4.4 $(N_t)_{t\geq 0}$ – a Poisson process, $\lambda = 1.5$

$N_t \sim \text{Poisson}(\lambda t)$

$\Updownarrow$

$\mathbb{P}(N_t = k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, \quad k = 0, 1, 2, \ldots$

(a) $\mathbb{P}(N_1 = 2, N_4 = 6)$

$\cancel{\mathbb{P}(N_1 = 2)}$

$\cancel{\mathbb{P}(N_1 = 2, N_4 = 6) \stackrel{?}{=} \mathbb{P}(N_1 = 2) \cdot \mathbb{P}(N_4 = 6)}$

incorrect

$N_1 = 2$

$\cancel{N_4 = 1}$

$\mathbb{P}(N_1 = 2, N_4 = 6) = \mathbb{P}(N_1 = 2, N_4 - N_1 = 6 - 2)$

$= \mathbb{P}(N_1 - N_0 = 2 - 0, N_4 - N_1 = 6 - 2) =$

independent

independ.

$= \mathbb{P}(N_1 - N_0 = 2 - 0) \cdot \mathbb{P}(N_4 - N_1 = 6 - 2) =$

$= \mathbb{P}(N_1 = 2) \cdot \underbrace{\mathbb{P}(N_4 - N_1 = 4)}_{\mathbb{P}(N_{4-1} = 4)} = \mathbb{P}(N_1 = 2) \cdot \mathbb{P}(N_3 = 4)$

stationarity of increments

$= \frac{(1 \cdot 1.5)^2}{2!} e^{-1.5 \cdot 1} \cdot \frac{(3 \cdot 1.5)^4}{4!} e^{-1.5 \cdot 3}$

b) $\mathbb{P}(N_5 \leq 1, N_6 = 2) =$
$= \mathbb{P}(N_5 = 0, N_6 = 2) +$
$\mathbb{P}(N_5 = 1, N_6 = 2)$

~~$N_5 \leq 1$~~
the possible values of N_5 $0, 1, 2, \ldots$

$N_5 \leq 1 \Leftrightarrow N_5 = 0 \vee N_5 = 1$

c) $\mathbb{P}(N_5 \geq 3, N_6 = 2) = 0$
($(N_t)_t$ - is increasing)

$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$

d) $\mathbb{P}(N_4 = 6 \mid N_1 = 2) =$

$= \frac{\mathbb{P}(N_4 = 6, N_1 = 2)}{\mathbb{P}(N_1 = 2)} = \frac{\mathbb{P}(N_1 = 2, N_4 = 6)}{\mathbb{P}(N_1 = 2)}$

e) $\mathbb{P}(N_1 = 2 \mid N_4 = 6) = \frac{\mathbb{P}(N_1 = 2, N_4 = 6)}{\mathbb{P}(N_4 = 6)}$

4.6 N_t - # of calls up to time t
$(N_t)_t$ - a Poisson process, $\lambda = 5$ (time unit = minutes)

a) $\mathbb{P}(N_{t+\cancel{30}\,\frac{1}{2}} - N_t = 0) \overset{?}{=} \mathbb{P}(N_{\frac{1}{2}} = 0)$
stationarity

b) $\mathbb{P}(N_1 = 4, N_2 = \underset{10}{\cancel{6}})$

c) $\mathbb{P}(N_1 = 6, N_5 = 25)$

4.7 N_t — # of customers up to time t (starting at 9)

time unit = hours

$(N_t)_t$ - Poisson process with $\lambda = 3$

(a) $\mathbb{P}(N_{\frac{1}{2}} \geq 2) = 1 - \left(\mathbb{P}(N_{\frac{1}{2}} = 0) + \mathbb{P}(N_{\frac{1}{2}} = 1)\right)$

(b) $\mathbb{P}(N_2 = 8, N_3 = 10)$

(c) $\mathbb{P}(N_{\frac{1}{4}} = 1 \mid N_1 = 6)$