Arun Kannan
1/6/15

Rotation Lecture Problem Solutions

5.1.1. Yes, there can be a torque (unless no force is acting), but the net torque must be 0. Same reasoning other way around.

5.1.2. $\vec{F} = \langle 7N, 3N, -4N \rangle$

$\vec{\tau} = \vec{r} \times \vec{F}$

a). $\vec{r} = \langle 2,5,6 \rangle_m - \langle 0,0,0 \rangle_m = \langle 2,5,6 \rangle m$

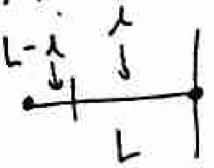
$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} 2 & 5 & 6 \\ 7 & 3 & -4 \\ i & j & k \end{vmatrix} Nm = \boxed{\langle -38, 50, -29 \rangle N \cdot m}$$

b) $\vec{r} = \langle 2,5,6 \rangle m - \langle 1,2,3 \rangle m = \langle 1,3,3 \rangle m$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} 1 & 3 & 3 \\ 7 & 3 & -4 \\ i & j & k \end{vmatrix} N \cdot m = \boxed{\langle -21, 25, -18 \rangle N \cdot m}$$

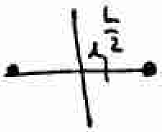
5.1.3. 0. Notice how it says "without slipping." What friction does is establishes $R\omega = v$. If v is too big then it acts against it. If v is too small ($R\omega > v$), then friction increases velocity and decreases angular velocity. Note — this only true when there are no other accelerations. If the sphere was put on an incline, friction would act, but it wouldn't necessarily be μF_N, but rather the ~~maximum~~ value ~~it could~~ to force $v = R\omega$.

5.2.1

a) 

$$m = \lambda \ell$$
$$dm = \lambda d\ell.$$

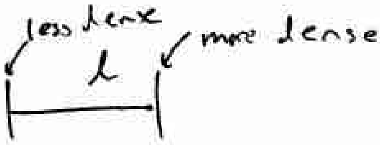
$$I = \int r^2 dm$$
$$= \int_0^L \ell^2 \lambda \, d\ell = \frac{L^3}{3}\lambda = \boxed{\frac{mL^3}{3}}$$

b) 

$$I = \int r^2 dm = 2\int_0^{\frac{L}{2}} \ell^2 \lambda \, d\ell = \frac{2\left(\frac{L}{2}\right)^3 \lambda}{3} = \boxed{\frac{ML^3}{12}}$$

5.2.2

Same thing except $\lambda(\ell) = k\ell^2$.



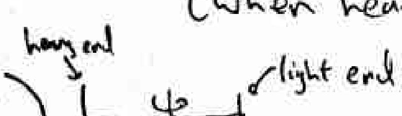
$$M = \int_0^L k\ell^2 d\ell = \frac{kL^3}{3}.$$

a) $$I = \int r^2 dm = \int_0^L \ell^2 k \ell^2 d\ell = \frac{kL^5}{5} = \frac{3L^2M}{5} \Rightarrow \boxed{\frac{3ML^2}{5}}$$

(when lighter end is closer)

b) $$I = \int r^2 dm = \int_0^L \ell^2 k (L-\ell)^2 d\ell = k\int_0^L L^2\ell^2 - 2\ell^3 L + \ell^4 d\ell$$
$$= k\left(\frac{L^5}{3} - \frac{L^5}{2} + \frac{L^5}{5}\right) = \boxed{\frac{3ML^2}{30}} = \boxed{\frac{ML^2}{10}}$$

(when heavier end is closer

c) 

heavy side

light side

$$I = \int r^2 dm = \int_0^{\frac{L}{2}} [\ \ + \ell^2 k\left(\tfrac{L}{2} - \ell\right)^2] \, d\ell.$$

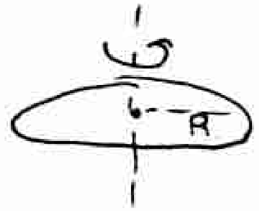
$$[\ell^2 k\left(\ell + \tfrac{L}{2}\right)^2$$

$$= k\int_0^{\frac{L}{2}} l^4 + \cancel{\frac{Ll^3}{2}} + \frac{L^2l^2}{4} + \frac{L^2l^2}{4} - \cancel{\frac{Ll^3}{2}} + l^4 \, dl$$

$$= k\int_0^{\frac{L}{2}} 2l^4 + \frac{L^2l^2}{2}\, dl = 2k\frac{L^5}{32\cdot 5} + \frac{k}{2}\cdot\frac{L^5}{8}\cdot\frac{1}{3}$$

$$= \boxed{\frac{ML^2}{30}}$$

5.2.3

a) 

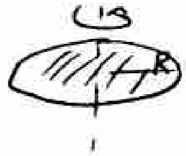
Let λ be linear density.

$M = 2\pi R\lambda$

$dm = 2\pi\lambda dr$

$$I = \int r^2 dm$$

$$= R^2\int dm = \boxed{MR^2}$$

b) 

Let σ be area density
(or λ = linear density of the rings)

It's like above, except several concentric rings

$$I = \int r^2 dm$$

r goes from 0 to R.

$m = \pi r^2\sigma$

$dm = 2\pi r\sigma dr$

length of rings.

$$= \int_0^R r^2\cdot 2\pi r\sigma\, dr$$

$$= \frac{2\pi R^4}{4} = \boxed{\frac{MR^2}{2}}$$