Simple Harmonic Motion

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1 Introduction

Forces give us a way to describe how a system will develop over time. Systems that exhibit periodic or harmonic behavior are especially fascinating. *Simple harmonic motion*, or SHM for short, is this periodic and repetitive motion of an object, such as a pendulum or a mass on a spring.

2 Definitions

- **Period** (T) - the time in seconds for an object in SHM to complete one full oscillation

$$T = \frac{total\,time}{\#\,of\,oscillations}$$

- **Frequency** (f) - the number of full oscillations that occur in 1 second

$$f = \frac{\#\,of\,oscillations}{total\,time} = \frac{1}{T}$$

- **Amplitude** (A) - the magnitude of displacement of an oscillating particle relative to its rest position

3 Important Examples

3.1 Mass on a Spring

Imagine that we hang a spring from the ceiling and attach a mass at the other end. We then will stretch the spring a certain amount and let go. Not surprisingly, this system will exhibit SHM. It turns out that the period of the resulting motion only depends on the mass of the object and spring constant

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

3.2 Pendulum

Pendulums are simple harmonic oscillators that take the form of a mass suspended at the end of a string. The period of a pendulum is the amount of time it takes for the mass to swing from an initial position back to the same position. This time, the period depends on the length of the string and the acceleration due to gravity

$$T_p = 2\pi\sqrt{\frac{L}{g}}$$

4 Problems

1. An oscillator completes 20 cycles in 50 seconds. Determine (a) the period and (b) the frequency of the oscillation.
2. A 4 kg mass is attached to a spring and lowered to equilibrium. During this process, the spring stretches 2 meters. Determine the spring constant and the period of a potential oscillation.