Space-Time Scalar Product Invariance Under Lorentz Transformations

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1 Space-time coordinates

in space-time we generally take the time coordinate to be $x^0 = ct$ where the superscript is an index of the variable (its contravariant)and the spacial coordinates are written $x^1 = x$, $x^2 = y$, and $x^3 = z$. We take the scalar product to be:

$$-a^0b^0 + a^1b^1 + a^2b^2 + a^3b^3$$

So here are our Lorentz transformations for reference (the a's are just 4-vector coordinates that also undergo Lorentz transformations). The bars are just the primes from before (but I used bars cause now we have superscripts)

$$\begin{aligned} \bar{a}^0 &= \gamma(a^0 - a^1 v/c) \\ \bar{a}^1 &= \gamma(a^1 - a^0 v/c) \\ \bar{a}^2 &= y \\ \bar{a}^3 &= z \end{aligned}$$

Your job is to show that:

$$-\bar{a}^0\bar{b}^0 + \bar{a}^1\bar{b}^1 + \bar{a}^2\bar{b}^2 + \bar{a}^3\bar{b}^3 = -a^0b^0 + a^1b^1 + a^2b^2 + a^3b^3$$

ie. these scalar products are invariant under Lorentz transformations.