

## Exponential Cluster Decay from Uniform Spectral Gap

Let  $H$  be a self-adjoint Hamiltonian on  $\mathcal{H}$  with

$$H \geq \gamma(I - P_0), \quad \gamma > 0,$$

where  $P_0$  is the vacuum projection.

**Theorem 1** (Exponential Decay of Connected Correlations). *Let  $A, B$  be bounded local observables with spatial separation  $r$ . Then*

$$|\langle \Omega, AB\Omega \rangle - \langle \Omega, A\Omega \rangle \langle \Omega, B\Omega \rangle| \leq C \|A\| \|B\| e^{-\gamma r},$$

where  $C$  depends only on locality constants.

**Sketch of Proof.** Using spectral calculus,

$$e^{-tH}(I - P_0) \leq e^{-\gamma t}(I - P_0).$$

For spacelike-separated operators one writes

$$\langle \Omega, AB\Omega \rangle - \langle \Omega, A\Omega \rangle \langle \Omega, B\Omega \rangle = \langle \Omega, Ae^{-rH}(I - P_0)B\Omega \rangle.$$

Applying the spectral bound yields exponential decay in  $r$ . □