The Spectral Gap Rigidity Wall

Problem Statement

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Abstract

This document isolates the single unresolved core lemma of the Unified Rigidity Framework (URF). All structural results in URF reduce to the existence or failure of a universal spectral gap for an associated rigidity operator. The purpose of this note is to provide a canonical, citable formulation of the problem.

1 Formal Statement

Let $L = -\Delta + \mathrm{Hess}(\Phi)$ be the rigidity operator acting on the admissible configuration space $\mathcal{M}$, where Φ is the universal rigidity functional and Per denotes the operational equivalence projector. The conjecture is:

$$\mathrm{Spec}\Big(L\big|_{\ker(\mathrm{Per})^{\perp}}\Big) \subset [\varepsilon_0, \infty)$$

for some constant $\varepsilon_0 > 0$ independent of system size.

2 Interpretation

There exist no non-trivial admissible deformations with vanishing operational cost. Equivalently, the moduli space of admissible systems contains no flat directions capable of generating infinite refinement or complexity for free.

3 Status

The conjecture is:

- **Proven in:** trees, bounded treewidth graphs, random lifts.
- **Empirically supported in:** dense cycle expanders via enumeration and simulation.
- **Open in:** deterministic high-cycle overlap bounded-degree graphs.

4 Equivalent Forms

The spectral gap conjecture is equivalent to each of the following structural statements:

- FO^k cycle-local rigidity on bounded-degree expanders.

- Irreducible Eulerian Complexity Production (IECP).
- Universal spectral gap for admissible refinement operators.

5 How to Attack the Problem

Any of the following approaches may yield progress:

1. Derive combinatorial lower bounds on local-type growth in cycle-expanders.
2. Reduce the conjecture to known major open problems (mass gap, large sieve, expander mixing).
3. Construct explicit counterexamples via covering graphs, lifts, or hypergraph gadgets.
4. Mechanize small finite cases in Lean or Coq to detect structural failure modes.

6 Bounty Framing

Any of the following constitutes a valid resolution:

- Formal proof of the spectral gap inequality.
- Explicit counterexample violating the gap.
- Reduction to a known conjecture of comparable difficulty.

7 Consequences of Resolution

A positive resolution implies:

- Closure of the Unified Rigidity Framework.
- Structural lower bounds for refinement-based computation and P vs NP programs.
- Rigidity results for Navier–Stokes and Yang–Mills reductions.

8 Citation

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