B-Magnetism Problem Set

1. a) $\boxed{\text{False}}$. The magnetic moment points from South to North by definition.

b) $\boxed{\text{False}}$ $\vec{\tau} = \vec{\mu} \times \vec{B}$

$\tau = \mu B \sin\theta$

If the plane of the loop is $\perp$ to $\vec{B}$, then $\vec{\mu} \parallel \vec{B}$, so $\theta = 0°$, and $\tau = \mu B \sin 0° = 0$. The maximum torque occurs when $\theta = 90°$, so this statement is false.

c) $B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{R}$, where R is the perpendicular distance to the source. R is the smallest at the surface of the wire, so B is maximized.

$\boxed{\text{True}}$

d) $\boxed{\text{False}}$. $B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{R}$ shows a linear decrease.

2. $\sum \vec{F}_{radial} = m\vec{a}$

$qvB\sin\theta = m\frac{v^2}{r}$ $\quad (\theta = 90°$, assume $\vec{v} \perp \vec{B})$

$qB = \frac{mv}{r}$

$r = \frac{mv}{qB}$ $\quad T = \frac{2\pi r}{v} = \frac{2\pi}{v}\left(\frac{mv}{qB}\right) = \frac{2\pi m}{qB}$

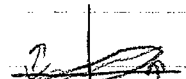
$$= \frac{2\pi (10.0\mu g)\left(\frac{1\,g}{10^6 \mu g}\right)\left(\frac{1\,kg}{10^3\,g}\right)}{0.300nC\left(\frac{1\,C}{10^9 nC}\right)(1.00\cdot 10^{-9}T)}\left(\frac{1\,hr}{3600\,s}\right)\left(\frac{1\,day}{24\,hr}\right)\left(\frac{1\,yr}{365\,days}\right) = \boxed{6.64\cdot 10^3\,yr}$$

$$\frac{kg}{C\cdot T} = \frac{kg}{C\frac{N}{C\,m/s}} = \frac{kg\,m/s}{kg\,m/s^2} = s \checkmark$$

3. $U_i + K_i = U_f + K_f$

$-\mu B\cos\theta = -\mu B \cos 0° + K_f$

$$\theta = \cos^{-1}\left(\frac{-\mu B + K_f}{-\mu B}\right) = \cos^{-1}\left(\frac{-0.025J/T\,(58mT) + 0.70mJ}{-0.025J/T\,(58mT)}\right) = \boxed{58.9°}$$

4. 

SHM since the loop tends towards equilibrium, when $\tau_{net}=0$.

restoring

$\tau = \mu B \sin\theta$

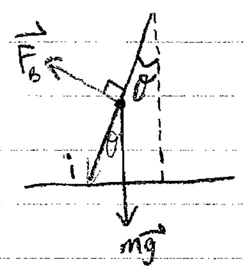
$$-IAB\sin\theta = I_{inertia}\frac{d^2\theta}{dt^2}$$

$\sin\theta \approx \theta$

$$-IAB\theta = I_{inertia}\frac{d^2\theta}{dt^2}$$

$$\frac{-IAB\theta}{I_{inertia}} = \frac{d^2\theta}{dt^2} = \frac{-I(\pi R^2)B\theta}{\frac{1}{2}mR^2} = -\frac{2I\pi B}{m}\theta = -\omega^2\theta \qquad \alpha \sim a = -\omega^2 x$$

$$\omega = \sqrt{\frac{2I\pi B}{m}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2I\pi B}} = \sqrt{\frac{2\pi m}{IB}}$$

5. 

$\vec{F} = q\vec{v}\times\vec{B}$

set $\vec{B}$ going up out of the page.

$\vec{F}_B = I\vec{\ell}\times\vec{B}$, $F_B = ILB\sin\phi$

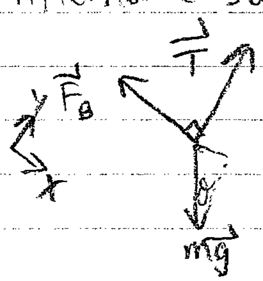
$\tau_B = \tau_g \qquad \tau = rF\sin\phi = \frac{L}{2}F\sin\phi$

$\frac{L}{2}I\ell B\sin\phi = \frac{L}{2}mg\sin\phi$

$\ell IB\sin 90^\circ = mg\sin\theta$

$$B = \boxed{\frac{mg\sin\theta}{\ell I}}$$

Alternate Solution:



$\sum\vec{F} = m\vec{a}$

$\sum F_x = ma_x \qquad \sum F_y = ma_y$

$mg\sin\theta = F_B \qquad T - mg\cos\theta = 0$

$mg\sin\theta = I\ell B$

$$B = \boxed{\frac{mg\sin\theta}{I\ell}}$$