Solution to IPhO 1967, Problem 1

This problem may be pretty straightforward, but usually IPhO problems don't just give up so easily. So in general, for a problem that might be hard, you try to figure out what's going on first; from there, you list the given parameters and the equations that you know, and see if you have enough information to solve the problem.

Our barebones intuitive understanding runs like this: before the collision, we know everything; after the collision, we have no clue what the velocities are, and have to infer them based on the information that we get about how far they fall and travel (e.g., s and h). Once we get the velocities, finding the kinetic energy lost to heat (read: the change in kinetic energy) and the distance traveled by the bullet after the collision (read: kinematics) should be no problem to find. Unfortunately, kinematics alone won't solve the problem, since we don't have enough information about the bullet's trajectory to infer its initial velocity. We need a conservation law. Mechanical energy is definitely not conserved– contact forces are almost never, even in Olympiad problems, conservative forces, and the problem even asks you for the amount of energy lost to heat. As between linear momentum and angular momentum, you can actually take your pick, but for ease of understanding we look at linear momentum. At the moment of collision, the ball remains stationary on the pole *despite* the fact that some momentum is being imparted to the ball. This is a very sticky concept, because it bases in the idea that the ball doesn't move off of the pole at the same instant that momentum is being imparted. Thus, the normal force from the pole balances the force of gravity at the instants before and after momentum exchange, and so there are no external forces for the system of the bullet and the ball– i.e., there is conservation of linear momentum.

Let's call the velocity of the ball after collision v_1 and the velocity of the bullet after collision v_2. Conservation of linear momentum implies:

$$mv_0 = Mv_1 + mv_2 \tag{1}$$

Straightforward kinematics says that

$$h = \frac{1}{2}gt^2 \tag{2}$$

Note that, since no velocity is imparted in the y-direction, the time it takes for the bullet and the ball to fall is the same.

$$s = v_1 t \tag{3}$$

We solve for v_1 using Eqns. 2 and 3 to get

$$v_1 = s\sqrt{\frac{g}{2h}} \tag{4}$$

We solve for v_2 using Eqns. 1 and 4 to get

$$v_2 = \frac{m}{M}(v_0 - s\sqrt{\frac{g}{2h}}) \tag{5}$$

Now that we have the two velocities after the collision, we can get almost anything else we want. This problem asks for the distance travelled by the bullet after the collision

$$S = v_2 t = \frac{m}{M}(v_0 - s\sqrt{\frac{g}{2h}})\sqrt{\frac{2h}{g}}$$

and the heat lost during the collision,

$$\Delta E = KE_i - KE_f = \frac{1}{2}mv_0^2 - (\frac{1}{2}mv_2^2 + \frac{1}{2}Mv_1^2)$$

Feel free to simplify that last one and actually plug in numbers to get the numerical answers.