Special Relativity Kinematics Problem Set 1 Solutions

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1 Stars observed moving faster than light?

Light leaves the star at time t'_a and it gets to earth in $t_a = t'_a + d_a/c$, and the same is true of the light from point b, $t_b = t'_b + d_b/c$. So the time difference that you measure on earth is $\Delta t = t_a - t_b = t'_a - t'_b + \frac{d_a - d_b}{c} = \Delta t' + \frac{-v\Delta t' \cos\theta}{c} = \Delta t' \left[1 - \frac{v}{c}\cos\theta\right]$

So we want $u = \frac{\Delta s}{\Delta t} = v\frac{\Delta t'}{\Delta t}\sin\theta = \frac{v \sin\theta}{1 - v/c\cos\theta}$

We all know how to take a derivative of u with respect to θ set it equal to zero blah blah... and the answer for the θ that maximizes the apparent velocity is:

$$\theta_{\max} = \arccos\frac{v}{c}$$

and as $v \to c$ $u \to \infty$

2 Sailboat's Mass Problem

Length contraction only occurs along the direction of motion so projecting we get for the length of the mass L:

$$\tan\bar{\theta} = \frac{L\sin\theta}{\frac{1}{\gamma}\cos\theta} = \gamma\tan\theta$$

3 Velocity Addition a la Einstein

3.1 2 frames

If a particle moves dx in S in dt time then:

$$\begin{aligned} u &= \frac{dx}{dt} \\ d\bar{x} &= \gamma(dx - vdt) \\ d\bar{t} &= \gamma\left(dt - \frac{v}{c^2}dx\right) \\ \bar{u} &= \frac{d\bar{x}}{d\bar{t}} = \frac{\gamma(dx - vdt)}{\gamma(dt - v/c^2 dx)} = \frac{u - v}{1 - uv/c^2} \end{aligned}$$

3.2 N frames

This is just done by induction. I'll put a full proof up later.