Introduction to Special Relativity

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1 Velocity Addition Rules

1. **The principle of relativity.** The laws of physics apply in all inertial reference systems.
2. **The universal speed of light.** The speed of light in vacuum is the same for all inertial observers, regardless of the motion of the source.
3. **Galileo's velocity addition rule.**

$$v_{AC} = v_{AB} + v_{BC}$$

4. **Einstein's velocity addition rule.**

$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + (v_{AB}v_{BC}/c^2)}$$

5. **Problem.** As the outlaws escape in their getaway car, which goes $\frac{3}{4}c$, the police officer fires a bullet from the pursuit car, which only goes $\frac{1}{2}c$. The muzzle velocity of the bullet (relative to the gun) is $\frac{1}{3}c$. Does the bullet reach its target (a) according to Galileo, (b) according to Einstein?

2 Time Dilation

1. Moving clocks run slow.

$$\Delta\bar{t} = \sqrt{1 - v^2/c^2}\Delta t$$

2. **Problem.** A rocket ship leaves earth at a speed of $\frac{3}{5}c$. When a clock on the rocket says 1 hour has elapsed, the rocket skip sens a light signal back to earth. (a) According to earth clocks, when was the signal sent? (b) According to earth clocks, how long after the rocket left did the signal arrive back on earth?

3 Lorentz Contraction

1. Moving objects are shortened.

$$\Delta\bar{x} = \frac{1}{\sqrt{1 - v^2/c^2}}\Delta x$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

2. **Problem.** A Lincoln Continental is twice as long as a VW Beetle, when they are at rest. As the Continental overtakes the VW, going through a speed trap, a (stationary) policeman observes that they both have the same length. The VW is going at half the speed of light. How fast is the Lincoln going? (Leave your answer as a multiple of c.)

3. **Problem.** A sailboat is manufactured so that the mast leans at an angle θ with respect to the deck. An observer standing on the dock sees the boat go by at speed v. What angle does this observer say the mast makes?