Taylor Approximations

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Taylor's (or Maclaurin's) approximations are another trick to simplify arithmetic, because they can change a weird trig, exponential, or square-rootish function into a polynomial.

Let's call your function f(x) and assume that it is infinitely differentiable. Furthermore, we claim we can represent it as an infinite polynomial:

$$f(x) = \sum_{n=0}^{\infty} a_n x^n$$

where a_n is a constant with respect to x. We need a systematic way to get these constant coefficients. One way to do this is to notice that setting $x = 0$ gets rid of all but the first term:

$$f(0) = a_0 + a_1(0) + a_2(0^2) + ... = a_0$$

That's nice, but we don't foresee anything else being done on this equation that would easily get the coefficients– there's no other way to get rid of a lot of other terms. So we try changing the equation by differentiating it:

$$f'(x) = \sum_{n=0}^{\infty} n a_n x^n = a_1 + 2a_2 x + 3a_3 x^2 + ...$$

Now if we set $x = 0$, we get that

$$f'(0) = a_1$$

We similarly get the other coefficients to finally find out that

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(0)}{n!} x^n \tag{1}$$

A similar more general analysis gets the more general equation of

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(x_0)}{n!} (x - x_0)^n \tag{2}$$