A Cumulative Thermodynamics Problem Set

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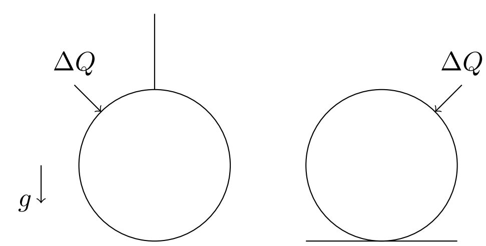
Each if these problems are rated on a scale of $1-5$, 1 being the easiest, and 5 being the most difficult.

1 Thermal Properties of Matter

1. **5** Two identical spherical containers are isolated. One is suspended by a thread, and the other is placed on a flat surface. Assume that the containers are not deformed. The same amount of heat is added to both containers, ΔQ, and the containers' coefficient of linear expansion, specific heat, radius, and mass are α, c, R, and m respectively. Determine the difference in both containers' final temperatures after the addition of heat. The acceleration due to gravity is g.

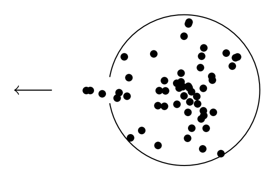


2. **3** When the air temperature T is below 0 degrees C, the water at the surface of a lake freezes to form a sheet of ice. Calculate the time it takes for a layer of thickness d to develop. Assume that the latent heat of melting/cooling is L, the conductivity of ice is κ, and the ice has a density ρ.

3. **3** Prove that the coefficient of volumetric expansion is thrice the coefficient of linear expansion. Hint...Consider a cube composed of three *linear* dimensions

2 Statistical Mechanics

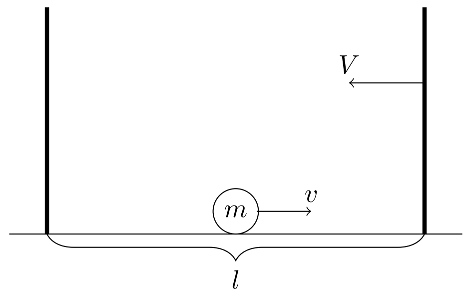
1. **5** Consider a spherical balloon with an initial mass of air M. The balloon is initially at a temperature T_0 when a small orifice is impressed upon the balloon. The orifice has an area A and the ideal gas inside of the balloon starts to diffuse outwards.



a. Assuming the pressure outside of the balloon is atmospheric (P_0), determine an integral expression for the mass inside of the balloon as a function of time. Assume the process occurs slowly, and is thus adiabatic. You may also use the density of the gas as a variable (ρ), which we will assume to be roughly constant.

b. Now, evaluate the integral for the case in which the gas expands into a vacuum. Determine the time it takes for all the gas to disperse out of the balloon. Assume the gas has a molar mass $\mathcal{M}$.

c. Now, consider the balloon (of mass m) as a dynamic object, subject to motion due to the expulsion of gas. Determine a differential equation as a function of time, assuming that the balloon moves through a vacuum. Ignore any pressure differences caused inside the balloon (i.e. the pressure is isotropic).



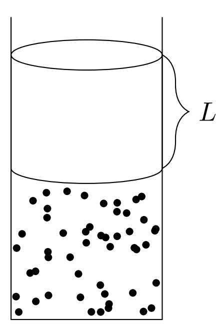
2. **4** A "superball" of mass m bounces back and forth between two surfaces at a speed v_0. All collisions are perfectly elastic, and the environment has zero gravity. If one surface is moved toward the other at a speed $V \ll v$, the ball will speed up. Determine the average force on each wall when they are a distance x from each other. Assume that the initial distance between the walls is l.

   a. From this example we introduced many months ago, determine the average pressure due to the single particle

   b. Using the ideal gas law for n mols of such a gas, determine the temperature of the gas as a function of distance.

3 The First Law of Thermodynamics

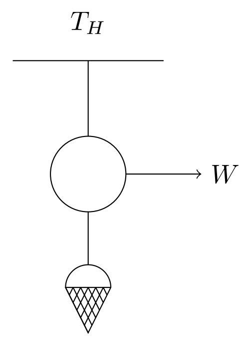
1. **4** A gas engine is composed of a solid piston of length L that is constrained to move in a glass container. Furthermore, the gas enclosed in the jar is initially at a pressure P_0 and temperature T_0 when the piston is held a height y_0 above the base of the container. The pressure and temperature outside the piston are also P_0 and T_0. When the piston is released, it behaves like a spring, reaching a minimum distance above the base of $y_0/8$ before oscillating and eventually reaching an equilibrium height y_2. Determine the ratio y_2/y_0 and the density of the piston if the process is adiabatic. Assume that the piston falls under the influence of gravity.



2. **3** A piston is operated with n mols of ideal gas at temperature T_0 and pressure P_0. The piston's area and mass are A, M respectively. Determine the period of oscillation if the piston is slightly disturbed from equilibrium and the gas undergoes an isothermal expansion.

4 The Second Law of Thermodynamics

1. **4** A student at TJ decides to celebrate 4 long years of hard work with some ice cream. His Carnot engine operates at a hot reservoir of temperature T_h and a cold reservoir (a jar full of ice cream) at initial temperature T_0. He extracts heat from the hot reservoir at a rate $\frac{dQ}{dt} = P_h$. As you may know, some of this heat goes into work, and the rest of it is sent into the cold reservoir. Initially, the ice cream is at the start of a phase change ($t = 0$), so the power output is constant. Find the time it takes, t_0, for the ice cream to completely change from a solid to liquid state. After the phase change, the power output varies as a function of time. Find this power output in terms of all the given variables (including t_0), and an arbitrary time $t > t_0$. Assume the ice cream to have a mass m, a latent heat of melting l, and a specific heat (at liquid state) c.



2. **4** Two identical bodies have a constant pressure heat capacity c_p are used as reservoirs to drive a heat engine. They remain at constant pressure, but their initial temperatures T_1 and T_2 change and reach a final, common temperature T_f.

   a. Determine the work done by the heat engine

   b. Determine an inequality relating T_f, T_1, and T_2.

   c. Determine the maximum obtainable work from the engine

5 Miscellaneous

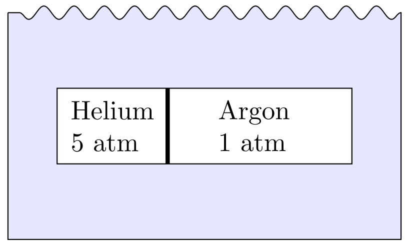
1. **4** Consider the figure below. A cylindrical container 80 cm long is separated into two compartments by a thin piston, clamped 30 cm from one end. The left side is filled with helium gas at 5 atm. The right is filled with argon gas at 1 atm. The cylinder is submerged in 1 liter of water, which is initially at a temperature of 25 degrees C. When the piston is released, a new equilibrium position will be reached.

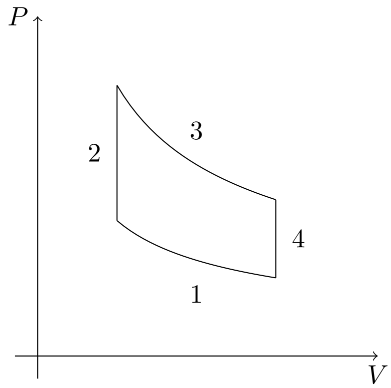
   a. What is the corresponding increase in the temperature of the water?

   b. What is the new position of the piston?

   c. What is the increase in the system's entropy?

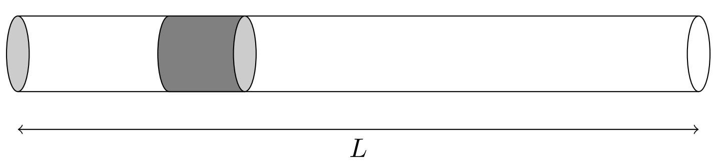
2. **3** The following cycle is called the Otto cycle: An adiabatic compression (1), an isochoric compression (2), and adiabatic expansion (3), and an isochoric expansion (4), as illustrated in the diagram below. Determine the efficiency of the cycle in terms of γ, and the ratio of the volumes α.





3. **4** A glass bulb contains air at room temperature and a pressure of 1 atm. It is placed in a *large* chamber filled with helium gas of the same pressure and temperature. The glass bulb is known to only be permeable to helium atoms. After a long time, once equilibrium has been reached, what will be the new gas pressure in the bulb?

4. **5** (USAPHO 2009) A potato gun fires a potato horizontally down a half-open cylinder of cross-sectional area A. When the gun is fired, the potato slug is at rest, the volume between the end of the cylinder and the potato is V_0, and the pressure of the gas in this volume is P_0. The atmospheric pressure is P_{atm}, where $P_0 > P_{atm}$. The gas in the cylinder is diatomic; this means that $c_v = 5R/2$ and $c_p = 7R/2$. The potato moves down the cylinder quickly enough that no heat is transferred to the gas. Friction between the potato and the barrel is negligible and no gas leaks around the potato. The potato has a mass m.



a. What is the maximum kinetic energy E_{max} with which the potato can exit the barrel? Express your answer in terms of P_0, P_{atm}, V_0, and m.

b. What is the length L in this case? Express your answer in terms of P_0, P_{atm}, V_0, A, and m.

Hint: You will need to solve a linear differential equation of a first order.