Shell Shock

A shell is shot from the edge of cliff of height H with initial velocity v_1 at an angle of θ degrees. It lands a distance d from the base of the cliff. What is v_1?

A second shell is launched with the same initial velocity. At the top of its trajectory it explodes into two fragments. One fragment of mass m_1 has zero speed immediately after the explosion and falls vertically. The other fragment of mass m_2 lands a distance s from the base of the cliff. Find $\frac{m_1}{m_2}$.

Answer

First question is just the usual kinematics stuff:

$$-H = v_1 \sin(\theta) t - \frac{1}{2} g t^2$$

$$d = v_1 \cos(\theta) t$$

Solve for v_1 to get

$$v_1 = \frac{d}{\cos(\theta)} \frac{1}{\sqrt{\frac{2}{g}(H + d\tan(\theta))}}$$

If you come up with a cleverer way than this brute force, let me know.

The next question we should answer by being clever. An unexploded shell would follow the path of the center of mass of the other two pieces of which it was composed. Let Δx be the distance the thing travels before exploding, which we can trivially find as $\Delta x = \frac{v_1^2 \sin(\theta)\cos(\theta)}{g}$. Then

$$d = \frac{m_1 \Delta x + m_2(\Delta s)}{m_1 + m_2}$$

Solving for $\frac{m_1}{m_2}$ gives

$$\frac{m_1}{m_2} = \frac{x + \Delta x - d}{d - \Delta x}$$