Here's a Solution to the Fun Problem

Let's work in the accelerated frame (gasp!) of the car. There appears to be a pseudoforce equal to the mass of the door times the car's acceleration pulling the car door backwards. Call the mass of a thin horizontal strip of the car door M, so that the total torque on this strip in the upwards direction is

$$\tau_z = -Ma\frac{L}{2}\sin(\theta) = I\alpha \tag{1}$$

Since the door is spinning around its hinge, the inertia of this thin strip is taken around its endpoint, which makes

$$I = \frac{1}{3}ML^2 \tag{2}$$

Memorize this fact (inertia of a strip around its endpoint is $\frac{1}{3}$ the product of its mass and length squared) or use the Parallel Axis Theorem. Plug Eqn. 2 into Eqn. 1 to get

$$-\frac{1}{2}MaL\sin(\theta) = \frac{1}{3}ML^2\frac{d^2\theta}{dt^2} \tag{3}$$

Since $\theta << 1$ (it starts out pretty small and only gets smaller) let's approximate $\sin(\theta) = \theta$. Rearranging terms in Eqn. 3 gives

$$\frac{d^2\theta}{dt^2} = -\frac{3a}{2L}\theta \tag{4}$$

This looks like the equation for a harmonic oscillator! (Yes, there's a good reason for why everything looks harmonic at some point. I'll tell you later, but try to guess why.) Anyway, remembering our earlier trick for finding solutions to differential equations, we get that

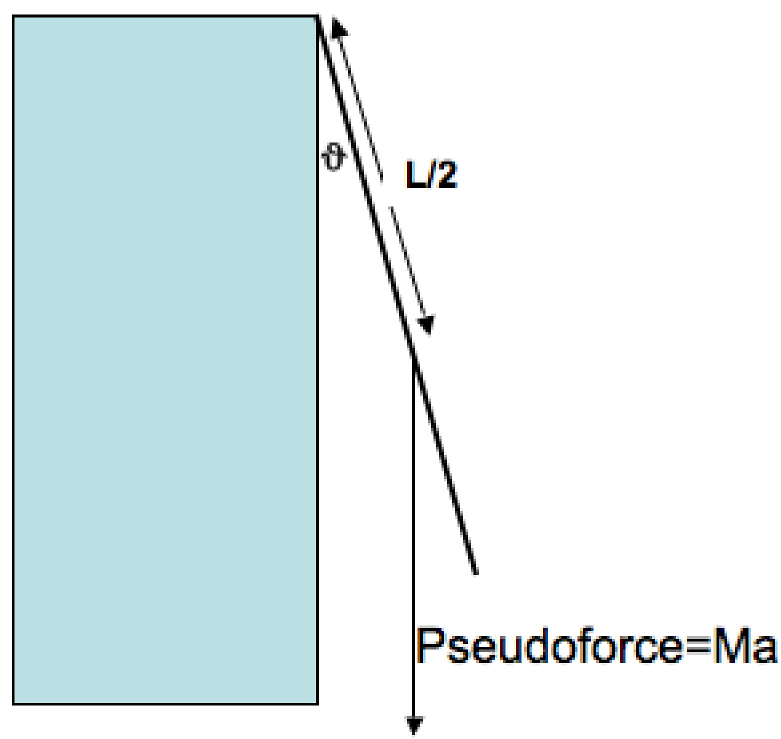
$$\theta = Ae^{i\sqrt{\frac{3a}{2L}}t} + Be^{-i\sqrt{\frac{3a}{2L}}t} \tag{5}$$

where A and B are arbitrary constants determined by initial conditions. Initially, $\theta = \frac{\pi}{180}$, and $\omega = \frac{d\theta}{dt} = 0$. Plugging these into Eqn. 5 gives

$$\theta = \frac{\pi}{180}\cos(\sqrt{\frac{3a}{2L}}t) \tag{6}$$

We want to find how much time it takes for the door to reach $\theta = 0$, so plugging that in gives

$$t_f = \frac{\pi}{2}\sqrt{\frac{2L}{3a}} \tag{7}$$


θ
L/2
Pseudoforce=Ma