Schutz A first course in GR

Hartle Gravity

Lancaster GR for the gifted Amateur

D'Inverno Einstein's Relativity

Bert Janssen Gravitación y Geometría
(ESP)

Lambourne* Gravitation & Cosmology.

PRS Las leyes de la FÍSICA son las mismas en todos los sist. de ref INERCIAL

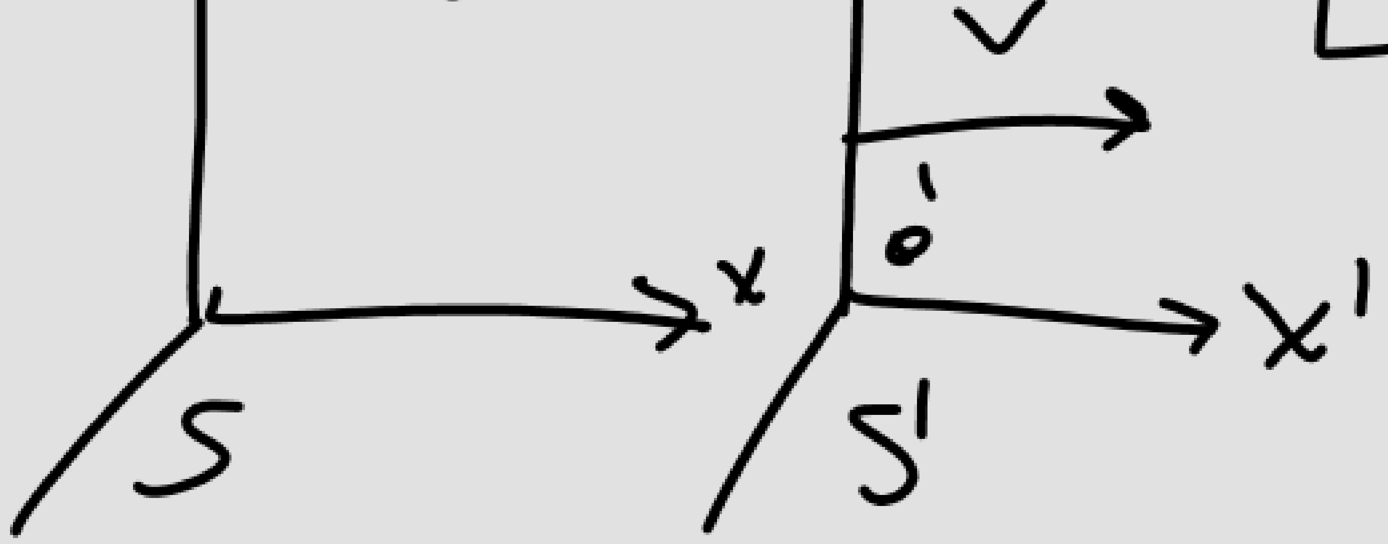
$C = cte$ es una ley de la física

$$x' = \gamma(x - vt)$$
$$t' = \gamma(t - vx)$$
$$y = y' ; \quad z = z'$$

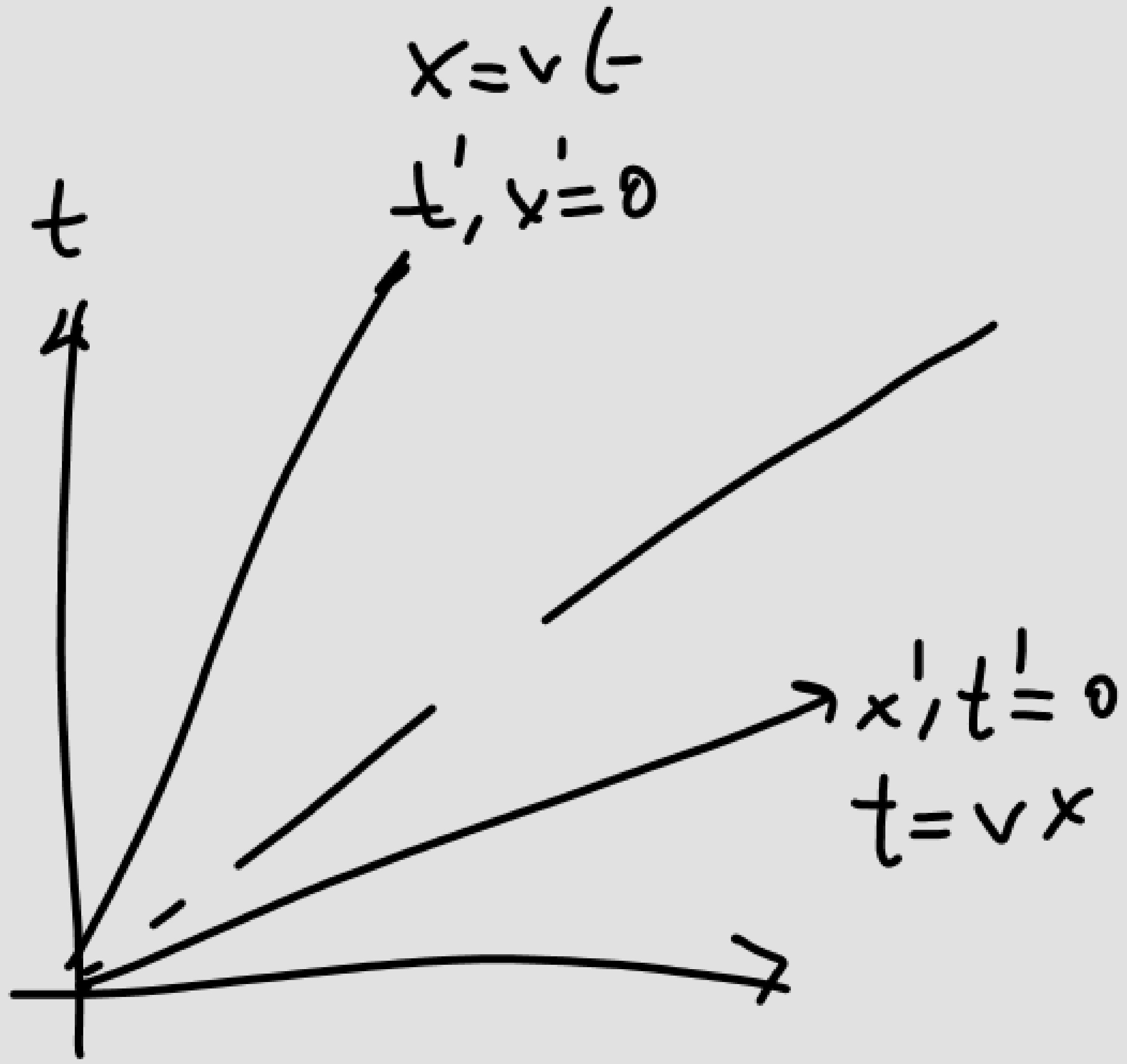
$$x = \gamma(x' + vt')$$
$$t = \gamma\left(t + \frac{vx}{c^2}\right)$$

$$\boxed{c = 1}$$

$$ct = \gamma\left(ct + \frac{v}{c}x\right)$$



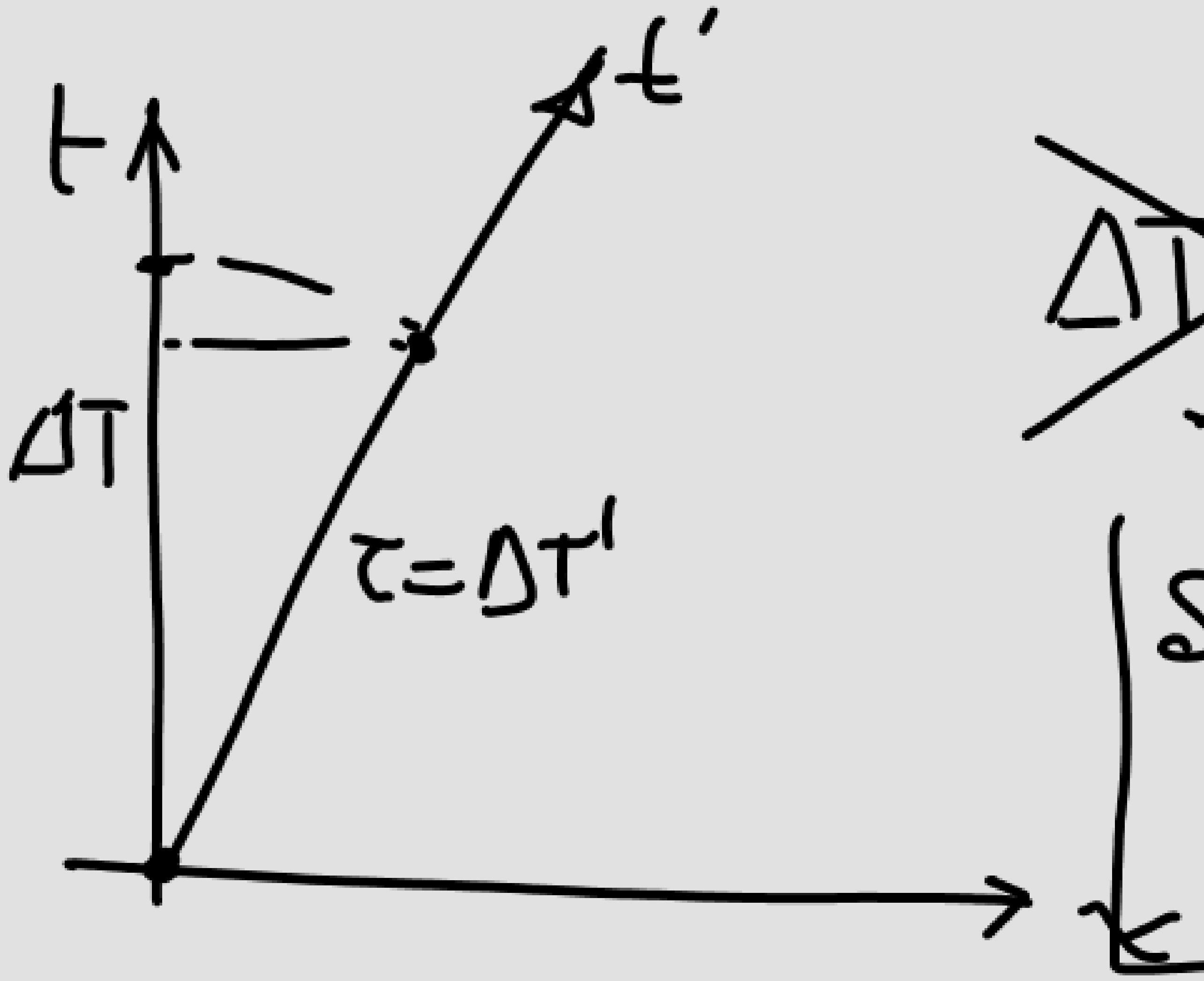
$$\gamma = \frac{1}{\sqrt{1 - v^2}} \qquad \gamma > 1$$



$$L' = \frac{L}{\gamma} \rightarrow L' < L$$

$$T = \gamma T' \qquad T > T'$$

$$\Delta x = 0$$

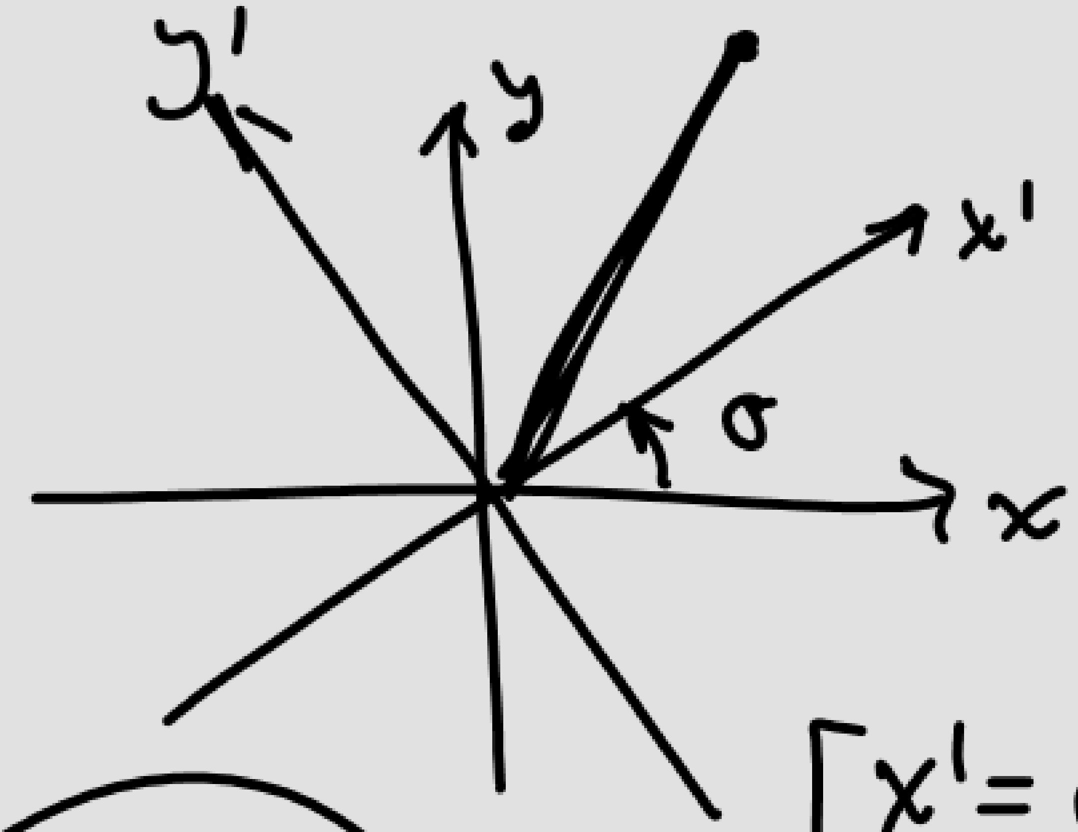


$$\cancel{\Delta T \quad \Delta T'}$$

$$\boxed{\cancel{\begin{array}{c} S \quad d(x,t)^2 = x^2 + t^2 \\ \| \\ d(x',t') = x'^2 + t'^2 \end{array}}}$$

$$\Delta T = \gamma \Delta T' \quad \gamma > 1 \Bigg\} \quad \cancel{\Delta T' > \Delta T} \quad \underline{\underline{\Delta T > \Delta T'}}$$

$$\frac{v << c}{t = t'}$$



$$\begin{cases} x' = \cos\theta\, x + \sin\theta\, y \\ y' = -\sin\theta\, x + \cos\theta\, y \end{cases}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

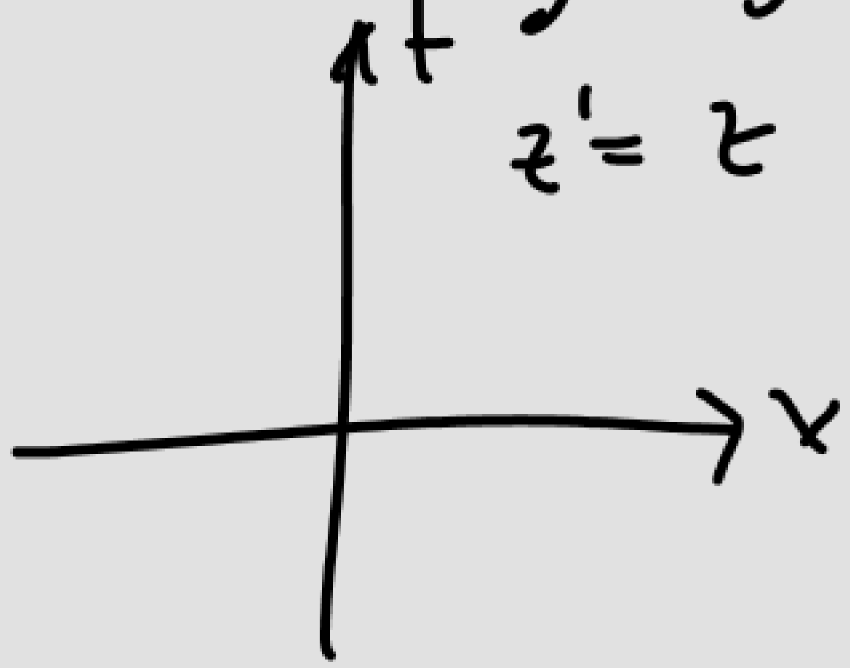
$$\downarrow$$

$$R(\theta)$$

$$x'^2 + y'^2 = \cos^2\theta\, x^2 + \sin^2\theta\, y^2 \cancel{+ 2\cos\theta \sin\theta\, xy}$$
$$+ \sin^2\theta\, x^2 + \cos^2\theta\, y^2 \cancel{- 2\sin\theta \cos\theta\, xy}$$
$$= (\cos^2\theta + \sin^2\theta)\, x^2 + (\cos^2\theta + \sin^2\theta)\, y^2$$
$$= x^2 + y^2$$

$$\begin{cases} x' = \gamma(x - vt) \\ t' = \gamma(t - vx) \end{cases}$$

$$\begin{cases} x'^2 = \gamma^2\left(x^2 + v^2t^2 - \underline{2vxt}\right) \\ t' = \gamma^2\left(t^2 + v^2x^2 - \underline{2vxt}\right) \\ y' = y \\ z' = z \end{cases}$$



$$\underline{x'^2 - t'^2} = \gamma^2\left[(x^2 - t^2) + v^2(t^2 - x^2) \cancel{-2vxt + 2vxt}\right]$$

$$\underbrace{y'^2 + z'^2}$$

$$= \gamma^2(x^2 - t^2)(1 - v^2)$$

$$= \gamma^2\underbrace{(1 - v^2)}(x^2 - t^2) = \frac{1 - v^2}{1 - v^2}(x^2 - t^2) = \frac{\overline{(x^2 - t^2)}}{y^2 + z^2} \quad \checkmark$$

$$\underline{t'^2 - x'^2 = t^2 - x^2}$$

$$x'^2 + t'^2 =$$

$$\gamma^2\left[\underbrace{(x^2 + t^2)} + v^2\underbrace{(x^2 + t^2)} - 4vxt\right]$$

$$\cancel{= \gamma^2\underbrace{(x^2 + t^2)}\underbrace{[1 + v^2 - 4vxt]}}$$

$$\gamma^2\underbrace{\left[(x^2 + t^2)(1 + v^2) - 4vxt\right]}$$

¿$= x^2 + t^2$?

$$\boxed{(- + + +)}$$

$$(+ - - -)$$

$t \quad x \quad y \quad z$

Intervalo espacio-temporal

$$s^2 = -t^2 + x^2 + y^2 + z^2$$ — INVARIANTE

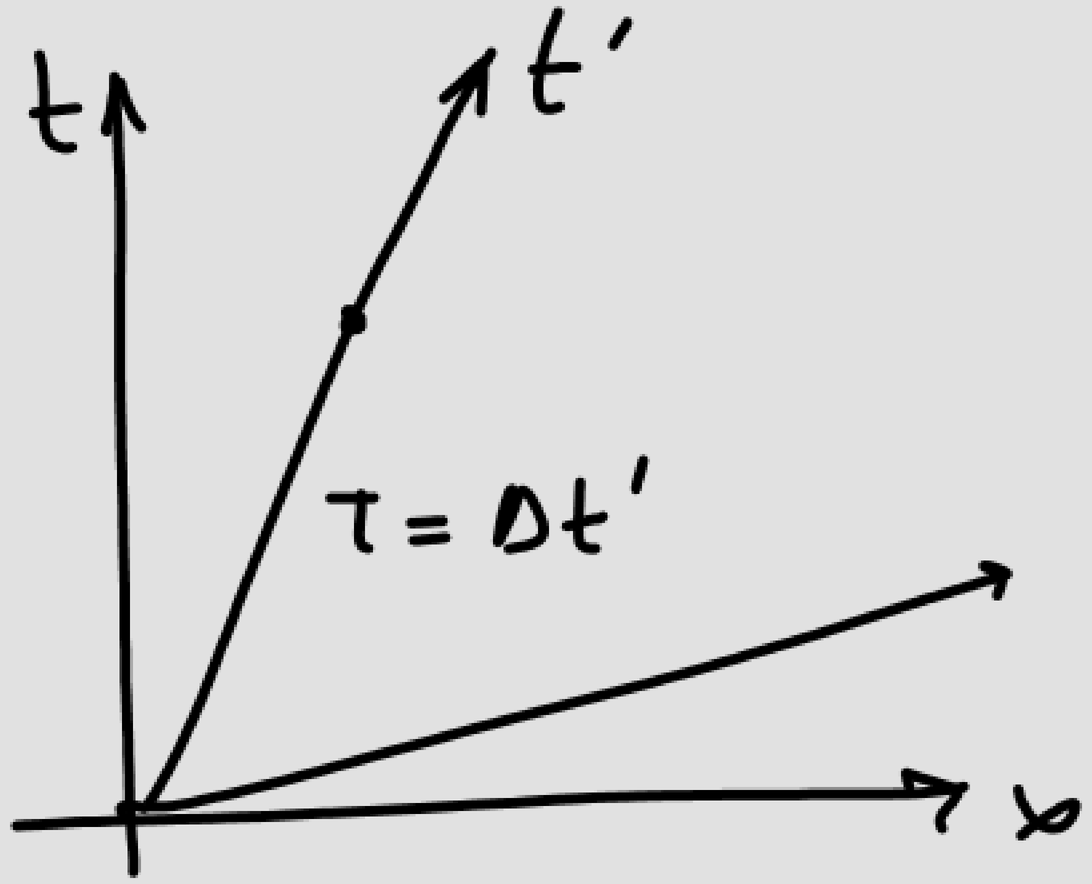
$$x' = \gamma(x - vt) < \begin{matrix} x'_1 = \gamma(x_1 - v t_1) \\ x'_2 = \gamma(x_2 - v t_2) \end{matrix} \Big\} \Delta x' = \gamma(\Delta x - v\Delta t)$$

$$\downarrow$$

$$\Delta x' = \gamma(\Delta x - v\Delta t)$$

$$\boxed{\begin{aligned} \Delta s^2 &= -\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 \\ &= -\Delta t'^2 + \Delta x'^2 + \Delta y'^2 + \Delta z'^2 \end{aligned}}$$

Definición de la "DISTANCIA" en el ESPACIO-TIEMPO



$$\Delta t' = \gamma(\Delta t - v\Delta x)$$

$$\Delta t = \gamma(\Delta t' + v\Delta x')$$

$$\Delta x' = 0 \rightarrow \Delta t = \gamma \Delta t'$$

Tiempo propio $\tau \equiv \boxed{\Delta t' = \sqrt{1-v^2}\,\Delta t}$

$$\Delta t' < \Delta t$$

$$\Delta s^2 = -c\Delta t^2 + \Delta x^2$$

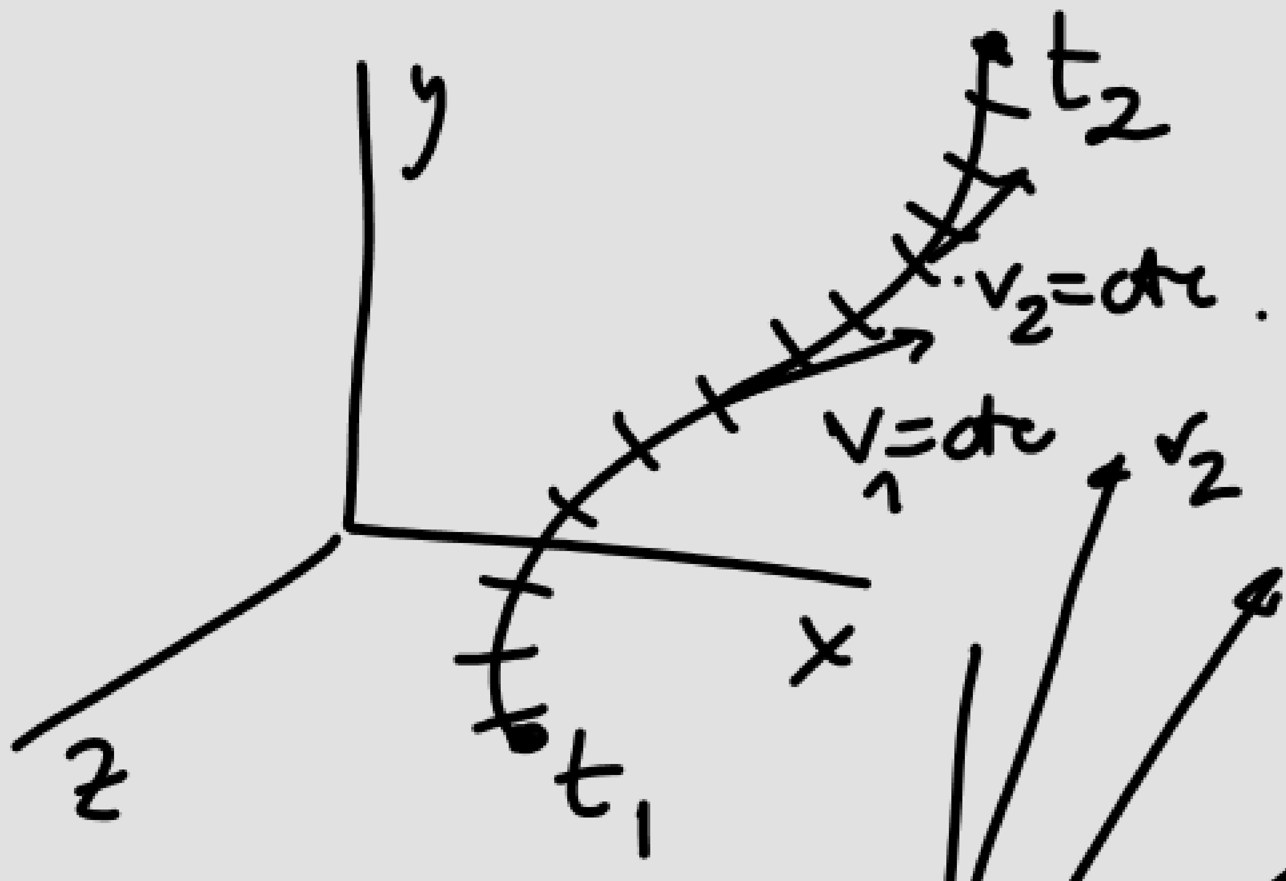
$c = 1$

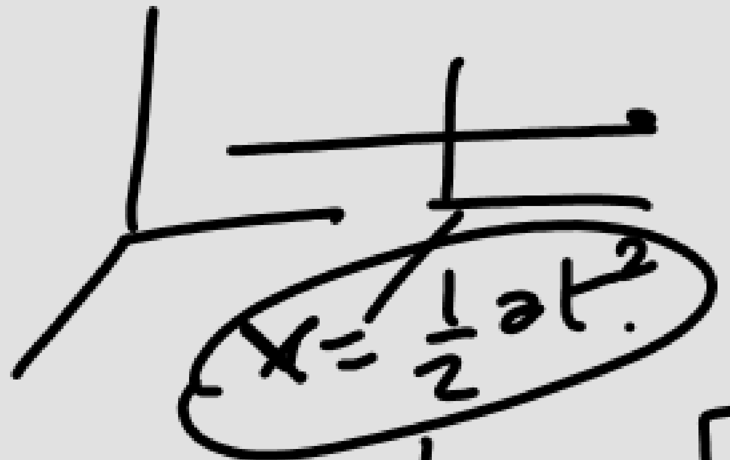
$$\Delta s^2 = -\Delta t^2 + \Delta x^2$$

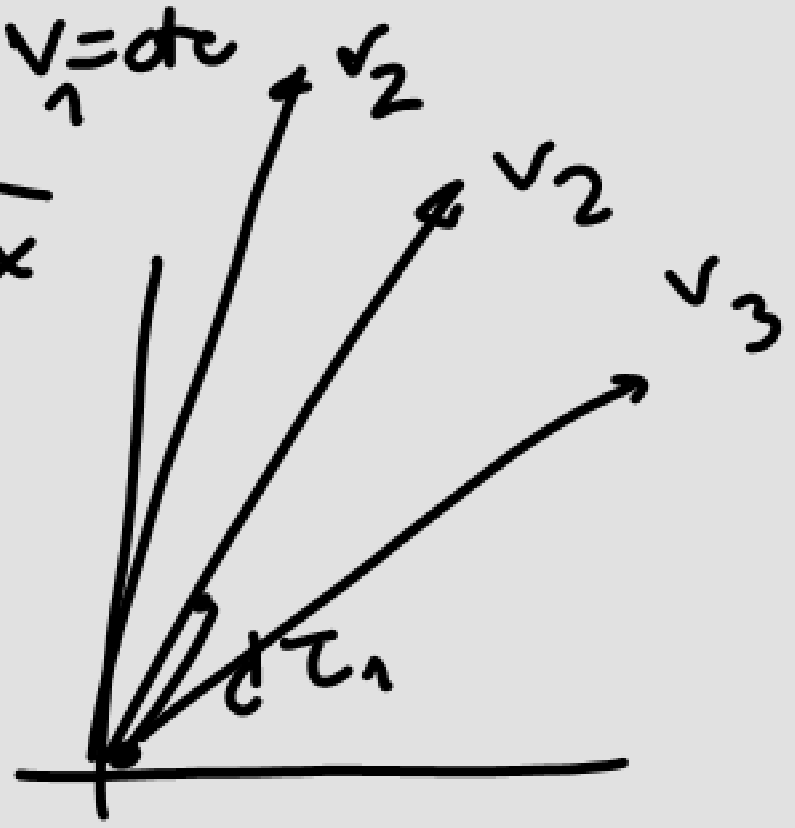
$$\Delta s^2 = -\Delta t'^2 + \Delta x'^2 = -\Delta t + \Delta x^2$$

$$\Delta t' = \Delta s^2$$

$$\boxed{\tau = \int_{t_1}^{t_2} \sqrt{1-v(t)^2}\,dt}$$





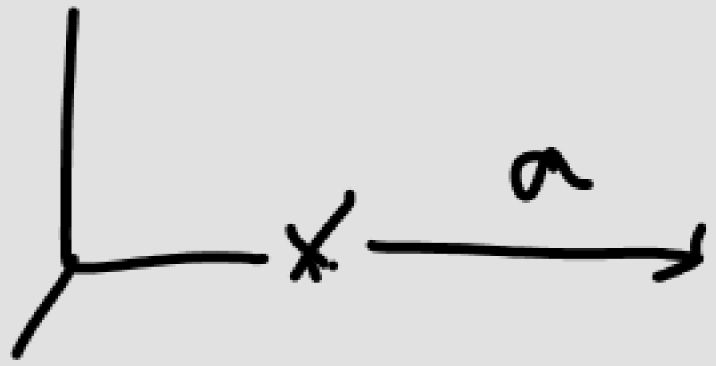


$$d\tau = \sqrt{1-v^2}\,dt$$

$f(t)$

$$\int d\tau = \int_{t_1}^{t_2} \sqrt{1-v(t)^2}\,dt$$

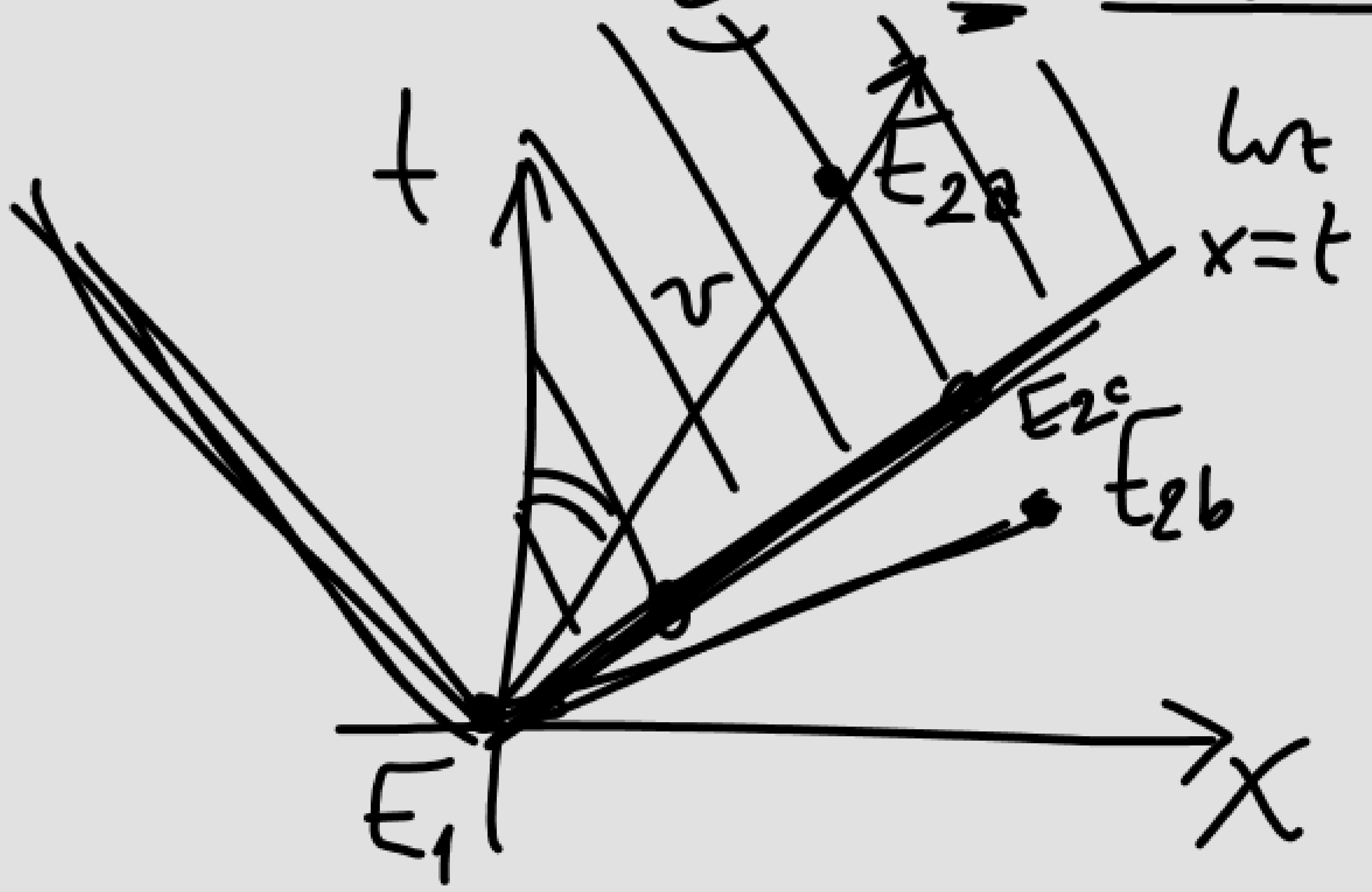
$ch(1-v^2)$

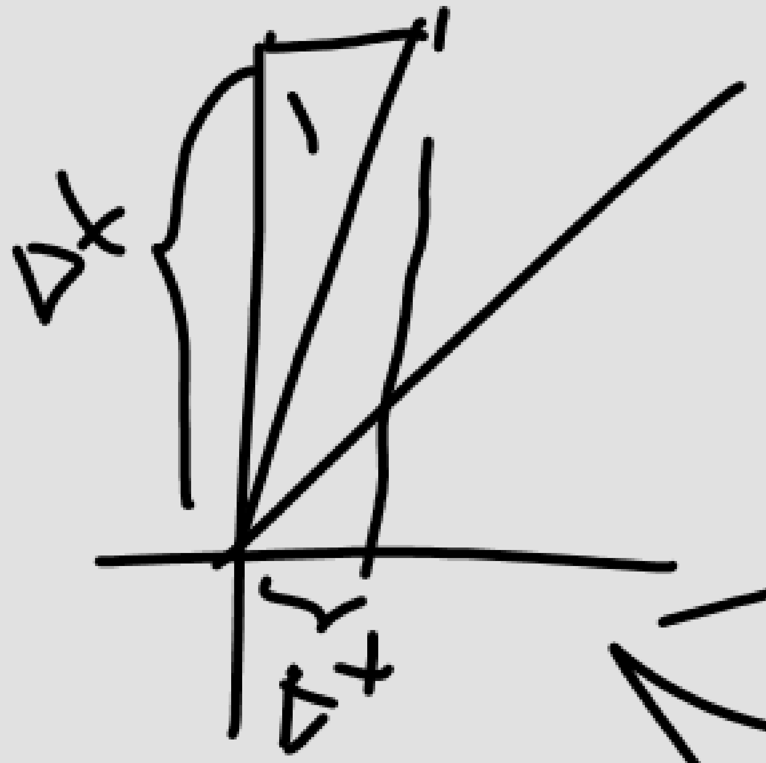


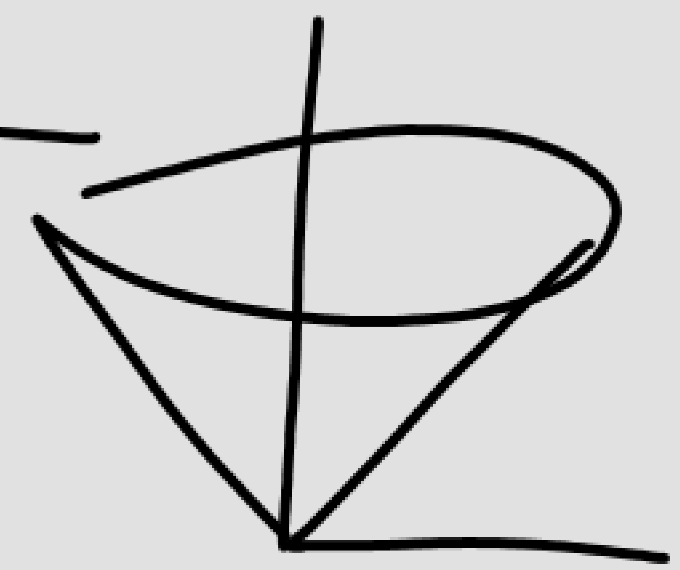
$(- + + +)$
$(+ - - -)$ $\quad \Delta s^2 = \overbrace{-\Delta t^2}^{<0} + \overbrace{\Delta x^2 + \Delta y^2 + \Delta z^2}^{>0}$

$E_1 = (t_1, x_1, y_1, z_1)$

$E_2 = (t_2, x_2, y_2, z_2)$







$\boxed{\Delta s^2 < 0}$ — E_1/E_{2a} Causalidad

$$\Delta x = 0$$
$$\boxed{\Delta t > 0 \quad \Delta t' > 0}$$

Causa-Efecto

TIMELIKE INTERVAL

$\Delta s^2 < 0$

$\Delta s^2 = 0 \rightarrow \Delta x^2 = \Delta t^2$

$\Delta x = \pm \Delta t$

E_1/E_{2c}

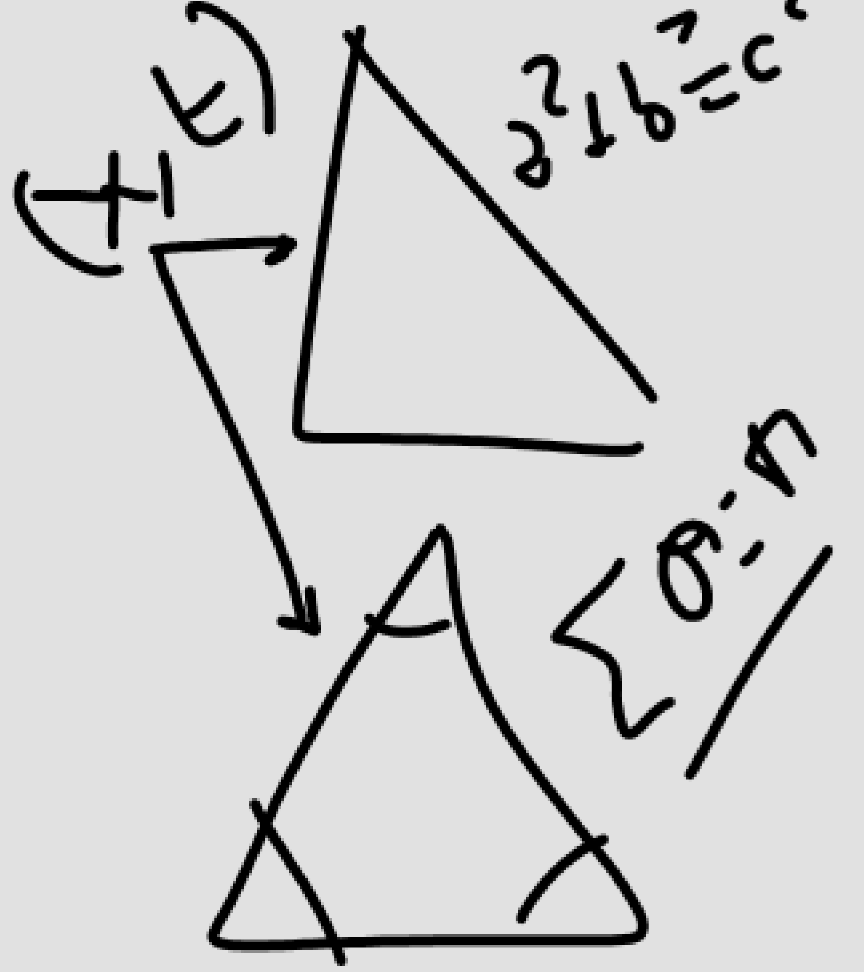
LIGHT LIKE
NULL LIKE

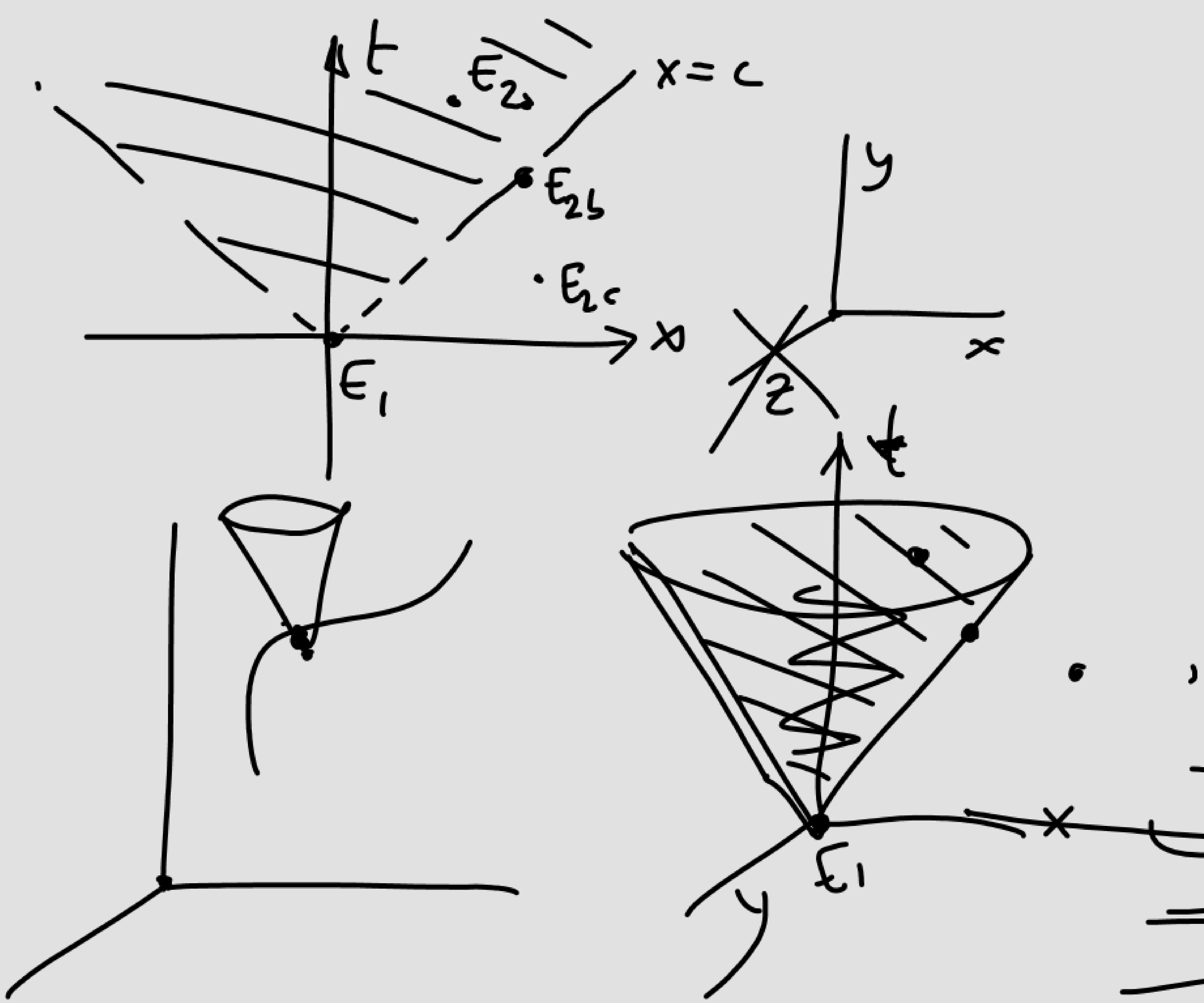
$\Delta s^2 > 0$ → No hay causalidad.

E_1/E_{2b}. $\Delta t = 0$

$\Delta x \neq 0$

SPACELIKE

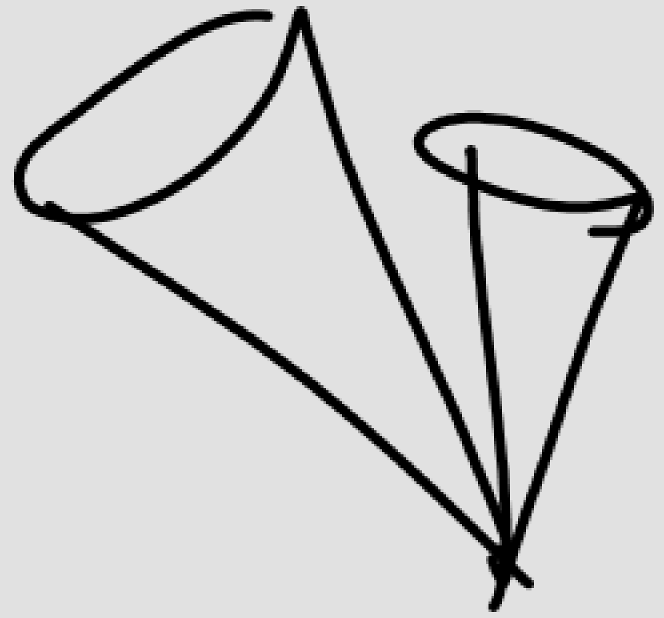


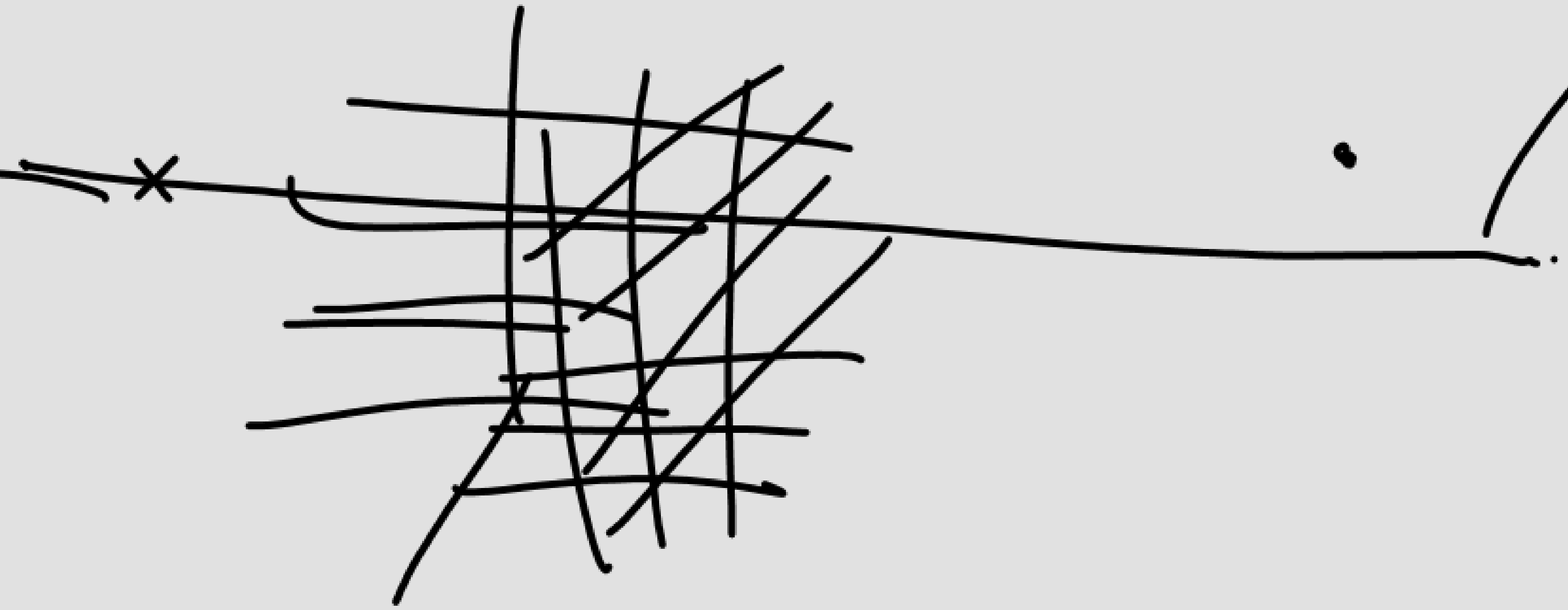


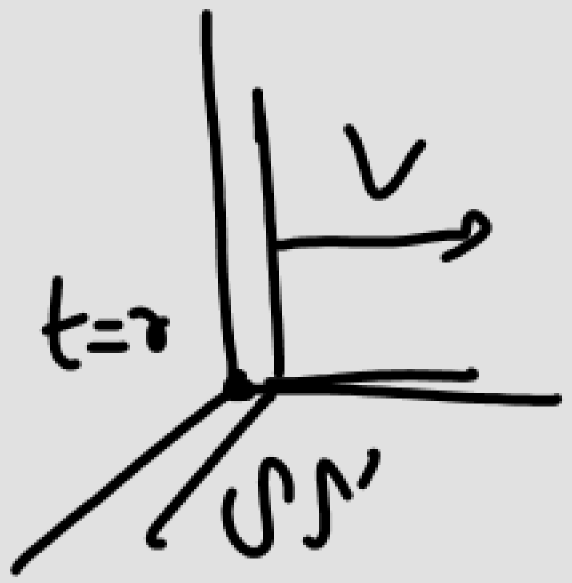
$\boxed{\Delta s^2 < 0}$

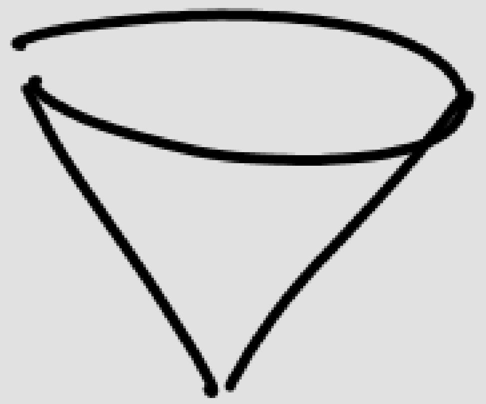
$S \quad \Delta t > 0 \quad t_2 > t_1$

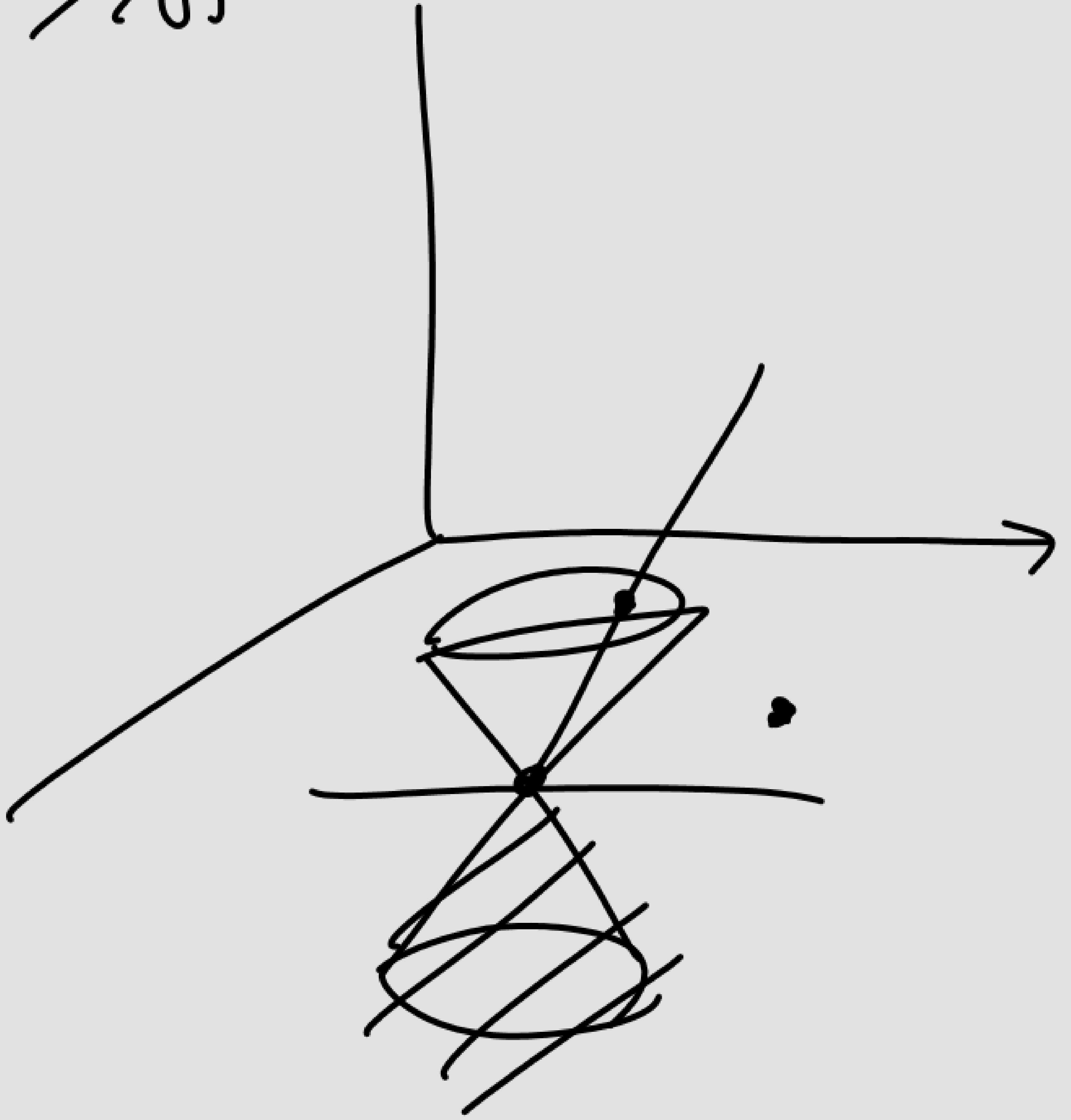
$\Delta t' > 0 \quad t_2' > t_1'$

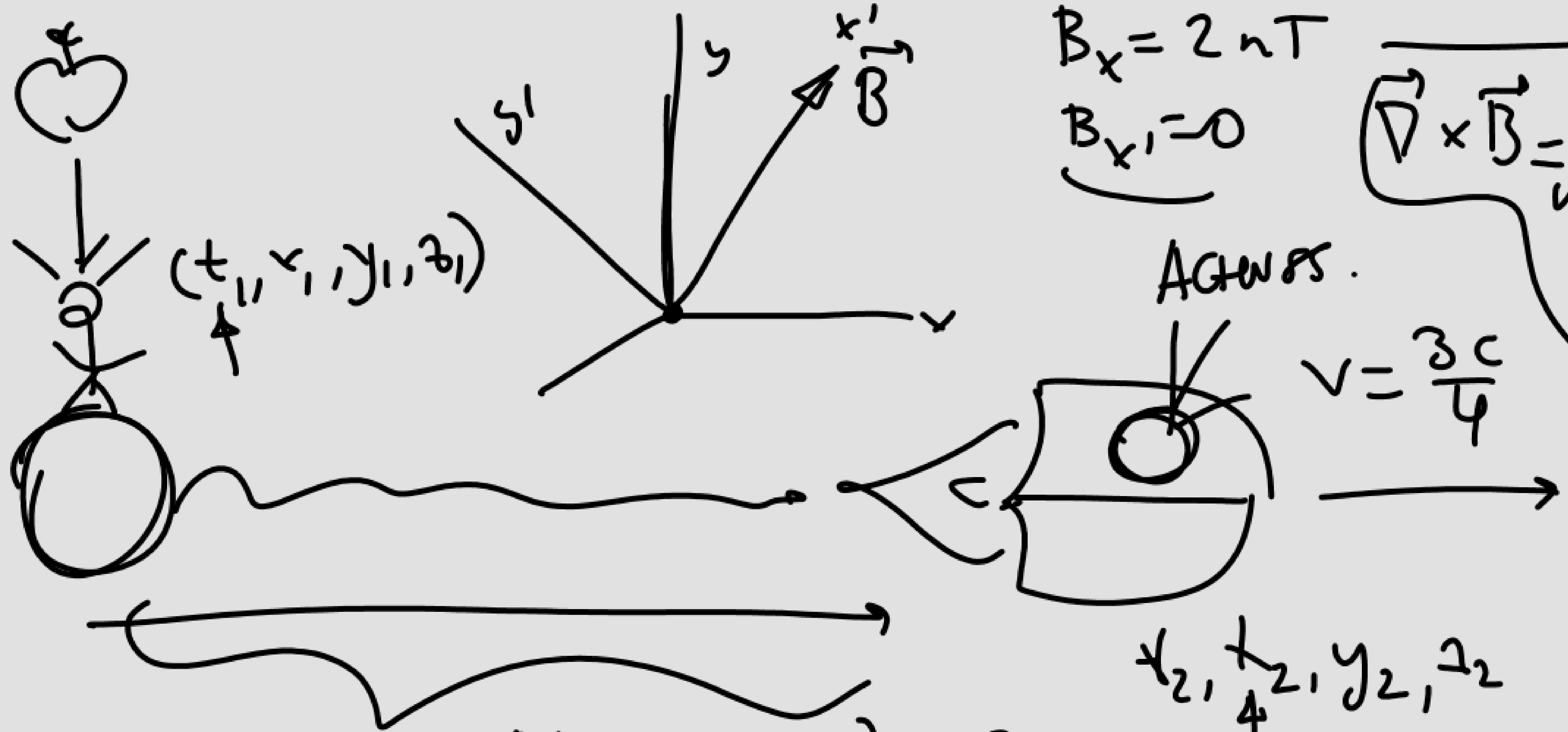












$B_x = 2\,nT$

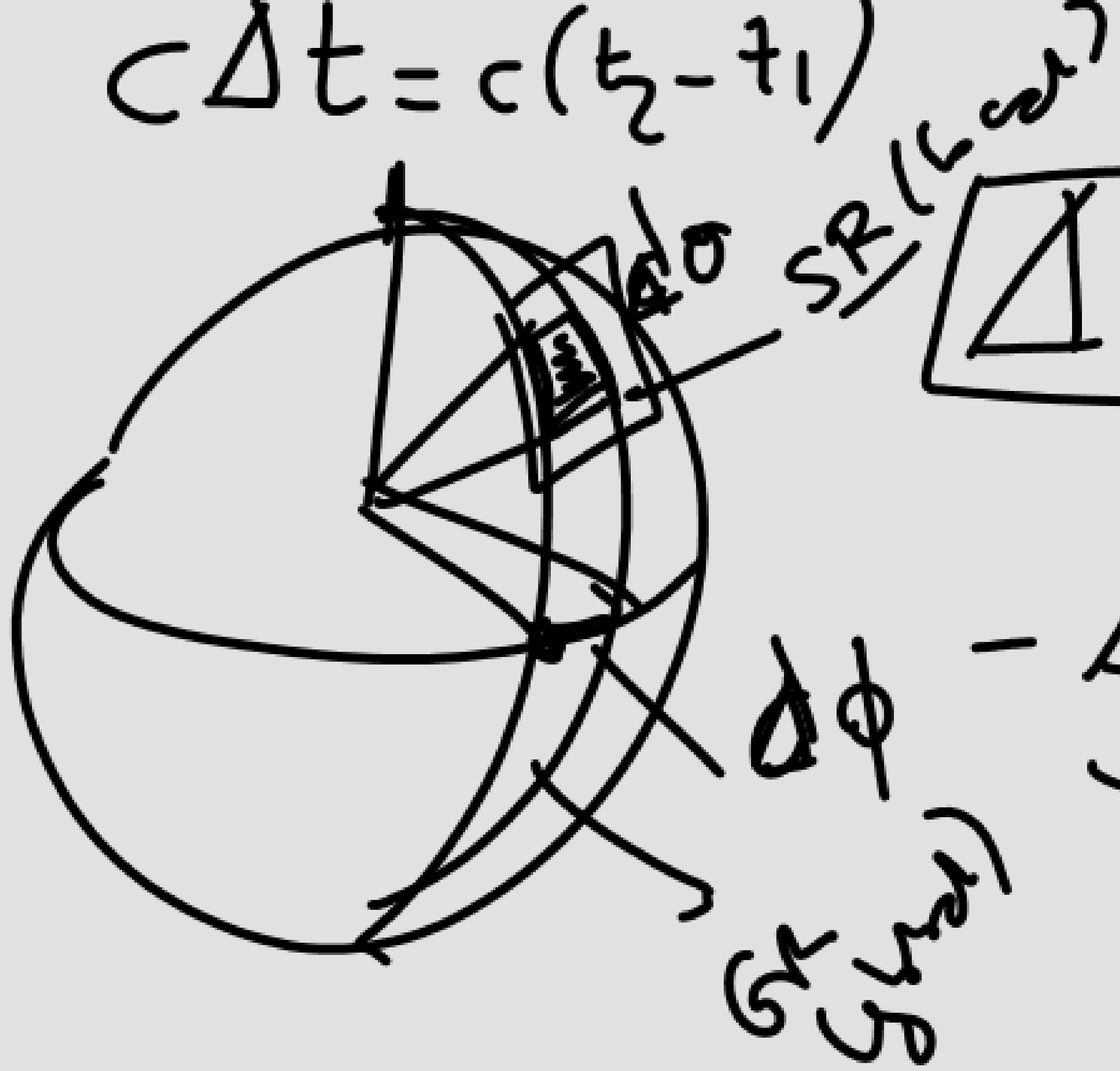
$B_{x'} = 0$

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{\partial E}{\partial t}}$$

$v = \frac{3c}{4}$

t_2, x_2, y_2, z_2

$c\Delta t = c(t_2 - t_1)$



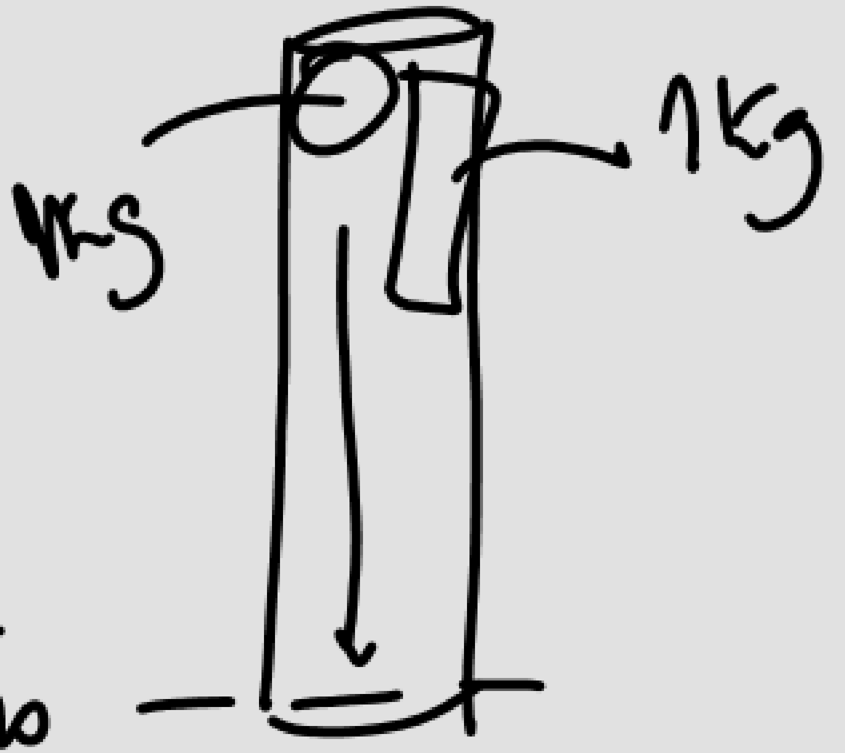
$$\boxed{\Delta s^2 > 0}$$

$$-\Delta t^2 + \underbrace{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

PRINCIPIO EQUIVALENCIA

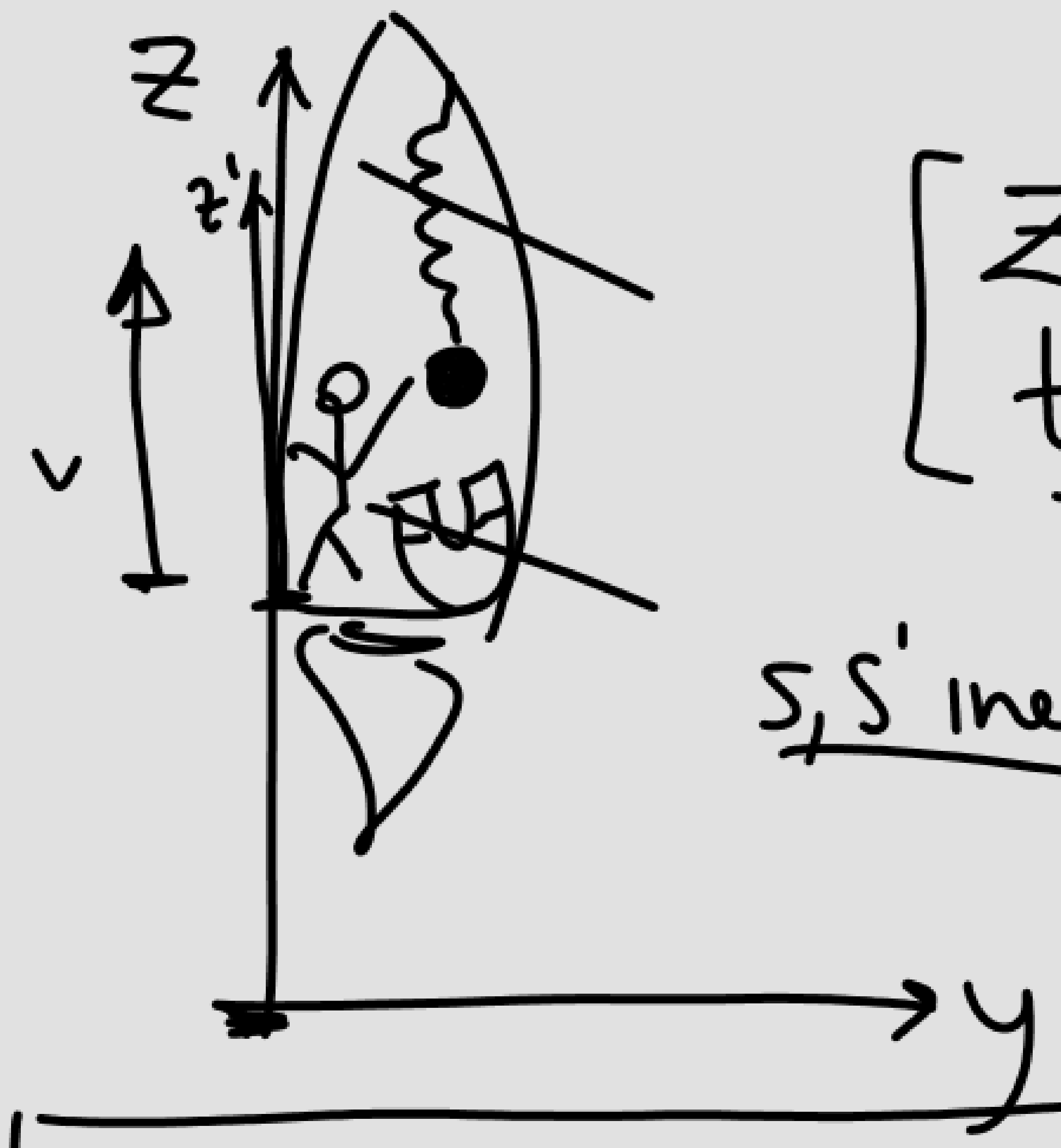
- Débil: masa inercial, masa gravitatoria } son lo mismo.
- Fuerte

En un espacio-tiempo local (ó suficientemete pequeño) los efectos de un campo gravitatorio no se pueden distinguir de los de un sisteme de referencia NO inercial (acelerado)



$$F = m_i a$$

$$\left.\begin{array}{l} m_g \cdot g = m_i \cdot a \\ a = g \end{array}\right\} m_i = m_g$$



$$\begin{cases} z = z' + vt \\ t = t' \end{cases}$$

$\downarrow d/dt$

$$u = u' + v$$

$\downarrow d/dt$

$$\boxed{a = a'}$$

S, S' inercial.

$$F = ma = ma'$$

$$F = 0 \rightarrow a = a'$$

$v = cte$

$v' = cte$

No hay gravedad.

$v << c \rightarrow$ No hay espacio-tiempo.

S' ES NO INERCIAL

$$z = z' + \frac{1}{2} g t'^2 \quad (t'' = t^2)$$

d/dt

$$u = u' + gt$$

d/dt'

$$a = a' + g$$

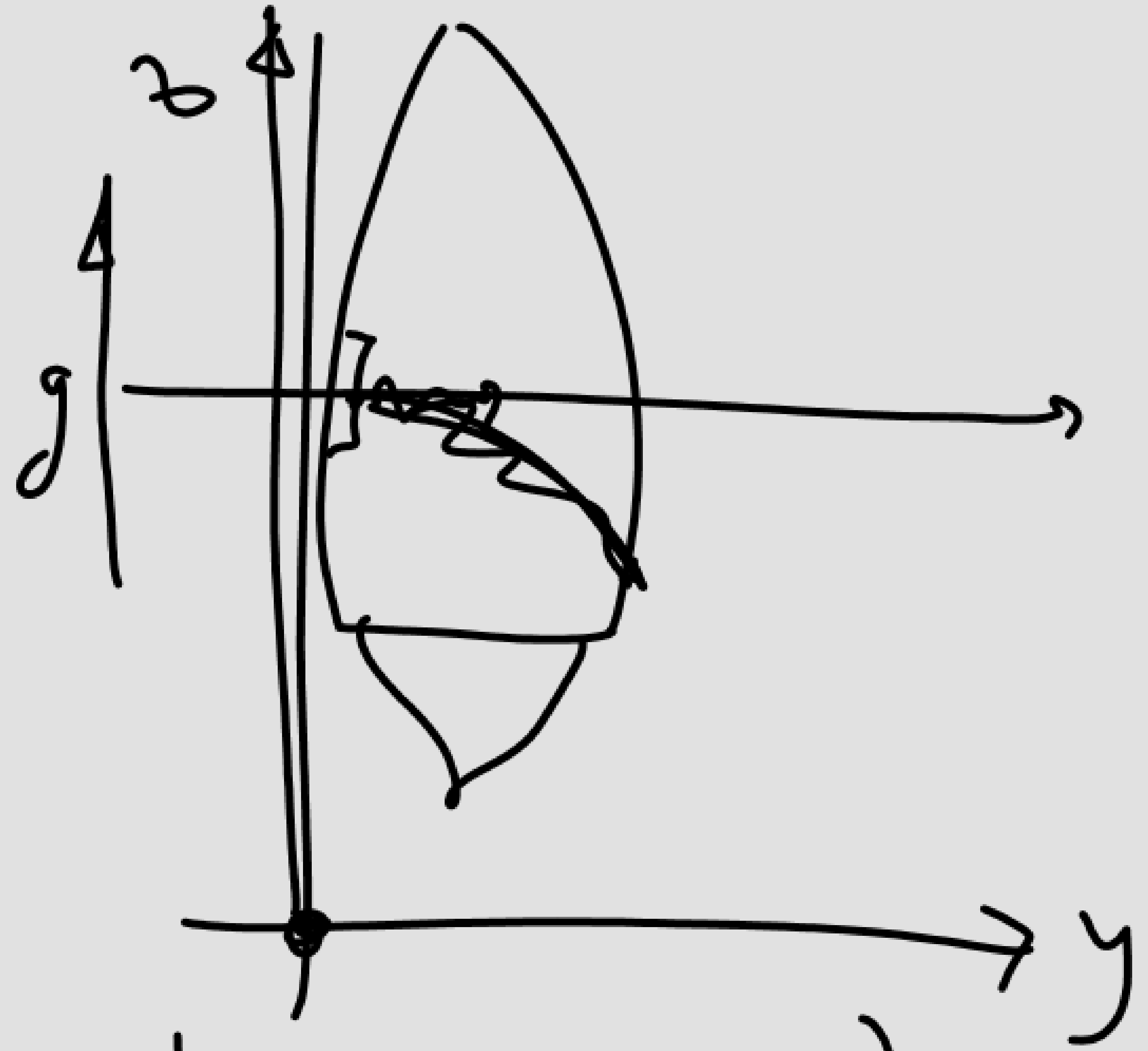
$$\boxed{a - g = a'}$$

$$\underbrace{ma}_{F} \boxed{-mg} = \underbrace{ma'}_{F'}$$

$$F = 0 \rightarrow F' = -mg$$

$\cancel{t' = \gamma(t - vx)}$

$\cancel{d/dt' = d/dt}$



$\boxed{\text{NO SR } (v \ll c) \quad \text{NO GRAVEDAD}}$

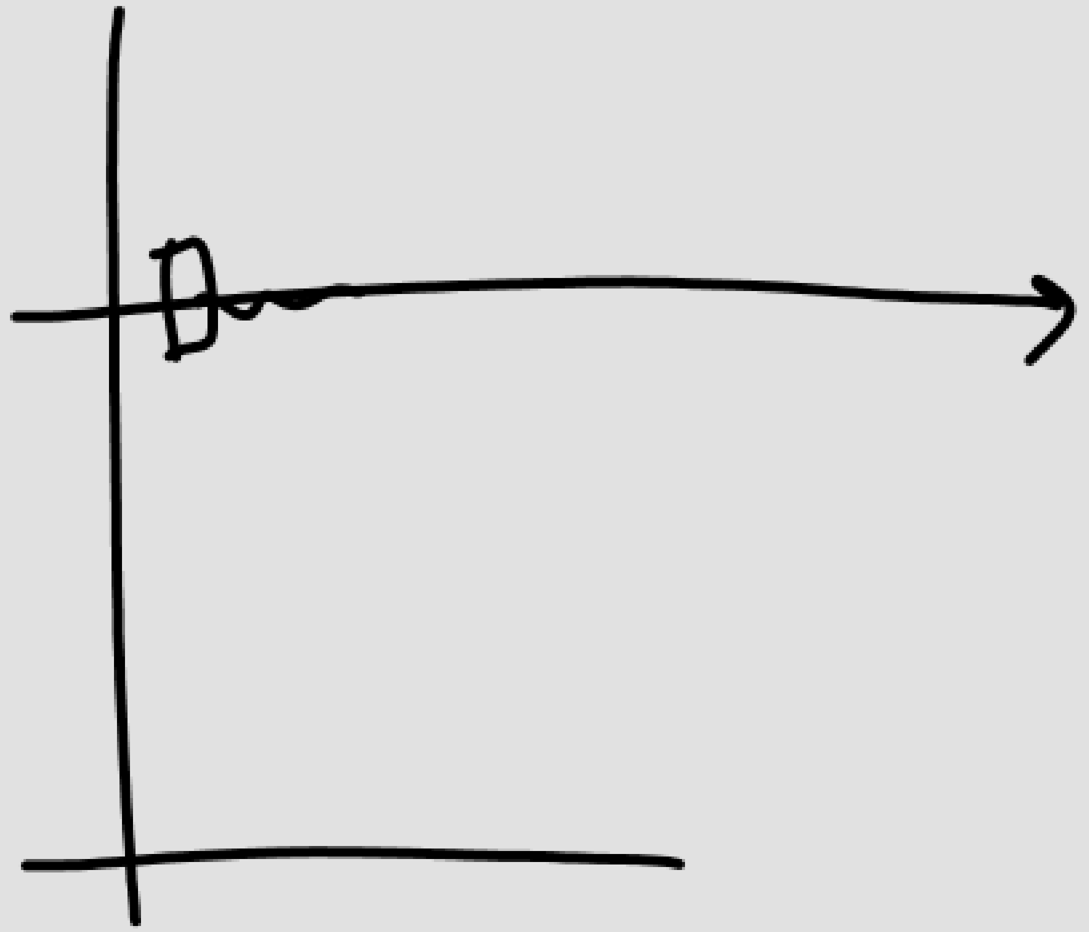
$$S: y = ct \rightarrow \cancel{t = y/c}$$

$$S': y' = ct' = ct \qquad t = y'/c$$

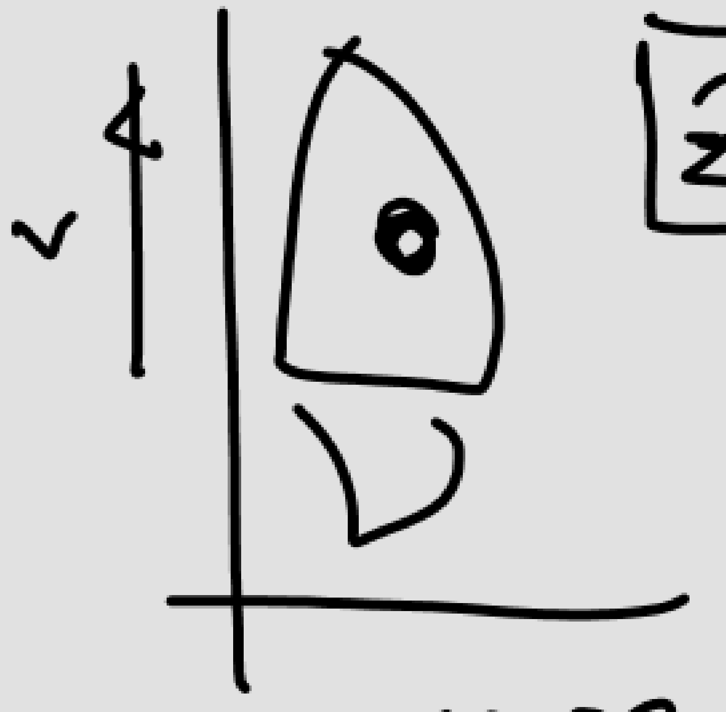
$$z' = \cancel{z}_{\to 0} + \frac{1}{2} g t^2 \qquad z' = \frac{1}{2} g \frac{y'^2}{c^2}$$

$t = 0, z = 0$

$$\boxed{z' = \frac{g}{2c^2} y'^2} \qquad y^2 = a x^2$$



$$\boxed{S, S'}$$



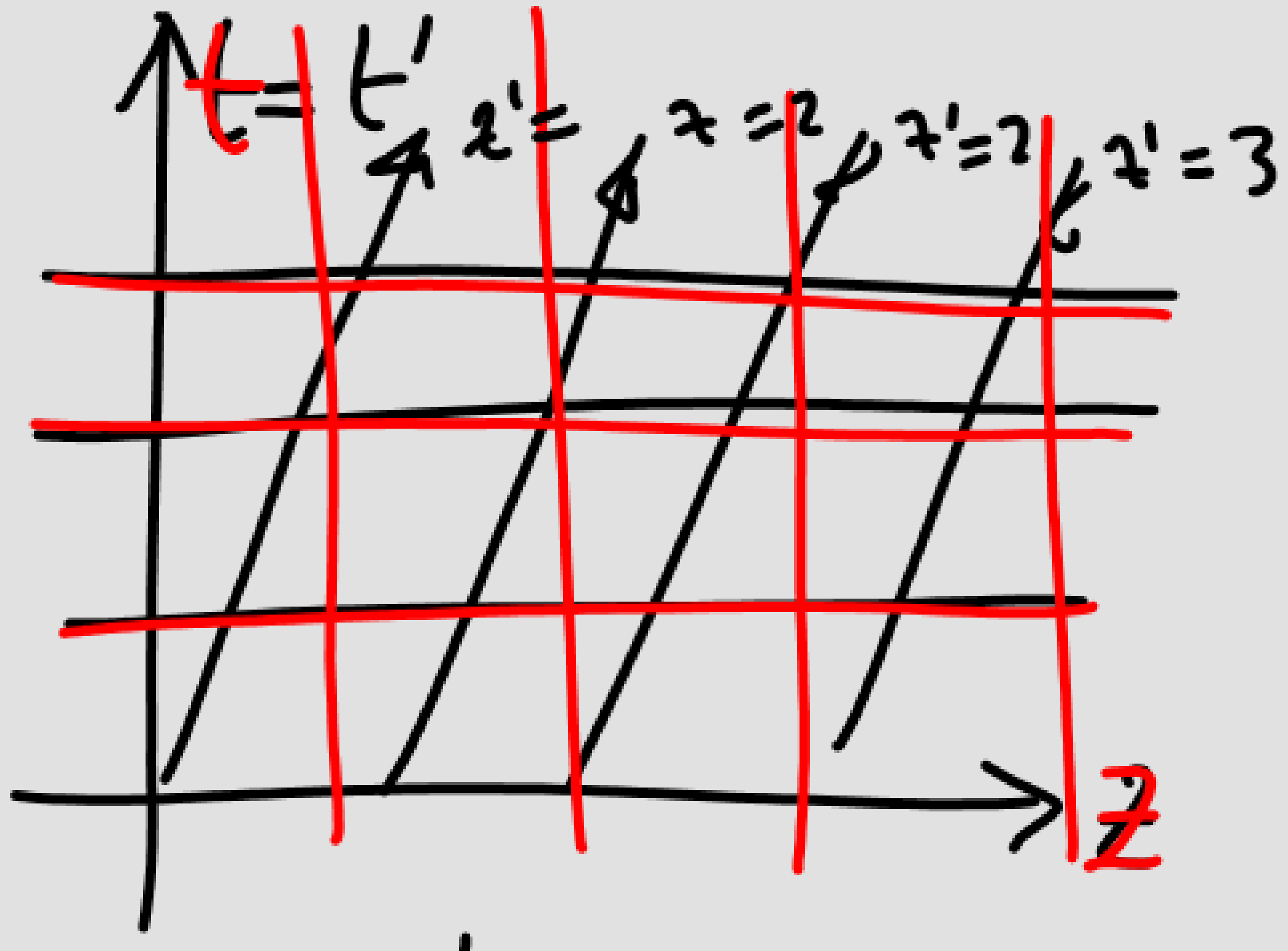
NO SR → $t = t'$

NO $\vec{g}$

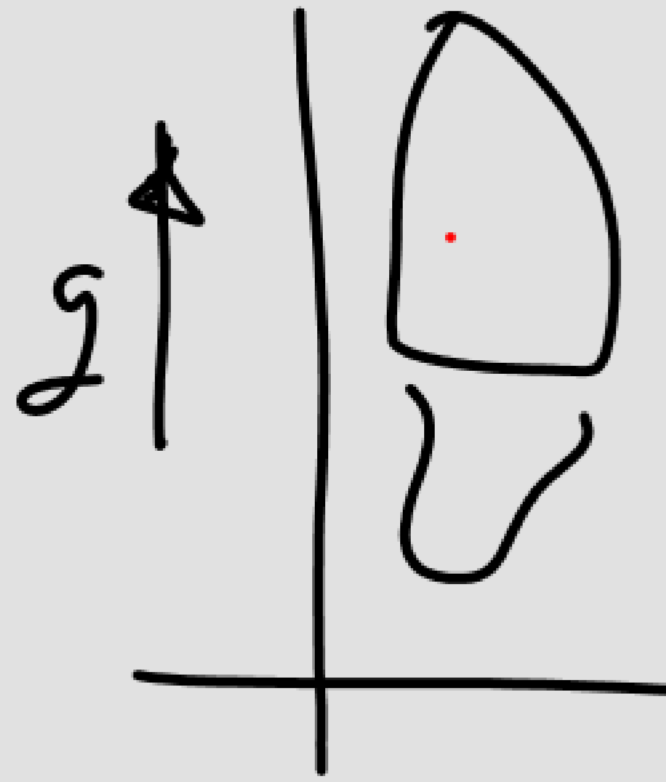
$$\boxed{z = z' + vt}$$

$z' = 1$

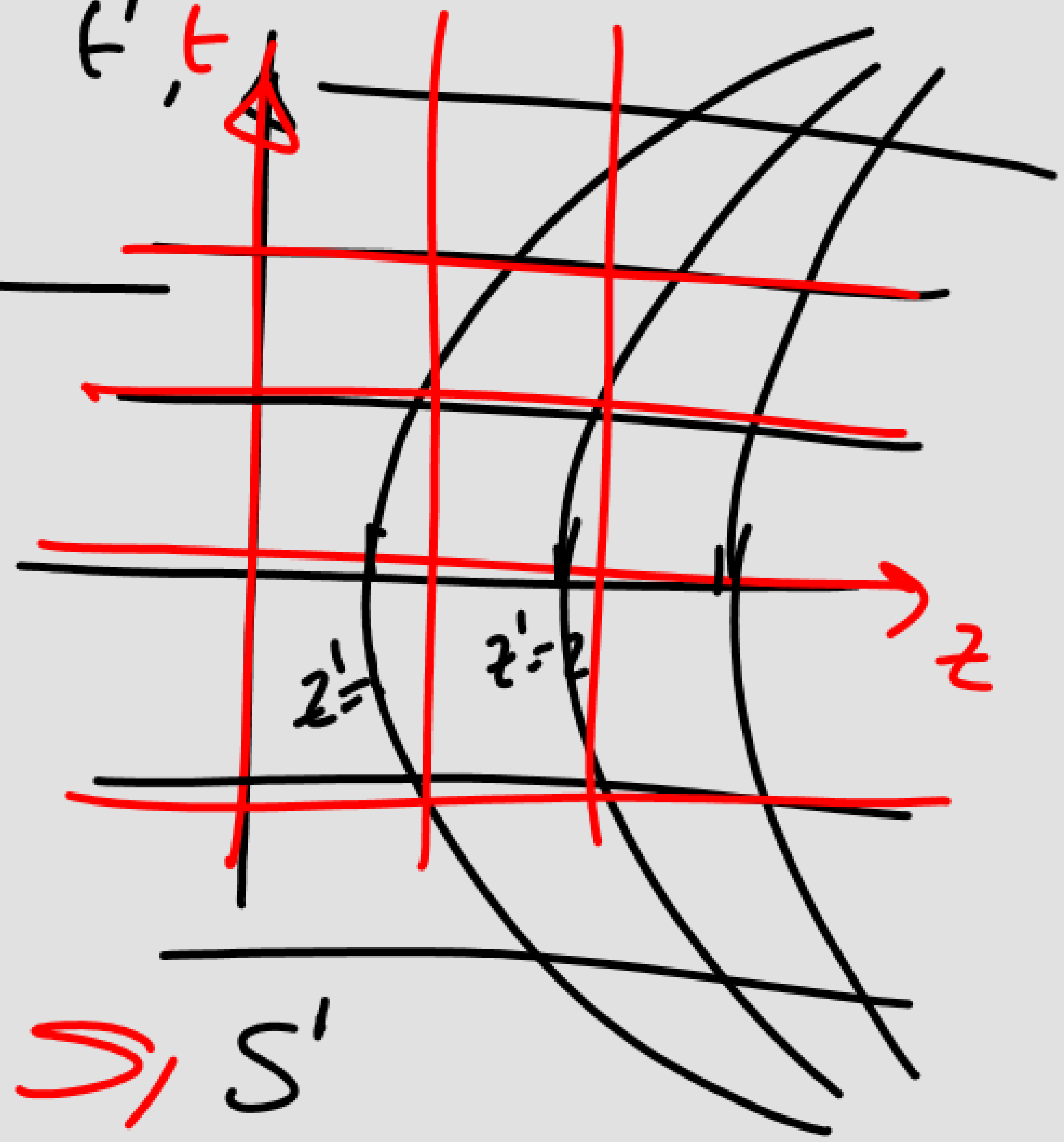
$z' = 0$



S, S'



$$z^* = z' + \frac{1}{2} g t^2$$



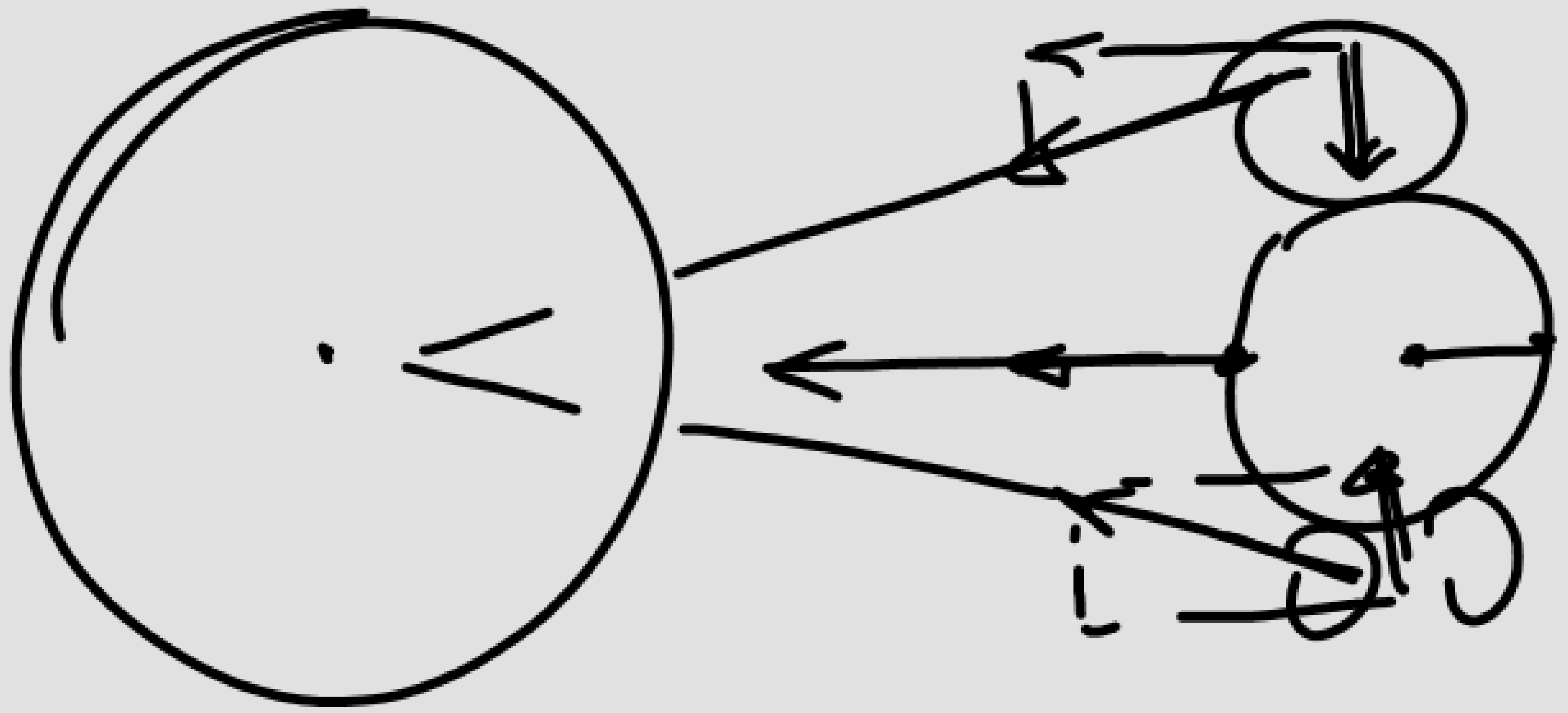
S, S'

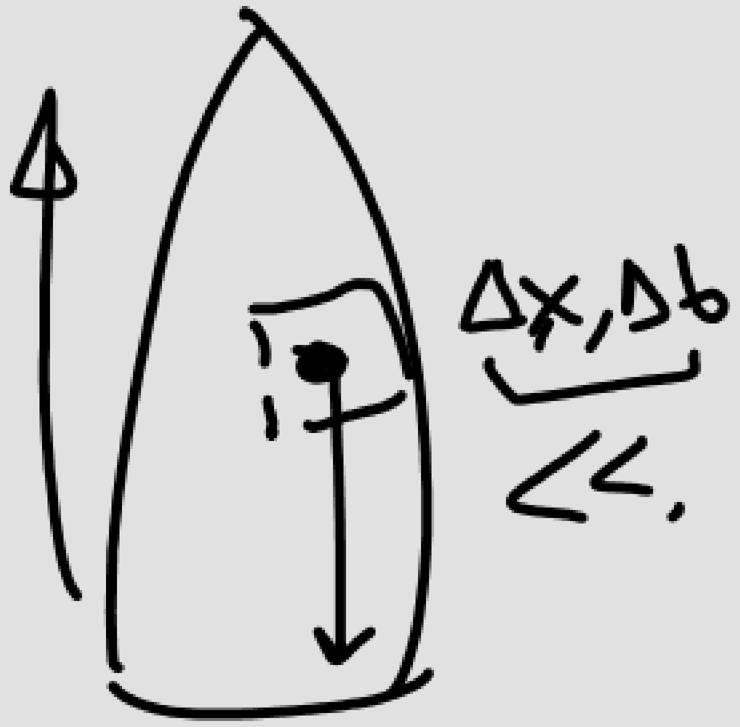
Coordenadas → curvilíneas / planas

↕

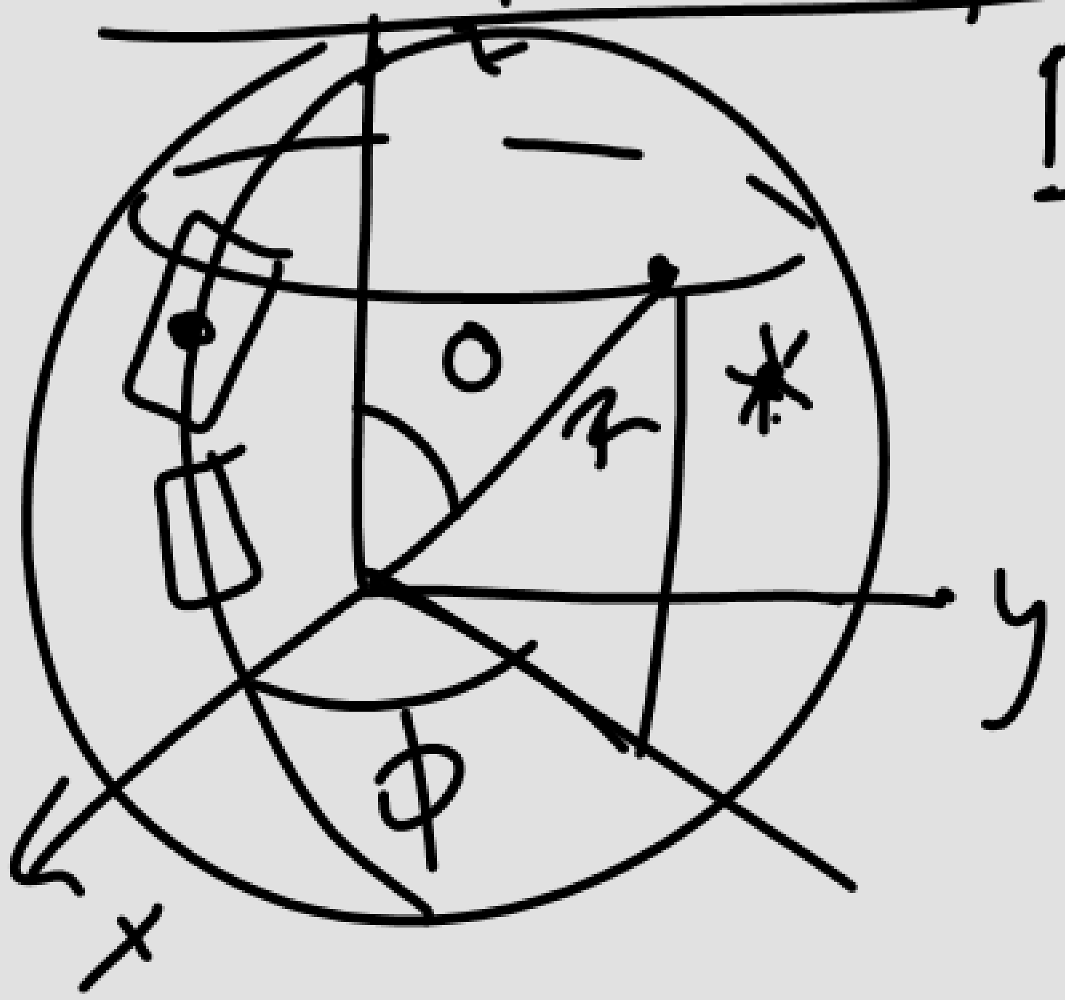
Gravedad → Campo grav / ~~Camp~~ No Inercial

2000 Miles-man





Coordenadas esféricas



$\mathbb{R}^2$

2D

(r, θ, ϕ)

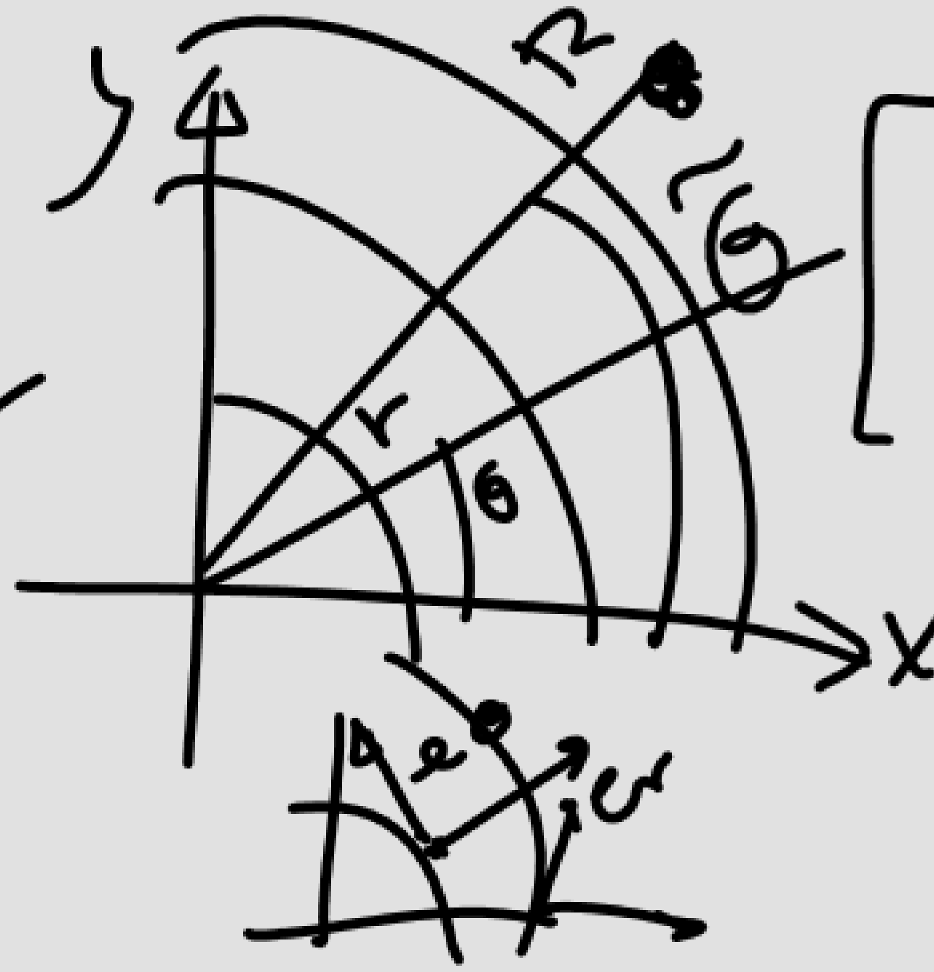
R

(x, y, z)

$$x = R \sin\theta \cos\phi$$
$$y = R \sin\theta \sin\phi$$
$$z = R \cos\theta$$

$(\theta, \phi) \longrightarrow \cancel{(x, y)}$

Coordenadas polares



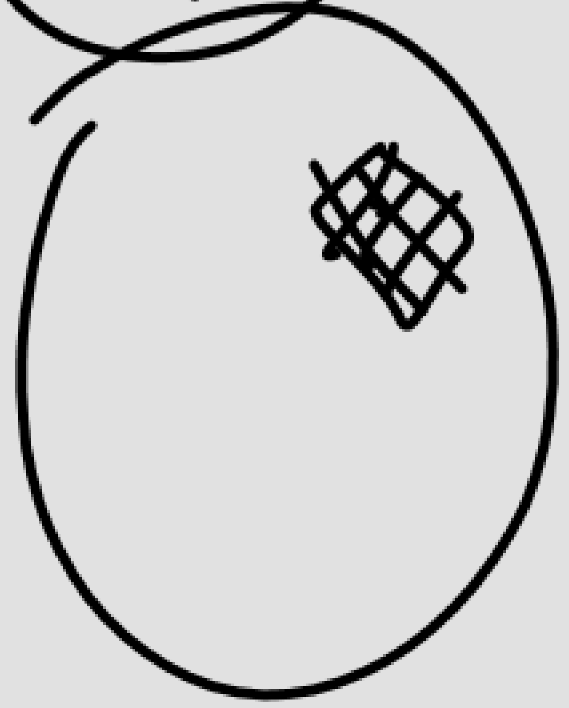
$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$(r, \theta) \longrightarrow (x, y)$

$R^{\alpha}{}_{\beta\gamma\delta} \neq 0$ La gravedad NO es un campo $\vec{F}, \nabla\phi$

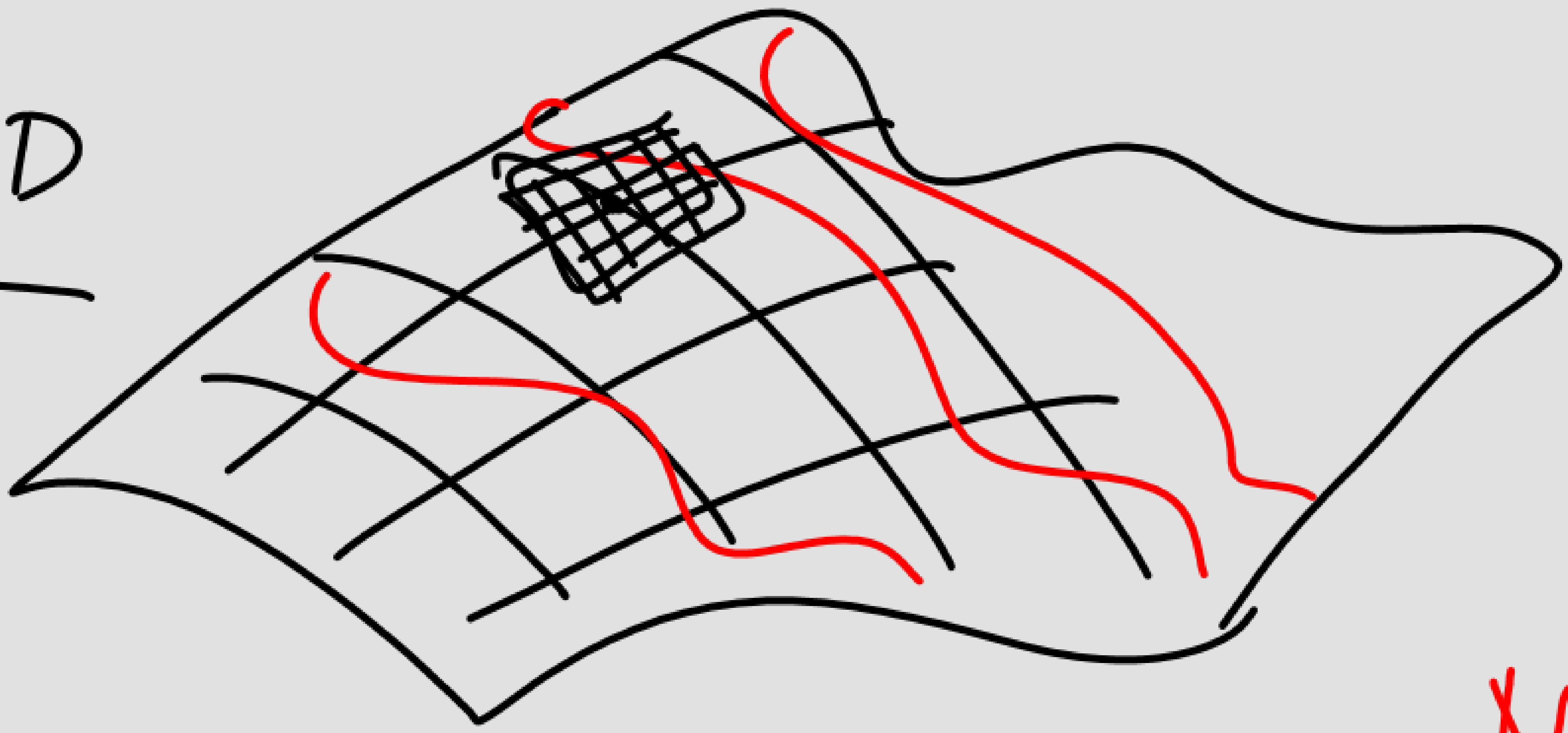
Es la curvatura del espacio-tiempo

- Aceleración
- Curvatura.
- Cambio de coorden.
- Curvilíneas

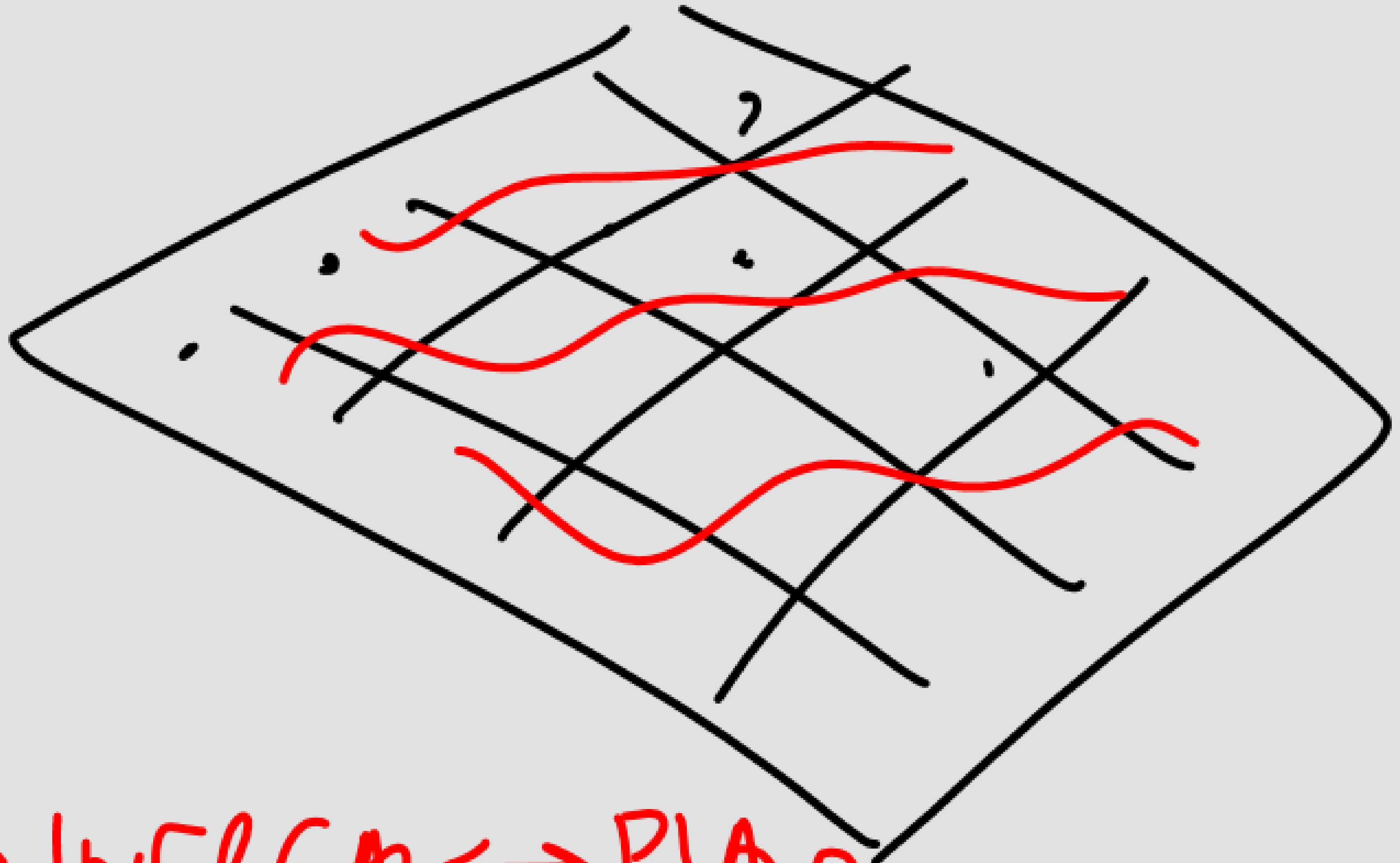


3D, 2D

$g_{\mu\nu}(x)$

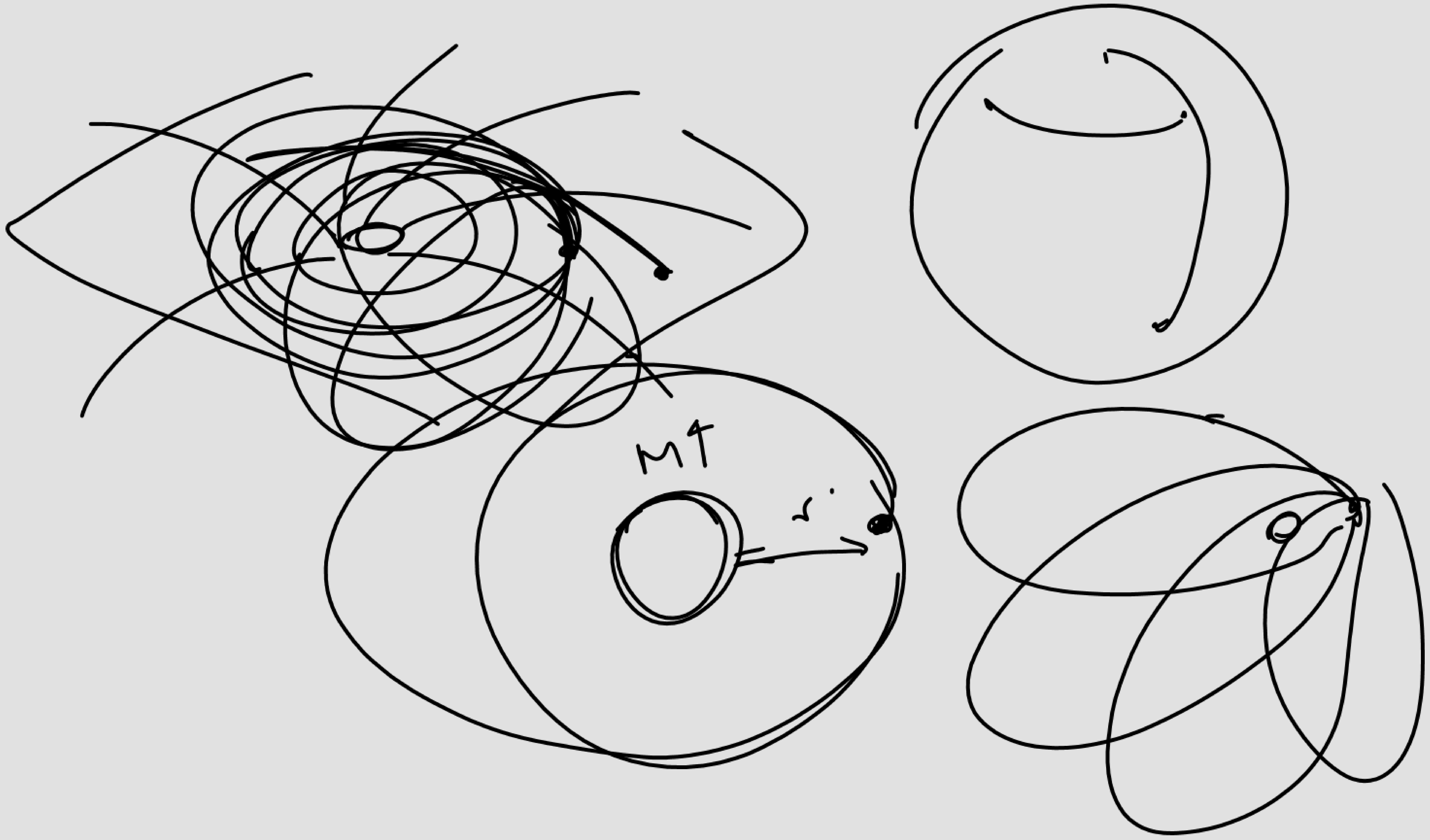


Espacio tiempo (curvado)

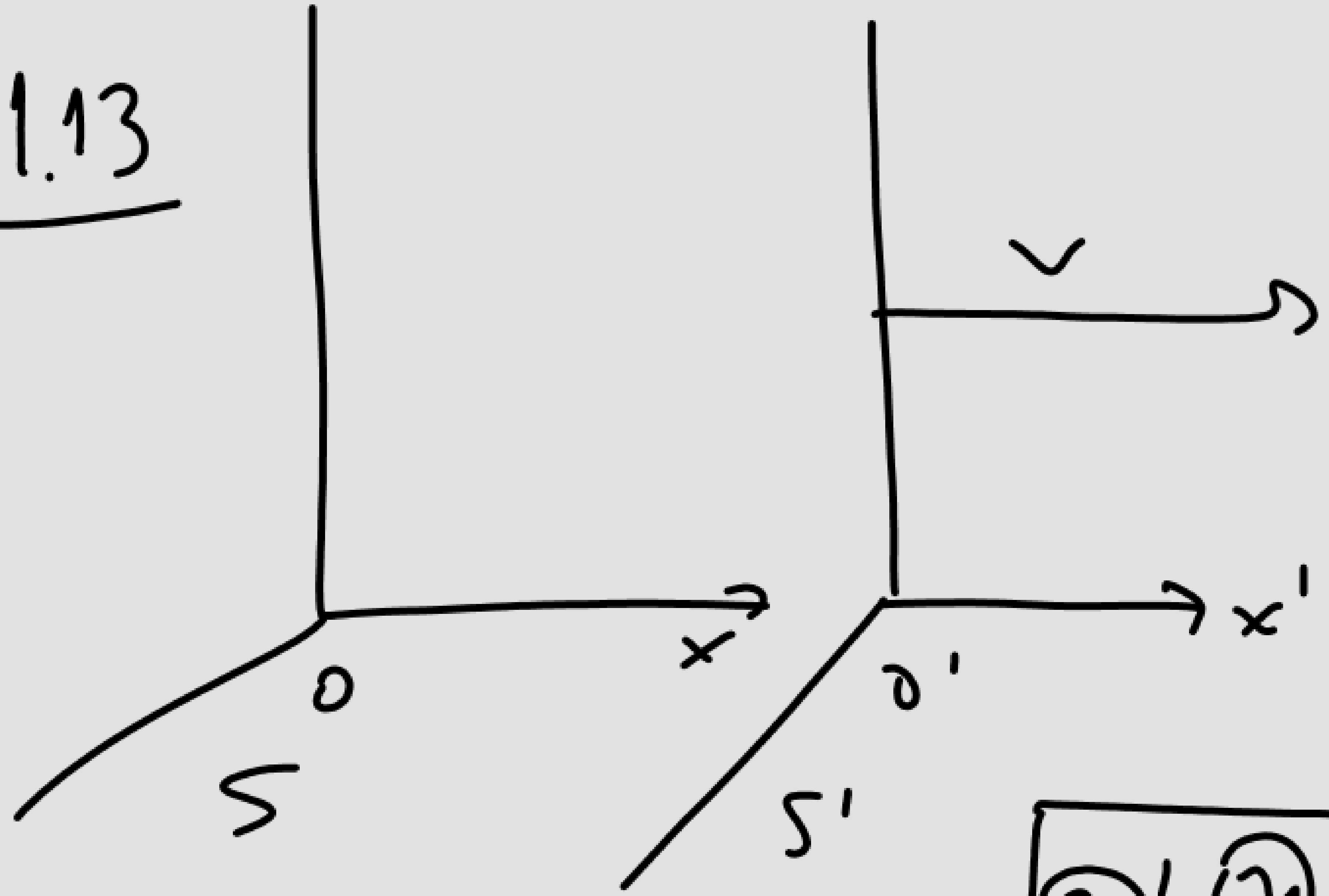


NO INERCIA $\leftrightarrow$ PLANO

GRAVITATORIO $\leftrightarrow$ CURVO.


M4

1.13



$$\begin{cases} x' = \gamma(x - vt) \xrightarrow{d} dx' = \gamma(dx - v\,dt) \\ t' = \gamma(t - vx) \longrightarrow dt' = \gamma(dt - v\,dx) \end{cases}$$

$$\left. \begin{array}{l} u' = \dfrac{dx'}{dt'} \\ u = \dfrac{dx}{dt} \end{array} \right\}$$

$$u' = \frac{dx'}{dt'} = \frac{dx - v\,dt}{dt - v\,dx}$$

$$u' = \frac{\frac{dx}{dt} - v\cancel{\frac{dt}{dt}}^{\,1}}{\cancel{\frac{dt}{dt}}_{\,1} - v\frac{dx}{dt}} = \frac{u - v}{1 - uv}$$

$$\boxed{u' = \frac{u - v}{1 - v^2 u}} \longleftrightarrow \boxed{u' = u - v}$$

Lorentz $\quad v/c \quad v<<c$ $\quad$ Galileo.

$$\boxed{c = 1}$$

$$\boxed{u' = \frac{u - v}{1 - \frac{uv}{c^2}}}$$

1.14

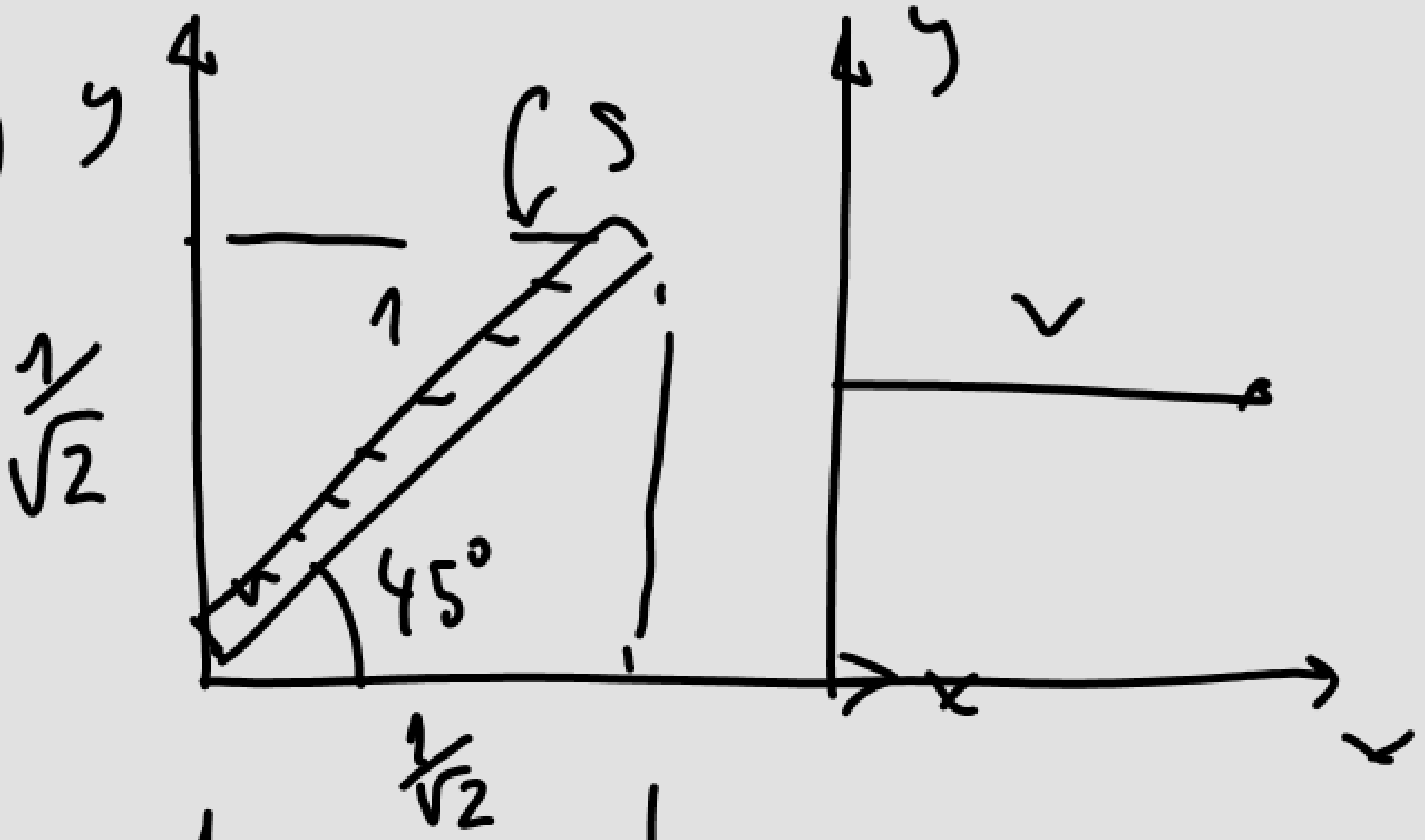
$$u' = \frac{u-v}{1-\frac{uv}{c^2}}$$

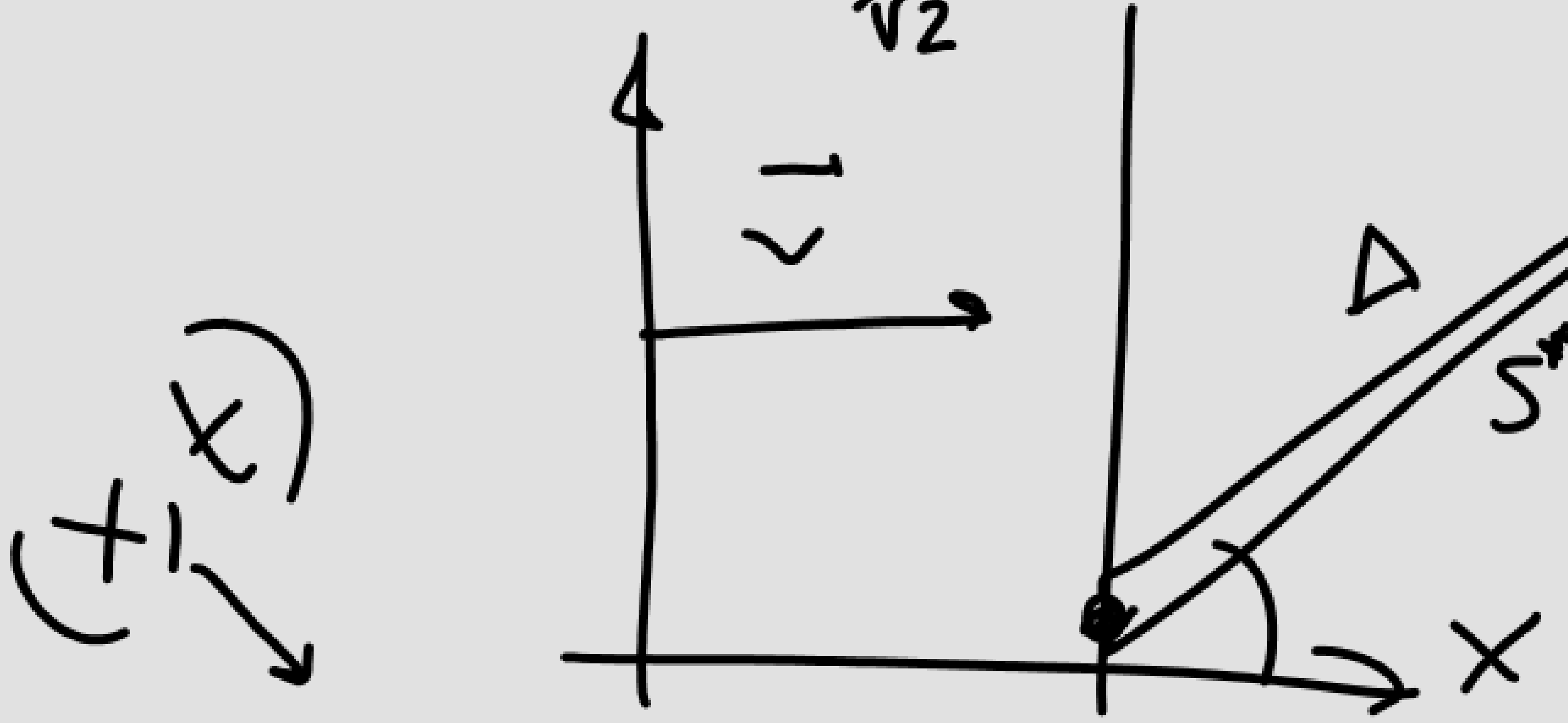
$c=1$

$$u' = \frac{u-v}{1-uv} \Rightarrow \frac{u-v}{1-uv} = \frac{1-v}{1-v} = 1$$

Si $\underline{u=1} \longrightarrow \underline{u'=1}$

1.4





$v = \sqrt{\frac{2}{3}}\,c$

$x = \frac{1}{\sqrt{2}}$

$y = \frac{1}{\sqrt{2}}$

$\Delta x' = \gamma(\Delta x - v\Delta t)$

* $\Delta x = \gamma(\Delta x' + v\cancel{\Delta t'})$ 0

$\Delta t' = 0$

$\Delta x = \gamma\,\Delta x'$

$\Delta y = \Delta y'$

$\Delta y' = \frac{1}{\sqrt{2}}$

$\Delta x' = \frac{\Delta x}{\gamma}$

$\gamma = \frac{1}{\sqrt{1-\frac{2}{3}}} \rightarrow \gamma = \frac{1}{\sqrt{\frac{1}{3}}}$

$\gamma = \sqrt{3}$.

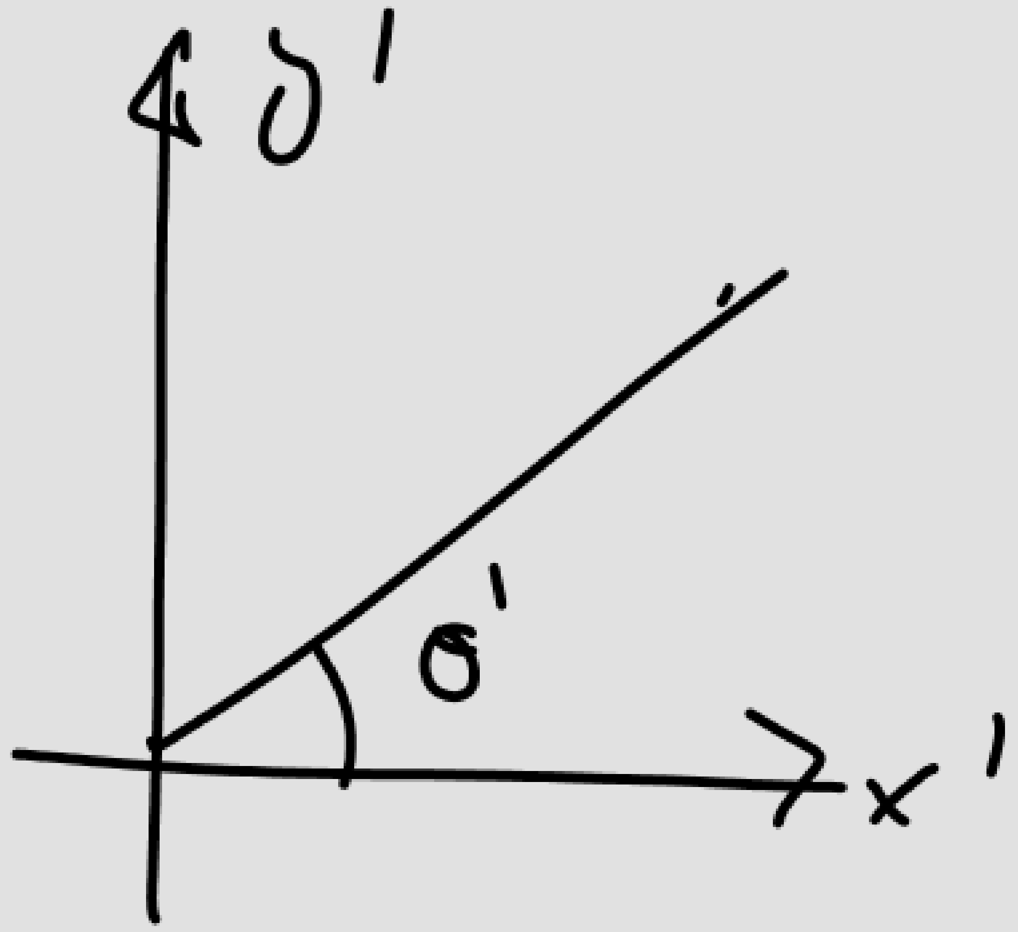
$\Delta x' = \frac{\Delta x}{\sqrt{3}} = \frac{1}{\sqrt{6}}$

$\Delta x = \frac{1}{\sqrt{2}}$

$L'^2 = \Delta y'^2 + \Delta x'^2$

$= \frac{1}{2} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

$\boxed{L' = \sqrt{\frac{2}{3}}\ m}$

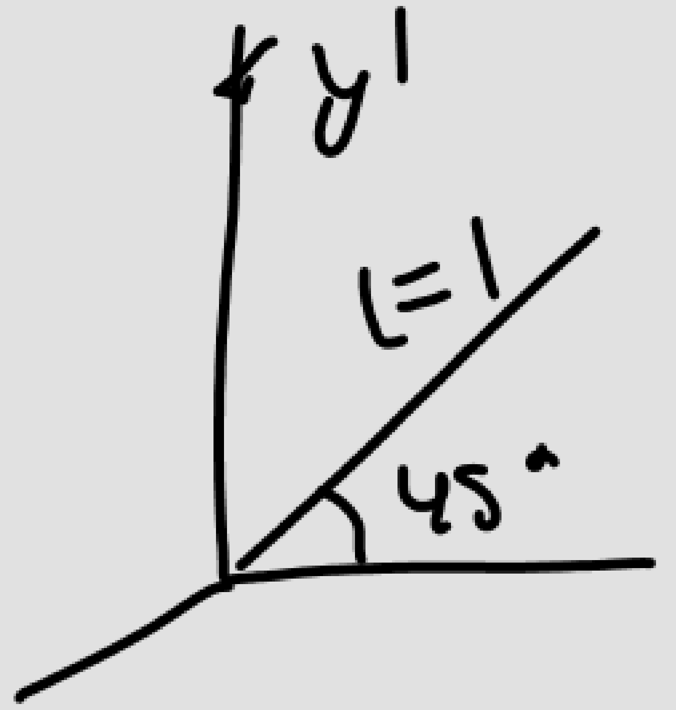


$$\tan\theta' = \frac{\Delta y'}{\Delta x'}$$

$$\theta' = \arctan\frac{\Delta y'}{\Delta x'} = \mathrm{arc}\sqrt{3}$$

$$\underline{\theta'} = \frac{\pi}{3} = 60^\circ$$

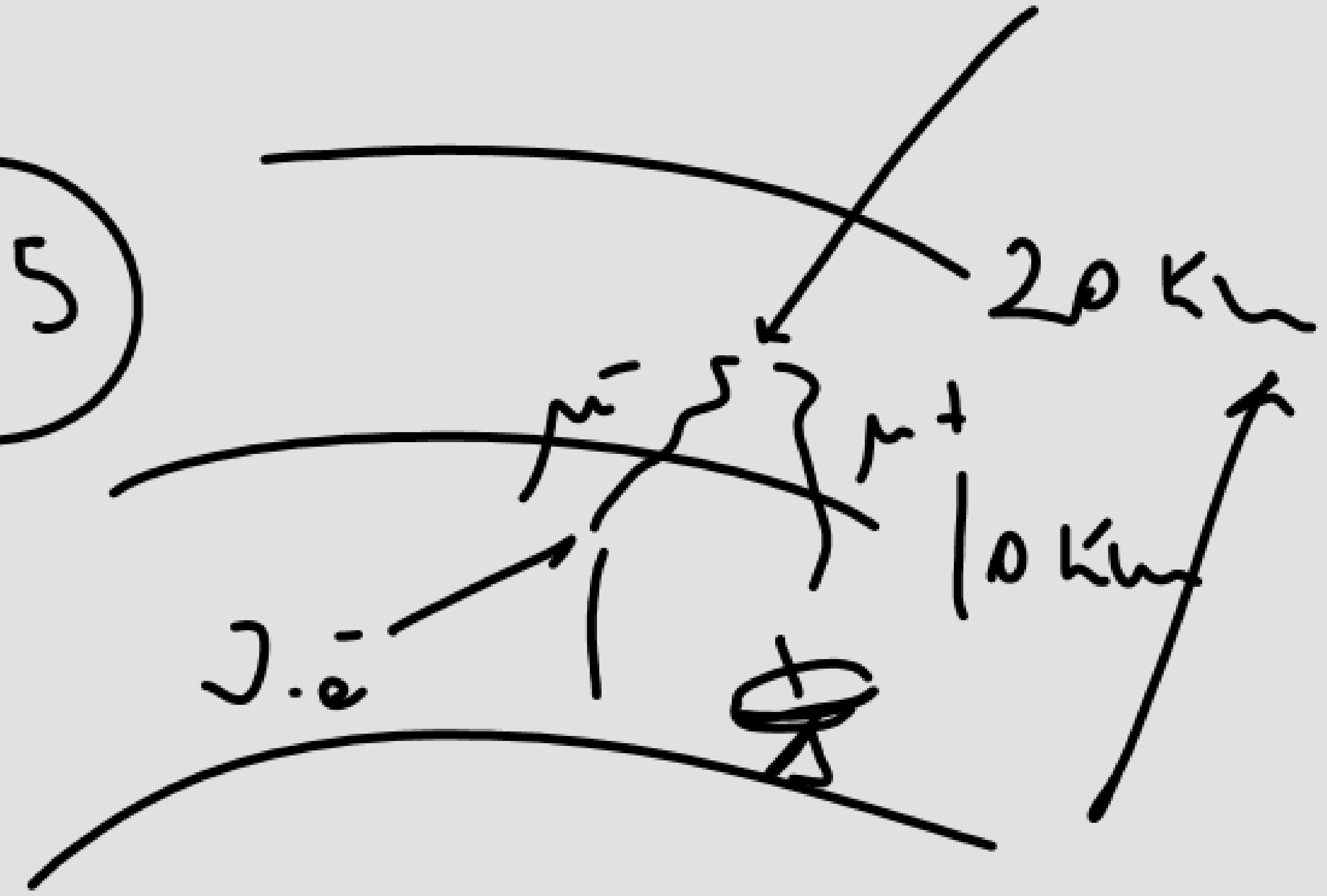
$$L' = \sqrt{\frac{2}{3}}$$



$$y' = y$$

$$y = \frac{1}{\sqrt{2}}$$

T. 5



$t_{\frac{1}{2}} = 2.2\,\mu s$

$V = 0{,}999\,c$

$v = 0.999$

$\Delta t' = \gamma(\Delta t - v\Delta x)$

$\Delta t = \gamma(\Delta t' + \cancel{v\Delta x'}) \rightarrow \circ\ S'$

$\Delta x' = 0$

$$\boxed{\Delta t = \gamma \Delta t'}$$

$\gamma > 1 \quad \Delta t > \Delta t'$

$\Delta t' = 2.2\,\mu s$

$$\Delta t = \frac{1}{\sqrt{1 - v^2}}\, 2.2 \simeq 49.21\,\mu s$$

$v = 0.999$

10^2

$$\Delta x = v\Delta t = 0.999 \cdot 3 \cdot 10^{8}\,\frac{m}{s} \cdot 49.21 \cdot 10^{-6}\,s \sim \cancel{4900.21}$$

~ 1

$= 3 \cdot 10^8\, m/s$

$15.000\,m \sim 15\,Km$

$\cancel{15000}$