XYSTEM Terminal Obstruction

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January 31, 2026

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Stress Test of Local Automorphism Groups Inacio F. Vasquez

Abstract

We give a complete stress test of the local–to–global presentation program for bounded–support automorphism groups. We show that for any infinite bounded–degree base graph G, the group of radius–R supported automorphisms $\Gamma_0(R)$ is an infinite–rank locally finite group. As a consequence, $\Gamma_0(R)$ is not finitely generated, not finitely presented, admits no uniform Dehn function, and cannot support any canonical holonomy or functorial presentation transfer. This yields a structural impossibility theorem for all XYSTEM–type locality principles.

1 Definition of the Local Automorphism Group

Let G be an infinite graph of uniformly bounded degree.

Definition 1. *For fixed $R \geq 1$, define*

$$\Gamma_0(R) := \{\sigma \in \mathrm{Aut}(G) \mid \mathrm{supp}(\sigma) \subseteq B_R(x) \textit{ for some } x \in V(G)\}.$$

This is the group of finitary automorphisms whose support lies in a ball of radius R.

2 Structural Decomposition

Proposition 1. *For any infinite bounded–degree graph G,*

$$\Gamma_0(R) \cong \bigoplus_{x \in V(G)} \mathrm{Sym}(B_R(x)).$$

Proof. Each automorphism in $\Gamma_0(R)$ has support contained in exactly one translate of $B_R(e)$. Distinct translates are disjoint and commute. Hence the group decomposes as a direct sum of finite symmetric groups. □

3 Test 1: Finite Generation

Theorem 1. $\Gamma_0(R)$ *is not finitely generated.*

Proof. Assume $S \subset \Gamma_0(R)$ finite generates $\Gamma_0(R)$. Then the union of supports of elements of S is finite. Choose y outside this union. Then $\mathrm{Sym}(B_R(y))$ is a subgroup disjoint from $\langle S \rangle$, contradiction. □

4 Test 2: Finite Presentability

Theorem 2. $\Gamma_0(R)$ *is not finitely presented.*

Proof. $\Gamma_0(R)$ is a strictly increasing union of finite groups. Any finitely presented locally finite group must stabilize at a finite stage. Hence $\Gamma_0(R)$ cannot be finitely presented. $\square$

5 Test 3: Dehn Function

Theorem 3. $\Gamma_0(R)$ *admits no finite Dehn function.*

Proof. Any presentation must encode infinitely many independent symmetric factors. Let w_n be a commutator in $\mathrm{Sym}(B_R(x_n))$. These cannot be filled using bounded relations. Thus filling area diverges faster than any recursive function. $\square$

6 Test 4: Holonomy Failure

Assume a surjective holonomy map

$$\Phi : \pi_1(X_R/\Gamma_0(R)) \twoheadrightarrow \mathrm{Isom}(Lk_R(e)).$$

But $\pi_1(X_R/\Gamma_0(R))$ is finitely generated while $\mathrm{Isom}(Lk_R(e))$ has infinite rank. No such surjection exists.

7 Test 5: Functoriality Failure

Suppose a section $s : E(Lk_R(e)) \to \Gamma_0(R)$ exists. Two embeddings of the same edge into disjoint translates yield different group elements representing the same geometric edge. Hence no naturality condition can hold.

8 Test 6: Spectral Obstruction

Even if each local factor is CAT(0), the global Laplacian decomposes as

$$\Delta = \bigoplus_x \Delta_x.$$

Hence $\lambda_1(\Delta) = 0$ with infinite multiplicity. Local curvature does not propagate globally.

9 Terminal Theorem

Theorem 4 (Structural Impossibility)**.** *No local–to–global presentation principle exists for bounded–support automorphism groups on infinite graphs. In particular, XYSTEM, LPL, and all holonomy–based locality frameworks are algebraically impossible in ZFC.*

10 Conclusion

The obstruction is not technical but structural: $\Gamma_0(R)$ is an infinite direct sum of finite groups and therefore too large to admit any finite, canonical, or functorial presentation theory.