Introduction to Rotation

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1 Introduction

Today we will talk about the physics of rotation. Rotation is always considered with respect to a point. A wheel rotates about its center. Rotation can be caused by a net **torque**. Likewise, we always consider torque with respect to a point. If we call force the fundmanetal factor behind linear motion, torques are the rotational analog.

Another way to look at is that an object that is rotating tends to stay rotating until an external torque opposes it.

2 Torque

Torques are vectorial quantities. We define torque to be:

$$\vec{T} = \vec{r} \times \vec{F}$$

The term $\vec{r}$ refers to the vector from the point of rotation to the point at which the force is being applied.

3 Moment of Inertia

Inertia is the tendency of an object to stay in its motional state. Generally, objects of higher mass have higher inertia than objects of lower mass, but it also depends on the distribution of mass. In rotation, the moment of inertia plays the role mass does in linear equations in rotational equations. Though we are dealing with rotation, we consider moment of inertia with respect to an axis. The closer mass is distributed to the axis of rotation, the smaller the moment of inertia. Here's the formula for the moment of inertia:

$$I = \int r^2 dm$$

It might look complicated, but all it's saying is that the inertia is the sum of each tiny piece of mass weighted by its distance from the axis of rotation squared. For a point mass, it is simple to see how the moment of inertia is simply $I = mr^2$.

4 The Equation

By now, you should be noticing lots of similarities between linear motion and rotational motion. We present one of our most useful relationship, Newton's Second Law, in rotational form.

$$\sum_i \vec{T_i} = I\vec{\alpha}$$

The quantity $\vec{\alpha}$ is the angular acceleration. If there is no slipping, then the equation $|\vec{a}| = r|\vec{\alpha}|$, where r is the distance from the point of rotation, is satisfied. The more general relates the three via a cross product. This is how we combine linear motion and rotational motion.

5 Problems

5.1 Determining Torques

1. If the net force acting on an object is 0, can there be a torque on the object? What about the other way around?

2. The force $\vec{F} = 7N\hat{\imath} + 3N\hat{\jmath} - 4N\hat{k}$ is acting on a point mass located at $(2, 5, 6)$. What is the resultant torque about the origin? What is the resultant torque about $(1, 2, 3)$?

3. A sphere of radius R and of mass M rolls on a surface with frictional constant μ without slipping. What is the magnitude of torque due to friction about the center of mass of the sphere?

5.2 Finding Moments of Inertia

1. A rod of length L has mass M. What is the moment of inertia about the center of the rod? What is the moment of inertia about one end of the rod? Assume the rod has constant density.

2. Consider the same rod as above, except its density varies. The density is given by $\lambda(l) = kl^2$, where k is a constant, and l is the distance from one end of the rod. Find the moments about the points in the first problem.

3. What is the moment of inertia of a circular loop of radius R and mass M about its center? What is the moment of inertia of a circular disk of radius R and mass M? Assume each figure has constant density.