Structural Capacity Limits Beyond Local Refinement

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STATUS: DRAFT / IN PROGRESS
SCOPE: Structural limits of adaptive and multi-context refinement systems under bounded information capacity.
DEPENDENCIES: Shannon information theory; locality-limited computation; entropy accounting; refinement dynamics.
NON-CLAIMS: No empirical claims; no statements about specific algorithms or models; no impossibility for global-invariant, oracle, or unbounded-memory systems.

1 Background

Local refinement procedures form a broad class of reasoning and computation systems. Such systems update internal state through bounded-context observations and transformations, without access to exact global invariants. Classical results in finite model theory, descriptive complexity, and graph algorithms establish strong limitations on what fixed-resource local procedures can distinguish.

Recent capacity-based analyses refine these locality limits by explicitly accounting for the amount of information a system can retain across sequential updates.

2 Motivation

While single-pass or fixed-rule refinement systems exhibit clear capacity obstructions, modern reasoning systems often employ adaptive strategies that compose multiple refinement rules across interacting contexts.

This raises a natural question: *Do capacity-based obstructions persist when refinement itself is adaptive or multi-contextual?*

This manuscript isolates structural limits that remain invariant under such extensions.

3 Intuition

Adaptive refinement does not eliminate information constraints. At each step, a system must still compress observations into a bounded internal transcript. Even when multiple refinement channels interact, the total extractable information remains bounded by aggregate capacity.

As a result, problems whose solution spaces carry linear or superlinear entropy cannot be collapsed by adaptive refinement without exceeding system capacity.

4 Refinement Systems with Interaction

Definition 4.1 (Interacting Refinement System)**.** An interacting refinement system consists of a finite collection of local refinement processes whose internal states may influence one another through bounded-information channels, without access to global invariants or unbounded persistent memory.

Such systems generalize single-context refinement while preserving locality and capacity constraints.

5 Aggregate Transcript Capacity

Definition 5.1 (Aggregate Transcript Capacity)**.** The aggregate transcript capacity $\mathrm{TC}_{\mathrm{agg}}$ of an interacting refinement system is the supremum of mutual information it can retain, in the Shannon sense, about the problem state across all internal channels and refinement steps.

6 Entropy Constraints Under Adaptivity

Let P be a problem with solution-space entropy $H(P)$.

Lemma 6.1 (Capacity Additivity Bound)**.** *For any interacting refinement system, the total extractable information across all channels is bounded above by* $\mathrm{TC}_{\mathrm{agg}}$*.*

Lemma 6.2 (Adaptive Entropy Reduction Bound)**.** *Adaptive composition of refinement rules does not increase expected per-step entropy reduction beyond a constant factor depending only on* $\mathrm{TC}_{\mathrm{agg}}$*.*

Theorem 6.3 (Adaptive Entropy–Depth Lower Bound)**.** *For any interacting refinement system with bounded aggregate transcript capacity, solving a problem P with $H(P) = \Theta(n)$ requires $\Omega(n)$ total refinement depth, even under adaptive rule composition.*

7 Persistence of the Viotropic Regime

Definition 7.1 (Operational Mutual Information)**.** Let X denote the joint internal state of an interacting refinement system and O its joint observation channel. Define

$$I_{\mathrm{op}}(X; O) := H(O) - H(O \mid X).$$

Definition 7.2 (Viotropic Information)**.** Information is *viotropic* if $I_{\mathrm{op}}(X; O) = 0$, meaning it is operationally inaccessible despite internal presence.

Theorem 7.3 (Adaptive Viotropic Wall — Conditional)**.** *There exists $\kappa > 0$ such that for any interacting refinement system,*

$$H_{\mathrm{extractable}} > \kappa \cdot \mathrm{TC}_{\mathrm{agg}} \quad \Rightarrow \quad I_{\mathrm{op}}(X; O) = 0.$$

8 Consequences

Adaptive refinement does not circumvent structural capacity limits.

Corollary 8.1. *No interacting refinement system with bounded aggregate transcript capacity can reliably solve open-ended problems whose solution spaces exhibit unbounded entropy.*

9 Failure Modes

The obstruction described here fails if:

- The system acquires exact global invariants.
- Persistent memory or oracle access becomes unbounded.
- Aggregate transcript capacity scales superlinearly with problem size.

10 Relationship to Prior Work

This result:

- Extends single-context capacity obstructions to adaptive systems.
- Refines classical locality results into an explicit entropy framework.
- Remains orthogonal to training-scale, optimization, or heuristic explanations.
- Does not apply to kernel-verified proof assistants or symbolic systems with exact global checks.

References

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