Cells Downwards and Spectral Rigidity toward RH

1 Spectral Setup

Let $\xi(s)$ denote the completed Riemann xi-function. It satisfies the functional equation

$$\xi(s) = \xi(1 - s).$$

Define the Hilbert space

$$\mathcal{H} = L^2 \left(\mathbb{R}, w(t)\, dt\right)$$

for an appropriate weight w induced by the Mellin transform of ξ.

2 Cell Decomposition

We decompose $\mathcal{H}$ into spectral cells

$$\mathcal{H} = \bigoplus_k \mathcal{C}_k$$

indexed by imaginary height bands.

3 Downward Transfer Operator

Define a transfer operator T mapping cells downward in height.

The key obstruction statement is:

Lemma 1 (Downward Rigidity Target)**.** *If a zero of $\xi(s)$ satisfies $\Re(s) \neq \frac{1}{2}$, then there exists a non-decaying mode in some cell $\mathcal{C}_k$ that is invariant under T.*

4 Missing Inequality

To complete the proof it suffices to prove a coercive estimate of the form

$$\|Tf\|^2 \leq \|f\|^2 - \delta \|f_\perp\|^2$$

for some $\delta > 0$ unless f is symmetric under $s \mapsto 1 - s$.