2D Kinematics

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1 Introduction

2D Kinematics involves objects that move along the x-axis and y-axis simultaneously.

2 Kinematic Definitions

2.1 Position and Displacement

A position vector $\vec{r}$ can be written as $\vec{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$. Since displacement is defined as the change in the position vector, $\Delta\vec{r} = \Delta x\hat{\mathbf{i}} + \Delta y\hat{\mathbf{j}}$.

2.2 Velocity

Average velocity is defined as change in position (displacement) over change in time, or $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$. Using components, we can also express this as $\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{\mathbf{i}} + \frac{\Delta y}{\Delta t}\hat{\mathbf{j}}$.

2.3 Acceleration

Average acceleration is defined as change in velocity over change in time, or $\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$. Using components, we can also express this as $\vec{a}_{avg} = \frac{\Delta v_x}{\Delta t}\hat{\mathbf{i}} + \frac{\Delta v_y}{\Delta t}\hat{\mathbf{j}}$.

3 2D Kinematic Equations

The three 1D Kinematics Equations also apply in 2D, however we must independently consider motion in the x-direction and the y-direction.

Table 4.2 Kinematic Equations in One and Two Dimensions

One-dimensional Kinematics Equations	Two-dimensional Kinematic Equations	
	x-direction	*y*-direction
$x = v_i t + \frac{1}{2} at^2$	$x = v_{ix} t + \frac{1}{2} a_x t^2$	$y = v_{iy} t + \frac{1}{2} a_y t^2$
$v_f^2 = v_i^2 + 2ax$	$v_{fx}^2 = v_{ix}^2 + 2a_x x$	$v_{fy}^2 = v_{iy}^2 + 2a_y y$
$v_f = v_i + at$	$v_{fx} = v_{ix} + a_x t$	$v_{fy} = v_{iy} + a_y t$

Checkpoint Problem: A particle with velocity $\vec{v_0} = -2.0\hat{i} + 4.0\hat{j}$ (in meters per second) at $t = 0$ undergoes a constant acceleration $\vec{a}$ of magnitude $a = 3.0\,\mathrm{m/s^2}$ at an angle $\theta = 130°$ from the positive direction of the x-axis. What is the particle's velocity $\vec{v}$ at $t = 5.0\,\mathrm{s}$, in unit-vector notation?

4 Projectile Motion

A **projectile** is an object that is launched with an initial velocity, has no power, and is assumed to not be affected by air. For example, a tennis ball in flight is a projectile, whereas a rocket is not a projectile.

4.1 Horizontal Motion

A projectile has *no horizontal acceleration*, so a projectile's horizontal velocity is unchanged from it's initial horizontal velocity v_{0x}. We know from the table above that $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$.

Since there is no horizontal acceleration, we can simplify: $x - x_0 = v_{0x}t$.

Because $v_{0x} = v_0 cos\theta_0$, this becomes

$$x - x_0 = (v_0 cos\theta_0)t \tag{1}$$

4.2 Vertical Motion

In the vertical direction, a projectile experiences constant downward vertical acceleration (gravity). Switching to y-notation, we know that $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$.

Since projectiles experience gravity, we can substitute -g for a: $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$.

Because $v_{0y} = v_0 sin\theta_0$, this becomes

$$y - y_0 = (v_0 sin\theta_0)t - \frac{1}{2}gt^2 \tag{2}$$

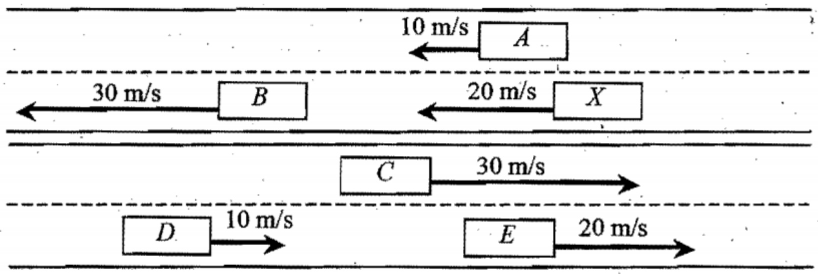
Checkpoint Problem: A convertible car with an open top is driving at constant velocity when a ball is thrown straight upward by one of the passengers. If there is no air resistance, where will the ball land?
(A) In front of the car
(B) In the car
(C) Behind the car
(D) The answer depends on the speed of the projectile
(E) The answer depends on the speed of the car

5 Relative Motion

The velocity of an object depends on the observer's **reference frame** (a physical object to which we attach our coordinate system). For example, a car observed to be moving at $30\,\mathrm{m/s}$ by a stationary observer will appear to be moving at $10\,\mathrm{m/s}$ by an observer in a car traveling alongside at $20\,\mathrm{m/s}$.

The reference frame of an object has to have constant velocity (ex. Earth). Common relative motion problem include a boat moving across a river and an airplane flying through the air.

Checkpoint Problem:



In the diagram above, six cars are shown driving on a four-lane highway. Two observers are riding in car *X*. When compared with the observers, which car appears to have a velocity of 10 meters per second in the opposite direction?

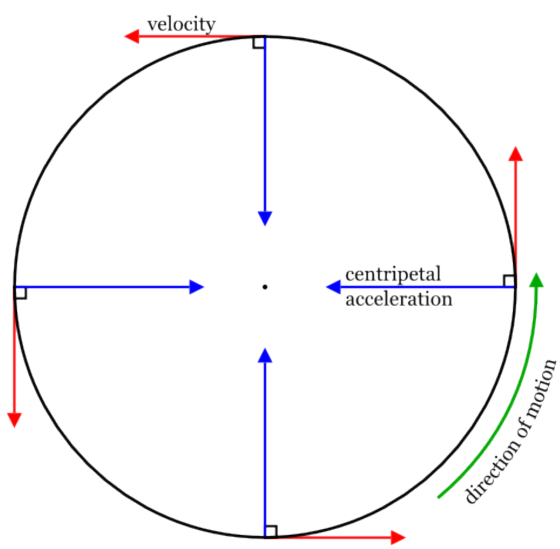
(A) *A*
(B) *B*
(C) *C*
(D) *D*
(E) *E*

6 Uniform Circular Motion

Uniform Circular Motion involves objects moving around a circle with a constant (*uniform*) speed.

However, the velocity's direction still changes, despite having a constant magnitude. Since velocity changes, there is also an acceleration that points to the center of the circle (called *centripetal acceleration*). The magnitude of the acceleration vector is defined by $a = \frac{v^2}{r}$, where v is the speed of the particle, and r is the radius of the circle.

In addition, the time to complete one revolution is called a *period*, T. This is defined by $T = \frac{2\pi r}{v}$.



7 Problems

1. (Projectile Motion) A projectile is fired horizontally from a gun that is 45.0 m above flat ground, emerging from the gun with a speed of 250 m/s. (a) How long does the projectile remain in the air? (b) At what horizontal distance from the firing point does it strike the ground? (c) What is the magnitude of the vertical component of its velocity as it strikes the ground?

2. (Projectile Motion) A dart is thrown horizontally with an initial speed of 10 m/s toward point P, the bull's-eye on a dart board. It hits at point Q on the rim, vertically below P, 0.19 s later. (a) What is the distance PQ? (b) How far away from the dart board is the dart released?

3. (Relative Motion) Snow is falling vertically at a constant speed of 8.0 m/s. At what angle from the vertical do the snowflakes appear to be falling as viewed by the driver of a car traveling on a straight, level road with a speed of 50 km/h?

4. (Relative Motion) A swimmer heads directly across a river, swimming at 1 m/s relative to the water. She arrives at a point 39 m downstream from the point directly across the river, which is 81 m wide. (a) What is the speed of the river current? (b) What is the swimmer's speed relative to the shore?

5. (Uniform Circular Motion) Find the centripetal acceleration for an object on the surface of a planet (at the equator) with with the following characteristics: radius $= 4 \times 10^6$m and 1 day = 100000 seconds.