Jeonghyeon Woo

Math 301

HW 4

1. For every integer n, if n is odd then $\mathrm{n}^2 + n + 1$ is odd

Proof.

$\mathrm{n}^2 + n + 1 = (2k_1 + 1)^2 + (2k_1 + 1) + 1$ by the definition of odd integer.

$= 4\mathrm{k}_1^2 + 4\mathrm{k}_1 + 1 + 2\mathrm{k}_1 + 1 + 1$

$= 4\mathrm{k}_1^2 + 6k_1 + 2 + 1$

Suppose $\mathrm{k}_2 = 2\mathrm{k}_1^2 + 3k_1 + 1$, then the format above can be expressed in

$\mathrm{n}^2 + n + 1 = 2(2k_1^2 + 3k_1 + 1) + 1$

$= 2\mathrm{k}_2 + 1$

Therefore, by the definition of odd integer, $\mathrm{n}^2 + n + 1$ is odd

2. For all real numbers r and s, if both r and s are rational then rs is also rational.

Proof.

$\mathrm{r} = \frac{\mathrm{a}}{\mathrm{b}}\ and\ s = \frac{c}{d}\ while\ b \neq 0\ and\ d \neq 0$ By the definition of rational number.

$\mathrm{rs} = \frac{\mathrm{ac}}{\mathrm{bd}}, \text{and bd} \neq 0$ since $\mathrm{xy} = 0 \to \mathrm{x} = 0 \text{ or } \mathrm{y} = 0$ and by property of implication $\mathrm{x} \neq 0 \text{ and } \mathrm{y} \neq 0 \text{ then } \mathrm{xy} \neq 0.$

Suppose $\mathrm{p} = \mathrm{ac} \text{ and } \mathrm{q} = \mathrm{bd}.$

$\mathrm{rs} = \frac{\mathrm{p}}{\mathrm{q}} \text{ and } \mathrm{q} \neq 0$

Therefore, rs is rational by the def. of rational number.