Regarding 2^{nd} Order Differential Equations: Techniques

We'll deal with linear, constant coefficient, homogeneous equations for now. (This means equations of the form

$$a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = 0 \tag{1}$$

A general solution to this differential equation is a linear combination of two functions $x_1(t)$ and $x_2(t)$. Why is unfortunately beyond the scope of this lecture.

$$x(t) = Ax_1(t) + Bx_2(t) \tag{2}$$

So as soon as we've found two linearly independent solutions to Eqn. 1, we've found the general formula for a solution to Eqn. 1. The meat of this lecture comes in telling you to guess

$$x(t) = e^{\alpha t}$$

for Eqn. 1. This gives us

$$a(\alpha^2 e^{\alpha t}) + b(\alpha e^{\alpha t}) + c(e^{\alpha t}) = 0 \tag{3}$$

Or, canceling the factor of $e^{\alpha t}$,

$$a\alpha^2 + b\alpha + c = 0 \tag{4}$$

Barring the special case of multiple roots, this gives us two values for α, and thus two linearly independent solutions to Eqn. 1. Initial conditions for the motion (usually $x(0)$ and $x'(0)$) determine A and B of Eqn. 2.