Introduction and Problems in Forces

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1 Introduction

In simple terms, a force is a "push" or a "pull". Unfortunately, this definition was not enough for scientists and engineers. So, in the late 1600's, Newton rigorized the definition of a force by introducing three postulates, now known as *Newton's Laws*.

1. Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it.
2. We will mathematically define force by the following equation: $\sum \vec{F} = m\vec{a}$, where a is the acceleration of the object, and m is an intrinsic property of the object known as its mass.
3. For every action there is an equal and opposite reaction.

These three laws will guide us through all the different problems that we can solve. Don't believe us? See for yourself...

2 Kinematics Problems

1. Consider a projectile launched at an angle θ from a horizontal ground with velocity v_0. Find the angle θ that maximizes
   (a) The horizontal distance traveled (the range)
   (b) The time of flight
   (c) the area enclosed by the ballistic trajectory
   (d) The total distance traveled (the arc length)
2. Repeat problem 1 for the case of a ground inclined at an angle Φ downwards with the horizontal.
3. A hot air balloon begins accelerating upwards from the ground with acceleration a. At the same time, a cannonball is projected from a cannon a distance L away at an inclination θ. Find the initial velocity v_0 required to impact the balloon.
4. Consider a region where the gravitational acceleration is g below a height h and $g/2$ above that height. A ball is launched with velocity v_0 at a height of $3h/2$. Find the launch angle θ that maximizes the time of flight.

3 Introductory Problems

1. An object under free fall suddenly experiences a retarding force bv, where v is the objects instantaneous velocity. If the mass of the object is m, determine its velocity as a function of time (assuming that it initially has a velocity v_0).
2. An object on a frictionless ramp is sliding down under the influence of gravity. If the ramp is inclined by an angle θ, determine the *downward* acceleration of the object.
3. A series of masses are attached together by strings. If all the masses are the same, and the tension applied on the first mass is T_0, show that the tension linearly increases as you go farther down the series.

4 Intermediate Problems

1. A pendulum consists of a mass (called a bob... who came up with that?) m suspended to a ceiling by a massless rope with a length L. The pendulum is displaced by an angle θ from the vertical. Determine the mass's equation of motion as an integral. Then, assuming that small-angle approximations are valid, evaluate the integral.

2. Assume that the rope is shortened at a constant, slow rate V. Determine the mass's equation of motion as an integral again. If possible, employ small angle approximations to simplify the integral.

3. Determine at what point the mass will swing in a horizontal circle rather than oscillate. The small angle approximation does not necessarily have to hold. This is more of a challenging problem, but it helps you get into the mode of thinking logically. (Answer: the height at which this occurs is $\frac{4V^2}{g}$).