Conditional Area Law from a Uniform Bakry–Émery Gap

1 Setting

Let $\mu_{a,L}^{Q}$ denote the finite-volume Wilson lattice Yang–Mills measure on the gauge quotient Q. Let Γ denote the carré-du-champ of the associated reversible generator $\mathcal{L}_{a,L}^{Q}$.

2 Uniform Gap Assumption

Lemma 1 (Uniform Bakry–Émery Gap)**.** *There exists $\kappa > 0$ such that for all lattice spacings a, volumes L, and all F in the domain of Γ,*

$$\int \Gamma(F)\, d\mu_{a,L}^{Q} \;\geq\; \kappa \operatorname{Var}_{\mu_{a,L}^{Q}}(F).$$

Equivalently,

$$\inf_{a,L} \lambda_{\min}\Big(-\mathcal{L}_{a,L}^{Q} \restriction (\mathbf{1})^{\perp} \Big) \geq \kappa > 0.$$

3 Clustering Consequence

Theorem 1 (Clustering from Uniform Gap)**.** *Assume Lemma 1. Then correlations of bounded local gauge-invariant observables decay exponentially with separation, uniformly in L.*

4 Chessboard Input

Lemma 2 (Chessboard Estimate)**.** *Assume a reflection-positivity / chessboard inequality sufficient to convert exponential clustering of plaquette observables into an area law bound for Wilson loops.*

5 Conditional Area Law

Theorem 2. *Assume Lemma 1 and Lemma 2. Then Wilson loop expectations satisfy an area law bound uniformly in L.*

6 Numerical Note

Discrete Laplacian tests showing L^{-2} scaling establish that derivative contributions alone cannot produce a uniform spectral gap. Any observed L^0 lower bound arises from an explicit constant shift (mass term). The existence of a true mass mechanism therefore reduces to Lemma 1.