EDRP: Discrete Random Processes
Problem set 1

1.1 We keep flipping a fair coin. Let X_n be -1 if a head comes up, and 1 if the coin comes up tail. Is $(X_n)_n$ a Markov chain? If so, find its transition matrix.

1.2 Solve the previous problem assuming that the coin is not necessarily fair - say, the probability of heads is $p \in (0, 1)$.

1.3 A Markov chain $(X_n)_n$ has transition matrix

$$\mathbf{P} = \begin{bmatrix} 0.1 & 0.3 & 0.6 \\ 0 & 0.4 & 0.6 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$$

with initial distribution $\alpha = (0.2, 0.3, 0.5)$. Find the following:

(a) $\mathbb{P}\left(X_7 = 3 \middle| X_6 = 2\right)$
(b) $\mathbb{P}\left(X_9 = 2 \middle| X_1 = 2, X_5 = 1, X_7 = 3\right)$
(c) $\mathbb{P}\left(X_0 = 3 \middle| X_1 = 1\right)$
(d) $\mathbb{E}X_2$

1.4 Let $X_0, X_1, \ldots$ be a Markov chain with transition matrix

$$\begin{bmatrix} 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

and initial distribution $\alpha = (1/2, 0, 1/2)$. Find the following:

(a) $\mathbb{P}\left(X_2 = 1 \middle| X_1 = 3\right)$
(b) $\mathbb{P}(X_1 = 3, X_2 = 1)$
(c) $\mathbb{P}\left(X_1 = 3 \middle| X_2 = 1\right)$
(d) $\mathbb{P}\left(X_9 = 1 \middle| X_1 = 3, X_4 = 1, X_7 = 2\right)$

1.5 For a two-state chain with transition matrix

$$P = \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix}, \quad p, q \in [0, 1],$$

and initial distribution $\alpha = (\alpha_1, \alpha_2)$, find the following:

(a) the two-step transition matrix
(b) the distribution of X_1