EDRP Lab 4

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1 Simulations of branching processes

1. Write a function simulating a given number of generations of a branching process with a given offspring distribution **a** (with a finite number of non-zero values). Your function, when called with parameters n and a, should return a vector of the length $n+1$ of the form

$$(1, z_1, \ldots, z_n),$$

   where z_i is the simulated size of the i-th generation of the branching process with the offspring distribution a.

2. Use your function to run simulations in some

- subcritical,
- critical,
- and supercritical cases.

2 Mean and variance of generation size

1. Fix an offspring distribution

```
a<-c(0.6,0.2,0.2)
```

for subcritical case. Run $nsim = 10000$ simulations of 10 generations $Z_0, \ldots, Z_{10}$ of a branching process with offspring distribution **a**.

> *Hint: You can use* `replicate` *function:*
>
> `simlist<-replicate(nsim, YourFunction(10,a))`
>
> *where* `YourFunction` *stands for the name of the function you wrote when solving Problem 1 to simulate 10 generations of a brancing process with offspring distribution* **a**.

2. Use your simulations to estimate means and variances of Z_1, Z_5 and Z_{10}. Compare your findings with theoretical values.

3. Repeat the same steps for the critical case with

```
a<-c(0.2,0.6,0.2)
```

and supercritical case with

```
a<-c(0.2,0.2,0.6)
```

3 Estimation of extinction probability

Consider a branching process with offspring distribution $\mathbf{a} = (1/4, 1/4, 1/2)$. Run 100 simulations of 20 generations $Z_0, \ldots, Z_{20}$. Use simulations to estimate the extinction probability of the process. Compare the estimate with the theoretical value, obtained by solving a quadratic equation.

4 Numerical computation of extinction probability

1. Consider a branching process with offspring distribution $\mathbf{a} = (1/4, 1/4, 1/2)$. Write a program, which implements the following algorithm (explained in the lectures), solving the equation $s = G(s)$, where G is the pgf of $\mathbf{a}$.

- Initialize with $s_0 \in (0, 1)$.
- Successively compute $s_n := G(s_{n-1})$ for $n \geq 1$. Print the values of s_n to see the speed of convergence.
- Set $s = s_n$ for large n.

5 Finding extinction probabilities in various cases

1. Use simulations to estimate the extinction probabilities of the branching processes specified below. Compare the estimates with the values obtained from the numerical solution of the equation $s = G(s)$ (see the previous problem).

- $a_0 = 0.8$, $a_4 = 0.1$, $a_9 = 0.1$,
- offspring distribution is uniform on $\{0, 1, \ldots, 10\}$,
- $a_0 = 0.6$, $a_3 = 0.2$, $a_6 = 0.1$, $a_{12} = 0.1$.