Energy, Work, and Power

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1 Introduction

Forces presented us a way of analyzing systems to see how they would evolve. Another tool that we can use is *energy*. It is rather difficult to define the term due to its abstraction, and so it is best understood in example situations. In this lecture we will be examining *mechanical energy*, which consists of two parts - kinetic and potential energy. Kinetic energy is derived from movement, while potential energy comes from being at a particular position. Note that energy is measured in *joules*.

2 Types of Energy

- Kinetic Energy (energy due to movement) - $\frac{1}{2}mv^2$
- Gravitational Potential Energy - mgh
- Spring Potential Energy - $\frac{1}{2}kx^2$

3 Work and Power

Work is the process of applying a force through a distance to change the energy of an object, or

$$W = F_{avg}\Delta d_{parallel}$$

where $d_{parallel}$ is the component of displacement parallel to the force F. We can see from this definition that we can also define work to be the area under a displacement-force graph. The concept of work can also be defined using the **work-kinetic energy theorem** which states that the work done is equal to the change in kinetic energy, or

$$W_{net} = \Delta K = -\Delta U$$

Power is simply defined to be the amount of work done over a period of time measured in joules per second or *watts*

$$P = \frac{W}{t} = \frac{F\Delta d}{t} = Fv$$

4 Conservation of Energy

Now that we understand how to compute energy, we now need to figure out how to use it to describe how a system evolves. When **conservative forces**, such as gravity and the spring force, act on a system, the total mechanical energy in the system remains constant, or

$$K_i + U_i = K_f + U_f$$

When **nonconservative forces**, such as friction, act on the system, some mechanical energy is lost, and so energy is not conserved. Note that the *total* energy is conserved, but not mechanical. In the case of friction, kinetic energy is converted to thermal energy.

5 Problems

1. A 1000 kilograms satellite completes a uniform circular orbit of radius 8.79×10^6 meters as measured from the center of Earth. The mass of Earth is approximately 6.0×10^{24} kilograms, and the universal gravity constant is approximately 6.67×10^{-11} $N \cdot m^2/kg^2$. Determine the work done by gravity as the satellite completes one full orbit around Earth.

2. A 3.0 kg block slides down a frictionless 30 degree incline. The mass is initially at rest at the top of the incline. Determine (a) the work done by gravity and (b) the final speed of the mass at the bottom of the incline.

3. Stretching a spinr g adistance of x requires a force of F. In the process, potential energy, U, is stored in the spring. How much force is required to stretch the spring a distance of $2x$, and what potential energy is stred in the spring as a result?

   A. F and $2U$

   B. $2F$ and $2U$

   C. $2F$ and $4U$

   D. $4F$ and $4U$

   E. $4F$ and $8U$

4. You want to life an object 1 meter. Pushing the object up a 30 degree inclines takes twice as long as lifting it straight up. When you use the incline, compared to lifting it straight up, you do

   A. half the work and use half the power

   B. half the work and use the same power

   C. the same work and use half the power

   D. the same work and use the same power

   E. twice the work and use half the power