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Math 301

HW 3

1. $\sim(P \rightarrow Q)$

P	Q	P->Q	~(P->Q)
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

2. $P \rightarrow \sim Q \text{ and } Q \rightarrow \sim P, \text{Logically equivalent?}$

P	Q	~P	~Q	P->~Q	Q->~P
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

It appears $P \rightarrow \sim Q \text{ and } Q \rightarrow \sim P$ share the truth value for the same P and Q values. Therefore, $P \rightarrow \sim Q \text{ and } Q \rightarrow \sim P$ are logically equivalent.

3. $(R \rightarrow S) \leftrightarrow (P \wedge Q)$ and P is false

   a. $P \wedge Q$ is false as P is false, so $(R \rightarrow S)$ is false. The only possible way it could be false is when R is true and S is false. R=true, S=false

   b. Q could be either true or false since truth value of Q does not matter.

4. $\left((\mathrm{P} \to \mathrm{Q}) \wedge \sim\mathrm{P}\right) \to \sim\mathrm{Q}$

P	Q	(P→ Q)	((P → Q) ∧~P)	~Q	$\left((\mathrm{P} \to \mathrm{Q}) \wedge \sim\mathrm{P}\right) \to \sim\mathrm{Q}$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

When P is false and Q is true, the inference does not work. Therefore, the inference is not valid.

5. a. $\forall \mathrm{r} \in \mathbb{R}, \mathrm{r} - \mathrm{r} = 0$ For all real number r, $\mathrm{r} - \mathrm{r} = 0$.

   b. $\exists \mathrm{n} \in \mathbb{Z}: 0 < \mathrm{n} < 1$ There is an integer n which is bigger than 0 and smaller than 1.

   c. $\forall \mathrm{x}, \mathrm{y} \in \mathbb{R}, \mathrm{xy} = \mathrm{yx}$ For all real number x and y, $\mathrm{xy} = \mathrm{yx}$.

   d. $\exists \mathrm{q} \in \mathbb{Q}: \forall \mathrm{r} \in \mathbb{Q}, \mathrm{qr} = 0$ There exists rational number q such that every rational number r, multiplication of q and r is equal to zero.

   e. $\forall \mathrm{r} \in \mathbb{Q}^+, \exists q \in \mathbb{Q}^+: qr = 1$ For all positive rational number r, there is a positive rational number q such that $\mathrm{qr} = 1$.

   f. $\forall \mathrm{X} \in \mathcal{P}(\mathbb{W}), |X| \in \mathbb{W}$ For all sets X in powerset of W, cardinality of X is in W.

6. a. Every non-negative real number has a square root

$\forall \mathrm{x} \in \mathbb{R} - \mathbb{R}^-,\ \exists \mathrm{y} \in \mathbb{R} - \mathbb{R}^-, \mathrm{y} = \sqrt{\mathrm{x}}$.

b. The sine of any real number is between -1 and 1, inclusive

$\forall x \in \mathbb{R}, -1 \leq \sin x \leq 1$

c. There are two integers such that the square of the first is the cube of the second

$\exists x, y \in \mathbb{Z}: \ x^2 = y^3.$

d. For every integer n where $n \geq 2$, there is a prime number between n and 2n

$\forall n \in \mathbb{Z}^+ - \{1\}, \exists x \in \mathbb{P}: n < x < 2n$

e. The tangent of every nonzero rational number is not rational

$\forall x \in \mathbb{Q} - \{0\}, \tan x \notin \mathbb{Q}$

f. The cardinality of every finite set of integers is less than that of its power set

$\forall X \in \mathcal{P}(\mathbb{Z}), |X| < \infty \rightarrow \ |X| < |\mathcal{P}(X)|$

7. a. Every rational number has a rational square

negation: There is rational number that has no rational square

b. There is a real number whose square is negative

negation: For all real number, its square is not negative

c. For every integer there is a larger integer

negation: There is an integer that has no larger integer

d. The cube of every integer is positive negation: There is an integer that its cube is not positive

8. a.$\forall \mathrm{n} \in \mathbb{Z}, 2\mathrm{n} \geq \mathrm{n}$

   negation: $\exists \mathrm{n} \in \mathbb{Z}, 2\mathrm{n} < \mathrm{n}$

   b. $\exists \mathrm{n}, \mathrm{k} \in \mathbb{Z}: \mathrm{n} - \mathrm{k} = \mathrm{k} - \mathrm{n}$

   negation: $\forall \mathrm{n}, \mathrm{k} \in \mathbb{Z}, \mathrm{n} - \mathrm{k} \neq \mathrm{k} - \mathrm{n}$

   c. $\forall \mathrm{r} \in \mathbb{R}, \exists \mathrm{s} \in \mathbb{R}, \mathrm{rs} = 1$

   negation: $\exists \mathrm{r} \in \mathbb{R}, \forall \mathrm{s} \in \mathbb{R}, \mathrm{rs} \neq 1$

   d. $\forall \mathrm{n} \in \mathbb{Z}, n > 0 \rightarrow n^2 > 0$

   negation: $\exists \mathrm{n} \in \mathbb{Z}, n > 0 \wedge n^2 \leq 0$

   e. $\forall \varepsilon \in \mathbb{R}^+, \exists M \in \mathbb{R}^+: x > M \rightarrow |f(x) - b| < \varepsilon$

   negation: $\exists \varepsilon \in \mathbb{R}^+, \forall M \in \mathbb{R}^+: x > M \wedge |f(x) - b| \geq \varepsilon$