# Rotation Lecture

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## 1 Introduction

For simplicity, point masses are a useful way to model objects. Real world objects, however, are extended bodies that have the ability to rotate about an axis and thus cannot be adequately represented by a point. As such, it is necessary to be able to describe the rotational motion of these objects.

## 2 Angular Position, Velocity, and Acceleration

A rotating object's position is denoted by an angle theta measured from some reference point (usually the positive x-axis). While the coordinate theta specifies the rotational position of an object at one instant in time, it's rotational motion is described in terms of the rate of change of theta. This rotational motion is called angular velocity and is found by taking the derivative of the object's angular position with respect to time. Similarly, an object's angular acceleration is the instantaneous rate of change of its angular velocity.

$$AngularPosition \rightarrow \vec{\theta}$$

$$AngularVelocity \rightarrow \vec{\omega} = \frac{d\vec{\theta}}{dt}$$

$$AngularAcceleration \rightarrow \vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

$$\vec{s} = \vec{\theta} \times \vec{r}$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{a_t} = \vec{\alpha} \times \vec{r}$$

## 3 Constant Angular Acceleration Equations

Linear kinematics is particularly simple with constant acceleration as it allows us to derive useful equations for position and velocity. With constant angular acceleration, equations for angular position and angular velocity can be derived in precisely the same way.

$$\vec{\omega} = \vec{\omega_0} + \vec{\alpha}t$$

$$\vec{\theta} = \vec{\theta_0} + \vec{\omega_0}t + \frac{1}{2}\vec{\alpha}t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\vec{a_c} = -\vec{r}\omega^2$$

## 4 Moment of Inertia

The Moment of Inertia($I$) describes the tendency of an object to rotate. Every object has a Moment of Inertia which can be found through calculus for simple objects. For a list of moments, visit `http://en.wikipedia.org/wiki/List_of_moments_of_inertia`

$$I = mr^2$$

## 5 Rolling Without Slipping and Torque

In the special case of rolling without slipping, a relationship exists between the acceleration of the object's center of mass and the linear acceleration of a point on the object's surface.

$$\vec{F} = m\vec{a}$$

$$\vec{r} \times \vec{F} = \vec{r} \times (m\vec{a})$$

$$\vec{\tau} = \vec{r} \times m(\vec{\alpha} \times \vec{r})$$

$$\vec{\tau} = m(\vec{r} \times (\vec{\alpha} \times \vec{r}))$$

$$\vec{\tau} = m(\vec{\alpha}(\vec{r} \cdot \vec{r}) - \vec{r}(\vec{r} \cdot \vec{\alpha}))$$

$$\vec{\tau} = mr^2\vec{\alpha}$$

$$\vec{\tau} = I\alpha$$

## 6 Rotational Kinetic Energy

To determine the kinetic energy of an object rotating about a fixed axis, we consider the body as being made up of a collection of particles, each with its own kinetic energy, and then sum them up.

$$K = \frac{1}{2}I\omega^2$$

## 7 Parallel Axis Theorem

The Parallex Axis Theorem is used to calculate the Moment of Inertia about an arbitrary point of distance $D$ away from the center of mass.

$$I = I_{cm} + MD^2$$

## 8 Angular Momentum

$$\vec{p} = m\vec{v}$$

$$\vec{r} \times \vec{p} = \vec{r} \times (m\vec{v})$$

$$\vec{L} = I\vec{\omega}$$

$$\frac{d\vec{L}}{dt} = \Sigma\vec{\tau_{ext}}$$