EDRP: Discrete Random Processes
Problem set 10

10.1 A three-state Markov chain has distinct holding time parameters q_a, q_b, and q_c. From each state, the process is equally likely to transition to the other two states. Find the stationary distribution.

10.2 A Markov chain on $\{1, 2, 3, 4\}$ has nonzero transition rates

$$q_{12} = q_{23} = q_{31} = q_{41} = 1,\ q_{14} = q_{32} = q_{34} = q_{43} = 2.$$

(a) If the chain is at state 1, how long on average will it take before moving to a new state?

(b) Over the long term, what proportion of visits will be to state 2?

10.3 A machine is subject to failures of types $i = 1, 2, 3$, at rates $\lambda_1 = 1/2$, $\lambda_2 = 1/3$, and $\lambda_3 = 1/4$. A failure of type i takes an exponential amount of time to repair, with rate $\mu_1 = 1/4$, $\mu_2 = 1/3$, and $\mu_3 = 1/2$. Let X_t denote the type of failure the machine is subject to at time t. Then $(X_t)_t$ is a continuous-time Markov chain with state space $\{0, 1, 2, 3\}$ (with 0 denoting no failure of any type). Assume that only one type of failure can occur at a time, and that after repair, the machine goes to state 0 before entering any other state. Find the limiting fraction of time the machine is subject to each type of failure.

10.4 A businessman flies back and forth among the cities A, B and C as follows: he stays in each city for an exponential amount of time with mean 1/4 month if the city is A or B, but with mean 1/5 month if the city is C. From A he goes to B or C with probability 1/2 each; from B he goes to A with probability 3/4 and to C with probability 1/4; from C he always flies to A.

(a) Find the limiting fraction of time that the businessman spends in each city.

(b) What is the average number of trips per year from B to A?

Answers

10.1
$$\pi = \left[\frac{q_b q_c}{q_a q_b + q_a q_c + q_b q_c} \quad \frac{q_a q_c}{q_a q_b + q_a q_c + q_b q_c} \quad \frac{q_a q_b}{q_a q_b + q_a q_c + q_b q_c}\right]$$

10.2 (a) 1/3

(b) $19/39 = 0.487179$

10.3 no failure: 2/9, failures of types $1, 2, 3$: 4/9, 2/9, 1/9

10.4 (a) A - 1/2, B - 1/4, C - 1/4

(b) 9