Energy and Momentum Lecture (11/2013) Solutions

Shankar Balasubramian Ross Dempsey

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The first question will be omitted for obvious reasons.

1 Solutions

2. We know that momentum is conserved in the collision because there are no net external forces. We can use this to determine the velocity after collision:

$$\begin{aligned}(12\,\mathrm{kg})(4\,\mathrm{m\,s^{-1}}) + (6\,\mathrm{kg})(-2\,\mathrm{m\,s^{-1}}) &= (18\,\mathrm{kg})v_f \\ v_f &= 2\,\mathrm{m\,s^{-1}}\end{aligned}$$

Now we have all the information we need to directly compute the loss of kinetic energy:

$$\begin{aligned}-\Delta E &= \frac{1}{2}\left(m_1 v_1^2 + m_2 v_2^2 - (m_1+m_2)v_f^2\right) \\ &= (12\,\mathrm{kg})(4\,\mathrm{m\,s^{-1}})^2 + (6\,\mathrm{kg})(-2\,\mathrm{m\,s^{-1}})^2 - (18\,\mathrm{kg})(2\,\mathrm{m\,s^{-1}})^2 \\ &= 192\,\mathrm{J} + 24\,\mathrm{J} - 72\,\mathrm{J} \\ &= 144\,\mathrm{J}\end{aligned}$$

3. The first part of the question is fairly straightforward. Momentum is conserved in the horizontal direction, so we know the following is true

$$0 = Nm_p u - mv_f$$

because the initial momentum is 0 (everything is at rest). Thus, we obtain

$$v_f = \frac{Nm_p u}{m}$$

The second part to the question requires a bit more thinking. We need to develop a recurrence relationship for the velocity of the railcar after the nth person jumps off. Since each person jumps at a velocity u relative to the railcar, their velocity relative to a stationary observer is $\bar{u} = u - v_n$, where v_n is the instantaneous velocity of the railcar when the particular person jumps. We will consider conservation of horizontal momentum before and after the nth man jumps off the railcar. We get the following

$$((N-n)m_p + m)v_n = ((N-n-1)m_p + m)v_{n+1} - m_p(u - v_n)$$

Simplifying this expression gives us

$$\begin{aligned}((N-n-1)m_p + m)v_n + m_p u &= ((N-n-1)m_p + m)v_{n+1} \\ v_n + \frac{m_p u}{(N-n-1)m_p + m} &= v_{n+1}\end{aligned}$$

This recurrence relationship is fairly simple to solve, since it is just a function of n being added each time. Our general velocity solution is

$$v_n = \sum_{i=N-n}^{N} \frac{m_p u}{i m_p + m}$$
$$v_N = \sum_{i=0}^{N} \frac{m_p u}{i m_p + m}$$

We can bound the last sum as follows

$$v_N = \sum_{i=0}^{N} \frac{m_p u}{i m_p + m} \leq \sum_{i=0}^{N} \frac{m_p u}{m} = \frac{N m_p u}{m}$$

which was our answer to part a. Thus, we see that the final velocity for part b will always be less than that of part a.

4. At the beginning of the process, all the potential energy in the spring $\frac{1}{2}kx_0^2$ becomes the initial kinetic energy of the block. It then experiences friction in the opposite direction of its motion, so that the differential $\vec{F} \cdot \vec{d\ell} = -F\,dx$. We can express the force in terms of μ and integrate:

$$\begin{aligned}\Delta E &= \int_0^d -\left(\mu_0 + \mu_1 \frac{x}{d}\right) mg\,dx \\ &= -mg\left(\mu_0 x + \mu_1 \frac{x^2}{2d}\right)_0^d \\ &= -mg\left(\mu_0 d + \mu_1 \frac{d}{2}\right)\end{aligned}$$

Thus, after exiting the horizontal track, the total energy of the object is $\frac{1}{2}kx_0^2 - mg\left(\mu_0 d + \mu_1 \frac{d}{2}\right)$. At this point, the object enters the quarter turn and then rises to a height h, before falling down. At the peak of its trajectory, all its kinetic energy has been converted to gravitational potential energy. Thus, we can write our final equation and solve:

$$\begin{aligned}mgh &= \frac{1}{2}kx_0^2 - mg\left(\mu_0 d + \mu_1 \frac{d}{2}\right) \\ h &= \frac{1}{2}\frac{k}{mg}x_0^2 - \left(\mu_0 d + \mu_1 \frac{d}{2}\right)\end{aligned}$$

5. Because the spring has some potential energy stored in it, we must first determine the velocity of the ball after it is launched from the spring. Due to the conservation of energy

$$\begin{aligned}\frac{1}{2}kx^2 &= \frac{1}{2}mv^2 \\ v &= x\sqrt{\frac{k}{m}}\end{aligned}$$

The ball will now fly in a kinematic trajectory. According to basic kinematic principles

$$\begin{aligned}y(t) &= h - \frac{1}{2}gt^2 \\ x(t) &= vt\end{aligned}$$

Thus, when the ball hits the ground, it has a range of $v\sqrt{\frac{2h}{g}}$, or (in this case) $x\sqrt{\frac{2kh}{mg}}$. This falls a horizontal distance y short of the landing point. Thus, the second child must compress the spring such that $x'\sqrt{\frac{2kh}{mg}} = x\sqrt{\frac{2kh}{mg}} + y$, or $x' = x + y\sqrt{\frac{mg}{2kh}}$.