Machine Learning Regression Notes (Complete Revision)

These notes cover everything you asked: linear regression, formulas, R^2, adjusted R^2, polynomial regression, pipelines, cross-validation, residuals, plotting, and step-by-step explanations.
Use this as your **future revision notebook**.

📘 1. TRAIN–TEST SPLIT

Code:

```
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.25, random_state=42)
```

Meaning:

- **X** → Input features
- **y** → Target/output values
- **X_train** → Used to train model
- **y_train** → Correct answers for training
- **X_test** → New unseen data for testing
- **y_test** → Real answers to check model accuracy

Why needed:

To test model on **new unseen** data to avoid cheating.

📘 2. SIMPLE LINEAR REGRESSION

Prediction formula (very important):

$\hat{y} = \beta_0 + \beta_1 x$

- **β_0** = intercept
- **β_1** = slope
- **x** = input
- **$\hat{y}$** = predicted output

How LinearRegression() finds β_0 and β_1?

It uses **OLS (Ordinary Least Squares)**:

$\beta = (X^T X)^{-1} X^T y$

This formula finds the **best-fit line** with minimum squared error.

📘 3. PREDICTION

y_pred = regression.predict(X_test)

Uses:

$\hat{y} = \beta_0 + \beta_1 x$

Model checks performance using **y_test** vs **y_pred**.

📘 4. RESIDUALS (ERRORS)

residuals = y_test - y_pred

- **Positive** → model predicted low
- **Negative** → model predicted high
- **Zero** → perfect prediction

Residual = actual − predicted

📘 5. R-SQUARED (R^2)

Formula:

$R^2 = 1 - (SSres / SStot)$

Where:

$SSres = \Sigma (y_i - \hat{y}_i)^2$
$SStot = \Sigma (y_i - \bar{y})^2$

Meaning:

- **1** → perfect
- **0** → model useless (same as mean)
- **< 0** → model worse than mean

📘 6. ADJUSTED R-SQUARED

Formula (copy-friendly):

Adjusted $R^2 = 1 - (1 - R^2) * (n - 1) / (n - p - 1)$

Where:

R^2 = normal R-squared
n = number of samples used for testing
p = number of features

Python equivalent:

```
1 - (1 - score) * (len(y_test) - 1) / (len(y_test) - X_test.shape[1] - 1)
```

📘 7. CROSS-VALIDATION

```
validation_score = cross_val_score(regression, X_train, y_train,
                                   scoring='neg_mean_squared_error', cv=3)
```

Why needed?

- One train-test split may be lucky or unlucky
- CV tests the model **3 times (cv=3)**
- Average of results gives **true performance**

Why negative MSE?

Because sklearn wants **higher = better**.
MSE is **lower = better** → so sklearn flips the sign.

📘 8. STANDARDIZATION (Scaling)

Formula:

z = (x − mean) / std

Important Rule:

```
scaler.fit(X_train)      # learn mean,std from training
scaler.transform(X_test) # apply same scaling to test
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Never fit on test data → avoids data leakage.

📘 9. POLYNOMIAL REGRESSION

PolynomialFeatures(degree=n, include_bias=True)

Transforms:

$X \rightarrow [1, X, X^2, X^3, ...]$

So linear regression becomes:

$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + ... + \beta_n x^n$

include_bias=True → adds 1 column so model can learn β_0.

📘 10. WHY PIPELINE?

```
Pipeline([
   ("poly_features", PolynomialFeatures()),
   ("lin_reg", LinearRegression())
])
```

Pipeline links steps like LEGO:

1. Create polynomial features
2. Train linear regression

Benefits:

- Clean code
- No data leakage
- Works with cross-validation

📘 11. SMOOTH CURVE GENERATION

X_new = np.linspace(-3, 3, 200).reshape(200, 1)

Meaning:

- Make **200 smooth numbers** from −3 to 3
- Used to draw smooth polynomial curve.

📘 12. COMPLETE POLYNOMIAL REGRESSION FUNCTION

Explanation of how a function fits model, predicts curve, plots training/testing points.

- Fit pipeline
- Predict on X_new
- Draw curve
- Draw blue training dots
- Draw green testing dots

📘 13. HOW SYNTHETIC DATA (X, y) WAS CREATED

```
X = 6 * np.random.rand(100,1) - 3

y = 0.5 * X**2 + 1.5 * X + 2 + np.random.randn(100,1)
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Meaning:

- X values between −3 and 3
- y created using equation:

$$y = 0.5x^2 + 1.5x + 2 + \text{noise}$$

- Added noise for realistic data.

📘 14. PLOTTING KEY POINTS

```
plt.scatter(X_train, y_train)  # blue dots
plt.scatter(X_test, y_test)    # green dots
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Shows learning and predictions clearly.

📘 FINAL SUMMARY FOR REVISION

- **Linear Regression** → $\hat{y} = \beta_0 + \beta_1 x$
- **Polynomial Regression** → adds X^2, X^3, …

- **OLS formula** → $\beta = (X^T X)^{-1} X^T y$
- **R^2 measures fit**, Adjusted R^2 penalizes extra features
- **Residuals** → actual − predicted
- **Cross-validation** → more reliable model testing
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- **Pipeline** → keeps process clean and avoids mistakes
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Linear Regression works properly only when these assumptions hold:

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- **Independence:** Data points should be independent.
- **Homoscedasticity:** Residuals should have constant variance.
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- Sensitive to outliers
- Requires assumptions to hold

- Performs poorly on nonlinear data

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Use polynomial regression when:

- Data shows a **curve**
- Simple linear regression underfits
- You want to capture **nonlinear patterns**

Avoid very high degrees (overfitting).

📘 18. UNDERFITTING vs OVERFITTING (IMPORTANT)

- **Underfitting:** Model too simple (degree=1)
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Visualization:

- Low degree → straight line → underfits
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- High degree → zig-zag curve → overfits

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LinearRegression in sklearn uses OLS formula, BUT another way to find β values is **Gradient Descent**.
Gradient Descent updates β values using:

$\theta_{\square} = \theta_{\square} - \alpha * (\partial J/\partial \theta_{\square})$

Used in:

- Deep Learning
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- SGDRegressor, etc.

📘 20. SUPER-SIMPLE CHILD-FRIENDLY EXPLANATIONS (10-year-old level)

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A feature is like a **clue**.
Example: "How many hours you studied" is a clue to predict your marks.

⭐ What is a Target (y)?

The value we want to guess.
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A method that draws **the best straight line** through your data.
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Example:
More study hours → more marks.

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Sometimes data is **curved**, not straight.
Polynomial regression draws a **curvy line**.

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Jump height increases quickly at first, then slows down.

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Residuals = **Mistakes**.

Residual = Actual value − Predicted value

If residual = 0 → Perfect prediction.

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R^2 tells **how good the model is**, from 0 to 1.

- 1 → Perfect model
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It is a **fair version of R^2** when we have many features.
It punishes unnecessary features.

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Instead of checking the model one time, we check it **3–10 times** on different small parts of data.

Like giving your model many mini-tests.

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Make all numbers "fair" so the model doesn't get confused by large values.

z = (value − mean) / std

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A pipeline is like a **machine with steps**.
Example:

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Instead of doing each step separately, the pipeline does it all automatically.

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Model is too smart → memorizes even noise → bad at new data.

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Model learns the real pattern, not too simple, not too complex.

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Random little bumps or mistakes in data.
Example: Measurement errors.

⭐ Simple Memory Tricks

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- **Polynomial Regression:** best curve.
- **Residual:** prediction mistake.
- **R^2:** how good the model is.
- **Pipeline:** automatic process.
- **Standardization:** make numbers fair.
- **Train–Test Split:** study vs exam.
- **Cross-Validation:** many exams.

📘 21. COST FUNCTIONS (LOSS FUNCTIONS) — SIMPLE EXPLANATION

Cost functions tell the model how "bad" its predictions are. The model tries to make the cost as small as possible.

⭐ Why do we use a Cost Function?

Because the model must **measure its mistakes**.

- Without a cost function, the model has **no idea** if it's doing good or bad.
- The model learns by **reducing this cost**.
- Cost function = teacher checking homework mistakes.

We use cost functions to:

- Compare models
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Formula:

$MSE = (1/m) * \Sigma (y_i - \hat{y}_i)^2$

Child-version: "Square the mistakes, add them, and take the average." Squaring makes big mistakes hurt more.

When to use: Standard choice for regression and when you want large errors punished.

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⭐ Huber Loss (Best of both worlds)

Huber loss uses MAE for large errors and MSE for small errors. It is robust yet smooth.

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Imagine the model is a student and cost is the number of pages of homework to fix mistakes.
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Convergence means **the model has learned enough** and **stops improving**.

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Learning rate decides how big steps we take.

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- Big jumps
- May skip the best point
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- **Fast convergence:** cost drops quickly
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Imagine sliding down a slide.

- At the top → high cost
- Sliding down → cost decreasing
- When you reach bottom → convergence
  If you jump too fast (high learning rate), you fall off.
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⭐ Visual: Cost vs Iterations

Cost keeps dropping like:

100 → 50 → 20 → 10 → 5 → 4.5 → 4.4 → 4.39 → 4.389 → (flat)

When numbers stop improving → **converged**.

If you want, I can add a small diagram showing cost decreasing over iterations or show how convergence looks in real code. 😊

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Meaning:

- Make **200 smooth numbers** from −3 to 3
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Explanation of how a function fits model, predicts curve, plots training/testing points.

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Shows learning and predictions clearly.

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Cost functions tell the model how "bad" its predictions are. The model tries to make the cost as small as possible.

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