EDRP Problem Set 3

3.1

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{3}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{3}{4} & 0 & \frac{1}{4} \end{bmatrix}$$

$$\pi = (\pi_1, \pi_2, \pi_3, \pi_4)$$

$$\pi P = \pi$$

$$x = (1, x_2, x_3, x_4) \qquad xP = x$$

$$\begin{cases} \frac{1}{2} x_3 = 1 \implies \boxed{x_3 = 2} \\ \frac{1}{4} x_2 + \frac{3}{4} x_4 = x_2 \implies \boxed{x_2 = x_4} \\ 1 + \frac{1}{2} x_3 = x_3 \implies \boxed{x_3 = 2} \\ \frac{3}{4} x_2 + \frac{1}{4} x_4 = x_4 \implies \boxed{x_2 = x_4} \end{cases} \implies x = (1, x_2, 2, x_2)$$

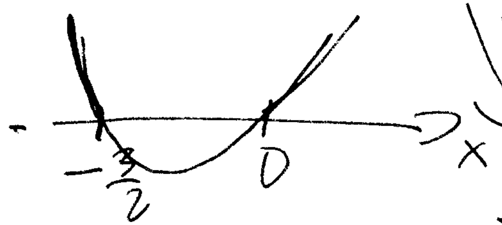
$$1 + x_2 + 2 + x_2 = 3 + 2x_2$$

$$\pi = \left(\frac{1}{3+2x_2}, \frac{x_2}{3+2x_2}, \frac{2}{3+2x_2}, \frac{x_2}{3+2x_2} \right) \qquad 3 + 2x_2 \neq 0$$

$$\begin{cases} \frac{1}{3+2x_2} > 0 \implies \boxed{x_2 > -\frac{3}{2}} \\ \frac{x_2}{3+2x_2} > 0 \iff x_2(3+2x_2) > 0 \end{cases}$$

$$x(3+2x) > 0$$

$$f(x) = x(3+2x)$$



$$\implies \boxed{x_2 < -\frac{3}{2} \ \vee \ x_2 > 0}$$

~~#2~~ Answer

$$\pi = \left(\frac{1}{3+2x_2}, \frac{x_2}{3+2x_2}, \frac{2}{3+2x_2}, \frac{x_2}{3+2x_2} \right)$$

$x_2 > 0$

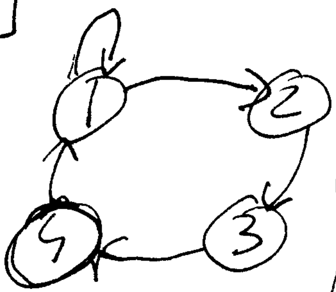
$$\pi P = \pi$$

3.2

$$\begin{bmatrix} 0.4 & 0.6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

P – stochastic matrix

$\forall_n$ P^n – again a stochastic matrix



$1 \to 1$ 1, 2, 3, 4, 5, ...

$1 \to 2$ 1, 2, 3, 4, 5, ...

$1 \to 3$ 2, 3, 4, 5, 6, ...

$1 \to 4$ 3, 4, 5, 6, ...

$2 \to 1$ 3, 4, 5, 6, ...

$2 \to 2$ 4, 5, 6, ...

$2 \to 3$ 1, 4, 5, 6, ...

$2 \to 4$ 2, 6, 7, 8, ...

$3 \to 1$ 2, 3, 4, 5, 6, ...

$3 \to 2$ 3, 4, 5, 6, ...

$3 \to 3$ 4, 5, 6, ...

$3 \to 4$ 1, 5, 6, ...

$4 \to 1$ 1, 2, 3, ...

$4 \to 2$ 2, 3, 4, 5, ...

$4 \to 3$ 3, 4, 5, 6, ...

$4 \to 4$ 4, 5, 6, ...

$$P^n = \left[p_n(i,j) \right]_{i,j}$$

probab. that you can go from i to j in n steps look at the smallest number which appears at every list

$$p_6(i,j) > 0 \quad \forall i,j \implies P^6 > 0$$

all entries of P^6 are strictly positive

P^5

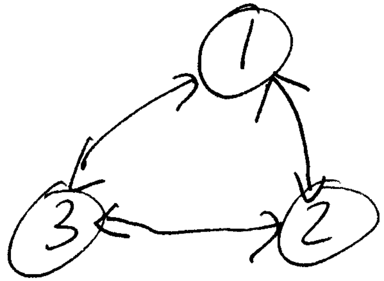
$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

$1 \to 1$ 2, 3, 4, ...

$1 \to 2$ 1, 2, 3, ...

$1 \to 3$ 1, 2, 3, ...

limiting ~~the~~ distribution



regular

$$P^{100} = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \end{bmatrix}$$

symmetry

$$\lim_{n\to\infty} P^n = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

stationary $\pi P = \pi$

$$\pi = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) \implies \text{limiting dist.} \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

uniform distribution