Rank Dichotomy for Finite-Dimensional CAT(0) Cube Complexes

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Setting

Let X be a finite-dimensional CAT(0) cube complex. Let Γ act properly, essentially, and cocompactly on X by cubical isometries.

Definitions

The rank of X is the maximal k such that $\mathbb{R}^k$ embeds isometrically in X. An isometry $g \in \Gamma$ is rank-one if it is hyperbolic and no axis of g bounds an isometrically embedded Euclidean half-flat.

Theorem 1 (Rank Dichotomy)**.** *Assume X is finite-dimensional and irreducible. Then exactly one of the following holds:*

1. *X contains an isometrically embedded Euclidean 2-flat.*
2. *Γ contains a rank-one isometry.*

Status

This document is a canonical statement boundary. No proof is claimed in this repository.