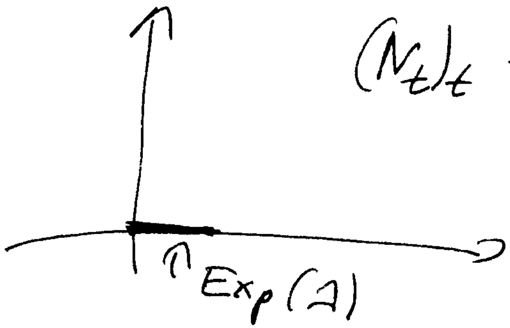
5.1 X_G, X_R, X_O – the times of the first green, red and orange subways that arrive at the station

we know the distributions

$X_R \sim Exp\left(\frac{1}{10}\right)$ (time unit = minutes)

$X_G \sim Exp\left(\frac{1}{15}\right)$

$X_O \sim Exp\left(\frac{1}{20}\right)$



$(N_t)_t$ – a Poisson process with parameter λ

$\cancel{X \sim Exp(\lambda)}$

(a) $M := \min(X_G, X_R, X_O)$

$\leftarrow$ the time of arrival of the 1st train

$$\mathbb{P}(M = X_G) = \mathbb{P}(\min(X_G, X_R, X_O) = X_G) = \frac{\frac{1}{15}}{\frac{1}{10} + \frac{1}{15} + \frac{1}{20}}$$

$M = \min(X_G, X_R, O) \sim Exp\left(\frac{1}{10} + \frac{1}{15} + \frac{1}{20}\right)$

(b) $$\mathbb{E}M = \mathbb{E}\big(\underbrace{\min(X_G, X_R, X_O)}_{\sim Exp\left(\underbrace{\frac{1}{10}+\frac{1}{15}+\frac{1}{20}}_{\lambda}\right)}\big) = \frac{1}{\frac{1}{10}+\frac{1}{15}+\frac{1}{20}}$$

$$\boxed{\begin{array}{l} X \sim Exp(\lambda) \\ \mathbb{E}X = \frac{1}{\lambda} \end{array}}$$

5.2 $(N_t^{(c)})_t$ – Poisson process with intensity $\lambda_c = 1$

$(N_t^{(T)})_t$ – — // — // — // – $\lambda_T = 2$

$N_t^{(c)}$ – # of cars that have arrived up to time t

$N_t^{(T)}$ – -//- trucks – // – // – // –

$(N_t^{(c)})_t$, $(N_t^{(T)})_t$ – independent

$N_t := N_t^{(c)} + N_t^{(T)}$ — # of vehicles up to time t

$\Downarrow$

$(N_t)_t$ – a Poisson process with intensity $\lambda = \lambda_c + \lambda_T = 3$

(a) $X^{(c)}$ – time of arrival of the 1st car

$X^{(T)}$ – // – // – truck

$X^{(c)} \sim Exp(\lambda_c)$, $X^{(T)} \sim Exp(\lambda_T)$

$M := \min(X^{(c)}, X^{(T)})$

$$\mathbb{P}(M = X^{(c)}) = \frac{\lambda_c}{\lambda_c + \lambda_T} = \frac{1}{1+2} = \frac{1}{3}$$

(b) $\mathbb{P}(N_5 \geq 1) = 1 - \mathbb{P}(N_5 = 0) =$
$= 1 - e^{-\boxed{15}}$

$N_t \sim \mathrm{Poiss}(3 \cdot t)$
$N_5 \sim \mathrm{Poiss}(15)$

$X \sim \mathrm{Poiss}(\lambda)$
$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$
$\mathbb{P}(X = 0) = e^{-\lambda}$

(c) $\min(X^{(c)}, X^{(t)}) \sim \mathrm{Exp}(1+2)$
the amount of time needed for the 1st vehicle to arrive

$$\boxed{X \sim \mathrm{Exp}(\lambda) \Rightarrow \mathbb{E}X = \frac{1}{\lambda}}$$

$\mathbb{E}\min(X^{(c)}, X^{(t)}) = \frac{1}{3}.$

(d) $\max(X^{(c)}, X^{(t)})$
the amount of time needed to ~~after~~ observe at least one car and one truck

$$\boxed{\begin{aligned}&\max(X, Y)\\ &= X + Y - \min(X, Y)\end{aligned}}$$

$X = 5,\ Y = 7$
$\max(X, Y) = 7$
$X + Y - \min(X, Y) = 5 + 7 - 5 = 7$

$\mathbb{E}\max(X^{(c)}, X^{(t)}) = \mathbb{E}(X^{(c)} + X^{(t)} - \min(X^{(c)}, X^{(t)}))$
$= \mathbb{E}X^{(c)} + \mathbb{E}X^{(t)} - \mathbb{E}\min(X^{(c)}, X^{(t)})$
$= \frac{1}{\lambda^{(c)}} + \frac{1}{\lambda^{(t)}} - \frac{1}{\lambda^{(c)} + \lambda^{(t)}}$

(5.3) $N_t^{(T)}$ – # of TRUCKS up to time t [4/pm]

$N_t^{(C)}$ – # of CARS up to time t

N_t – # of vehicles " – || –

$(N_t)_t$ – a Poisson process

THINNING

$\Downarrow$

$(N_t^{(T)})_t$ – a Poisson process with parameter: $\frac{2}{3} \cdot 60 \cdot 0.1 = 4$

our new time unit is hours

$(N_t^{(C)})_t$ – — || – || – $\frac{2}{3} \cdot 60 \cdot 0.9 = 36$

(a) $\mathbb{P}(N_1^{(T)} \geq 1) = 1 - \mathbb{P}(N_1^{(T)} = 0)$.

(b) $\mathbb{E} N_8^{(T)} = 4 \cdot 8$ $N_8^{(T)} \sim \text{Poisson}(8 \cdot 4)$

$X \sim \text{Poiss}(\lambda)$

$\mathbb{E}X = \lambda$

(c) $\mathbb{P}(N_3^{(T)} = 0, N_3^{(C)} \geq 1) \overset{\text{ind.}}{=} \mathbb{P}(N_3^{(T)} = 0) \cdot \mathbb{P}(N_3^{(C)} \geq 1)$

5.4 $N_t^{(r)}$ – # of red cars up to time t

$N_t^{(b)}$ – $\rightarrow$ independent

$N_t := N_t^{(b)} + N_t^{(r)}$ – # of cars SUPERPOSITION

$(N_t)_t$ – a Poisson process with parameter $(r+b)$

(a) $\mathbb{P}(N_1 = 0)$

(b) $\mathbb{P}(N_1^{(r)} = 1, N_1^{(b)} = 1 \mid N_1 = 2) =$

$\mathbb{P}(A|B) = \dfrac{\mathbb{P}(A, B)}{\mathbb{P}(B)}$

$$= \frac{\mathbb{P}(N_1^{(r)} = 1, N_1^{(b)} = 1, N_1 = 2)}{\mathbb{P}(N_1 = 2)} = \frac{\mathbb{P}(N_1^{(r)} = 1, N_1^{(b)} = 1)}{\mathbb{P}(N_1 = 2)} =$$

$$\overset{\text{ind.}}{=} \frac{\mathbb{P}(N_1^{(r)} = 1) \cdot \mathbb{P}(N_1^{(b)} = 1)}{\mathbb{P}(N_1 = 2)}$$