Special Relativity Problem Set #1

Shankar Balasubramanian   Ross Dempsey

October 2014

1 Problems

1. (Easy) Two trains, A and B, each have proper length L and move in the same direction. A's speed is $\frac{4c}{5}$ and B's speed is $\frac{3c}{5}$. The front of A starts out lined up with the back of B. How long, for an inertial observer, does it take for the back of A to pass the front of B?

2. (Easy) A train with a proper length L moves at a speed $\frac{5c}{13}$ with respect to the ground. A ball is thrown from the back of the train to the front. The speed of the ball with respect to the train is $\frac{c}{3}$. As viewed by someone on the ground, how much time does the ball spend in the air, how far does it travel, and what is its velocity as perceived by a stationary observer?

3. (Easy) A rod of proper length l_0 oriented parallel to the x axis moves with a speed u along the x-axis in a stationary reference frame. What is the length of the rod as perceived by an observer moving at a velocity v?

4. (Easy) A stick of length L passes you at a speed v. There is a particular time Δt you measure between the passing of the front end of the rod and the back end. What is this time interval in
   (a) your frame? (working in your frame)
   (b) your frame? (working in the stick's frame)
   (c) the stick's frame? (working in your frame)
   (d) the stick's frame (working in the stick's frame)

5. (Medium) A square with side L flies towards you at a velocity v in a direction parallel to two of it's sides. You stand in the plane of the square. When you see the square at it's nearest point, show that it appears to be rotated rather than contracted.

6. (Medium) A and B travel at $4c/5$ and $3c/5$ respectively with respect to the ground. C travels in between A and B. How fast should she travel so that she sees A and B approaching her at the same speed? What speed do they appear to approach her at?

2 Extensions

7. (Medium) Differentiate the Lorentz transformation formulae to determine the following velocity transformation formulae for a frame Σ' moving with velocity v in the positive x direction, where $\gamma = 1/\sqrt{1 - v^2/c^2}$:

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2}$$
$$u'_y = \frac{1}{\gamma}\frac{u_y}{1 - u_x v/c^2}$$
$$u'_z = \frac{1}{\gamma}\frac{u_z}{1 - u_x v/c^2}$$

8. (Medium) There is a notion of conserved momentum $p = mu$ in the above velocity transformation. However, the mass m is not Lorentz invariant. Let $mu_y = m'u'_y$ and derive the relationship between m and m'. You should find that there is a Lorentz invariant quantity $m\sqrt{1 - u_x^2/c^2}$, which we denote by m_0 and call the rest mass.

9. (Medium) Consider an angle ϕ such that $\tan\phi = -i\frac{v}{c}$. Show that upon rotation through ϕ, the complex vector $\mathbf{r} = \langle x, ict\rangle$ transforms into $\langle x', ict'\rangle$ where x' and t' are given by the Lorentz transformation formulae. ϕ is known as the rapidity.

10. (Medium) A vector such as the one in the previous problem, which has its length invariant under a Lorentz transformation, is called a world-vector[1]. Show that $\mathbf{p} = \langle p_x, ict \rangle$ is a world-vector.

11. (Medium) A cookie cutter stamps circular cookies as the dough rushes beneath it (at a speed v). Determine the orientation and the eccentricity of the resulting cookies.

12. (Hard) Consider an object accelerating at a proper acceleration $a(t)$. Determine the velocity of the spaceship as a function of time (in the spaceship's frame). Hint: Use the velocity addition formula to find the relative velocity of the spaceship with respect to the inertial frame after a acceleration for a small period of time. The answer should be

$$v(t) = c \tanh\left(\frac{1}{c}\int_0^t a(t)dt\right)$$

[1] They are more often called four-vectors, but since we are neglecting two spatial dimensions, we will call them world-vectors for now.