Energy Problems

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What you need to know to solve the problems
Conservation of Energy:

$$\Delta E = \Delta U + \Delta KE = 0$$

When energy is conserved,

$$\Delta U = -\int_{\mathbf{r}_0}^{\mathbf{r}_f} \mathbf{F} \cdot d\mathbf{s}$$

We will use the force of gravity as

$$\mathbf{F} = -mg\mathbf{j}$$

The spring force is

$$\mathbf{F} = -k\mathbf{r}$$

where $\mathbf{r}$ is a vector from the spring equilibrium position to the object.

Warmup problem
Which of the following paths will result in the largest velocity at the end of the path for a block starting from rest at the top of the path?
A. B.

Easy problem
You have a spring hanging from the ceiling with spring constant k. The equilibrium length of the **isolated** spring is L. You attach a block of mass M to the spring and pull it down a distance d from the ceiling. What is the velocity at the equilibrium position? (This is not the same as the equilibrium position of the isolated spring.)

Medium problem
You drop a block of mass M from a height h and it slips down a friction-less incline and through a loop of radius R. Then the block slides down an incline with $\mu_k = \alpha x$, where x is the length of the incline it has traversed and $\alpha = \frac{1}{\text{meters}}$. Finally, the block travels along a friction-less bump. Find a bound in h such that the block reaches the end of the track and does so with non-zero velocity and always stays on the track. *Yes, this is medium.*

Hard problem
Look at the figure below. You have a spring attached at a distance x below the center of a semicircular track with radius R. The spring has an equilibrium length $x + R > L \geq \sqrt{x^2 + R^2}$ and spring constant k. You attach a block of mass m to the spring. You shoot the block up at a velocity v_0 starting at a height x below the center. Find the condition for v_0 so that the block stays on the track.