EDRP Lab 5

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1 Problem - Various computations for branching processes

Consider a branching process with the offspring distribution $\mathbf{a} = (1/4, 3/4, 0, 0, \ldots)$.

1. Use simulations to estimate the probability that the process goes extinct by the third generation. Compare your results with the theoretical values.
2. Use simulations to estimate the average time of extinction. Compare your results with the theoretical values.
3. Simulate the total progeny for the branching process with the offspring distribution $\mathbf{a} = (3/4, 1/4, 0, 0, \ldots)$. Estimate the mean of the total progeny distribution. Compare the estimate with the theoretical value.

2 Problem - The mean and variance of the total progeny distribution

Simulate the total progeny for a branching process whose offspring distribution is Poisson with parameter $\lambda = 0.6$. Estimate the mean and variance of the total progeny distribution. Do you know the exact value of the mean? Can you explicitly find the total progeny distribution?

3 Problem - A branching process with immigration

1. Simulate the branching process with immigration, with the offspring distribution

$$\mathbf{a} = (1/4, 3/4, 0, \ldots), \ ,$$

and the immigration distribution is Poisson with parameter $\lambda = 1.2$.

2. In one of the problems from the course's Problem Sets, you were asked to guess the limiting distribution of the n-th generation size (with n going to infinity). Illustrate the limit result with $n = 100$. How can you verify it via simulations?

4 Problem - Spreading of an infectious disease

An infectious disease is spreading in the following way. At time 0, one person is infected. At each discrete unit of time, every person who has got infected "decides' ' how many people to infect according to the following mechanism. Each person flips a fair coin. If heads, they infect no one. If tails, they proceed to roll a fair die until 5 appears. The number of rolls needed determines how many people they will infect.

Answer the following questions by means of simulations:

1. After three generations, how many people, on average, have got infected?
2. Find the probability that the infection-spreading process will stop after three generations.
3. Find the probability that the infection-spreading process will eventually stop.