Solution to Nasty Relative Motion Problem #1

a. The solution to this problem rests almost entirely about understanding how to use relative motion and about *being careful.* The relative motion bit is this:

$$\mathbf{v}_{block-incline} + \mathbf{v}_{incline-ground} = \mathbf{v}_{block-ground} \tag{1}$$

which is the statement that if we know how the block moves relative to the incline and how the incline moves relative to the ground, we can find out how the block moves relative to the ground. In order to make use of this vector equation, we must set up a coordinate system. For problems in which there is no clear reason to choose one coordinate system over another, such as in this problem– do you want to do this in the rotated frame of reference of the incline, or in the frame of reference of the ground?– you generally choose a frame of reference based entirely upon the question being asked. Here, the question asks for the distance that the block travels parallel to the ground, and so we choose the reference frame of the ground, with the positive x-direction defined by the direction traveled by the incline on which the block is traveling (to the right). Then finally we are able to write our relative motion vector equation in terms of its x-component, which is what we're interested in for purposes of this problem:

$$v_{x,block-incline} + v_{x,incline-ground} = v_{x,block-ground} \tag{2}$$

The incline is traveling only in the x-direction with respect to (wrt) the ground, so its velocity in the x-direction is

$$v_{x,incline-ground} = S(t) = \frac{1}{2}g\theta(t)t$$

The block is traveling at an angle with respect to the incline, and so its velocity needs to be decomposed into its components through use of the useful $v_x = v\cos(\theta)$:

$$v_{x,block_incline} = v(t)\cos(\theta(t)) = -gt$$

Then we are told that θ starts at 15 degrees and goes down a degree every second. This means

$$\theta(t) = 15^\circ - \frac{1^\circ}{\text{sec}}t = \frac{\pi}{12} - \frac{\pi}{180}t$$

There is a very good reason for using radians rather than degrees, because that way you don't have to convert degrees to radians at the very end of the problem. (Degrees are a dimensionless unit, but a unit nonetheless. Radians are just numbers with a fancy name because they refer to a special circular system.) We plug these three formulas back into Eqn. 2 to get

$$v_{x,block-ground} = \frac{1}{2}gt(\frac{\pi}{12} - \frac{\pi}{180}t) - gt \tag{3}$$

We note that it takes $15°/\frac{1°}{s} = 15s$ to get to no incline. So to get the total distance traveled during that time by the block wrt to the ground, we use our calculus hammer

$$\Delta x = \int_{t=0}^{t=15} v_{x,block-ground} dt \tag{4}$$

You should get approximately 1006 meters, if g is considered to be $9.8\frac{m}{s^2}$, or about $102.7s^2$ g. Obviously this is a rather ridiculous answer for most normal sized systems, but I never said that this problem was in any way physical.

b. Similarly, but this is easier conceptually with more emphasis on the calculus (which you may not yet know but which you will learn soon in your calculus class and whatnot.) We skip right to the good part,

$$\Delta x_{block-incline} = \int_{t=0}^{t=15} v_{block-incline} dt = \int_0^{15} gt \sec(\frac{\pi}{12} - \frac{\pi}{180}t) dt \tag{5}$$

Plug this into your TI-89 or actually do it out (the horrors– try integration by parts with some u-substitution) and see if you get about 15 g.