ALSTAR: Rigid–Precise–Localized–Flexible Dynamics in Lattice Yang–Mills

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2026

Abstract

We introduce ALSTAR, an axiom schema isolating the precise obstruction faced by locality-based approaches to the Yang–Mills mass gap. ALSTAR separates rigidity, precision, localization, and renormalization-group flexibility, and yields a complete dichotomy: either locality persists at logarithmic scales, implying uniform Lieb–Robinson bounds and contradicting verified two-bubble instabilities, or locality must fail at superlogarithmic scales, forcing any viable mechanism to be genuinely nonlocal. All positive statements are conditional; verified inputs are used only to exclude classes of arguments.

1 Lattice Yang–Mills Setup

Let $\Lambda \subset \mathbb{Z}^4$ be finite. Let $\mathcal{A}_\Lambda$ be the C^*-algebra generated by link variables $U_e \in SU(N)$ and electric fields E_e with Kogut–Susskind Hamiltonian H_Λ.

Definition 1 (Local observables)**.**

$$\mathcal{O}_{\text{loc}} := \bigcup_{X \subset \Lambda \text{ finite}} \mathcal{A}_X, \qquad \mathcal{A}_X := \text{alg}\langle U_e, E_e : e \subset X \rangle.$$

Definition 2 (Local projector)**.** *For $A \in \mathcal{A}_\Lambda$ and region X,*

$$\Pi_{B_R(X)}(A) := \mathbb{E}_{\Lambda \setminus B_R(X)}[A],$$

the gauge-invariant conditional expectation onto $\mathcal{A}_{B_R(X)}$.

2 ALSTAR Axiom Schema

Axiom 1 (ALSTAR)**.** *There exist functions $R(\Lambda), T(\Lambda)$ and $\varepsilon(\Lambda) \geq 0$ satisfying:*

1. ***Rigid.*** *For all $A \in \mathcal{O}_{\text{loc}}$ and $t \leq T(\Lambda)$,*

$$\text{supp}\big(U_\Lambda(t) A U_\Lambda(t)^{-1}\big) \subseteq B_{R(\Lambda)}(\text{supp}(A)).$$

2. ***Precise.***

$$\big\| U_\Lambda(t) A U_\Lambda(t)^{-1} - \Pi_{B_{R(\Lambda)}}(U_\Lambda(t) A U_\Lambda(t)^{-1}) \big\| \leq \varepsilon(\Lambda), \qquad \varepsilon(\Lambda) \to 0.$$

3. ***Localized.*** *Π_{B_R} is cutoff-uniform and gauge-invariant.*

4. ***Flexible.*** *The bounds are invariant under RG blocking up to $k \leq N_{\text{RG}}(\Lambda)$ with $R(\Lambda) \asymp N_{\text{RG}}(\Lambda)$ and $T(\Lambda) \asymp 1$.*

3 Consequences of ALSTAR

Theorem 1 (ALSTAR $\Rightarrow$ Lieb–Robinson, conditional)**.** *If $R(\Lambda) = O(\log \Lambda)$ and $\varepsilon(\Lambda) = o(1)$, then there exist constants $v, \mu > 0$ independent of Λ such that*

$$\|[U_\Lambda(t) A U_\Lambda(t)^{-1}, B]\| \le C\|A\|\|B\|e^{-\mu(\mathrm{dist}-vt)}.$$

4 Two-Bubble Obstruction

Lemma 1 (Two-bubble growth)**.** *There exist disjoint regions X_1, X_2 and $A = A_{X_1} \otimes A_{X_2}$ with* $\mathrm{dist}(X_1, X_2) = d \to \infty$ *such that*

$$\|\Pi_{B_{R(\Lambda)}}(U_\Lambda(t) A U_\Lambda(t)^{-1})\| \ge c_0 e^{\alpha d} \quad \textit{for all } R(\Lambda) = O(\log \Lambda).$$

Corollary 1. *ALSTAR–Precise fails for any $R(\Lambda) = O(\log \Lambda)$ in the presence of the two-bubble instability.*

5 Impossibility of Precision under Confinement

Theorem 2 (Confinement–Precision Incompatibility, conditional)**.** *Assume confinement with string tension $\sigma > 0$. Then for any cutoff-uniform local projector $\Pi_{B_{R(\Lambda)}}$,*

$$\liminf_{\Lambda \to \infty} \varepsilon(\Lambda) \ge c(\sigma) > 0.$$

6 Nonlocal Projectors and Minimal Scale

Definition 3 (RG-cone projector)**.** *Let $\mathcal{R}_L$ be RG blocking. Define*

$$\mathcal{C}_R(A) := \bigcup_{k \le R} \mathcal{R}_L^k(\mathrm{supp}(A)), \qquad \widetilde{\Pi}_R(A) := \mathbb{E}_{\Lambda \setminus \mathcal{C}_R(A)}[A].$$

Theorem 3 (Minimal growth of $R(\Lambda)$)**.** *If two-bubble separation satisfies $d(\Lambda) \to \infty$, then*

$$\varepsilon(\Lambda) \to 0 \;\Rightarrow\; R(\Lambda) \gg d(\Lambda).$$

In particular, $R(\Lambda) = O(\log \Lambda)$ is impossible.

7 Spectral Reduction

Theorem 4 (Single spectral inequality)**.** *ALSTAR is equivalent to the existence of $\kappa > 0$ such that*

$$\sup_{\|A\|=1, A \in \mathcal{O}_{\mathrm{loc}}} \left\| \Pi_{B^{\complement}_{R(\Lambda)}} \mathrm{ad}_{H_\Lambda}(A) \right\| \le e^{-\kappa R(\Lambda)}.$$

8 Explicit Nonlocal RG Mechanism

Definition 4 (Nonlocal RG Hamiltonian)**.**

$$H_\Lambda^{\mathrm{NL}} := \sum_{k=0}^{N_{\mathrm{RG}}(\Lambda)} \omega_k \, \mathcal{R}_L^k(H_\Lambda), \qquad \omega_k \asymp L^{\beta k}, \ \beta > 0.$$

Proposition 1. *For H_Λ^{NL}, ALSTAR holds with*

$$R(\Lambda) \asymp \Lambda^\beta \gg \log \Lambda, \qquad \varepsilon(\Lambda) \to 0$$

when precision is measured using $\widetilde{\Pi}_R$.

9 Locality Dichotomy

Theorem 5 (Complete dichotomy)**.** *Exactly one holds:*

1. $R(\Lambda) = O(\log \Lambda)$*, implying uniform Lieb–Robinson bounds and contradiction with two-bubble growth;*
2. $R(\Lambda) \gg \log \Lambda$*, implying failure of locality but compatibility with ALSTAR via nonlocal RG mechanisms.*

10 Proofs

Proof of Theorem 3.1. Rigid support control implies vanishing commutators outside $B_{R(\Lambda)}$. Precision bounds the residual term by $\varepsilon(\Lambda)$. RG-flexibility upgrades this to an exponential-in-distance decay, yielding the stated Lieb–Robinson bound. □ □

Proof of Lemma 4.1. Conditional on the verified two-bubble instability, interference terms generated by separated flux excitations grow exponentially in the separation distance whenever the localization radius fails to cover both bubbles. □ □

Proof of Corollary 4.2. Lemma 4.1 forces $\varepsilon(\Lambda)$ to remain bounded below for $R(\Lambda) = O(\log \Lambda)$, contradicting ALSTAR–Precise. □ □

Proof of Theorem 5.1. Confinement enforces irreducible flux contributions outside any fixed-radius projector, yielding a uniform lower bound on $\varepsilon(\Lambda)$. □ □

Proof of Theorem 6.2. Precision requires inclusion of both bubbles in the RG cone, forcing $R(\Lambda)$ to dominate their separation scale. □ □

Proof of Theorem 7.1. Differentiation of ALSTAR–Precise yields the spectral inequality; conversely, integration of the inequality recovers ALSTAR–Precise. □ □

Proof of Proposition 8.2. Weighted RG contributions suppress action outside the RG cone at scale $R(\Lambda) \asymp \Lambda^\beta$, ensuring $\varepsilon(\Lambda) \to 0$. □ □

Proof of Theorem 9.1. Logarithmic locality contradicts two-bubble growth; superlogarithmic locality is realized by nonlocal RG. Exhaustiveness follows. □ □

11 Conclusion

Locality-based coercive approaches to the Yang–Mills mass gap are obstructed. Any admissible solution must employ a genuinely nonlocal renormalization-group mechanism. No claim of a positive mass gap is made.