



Intelligent Information Systems

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Lecture Notes: Module 3



HUMAN CAPITAL
HUMAN – BEST INVESTMENT!

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Short Note on Logics

Module 3

Prologue

Some quotes

“Logic is the anatomy of thought” — John Locke

“Logic is not a body of doctrine, but a mirror-image of the world. Logic is transcendental”

— Ludwig Wittgenstein

“The want of logic annoys. Too much logic bores. Life eludes logic, and everything that logic alone constructs remains artificial and forced” — André Gide

“Logic is the art of going wrong with confidence”

— J. Wood Krutch



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A Few Questions

Question 1

Is logic part of:

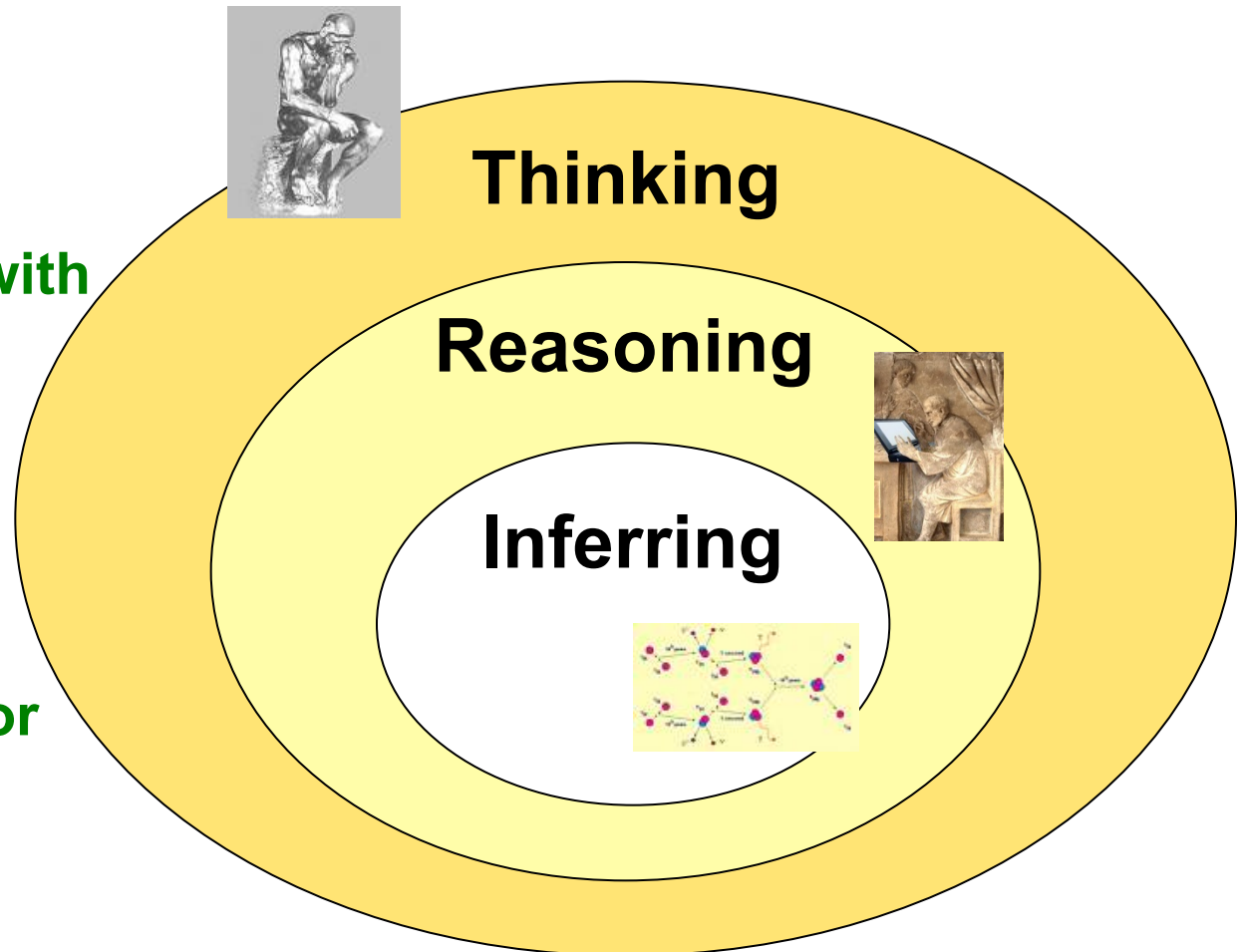
- mathematics**
- philosophy**
- art**
- ordinary discourse**
- ... ?**



Question 2

Does logic deal with
the validity of:

- thinking and/or
- reasoning and/or
- inferring?



Question 3

**Do we
discover
or
invent
logic?**



Question 4

Is logic an **in-born** feature of our mind
or it is a reasoning mechanism developed by our mind in order to cope with the real physical and social world?

	p	q	$p \rightarrow q$
1	0	0	1
2	0	1	1
3	1	0	0
4	1	1	1

Do we really cope?

(1,2) "If the moon is made of green cheese, then life exists on other planets."

(2,4) "If life exists on other planets, then life exists on earth."

Question 5

**Can different
co-existing universes be
governed by different logics?**
(quantum mechanics, macro-cosmos,
human habitat)





What's Logics?



Raffaello Santi:
School of Athens (1509-1511)
Vaticano, Rome

***"I saw the Master there of those who know,
Amid the philosophic family,
By all admired, and by all revered;
There Plato too I saw, and Socrates,
Who stood beside him closer than the rest."***

—Dante, The Divine Comedy

Father of Logic



Aristotle

Αριστοτέλης

384 BC – March 7, 322 BC

A student of Plato and teacher of
Alexander the Great.

For Aristotle logic was a tool for thinking -
"Organon".

He coined the term ***sylllogism***.

Aristotle's conception of logic was the
dominant form of logic up until the advances
in mathematical logic in the 19th century.

Kant himself thought that Aristotle had done
everything possible in terms of logic.

Etymology

λόγος logos;

meaning word, account,

reason or principle

Seven Liberal Arts

- **Trivium:**

- grammar

- **dialectic (logic)**

- rhetoric

- **Quadrivium:**

- arithmetic

- music

- geometry

- astronomy



source:
Hortus deliciarum,
Herrad von Landsberg,
12th century

Liberal arts made up the core curriculum of the medieval universities.

Plato & Aristotle's Principles of Thought

- **Law of Identity**

$$p=p$$

Aristotle notes: "*without the law of identity, there can be no responsibility for vice*"

- **Law of Contradiction**

$\sim(p \cap \sim p)$ once cannot say that p and not p
in the same respect at the same time;

- ***Tertium non datur*** (Law of Excluded Middle)

$(p \cup \sim p)$ either something exists or does not

Aristotle's Tertium non datur

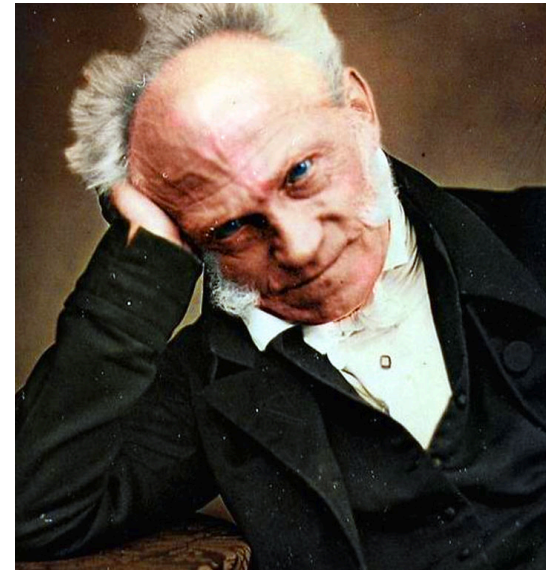
But on the other hand there cannot be an intermediate between contradictories, but of one subject we must either affirm or deny any one predicate. This is clear, in the first place, if we define what the true and the false are. To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, and of what is not that it is not, is true; so that he who says of anything that it is, or that it is not, will say either what is true or what is false

— Aristotle, Metaphysics, Book IV, Part 7
(translated by W.D. Ross)



Schopenhauer's Principle of Sufficient Reason

Truth is the reference of
a judgment to something
outside it as its sufficient
reason or ground.



1788 - 1860

Syllogisms

- **Categorical**

All human beings are mortal. Socrates is a human being.

Socrates is mortal.

- **Conditional** (*hypothetical*)

if A, then B

e.g. *If life is a struggle, then I am fully involved in life.*
since I'm struggling to understand logic, and
life is a struggle, so I am fully involved in life.

- **Disjunctive**

either A or B

e.g. *This is either a dictionary or a chemistry book.*
It is not a dictionary, so it is a chemistry book.

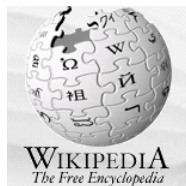
Logics – Informal View

**Rigorous reasoning (e.g. deduction)
based on formal structures, abstracting
from the content of the propositions
being made.**

**Used mainly for mathematical
type of reasoning.**

Logics – Informal View

**“Logic is the study of the
principles and criteria of
valid **inference** and
demonstration”**



<http://en.wikipedia.org/wiki/Logic>

Which logics?

<p>Philosophical Logic (since antiquity)</p>	<p>deals with the analysis and principles of sound thinking and valid reasoning.</p> <p><i>Aristotle, Descartes, I. Kant</i></p>
<p>Mathematical Logic (since XIX century)</p>	<p>deals with the analysis and principles of valid reasoning/ inference, mainly regarding mathematical entities.</p> <p><i>G. Bool, G. Frege, i B. Russell, K. Goedel, A. Tarski</i></p>
<p>Informal Logic (since XX century)</p>	<p>The study of arguments as presented in ordinary language and discourses</p> <p><i>K. Ajdukiewicz, J. A. Blair, R.H. Johnson, S Toulmin</i></p>



Great Logicians

Polish School of Logics



T. Kotarbiński



A. Tarski



K. Ajdukiewicz



S. Leśniewski

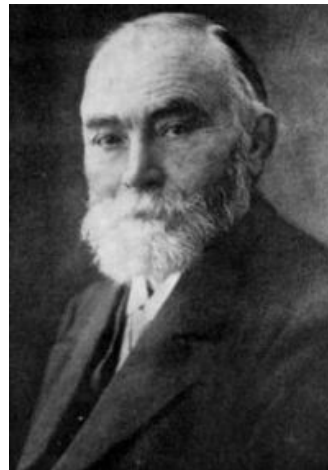


J. Łukasiewicz

German Logicians



G. Cantor



F. G. Frege



G. Gentzen



G. Leibniz



L. Kronecker

Other Giants



B. Bolzano



G. Boole



G. Chaitin



B. Russell



K. Gödel



A. Turing

***What about
you?***

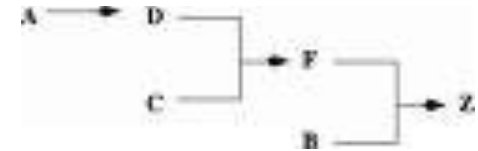




A Note on Formal Logic

Original Goals of Formal Logic

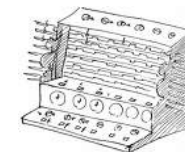
1. To discover, describe and examine reliable laws of reasoning.
2. To investigate/prove the soundness of mathematics, in particular to find the answer to *Entscheidungsproblem*, Hilbert, 1900--1917.
to produce a "fully automatic procedure" for deciding whether a given proposition (sentence) is true or false.]
3. Leibniz's dream to automate reasoning (calculus).



D. Hilbert



G. Leibniz



Vital Question

**Are the goals of
formal logic
achievable?**

Informal Definition

Logic includes:

- **language**
- **set of logical values**
- **inference/reasoning mechanism**



Language

Language is
a pair of:

- alphabet,
- grammar (syntax).

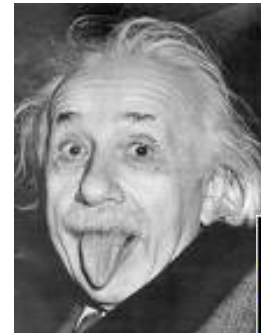
Example

$A = \{ *, \$, ! \}$ – *alphabet*

Allowed are 3-element strings
starting with * – *grammar*

***, *!!, *\$! – correct words

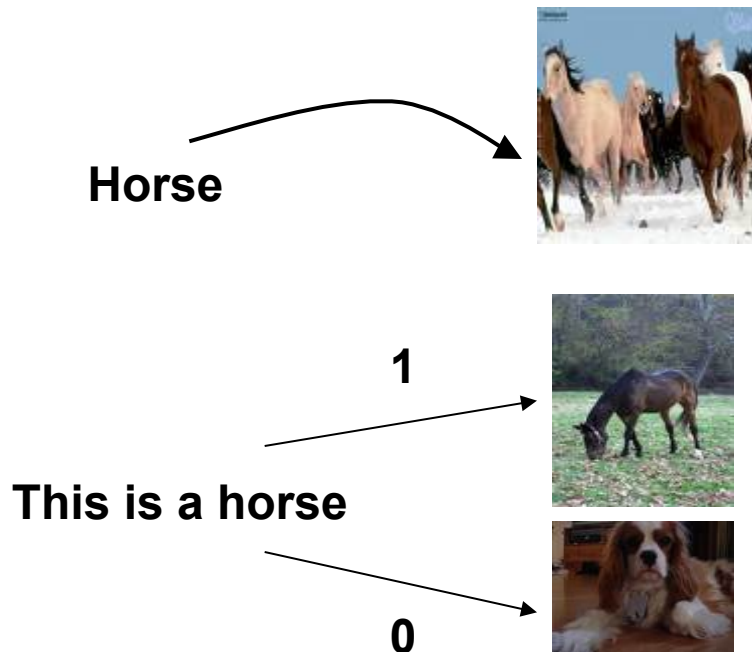
!** , &\$! , ** – wrong words



Language has
no semantics

Logical Values (“logical semantics”)

A **semantics** for a formal language is given by defining mappings from each syntactic category into suitable semantic domains.



Syntactically correct logical formulas are assigned **logical values**, i.e.

- **true** (1, T) or
- **false** (0, F)

Inference

Inference is the act or process of deriving a conclusion based solely on what one already knows.

Deduction (used in classical/formal logic)

- inference in which the conclusion is of no greater generality than the premises.
- finding the effect with the cause and the rule.



Sherlock Holmes

Mathematical induction

Method used to prove that every statement in an infinite sequence of statements is true.

Induction (used in informal logic)

Generalization.

... (other methods, e.g. analogy, statistics)



Examples of Inference Rules

An inference rule is a method of deriving conclusions from premises.

Modus Ponens $\alpha, \alpha \Rightarrow \beta / \beta$

Robinson's $A \vee B, C \vee \neg B / A \vee C$

Resolution *(indirect proof)*

Generalization $\alpha / \forall x \alpha$

Example of Inference

Let

A – I'm ok.

B – My friends are ok.

Given that (someone can guarantee that)

A

$A \rightarrow B$

are valid we can claim that

My friends are ok.

$$\frac{A, A \rightarrow B}{B}$$

Modus Ponens

Proof / Deduction

Suppose A_1, A_2, \dots, A_k are all the axioms and previously proved theorems. A **formal proof**, or deduction, of a sentence P is a sequence of statements S_1, S_2, \dots, S_n where

S_n is P , and one of the following holds:

- S_i is one of A_1, A_2, \dots, A_k , or
- S_i follows from the previous statements by a valid argument using the rules of reasoning.

Example of a Proof

Try to follow the inference chain (deduction)

Hypotheses:

- If you send me an email, then I'll finish writing the program.
- If you don't send me an email, then I will go to sleep early.
- If I go to sleep early, then I will wake up feeling refreshed.

Does it follow that:

- If I don't finish writing the program, then I will wake up feeling refreshed. ?

$$\begin{array}{c} t \rightarrow u \\ \text{R: } \text{-----} \\ \neg u \rightarrow \neg t \end{array}$$

p: "you send me an email"
q: "I'll finish writing the program"
r: "I'll go to sleep early"
s: "I'll wake up feeling refreshed"

- | | |
|--------------------------------|------------------------------------|
| 1. $p \rightarrow q$ | Hypothesis |
| 2. $\neg q \rightarrow \neg p$ | From 1 by rule R |
| 3. $\neg p \rightarrow r$ | Hypothesis |
| 4. $\neg q \rightarrow r$ | From 2,3 by Hypothetical syllogism |
| 5. $r \rightarrow s$ | Hypothesis |
| 6. $\neg q \rightarrow s$ | From 4,5 by Hypothetical syllogism |

T-Structure

T-Structure is a formalized description of the real world. It is a purely syntactical notion.

T = <

**Language,
{Logical Axioms,
Specific Axioms,
Inference Mechanism}**

>



Logical axioms (tautologies) and inference rules are given, therefore, the T-Structure is defined by specifying specific axioms.

Note: The notion of T-Structure is local in this module and has been coined for the sake of the present discourse only. Noteworthy, it refers to the notion of theory in logic. 39

Example of a T-Structure

Language

– natural language

Logical axioms

– Aristotle's axioms

Inference mechanism

– modus ponens

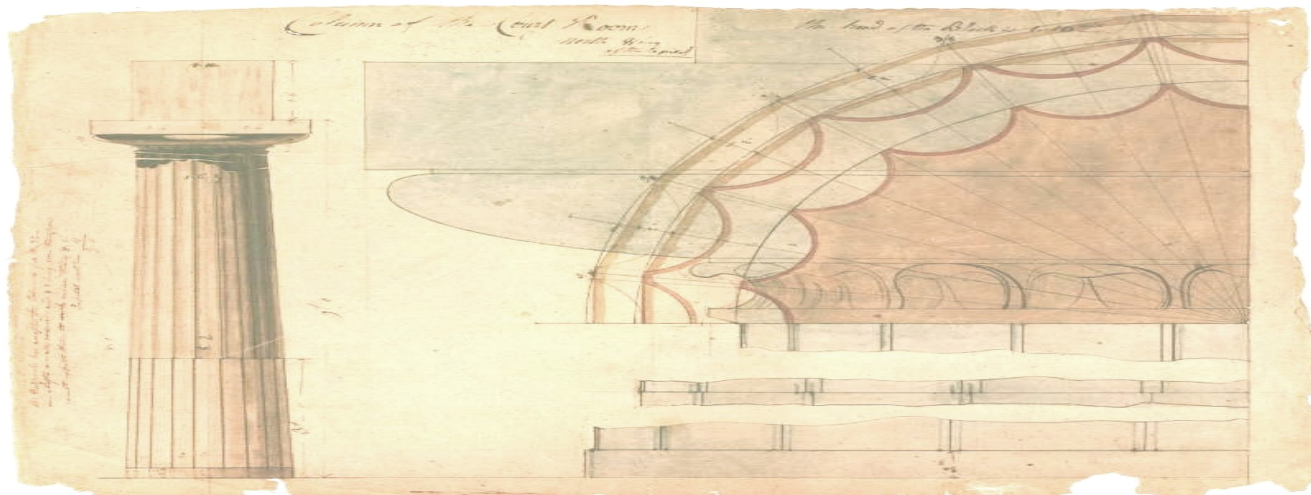
Specific axioms:

- *John is a brother of Adam*
- *Adam is a brother Bill*
- *If A is a brother of B and B is a brother of C, then A is a brother of C*
- *if A and B are brothers, then A and B get a brother's grant*
- ...

The above set is an information system!

Important Conclusion

Information systems “in logic” are T-Structures.



Quiz

**Can a T-Structure
include a false
statement?**

YES

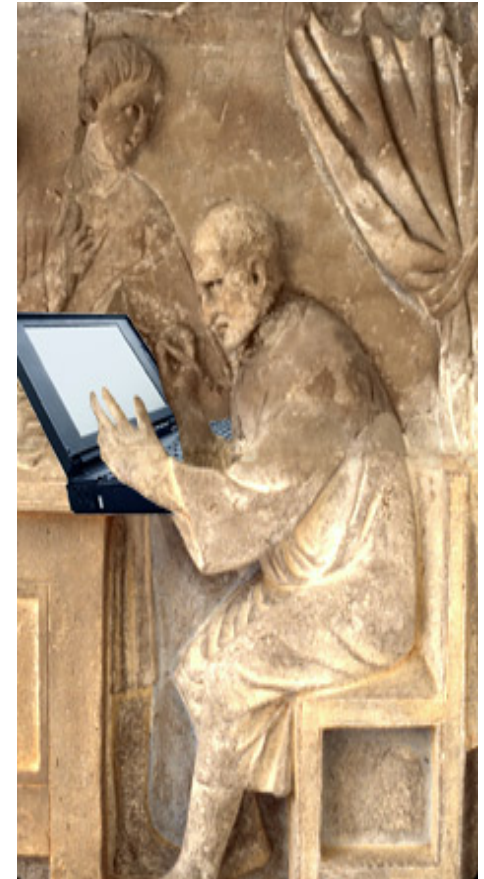
No



Logical Information System

Processing of Queries

**Answering queries in
logical information
systems consists in
treating the query as a
theorem and attempting
to prove it.**



Example

$$\frac{A, A \rightarrow B}{B}$$

Knowledge Base

Base of Facts

Mother(Anna, Kasia),
Mother(Anna, Tomek),
Siblings(Jacek, Kasia),
Siblings(Irena, Maria),
Father(Jan, Kasia),
Kid(Kasia, Anna, Jan),
...

Base of Rules

$Mother(x, y) \wedge Mother(x, z) \rightarrow Siblings(y, z),$
 $Mother(x, y) \wedge Father(z, y) \rightarrow Kid(y, x, z),$
...

QUERY

Are Tomek and Kasia siblings ?

FORMULA

Siblings(Tomek, Kasia)

ANSWER

$Mother(x, Tomek) \wedge Mother(x, Kasia) \rightarrow Siblings(Tomek, Kasia)$ valid owing to Base of Rules

**$Mother(Anna, Tomek),$
 $Mother(Anna, Kasia)$**
valid owing to Base of Facts

Hence by means of modus ponens

$Siblings(Tomek, Kasia)$ is valid, so Tomek and Kasia are siblings.

Meta-logic

Semantic Consequence

Let

α — formula

Φ — set of formulae

Notation:

$$\Phi \models \alpha$$

α is a **semantic consequence** of Φ

Syntactic Consequence

α is the **syntactic consequence** of a set Φ of iff α can be derived/
deduced from Φ (and general
axioms).

Notation:

$$\Phi \vdash \alpha$$

Syntactic = Semantic

Machine

Man

$$\mathbf{T} \vdash \phi \quad \text{iff} \quad \mathbf{T} \models \phi$$

**This is the theoretical basis for building
information systems in logic.**

Requested Properties

Consistency, which means that none of the theorems of the system contradict one another.

Soundness, which means that the system's rules of proof will never allow a false inference from a true premise. If a system is sound and its axioms are true then its theorems are also guaranteed to be true.

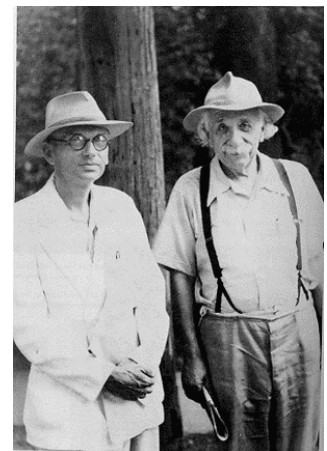
Completeness, which means that there are no true sentences in the system that cannot, at least in principle, be proved in the system.

Not all systems achieve all three virtues. The work of Kurt Gödel has shown that no useful system of arithmetic can be both consistent and complete.

Goedel's Theorem (GT)

GT proves that there exist meaningful mathematical statements that are neither provable nor disprovable, now or ever - neither provable nor disprovable.

That is, not simply because human thought or knowledge is insufficiently advanced, but because the very nature of logic renders them incapable of resolution, no matter how long the human race survives or how wise it becomes.



Summary

1. **Formal logical reasoning is not an absolute law which governs the universe.**
2. **Formal logic is not a set of rules which govern human behavior and discourse.**
3. **However, it helps describe, conceptualize, explain and understand the world of science and to some extent the world of people. It can also be used to build up information systems.**

Pros & Cons

Advantages of LIS

1. The LIS description language is similar to a natural language.
2. **Comprehensiveness of a logical language, which covers knowledge database description, manipulation, and querying.**
3. Declarative rather than procedural, which makes the use simpler.
4. **Stronger expressiveness than conventional systems** (e.g. "John has always cookies or fruits", "Miller is not an engineer").
5. Modularity.
6. **Inference**
(e.g. Robinson's resolution: $A \vee B, C \vee \neg B / A \vee C$).

Problems with LIS

1. Limits entailed by Gödel's Theorem.

First Order Theories, in general, are not decidable



2. Large computational complexity. ("exponential explosion").





Argumentation. Informal Logic.

Spheres of Argument



▪ **Private**



▪ **Public**



▪ **Technical**



Note: Migration of arguments from one sphere to another is quite common.

The origins of **argumentation**
and rhetoric go back to ancient
Greece; the 5th century B.C.



Through centuries the study
of argumentation has been
developed and now is a
subject to exciting and vibrant
studies.

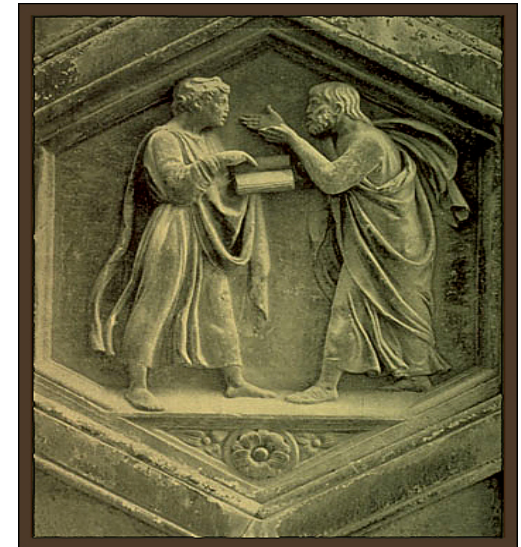


The Romans adopted Aristotelian rhetoric for pedagogical purposes; they used it as a means of instruction (liberal arts).

For the Romans a good citizen was the one who, as Quintilian said, is “*the good person, speaking well*”, and who can influence others through argument.

Canon

- invention
- arrangement
- style
- memory
- delivery



Invention

The process of discovery of what could be used in an argument.

Arrangement

This is the structure and organization of the speech.

Style

It focuses on the use of language. Selection of words, tones, repetitions, rhetoric figures.

Memory

To keep in mind what is to be said. It refers to mnemonics and other memory aids.

Delivery

Physical presentation – voice, body language.

Canon

- invention
- arrangement
- style
- memory
- delivery

In the 20th century it was realized that the Cartesian and Logical Positivism approaches to argumentation proved to be a blind alley.

The main reason was its abstraction from meaning.



Definitions

“The study of arguments as presented in **ordinary language**, as contrasted with the presentations of arguments in an artificial, formal, or technical language.”

http://en.wikipedia.org/wiki/Informal_logic



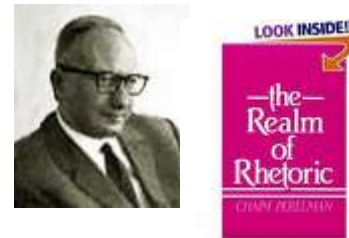
“A branch of logic whose task is to develop non-formal standards, criteria, procedures for the analysis, interpretation, evaluation, criticism and **construction of argumentation in everyday discourse.**”

*Johnson, Ralph H., and Blair, J. Anthony (1987),
"The Current State of Informal Logic",
•Informal Logic, 9(2–3), 147–151*

Stephen Toulmin
discourse about ethics



Chaim Perelman
discourse about justice



Charles Hamblin
fallacies (e.g. ad misericordiam, appeal to authority
ad baculum, ad personam)



Frans van Eemeren, Rob Grootendorst
logic of conversation – Pragma-Dialectics



Jürgen Habermas
role of communications



Recommended Readings:

- Casti J., de Pauli W., “Gödel: A Life of Logic ”, Perseus Books, 2007.
- Gensler H., “Introduction to Logic”, Routledge, 2001.
- Grayling A.C., “An Introduction to Philosophical Logic”, 3rd ed., Blackwell Publishing, 2001.
- Hofstadter D.R., “Godel, Escher, Bach: An Eternal Golden Braid“, Basic Books, 1999.
- Kant I., Abbot T.K., “Kant's Introduction to Logic and His Essay on the Mistaken Subtlety of the Figures (Key Texts)”, Thoemmes Press, 1998.
- Nagel E., “Goedel’s Proof”, New York University Press, 2001.
- Tarski A., “Introduction to Logic”, Dover Publications, 1995.



<http://en.wikipedia.org/wiki/Logic>

Epilogue

Worth Remembering

**“Pure logic is the
ruin of the spirit”**

— Antoine de Saint Exupery





Thank you!



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