Math 301 Fall 2018

Assignment 1 – Due Friday, August 24

In this assignment, you are to typeset the passage below using MS Word's equation editor or other mathematical typesetting software. The file is then to be saved as a PDF and then submitted via Blackboard. You are not expected to understand what the passage means mathematically, although you should make an effort to see how the language around the mathematics is structured. Pay particular attention to paragraph structure and visual effects such as "display mode" used for the sum and integral equation in the statement of the individual ergodic theorem. Mathematical text (including individual variables) is to be typeset in math mode.

Cutting and pasting the entire passage is not in the spirit of this assignment. Remember, all assignments in this course are to be typeset and submitted in this fashion, so practicing this skill early is to your advantage.

Suppose that $a_1, \dots, a_n$ is a finite sequence of real numbers and that m is a positive integer where $m \leq n$. A term a_k will be called an m-leader if there exists a positive integer p such that $1 \leq p \leq m$ and such that $a_k + \cdots + a_{k+p-1} \geq 0$. Thus, for instance, the 1-leaders are the nonnegative terms of the sequence; observe, however, that if $m > 1$, then an m-leader need not be nonnegative.

Lemma. The sum of the m-leaders is nonnegative.

Proof. If there are no m-leaders, then the assertion is true. Otherwise, let a_k be the first m-leader and let $a_k + \cdots + a_{k+p-1}$ be the shortest nonnegative sum it leads (here, $p \leq m$). We assert that every a_h in this sum is itself an m-leader and that $a_h + \cdots + a_{k+p-1} \geq 0$. If not, then $a_k + \cdots + a_{h-1} > 0$, contradicting the original choice of p. Proceed now inductively with the sequence $a_{k+p}, \dots, a_n$; the sum of the shortest nonnegative sums so obtained is exactly the sum of the m-leaders.

Individual Ergodic Theorem. If T is a measure-preserving (but not necessarily invertible) transformation on a space X (with possibly infinite measure) and if $f \in L_1$, then

$$\frac{1}{n}\sum_{j=0}^{n-1} f\left(T^j x\right)$$

converges almost everywhere. The limit function f^* is integrable and invariant (i.e., $f^*(Tx) = f^*(x)$ almost everywhere.) If $m(X) < \infty$, then

$$\int f^*(x)\,dx = \int f(x)\,dx$$
