

Exponential Cluster Decay from Uniform Spectral Gap

Let H be a self-adjoint Hamiltonian on \mathcal{H} with

$$H \geq \gamma(I - P_0), \quad \gamma > 0,$$

where P_0 is the vacuum projection.

Theorem 1 (Exponential Decay of Connected Correlations). *Let A, B be bounded local observables with spatial separation r . Then*

$$|\langle \Omega, AB\Omega \rangle - \langle \Omega, A\Omega \rangle \langle \Omega, B\Omega \rangle| \leq C \|A\| \|B\| e^{-\gamma r},$$

where C depends only on locality constants.

Sketch of Proof. Using spectral calculus,

$$e^{-tH}(I - P_0) \leq e^{-\gamma t}(I - P_0).$$

For spacelike-separated operators one writes

$$\langle \Omega, AB\Omega \rangle - \langle \Omega, A\Omega \rangle \langle \Omega, B\Omega \rangle = \langle \Omega, Ae^{-rH}(I - P_0)B\Omega \rangle.$$

Applying the spectral bound yields exponential decay in r . \square