Answers to 1^{st} part of Newton's Laws 1 Problem Set

You should always try to intuit the solution before you start plugging and chugging. That way, you can check if your answer is wildly off balance when you've gone through too many technical details. So our intuition says the following: each rope acts like it's pulling on everything behind it, independent of what's in front of it. So the more mass that remains behind a rope, the less its tension will be, in proportion to the total mass of the blocks behind it. But to justify this...

First we label the tension of the rope behind the first block as T_{2-1}, behind the second block as T_{3-2}, behind the i^{th} block as $T_{(i+1)-i}$, and so on down to $T_{(N)-(N-1)}$. Solving for a general $T_{(k+1)-k}$ solves the problem.

Now we apply what we know (Newton's Second Law) to every single block.

$$F_x = ma_x \tag{1}$$

for every single block gives the following:

$$F - T_{2-1} = m_1 a_1$$

$$T_{2-1} - T_{3-2} = m_2 a_2$$

...

$$T_{(i-1)-i} - T_{(i+1)-i} = m_i a_i$$

...

$$T_{(N)-(N-1)} = m_N a_N$$

So now we have $2N-1$ variables (all of the tensions and the accelerations) and only N equations. But we have a very powerful statement that we can make, based on the fact that the ropes are unstretchable. That means that the blocks always remain the same distance apart from one another; thus, their accelerations are the same.

$$a_1 = a_2 = ... = a_N \tag{2}$$

Now we have reduced the number of variables to N, and so we know that we can solve the system just with these equations. A clever way to do so is to notice that adding successive equations for each block results in the tension of the string in between the blocks canceling. (This is a roundabout way of defining a system which I assume Dr. Dell will tell you all about; really it's a way of looking at forces as either external or internal, and in this case, you're defining the system of two blocks, so that the internal force of the tension from the middle string cancels out.) So we first add all of the force law equations to get

$$F = \sum_{j=1}^{N} m_j a_j = a \sum_{j=1}^{N} m_j = Ma \tag{3}$$

where M is the total mass of all the blocks. Then, to get $T_{(i+1)-i}$ we add the force law equations from block $i+1$ to block N:

$$T_{(i+1)-i} = a \sum_{j=i+1}^{N} m_j = a(M - \sum_{j=1}^{i} m_j) \tag{4}$$

In solving for a and combining Eqns. 4 and 5, we get

$$T_{(i+1)-i} = F(1 - \frac{\sum_{j=1}^{i} m_j}{M}) \tag{5}$$

This answer is good because it satisfies our intuition, so we move right on.

To get an answer, you have to sum the masses. The problem stipulates what the mass of the j^{th} block is. In part a, the mass is constant; in part b, the mass is increased by δ for each successive block. So in general

$$m_j = m + (j-1)\delta$$

where δ is 0 for part a. We plug this into Eqn. 5 to get

$$T_{(i+1)-i} = F(1 - \frac{im + \delta\frac{i(i-1)}{2}}{Nm + \delta\frac{N(N-1)}{2}})$$

If you didn't get this answer, don't panic, because your answer may be the same. Try rearranging terms. If you still don't get this answer, talk to me, Pwang, or Freddy.