EDRP: Discrete Random Processes
Problem set 5

5.1 In a city there are three kinds of subway lines: the red, green and orange lines. Subways on each line arrive at a station according to three independent Poisson processes. On average, there is one red train every 10 minutes, one green train every 15 minutes, and one orange train every 20 minutes.

(a) When you arrive at the station what is the probability that the first subway that arrives is for the green line?

(b) How long will you wait, on average, before some train arrives?

5.2 Cars arrive at an intersection according to a Poisson process with parameter 1. Trucks arrive, independently of cars, according to a Poisson process with parameter 2. Time unit is minutes.

(a) What is the probability that a car arrives first?

(b) What is the probability that the first vehicle will arrive in the first five minutes of observation?

(c) How long will you wait, on average, before some vehicle arrives?

(d) How long will you wait, on average, before at least one vehicle of both kinds arrive?

Hint for (d): if X and Y are arbitrary random variables, then $\max(X, Y) = X + Y - \min(X, Y)$.

5.3 Traffic on a road follows a Poisson process with rate 2/3's of a vehicle per minute. 10% of the vehicles are trucks, the other 90% are cars.

(a) What is the probability that at least one truck passes in an hour?

(b) How many trucks on average pass the road in 8 hours?

(c) Find the probability that only cars were passing the road between 2 and 5 p.m.

5.4 Red cars arrive at an intersection according to a Poisson process with parameter r. Blue cars arrive, independently of red cars, according to a Poisson process with parameter b. The time unit is hours.

(a) Find the probability that no car will arrive at the intersection in the first hour of observation.

(b) Two cars arrived in the first hour. Find the probability that they were of different colors.

5.5 Starting at 6 a.m., red, blue and green cars arrive at a highway toll booth according to independent Poisson processes. Red cars arrive about once every 5 minutes. Blue cars arrive about once every 10 minutes. Green cars arrive about once every 30 minutes.

(a) Find the probability that in the first 20 minutes, exactly three cars – two red and one green – arrive at the booth.

(b) At the toll booth, the chance that a driver has exact change is 1/4, independent of car's color. Find the probability that no car has exact change in the first 10 minutes.

(c) What is the average time of arrival of the third red car?

(d) What is the average time of arrival of the third car (regardless the color)?

5.6 Starting at 9 a.m., customers arrive at a store according to a nonhomogeneous Poisson process with intensity function $\lambda(t) = t^2$, for $t > 0$, where the time unit is hours.

(a) What is the probability that 15 customers will enter the store by noon?

(b) What is the probability that 15 customers will enter the store between 10 a.m. and 11 a.m. ?

Answers

5.1 (a) 4/13

(b) 60/13

5.2 (a) 1/3

(b) $1 - \exp(-15)$

(c) 1/3

(d) 7/6

5.3 (a) $1 - \exp(-4)$

(b) 32

(c) $\exp(-12) - \exp(-120)$

5.4 (a) $\exp(-(r + b))$

(b) $\frac{2rb}{(r+b)^2}$

5.5 (a) $\frac{16}{3}\exp(-20/3)$

(b) $\exp(-5/6)$

(c) 15

(d) 9

5.6 (a) $\frac{9^{15}}{15!}\exp(-9)$

(b) $\frac{(7/3)^{15}}{15!}\exp(-7/3)$