters. Then, their weights are accordingly assigned, as follows:

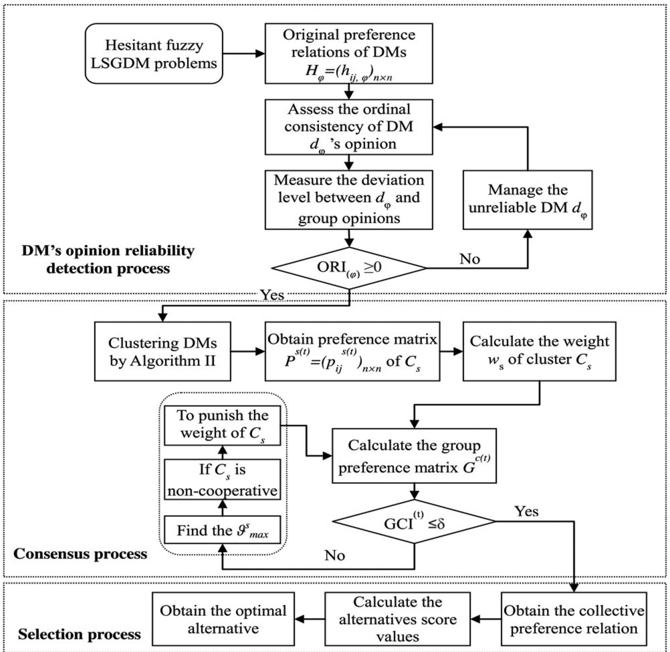

Fig. 2. Flowchart of the RI-CRP for hesitant fuzzy LSGDM problems.

$$w_{S-s}^{(t+1)} = w_{S-s}^{(t)} + \frac{(1-\zeta)w_s^{(t)}}{Q}. \tag{15}$$

set $t = t+1$ and go to Step 6.

Step 8: Let $\bar{G}^c = G^{c(t)}$, according to the (1), the alternatives score values of the collective opinion $\bar{G}^c$ can be calculated, and the best alternative(s) can be selected.

Step 9: End.

C. Flowchart of RI-CRP for Hesitant Fuzzy LSGDM Problems

Once the consensus among DMs is reached, the selection process based on the score function which was introduced in Section II is employed to obtain the group alternative ranking. Then, the final decision can be obtained. The detail RI-CRP for hesitant fuzzy LSGDM problems is depicted in Fig 2.

VI. Numerical Example and Analysis

In this section, a numerical example is provided to examine the utility and applicability of the proposed ARC method and the RI-CRP for hesitant fuzzy LSGDM problems. Meanwhile, the analyses of the ARC method are given in Section VI-B. Some comparisons and discussions are provided in Section VI-C.

A. Numerical Example

Suppose there are five alternatives $X = \{x_1, x_2, x_3, x_4, x_5\}$ for a hesitant fuzzy LSGDM problem, and 30 DMs $D = \{d_1, d_2, \ldots, d_{30}\}$ are involved. All the DMs use the normalized HFPRs to make the comparisons of alternatives and give their assessment information. Then, we have 30 original HFPRs

TABLE I
Original Preference Information Matrix of DMs

		x_1	x_2	x_3	x_4	x_5
d_1	x_1	{0.5}	{0.7,0.1,0.6}	{0.9,0.6,0.7}	{0.5,0.7,0.4}	{0.4,0.2,0.3}
	x_2	{0.3,0.9,0.4}	{0.5}	{0.6,0.8,0.7}	{0.7,0.4,0.3}	{0.1,0.2,0.3}
	x_3	{0.1,0.4,0.3}	{0.4,0.2,0.3}	{0.5}	{0.8,0.9,0.4}	{0.1,0.4,0.2}
	x_4	{0.5,0.3,0.6}	{0.3,0.6,0.7}	{0.2,0.1,0.6}	{0.5}	{0.2,0.4,0.3}
	x_5	{0.6,0.8,0.7}	{0.9,0.8,0.7}	{0.9,0.6,0.8}	{0.8,0.6,0.7}	{0.5}
d_2	x_1	{0.5}	{0.6,0.7,0.8}	{0.7,0.9,0.6}	{0.4,0.5,0.6}	{0.6,0.8,0.9}
	x_2	{0.4,0.3,0.2}	{0.5}	{0.5,0.6,0.7}	{0.3,0.4,0.2}	{0.8,0.7,0.6}
	x_3	{0.3,0.1,0.4}	{0.5,0.4,0.3}	{0.5}	{0.2,0.3,0.1}	{0.7,0.8,0.9}
	x_4	{0.6,0.5,0.4}	{0.7,0.6,0.8}	{0.8,0.7,0.9}	{0.5}	{0.7,0.9,0.6}
	x_5	{0.4,0.2,0.1}	{0.2,0.3,0.4}	{0.3,0.2,0.1}	{0.3,0.1,0.4}	{0.5}
				…		
d_{30}	x_1	{0.5}	{0.8,0.9,0.7}	{0.6,0.9,0.7}	{0.8,0.7,0.6}	{0.1,0.4,0.3}
	x_2	{0.2,0.1,0.3}	{0.5}	{0.5,0.7,0.8}	{0.6,0.8,0.7}	{0.1,0.3,0.2}
	x_3	{0.4,0.1,0.3}	{0.5,0.3,0.2}	{0.5}	{0.7,0.9,0.8}	{0.3,0.2,0.4}
	x_4	{0.2,0.3,0.4}	{0.4,0.2,0.3}	{0.3,0.1,0.2}	{0.5}	{0.2,0.4,0.1}
	x_5	{0.9,0.6,0.7}	{0.9,0.7,0.8}	{0.7,0.8,0.6}	{0.8,0.6,0.9}	{0.5}

TABLE II
Revised Preference Relation of the DM d_1

d_1	x_1	x_2	x_3	x_4	x_5
x_1	{0.5}	{0.7,0.1,0.6}	{0.9,0.6,0.7}	{0.6,0.7,0.4}	{0.4,0.2,0.3}
x_2	{0.3,0.9,0.4}	{0.5}	{0.6,0.8,0.7}	{0.7,0.6,0.3}	{0.1,0.2,0.3}
x_3	{0.1,0.4,0.3}	{0.4,0.2,0.3}	{0.5}	{0.8,0.9,0.4}	{0.1,0.4,0.2}
x_4	{0.4,0.3,0.6}	{0.3,0.4,0.7}	{0.2,0.1,0.6}	{0.5}	{0.2,0.4,0.3}
x_5	{0.6,0.8,0.7}	{0.9,0.8,0.7}	{0.9,0.6,0.8}	{0.8,0.6,0.7}	{0.5}

$H_\varphi = (h_{ij,\varphi})_{n\times n}$, $(\varphi = 1, 2, \ldots, 30)$, and the specific preference information can be seen in Table I.

Moreover, some related parameters are set as follows.

1) The maximum modification round is set to $T_{\max} = 3$, the acceptable ordinal consistency parameter is $\alpha = 0.5$, and the acceptable deviation threshold is preset as $\sigma = 0.12$ in the DM's opinion reliability detection process. Meanwhile, the SI parameter is $\mu = 0.5$ (as mentioned in Section IV).
2) The minimum level of consensus threshold is $\delta = 0.05$, and the maximum number of iterations is $t_{\max} = 3$ in the RI-CRP.
3) The punishment parameter for updating the weight is $\zeta = 0.5$.

Step 1: Apply Algorithm 1 to detect the DM's opinion reliability. Then, we have $\psi_1 = \{d_5, d_{10}, d_{12}\}$, $\psi_2 = \{d_1, d_3\}$, $\psi = \{d_\varphi\}$, $d_\varphi \in D$, and $d_\varphi \notin (\psi_1 \cup \psi_2)$.

Step 2: DMs in ψ_1 are rejected directly. A moderator is introduced to persuade the DMs in ψ_2 to make some adjustments. Only DM d_1 is willing to follow the advice of the moderator to revise his/her preference relation. At $T = 1$, d_1's revised opinion is ordinally consistent. Thus, DM d_1 is classified into ψ. In this step, we use the repairing ordinal inconsistency method provided in [37] to modify d_1. The specific steps of this method can be seen in Algorithm 2 from [37]. Furthermore, d_3's opinion is rejected directly due to he/she is a reluctance to make any modifications. The revised results of d_1 can be seen in Table II.

TABLE III
RESULTS OF DL AND ORI OF THE DMs $d_\varphi\,(\varphi \neq 3,5,10,12)$

d_φ	$DL_{(\varphi)}$	$ORI_{(\varphi)}$
d_1	0.1088	0.0112
d_2	0.2477	–0.1277
…	…	…
d_{30}	0.1000	0.0200

Step 3: The remaining 26 DMs' HFPRs are acceptably ordinally consistent and the weights of these DMs are $\lambda_\varphi = 1/26\ (\varphi \neq 3,5,10,12)$. By (5), the collective preference relation $H = (h_{ij})_{n\times n}$ can be obtained. Then, using (6) and (7), we have the $\mathrm{DL}_{(\varphi)}$ and $\mathrm{ORI}_{(\varphi)}$ of these DMs $d_\varphi\ (\varphi \neq 3,5,10,12)$. Furthermore, we have the unreliable DMs set $\psi_3 = \{d_2, d_4, d_7, d_{11}\}$. The detail results are shown in Table III.

Step 4: In ψ_3, the DMs d_2 and d_4 are not willing to change their opinions despite the guidance of the moderator. Thus, their opinions are rejected directly. The DMs d_7, and d_{11} are willing to make some modifications as suggested by the advice of moderator. In the $T_{\max} = 3$ limitation, their revised preference relations satisfy the reliability requirement. Then, $d_\varphi\ (\varphi = 7, 11)$ are classified into the reliable DMs set Ω. In this step, we take the principle which is provided in [38] to repair the deviation between the individual opinion and the collective opinion. We find the position i_τ and j_τ of the maximum elements $h_{i_\tau j_\tau,\varphi}$ of d_φ, where $h_{i_\tau j_\tau,\varphi} = \max_{i,j} d(h_{ij,\varphi}, h_{ij})(i,j \in N)$, and return H_φ to d_φ to construct a new HFPR $\bar{H}_\varphi = (\bar{h}_{ij,\varphi})_{n\times n}$ according to d_φ's new judgment, where

$$\bar{h}_{ij,\varphi} = \begin{cases} h_{ij}, & \text{if } i = i_\tau, j = j_\tau; \\ h_{ij,\varphi}, & \text{otherwise} \end{cases}.$$

This repairing method does not only can satisfy the reliability requirements, but also can preserve the initial DM's preference information as much as possible. The detail results are shown in Table IV.

Step 5: Applying Algorithm 2, the remaining 24 provide reliable opinions DMs are divided into four clusters

$$C_1 = \{d_1, d_7, d_9, d_{19}, d_{21}, d_{26}, d_{28}, d_{30}\},$$
$$C_2 = \{d_6, d_8, d_{13}, d_{16}, d_{17}, d_{22}, d_{23}, d_{27}, d_{29}\}$$
$$C_3 = \{d_{11}\},\ C_4 = \{d_{14}, d_{15}, d_{18}, d_{20}, d_{24}, d_{25}\}$$

By (9), we have the weight w_s of cluster C_s

$$w_1^{(0)} = 8/24, w_2^{(0)} = 9/24, w_3^{(0)} = 1/24, w_4^{(0)} = 6/24.$$

Then, the decision matrix $P^{s(0)} = (p_{ij}^{s(0)})_{n\times n}$ of cluster C_s can be obtained by (10). The detail results are omitted due to space constrictions.

Step 6: Utilizing (11), the collective decision matrix $G^{c(0)} = (g_{ij}^{c(0)})_{n\times n}$ can be obtained, and then by (12) and (13),

TABLE IV
REVISED RESULTS OF THE DMs d_7 AND d_{11}

d_φ	$h_{ij,\varphi}^{(T)}$	$ORI_{(\varphi)}^{(T)}$
d_7	$h_{34,7}^{(1)} \to \{0.6231, 0.7231, 0.5962\}$	$RI_{(7)}^{(1)} = -0.0085$
	$h_{43,7}^{(1)} \to \{0.3769, 0.2769, 0.4038\}$	$RI_{(7)}^{(2)} = 0.0229$
	$h_{24,7}^{(1)} \to \{0.6458, 0.7417, 0.7167\}$	
	$h_{42,7}^{(1)} \to \{0.3542, 0.2583, 0.2833\}$	
d_{11}	$h_{25,11}^{(1)} \to \{0.1731, 0.3615, 0.2962\}$	$RI_{(11)}^{(1)} = -0.0675$
	$h_{52,11}^{(1)} \to \{0.8269, 0.6385, 0.7038\}$	$RI_{(11)}^{(2)} = -0.0330$
	$h_{15,11}^{(2)} \to \{0.3958, 0.2125, 0.3167\}$	$RI_{(11)}^{(3)} = 0.00170$
	$h_{51,11}^{(2)} \to \{0.6042, 0.7875, 0.6833\}$	
	$h_{45,11}^{(3)} \to \{0.2167, 0.2125, 0.3125\}$	
	$h_{54,11}^{(3)} \to \{0.7833, 0.7875, 0.6875\}$	

we have the $\vartheta^{s(0)}$ and $GCI^{(0)}$ as follows:

$$\vartheta^{1(0)} = 0.0505, \vartheta^{2(0)} = 0.0440, \vartheta^{3(0)} = 0.1183, \vartheta^{4(0)} = 0.0560, GCI^{(0)} = 0.0523 > \delta = 0.05.$$

Then turn to Step 7.

Step 7: Cluster C_3 has the largest deviation from the collective, $\vartheta^{3(0)} = 0.1183$. DMs in C_3 in this round are unwilling to make any compromise. Thus, the weight of C_3 will be punished. After that we turn to Step 6, and we have $\mathrm{GCI}^{(1)} = 0.0509 > \delta = 0.05$, $\vartheta^{3(1)} = 0.1209$, so C_3 still needs to make adjustments. In the next round, the DMs in C_3 are willing to make some modifications. Similarly to the modification method in Step 4, we find the position i_τ and j_τ of the maximum elements $o_{i_\tau j_\tau,s}^{(t)}$ in C_s, which has the largest deviation from the group's opinion. That is, C_s has the maximal value of $\vartheta_{\max}^s$, where $o_{i_\tau j_\tau,s}^{(t)} = \max_{i,j} d(p_{ij}^{s(t)}, g_{ij}^{c(t)})\ (i,j \in N)$, return $P^{s(t)}$ to the cluster C_s to construct a new preference relation $P^{s(t+1)} = (p_{ij}^{s(t+1)})_{n\times n}$ according to C_s's new judgment, where

$$p_{ij}^{s(t+1)} = \begin{cases} g_{ij}^{c(t)}, & \text{if } i = i_\tau, j = j_\tau; \\ p_{ij}^{s(t)}, & \text{otherwise} \end{cases}.$$

Return to Step 6. After 3 modification rounds, we have $\mathrm{GCI}^{(3)} = 0.0498 < \delta = 0.05$, then the acceptable consensus is reached. The results of the RI-CRP are shown in Table V.

Finally, by (1), we can calculate the alternatives scores of $\mathrm{GCI}^{(3)}$ as follows:

$$s(x_1) = 2.4815, s(x_2) = 2.5277, s(x_3) = 2.3158$$
$$s(x_4) = 1.6844, s(x_5) = 3.4829.$$

Thus, we have $x_5 \succ x_2 \succ x_1 \succ x_3 \succ x_4$. Then, the optimal consensus alternative is x_5.

TABLE V
DETAIL RESULTS OF THE RI-CRP $(s = 1, 2, 3, 4)$

t	$w_s^{(t)}$	$p^{3(t)}$	$\vartheta^{s(t)}$, $GCI^{(t)}$
0	$w_1^{(0)} = 8/24$	$\cdots$	$\vartheta^{1(0)} = 0.0505$
	$w_2^{(0)} = 9/24$		$\vartheta^{2(0)} = 0.0440$
	$w_3^{(0)} = 1/24$		$\vartheta^{3(0)} = 0.1183$
	$w_4^{(0)} = 6/24$		$\vartheta^{4(0)} = 0.0560$
			$GCI^{(0)} = 0.0523$
1	$w_1^{(1)} = 0.3403$	$\cdots$	$\vartheta^{1(1)} = 0.0497$
	$w_2^{(1)} = 0.3819$		$\vartheta^{2(1)} = 0.0457$
	$w_3^{(1)} = 0.0208$		$\vartheta^{3(1)} = 0.1209$
	$w_4^{(1)} = 0.2569$		$\vartheta^{4(1)} = 0.0544$
			$GCI^{(1)} = 0.0509$
2	$w_1^{(2)} = 0.3403$	$p_{35}^{3(2)} \to \{0.2913, 0.2379, 0.3532\}$	$\vartheta^{1(2)} = 0.0497$
	$w_2^{(2)} = 0.3819$	$p_{53}^{3(2)} \to \{0.7087, 0.7621, 0.6468\}$	$\vartheta^{2(2)} = 0.0461$
	$w_3^{(2)} = 0.0208$		$\vartheta^{3(2)} = 0.0891$
	$w_4^{(2)} = 0.2569$		$\vartheta^{4(2)} = 0.0545$
			$GCI^{(2)} = 0.0504$
3	$w_1^{(3)} = 0.3403$	$p_{34}^{3(3)} \to \{0.6609, 0.7652, 0.6403\}$	$\vartheta^{1(3)} = 0.0492$
	$w_2^{(3)} = 0.3819$	$p_{43}^{3(3)} \to \{0.3391, 0.2348, 0.3597\}$	$\vartheta^{2(3)} = 0.0466$
	$w_3^{(3)} = 0.0208$		$\vartheta^{3(3)} = 0.0665$
	$w_4^{(3)} = 0.2569$		$\vartheta^{4(3)} = 0.0540$
			$GCI^{(3)} = 0.0498$

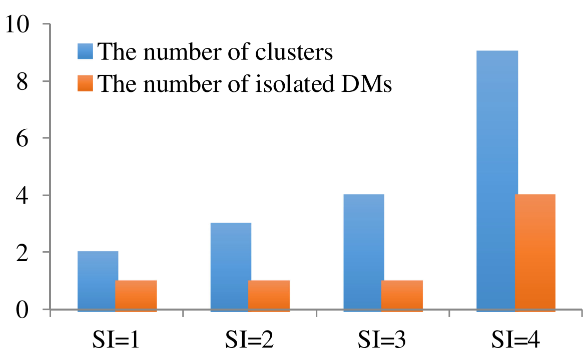

Fig. 3. ARC results with different SI.

B. Analysis of the ARC Method in the Numerical Example

In the numerical example of Section VI-A, considering the feasibility of the numerical example, we assume $\mu = 0.5$. By applying (8), we have $\mathrm{SI}(d_\varphi, d_m) = 3$. It denotes that d_φ and d_m shares the majority of same opinions, and can be classified into one group. Actually, all the $\mathrm{SI}(d_\varphi, d_m)$ possible values in this numerical example are $\{1, 2, 3, 4\}$. Thus, we carry out the same experiment with other different $\mathrm{SI} \in \{1, 2, 4\}$ in this section. Combining with the numerical example results in Section VI-A with $\mathrm{SI} = 3$, we draw the clustering results with $\mathrm{SI} \in \{1, 2, 4\}$ in Fig. 3.

From Fig. 3, we conclude that depending on the different values of SI, a different number of clusters can be obtained. Meanwhile, the higher the value of the SI, the greater the number of clusters will be. Obviously, the value of SI is directly affected by the selection of μ. Thus, in real hesitant fuzzy LSGDM problems, DMs can select the value of $\mu \in [1/n, (n-1)/n]$ based on the actual requirements, so as to obtain a reasonable clustering result as well as to improve the efficiency of the RI-CRP.

C. Comparisons and Discussions

In order to deal with the DM's hesitancy and fuzziness in LSGDM problems, we allow DMs use HFPRs to express their assessment information. And then we discuss and put forward a new DMs clustering method and consensus model, as well as to apply these methods to LSGDM problems. All of these aspects in existing literature need to be further improved.

Compared with the existing consensus models for LSGDM problems [4], [5], [7]–[10], the novel features in our proposed approach are described as follows.

1) We first present the idea to detect the reliability of DMs' assessment information for hesitant fuzzy LSGDM problems. As we previously mentioned, due to the large number of DMs participating in LSGDM problems, it is really hard to ensure that each DM's assessment information is reliable, and some of them may give dishonest or contradictory opinions. Once the unreliable opinions are used in the CRPs, the validity and reliability of the final decision will be greatly decreased for LSGDM problems. However, all the remaining CRPs are mostly based on the hypothesis that all the DMs' opinions provided for LSGDM problems are reliable, without detecting their reliability before the CRPs or the decision select process. For example, in the practical example in [9], the authors have given the DM d_{13}'s preference relations as follows:

$$P^{(13)} = \begin{pmatrix} 0.5 & 0.55 & 0.45 & 0.25 & 0.7 & 0.3 \\ 0.45 & 0.5 & 0.7 & 0.85 & 0.4 & 0.8 \\ 0.55 & 0.3 & 0.5 & 0.65 & 0.7 & 0.6 \\ 0.75 & 0.15 & 0.35 & 0.5 & 0.95 & 0.6 \\ 0.3 & 0.6 & 0.3 & 0.05 & 0.5 & 0.85 \\ 0.7 & 0.2 & 0.4 & 0.4 & 0.15 & 0.5 \end{pmatrix}.$$

Obviously, we can see that the DM d_{13} has given $p_{12}^{(13)} = 0.55 > 0.5$, and $p_{23}^{(13)} = 0.7 > 0.5$. Then, based on the ordinal consistency, i.e., transitivity condition, we should have $p_{13}^{(13)} > 0.5$. However, the DM d_{13} provides $p_{13}^{(13)} = 0.45 < 0.5$. Similarly, we have $p_{23}^{(13)} = 0.7 > 0.5$, and $p_{35}^{(13)} = 0.7 > 0.5$, but $p_{25}^{(13)} = 0.4 < 0.5$. Thus, we can conclude that DM d_{13} has contradictory views. That is, the opinion given by DM d_{13} is unreliable. However, this unreliable assessment information is directly utilized in the decision-making process, and then the reliability of the final decision will be decreased. In order to ensure the reliability of the final decision, the author needs to effectively deal with DMs' unreliable opinions. It is one of the goals of our model.

2) Different from the remaining clustering methods, we have proposed an innovative ARC clustering method, which is implemented based on DMs' alternative ranking, for hesitant fuzzy LSGDM problems. The ARC method can overcome the defect of some remaining clustering method, which is the clustering result(s) is less objective due to several clustering correlation coefficients need to preset.

VII. Conclusion

In this paper, we extend the HFPRs into LSGDM problems. Moreover, we present a novel DM's clustering method, as well as a new consensus model for hesitant fuzzy LSGDM. Compared with the existing methods for LSGDM problems, the improvements of this paper are given below.

1) A novel RI-CRP is proposed. By assessing the DMs' HFPRs ordinal consistency and measuring the deviation with from the collective opinion, the DM's ORI is proposed. It is used to easily detect unreliable DMs' opinions in the reliability detection process. For unreliable DMs, a moderator is introduced to give suggestions to them within the given number of rounds. By detecting the DM's opinion reliability and managing unreliable DMs, we can avoid the unreliable DMs involving the CRPs, thus ensuring that the final decision is reasonable and reliable.
2) The ARC method is given. To improve the objectiveness of clustering method, we present an ARC method, which is implemented to cluster DMs according to their alternative ranking. Moreover, the ARC method is utilized in the RI-CRP to improve the efficiency of hesitant fuzzy LSGDM problems.
3) Our RI-CRP for hesitant LSGDM problems implements a dynamic weight penalizing mechanism to deal with the noncooperative clusters. The implementation of the weight punishment makes the RI-CRP more efficient for hesitant LSGDM problems.

In real LSGDM problems, due to the different decision-making conditions and the more likely behaviors among DMs, the application efficiency of the proposed RI-CRP and ARC methods in the decision making processes will be affected. For example, in the emergency decision-making problems, an optimal decision is usually required to be made within a specific time frame. Thus, how to obtain a high level of consensus on the basis of ensuring that all the DMs' opinions are reliable is still a topic worth further discussion.

In addition, in some real LSGDM problems, due to time pressure, lack of knowledge, and the DM's limited experience related with the problem domain, DMs may provide the incomplete assessment information [20], [39]–[41]. Thus, in future work, we will extend the proposed ARC method and RI-CRP to the LSGDM with incomplete assessment information, as well as to further verify the validity of them.

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