Capacitance for C Team

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1 What is a Capacitor?

The ratio of charge Q to the potential V of an isolated conductor is called its c apacitance C:

$$C = \frac{Q}{V} \tag{1}$$

Capacitance is a measure of hte capacity to store charge ofr a given potential difference, and it only depends on the size and shape of the conductor. The SI unit of capacitance is the coulomb per volt, called a **farad** (F), where $1F = 1C/V$.

Capacitors

A system of two conductors carrying equal but opposite charges are called a **capacitor**; a common capacitor is the **parallel-plate capacitor** which utilizes two parallel conducting plates. This sandwhich can be rolled up into cylinders which many of us see on circuit boards. If we place a charge +Q on one plate and a -Q on the other, the electric field between them is approximately the same as the field between two infinite sheets of charge. For an infinite sheet, the electric field is $Q/A = \sigma/\epsilon_0$, so

$$V = Ed = \frac{\sigma}{\epsilon_0}d = \frac{Qd}{\epsilon_0 A} \tag{2}$$

In general, capacitance depends on the size, shape, and the medium between the two plates.

2 Storing Electrical Energy

When capacitor is being charged, positive charge is transferred from the negatively charged conductor to the positive charged conductor. Therefore, work is needed, which means that the work gets converted and stored into electrostatic potential energy.

The equation for finding the stored energy can be represented in many ways:

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \tag{3}$$

The derivation of this equation is shown in the following (using calculus):

Let q be the charge that has been transferred at some time during the charging process. The potential is then $V = q/C$. If a small amount of additional charge dq is now transferred from the negative conductor to the positive conductor through a potential increase of V, the potential energy of the charge is increased by

$$dU = Vdq = \frac{q}{C}dq \tag{4}$$

Integrating dU, you get:

$$U = \int dU = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C} \tag{5}$$

3 Combinations of capacitors

Capacitors are often used in combinations of each other to generate a unique capacitance value. They can be placed into parallel circuits or series circuits as shown below.

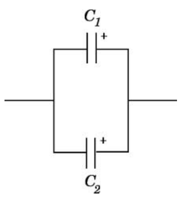
cap.jpeg


Figure 1: A simple parallel capacitor circuit

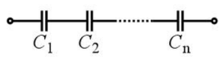


Figure 2: A simple series capacitor circuit

For parallel capacitor circuits, if we have n capacitors, charges of different amounts are flowing into each of the capacitors, so the total charge stored is

$$Q = Q_1 + Q_2 = C_1V + C_2V = (C_1 + C_2)V \tag{6}$$

The equivalent capacitance therefore is the **sum** of all the capacitors. Therefore, for parallel circuits,

$$C_{eq} = C_1 + C_2 + C_3 + ... \tag{7}$$

For series circuits, the total voltage must sum up to the the emf, so

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q(\frac{1}{C_1} + \frac{1}{C_2}) \tag{8}$$

Since $C_{eq} = Q/V$,the equivalent capacitance for a series circuit is:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... \tag{9}$$

Dielectrics

A nonconducting material, such as air, glass, paper, or wood, is called a **dielectifc**. The electric field between the plates of a capacitor is weakened by the dielectric, so the potential difference is reduced and capacitance is increased. If the original field is noted as E_0, the field of the dielectric is

$$E = \frac{E_0}{\kappa} \tag{10}$$

where κ is the **dielectric constant**. Therefore,

$$C = \kappa C_0 = \frac{\kappa \epsilon_0 A}{d} \tag{11}$$

4 Exercises

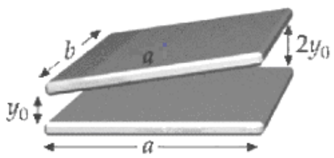
1. A parallel-plate capacitor has square plates of side 10cm separated by 1mm. a)Calculate the capacitance of this system. b) If this capcitor is charged to 12V, how much charge is transferred from one plate to another?

2. A $2\mu F$ capacitor and a $4\mu F$ are connected in series across an 18 V battery. Find the charge on the capacitors and the potential difference across each.

3. True or false: A dielectric inserted into a capcitor increases the capacitance.

4.**Challenge:** A parallel-plate capacitor has plates separated by a distance s. The space between the plates is filled with two dielectrics, one of thickness $\frac{1}{4}s$ and dielectric constantκ_1, the other with thickness $\frac{3}{4}s$ and dielectric constantκ_2. Find the capacitance of this capacitor in terms of C_0, the capacitance with no dielectrics.

5.**Challenge:** A capacitor has rectangular plates of length a and width b. The top plate is inclined at a small angle as shown below. The plate separation varies from $d = y_0$ at the left to $d = 2y_0$ at the right, where y_0 is much less than a or b. Calculate the capacitance using strips of width dx and length b to approximate differential capacitors of area bdx and separation $d = y_0 + (y_0/a)x$ that are connected in parallel. (Calculus needed!) parallel circuits or series circuits as shown below.



Solutions to Exercises

1.a)88.5pF b) 1.06nC

2.Charge=24μC, Potential difference: V_2=12V, V_4=6V

3. True

4.$\dfrac{4\kappa_1\kappa_2}{3\kappa_1 + \kappa_2}$

5.$\dfrac{\epsilon_0 ab}{y_0} ln2$