

(9.1)

$$Q = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 2 & 2 & -4 \end{bmatrix}$$

[1/3]

Holding time parameters:  $q_1 = 1, q_2 = 2, q_3 = 4$

$$S = \{1, 2, 3\}$$

$$T_1 \sim \text{Exp}(q_1=1) \quad \mathbb{E}T_1 = \frac{1}{q_1} = 1$$

$$T_2 \sim \text{Exp}(q_2=2) \quad \mathbb{E}T_2 = \frac{1}{2}$$

$$T_3 \sim \text{Exp}(q_3=4) \quad \mathbb{E}T_3 = \frac{1}{4}$$

embedded chain

$\tilde{P}$  - the transition matrix of the embedded chain

$$p_{ij} = \frac{q_{ij}}{q_i}, \quad i \neq j$$

$$\tilde{P} = [p_{ij}]_{i,j} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

$$p_{21} = \frac{q_{21}}{q_2} = \frac{1}{2}$$

$$Q = \begin{bmatrix} -q_1 & q_{12} & q_{13} \\ q_{21} & -q_2 & q_{23} \\ q_{31} & q_{32} & -q_3 \end{bmatrix}$$

$$p_{12} = \frac{q_{12}}{q_1} = \frac{1}{1} = 1$$

$$p_{13} = \frac{q_{13}}{q_1} = 0$$

9.2  $S = \{1, 2, 3, 4\}$   $q_{12} = q_{23} = q_{31} = q_{41} = 1$  [2/3]

$$Q = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} -3 & 1 & 0 & 2 \\ 0 & -1 & 1 & 0 \\ 1 & 2 & -5 & 2 \\ 1 & 0 & 2 & -3 \end{bmatrix} \end{matrix}$$

$$q_{14} = q_{32} = q_{34} = q_{43} = 2$$

$$p_{ij} = \frac{q_{ij}}{q_i}, \quad i \neq j$$

$$\tilde{P} = \begin{bmatrix} 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 1 & 0 \\ \frac{1}{5} & \frac{2}{5} & 0 & \frac{2}{5} \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 \end{bmatrix}$$

$$\begin{aligned} q_1 &= 3 \\ q_2 &= 1 \\ q_3 &= 5 \quad q_4 = 3 \end{aligned}$$

10.2  $\boxed{\pi Q = 0}$   
 $(0, 0, 0, 0)$

$$\boxed{\phantom{0000}} = \boxed{\phantom{0000}}$$

$$\begin{aligned} \pi &= (\pi_1, \pi_2, \pi_3, \pi_4) \\ \pi_i &\geq 0 \quad \sum_{i=1}^4 \pi_i = 1 \end{aligned}$$

$$(p_1, p_2, p_3, p_4) \begin{bmatrix} -3 & 1 & 0 & 2 \\ 0 & -1 & 1 & 0 \\ 1 & 2 & -5 & 2 \\ 1 & 0 & 2 & -3 \end{bmatrix} = [0, 0, 0, 0]$$

$$\begin{aligned} p_2 &= \frac{12}{5} p_1 \\ p_3 &= \frac{7}{5} p_1 \\ p_4 &= \frac{8}{5} p_1 \end{aligned}$$

$$\begin{aligned} p_1 + p_2 + p_3 + p_4 &= 1 \\ p_1 + \frac{12}{5} p_1 + \frac{7}{5} p_1 + \frac{8}{5} p_1 &= 1 \\ p_1 &= \frac{5}{39} \end{aligned}$$

$$\pi = \left[ \frac{5}{39}, \frac{19}{39}, \frac{7}{39}, \frac{8}{39} \right]$$

the stationary distrib.

$$q_1 = 3 \quad T_1 \sim \text{Exp}(3) \quad [3/3]$$

$$\mathbb{E}T_1 = \frac{1}{3}.$$

$$\boxed{X \sim \text{Exp}(\lambda) \\ \mathbb{E}X = \frac{1}{\lambda}}$$

the longterm proportion of visits to  
states 2  
is  $\frac{19}{39}$