Electrostatics solutions 11/14/06

1. Let the charge of the first two spheres be Q. Then, by Coulomb's Force Law,

$$F = \frac{kQ^2}{d^2} \tag{1}$$

When the first conducting sphere touches the uncharged conducting sphere, charge transfers so that they charge on each is equal– i.e., the first conducting sphere now has a charge of $\frac{1}{2}Q$ and the formerly uncharged conducting sphere now has a charge of $\frac{1}{2}Q$. The same equilibrium process occurs when the third sphere touches the second sphere, so now both the second and third sphere have charges of $\frac{3}{4}Q$. Then the force between the first and second spheres is

$$F_{new} = \frac{k(\frac{1}{2}Q)(\frac{3}{4}Q)}{d^2} = \frac{3}{8}F \tag{2}$$

2. The graph doesn't have to be perfect, but the electric field should start at 0 from the center of the disk, increase until it hits $\frac{a\sqrt{2}}{2}$, and then decrease to 0 as the distance from the disk increases. See Freddy's lecture for details on how to figure out what the electric field is for continuous charge distributions.

3. The proton. Electric potential is inversely proportional to distance, and is positive when two things are repelled from one another and vice versa. So the proton has a very large positive potential contribution from the repelling positively charged plate, whereas the electron has a smaller positive potential contribution from the farther away negatively charged plate. Similarly for the negative potential contribution.

4a. The key to this is that the sphere doesn't want to have any external electric field between its inner and outer shells. There is a charge of $-Q$ on the inner part of the sphere to counteract the electric field from the point charge of $+Q$. No charge is placed between the inner and outer edge of the sphere, since that would induce an electric field within the sphere. The remaining charge is placed on the outer shell, $-q + Q$.

4b. The electrostatic potential is only affected by the inner shell's charge, since the contribution from the outer shell cancels itself out (a.k.a., the Shell Theorem). The inner shell acts like a point charge,

$$V = \frac{kQ(-Q)}{R^2} = -\frac{kQ^2}{R^2} \tag{3}$$

5. Bring in the charges in any order, because it doesn't matter. First step is to bring in one charge- let's say the negatively charged particle- to one corner of the square. This takes no work since there's no other charge there to repel/attract it.

Second step is to bring in the next charge- by default a positively charged particle to an adjacent corner. (Could be opposite corner; doesn't matter.)

$$W_2 = -\frac{kq^2}{a^2} \tag{4}$$

Third step is to bring in the next charge to either of the remaining corners. Let's choose the corner opposite the negatively charged particle.

$$W_3 = \frac{kq^2}{a^2} - \frac{kq^2}{(a\sqrt{2})^2} \tag{5}$$

Finally, let's bring the last charge up to the remaining corner.

$$W_4 = -\frac{kq^2}{a^2} + \frac{kq^2}{(a\sqrt{2})^2} + \frac{kq^2}{a^2} \tag{6}$$

Total work is the sum of these works, which totals to

$$W^{tot} = W_2 + W_3 + W_4 = 0 \tag{7}$$

Surprise! It's a nice answer.