# Forces

Ray Liu and Akhil Waghmare

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## 1 Introduction

Kinematics describes the motion of objects. Dynamics accesses the actual cause of motions. What can cause an object to change its velocity, either magnitude or direction, or both, is called force. Force is the interaction between objects. So if an object receives a force, there must be another object that exerts that force.

## 2 Force

Forces are vector quantities. Their magnitudes are defined in terms of the acceleration they would give to an object of known mass (e.g. 1 kg). A force that accelerates 1 kg of mass by exactly 1 $m/s^2$ is defined to have a magnitude of 1 Newton. The direction of a force is the direction of the acceleration it causes. Forces are combined according to the rules of vector algebra.The net force on a body is the vector sum of all the forces acting on the body.

### 2.1 Gravitational force

Pull from the earth:

$$F_g = mg. \tag{1}$$

### 2.2 Normal force

A normal force $\vec{F}_N$ is the force on a body from a surface against which the body presses.The normal force is always perpendicular to the surface.

### 2.3 Frictional force

A frictional force $\vec{F}_f$ is the force on a body when the body slides or attempts to slide along a surface. The force is always parallel to the surface and directed so as to oppose the sliding. The maximum force an object can withstand before moving is the static friction force:

$$F_s = \mu_s N. \tag{2}$$

The friction force that is against an object's motion is the kinetic friction force:

$$F_k = \mu_k N. \tag{3}$$

Both static and kinetic friction forces are proportional to normal force. $\mu_s$ and $\mu_k$ are the coefficient of friction. Generally, $\mu_s > \mu_k$.

### 2.4 Tension

Tension is a force exerted by ropes and strings in the direction of the ropes and strings.

### 2.5 Spring Restoring Force

Deformation of a srping by amount $x$ provides a restoring force that is proportional to the deformation but in the opposite direction (Hooke's law):

$$F_s = -kx. \tag{4}$$

$k$ is the spring constant.

## 3 Newtonian Mechanics

The velocity of an object can change (the object can accelerate) when the object is acted on by one or more forces (pushes or pulls) from other objects. Newtonian mechanics relates accelerations and forces.

### 3.1 Newton's First Law

If there is no net force (sum of all forces) on a body, the body remains its velocity constant (magnitude and direction). If $\vec{v} = 0$ initially, $\vec{v} = 0$ later.

### 3.2 Mass

Mass is a quantity of inertia of an object and represent the ability to resist the change in motion.

### 3.3 Newton's Second Law

The net force $\sum \vec{F}$ on a body with mass $m$ is related to the body's acceleration $\vec{a}$ by

$$\sum \vec{F} = m\vec{a} \tag{5}$$

which in 3D coordiante system,

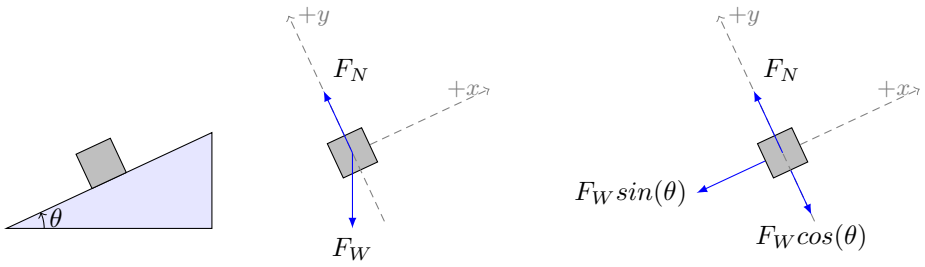
$$\sum F_x = ma_x, \sum F_y = ma_y, \sum F_z = ma_z, \tag{6}$$

### 3.4 Inertial Reference Frames

Reference frames in which Newtonian mechanics holds are called inertial reference frames. Reference frames are those at rest or moving in constant velocity. An accelerating car or a rotating disk are not inertial reference frames!

### 3.5 Free-Body Diagrams

A free-body diagram is a stripped-down diagram in which only one body is considered.That body is represented by a dot. The external forces on the body are drawn as "pulling forces", and a coordinate system is chosen, oriented to simplify the solution.



### 3.6 Newton's Third Law

If a force $\vec{F}_{BC}$ acts on body B due to body C, then there is a force $\vec{F}_{CB}$ on body C due to body B:

$$\vec{F}_{BC} = \vec{F}_{CB}. \tag{7}$$

## 4 Problems

1. A 3 kg mass is traveling at 15 $\frac{m}{s}$ when friction begins to decelerate it to a stop in 5 seconds. What is the magnitude of the force of friction?

2. A 20-newton force is pushing two blocks, with masses 2 kg and 8 kg, together horizontally along a frictionless floor. What is the force that the 8 kg mass exerts on the 2 kg mass?

3. The setup below is called an *atwood machine*. It is composed of two masses attached by a string over a frictionless pulley. Determine the acceleration of each mass. Assume masses $m_1$ and $m_2$.