$$
\begin{cases}
\dfrac{dS}{dt} = \mu(E+I+R) - \beta\dfrac{IS}{N} \\[2mm]
\dfrac{dE}{dt} = \beta\dfrac{IS}{N} - (\mu+\alpha)E \\[2mm]
\dfrac{dI}{dt} = \alpha E - (\gamma+\mu)I \\[2mm]
\dfrac{dR}{dt} = \gamma I - \mu R
\end{cases}
$$

$$f_1 = \mu(E+I+R) - \beta\frac{IS}{N}$$

$$\frac{\partial f_1}{\partial S} = -\beta\frac{IN - IS}{N^2}\Big|_{t_0} = -\beta\frac{I_0(N_0 - S_0)}{N_0^2}$$

$$\frac{\partial f_1}{\partial E_1} = \mu + \beta\frac{IS}{N^2}\Big|_{t_0} = \mu + \beta\frac{I_0 S_0}{N_0^2}$$

$$\frac{\partial f_1}{\partial I_2} = \mu - \beta\frac{SN - IS}{N^2}\Big|_{t_0} = \mu - \beta\frac{S_0(N_0 - I_0)}{N_0^2}$$

$$\frac{\partial f_1}{\partial R} = \mu + \beta\frac{IS}{N^2}\Big|_{t_0} = \mu + \beta\frac{I_0 S_0}{N_0^2}$$

$$f_1 \approx \left(\mu(E_0 + I_0 + R_0) - \beta\frac{I_0 S_0}{N_0}\right) - \beta\frac{I_0(N_0 - S_0)}{N_0^2}(S - S_0) +$$
$$+ \left(\mu + \beta\frac{I_0 S_0}{N_0^2}\right)(E - E_0) + \left(\mu - \beta\frac{S_0(N_0 - I_0)}{N_0^2}\right)(I - I_0) +$$
$$+ \left(\mu + \beta\frac{I_0 S_0}{N_0^2}\right)(R - R_0)$$

$$f_2 = \beta\frac{IS}{N} - (\mu+\alpha)E$$

$$\frac{\partial f_2}{\partial S} = \beta\frac{IN - IS}{N^2}\Big|_{t_0} = \beta\frac{I_0(N_0 - S_0)}{N_0^2}$$

$$\frac{\partial f_2}{\partial E} = -\beta\frac{IS}{N^2} - (\mu+\alpha)\Big|_{t_0} = -\beta\frac{I_0 S_0}{N_0^2} - (\mu+\alpha)$$

$$\frac{\partial f_2}{\partial I} = \beta \frac{SN - IS}{N^2}\Big|_{t_0} = \beta \frac{S_0(N_0 - I_0)}{N_0^2}$$

$$\frac{\partial f_2}{\partial R} = -\beta \frac{IS}{N^2}\Big|_{t_0} = -\beta \frac{I_0 S_0}{N_0^2}$$

$$f_2 \approx \left(\beta \frac{I_0 S_0}{N_0} - (\mu+\alpha)E_0\right) + \beta \frac{I_0(N_0 - S_0)}{N_0^2}(S - S_0) -$$
$$-\left(\beta \frac{I_0 S_0}{N_0^2} + (\mu+\alpha)\right)(E - E_0) + \beta \frac{S_0(N_0 - I_0)}{N_0^2}(I - I_0) -$$
$$-\beta \frac{I_0 S_0}{N_0^2}(R - R_0)$$

$$f_3 = \alpha E - (\gamma+\mu) I$$

$$\frac{\partial f_3}{\partial S} = 0$$

$$\frac{\partial f_3}{\partial I} = -\gamma - \mu$$

$$\frac{\partial f_3}{\partial E} = \alpha$$

$$\frac{\partial f_3}{\partial R} = 0$$

$$f_3 = -(\gamma+\mu)(I - I_0) + \alpha(E - E_0) + \alpha E_0 - (\gamma+\mu) I_0$$

$$f_4 = \gamma I - \mu R$$

$$\frac{\partial f_4}{\partial S} = 0$$

$$\frac{\partial f_4}{\partial E} = 0$$

$$\frac{\partial f_4}{\partial I} = \gamma$$

$$\frac{\partial f_4}{\partial R} = -\mu$$

$$f_4 \approx \gamma (I - I_0) - \mu (R - R_0) + \gamma I_0 - \mu R_0$$

$$A_1 = \frac{I_0 N_0}{N_0^2}, \quad A_2 = \frac{I_0 S_0}{N_0^2}, \quad A_3 = \frac{S_0 N_0}{N_0^2}$$

$$\frac{dS}{dt} = -\beta(A_1-A_2)(S-S_0)+(\mu+\beta A_2)(E-E_0)+f_{11}(t)$$

$$A_1 = \quad -\beta(A_1-A_2) \quad \mu+\beta A_2$$

$$\begin{cases} \dfrac{dS}{dt} = -\beta(A_1-A_2)(S-S_0)+(\mu+\beta A_2)(E-E_0)+f_{12}(t) \\ \dfrac{dE}{dt} = \beta(A_1-A_2)(S-S_0)+(-\beta A_2-\mu-\alpha)(E-E_0)+f_{22}(t) \end{cases}$$

$$A_1 = \begin{bmatrix} -\beta(A_1-A_2) & (\mu+\beta A_2) \\ \beta(A_1-A_2) & -\beta A_2-\mu-\alpha \end{bmatrix}$$

$$|A_1-\lambda E| = \begin{vmatrix} -\beta(A_1-A_2)-\lambda & \mu+\beta A_2 \\ \beta(A_1-A_2) & -\beta A_2-\mu-\alpha-\lambda \end{vmatrix} =$$

$$= \begin{vmatrix} -\beta(A_1-A_2)-\lambda & \mu+\beta A_2 \\ -\lambda & -\alpha-\lambda \end{vmatrix} =$$

$$= \alpha\beta(A_1-A_2)+\lambda(\alpha+\beta(A_1-A_2))+\lambda^2+\mu\lambda+\lambda\beta A_2 =$$

$$= \lambda^2+\lambda(\alpha+\beta A_1-\cancel{\beta A_2}+\mu+\cancel{\beta A_2})+\alpha\beta(A_1-A_2) =$$

$$= \lambda^2+\lambda(\alpha+\beta(A_1-\cancel{2A_2})+\mu)+\alpha\beta(A_1-A_2)$$

$$D = \alpha^2+\beta^2(A_1-2A_2)^2+\mu^2+2\alpha\beta(A_1-2A_2)-$$
$$-2\alpha\mu-2\beta\mu(A_1-2A_2)-4\alpha\beta(A_1-A_2)-$$
$$-4\alpha\beta A_2+4\alpha\beta A_2-4\alpha\beta(A_1-2A_2)+4\alpha\beta(A_1-2A_2)=$$
$$= \alpha^2+\beta^2(A_1-2A_2)^2+\mu^2-2\alpha\beta(A_1-2A_2)-$$
$$-2\alpha\mu-2\beta\mu(A_1-2A_2)+4\alpha\beta A_2 =$$
$$= (\alpha-\beta(A_1-2A_2)-\mu)^2-4\alpha\beta A_2$$

$$\mathcal{D} = \alpha^2 + \beta^2 A_1^2 + \mu^2 + \overset{-2\alpha\beta A_1}{2\alpha\beta A_1} + \overset{-2\alpha\mu}{2\alpha\mu} + 2\beta\mu A_1 -$$

$$- 4\alpha\beta(A_1 - A_2) - 4\alpha\mu + 4\alpha\mu =$$

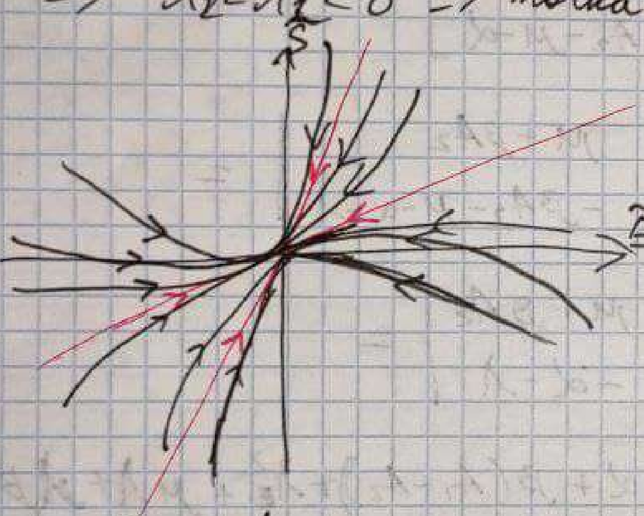
$$= ((\beta A_1 - \alpha + \mu)^2 + 4\alpha\beta A_2 + 4\alpha\mu)$$

$$\lambda_{1,2} = \frac{-(\alpha + \beta A_1 + \mu) \pm \sqrt{(\beta A_1 - \alpha + \mu)^2 + 4\alpha\beta A_2 + 4\alpha\mu}}{2} \leq 0$$

$$\cancel{|A_1 - \lambda E| = \lambda^2 + \lambda(\alpha + \beta A_1 + \mu) + \alpha\beta(A_1 - A_2)}$$

Если $\det A_1 \neq 0$ $(\alpha \neq 0, \beta \neq 0, A_1 \neq A_2)$

$\Rightarrow \lambda_2 < \lambda_1 < 0 \Rightarrow$ точка покоя - устойчивый узел



$$\begin{cases} \tilde{S} = S - S_0 \\ \tilde{E} = E - E_0 \end{cases}$$

Если $\det A_1 = 0$

1) $\beta = 0$

$$A_1 = \begin{bmatrix} 0 & \mu \\ 0 & -\mu-\alpha \end{bmatrix}$$

$|A_1 - \lambda E| = \lambda^2 + (\mu + \alpha)\lambda = 0$ $\quad \lambda_1 = 0, \lambda_2 = -\mu - \alpha < 0$

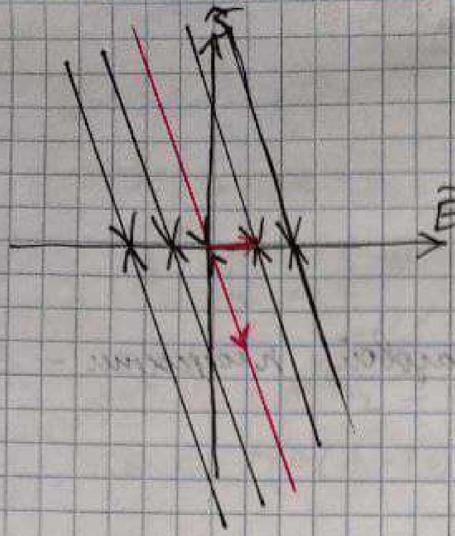
~~СВ~~:

$\cancel{0V_{11} + \mu V_{12} = 0}$

СВ

$$\begin{cases} 0V_{11} + \mu V_{12} = 0 \\ 0V_{11} - (\mu + \alpha)V_{12} = 0 \end{cases} \quad V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{cases} (\mu + \alpha) V_{21} + \mu V_{22} = 0 \\ 0V_{21} + 0V_{22} = 0 \end{cases} \quad V_2 = \begin{bmatrix} 1 \\ -(1 + \frac{\alpha}{\mu}) \end{bmatrix}$$



$$\begin{cases} \tilde{S} = S - S_0 \\ \tilde{E} = E - E_0 \end{cases}$$

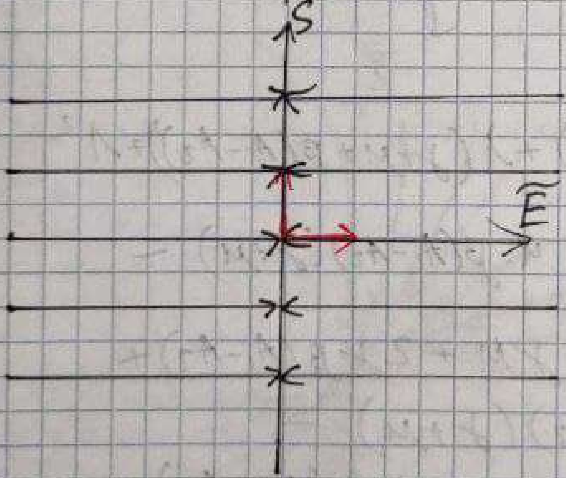
г) $\begin{cases} \beta = 0 \\ \mu = 0 \end{cases}$ или $\begin{cases} \beta = 0 \\ \alpha = 0 \end{cases}$

$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\alpha \end{bmatrix}$

$|A_1 - \lambda E| = \lambda^2 + \alpha\lambda = 0$

$\lambda_1 = 0, \quad \lambda_2 = -\alpha < 0$

$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



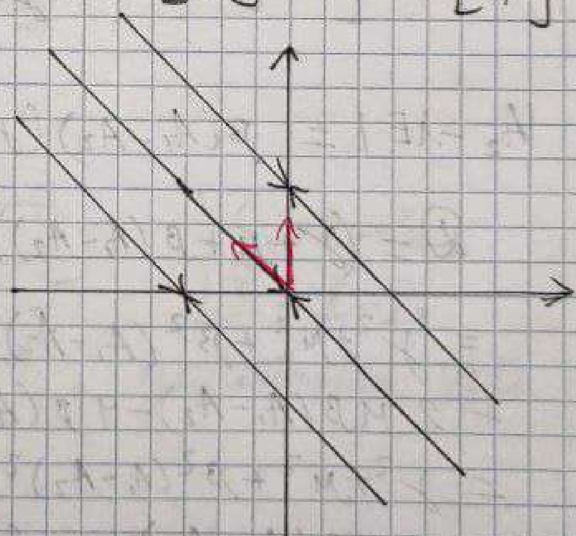
$$\begin{cases} \tilde{S} = S - S_0 \\ \tilde{E} = E - E_0 \end{cases}$$

$A_1 = \begin{bmatrix} 0 & \mu \\ 0 & -\mu \end{bmatrix}$

$|A_1 - \lambda E| = \lambda^2 + \mu\lambda = 0$

$\lambda_1 = 0, \quad \lambda_2 = -\mu < 0$

$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$



3) $\begin{cases} \beta \neq 0 \\ \alpha = 0 \\ \mu = 0 \end{cases}$ $\quad A_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\lambda_1 = \lambda_2 = 0 \Rightarrow$ Каждая $(\cdot)$ фазовой плоскости — является $(\cdot)$ покоя

4) ~~$\lambda_1 = \lambda_2 = 0$~~

$$\begin{cases} \dfrac{dS}{dt} = -\beta(A_1 - A_2)(S - S_0) + (\mu - \beta(A_3 - A_2))(I - I_0) + f_{23}(t) \\ \dfrac{dI}{dt} = -(\gamma + \mu)(I - I_0) + f_{31}(t) \end{cases}$$

$$A_2 = \begin{bmatrix} -\beta(A_1 - A_2) & \mu - \beta(A_3 - A_2) \\ 0 & -\gamma - \mu \end{bmatrix}$$

$$|A_2 - \lambda E| = \beta(A_1 - A_2)(\gamma + \mu) + \lambda(\gamma + \mu + \beta(A_1 - A_2)) + \lambda^2$$

$$\mathcal{D} = (\gamma + \mu + \beta(A_1 - A_2))^2 - 4\beta(A_1 - A_2)(\gamma + \mu) =$$

$$= \gamma^2 + \mu^2 + \beta^2(A_1 - A_2)^2 + 2\gamma\mu + 2\gamma\beta(A_1 - A_2) +$$
$$+ 2\mu\beta(A_1 - A_2) - 4\beta(A_1 - A_2)(\gamma + \mu) =$$
$$= \gamma^2 + \mu^2 + \beta^2(A_1 - A_2)^2 + 2\gamma\mu - 2\gamma\beta(A_1 - A_2) -$$
$$- 2\mu\beta(A_1 - A_2) = (\gamma + \mu - \beta(A_1 - A_2))^2$$

$$\lambda_{1,2} = \frac{-(\gamma + \mu + \beta(A_1 - A_2)) \pm \sqrt{(\gamma + \mu - \beta(A_1 - A_2))^2}}{2}$$

$$\lambda_1 = \frac{-\gamma - \mu - \beta(A_1 - A_2) + \gamma + \mu - \beta(A_1 - A_2)}{2} = -\beta(A_1 - A_2) \lessgtr 0$$

$$\lambda_2 = \frac{-\gamma - \mu - \beta(A_1 - A_2) - \gamma - \mu + \beta(A_1 - A_2)}{2} = -(\gamma + \mu) < 0$$

1) $\lambda_2 < \lambda_1 < 0$ (при $\beta \neq 0$, $A_1 \neq A_2$, $\gamma \neq \mu$)

$$\lambda_1 = -\beta(A_1 - A_2)$$

$$A_2 - \lambda_1 E = \begin{bmatrix} 0 & \mu - \beta(A_3 - A_2) \\ 0 & \beta(A_1 - A_2) - \gamma - \mu \end{bmatrix}$$

$$\begin{cases} 0\, V_{11} + \mu V_{12} = 0 \\ 0\, V_{11} + (\beta(A_1 - A_2) - \gamma - \mu) V_{12} = 0 \end{cases} \Rightarrow V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

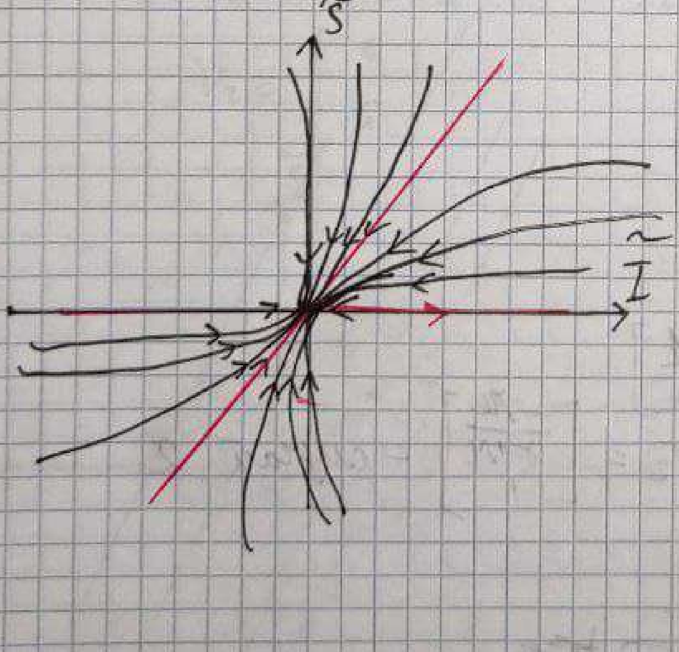
$$\lambda_2 = -(\gamma + \mu)$$

$$A_2 - \lambda_2 E = \begin{bmatrix} \gamma + \mu - \beta(A_1 - A_2) & \mu - \beta(A_3 - A_2) \\ 0 & 0 \end{bmatrix}$$

$$(\gamma + \mu - \beta(A_1 - A_2)) V_{21} + (\mu - \beta(A_3 - A_2)) V_{22} = 0$$

$$V_{22} = \frac{\gamma + \mu - \beta(A_1 - A_2)}{\beta(A_3 - A_2) - \mu} V_{21} \Rightarrow \cancel{V_2 = \begin{bmatrix} \\ 1 \end{bmatrix}}$$

$$V_2 = \begin{bmatrix} \dfrac{\beta(A_3 - A_2) - \mu}{\gamma + \mu - \beta(A_1 - A_2)} \\ 1 \end{bmatrix}$$



$$\begin{cases} \tilde{S} = S - S_0 \\ \tilde{I} = I - I_0 \end{cases}$$

устойчивый узел

2) $\beta = 0 \Rightarrow \lambda_1 = 0,\ \lambda_2 = -(\gamma+\mu)$

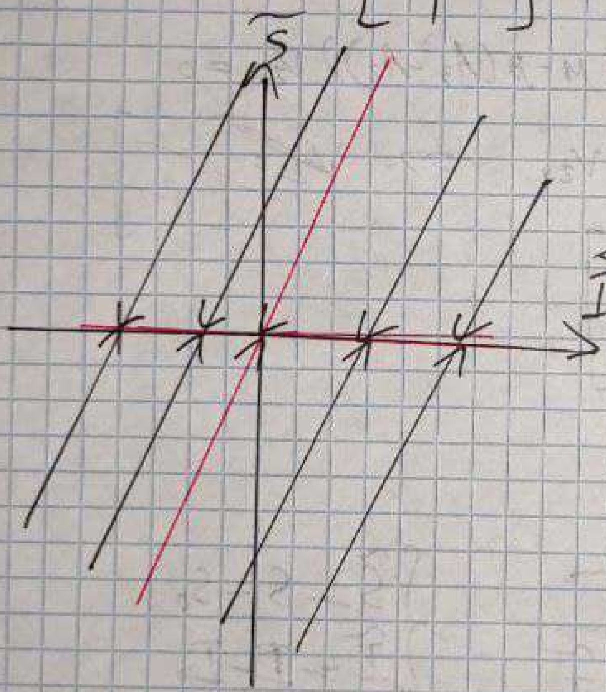
$\lambda_1 = 0$

$$A_2 - \lambda_1 E = \begin{bmatrix} 0 & \mu \\ 0 & -\gamma-\mu \end{bmatrix}$$

$$\begin{cases} 0V_{11} + \mu V_{12} = 0 \\ 0V_{11} + (-\gamma-\mu) V_{12} = 0 \end{cases} \Rightarrow V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\lambda_2 = -(\gamma+\mu) < 0$

$$V_2 = \begin{bmatrix} \frac{-\mu}{\gamma+\mu} \\ 1 \end{bmatrix}$$



$$\begin{cases} \tilde{S} = S - S_0 \\ \tilde{I} = I - I_0 \end{cases}$$

3) $A_1 = A_2$

$\lambda_1 = 0 \Rightarrow V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\lambda_2 = -(\gamma+\mu) \Rightarrow V_2 = \begin{bmatrix} \frac{-\mu}{\gamma+\mu} \\ 1 \end{bmatrix}$ – случай 2

4) $\gamma = -\mu$

$\lambda_1 = -\beta(A_1 - A_2) \Rightarrow$

$$A_2 - \lambda_1 E = \begin{bmatrix} \overset{0}{\cancel{-\beta(A_1-A_2)}} & \mu - \beta(A_1 - A_2) \\ 0 & \beta(A_1 - A_2) \end{bmatrix} \Rightarrow V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\lambda_2 = 0$

$$A_2 - \lambda_2 E = \begin{bmatrix} -\beta(A_1 - A_2) & \mu - \beta(A_3 - A_2) \\ 0 & 0 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} \frac{\mu - \beta(A_3 - A_2)}{\beta(A_1 - A_2)} \\ 1 \end{bmatrix}$$ – случай 2

5) $\beta = 0$ и $\gamma = -\mu$

$$A_2 = \begin{bmatrix} 0 & \mu \\ 0 & 0 \end{bmatrix}$$

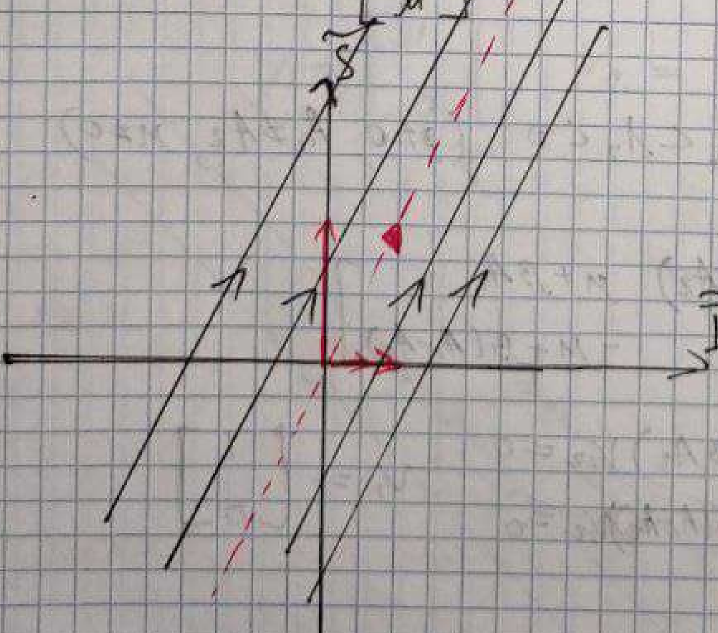
$\lambda_1 = \lambda_2 = 0$

$$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \mu \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$0 V_{21} + \mu V_{22} = 1 \Rightarrow V_{22} = \frac{1}{\mu}$

$$V_2 = \begin{bmatrix} 0 \\ \frac{1}{\mu} \end{bmatrix}$$



$$\begin{cases} \tilde{S} = S - S_0 \\ \tilde{I} = I - I_0 \end{cases}$$

6) $\beta = 0$, $\gamma = -\mu$, $\mu = 0$

$$A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\lambda_1 = \lambda_2 = 0 \Rightarrow \forall (\cdot)$ фаз. плоск-ти - $(\cdot)$ покоя

$$\begin{cases} \frac{dS}{dt} = -\beta(A_1 - A_2)(S - S_0) + (\mu + \beta A_2)(R - R_0) + f_{13}(t) \\ \frac{dR}{dt} = -\mu(R - R_0) + f_{41}(t) \end{cases}$$

$$A_3 = \begin{bmatrix} -\beta(A_1 - A_2) & \mu + \beta A_2 \\ 0 & -\mu \end{bmatrix}$$

$$|A_3 - \lambda E| = \lambda^2 + \lambda(\mu + \beta(A_1 - A_2)) + \mu\beta(A_1 - A_2)$$

$$\mathcal{D} = \mu^2 + \beta^2(A_1 - A_2)^2 + 2\mu\beta(A_1 - A_2) - 4\mu\beta(A_1 - A_2) =$$
$$= (\mu - \beta(A_1 - A_2))^2$$

$$\lambda_{1,2} = \frac{-(\mu + \beta(A_1 - A_2)) \pm (\mu - \beta(A_1 - A_2))}{2}$$

$$\lambda_1 = -\beta(A_1 - A_2)$$
$$\lambda_2 = -\mu$$

1) $\cancel{A_1 = A_2 \ldots}$ $\lambda_2 < \lambda_1 < 0$ $(\beta \neq 0, A_1 \neq A_2, \mu \neq 0)$

$\lambda_1 = -\beta(A_1 - A_2)$

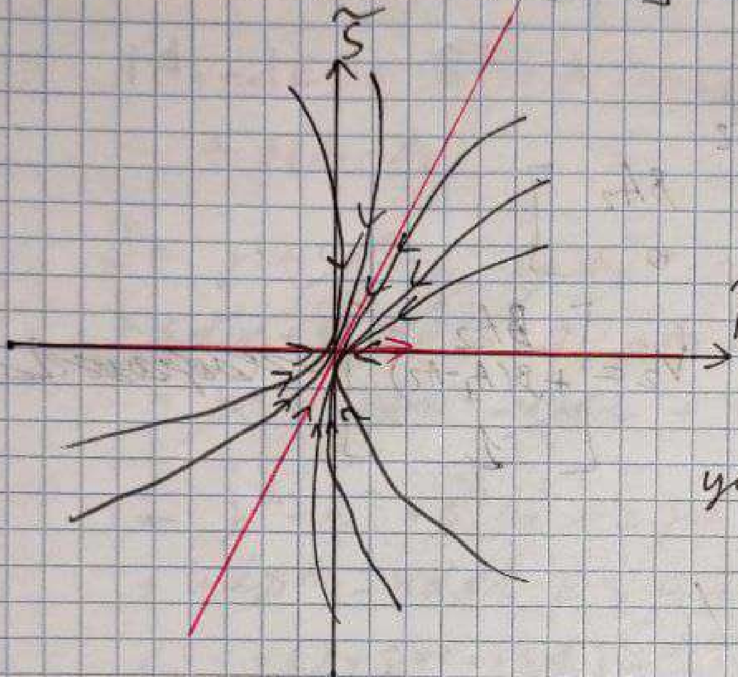
$$A_3 - \lambda E = \begin{bmatrix} \cancel{-2\beta(A_1 - A_2)}\ 0 & \mu + \beta A_2 \\ 0 & -\mu + \beta(A_1 - A_2) \end{bmatrix}$$

$$\begin{cases} \cancel{-2\beta(A_1 - A_2)V_{11} + (\mu + \beta A_2)V_{12} = 0} \\ \cancel{-\beta \quad (-\mu - \beta(A_1 - A_2))V_{12} = 0} \end{cases} \qquad V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\lambda_2 = -\mu$

$$A_3 - \lambda_2 E = \begin{bmatrix} -\beta(A_1 - A_2) + \mu & \mu + \beta A_2 \\ 0 & 0 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} \frac{\beta(A_1 - A_2) - \mu}{\mu + \beta A_2} \\ 1 \end{bmatrix}$$



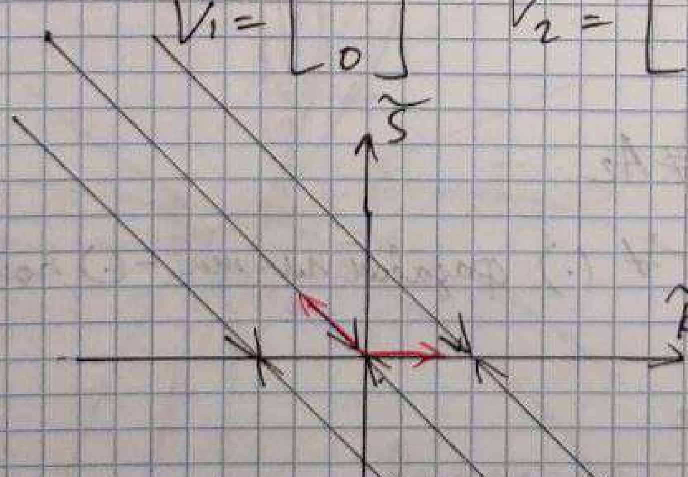
$$\begin{cases} \tilde{S} = S - S_0 \\ \tilde{R} = R - R_0 \end{cases}$$

устойчивый узел

2) $\beta = 0$ ~~или $A_1 = A_2$~~

$$A_3 = \begin{bmatrix} 0 & \mu \\ 0 & -\mu \end{bmatrix}$$

$$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$



$$\begin{cases} \tilde{S} = S - S_0 \\ \tilde{R} = R - R_0 \end{cases}$$

3) $A_1 = A_2$, $\beta \neq 0$, $\mu \neq 0$

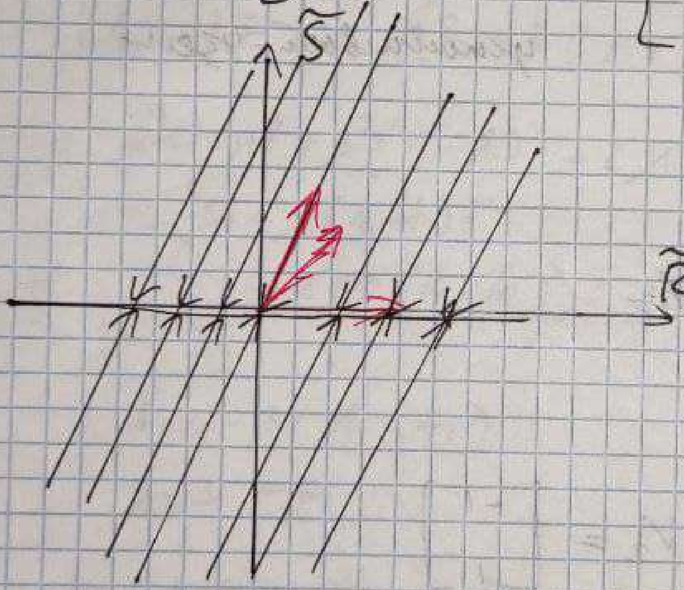
$$A_3 = \begin{bmatrix} 0 & \mu + \beta A_2 \\ 0 & -\mu \end{bmatrix}$$

$$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad V_2 = \begin{bmatrix} -\frac{\mu}{\mu + \beta A_2} \\ +1 \end{bmatrix}$$ – случай 2

4) $\mu = 0$, $\beta \neq 0$, $A_1 \neq A_2$

$$A_3 = \begin{bmatrix} -\beta(A_1 - A_2) & \beta A_2 \\ 0 & 0 \end{bmatrix}$$

$$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad V_2 = \begin{bmatrix} \frac{\beta A_2}{+\beta(A_1 - A_2)} \\ 1 \end{bmatrix}$$ ~~случай 2~~



$$\begin{cases} \tilde{S} = S - S_0 \\ \tilde{R} = R - R_0 \end{cases}$$

5) $\beta = 0$, $\mu = 0$, $A_1 \neq A_2$

$$A_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ – $\forall$ (·) фазовой пл-ти – (·) покоя

6) $\beta = 0$, $\mu = 0$, $A_1 = A_2$

$$A_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ – $\forall$(·) фаз. плоск-ти – (·) покоя

$$\begin{cases} \dfrac{dE}{dt} = -(\beta A_2+\mu+\alpha)(E-E_0) + \beta(A_3-A_2)(I-I_0) + f_{13}(t) \\ \dfrac{dI}{dt} = \alpha(E-E_0) - (\gamma+\mu)(I-I_0) + f_{12}(t) \end{cases}$$

$$A_4 = \begin{bmatrix} -(\beta A_2+\mu+\alpha) & \beta(A_3-A_2) \\ \alpha & -\gamma-\mu \end{bmatrix}$$

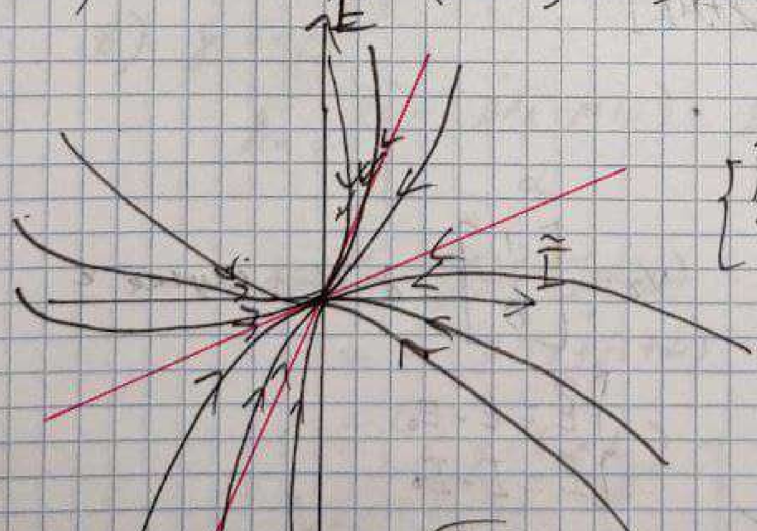
$$|A_4-\lambda E| = \lambda^2+\lambda(\gamma+2\mu+\alpha+\beta A_2) + (\gamma+\mu)(\beta A_2+\mu+\alpha) - \\ - \alpha\beta(A_3-A_2)$$

$$\mathcal{D} = \gamma^2+\cancel{4\mu^2}+\alpha^2+\beta^2A_2^2+\cancel{4\gamma\mu}+2\gamma\alpha+2\gamma\beta A_2 + \\ + 4\cancel{\alpha\mu}+\cancel{4\mu\beta A_2}+2\alpha\beta A_2-4\gamma\beta A_2-\cancel{4\gamma\mu}-4\gamma\alpha - \\ - \cancel{4\mu\beta A_2}-\cancel{4\mu^2}-\cancel{4\mu\alpha}+4\alpha\beta A_3-4\alpha\beta A_2 -$$

$$= \gamma^2+\alpha^2+\beta^2A_2^2-2\gamma\alpha \mp 2\gamma\beta A_2-2\alpha\beta A_2+4\alpha\beta A_3 + \\ + 4\alpha\beta A_2-4\alpha\beta A_2 = (\gamma-\alpha-\beta A_2)^2+4\alpha\beta(A_3-A_2)$$

$$\lambda_{1,2} = \frac{-(\gamma+2\mu+\alpha+\beta A_2) \pm \sqrt{(\gamma-\alpha-\beta A_2)^2+4\alpha\beta(A_3-A_2)}}{2}$$

1) $\lambda_2 < \lambda_1 < 0$ $(\alpha \neq 0,\ \beta \neq 0,\ \mu \neq 0,\ A_2 \neq A_3,\ A_2 \neq 0,\ \gamma \neq 0)$



$$\begin{cases} \hat{E} = E - E_0 \\ \hat{I} = I - I_0 \end{cases}$$

устойчивый узел

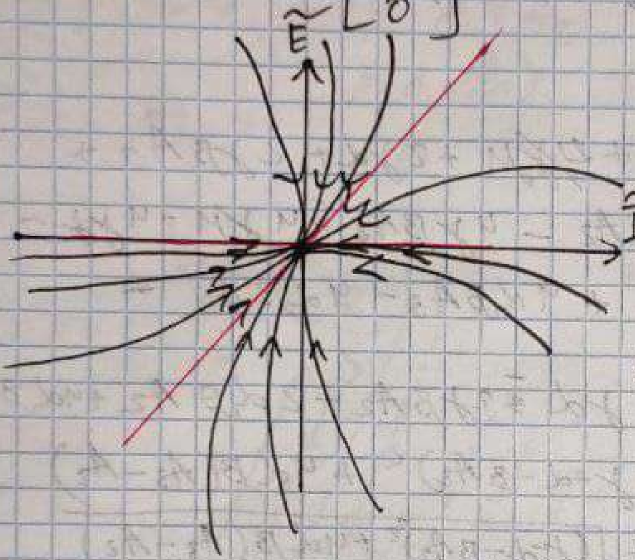
~~То же самое, если $\alpha=0$ $\beta=0$ $A_2=A_3$ $A_2=0$ и любые их комбинации или~~ комбинация их подмн-ва с $\gamma \neq 0$

2) $\alpha = 0$

$$A_4 = \begin{bmatrix} -(\beta A_2 + \mu + \alpha) & \beta(A_3 - A_2) \\ 0 & -\gamma - \mu \end{bmatrix}$$

$$\lambda_1 = -(\beta A_2 + \mu + \alpha) < 0$$
$$\lambda_2 = -\gamma - \mu < 0$$

$$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad V_2 = \begin{bmatrix} \dfrac{\beta(A_3 - A_2)}{\beta A_2 - \gamma + \alpha} \\ 1 \end{bmatrix}$$



$$\begin{cases} \tilde{E} = E - E_0 \\ \tilde{I} = I - I_0 \end{cases}$$

устойчивый узел

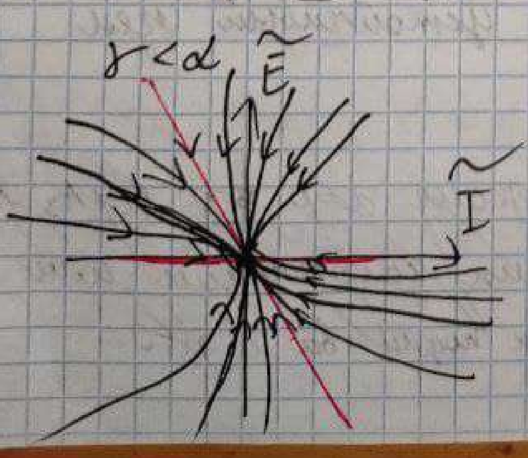
3) $\gamma = 0$

$$A_4 = \begin{bmatrix} -(\mu + \alpha) & 0 \\ \alpha & -(\gamma + \mu) \end{bmatrix}$$

$$\lambda_1 = -(\mu + \alpha)$$
$$\lambda_2 = -(\gamma + \mu)$$

$$V_1 = \begin{bmatrix} \dfrac{\gamma - \alpha}{\alpha} \\ 1 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$ $\gamma > \alpha$ - случай 2

$\gamma < \alpha$



$$\begin{cases} \tilde{E} = E - E_0 \\ \tilde{I} = I - I_0 \end{cases}$$

устойчивый узел

4) $\alpha = 0,\ \beta = 0$

$$A_4 = \begin{bmatrix} -\mu & 0 \\ 0 & -\gamma-\mu \end{bmatrix}$$

$\lambda_1 = -\mu$

$\lambda_2 = -\gamma-\mu$

$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ – случаи 2, 3

5) $A_2 = A_3$

$$A_4 \begin{bmatrix} -\beta A_2 - \mu - \alpha & 0 \\ \alpha & -\gamma-\mu \end{bmatrix}$$ – случай 3

6) $A_2 = 0$

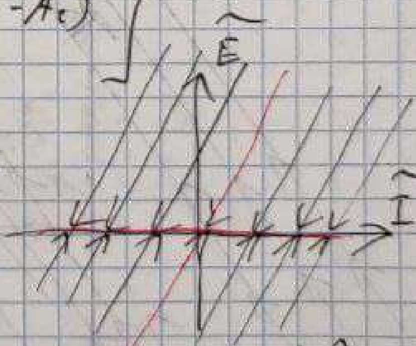
$$A_4 = \begin{bmatrix} -(\mu+\alpha) & \beta A_3 \\ \alpha & -\gamma-\mu \end{bmatrix}$$ – случай 1

7) $A_2 = A_3 = 0$

$$A_4 = \begin{bmatrix} -(\mu+\alpha) & 0 \\ \alpha & (-\gamma-\mu) \end{bmatrix}$$ – случай 3

8) $\gamma = -\mu$

$$A_4 = \begin{bmatrix} -(\beta A_2 + \mu + \alpha) & \beta(A_3 - A_2) \\ \alpha & 0 \end{bmatrix}$$



$\cancel{\lambda_{1,2} = -(\beta A_2 + \mu + \alpha)}$

$\lambda_1 = -(\beta A_2 + \mu + \alpha)$

$\lambda_2 = 0$

$$V_2 = \begin{bmatrix} \dfrac{\beta(A_3 - A_2)}{\beta A_2 + \mu} \\ 1 \end{bmatrix} \qquad V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \begin{cases} \tilde{E} = E - E_0 \\ \tilde{I} = I - I_0 \end{cases}$$

9) $\gamma = -\mu$, $\alpha = 0$

$$A_4 = \begin{bmatrix} -(\beta A_2 + \mu + \cancel{\alpha}) & \beta(A_3 - A_2) \\ 0 & 0 \end{bmatrix}$$

$\lambda_1 = 0$

$\lambda_2 = -(\beta A_2 + \mu + \alpha)$

$$V_1 = \begin{bmatrix} \frac{\beta(A_3 - A_2)}{\beta A_2 + \mu} \\ 1 \end{bmatrix} \quad V_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ - случай 8}$$

10) $\gamma = -\mu$, $\mu = 0$, $\alpha = 0$ $A_2 = 0$

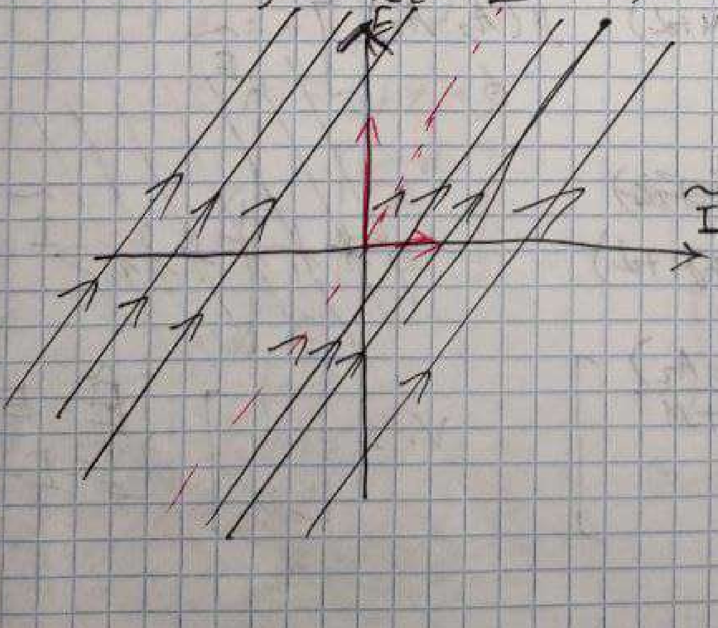
$$A_4 = \begin{bmatrix} 0 & \beta A_3 \\ 0 & 0 \end{bmatrix}$$

$\lambda_1 = \lambda_2 = 0$

$$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \beta A_3 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\beta A_3 V_{22} = 1 \Rightarrow V_2 = \begin{bmatrix} 0 \\ \frac{1}{\beta A_3} \end{bmatrix}$$



$$\begin{cases} \widehat{E} = E - E_0 \\ \widehat{I} = I - I_0 \end{cases}$$

11) $\gamma = -\mu$, $\mu = 0$, $\alpha = 0$, $\beta = 0$

$$A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \forall (\cdot) \text{ фаз. плоскости} - (\cdot) \text{ покоя}$$

$$\begin{cases} \frac{dE}{dt} = -(\beta A_2 + \mu + \alpha)(E - E_0) - \beta A_2 (R - R_0) + f_{24}(t) \\ \frac{dR}{dt} = -\mu (R - R_0) + f_{42}(t) \end{cases}$$

$$A_5 = \begin{bmatrix} -(\beta A_2 + \mu + \alpha) & -\beta A_2 \\ 0 & -\mu \end{bmatrix}$$

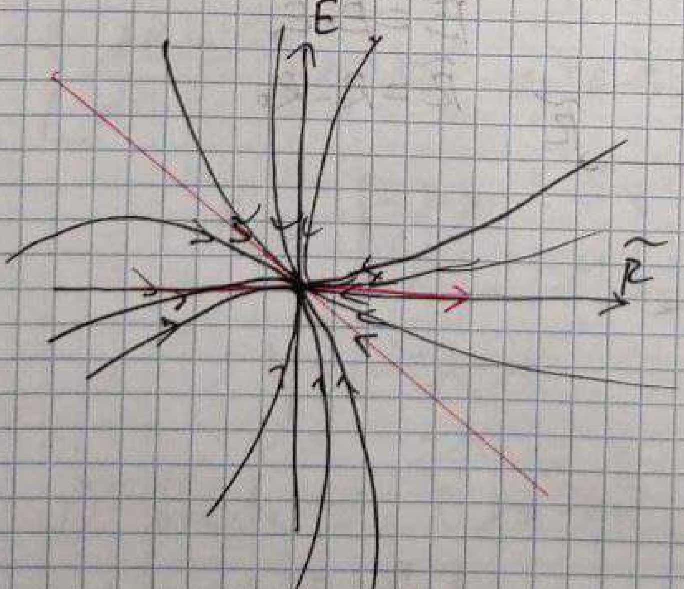
$\lambda_1 = -(\beta A_2 + \mu + \alpha) < 0$

$\lambda_2 = -\mu < 0$

1) $\lambda_1 < \lambda_2 < 0$ ($\mu \neq 0$, $\beta \neq 0$, $A_2 \neq 0$)

$$A_5 - \lambda_1 E = \begin{bmatrix} 0 & -\beta A_2 \\ 0 & -\mu \end{bmatrix}$$

$$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad V_2 = \begin{bmatrix} \frac{-\beta A_2}{\beta A_2 + \mu + \alpha} \\ 1 \end{bmatrix}$$



$$\begin{cases} \tilde{E} = E - E_0 \\ \tilde{R} = R - R_0 \end{cases}$$

$\lambda_1 = -(\beta A_2 + \alpha)$

$\lambda_2 = 0$

$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $V_2 = \begin{bmatrix} -\frac{\beta A_2}{\beta A_2 + \mu} \\ 1 \end{bmatrix}$ ~~[illegible]~~

рассмотрим позже

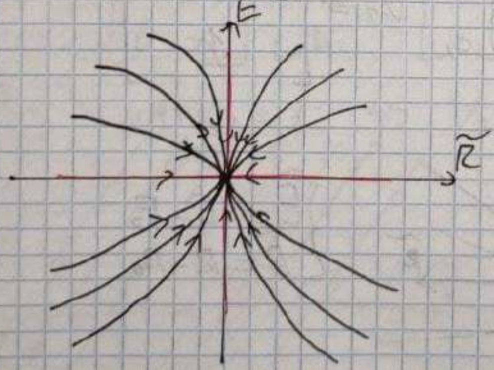
3) $\beta = 0$ или $A_2 = 0$

$$A_5 = \begin{bmatrix} -(\mu+\alpha) & 0 \\ 0 & -\mu \end{bmatrix}$$

$\lambda_1 = (-\mu - \alpha)$

$\lambda_2 = -\mu$

$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $V_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



$\tilde{E} = E - E_0$

$\tilde{R} = R - R_0$

4) $\mu = 0$, $\alpha = 0$, $A_2 = 0$

$\begin{bmatrix} 0 & -\mu \end{bmatrix}$

$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad V_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

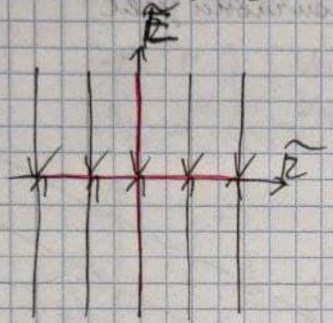


рис. к случ. 5

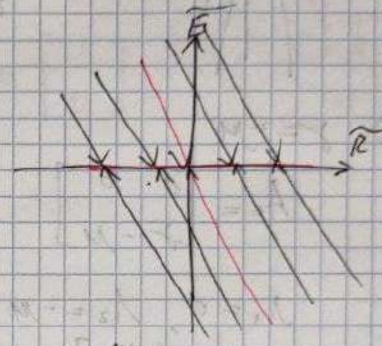


рис. к. случ. 2

$\hat{E} = E - E_0$

$\hat{R} = R - R_0$

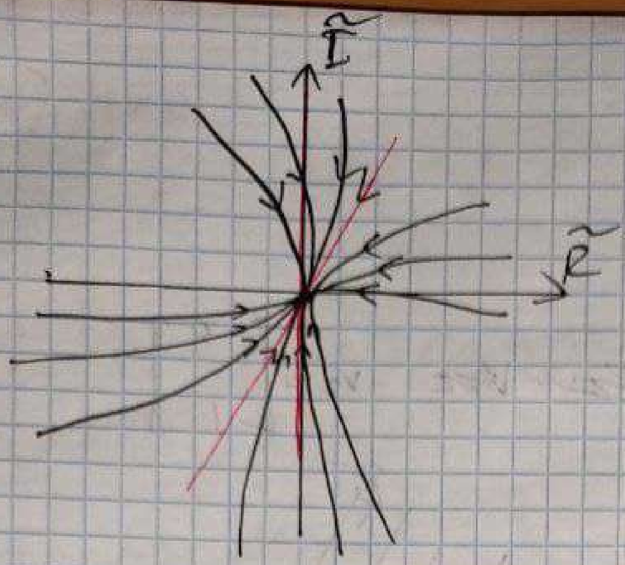
$$\begin{cases} \dfrac{dE}{dt} = -(\gamma+\mu)(I - I_0) + f_{34}(t) \\ \dfrac{dR}{dt} = \gamma(I - I_0) - \mu(R - R_0) + f_{43}(t) \end{cases}$$

$$A_6 = \begin{bmatrix} -\gamma-\mu & 0 \\ \gamma & -\mu \end{bmatrix}$$

$\lambda_1 = -(\gamma+\mu)$

$\lambda_2 = -\mu$

1) $\lambda_1 < \lambda_2 < 0 \quad (\gamma \neq -\mu), \mu \neq 0, \gamma \neq 0)$



$$\begin{cases} \tilde{I} = I - I_0 \\ \tilde{R} = R - R_0 \end{cases}$$

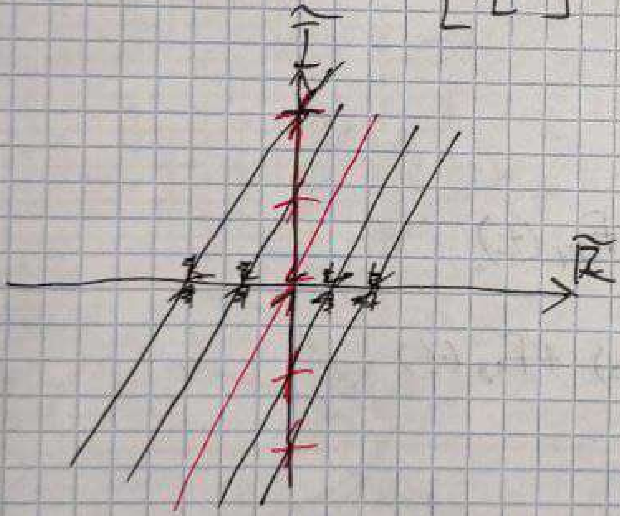
устойчивый узел

2) $\gamma = -\mu$

$$A_6 = \begin{bmatrix} 0 & 0 \\ \gamma & -\mu \end{bmatrix}$$

$\lambda_1 = 0, \ \lambda_2 = -\mu$

$$V_1 = \begin{bmatrix} \frac{\mu}{\gamma} \\ 1 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



$$\begin{cases} \tilde{I} = I - I_0 \\ \tilde{R} = R - R_0 \end{cases}$$

3) $\gamma = -\mu, \ \mu = 0$

$$A_6 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \forall (\cdot) \text{ фаз. пл-ти} - (\cdot) \text{ покоя}$$

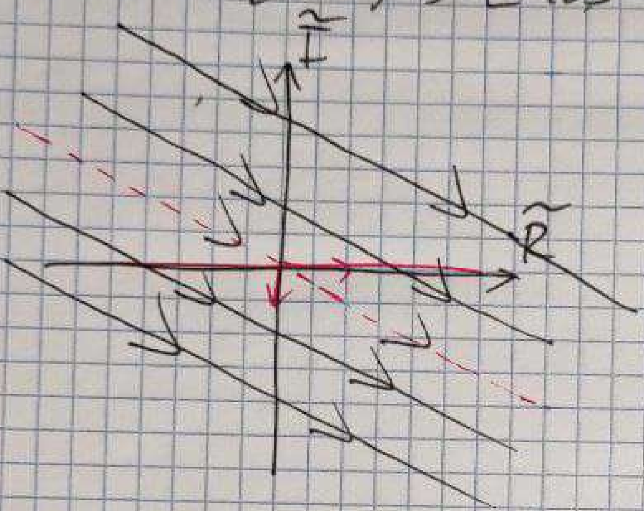
4) $\gamma = 0$

$$A_6 = \begin{bmatrix} -\mu & 0 \\ 0 & -\mu \end{bmatrix}$$

$$\lambda_1 = \lambda_2 = -\mu$$

$$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\mu & 0 \\ 0 & -\mu \end{bmatrix} \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow V_2 = \begin{bmatrix} -\frac{1}{\mu} \\ 0 \end{bmatrix}$$

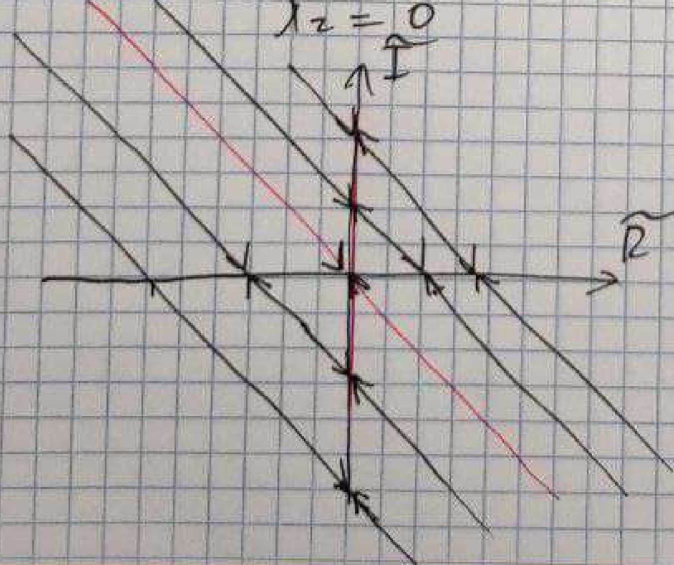


5) $\mu = 0$

$$A_0 = \begin{bmatrix} -\gamma & 0 \\ \gamma & 0 \end{bmatrix}$$

$$\lambda_1 = -\gamma \qquad V_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\lambda_2 = 0$$



$$\begin{cases} \tilde{I} = I - I_0 \\ \tilde{R} = R - R_0 \end{cases}$$