# Momentum and Collisions

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## 1 Center of Mass

The position of the center of mass of a system of particles is defined as a weighted sum of the particles' individual positions with respect to mass: $\mathbf{M} = \frac{\sum m_i r_i}{\sum m_i}$. It can also be written as $M\overrightarrow{r} = \int \overrightarrow{r}\, dm$ where $\overrightarrow{r}$ is the position vector of the particle.

### 1.1 Exercise

A 59kg woman and 71kg man sit on a seesaw long. What is their center of mass?

## 2 Momentum

Momentum is defined as $\overrightarrow{\mathbf{p}} = m\overrightarrow{\mathbf{v}}$, the product of its mass and velocity. The total momentum $\overrightarrow{\mathbf{P}}$ of a system of many particles is the sum of the momenta of individual particles: $\overrightarrow{\mathbf{P}} = \sum m_i \overrightarrow{v_i} = \sum \overrightarrow{p_i}$.

### 2.1 Conservation of Momentum

If the net external force on a system is zero, the total momentum of the system remains constant. $\overrightarrow{\mathbf{P}} = \sum m_i \overrightarrow{v_i} = M\overrightarrow{v_{cm}} =$ constant where $v_{cm}$ is the velocity of the center of mass.

## 3 Collisions

A collision is the meeting of particles or of bodies in which each exerts a force upon the other, causing the exchange of energy or momentum.

### 3.1 Elastic Collisions

Elastic collision is a collision in which the total kinetic energy of the colliding bodies or particles is the same after the collision as it was before.

### 3.2 Inelastic Collisions

Inelastic collision is a collision in which the total kinetic energy of the colliding bodies or particles is not the same after the collision as it was before. An extreme case is the perfectly inelastic collision, in which all of the kinetic energy relative to the center of mass is converted to thermal or internal energy of the system, and the two objects stick together after the collision.

## 4 Problems

1. A can in the shape of a symmetrical cylinder with mass M and height H is filled with water. The initial mass of the water is M, the same mass as the can. A small hole is punched in the bottom of the can, and the water drains out. (a) If the height of the water in the can is x, what is the height of the center of mass of the can plus the water remaining in the can? (b) What is the minimum height of the center of mass as the water drains out?

2.A bullet of mass m1 is fired with a speed v into the bob of a ballistic pendulum of mass m2. Find the maximum height h attained by the bob if the bullet passes through the bob and emerges with a speed v/3.

3. A steel ball of mass 0.400 kg is fastened to a cord that is 50.0 cm long and fixed at the far end. The ball is then released when the cord is horizontal. At the bottom of its path, the ball strikes a 2.00 kg steel block initially at rest on a frictionless surface. The collision is elastic. (a) Find the speed of the ball just after collision. (b) Find the speed of the block just after collision.