Momentum Problem Set Solutions

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1. Because of the elastic collision, we can use both conservation of energy and conservation of ~~energy~~ momentum.

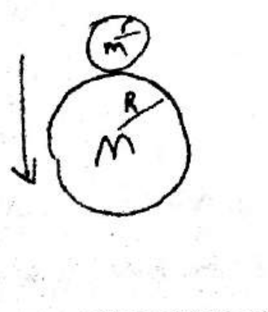
$$mv_0 = mv_m + Mv_M$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_m^2 + \frac{1}{2}Mv_M^2$$

This is a system of equations in two-variables (V_m and V_M), so it can be solved. We'll need to use the quadratic formula. After doing so, we have:

$$\boxed{\begin{aligned} V_m &= \left(\frac{m-M}{m+M}\right)v_0 \\ V_M &= \left(\frac{2m}{m+M}\right)v_0 \end{aligned}}$$

2.



When the large ball hits the ground, it changes direction. At this instant, both balls have a velocity $\sqrt{2gh}$, but in opposite directions. Now, we have to use conservation of energy and momentum, due to inelastic collisions

$$-m\sqrt{2gh} + M\sqrt{2gh} = mv_m + MV_M$$

$$\frac{1}{2}m(2gh) + \frac{1}{2}M(2gh) = \frac{1}{2}mv_m^2 + \frac{1}{2}Mv_M^2$$

Once again, we have a system of equations in v_m and V_M. Solving the resulting quadratic, we find that:

$$v_m = \left(\frac{m-M}{m+M}\right)\left(\sqrt{2gh}\right) + \left(\frac{2M}{m+M}\right)\left(\sqrt{2gh}\right)$$

$$= \left(\frac{3M-m}{M+m}\right)\sqrt{2gh}.$$

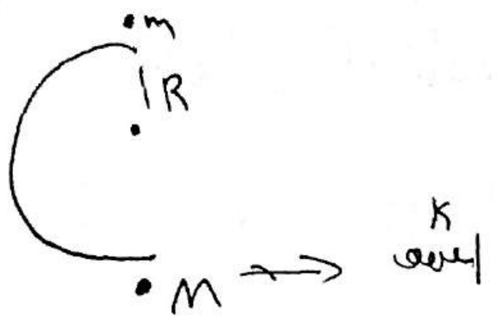
Now, it's a simple kinematics problem with an initial velocity upward of $\left(\frac{3M-m}{M+m}\right)\sqrt{2gh}$ and acceleration g. The maximum height, therefore, is: $\frac{v_0^2}{2g}$. So we have

$$\left(\frac{3M-m}{M+m}\right)^2 h.$$

However, we need to add in the ~~radius~~ diameter of the larger ball because the collision happened $2R$ above the ground. Then we add r for the smaller mass' center of mass's height.

$$\boxed{\left(\frac{3M-m}{M+m}\right)^2 h + 2R + r}$$

3. This problem synthesizes everything we've covered so far.



<u>of mass m</u>

$PE_i = 2mgR$

$KE_f = \frac{1}{2}mv^2$

Equate the two, and the mass m hits mass M with velocity $v = 2\sqrt{gR}$.

Then we use conservation of momentum:

$$m\,2\sqrt{gR} = (m+M)v_f \implies v_f = \frac{2m\sqrt{gR}}{m+M}.$$

As for the oscillation of the spring that only depends on the mass and spring constant. So!

$$\boxed{T = 2\pi\sqrt{\frac{k}{M+m}}}$$

For the second question, we use conservation of energy

$$\frac{1}{2}(m+M)v_f^2 = \frac{1}{2}k\Delta x^2$$

$$\Delta x = v_f\sqrt{\frac{m+M}{k}} \implies \boxed{\Delta x = \frac{2m}{m+M}\sqrt{\frac{gR(m+M)}{k}}}$$

As an extra fun fact, if there is an elastic ~~collision~~ collision between a mass m and mass M moving with velocities V_{om} and V_{oM}, then their final velocities are given by: (in 1-D!!)

$$V_{fm} = \left(\frac{m-M}{m+M}\right)V_{om} + \left(\frac{2M}{m+M}\right)V_{oM}$$

$$V_{fM} = \left(\frac{M-m}{m+M}\right)V_{oM} + \left(\frac{2m}{m+M}\right)V_{om}$$