EDRR Tutorials

$X_0, X_1, X_2, \dots$ – random variables taking values in state space S

$(X_n)_{n=0,1,2,\dots}$

<u>Markov chain</u> $x_0, x_1, \dots, x_{n+1} \in S$

$$\mathbb{P}(X_{n+1} = x_{n+1} \mid X_0 = x_0, X_1 = x_1, \dots, X_n = x_n)$$
$$= \mathbb{P}(X_{n+1} = x_{n+1} \mid X_n = x_n)$$ <u>Markov property</u>

$$X_n = \begin{cases} 1, & \text{with prob. } \frac{1}{2} \\ -1, & \text{with prob. } \frac{1}{2} \end{cases}$$

X_n = "the result of the n-th toss of a symmetric coin"

$(X_n)_n$ – is it a Markov chain?

"H" $\longrightarrow$ 1

"T" $\longrightarrow$ -1

$$(*)\ \mathbb{P}(X_4 = 1 \mid X_0 = 1, X_1 = -1, X_2 = -1, X_3 = 1) = \mathbb{P}(X_4 = 1) = \frac{1}{2}$$

the tosses are independent

A, B – two random events

A, B – <u>independent</u> $\qquad \mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$

conditional probab.

$\Rightarrow$ if A, B are independent

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(A) \cdot \cancel{\mathbb{P}(B)}}{\cancel{\mathbb{P}(B)}}$$

$\Downarrow$

$$\mathbb{P}(A|B) = \mathbb{P}(A)$$

$$(**) \quad \mathbb{P}(X_4 = 1 \mid X_3 = 1) = \mathbb{P}(X_4 = 1) = \frac{1}{2}$$

$$\mathbb{P}(X_4 = 1 \mid X_0 = 1, X_1 = -1, X_2 = -1, X_3 = 1) = \mathbb{P}(X_4 = 1 \mid X_3 = 1)$$

$(X_n)_n$ – a Markov chain

$S = \{-1, 1\}$

$$\mathbb{P}(X_{n+1} = 1 \mid X_n = 1) = \mathbb{P}(X_{n+1} = 1) = \frac{1}{2}$$

$$\mathbb{P}(X_{n+1} = -1 \mid X_n = 1) = \frac{1}{2}$$

$$\mathbb{P}(X_{n+1} = 1 \mid X_n = -1) = \frac{1}{2}$$

$$\mathbb{P}(X_{n+1} = -1 \mid X_n = -1) = \frac{1}{2}$$

transition matrix

$$\mathbf{P} = \begin{array}{c|c} & \begin{array}{cc} -1 & 1 \end{array} \\ \begin{array}{c} -1 \\ 1 \end{array} & \begin{bmatrix} \mathbb{P}(X_{n+1}=-1 \mid X_n=-1) & \mathbb{P}(X_{n+1}=1 \mid X_n=-1) \\ \mathbb{P}(X_{n+1}=-1 \mid X_n=1) & \mathbb{P}(X_{n+1}=1 \mid X_n=1) \end{bmatrix} \end{array} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

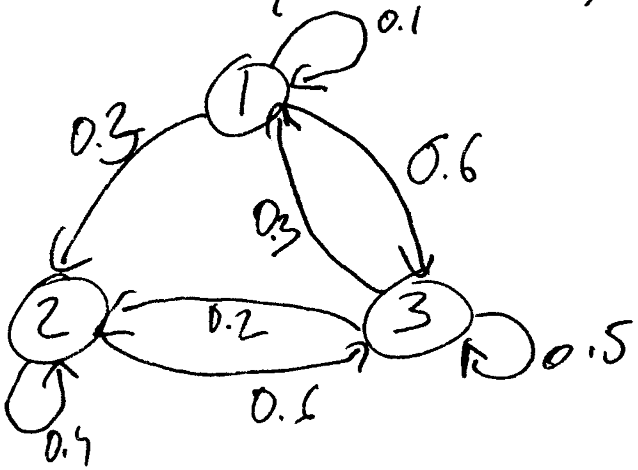
stochastic matrix

1.3

$$P = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.1 & 0.3 & 0.6 \\ 0 & 0.4 & 0.6 \\ 0.3 & 0.2 & 0.5 \end{bmatrix} \end{array}$$

$S = \{1, 2, 3\}$

$0.1 = \mathbb{P}(X_{n+1} = \boxed{1} \mid X_n = \boxed{1})$



i-th row

j-th column

$\mathbb{P}(X_{n+1} = j \mid X_n = i)$

X_0 - initial distribution

$\alpha = (0.2, 0.3, 0.5)$

$0.2 = \mathbb{P}(X_0 = 1)$
$0.3 = \mathbb{P}(X_0 = 2)$
$0.5 = \mathbb{P}(X_0 = 3)$

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$X_0 \quad X_1 \quad X_2 \quad \ldots$

$2 \quad 3 \quad 1$