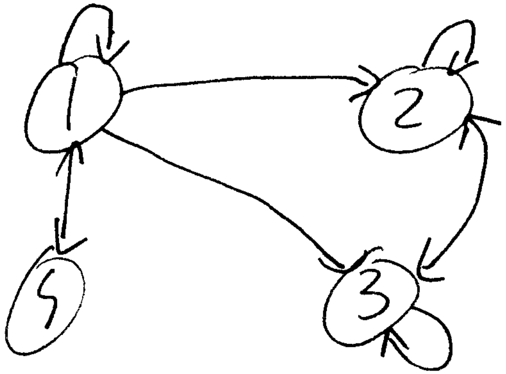
3.4



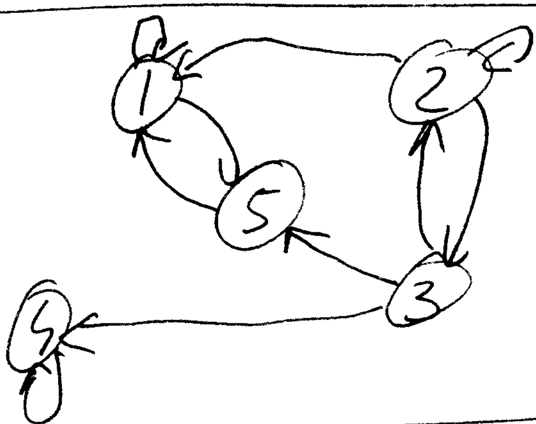
$j \in S$ is accessible from $i \in S$ $\iff \exists n \geq 0: (P^n)_{i,j} > 0$

i and j communicate if i is accessible from j and j is accessible from i

$\{1, 4\}$

$\{2, 3\}$

3.5



$\{1, 5\}, \{2, 3\}, \{4\}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3.6

$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{bmatrix}$ $\quad P^n = ?$

$P \cdot P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \\ 0 & 1 \end{bmatrix}$

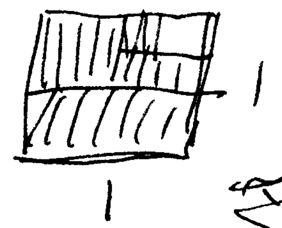
$P^n = \begin{bmatrix} \frac{1}{2^n} & 1 - \frac{1}{2^n} \\ 0 & 1 \end{bmatrix}$

$\sum_{n=0}^{\infty} P^n = \sum_{n=0}^{\infty} \begin{bmatrix} \frac{1}{2^n} & 1 - \frac{1}{2^n} \\ 0 & 1 \end{bmatrix} =$

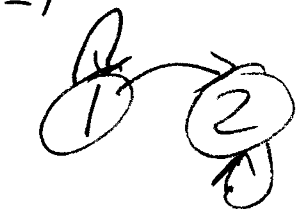
$= \begin{bmatrix} \sum_n \frac{1}{2^n} & \sum (1 - \frac{1}{2^n}) \\ \sum 0 & \sum 1 \end{bmatrix} = [$

$$= \begin{bmatrix} 2 & +\infty \\ 0 & +\infty \end{bmatrix} = \sum_{n=0}^{\infty} P^n$$

$$\sum_{n=0}^{\infty} \frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 2$$



$$\sum_{n=1}^{\infty} 1$$



$j \in S$ <u>recurrent</u> = the chain started in j eventually revisits j

<u>transient</u> – there is a positive prob. that the chain started in j never returns to j

<u>$j \in S$ is recurrent</u> $\iff \sum_{n=0}^{\infty} (P^n)_{j,j} = \infty$

<u>transient</u> $\iff \sum_{n=0}^{\infty} (P^n)_{j,j} < \infty$

<u>state</u> 1 $\quad \sum_{n=0}^{\infty} (P^n)_{1,1} = 2 < \infty \implies$ 1 is transient

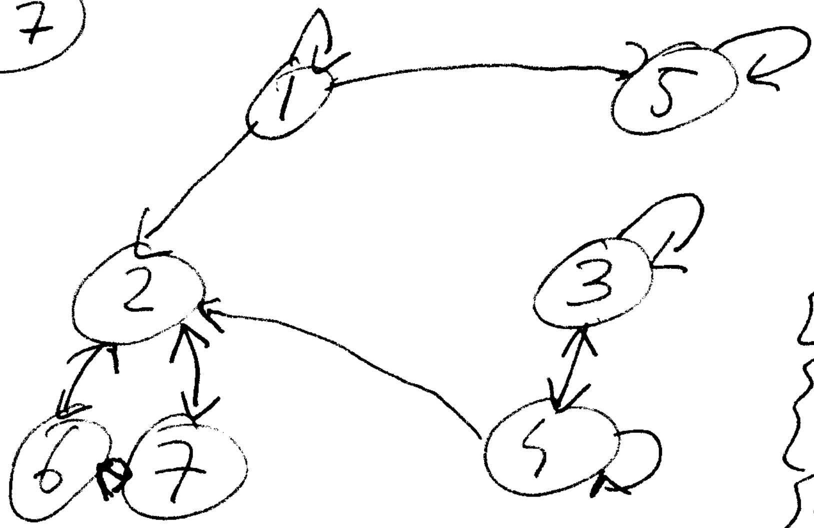
2 $\quad \sum_{n=0}^{\infty} (P^n)_{2,2} = +\infty \implies$ 2 is recurrent

$$\sum_{n=0}^{\infty}\left(1-\frac{1}{2^n}\right) = 0 + \left(1-\frac{1}{2}\right) + \left(1-\frac{1}{4}\right) + \dots$$

$$= 0 + \frac{1}{2} + \frac{3}{4} + $$

$$\lim_{n\to\infty}\left(1-\underbrace{\frac{1}{2^n}}_{0}\right) = 1$$

3.7



Communication classes

$\{1\}$, transient, period $=1$

$\{2, 6, 7\}$, recurrent, period $=1$

$\{3, 4\}$, transient, period $=$

$\{5\}$, recurrent, period $=$

$$\{n>0 : (P^n)_{1,1} > 0\} = \{1, 2, 3, 4, \dots\}$$

$$d(1) = \gcd\{n>0 : (P^n)_{1,1} > 0\} = 1$$

$\{n>0 : (P^n)_{2,2} > 0\} = \{2, 3, \ldots\}$ [3/5]

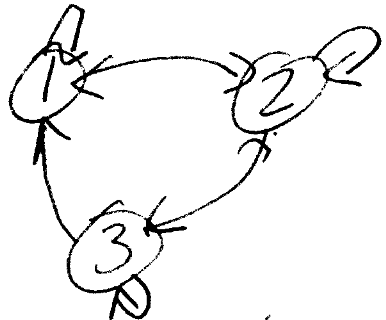
$d(2) = \gcd\{n>0 : (P^n)_{2,2} > 0\} = 1$

$\Pi = [\Pi_a, \Pi_b, \Pi_c]$

$\Pi_a, \Pi_b, \Pi_c > 0$

$\Pi_a + \Pi_b + \Pi_c = 1$

(3.8) is it irreducible?



$\{1, 2, 3\} \Rightarrow$ it is irreducible

find the stationary ~~flow~~ distrib.

$\Pi = \left(\frac{19}{85}, \frac{48}{85}, \frac{18}{85}\right)$

expected return times

$\frac{85}{19}, \frac{85}{48}, \frac{85}{18}$

3.10

(a) ~~xx~~ stationary distr. = limiting distr.

$$\Pi = \begin{array}{ccc} r & s & c \\ \left(\frac{1}{9},\right. & \frac{3}{4}, & \left.\frac{5}{36}\right) \end{array}$$

$\mathbb{P}$(a given day will be a rainy) $= \frac{1}{9}$

(b) long term proportion of clear days $= \frac{5}{36}$

$\frac{5}{36} \cdot 365 = \ldots$

(c)

expected time between visits to a state

= expected returned time

$$\frac{1}{\frac{3}{4}} = \frac{4}{3}$$