

# Area Law and Transfer Hamiltonian: Structural Bridge

## 1 Wilson Loop as Transfer Matrix Expectation

Let  $W(R, T)$  be a rectangular Wilson loop in  $R \times T$  geometry.

By reflection positivity and the transfer-matrix construction,

$$\langle W(R, T) \rangle = \langle \Omega, \Pi_R e^{-TH} \Pi_R \Omega \rangle,$$

where  $H$  is the transfer Hamiltonian.

### 1.1 Spectral Decomposition

Using the spectral theorem:

$$\langle W(R, T) \rangle = \sum_n |\langle n | \Pi_R \Omega \rangle|^2 e^{-E_n T}.$$

## 2 Exponential Decay and Spectrum

If for fixed  $R$  one has

$$\langle W(R, T) \rangle \leq C e^{-mT}$$

for all  $T \geq 0$ , then

$$\inf(\sigma(H) \setminus \{0\}) \geq m.$$

## 3 Area Law Assumption

Assume spatial area law:

$$\langle W(R, T) \rangle \leq e^{-\sigma R T}.$$

Define the static potential  $V(R)$  via

$$\langle W(R, T) \rangle \sim e^{-V(R)T}.$$

Area law implies

$$V(R) \geq \sigma R.$$

## 4 Target Inequality

We attempt to derive

$$\inf(\sigma(H) \setminus \{0\}) \geq \sigma.$$