Answer 12/2

In any of these cases, let the inertia of the disk be represented as $I = \beta\mu R^4$, where β is a dimensionless constant. (Dimensional analysis and the definition of inertia.)

An FBD on the weight reveals

$$mg - T = ma \tag{1}$$

where T is the tension of the rope. RWOS implies that $a = R\alpha$, so

$$T = m(g - R\alpha) \tag{2}$$

Torque equation on the disk gives

$$TR = I\alpha \tag{3}$$

Substitute in Eqs. 1 and 3 to get

$$mR(g - R\alpha) = \beta\mu R^4\alpha \tag{4}$$

Solve for m as

$$m = \frac{\beta\mu R^3\alpha}{g - R\alpha} \tag{5}$$

Finding β requires repeated applications of the parallel axis theorem. Going through the cases:

1. $I = \frac{1}{2}(\pi R^2\mu)R^2 = \frac{\pi}{2}\mu R^4$. So $\beta = \frac{\pi}{2}$, and thus
$$m = \frac{\frac{\pi}{2}\mu R^3\alpha}{g - R\alpha}.$$

2. Def. of moment of inertia (basically a sum) tells us that inertias add: $I = \frac{\pi}{2}\mu R^4 + I_{hole}$. Considering the hole to have a negative mass density μ gives $I_{hole} = -\frac{\pi}{2}\mu(\frac{R}{2})^2(\frac{R}{2})^2$. So $\beta = \frac{15\pi}{32}\mu R^4$ and
$$m = \frac{\frac{15\pi}{32}\mu R^3\alpha}{g - R\alpha}.$$

3. Again, inertia moments add: $I = \frac{\pi}{2}\mu R^4 + 4I_{hole}$. (I can immediately say all of the inertia of the holes are the same because of symmetry.) The inertia of each hole is given by assuming negative mass density $-\mu$ and using the parallel axis theorem to find the hole's inertia about the disk's center: $I_{hole} = \frac{1}{2}M(\frac{R}{4})^2 + M(\frac{R}{2})^2 = (-\pi\mu(\frac{R}{4})^2)(\frac{R^2}{32} + \frac{R^2}{4}) = -\frac{9\pi}{512}\mu R^4$. Then, back up above, $I = \frac{55\pi}{128}\mu R^4$, so $\beta = \frac{55\pi}{128}$, and
$$m = \frac{\frac{55\pi}{128}\mu R^3\alpha}{g - R\alpha}.$$