Structural Rigidity under Bounded Information Refinement

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STATUS: VERIFIED / STRUCTURAL STATEMENT
SCOPE: Information-theoretic rigidity of refinement-based inference under locality, capacity, and extensivity constraints.
DEPENDENCIES: Shannon information theory; locality; thermodynamic scaling.
NON-CLAIMS: No algorithmic optimality; no convergence guarantees; no dynamics-specific results.

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1 Background

Refinement-based inference procedures arise in measurement, control, and algorithmic reasoning. Such procedures operate by adaptively querying local observables and accumulating a transcript over time. When information gain per step is bounded, global reconstruction may be obstructed by structural limits independent of dynamics or geometry.

This manuscript isolates a minimal abstract spine capturing this obstruction.

2 Motivation

Under what conditions does bounded local refinement impose an unavoidable limit on globally extractable information?

3 Intuition

Each refinement step contributes only a bounded increment of mutual information. If the total information required by a task exceeds the cumulative capacity of the transcript, no adaptive strategy can succeed. This limitation persists regardless of computational power or dynamical sophistication.

4 Local Distinguishability

Definition 4.1 (State Space)**.** Let V be a finite vertex set and $\mathcal{H} = \bigotimes_{v\in V} \mathcal{H}_v$ a finite-dimensional Hilbert space. For $\Lambda \subseteq V$ and a density matrix ρ on $\mathcal{H}$, define the reduced state

$$\rho_\Lambda := \mathrm{Tr}_{V\setminus\Lambda}(\rho).$$

Definition 4.2 (Local Distinguishability (LD))**.** There exist constants $R < \infty$ and $\delta > 0$ such that for all distinct sites $x \neq y$,

$$\|\rho_{B_R(x)} - \rho_{B_R(y)}\|_1 \geq \delta.$$

5 Bounded Information Refinement

Definition 5.1 (Transcript Model)**.** Let X denote a global microstate. A refinement process produces a transcript $\tau_t = (O_1, \dots, O_t)$, where each O_t is the outcome of an adaptive local query.

Assumption 5.2 (Per-step Capacity Bound (CB))**.** There exists a constant $C < \infty$ such that for all $t \geq 1$,

$$I(X; O_t \mid \tau_{t-1}) \leq C.$$

Definition 5.3 (Cumulative Refinement Capacity)**.** For $T \geq 1$, define

$$I_T := \sum_{t=1}^{T} I(X; O_t \mid \tau_{t-1}), \qquad I := \sup_{T \geq 1} I_T.$$

Lemma 5.4 (Alphabet Bound)**.** *If each O_t takes values in a finite alphabet Σ, then*

$$I(X; O_t \mid \tau_{t-1}) \leq \log |\Sigma|.$$

Lemma 5.5 (Telescoping Bound)**.** *For any $T \geq 1$,*

$$I(X; \tau_T) = I_T \leq TC.$$

6 Thermodynamic Extensivity

Assumption 6.1 (Amenable Geometry)**.** Let (Λ_N) be an increasing exhaustion with $|\Lambda_N| \to \infty$ and finite-range Hamiltonians H_{Λ_N}. Assume the limit

$$e^* := \lim_{N \to \infty} \frac{1}{|\Lambda_N|} \lambda_{\max}(H_{\Lambda_N})$$

exists.

Assumption 6.2 (Non-Amenable Geometry)**.** In a non-amenable setting, assume there exists a subsequence (Λ_{N_k}) such that

$$e^* := \lim_{k \to \infty} \frac{1}{|\Lambda_{N_k}|} \lambda_{\max}(H_{\Lambda_{N_k}})$$

exists.

7 Structural Rigidity

Theorem 7.1 (Capacity Obstruction)**.** *Under LD and CB, for any horizon T and any derived observable $O = f(\tau_T)$,*

$$I(X; O) \leq I(X; \tau_T) \leq TC.$$

Corollary 7.2 (Infeasibility Beyond Demand)**.** *If a task requires $I(X; O) \geq D$ for some $D > 0$, then any refinement process with per-step capacity C requires*

$$T \geq \frac{D}{C}.$$

8 Relationship Map

This rigidity principle:

- isolates a universal information-theoretic obstruction,
- is independent of dynamics and geometry,
- serves as a reusable structural spine for domain-specific results.

References

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