Typing rules

Product type

$$\frac{\Gamma, x : A \vdash B \ : \ \text{Type}}{\Gamma \vdash (x : A) \to B \ : \ \text{Type}}$$

Application

$$\frac{\Gamma \vdash c \ : \ (x : A) \to B \quad \Gamma \vdash a \ : \ A}{\Gamma \vdash c(a) \ : \ B\{x := a\}}$$

Abstraction

$$\frac{\Gamma, x : A \vdash b \ : \ B}{\Gamma \vdash \lambda x : A \ .b \ : \ (x : A) \ \to \ B}$$

Record type

$$\frac{\Gamma \vdash A_1 \ : \ \text{Type} \quad \ldots \quad \Gamma, x_1 : A_1, \ldots, x_{n-1} : A_{n-1} \vdash A_n \ : \ \text{Type}}{\Gamma \quad \vdash \quad \begin{bmatrix} x_1 & : & A_1 \\ \ldots & & \\ x_n & : & A_n \end{bmatrix} \quad : \quad \text{RecType}}$$

Projection

$$\frac{\Gamma \vdash r \ : \ \begin{bmatrix} x_1 & : & A_1 \\ \ldots & & \\ x_n & : & A_n \end{bmatrix}}{\Gamma \quad \vdash \quad r.x_i \quad : \quad A_i\{x_1 := r.x_1, \ldots, x_{i-1} := r.x_{i-1}\}}$$

Record

$$\frac{\Gamma \vdash a_1 \ : \ A_1 \quad \ldots \quad \Gamma \vdash a_n \ : \ A_n\{x_1 := a_1, \ldots, x_{n-1} := a_{n-1}\}}{\Gamma \quad \vdash \quad \begin{bmatrix} x_1 & = & a_1 \\ \ldots & & \\ x_n & = & a_n \end{bmatrix} \quad : \quad \begin{bmatrix} x_1 & : & A_1 \\ \ldots & & \\ x_n & : & A_n \end{bmatrix}}$$

Union

$$\frac{A \ : \ \text{Type} \quad B \ : \ \text{Type}}{A \cup B \ : \ \text{Type}} \qquad \frac{a \ : \ A}{a \ : \ A \cup B} \qquad \frac{b \ : \ B}{b \ : \ A \cup B}$$

Intersection

$$\frac{A \ : \ \text{Type} \quad B \ : \ \text{Type}}{A \cap B \ : \ \text{Type}} \qquad \frac{a \ : \ A \quad a \ : \ B}{a \ : \ A \cap B}$$

Subtyping

Main rule

$$\frac{a \,:\, A \quad A \subseteq B}{a \,:\, B}$$

Derived rules for union and intersection

$$A \subseteq A \cup B \quad B \subseteq A \cup B \quad A \cap B \subseteq A \quad A \cap B \subseteq B$$

$$R + [x : A] \subseteq R$$