EDRP: Discrete Random Processes
Problem set 2

2.1 Consider the transition matrix

$$\mathbf{P} = \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix}$$

for a general two-state chain ($0 \le p, q \le 1$).

(a) Find the limiting distribution (if it exists) if $p + q = 1$.

(b) Find the limiting distribution (if it exists) if $p + q \neq 1$.

Hint for (b): use mathematical induction to prove that

$$\mathbf{P}^n = \frac{1}{p+q}\begin{bmatrix} q & p \\ q & p \end{bmatrix} + \frac{(1-p-q)^n}{p+q}\begin{bmatrix} p & -p \\ -q & q \end{bmatrix}.$$

2.2 Consider a Markov chain with transition matrix

$$\mathbf{P} = \begin{bmatrix} 1/2 & 1/4 & 0 & 1/4 \\ 0 & 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 & 0 \\ 0 & 1/4 & 1/2 & 1/4 \end{bmatrix}.$$

Find the stationary distribution.

2.3 Consider a Markov chain with the transition matrix

$$\mathbf{P} = \begin{bmatrix} 1-a & a & 0 \\ 0 & 1-b & b \\ c & 0 & 1-c \end{bmatrix},$$

where $0 < a, b, c < 1$. Find the stationary distribution.

2.4 A Markov chain has transition matrix

$$\mathbf{P} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1/4 & 0 & 3/4 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 3/4 & 0 & 1/4 \end{bmatrix}.$$

Find the set of all stationary distributions.

2.5 Determine which of the following matrices are regular:

$$\begin{bmatrix} 0.4 & 0.6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ p & 1-p \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 & 0 \\ 0.25 & 0.5 & 0.25 \\ 1 & 0 & 0 \end{bmatrix}.$$

Hint: you don't have to compute any powers of the matrices. Think about the possibilities of reaching states from other states.

2.6 Assume that a Markov chain has transition matrix

$$\mathbf{P} = \begin{bmatrix} 0 & 1-p & p \\ p & 0 & 1-p \\ 1-p & p & 0 \end{bmatrix}$$

for $0 < p < 1$. Find the limiting distribution.