Inácio Rodrigues - 485178

01. a) $F = K\dfrac{q_1 q_2}{r^2}$ $\qquad K = 9\cdot 10^9 \dfrac{N\cdot m^2}{C^2}$

$q_1 = 1nC$
$q_2 = 5nC$ $\Rightarrow$ $r_1 = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$= \sqrt{(2-1)^2+(0-(-3))^2+(4-7)^2}$

$= \sqrt{1^2+3^2+(-3)^2} = \sqrt{1+9+9} = \sqrt{19}$

$q_1 = 1nC$
$q_2 = -2nC$ $\Rightarrow$ $r_2 = \sqrt{(-3-1)^2+(0-(-3))^2+(5-7)^2}$

$= \sqrt{(-4)^2+3^2+(-2)^2} = \sqrt{16+9+4} = \sqrt{29}$

$$F_1 = \frac{(9\cdot 10^9)(1\cdot 10^{-9})(5\cdot 10^{-9})}{19} = \frac{45\cdot 10^{-9}}{19} = 2{,}37\cdot 10^{-9}\,N$$

$$F_2 = \frac{(9\cdot 10^9)(1\cdot 10^{-9})(-2\cdot 10^{-9})}{29} = \frac{-18\cdot 10^{-9}}{29} = -0{,}62\cdot 10^{-9}\,N$$

$$F_T = F_1 + F_2 = 1{,}75\cdot 10^{-9}\,N$$

b) $$E_1 = K\frac{q_1}{r_1^2} = \frac{(9\cdot 10^9)(5\cdot 10^{-9})}{19} = \frac{45\cdot 10^0}{19} = 2{,}37\,\frac{N}{C}$$

$$E_2 = K\frac{q_2}{r_2^2} = \frac{(9\cdot 10^9)(-2\cdot 10^{-9})}{29} = \frac{-18\cdot 10^0}{29} = -0{,}62\,\frac{N}{C}$$

$$E_T = E_1 + E_2 = 1{,}75\,\frac{N}{C}$$

06. $4 < r < 5$

$0 < \theta < 25^0$

$0,9\pi < \phi < 1,1\pi$

$\rho V = 10(r-4)(r-5)\,\text{sen}\,\theta \cdot \text{sen}\left(\frac{\phi}{2}\right)$

fora da região $\rho V = 0$

(r, θ, ϕ)

$$Q = 10 \int_{0,9\pi}^{1,1\pi} \int_{0}^{25^0} \int_{4}^{5} (r-4)(r-5)\,\text{sen}\,\theta \cdot \text{sen}\left(\frac{\phi}{2}\right) r^2\,\text{sen}\,\theta\, dr\, d\theta\, d\phi$$

$$= 10 \left[\left[\frac{r^5}{5} - \frac{9r^2}{4} + \frac{20r^3}{3} \right]_4^5 \cdot \left[\frac{\theta}{2} - \frac{1}{4}\,\text{sen}\,2\theta \right]_0^{25^0} \cdot \left[-2\cos\left(\frac{\theta}{2}\right) \right]_{0,9\pi}^{1,1\pi} \right]$$

$$= 10(3,39) \cdot ((0,0266) \cdot (0,626)$$

$Q = 0,5644\,C$

$\cong 0,57\,C$

07. $F = k \dfrac{q_1 q_2}{r^2} \hat{r}$

$Q_1 = 2\mu C$, $P_1 = (1, 2, 1)$

$Q_2 = -4\mu C$, $P_2 = (-1, 0, 2)$

$Q_3 = -3\mu C$, $P_3 = (2, 1, 3)$

$F_T = F_{13} + F_{23}$

$r_{13} = P_3 - P_1 = (2-1, 1-2, 3-1) = (1, -1, 2)$

$r_{13} = |r_{13}| = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}$

$\hat{r}_{13} = \left(\dfrac{1}{\sqrt{6}}, \dfrac{-1}{\sqrt{6}}, \dfrac{2}{\sqrt{6}} \right)$

$r_{23} = P_3 - P_2 = (2-(-1), 1-0, 3-2) = (3, 1, 1)$

$r_{23} = |r_{23}| = \sqrt{3^2 + 1^2 + 1^2} = \sqrt{9+1+1} = \sqrt{11}$

$\hat{r}_{23} = \left(\dfrac{3}{\sqrt{11}}, \dfrac{1}{\sqrt{11}}, \dfrac{1}{\sqrt{11}} \right)$

$F_{13} = \dfrac{(9 \cdot 10^9)(2 \cdot 10^{-6})(3 \cdot 10^{-6})}{6} = 9 \cdot 10^{-3} \left(\dfrac{1}{\sqrt{6}}, \dfrac{-1}{\sqrt{6}}, \dfrac{2}{\sqrt{6}} \right) N$

$F_{23} = \dfrac{(9 \cdot 10^9)(4 \cdot 10^{-6})(3 \cdot 10^{-6})}{11} = 9{,}82 \cdot 10^{-3} \left(\dfrac{3}{\sqrt{11}}, \dfrac{1}{\sqrt{11}}, \dfrac{1}{\sqrt{11}} \right) N$

$F_{Tx} = \dfrac{9}{\sqrt{6}} + \dfrac{29{,}46}{\sqrt{11}} = 12{,}56 \ N$

$F_{Ty} = \dfrac{-9}{\sqrt{6}} + \dfrac{9{,}82}{\sqrt{11}} = -0{,}71 \ N$

$F_{Tz} = \dfrac{18}{\sqrt{6}} + \dfrac{9{,}82}{\sqrt{11}} = 10{,}31 \ N$

09. a) $\rho_L = 12x^2$ $\qquad Q = \int_0^5 12x^2\,dx$

$0 < x < 5$

b) $Q \int_0^4 \rho_s \cdot 2\pi\rho\, dz = \int_0^4 (3z^2) \cdot 2\pi(3)\,dz$

$Q = 18\pi \int_0^4 z^2\,dz = 18\pi \left[\frac{z^3}{3}\right]_0^4 = 18\pi\left[\frac{64}{3}\right] = 384\pi$ nC

c) $Q = \int_0^{2\pi}\int_0^{\pi}\int_0^4 10r\,\text{sen}(\theta) \cdot r^2\,\text{sen}(\theta)\,dr\,d\theta\,d\phi$

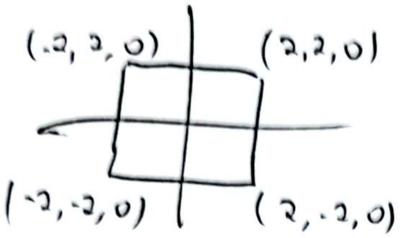
$\int_0^4 r^3\,dr = \left[\frac{r^4}{4}\right]_0^4 = \frac{256}{4} = 64$

$\int_0^{\pi} \text{sen}^2(\theta)\,d\theta = \frac{\pi}{2}$

$\int_0^{2\pi} d\phi = 2\pi$

$Q = 10 \cdot 64 \cdot \frac{\pi}{2} \cdot 2\pi = 640\pi^2$ C

10.



$$E = \frac{Kq}{r^2}\hat{a}_r \qquad K = \frac{1}{4\pi\varepsilon_0}$$

$P(0,0,6)$

$A(2,2,0)$ $\qquad |\bar{r}| = \sqrt{(0-2)^2 + (0-2)^2 + (6-0)^2} = 2\sqrt{11}$

$|r|^2 = 44$

$$\hat{a}_r = \frac{-2\hat{i} - 2\hat{j} + 6\hat{k}}{2\sqrt{11}} \qquad E_1 = \frac{K(2Q)}{44}\left[\frac{-2\hat{i} - 2\hat{j} + 6\hat{k}}{2\sqrt{11}}\right]$$

$$= \frac{KQ}{44\sqrt{11}}\left[-2\hat{i} - 2\hat{j} + 6\hat{k}\right]$$

$P(0,0,6)$

$B(-2,2,0)$ $\qquad |\bar{r}| = 2\sqrt{11}$

$|r|^2 = 44$

$$E_2 = \frac{KQ}{44}\left[\frac{2\hat{i} - 2\hat{j} + 6\hat{k}}{2\sqrt{11}}\right]$$

$$= \frac{KQ}{44\sqrt{11}}\left[\hat{i} - \hat{j} + 3\hat{k}\right]$$

$P(0,0,6)$

$C(-2,-2,0)$ $\qquad |\bar{r}| = 2\sqrt{11}$

$|r|^2 = 44$

$$E_3 = \frac{K(-2Q)}{44}\left[\frac{2\hat{i} + 2\hat{j} + 6\hat{k}}{2\sqrt{11}}\right]$$

$$= \frac{-KQ}{44\sqrt{11}}\left[2\hat{i} + 2\hat{j} + 6\hat{k}\right]$$

$P(0,0,6)$

$D(2,-2,0)$ $\qquad |\bar{r}| = 2\sqrt{11}$

$|r|^2 = 44$

$$E_4 = \frac{K(-Q)}{44}\left[\frac{-2\hat{i} + 2\hat{j} + 6\hat{k}}{2\sqrt{11}}\right]$$

$$= \frac{-K \cdot Q}{44\sqrt{11}}\left[-\hat{i} + \hat{j} + 3\hat{k}\right]$$

$$E_T = E_1 + E_2 + E_3 + E_4$$

$$= \frac{KQ}{44\sqrt{11}}\left[-\hat{i} - 3\hat{j} + 9\hat{k}\right] - \frac{KQ}{44\sqrt{11}}\left[\hat{i} + 3\hat{j} + 9\hat{k}\right]$$

$$= \frac{KQ}{44\sqrt{11}}\left[-2\hat{i} - 6\hat{j}\right]$$

$$= \frac{9 \cdot 10^{9} \cdot 15 \cdot 10^{-6}}{44\sqrt{11}}\left[-2\hat{i} - 6\hat{j}\right]$$

$$= 925{,}0916\left(-2\hat{j} - 6\hat{j}\right)$$

$$= -1850{,}18\left(\hat{i} + 3\hat{j}\right)\ \frac{N}{m}$$

12. $F(x,y,z) = \frac{k \cdot Q}{r^3} \cdot \hat{r}$

$k = 8{,}99 \cdot 10^9 \ \frac{Nm^2}{C^2}$

$Q = -10^{-8} C$

$$E(x,y,z) = \frac{k \cdot Q}{(x^2+y^2+z^2)^{\frac{3}{2}}} \cdot (x,y,z)$$

$P(1,1,2)$

$x = 1$

$y = 1$

$z = 2$

$$r = \sqrt{1^2+1^2+2^2} = \sqrt{6}$$

$$E(1,1,2) = \frac{8{,}99 \cdot 10^9 \cdot (-10^{-8})}{(\sqrt{6})^3} \cdot (1,1,2)$$

$$= \frac{-8{,}99 \cdot 10}{6\sqrt{6}} \cdot (1,1,2)$$

$$E(1,1,2) = \left(\frac{-8{,}99 \cdot 10}{6\sqrt{6}}, \frac{-8{,}99 \cdot 10}{6\sqrt{6}}, \frac{-17{,}98 \cdot 10}{6\sqrt{6}} \right) \frac{N}{C}$$