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Math 301

HW 7

1. a.

$\frac{\mathrm{a}}{\mathrm{b}} \sim \frac{c}{d}$ iff b=d, where both are in lowest terms.

Reflexivity

Suppose $\mathrm{a} = \mathrm{c}$. $\frac{\mathrm{a}}{\mathrm{b}} \sim \frac{\mathrm{a}}{\mathrm{b}}$, so it is reflexive.

Symmetry

Let $\mathrm{k} = \mathrm{b} = \mathrm{d}$. $\frac{\mathrm{a}}{\mathrm{k}} \sim \frac{c}{k}$, and $\frac{\mathrm{c}}{\mathrm{k}} \sim \frac{\mathrm{a}}{\mathrm{k}}$, so it is symmetric

Transitivity

Let $\mathrm{x} \in \mathbb{Z}, \mathrm{y} \in \mathbb{Z},\ \mathrm{d} = \mathrm{y}$. $\frac{\mathrm{c}}{\mathrm{d}} \sim \frac{\mathrm{x}}{\mathrm{y}}$. Since $\mathrm{b} = \mathrm{d} = \mathrm{y}$, $\frac{\mathrm{a}}{\mathrm{b}} = \frac{\mathrm{x}}{\mathrm{y}}$, so it is transitive.

Thus, the relation is an equivalence relation.

b.

$$[\mathrm{b}] = \left\{\mathrm{a} \in \mathbb{Z}, \mathrm{k} \in \mathbb{Z}: \frac{\mathrm{a}}{\mathrm{k}} \sim \frac{b}{k}\right\} = \{\mathrm{b} \in \mathbb{Z}\}$$

$$[\mathrm{x}] = \left\{\mathrm{b} \in \mathbb{Z}, \mathrm{k} \in \mathbb{Z}, \frac{\mathrm{b}}{\mathrm{k}} \sim \frac{\mathrm{x}}{\mathrm{k}}\right\} = \{\mathrm{x} \in \mathbb{Z}\}$$

2. $\mathrm{r} \sim \mathrm{s}$ iff $|\mathrm{r} - \mathrm{s}| < 1$ possesses reflexivity and symmetry but no transitivity.

Reflexivity $\quad |\mathrm{r} - \mathrm{r}| < 1 \rightarrow 0 < 1$

Symmetry $\quad |\mathrm{r} - \mathrm{s}| = |\mathrm{s} - \mathrm{r}| < 1$

Transitivity, counter example.

$$|0.1 - 0.2| < 1 \text{ and } |0.2 - 1.1| < 1 \text{ but } |0.1 - 1.1| \nless 1$$

3. $\mathrm{a} \sim \mathrm{b}$ iff $|\mathrm{a} - \mathrm{b}| \neq 0$

Symmetry $|\mathrm{a} - \mathrm{b}| \neq 0 \rightarrow |\mathrm{b} - \mathrm{a}| \neq 0$

Transitivity $|\mathrm{a} - \mathrm{b}| \neq 0$ and $|\mathrm{b} - \mathrm{c}| \neq 0 \rightarrow |\mathrm{a} - \mathrm{c}| \neq 0$

Reflexivity, counter example

$|\mathrm{a} - \mathrm{a}| \neq 0$ when $\mathrm{a} = 1$ is false

4. a. Domain: $\mathbb{Z}$

b. Codomain: $\mathbb{Z}$

c. Range: $\{\mathrm{f(n)} = 10\mathrm{n} + 5\} \subseteq \mathbb{Z}$

d. Let $\mathrm{f(a)} = 10\mathrm{a} + 5 \text{ and } \mathrm{f(b)} = 10\mathrm{b} + 5$. Assume f is an injunction.

$$\mathrm{f(a)} = \mathrm{f(b)}$$

$$10\mathrm{a} + 5 = 10\mathrm{b} + 5$$

$$\mathrm{a} = \mathrm{b}$$

Thus, f is an injunction.

e. Assume f is a surjection. Let $\mathrm{n} \in \mathbb{Z}$, such that $\mathrm{f(n)} = 1$

$$\mathrm{f(n)} = 10\mathrm{n} + 5$$

$$10\mathrm{n} + 5 = 1$$

$$\mathrm{n} = -\frac{4}{10}$$

Contradiction.

Thus, f is not a surjection.

5. $\mathrm{g}(\mathrm{n} \in \mathbb{Z}^+, \mathrm{k} \in \mathbb{Z}^+) \rightarrow \frac{\mathrm{n}}{\mathrm{k}} \in \mathbb{Q}^+$

a. Let $\mathrm{g(a, b)} = \frac{\mathrm{a}}{\mathrm{b}} \; and \; g(x, y) = \frac{x}{y}$. Assume f is an injection.

$$\mathrm{g(a, b)} = \mathrm{g(x, y)}$$

$$\frac{\mathrm{a}}{\mathrm{b}} = \frac{\mathrm{x}}{\mathrm{y}}$$

$$\frac{1}{2} = \frac{2}{4}$$

$$\mathrm{a} = \mathrm{x} \text{ and } \mathrm{b} = \mathrm{y}$$

Contradiction.

Thus, g is not an injection.

b. Let $\mathrm{n} \in \mathbb{Z}^+, k \in \mathbb{Z}^+$. Assume g is a surjective. We want $\{\mathrm{g(n, k)}\} = \mathbb{Q}^+$.

$$\mathrm{g(n, k)} = \frac{\mathrm{n}}{\mathrm{k}}$$

$$x = \frac{n}{k}$$

x is always rational by def. of rational number, and x is always positive since both n and k are always positive. Thus, g is a surjective.

6. $h: \mathbb{R} \to \mathbb{R}$ $h(r) = 10r + 5$

a. h is an injection as the same way as $f(n)$ of #4 is an injection.

b. Let $r \in \mathbb{R}$. Assume h is a surjective. We want $\{h(r)\} = \mathbb{R}$

$$h(r) = 10r + 5$$

$$x = 10r + 5$$

$$r = \frac{x - 5}{10}$$

It shows, $h\left(\frac{x-5}{10}\right) = x$ for any x, so $\{x\} = \mathbb{R}$

Thus, it is a surjective.