(5.5) $N_t^{(R)}$ – # of red cars up to time t [115]

$N_t^{(B)}$ – # of blue cars

$N_t^{(G)}$ – # of green cars

$(N_t^{(R)})_t$ – a Poisson process, $\lambda_R = \frac{1}{5}$ (time unit = minutes)

$(N_t^{(B)})_t$ – a Poisson process, $\lambda_B = \frac{1}{10}$ independent

$(N_t^{(G)})_t$ – a Poisson process, $\lambda_G = \frac{1}{30}$

(a) $\mathbb{P}(N_{20}^{(R)} = 2, N_{20}^{(G)} = 1, N_{20}^{(B)} = 0) \overset{\text{ind.}}{=} \mathbb{P}(N_{20}^{(R)}=2) \cdot \mathbb{P}(N_{20}^{(G)}=1) \cdot \mathbb{P}(N_{20}^{(B)}=0)$

(b) $N_t = N_t^{(R)} + N_t^{(B)} + N_t^{(G)}$

$\Downarrow$

$(N_t)_t$ – a Poisson process, $\lambda = \lambda_R + \lambda_B + \lambda_G$

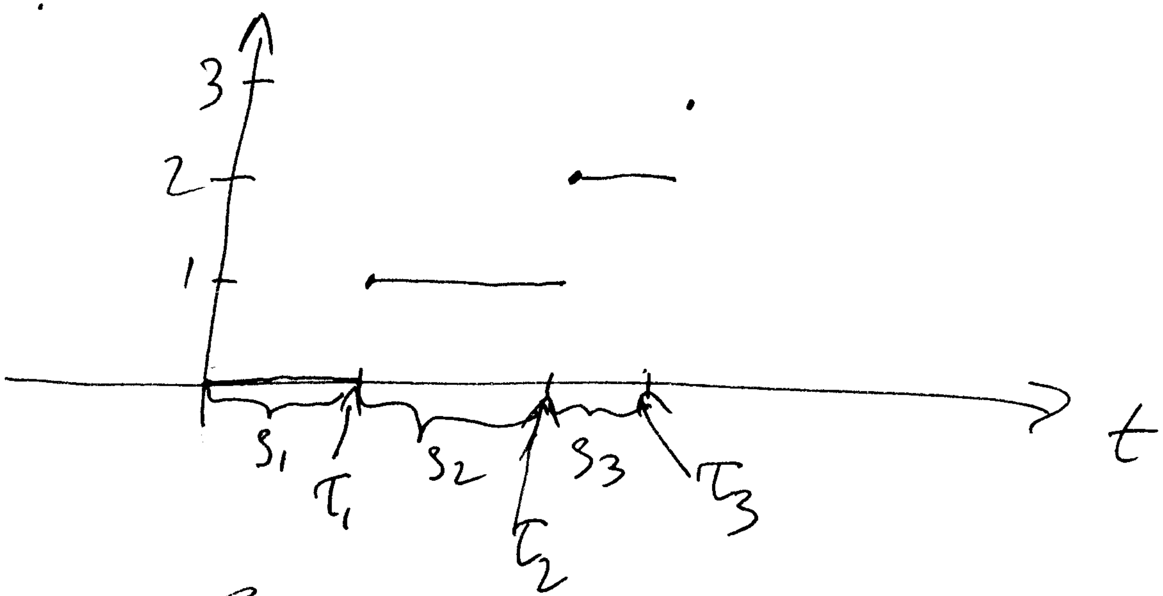
B_t – # of drivers with exact change $p = \frac{1}{4}$

$(B_t)_t$ – a Poisson process with intensity $p \cdot \lambda = p(\lambda_R + \lambda_B + \lambda_G)$

$\mathbb{P}(B_{10} = 0)$

(c) C – time of arrival of the 3rd red car

$\mathbb{E}C = ?$



$s_n \sim Exp(\lambda)$

$\mathbb{E} s_n = \frac{1}{\lambda}$

$\tau_1 = s_1$

$\tau_2 = s_1 + s_2$

$\vdots$

$\tau_n = s_1 + \ldots + s_n$

$\mathbb{E}\,\tau_n = \mathbb{E}(s_1 + \ldots + s_n)$

$= n \cdot \frac{1}{\lambda} = \frac{n}{\lambda}$

$\mathbb{E}C = \frac{3}{\lambda_R} = 3 \cdot 5 = 15$

$\lambda_R = \frac{1}{5}$

(d) D – time of arrival of the 3rd car

$\mathbb{E}D = \frac{3}{\lambda}$

(5.6) $\lambda(t) = t^2, \ t \geq 0$

N_t – # of customerss tht arrived from 9AM up to t

(a) $\mathbb{P}(N_3 = 15) = ?$

N_3 – a poisson distributed random varible with the mean $\mathbb{E} N_3 = \int_0^3 t^2 dt = \frac{1}{3} t^3 \Big|_0^3 = \frac{1}{3} \cdot 3^3 = 3^2 = 9$

$N_3 \sim \text{Poiss}(9)$

$$\underbrace{\mathbb{P}(N_3 = 15) = \frac{9^{15}}{15!} e^{-9}}$$

$$\boxed{\mathbb{P}(X=k) = \frac{\lambda^k}{k!} e^{-\lambda} \quad k = 0, 1, 2, \ldots} \quad \boxed{X \sim \text{Poiss}(\lambda) \quad \mathbb{E}X = \lambda}$$

(b) $\mathbb{P}(N_2 - N_1 = 15)$

$$\underbrace{N_2 - N_1 \sim \text{Poiss}\left(\int_1^2 t^2 dt\right)}$$

probability generating function

X – a random variable taking values in $\{0,1,2,3,\dots\}$

$$\underbrace{G_X(s)} = \mathbb{E}s^X = \sum_{k=0}^{\infty} s^k \cdot \mathbb{P}(X=k) =$$

$$= \underline{\mathbb{P}(X=0) + s\cdot\mathbb{P}(X=1) + s^2\cdot\mathbb{P}(X=2) + \dots}$$

$$G_X(s) = G_Y(s) \quad \forall s \implies \mathbb{P}(X=k) = \mathbb{P}(Y=k) \quad \forall k$$

6.1

X	0	1	2	5
	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{5}{8}$

$$G_X(s) = \mathbb{P}(X=0) + s\cdot\mathbb{P}(X=1) + s^2\cdot\mathbb{P}(X=2) + s^5\cdot\mathbb{P}(X=5)$$

$$= \underbrace{\frac{1}{8} + s\cdot\frac{1}{8} + s^2\cdot\frac{1}{8} + s^5\cdot\frac{5}{8}}$$

$$G_X'(s) = \frac{1}{8} + 2\cdot s\cdot\frac{1}{8} + 5\cdot s^4\cdot\frac{5}{8}$$

$$G_X''(s) = 2\cdot\frac{1}{8} + 5\cdot 4\cdot\frac{5}{8}s^3$$

6.2 $G^{(j)}(0)/j!$

$$G^{(0)} = G$$

$$\frac{G^{(0)}(0)}{0!} = \frac{G(0)}{0!} = \frac{\frac{1}{8}}{1} = \frac{1}{8}$$

$$\frac{G^{(1)}(0)}{1!} = \frac{\frac{1}{8}}{1} = \frac{1}{8}$$

$$\frac{G^{(2)}(0)}{2!} = \frac{2\cdot\frac{1}{8}}{2} = \frac{1}{8}$$

$$\frac{G^{(3)}(0)}{3!}, \quad \frac{G^{(4)}(0)}{4!}, \quad \frac{G^{(5)}(0)}{5!}$$

6.3 [5/5]

pgf $\boxed{G_X(s) = \underbrace{1-p} + ps}$

$\Downarrow$

$\mathbb{P}(X=t) = ?\quad \forall t = 0,1,2,\ldots$

$$\boxed{G_X(s) = \mathbb{P}(X=0) + s\cdot\mathbb{P}(X=1) + s^2\cdot\mathbb{P}(X=2) + \ldots}$$

$\mathbb{P}(X=0) = 1-p$

$\mathbb{P}(X=1) = p$

$\mathbb{P}(X=2) = 0 = \mathbb{P}(X=t) \quad \forall t \geq 2$