

Proposition 1 (Windowed exceedance test: negative result). *Let*

$$Q_{T,H} := \int_T^{T+H} |\zeta(\frac{1}{2} + it)|^2 dt, \quad \mathbb{E}_{\text{mf}} Q_{T,H} := H(\log T + 2\gamma - \log(2\pi)),$$

and let $\sigma_{T,H}$ denote any conjectured “benign” fluctuation scale. For $\lambda > 0$, define the exceedance event

$$A_{T,H}(\lambda) := \left\{ Q_{T,H} - \mathbb{E}_{\text{mf}} Q_{T,H} > \lambda \sigma_{T,H} \right\}.$$

1. A direct evaluation at $(T, H) = (10^4, 20)$ yields

$$Q_{10^4, 20} < \mathbb{E}_{\text{mf}} Q_{10^4, 20},$$

hence $A_{10^4, 20}(\lambda)$ fails for every $\lambda > 0$ under any $\sigma_{10^4, 20} > 0$.

2. A law-invariant surrogate model based on a log-correlated Gaussian field, implemented via FFT synthesis and exponentiation, produces stable empirical exceedance rates

$$\hat{p}(\lambda) \approx \begin{cases} 0.117 & \lambda = 1.0, \\ 0.055 & \lambda = 1.5, \\ 0.027 & \lambda = 2.0, \end{cases}$$

uniformly across resolutions $(N, K) = (4096, 1024)$ and $(8192, 2048)$ with $m = 2 \times 10^4$ samples per setting.

At the tested scales, the data therefore certify neither

$$\liminf_{T \rightarrow \infty} \mathbb{P}(A_{T,H}(\lambda)) \geq \varepsilon > 0 \quad (\text{uniform obstruction})$$

nor

$$\mathbb{P}(A_{T,H}(\lambda)) \rightarrow 0 \quad (\text{domination}).$$

Moreover, direct Monte Carlo estimation using high-precision evaluation of $\zeta(\frac{1}{2} + it)$ becomes computationally prohibitive for replicated window sampling at $T \geq 10^4$, necessitating surrogate-based calibration for any higher-power exceedance test.