Dumbbell Problem

A dumbbell of length $2L$ is lying at rest on a frictionless horizontal surface. At each dumbbell end is a small cube of mass m, but the rod connecting these two cubes is massless. An object with mass M and velocity v perpendicular to the dumbbell strikes the dumbbell at a distance d from its center of mass. They collide elastically. After the collision, the mass M object is at rest. Find M in terms of d, L, v, and m.

Answer

Hey, let's engage in some dimensional analysis before we start the problem. M needs to be linear to m and the answer should include the ratio $\frac{d}{L}$, but the answer can't have v in it because there is no other parameter to nullify v's time dimension.

An elastic collision means that linear momentum and kinetic energy is conserved. Linear momentum conservation:

$$Mv = p_i = p_f = 2mv_{cm}$$

Kinetic energy conservation:

$$Mv^2 = 2KE_i = 2KE_f = I\omega^2 + (2m)v_{cm}^2$$

Angular momentum is conserved about the center of mass of the dumbbell (do you see any external torques?)

$$Mvd = L_i = L_f = I\omega$$

Finally, the inertia of the dumbbell about its center of mass is

$$I = 2mL^2$$

We have 4 equations and 4 variables– v_{cm}, M, ω, and I. Solving for M gives

$$M = \frac{2m}{1 + \frac{d^2}{L^2}}$$