

$$P = \begin{bmatrix} 0.1 & 0.3 & 0.6 \\ 0 & 0.4 & 0.6 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$$

~~1~~ ~~2~~ ~~3~~

stochastic matrix

$\alpha = (0.2, 0.3, 0.5) \leftarrow$ initial distribution

$(X_0), X_1, X_2, \dots$

1, 2, 3 - three states

$$P(X_0 = 1) = 0.2$$

$$P(X_0 = 2) = 0.3$$

$$P(X_0 = 3) = 0.5$$

$X_0 \quad X_1 \quad X_2$

$\textcircled{2} \quad \textcircled{3}$

$$P(X_7 = 3 | X_6 = 2) = 0.6$$

time-homogeneous

$$P(X_1 = 1 | X_0 = 2) = 0$$

$$P(X_1 = 2 | X_0 = 2) = 0.4$$

$$P(X_1 = 3 | X_0 = 2) = 0.6$$

$$\begin{aligned} P(X_{n+1} = j' | X_n = i') &= P(X_1 = j' | X_0 = i') \\ &= P(X_3 = j' | X_2 = i') \\ &= P(X_7 = j' | X_6 = i') \end{aligned}$$

$$P(X_9 = 2 \mid X_1 = 2, X_5 = 1, X_7 = 3)$$

$$= P(X_9 = 2 \mid X_7 = 3) = (P^2)_{3,2}$$

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$$P(X_{15} = 1 \mid X_{10} = 2) = (P^5)_{2,1}$$

$$P^2 = \begin{bmatrix} 0.19 & 0.27 & 0.54 \\ 0.18 & 0.28 & 0.54 \\ 0.18 & 0.27 & 0.55 \end{bmatrix} \Rightarrow$$

$$P(X_9 = 2 \mid X_1 = 2, X_5 = 1, X_7 = 3) \\ = P(X_9 = 2 \mid X_7 = 3) \\ = (P^2)_{3,2} = 0.27$$

$$\begin{aligned} c) P(X_0 = 3 \mid X_1 = 1) &= \\ &= \frac{P(X_0 = 3, X_1 = 1)}{P(X_1 = 1)} = \frac{P(X_1 = 1, X_0 = 3)}{P(X_1 = 1)} \\ &= \frac{P(X_1 = 1 \mid X_0 = 3) \cdot P(X_0 = 3)}{P(X_1 = 1)} = \frac{0.3 \cdot 0.5}{0.17} \end{aligned}$$

$$P(A|B) = \frac{P(A, B)}{P(B)}$$

$$\downarrow \\ P(A \cap B) = P(A, B) = P(A|B) \cdot P(B)$$

$$P(X_1=1|X_0=3)=0.3$$

$$P(X_0=3)=0.5$$

$$P(X_0=1)$$

$$P(X_n=j) = (\alpha P^n)_j$$

$$\alpha P^n =$$

$$(0.2 \ 0.3 \ 0.5) \begin{pmatrix} 0.1 & 0.3 & 0.6 \\ 0 & 0.4 & 0.6 \\ 0.3 & 0.2 & 0.5 \end{pmatrix} = (0.02+0.15, \dots, \dots) \\ = (0.17, \dots, \dots)$$

$$P(X_1=1)=0.17$$

$$EX_2$$

X	1	2	3
	0.1	0.8	0.1

expected
value

$$EX = 1 \cdot 0.1 + 2 \cdot 0.8 + 3 \cdot 0.1$$

mean

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$$P(X_2 = j) = (\alpha p^2)_j.$$

$$(0.2 \quad 0.3 \quad 0.5) \begin{pmatrix} 0.19 & 0.27 & 0.55 \\ 0.18 & 0.28 & 0.55 \\ 0.18 & 0.27 & 0.55 \end{pmatrix} = (p_1 \quad p_2 \quad p_3)$$

X_2	1	2	3
	p_1	p_2	p_3

$$E X_2 = 1 \cdot p_1 + 2 \cdot p_2 + 3 \cdot p_3.$$

$$\begin{aligned} P(X_1 = 3, X_2 = 1) &= \\ &= \underbrace{P(X_2 = 1 \mid X_1 = 3)} \cdot \underbrace{P(X_1 = 3)} \end{aligned}$$

$$P(A \mid B) = \frac{P(A, B)}{P(B)}$$

$$\Downarrow$$

$$P(A, B) = P(A \mid B) \cdot P(B)$$

$$P(X_1=3/X_2=1) = \frac{P(X_1=3, X_2=1)}{P(X_2=1)} = \frac{P(X_2=1/X_1=3) \cdot P(X_1=3)}{P(X_2=1)} \quad [5/5]$$

$$\boxed{P(X_n=j) = (\alpha P^n) j}$$

\downarrow \downarrow
 $P(X_2=1)$ $P(X_1=3)$