A Short Guide to the $F = ma$ Exam

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1 Introduction

The $F = ma$ is a 25-question, 75-minute multiple-choice exam typically given toward the end of January, covering most of elementary mechanics. Despite its limited time constraints, the $F = ma$ actually tests mechanics knowledge to a fairly deep level, and scoring well on the exam can be a difficult task even for those who are well-versed in mechanics.

Most students take the $F = ma$ with the goal of qualifying for the USAPhO, a more comprehensive 3-hour physics exam that about 500 students take during April. Typically, students need 14-18 questions correct on the exam to qualify, depending on the difficulty. No deductions are given for guessing. Typically, around 10-20 students from TJHSST qualify for the USAPhO.

I prepared this guide because in my experience, the $F = ma$ seems to have less resources available for preparation than it should (especially compared to the AMC competitions), and students do not generally have a good idea of what to expect. This guide was written for students on the TJHSST Physics Team, although it will be useful for anyone attempting to score well.

2 Content

Here I'll address how to learn most of the content covered on the exam, which should probably be done over a significant period of time.

A **rigorous** AP Physics C Mechanics class *technically* teaches most of the content needed to succeed on the $F = ma$. Very few students perform well on the $F = ma$ and qualify for the USAPhO without taking a physics class in school — differing starkly from the AMC competitions, where performance is rarely affected by school courses. The AP Physics C class, however, is not enough to guarantee qualification for the USAPhO. Mechanics has a great deal of subtleties, and many physics problems require deep *understanding* beyond the basic principles — they are often more of an art to be learned.

The two main content areas not covered in a typical AP Physics class are fluids and error: for the former, most good physics textbooks have a comprehensive chapter on fluids with good practice problems. A deep understanding of Archimedes' principle and gauge pressure should be enough to solve most fluids questions. See the end of this document for a quick treatment of error.

Those not taking an AP Physics class who want to self-study for the exam will have a great deal of content to learn. I'd recommend working through a book like Halliday, Resnick, Krane or Tipler and learning the following topics (this should also serve as a good review list for those who already have a mechanics foundation):

- Kinematics
- Dynamics (Newton's Laws, friction, springs, etc.)
- Momentum
- Work, Energy (kinetic, potential, conservation)

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- Gravitation
- Rotational Kinematics
- Angular momentum, torque
- Waves
- Fluids

3 Exam Prep

Doing well on the $F = ma$ requires a deep knowledge of basic concepts, an ability to apply them in a wide variety of scenarios, and good physical intuition.

My experience has been that doing practice $F = ma$ tests is a very good way to prepare for the exam. In the past, I actually learned a good amount of new physics by doing these practice tests and looking up concepts that I didn't understand. I'd recommend doing about 5 practice tests (some under time pressure) before the exam, and carefully reviewing solutions for problems that you miss (Eric Zhang recently published a great set of $F = ma$ solutions here).

4 Exam Strategy

Very few students taking the $F = ma$ will be able to solve all of the questions with a high degree of confidence within the time constraints. Thus, speed is very important. The questions are not technically ordered by difficulty, and often a few questions toward the end of the exam will be quite easy. Similarly, there are often very difficult questions at the beginning of the test (hello 2017 problem 2!).

I recommend going through the test at a decent speed, marking tricky questions for later, and returning to these questions after getting through most of the test. But don't rush too much, for some questions are tricky and silly mistakes are common. Reading the problem statements carefully is essential.

Even if you prepare well, there will be a few curveballs. Don't worry too much — just use your intuition and do your best!

5 Other Things

A few random things related to the test:

- Most years, the $F = ma$ has a dimensional analysis question. The question will probably be really complicated, and you won't have any idea how to do it until you realize that only one of the answer choices has the right units :)
- Simple harmonic motion is highly emphasized! Know that for a quantity x, if $\ddot{x} = ax$ ($\ddot{x}$ is notation for $\frac{d^2x}{dt^2}$), then $\omega^2 = a$, $f = \frac{\omega}{2\pi}, T = \frac{1}{f}$. Be able to apply this in a wide variety of scenarios.
- Sometimes (actually I can only remember one time), the vis-viva equation is useful: for an orbit, $v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$ where v is velocity, r is distance between the two bodies, a is semi-major axis, and M is the mass of the body in the center.
- The exam uses $g = 10\,\mathrm{m/s^2}$ to simplify calculations (the test booklet will remind you!)
- Know that springs, when put together in circuits, behave like capacitors (spring constants add in parallel, and $k = \frac{k_1 k_2}{k_1 + k_2}$ in series). Also, if you cut a spring into pieces, the constant varies inversely with the length of the piece (cutting a spring in half gives two springs with twice the spring constant, etc.)
- Although calculus is not necessary for the exam, it is often useful. In particular, Taylor series and the approximations that result from them (such as $\sin x \approx x, \cos x \approx 1 - \frac{x^2}{2}$ for small x) occasionally surface. Even in cases where calculus cannot be used explicitly, it provides helpful intuition.

6 Error

A good understanding of error propagation has recently become important on the exam, and since few students encounter this in their physics classes, I have put together a short summary here:

When adding quantities x, y with uncertainties $\delta x, \delta y$, respetively, the uncertainties *add in quadrature*, meaning $\delta(x+y) = \sqrt{(\delta x)^2 + (\delta y)^2}$. This applies when adding any number of quantities:

$$\delta\left(\sum_{i=1}^{n} a_i\right) = \sqrt{\sum_{i=1}^{n} (\delta a_i)^2}.$$

When multiplying quantities, proportional uncertainties add in quadrature. This means

$$\frac{\delta\left(\prod_{i=1}^{n} a_i\right)}{\left(\prod_{i=1}^{n} a_i\right)} = \sqrt{\sum_{i=1}^{n}\left(\frac{\delta a_i}{a_i}\right)^2}.$$

An example:

Example 6.1. Suppose Ryan is measuring the surface area of a long rod. He uses a meter stick 3 times with measurements of $1.0\,\mathrm{m} \pm 0.1\,\mathrm{m}$, $1.0\,\mathrm{m} \pm 0.1\,\mathrm{m}, 0.5\,\mathrm{m} \pm 0.1\,\mathrm{m}$ to find that the length is $2.5\,\mathrm{m}$, and finds that the radius is $1.0\,\mathrm{cm} \pm 0.2\,\mathrm{cm}$ using a ruler. What is the uncertainty in the value he calculates for the surface area?

Solution. Let the three length measurements be l_1, l_2, l_3, and the radius measurement be r. Then the quantity is $2\pi r \cdot (l_1 + l_2 + l_3)$. The uncertainty in the length measurement is

$$\delta l = \sqrt{(\delta l_1)^2 + (\delta l_2)^2 + (\delta l_3)^2}$$

and if the area is A, we have

$$\frac{\delta A}{A} = \sqrt{\left(\frac{\delta l}{l}\right)^2 + \left(\frac{\delta r}{r}\right)^2}.$$

We know $A = 2\pi \cdot 2.5\,\mathrm{m} \cdot 0.010\,\mathrm{m} = 0.16\,\mathrm{m}^2$, so the answer is

$$\delta A = 0.16\,\mathrm{m}^2 \cdot \sqrt{\left(\frac{\delta l}{l}\right)^2 + \left(\frac{\delta r}{r}\right)^2} = 0.16\,\mathrm{m}^2 \cdot \sqrt{\frac{(0.1\,\mathrm{m})^2 + (0.1\,\mathrm{m})^2 + (0.1\,\mathrm{m})^2}{(2.5\,\mathrm{m})^2} + \left(\frac{0.2\,\mathrm{cm}}{1\,\mathrm{cm}}\right)^2} = 0.03\,\mathrm{m}^2$$

so the measurement is $\boxed{0.16\,\mathrm{m}^2 \pm 0.03\,\mathrm{m}^2}$.

Finally, one concept that appeared on the 2018 exam was the following: when one makes n independent measurements of the same quantity, the uncertainty decreases by a factor of $\sqrt{n}$. This is a consequence of the additivity of variance.

7 Conclusion

Good luck! Get a good night's sleep, and make sure to be careful and neat with your work during the test. Don't worry too much about the results, and have fun with the problems!