1D Kinematics

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1 Introduction

Kinematics involves the mathematical relationships between the key quantities that define the motion of an object - displacement, velocity, and acceleration.

2 Kinematic Definitions

2.1 Displacement

Displacement, often denoted as $\vec{S}$, is defined as the change in position vector measured in meters. Because we will be working in 1D, the vector will be one-dimensional. For example, the displacement vector for an object that moves between the points $x = 1$ and $x = 5$ is $4\hat{\mathbf{i}}$. Now if the object moves back to $x = 1$, the displacement vector for the overall trip would be the zero vector. This is because the the vector only depends on the initial and final positions.

2.2 Velocity

Velocity, denoted as $\vec{v}$ is defined as the change in displacement over a certain time, or written mathematically

$$\vec{v} = \frac{\Delta \vec{S}}{\Delta t}$$

From the definition, we see that velocity is also a vector. It is measured in meters per second or $\frac{m}{s}$. It is important to know the difference between *velocity* and *speed*. Velocity is a vector quantity and therefore has a direction, whereas speed is the magnitude of velocity and is a scalar quantity.

2.3 Acceleration

Acceleration, denoted as $\vec{a}$ is defined as the change in acceleration over a certain time, or written mathematically

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

Just like displacement and velocity, acceleration is a vector. It is measured in $\frac{m}{s^2}$. One important acceleration value to know is that of gravity, $g = 9.81\ \frac{m}{s^2}$, since it comes up in many problems.

3 Kinematic Equations

There are three equations that govern kinematics. Note that because we are working in 1D, we can drop the vector notation as long as we know what direction we are working in. We must take note that these equations only work when *acceleration is constant*. The three equations are

$$v_f = v_0 + at \tag{1}$$

$$S_f = S_0 + v_0 t + \frac{1}{2} a t^2 \tag{2}$$

$$v_f^2 - v_0^2 = 2aS \tag{3}$$

4 Reading Kinematic Graphs

Many times on the SAT Subject Test, you are given a graph of some kinematic quantity against time and are asked to solve for another quantity.

When given a displacement vs time graph, the velocity between any two points in time is the slope of the graph between the points.

When given a velocity vs time graph, the acceleration between any two points in time is the slope of the graph between the points, and the displacement between the two points is the signed area under the graph between the points.

When given an acceleration vs time graph, the change in velocity between any two points in time is the signed area under the graph between the points.

5 Exercises

1. Can an object have constant speed and non-zero acceleration?

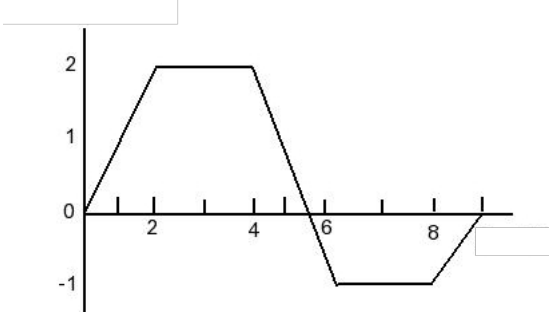
2. Which will hit the ground first if dropped from the same height: a feather or a bowling ball?

3. A toy car starts moving on a horizontal track with an initial speed of 2 $\frac{m}{s}$ and an acceleration of 3 $\frac{m}{s^2}$ due to rocket boosters attached to the car. How fast is the car moving after 5 seconds? How far has the car travelled after 10 seconds?

4. A boy drops a rock from a ledge that is 20 meters above the ground. How long does it take for the rock to hit the ground? What is its speed at that moment?

5. Newton wanted to see how gravity worked. He stood on a tree branch that was 10 meters above the ground and threw an apple up with a speed of 10 $\frac{m}{s}$. When will the apple be at its highest height? What is the speed and acceleration of the apple at this point? How long will it take for the apple to hit the ground?

6. The graph below is a velocity vs time graph for a ball moving on a horizontal surface. What is the acceleration of the ball between times $t = 4$ and $t = 6$? Between times $t = 2$ and $t = 4$? How far has the ball traveled from its starting position at time $t = 4$? What is the total distance travelled by the ball at time $t = 9$?



6 Problems

1. A mass initially at rest experiences a uniform acceleration a. During time interval t, the mass is displaced by a distance x. What is the displacement of the mass when the elapsed time doubles and equals $2t$?

2. A runner is training on an oval-shaped track where each lap has a length of 400 meters. The runner maintains a constant speed and completes two laps, returning to the starting point in 100 seconds. What is the velocity of the runner for this time interval?

3. A person throws a ball upward when standing on top of a hill. The ball slows on the way up, reaches an instantaneous stop at maximum height, increases speed on the way down, and finishes the motion at a lower final height. Draw a graph that represents the vertical displacement of the ball vs time.

4. Joe and his brother John want to play a game. Joe stands on a balcony that is 50 meters above the ground while John stands on another balcony that is 25 meters above the ground. Joe drops an orange from his balcony, while John must throw his orange upward. With what speed must John throw his orange so that the oranges collide at exactly the midway point between the balconies?

5. The driver of a car moving at 28 $\frac{m}{s}$ notices that he is driving straight towards a brick wall and so he starts to brake when he is 56 meters from the wall. Assuming that the car stops directly in front of the wall, what is the acceleration of the car during this time?

6. Two teams of movers are lowering a piano from the window of a 10 floor apartment building. The rope breaks when the piano is 30 meters above the ground. The movers on the ground, alerted by the shouts of the movers above, first notice the piano when it is 14 meters above the ground. How long do they have to get out of the way before the piano hits the ground? (2010 $F = ma$)