Rotation Problem Set Solutions:

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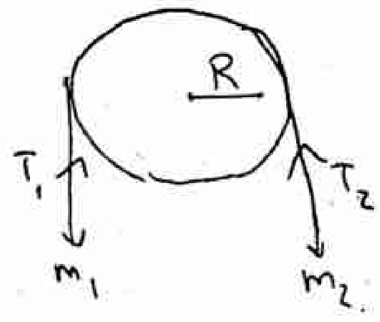
1/6/15

1.



$I = \frac{1}{2} MR^2$ — using circular rings

Proof:

$$I = \int_0^R r^2 dm = \int_0^R r^2 \cdot 2\pi r \lambda \, dr = \frac{\pi r^4 \lambda}{2}$$
$$= \frac{1}{2} MR^2$$

Torque:

$$T_2 R - T_1 R = \frac{1}{2} MR^2 \alpha$$
$$= \frac{1}{2} MaR$$

$R\alpha = a$ ⟸ doesn't slip, so that's true

Forces:

$$-T_2 + m_2 g = m_2 a \Rightarrow m_2(g-a) = T_2$$

$$T_1 - m_1 g = m_1 a \qquad m_1(g+a) = T_1$$

System of equations in three variables.

$$m_2(g-a)\cancel{R} - m_1(g+a)\cancel{R} = \frac{1}{2} M\cancel{R}a.$$

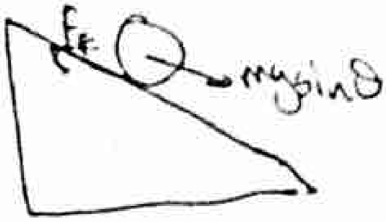
$$m_2 g - m_1 g = m_1 a + m_2 a + \frac{1}{2} Ma.$$

$$\boxed{a = \frac{(m_2 - m_1) g}{m_1 + m_2 + \frac{M}{2}}}$$

⟸ Notice how this simplifies to the expected answer for the massless pulley ($M=0$).

Since $\alpha = \frac{a}{R}$, the angular acceleration of the disk is:

$$\boxed{\alpha = \frac{(m_2 - m_1) g}{R\left(m_1 + m_2 + \frac{M}{2}\right)}}$$

2. 

Note that F_f is not necessarily equal to μF_N in this problem. It's never ~~true~~ safe to assume that for rolling as you might end up with too many constraints. See last week's lecture's solutions for a detailed explanation.

Torques:

$$F_f R = \frac{2}{5} MR^2 \alpha = \frac{2}{5} MRa. \quad \Rightarrow F_f = \frac{2}{5} Ma$$

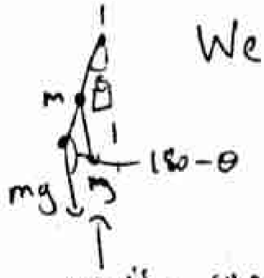
Forces:

$$Mg\sin\theta - F_f = Ma.$$

$$\longrightarrow \quad Mg\sin\theta = \frac{7}{5} Ma.$$

$$\frac{5g\sin\theta}{7} = \frac{dv}{dt}.$$

$$\boxed{v(t) = \frac{5g\sin\theta}{7} t \qquad \omega(t) = \frac{5g\sin\theta}{7R} t}$$

3. 

We can't treat this as a point mass.

$$\vec{\tau} = r \times \vec{F} \Rightarrow |\vec{\tau}| = rF\sin\theta = \frac{L}{2} mg \sin(180-\theta)$$

$$= \frac{L}{2} mg\sin\theta.$$

gravity can be simulated as if it only acted on the center of mass (at $\frac{L}{2}$)

Torque:

$$-\frac{Lmg\sin\theta}{2} = \frac{mL^3}{3} \frac{d^2\theta}{dt^2}$$

(negative because torque opposes)

$$I = \int r^2 dm = \int_0^L \ell^2 \lambda d\ell = \frac{L^3}{3}\lambda = \frac{mL^2}{3}.$$

For small theta: $\sin\theta \approx \theta \Rightarrow \frac{3g\theta}{2L} + \frac{d^2\theta}{dt^2} = 0 \Rightarrow \boxed{T = 2\pi\sqrt{\frac{2L}{3g}}}$