URF Axiomatics: Numbered Laws, Entropy Ceiling, and Terminality

Inacio F. Vasquez

Gödel-Style Numbered Laws

Axiom 1 (URF-0.0 Finite Realizability). All information-bearing objects admit a finite realization.

Axiom 2 (URF-0.1 Locality). There exists r such that

$$Y_t = f_t(N_r(X), Y_{<t})$$

for all t.

Axiom 3 (URF-0.2 Capacity). There exists $C < \infty$ such that for all T,

$$I(X; Y_{1:T}) \leq C.$$

Axiom 4 (URF-0.3 Entropy Non-Amplification). For all t,

$$I(X; Y_t \mid Y_{<t}) \leq K$$

for a universal constant K.

Axiom 5 (URF-0.4 No Global Reconstruction). No admissible refinement reconstructs unbounded global structure from local views.

Axiom 6 (URF-0.5 Exchange Correlation Capacity Bound). For any systems A, B interacting only via a mediator M, if M is finite-dimensional or quasi-local with exponential clustering, then there exists a constant $\mathrm{Cap}(M) < \infty$ such that

$$\forall t : \quad I(A : B)_t \leq \mathrm{Cap}(M).$$

Axiom 7 (URF-0.6 Support Rigidity). For all r there exists R such that

$$\mathrm{FO}_r^k(x) \equiv \mathrm{FO}_r^k(y) \;\Rightarrow\; \mathrm{FO}_R^k(N_R(x)) \equiv \mathrm{FO}_R^k(N_R(y)).$$

Axiom 8 (URF-0.7 Irreversibility). Information erasure produces entropy at least $k_B \ln 2$ per bit.

Axiom 9 (URF-0.8 Non-Viotropy). No admissible process satisfies

$$\lim_{E \to 0} \frac{I}{E} = \infty.$$

Axiom 10 (URF-0.9 Rigidity–Collapse). Persistent local indistinguishability implies global structural collapse.

Law 3 from Law 2

Theorem 1 (URF-T.3 Entropy Ceiling)**.** *Assume URF-0.2. Then for all T and all $t \leq T$,*

$$I(X; Y_t \mid Y_{<t}) \leq C.$$

Proof. By the chain rule,

$$I(X; Y_{1:T}) = \sum_{i=1}^{T} I(X; Y_i \mid Y_{<i}),$$

with all summands nonnegative. Since the sum is at most C, each term is at most C. □

Support Rigidity as Conjecture

Conjecture 1 (URF-C.6)**.** *Axiom URF-0.6 holds for bounded-degree graphs and fixed k.*

Categorical Terminality

Definition 1. *Let* Obs *be the category whose objects are finite-capacity observers and whose morphisms are locality-preserving refinements.*

Definition 2. *Define $F : \mathrm{Obs} \to \mathbf{Set}$ by*

$$F(O) = \{\textit{global invariants extractable from } O \textit{ under capacity } C\}.$$

Theorem 2 (URF-T.0 Terminal Extractable Functor)**.** *For any functor $G : \mathrm{Obs} \to \mathbf{Set}$ respecting refinement and capacity, there exists a unique natural transformation $G \Rightarrow F$.*

Proof. Every element of $G(O)$ induces an extractable functional on transcripts. Capacity forces this functional to lie in $F(O)$. Refinement invariance gives naturality; uniqueness follows from agreement on all admissible transcripts. □

Corollary 1 (URF-T.1 Conditional Collapse)**.** *Assume URF-0.4, URF-0.6, and URF-C.6. Then URF-0.9.*