Hard Problem Solution

Let's use the work-energy theorem between the starting position and the top of the curve:

$$W = W_{spring} + W_{gravity} = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \tag{1}$$

First let's find what v_f is, because that's not as hard finding the work. If the particle were to at some point barely hold contact with the track, it would be at the top of the silo. (At the top of the silo, both the spring force and gravity are contributing fully to centripetal acceleration, so the additional contribution of the normal force is not needed to keep the thing in circular motion.) Then

$$k(X + R - L) + mg + N = m\frac{v_f^2}{R} \tag{2}$$

where $N \geq 0$. This becomes

$$m\frac{v_f^2}{R} - mg - k(X + R - L) \geq 0 \tag{3}$$

Plugging (1) into (3) gives

$$\frac{2W + mv_0^2}{R} - mg - k(X + R - L) \geq 0 \tag{4}$$

Now to find the work. The work of gravity is pretty easy, since we're just looking at vertical displacement:

$$W_{gravity} = -mg(X + R) \tag{5}$$

The work of the spring is not as clear cut because the direction of the spring force constantly changes. Note that because $\sqrt{X^2 + R^2} < L < X + R$ the spring force starts pulling the particle down rather than pushing it up at the juncture of the line and the semicircle. So we'll divide the work of the spring into two different calculations, first along the line and then along the semicircle.

Along the line (ref. the drawing):

$$W_{spring}^{line} = \int_{s=0}^{s=X} -k(\sqrt{s^2 + R^2} - L)\frac{s}{\sqrt{s^2 + R^2}}ds \tag{6}$$

Have fun calculating that integral.

Along the semicircle (ref. the drawing):

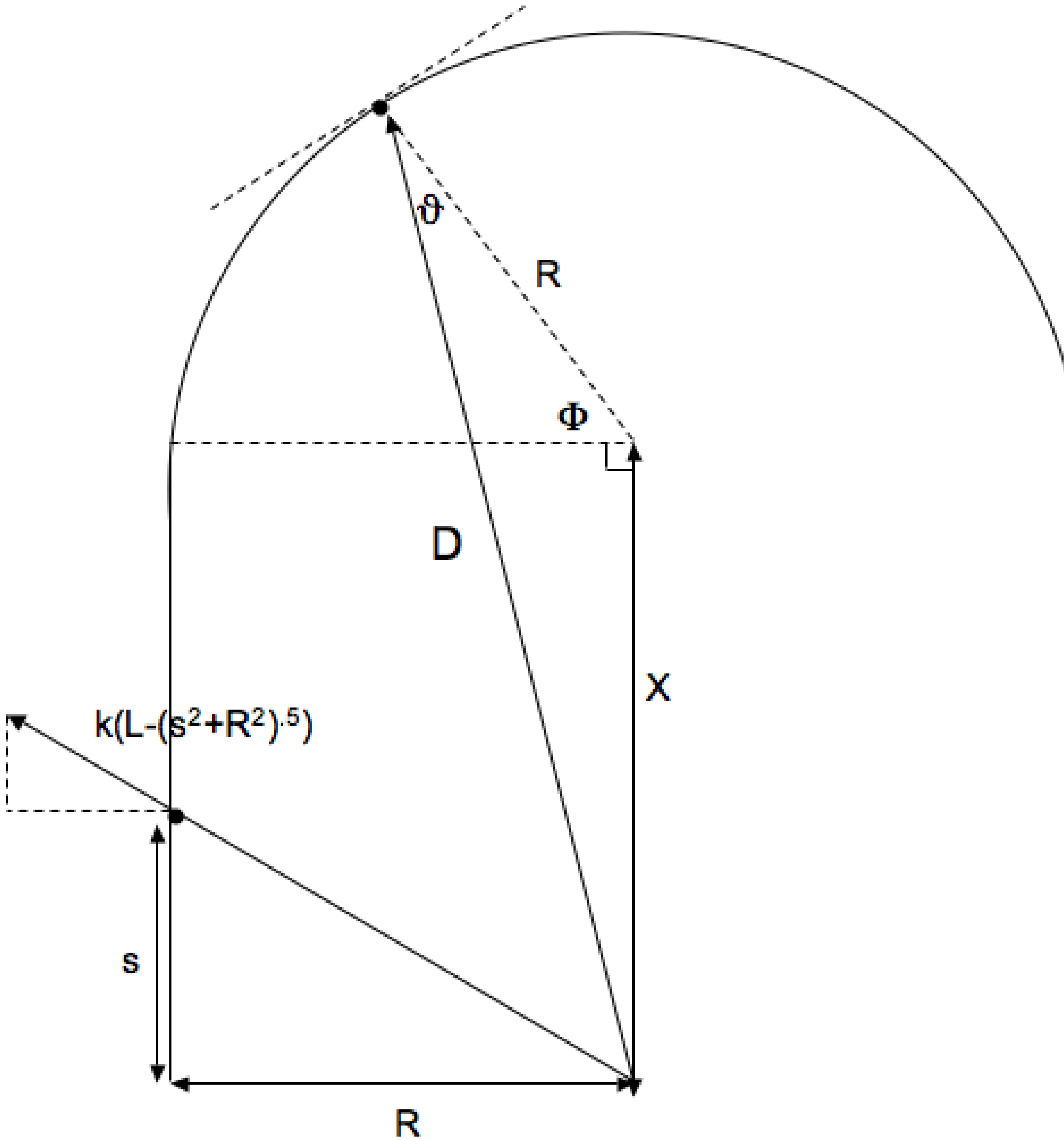
$$W_{spring}^{semi} = \int_{\phi=0}^{\phi=\frac{\pi}{2}} -k(D - L)\sin(\theta)(Rd\phi) \tag{7}$$

Now we use the Law of Cosines twice to get D and θ in terms of ϕ:

$$D^2 = X^2 + R^2 - 2XR\cos(\phi + \frac{\pi}{2}) \tag{8}$$

$$X^2 = R^2 + D^2 - 2RD\cos(\theta) \tag{9}$$

Plug (9) and (8) into (7), and plug that, (6), and (5) into (4) to get your answer.


ϑ
R
Φ
D
X
k(L-(s2+R2).5)
s
R

Some of you might be asking, "What the heck? I thought physics was about intuition." It is. You need to choose the correct points to which to apply the work-energy theorem– make sure you figure out where the particle is most likely to lose contact. But unfortunately or fortunately, depending on what you like and what you're good at, a lot of physics is doing the grunt work after making the brilliant insight. Usually, Olympiad problems are designed to minimize the grunt work, but of course, that makes them much less realistic than this problem in terms of actually doing physics.