

Area Law and Transfer Hamiltonian: Structural Bridge

1 Wilson Loop as Transfer Matrix Expectation

Let $W(R, T)$ be a rectangular Wilson loop in $R \times T$ geometry.

By reflection positivity and the transfer-matrix construction,

$$\langle W(R, T) \rangle = \langle \Omega, \Pi_R e^{-TH} \Pi_R \Omega \rangle,$$

where H is the transfer Hamiltonian.

1.1 Spectral Decomposition

Using the spectral theorem:

$$\langle W(R, T) \rangle = \sum_n |\langle n | \Pi_R \Omega \rangle|^2 e^{-E_n T}.$$

2 Exponential Decay and Spectrum

If for fixed R one has

$$\langle W(R, T) \rangle \leq C e^{-mT}$$

for all $T \geq 0$, then

$$\inf(\sigma(H) \setminus \{0\}) \geq m.$$

3 Area Law Assumption

Assume spatial area law:

$$\langle W(R, T) \rangle \leq e^{-\sigma RT}.$$

Define the static potential $V(R)$ via

$$\langle W(R, T) \rangle \sim e^{-V(R)T}.$$

Area law implies

$$V(R) \geq \sigma R.$$

4 Target Inequality

We attempt to derive

$$\inf(\sigma(H) \setminus \{0\}) \geq \sigma.$$