Gravitation

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1 Introduction

The question of what gravity actually is has stymied physicists for centuries for the simple reason that the Gravitational Constant (G) has never been derived. Instead of thinking about the philosophical implications of the bending of space-time, we will just treat gravity as a force "pulling" together different objects, and use a few different laws to help us solve problems related to gravitation. Gravity is one of the four fundamental forces along with the strong and weak nuclear forces and electromagnetism. Gravity was initially described in the universe by Newton though he was helped by his predecessors. Brahe recorded the data while Kepler modelled the data. Finally, Newton attempted to describe why Kepler's laws worked.

2 Kepler's 3 Laws

2.1 Kepler's First Law

Kepler's first law deals with the paths that planets, comets, and other astronomical bodies take around the sun. Specifically, all of the planets take elliptical paths around the sun, with the sun situated at one of the two foci. For most of the planets, the eccentricity of the path is so small that the path appears to be a circle. All orbital paths are conic sections.

2.2 Kepler's Second Law

Kepler's second law says that the area that an object in orbit sweeps out is equal for any equal interval of time. This implies that the object travels faster when it is nearer to the central mass. This can be further proved by conservation of angular momentum. Angular momentum is conserved because there is no external torque in the system.

2.3 Kepler's Third Law

Kepler's third law states that the square of the period of a planet's revolution is proportional to the cube of the semimajor axis of the orbit. Taking into account the constant of proportionality, it can be stated as:

$$T^2 = \frac{4\pi^2 a^3}{GM}$$

Where T is the period of a planet's revolution about the sun, a is the semimajor axis of the planet's orbit, M is the mass of the planet in question, and G is the empirically determined gravitational constant ($G = 6.67 \times 10^{-11} N \cdot (m/kg)^2$)

3 Newton's Law of Universal Gravitation

More general than Kepler's Laws, which solely hold for objects in orbit around others, Newton's Law of Universal Gravitation relates the masses of two objects and their distances to the mutual gravitational attraction shared between them. Specifically, it states that

$$F(r) = -\frac{GM_1M_2}{r^2}\hat{r}$$

4 Energy

While kinetic energy is esentially the same, potential energy changes quite a bit in the universal scale. When $E_{total} < 0$, the orbit is bound. When $E_{total} > 0$, the orbit is unbound.

4.1 The Gravitational Potential

Gravitational potential energy is 0 when the distance between the objects approaches infinity and decreases as the two objects approach each other, becoming more negative. In other words, as the objects are "pulled" apart, the energy stored increases, similar to a spring.

$$U(r) = -\frac{GM_1M_2}{r}$$

5 Gravitational Field

The gravitational field describes the strength and direction of gravitational attraction due to a specific mass at any point in space. As such, it is an example of a "vector field", a construct which assigns a vector to every point in space.

$$\vec{g} = \frac{GM}{r^2}\hat{r}$$

6 Shell Theorem

1. Inside a spherical shell, $|\vec{g}| = 0$
2. Outside a shell, the gravitational field is equivalent to that obtained by treating the shell as a point mass at its center
3. A spherically uniform ball, being a series of infinitesimally thick spherical shells, can also be effectively treated as a point mass at its center

7 Problems

1. Two particles of masses m_1 and m_2 are released from rest at a large separation distance. Find their speeds v_1 and v_2 when their separation distance is r. (Tipler)
2. Two widely separated uniform solid spheres, S_1 and S_2, have equal masses, but different radii, R_1 and R_2. If the gravitational field strength on the surface of S_1 is g_1 what is the gravitational field strength on the surface of S_2? (Tipler)
3. In a double star system, two stars of mass M each rotate about the system's center of mass at a radius of R. What is their common angular speed? If a meteoroid passes through the system's center of mass perpendicular to their orbital plane, what minimum speed must it have at the center of mass if it is to escape to "infinity" from the two-star system? (Tipler)