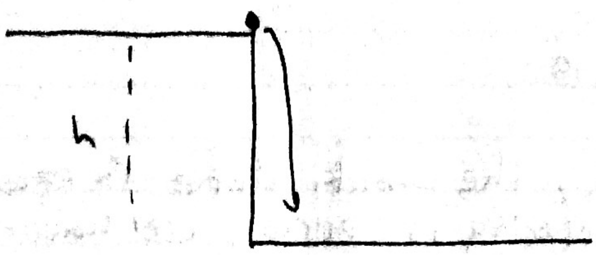
WORK AND ENERGY
LECTURE PROBLEMS: SOLUTIONS

10/24/14
ARUN KANNAN

1. The work I did is simply the force I applied times the distance over which I applied that force.

The work done on the box is zero, since by the work - kinetic energy theorem, since there was no net change in K_E, $W=0$. The work I did was released as thermal energy because of friction doing negative work on the box.

2.



Let's define the ground to have zero gravitational potential energy. This means the ball, when at the top, has gravitational $P_E = mgh$.

Since gravity is conservative and the system is closed, we can use the law of conservation of energy:

$$K_i + P_i = K_f + P_f.$$

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 + mg(0).$$

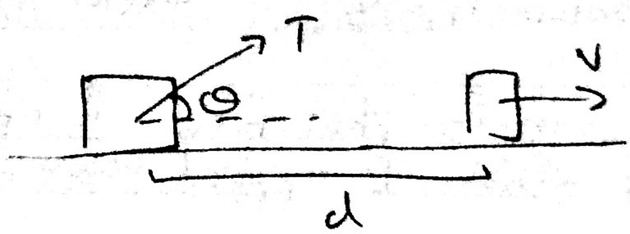
Since $v_i = 0$, we have:

$$mgh = \frac{1}{2}mv_f^2.$$

So the ball hits the ground with velocity

$$\boxed{v_f = \sqrt{2gh}}$$

3.



Since there is no friction on the box, the work done by the tension is the component of tension parallel to motion times the distance travelled in the direction of motion.

Component parallel: $T\cos\theta$
Distance: d
Work: $Td\cos\theta$

Since energy is conserved, the work done is equal to ΔK. Since starting velocity is zero, we have:

$$Td\cos\theta = \frac{1}{2}mv_f^2$$

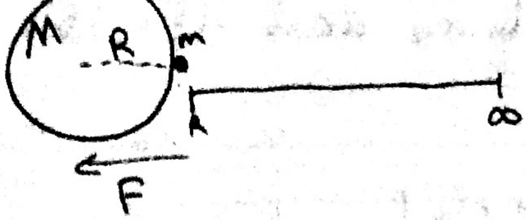
$$\boxed{V_f = \sqrt{\frac{2Td\cos\theta}{m}}}$$

4. We will go to the definition of work to solve this problem.

$$W = \int_c \vec{F} \cdot d\vec{r}$$

In order to calculate the minimum work needed to send this object out of orbit is, we need to determine what C is. However, gravity is a conservative force, even this new form. Remember that if a force is a power function in one variable, it MUST be conservative since we can easily find an antiderivative for it (this would be the potential function ~~M~~ times negative 1).

This means we operate independent of path. Our starting point will be $r = R$, and our finishing point will be very far away. For all intents and purposes, this point is $r = \infty$. Because the path doesn't matter, we will let it be a straight line.



$$W = \int_R^\infty \left(-\frac{GMm}{r^2}\hat{r}\right) \cdot d\vec{r}$$

$\hat{r}$ starts at the center of the planet and points to the object, but is of magnitude 1. $d\vec{r}$ points in the change of $\vec{r}$, which points from the center of the planet to the object, and its magnitude is that of that distance. This means:

$$\hat{r} \cdot d\vec{r} = |\hat{r}||d\vec{r}|\cos\theta \quad \theta = 0 \quad = dr$$

$$W = \int_R^\infty -\frac{GMm}{r^2} dr = -GMm\left(-\frac{1}{\infty} + \frac{1}{R}\right) = -\frac{GMm}{R}$$

This is the work done by gravity. In order to overcome this, we must supply at least that energy so that the object doesn't come back. Our answer is therefore the negative of that.

$$\boxed{E_{MSW} = \frac{GMm}{R}}$$

Since we need to supply at least that much energy, our escape velocity is given by:

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

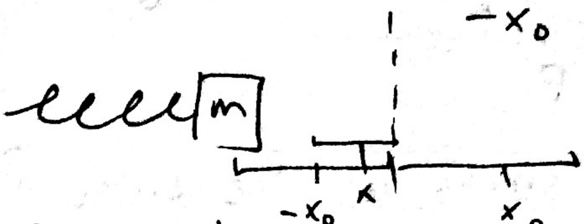
$$\boxed{v_e = \sqrt{\frac{2GM}{R}}}$$

5. When the spring is compressed x_0 and the block isn't moving:

$$P_{B,i} = \frac{1}{2}kx_0^2$$

$$K_{B,i} = 0$$

$-x_0 < x < x_0$



At any given time, the potential energy is given by $\frac{1}{2}kx(t)^2$, and the kinetic energy is given by $\frac{1}{2}mv(t)^2$. By the law of conservation of energy:

$$P_i + K_i = P(t) + K(t)$$

$$\frac{1}{2}kx_0^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$\boxed{\frac{1}{2}kx_0^2 = \frac{1}{2}kx^2 + \frac{1}{2}m\left(\frac{dx}{dt}\right)^2}$$