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Math 301

Assignment 2

1. U = {1, 2, 3, 4, 5}, A={1, 2, 3}, B={1, 3, 5}.
   a. $A \cup B = \{1, 2, 3, 5\}$
   b. $A \cap \bar{B} = \{2\}$
   c. $\overline{A - B} = \{1, 3, 4, 5\}$
   d. $\bar{A} \times \bar{B} = \{4, 5\} \times \{2, 4\} = \{(4,2), (4, 4), (5, 2), (5,4)\}$

2. Same sets as above
   a. $|A \cup B|$ = 4
   b. $|A \times B|$ = 3*3 = 9
   c. $|\overline{\mathrm{B}}|$ = 2
   d. $|A^2 \times B \times \emptyset|$ = 0
   e. $|\mathcal{P}(\{1,2,3,5,8\})| = 2^5 = 32$

3. a. $\{x \in \mathbb{Z} : x^2 < 10\} = \{-3, -2, -1, 0, 1, 2, 3\}$
   b. $\{1\} \times \{2,3\} \times \{4,5,6\}$ = {(1,2,4),(1,2,5),(1,2,6),(1,3,4),(1,3,5),(1,3,6)}
   c. $\{0,2\}^3 = \{0,2\} \times \{0,2\} \times \{0,2\}$ = {(0,0,0),(0,0,2),(0,2,0),(0,2,2),(2,0,0),(2,0,2),(2,2,0),(2,2,2)}
   d. $\mathcal{P}(\mathcal{P}(\emptyset)) = \wp(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$

4. T or F when $\mathbb{Z}$ is a set of all integers
   a. $1 \in \mathbb{Z}$ T
   b. $\{1\} \in \mathbb{Z}$ F
   c. $\{1\} \subseteq \mathbb{Z}$ T
   d. $\emptyset \in \mathbb{Z}$ F
   e. $\emptyset \subseteq \mathbb{Z}$ T

f. $\{\emptyset\} \subseteq \mathbb{Z}$ F

g. $\{1,2,3\} \in \wp(\mathbb{Z})$ T

h. $\{1,2,3\} \subseteq \wp(\mathbb{Z})$ F

5. $A_1 = \{0,1,2\}$ $A_2 = \{0,2,4\}$ $A_3 = \{0,3,6\}$ $A_4 = \{0,4,8\}$ $A_5 = \{0,5,10\}$

    a. $\bigcup_{i=1}^{5} A_i = \{0,1,2,3,4,5,6,8,10\}$

    b. $\bigcap_{i=1}^{5} A_i = \{0\}$

6. Statements, then T or F

    a. Stop, look, and listen

        Not a statement

    b. Every real number is an even integer.

        False statement

    c. Every even integer is a real number.

        True statement

    d. For any real numbers x and y, if $5x = 5y$, then $x = y$.

        True statement

    e. For any real numbers x and y, if $x^2 > y^2$, then $x > y$.

        False statement

    f. There exist sets that are finite.

        True statement

    g. There exist sets that are infinite.

        True statement

    h. 42

        Not a statement

    i. The answer is 42.

        Not a statement (but is true according to Deep Thought)

    j. For every real number x, $-2 \le \cos x \le 2$.

        True statement

7. Expression of $P \wedge Q$, $P \vee Q$, $\sim P$

   a. The integer 8 is both even and a power of 2.

      8 is even and power of 2 = 8 is even and 8 is power of 2 , So, $P \wedge Q$

   b. The identity matrix is not invertible. $\sim P$

   c. The first test is either on Wednesday or Friday.

      Test is either on Wednesday or Friday = Test is on Wednesday or test is on Friday, so $P \vee Q$

   d. $x \neq y$

      $\sim P$

   e. $x \leq y$

      $(x \leq y) = (x < y \text{ } or \text{ } x = y)$, so $P \vee Q$

   f. $x \in A \cap B$

      .$P \wedge Q$

   g. $x \in A \cup B$

      .$P \vee Q$