Jeonghyeon Woo

Math 301

Assignment 1

Suppose that $a_1 \ldots, a_n$ is a finite sequence of real numbers and that m is a positive integer where $m \leq n$. A term a_k will be called an m-leader if there exists a positive integer p such that $1 \leq p \leq m$ and such that $a_k + \cdots + a_{k+p-1} \geq 0$. Thus, for instance, the 1-leaders are the nonnegative terms of the sequence; observe, however, that if $m > 1$, then an m-leader need not be nonnegative.

Lemma. The sum of the m-leaders is nonnegative.

Proof. If there are no m-leaders, then the assertion is true. Otherwise, let a_k be the first m-leader and let $a_h + \cdots + a_{k+p-1}$ be the shortest nonnegative sum it leads (here, $p \leq m$). We assert that every a_h in this sum is itself an m-leader and that $a_h + \cdots + a_{k+p-1} \geq 0$. If not, then $a_k + \cdots + a_{h-1} > 0,$ contradicting the original choice of p. Proceed now inductively with the sequence $a_{k+p}, \ldots, a_n$; the sum of the shortest nonnegative sums so obtained is exactly the sum of the m-leaders.

Individual Ergodic Theorem. If T is a measure-preserving (but not necessarily invertible) transformation on a space X (with possibly infinite measure) and if $f \in L_1$, then

$$\frac{1}{n}\sum_{j=0}^{n-1} f(T^j x)$$

converges almost everywhere. The limit function f^* is integrable and invariant (i.e.,

$f^*(Tx) = f^*(x)$ almost everywhere.) If $m(X) < \infty$, then

$$\int f^*(x)dx = \int f(x)dx$$