Review Problem Set 1

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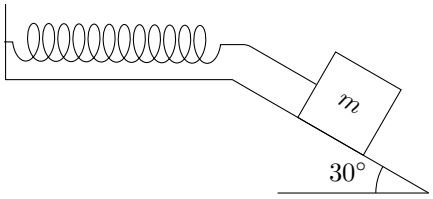
Here are a series of problems, many of which are from past F=ma exams, and one from a past quarterfinal exam. (Yes, USAPhO used to be broken down into 3 exams).

1 Problems

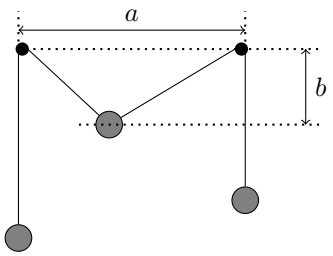
1. (2007 F = ma) **EASY** At time $t = 0$ a drag racer starts from rest at the origin and moves along a straight line with velocity given by $v = 5t^2$, where v is in m/s and t in s. The expression for the displacement of the car from $t = 0$ to time t is?

2. (2007 F=ma) **MEDIUM** A non-Hookian spring has force $F = -kx^2$ where k is the spring constant and x is the displacement from its unstretched position. For the system shown of a mass m connected to an unstretched spring initially at rest, how far does the spring extend before the system momentarily comes to rest? Assume that all surfaces are frictionless and that the pulley is frictionless as well.

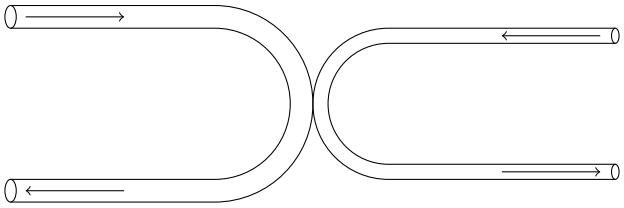


3. (2007 F=ma) **MEDIUM** If a planet of radius R spins with an angular velocity ω about an axis through the North Pole, what is the ratio of the normal force experienced by a person at the equator to that experienced by a person at the North Pole? Assume a constant gravitational field g and that both people are stationary relative to the planet and are at sea level.

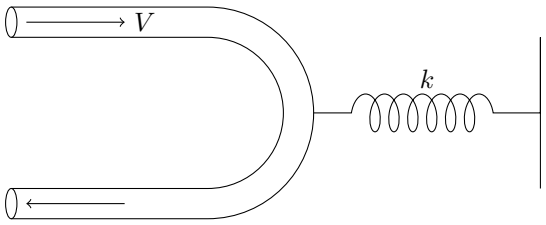
4. (2010 F=ma) **MEDIUM** The three masses shown in the accompanying diagram are equal. The pulleys are small, the string is lightweight, and friction is negligible. Assuming the system is in equilibrium, what is the ratio a/b? The figure is not drawn to scale!



5. (a) (2010 F=ma) **MEDIUM** Two streams of water flow through the U shaped tubes shown. The tube on the left has cross-sectional area A, and the speed of the water flowing through it is v; the tube on the right has cross-sectional area $A = 1/2A$. If the net force on the tube assembly is zero, what must



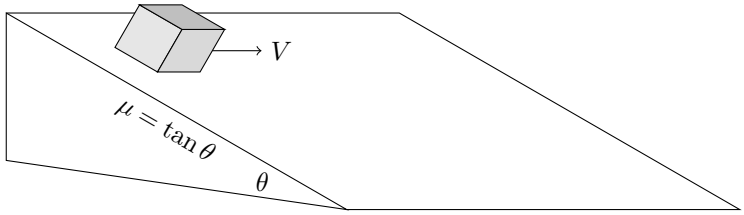
be the speed v of the water flowing through the tube on the right? Neglect gravity, and assume that the speed of the water in each tube is the same upon entry and exit.
(b) **MEDIUM** Consider the same situation as in part (a) except with the U-tubes moving with a constant velocity V.
(c) **MEDIUM** Consider the same situation as in part (a) except with the U-tubes moving at a constant acceleration a.
(d) **HARD** Consider a single U-tube with cross sectional area A. The tube is attached to a spring with a constant k, and a fluid is sprayed into the tube at a rate V with respect to the fixed ground. Determine the equation of motion of the tube if it is displaced from its equilibrium position, if the total length of the tube is ℓ.



(e) **MEDIUM** Consider a U-tube perturbed on one end (by perturbed, I mean that fluid on one end is a bit higher up than on the other end). Determine the frequency of oscillation around the equilibrium position. Hint: consider the imbalance of gravitational forces on both sides of the tube.

6. (2012 F=ma) **MEDIUM** Three point masses m are attached together by identical springs. When placed at rest on a horizontal surface the masses form a triangle with side length ℓ. When the assembly is rotated about its center at angular velocity ω, the masses form a triangle with side length 2ℓ. What is the spring constant k of the springs?

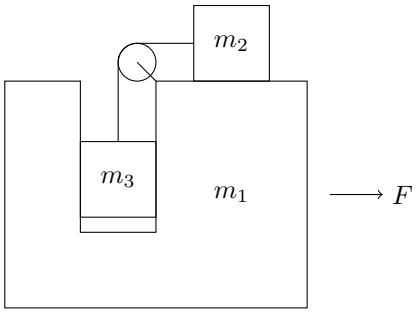
7. **VERY HARD** Consider a small block moving on a wedge inclined by an angle θ. The coefficient of friction is $\mu = \tan\theta$. The block is initially launched perpendicular to the $g\sin\theta$ component at an initial velocity V. After a long time, what is the block's final speed?



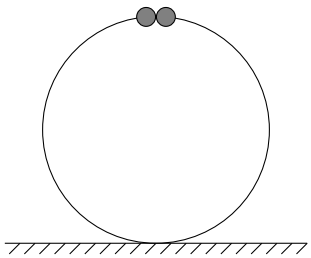
8. **HARD** A block is launched at an angle θ to the horizontal, and it lands face down on a level surface. The surface is rough and has a coefficient of friction μ. Determine the angle the block should be launched as to maximize the its total horizontal displacement when it stops moving.

9. **MEDIUM** A uniform rope of weight W hangs between two trees. The ends of the rope are at the same height and they each make an angle θ with the trees. Determine the tension at either end of the ropes and at the middle of the rope. Hint: there's a simple solution.

10. **HARD** Consider a system of three blocks (with masses m_1, m_2 and m_3) with frictionless surfaces. Block 2 is free to slide horizontally on top of block 1, which in turn slides on a frictionless table. Block 3 is free to slide up and down relative to block 1, i.e. it is an elevator car constrained to slide in a vertical shaft cut in to block 1. There is a massless string and pulley connecting blocks 2 and 3.
(a) If an external force F is applied to block 1, what value of F will allow m_3 to remain at the same height?
(b) With no external force ($F = 0$), what is the acceleration of block 1?



11. (Quarterfinal Exam 2008) **HARD** Two beads (both mass m) are slid into a larger ring with mass M. The beads are initially at the top of the ring, and are slightly disturbed from this position. Determine the maximum value of m such that the ring will rise. Determine the location of the beads when the ring rises.



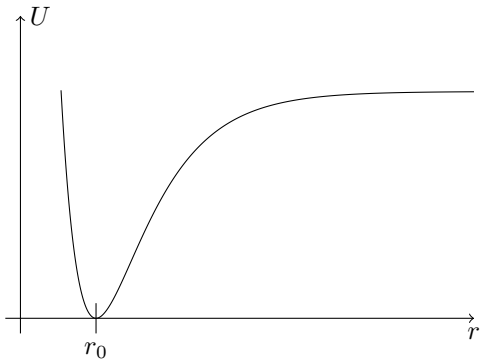
12. **VERY HARD** A ball is launched with an initial velocity v_0 normal to a ramp inclined at an angle ϕ. The coefficient of restitution of the ramp is $\gamma < 1$. The ball will repeatedly bounce along the ramp for a finite distance. Assuming that the ramp is infinitely long, determine this distance.

13. (2013 F=ma) A simple pendulum experiment is constructed from a point mass m attached to a pivot by a massless rod of length L in a constant gravitational field. The rod is released from an angle $\theta_0 < \pi/2$ at rest. Ignore air resistance and friction.
(a) **MEDIUM** What is the maximum value of the tension in the rod?
(b) **HARD** Now consider a force F of constant magnitude which acts opposite to the motion of the ball. Determine the maximum tension in the rod, as well as the angle at which this maximum tension occurs.

14. (a) **MEDIUM** Consider a potential that looks like the graph below. As we know, the equilibrium point r_0 can be approximated by a local quadratic. Using the Taylor series of U, show that, for a particle under this potential, slightly perturbed from equilibrium, Newton's second law looks like

$$-\frac{d^2U}{dr^2}\bigg|_{r_0}(r - r_0) = m\frac{d^2(r - r_0)}{dt^2}$$

which is simple harmonic in nature. Using this fact, determine the effective spring constant k.
(b) **MEDIUM** Consider the Lennard-Jones potential, given by



$$U = \epsilon \left[\left(\frac{r_0}{r} \right)^{12} - 2 \left(\frac{r_0}{r} \right)^{6} \right]$$

Determine the radius and "depth" of the potential well dictated by this potential. Find the frequency of small oscillations about this depth.