Momentum/Energy Problem Set 2

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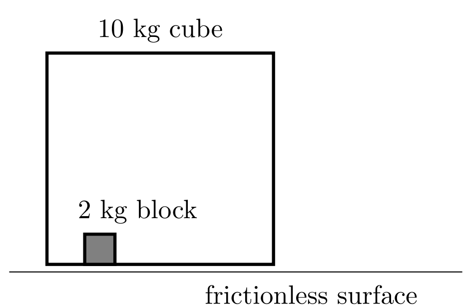
Most of these problems are extremely difficult in nature, so don't lose hope if you cannot solve all of them. We will post detailed solutions to these problems soon, but it might take several weeks.

1 Problems

1. **EASY**(F=ma 2008) A toboggan sled is traveling at 2.0 m/s across the snow. The sled and its riders have a combined mass of 120 kg. Another child (40 kg) headed in the opposite direction jumps on the sled from the front. She has a speed of 5.0 m/s immediately before she lands on the sled. What is the new speed of the sled? Neglect any effects of friction.

2. **EASY** (F=ma 2013) Jordi stands 20 m from a wall and Diego stands 10 m from the same wall. Jordi throws a ball at an angle of 30 degrees above the horizontal, and it collides elastically with the wall. How fast does Jordi need to throw the ball so that Diego will catch it? Consider Jordi and Diego to be the same height, and both are on the same perpendicular line from the wall.

3. **EASY** (F=ma 2014) A cubical box of mass 10 kg with edge length 5 m is free to move on a frictionless horizontal surface. Inside is a small block of mass 2 kg, which moves without friction inside the box. At time $t = 0$, the block is moving with velocity 5 m/s directly towards one of the faces of the box, while the box is initially at rest. The coefficient of restitution for any collision between the block and box is 90%, meaning that the relative speed between the box and block immediately after a collision is 90% of the relative speed between the box and block immediately before the collision.



After 1 minute, the block is a displacement x from the original position. Which of the following is closest to x?

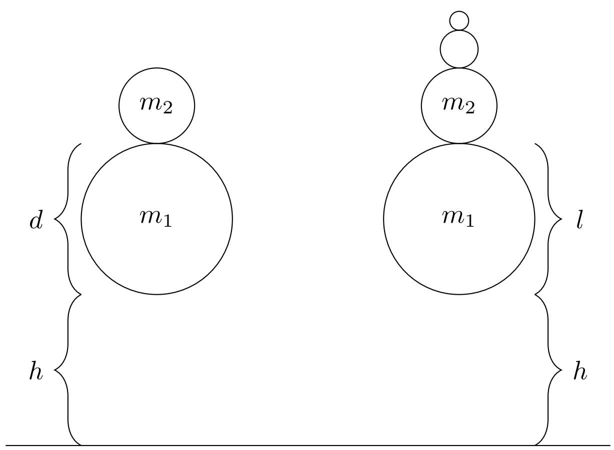
(A) 0 m
(B) 50 m
(C) 100 m
(D) 150 m

(E) 200 m

4. **EASY** Sand falls from a stationary "hopper" onto a freight car which is moving at a uniform velocity v. The sand falls at a rate $\frac{dm}{dt}$. How much force is needed to keep the car moving at a speed v. Now, consider the case when the freight car is leaking sand at a rate $\frac{dm}{dt}$ which moving at a speed v. What force is required for the car to move uniformly at a speed v?

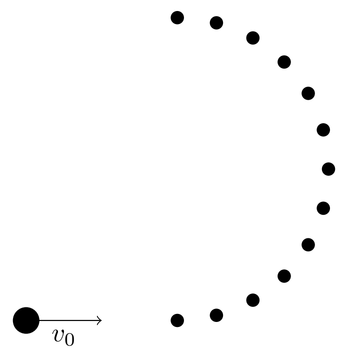
5. **MEDIUM** Two billiard balls collide elastically (but not head on, causing them to splay out at angles). Show that the angle between the resulting velocities of the two balls is 90 degrees.

6. The following problem discusses about the center of mass reference frame, which is a helpful tool for analyzing collisions. The center of mass reference frame is self explanatory: we essentially picture ourselves riding along the trajectory of the center of mass, viewing the collision in that particular orientation. Using this fact, answer the following questions about the center of mass frame with respect to elastic collisions
(a) **MEDIUM** Velocities are conserved during elastic collisions (!), but are rotated by a "scattering" angle Θ.
(b) **MEDIUM** Show that, through the vector geometry of the situation, $\tan\theta = \frac{v'\sin\Theta}{V+v'\cos\Theta}$, where v' is the final velocity of a particle in the center of mass frame, θ is the lab frame scattering angle, Θ is the scattering angle in the center of mass frame, and V is the velocity of the center of mass of the system.
(c) **MEDIUM** Show that the expression in part (b) simplifies to $\tan\theta = \frac{\sin\Theta}{(m_1/m_2)+\cos\Theta}$.
(d) **EASY** Find the maximum value of (c) and interpret the result geometrically



7. (a) **MEDIUM** A tennis ball with a small mass m_2 sits on top of a basketball with a large mass m_1. The bottom of the basketball is a height h above the ground, and the bottom of the tennis ball is a height $h + d$ above the ground. The balls are dropped. To what height does the tennis ball bounce? Note: Work in the approximation where m_1 is much larger than m_2, and assume that the balls bounce elastically. Also assume, for the sake of having a nice clean problem, that the balls are initially separated by a small distance, and that the balls bounce instantaneously.
(b) **HARD** Now consider n balls having masses $m_1, m_2, \cdots, m_n$ (with $m_1 \gg m_2 \gg \cdots \gg m_n$), standing in a vertical stack. The bottom of the first ball is a height h above the ground, and the bottom of the second ball is a height $h + l$ above the ground. The balls are dropped. In terms of n, to what height does the top ball bounce? Note: Make assumptions and approximations similar to the ones in part (a). Assume that the balls still bounce elastically (which is a bit absurd here), and ignore wind resistance, etc., and assume that l is negligible.

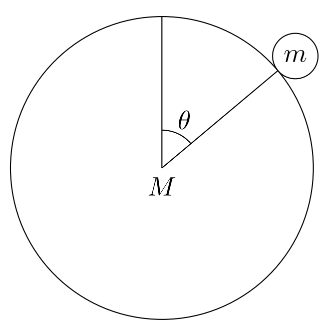
8. (Semifinal Exam 2009) **HARD** Two balls, are dropped from a height h, the smaller of the two placed on the larger, making an angle ϕ with the vertical. Assuming that $M \gg m$, and the resulting collision is elastic, determine the range of the resulting motion of the small ball. At what value of ϕ is this range maximized? What is the range's maximum value?



9. **VERY HARD** N identical balls lie equally spaced in a semicircle on a frictionless horizontal table, as shown. The total mass of these balls is M. Another ball of mass m approaches the semicircle from the left, with the proper initial conditions so that it bounces (elastically) off all N balls and finally leaves the semicircle, heading directly to the left. (a) In the limit $N \to \infty$ (so the mass of each ball in the semicircle, M/N, goes to zero), find the minimum value of M/m that allows the incoming ball to come out heading directly to the left. Hint: Consider the center of mass reference frame, and what the scattering angle should be. (b) In the minimum M/m case found in part (a), show that the ratio of m's final speed to initial speed equals $e^{-\pi}$.

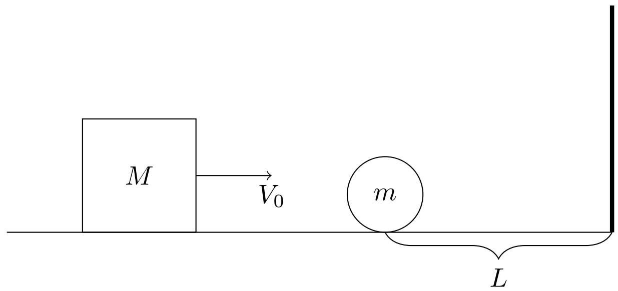
10. **VERY HARD** A small ball is attached to a massless string of length L, the other end of which is attached to a thin, vertical pole. The ball is thrown such that it initially moves in a circular path, with the string making an angle θ_0 with the vertical. As time passes, the string will wrap itself around the pole. Assume that the process is adiabatic (very slow in nature), so the mass's motion can be always approximated as a circle. Neglect wind resistance and assume that the string does not slip on the pole once they are in contact. Determine the point when the ball hits the pole, as well as the ratio of the final and initial velocity of the ball.

    Hint: Consider the quantity $E = K + U$ and notice that $dE = 0$.



11. **VERY HARD** This is a classic example of the Conservation of Energy and Momentum: If we consider a small mass placed on top of a humongous hemisphere, and we perturb the small mass, it will slide down the hemisphere. The angle at which the small mass loses contact with the hemisphere is $\cos\theta = \frac{2}{3}$.

    Now, consider the case when the hemisphere's mass is NOT much larger than the mass of the block. If the mass of the block is m and the mass of the sphere is M, determine the point at which the small mass will lose contact with the larger mass. You may express your answers in terms of g and the radius of the hemisphere, R. You can write your solution as a cubic equation; however, the solution to the case $m = M$ is quite simple.



12. **EXTREMELY HARD** A block with large mass M slides with speed V_0 on a frictionless table towards a wall. It collides elastically with a ball with small mass m, which is initially at rest at a distance L from the wall. The ball slides towards the wall, bounces elastically, and then proceeds to bounce back and forth between the block and the wall.

    (a) How close does the block come to the wall?

    (b) How many times does the ball bounce off the block, by the time the block makes its closest approach to the wall? Assume that $M \gg m$, and give your answers to leading order in m/M.

    This question is out of the scope of the USAPhO curriculum, but the general method of solution is very interesting.