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# 1 Template

## 1.1 Macros

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i=(a); i<(b); i++)
#define pb push_back
#define mp make_pair
#define debug(x) cout<<__LINE__<<": "<<#x<<" = "<<x<<endl;
#define debug2(x, y) cout<<__LINE__<<": "<<#x<<" = "<<x<<\
" "<<#y<<" = "<<y<<endl;

#define all(c) (c).begin(), (c).end()
#define F first
#define S second
#define UNIQUE(c) \
    sort(all(c)); \
    (c).resize(unique(all(c))-c.begin());

#define PI 3.1415926535897932384626433832795028841971

typedef long long ll;
typedef pair<int, int> ii;
typedef vector<int> vi;

const int INF = 0x3f3f3f3f;
const double EPS = 1e-9;

inline int cmp(double x, double y = 0, double tol = EPS) {
    return ((x <= y+tol) ? (x+tol < y) ? -1:0:1);
}
```

# 2 Numerical algorithms

## 2.1 Triângulo de Pascal

```
// Calcula os numeros binomiais (N,K) = N!/(K!(N-K)!). (N,K)
// representa o numero de maneiras de criar um subconjunto de tamanho
// K dado um conjunto de tamanho N. A ordem dos elementos nao
// importa.
const int MAXN = 50;
long long C[MAXN][MAXN];
void calc_pascal() {
    memset(C, 0, sizeof(C));
    for (int i = 0; i < MAXN; ++i) {
        C[i][0] = C[i][i] = 1;
        for (int j = 1; j < i; ++j)
            C[i][j] = C[i - 1][j - 1] + C[i - 1][j];
    }
}
/*
Pascal triangle elements:
C(33, 16) = 1.166.803.110 [int limit]
C(34, 17) = 2.333.606.220 [unsigned int limit]
C(66, 33) = 7.219.428.434.016.265.740 [int64_t limit]
C(67, 33) = 14.226.520.737.620.288.370 [uint64_t limit]
```

```
Fatorial
12 ! = 479.001.600 [(unsigned)int limit]
20 ! = 2.432.902.008.176.640.000 [(unsigned)int64_t limit]
*/
```

## 2.2 GCD-LCM

```
// Calcula o maior divisor comum entre A e B
ll A, B;
cin >> A >> B;
cout << __gcd(A, B);

// Calcula o menor multiplo comum entre A e B
ll lcm(ll A, ll B) {
    if (A and B)
        return abs(A) / __gcd(A, B) * abs(B);
    else
        return abs(A | B);
}
```

## 2.3 Bezout Theorem

```
// Determina a solucao da equacao a*x+b*y = gcd(a, b), onde a e b sao
// dois numeros naturais. Como chamar: egcd(a, b), Retorna: a tupla
// {gcd(a, b), x, y}. Determina tambem o Inverso Modular.
struct Triple {
    ll d, x, y;
    Triple(ll q, ll w, ll e) : d(q), x(w), y(e) {}
};
Triple egcd(ll a, ll b) {
    if (!b)
        return Triple(a, 1, 0);
    Triple q = egcd(b, a % b);
    return Triple(q.d, q.x - a / b * q.y);
}
// Retorna o inverso modular de A modulo N
// O inverso modular de um numero A em relacao a N eh um numero X tal
// que (A*X)%N = 1
ll invMod(ll a, ll n) {
    Triple t = egcd(a, n);
    if (t.d > 1)
        return 0;
    return (t.x % n + n) % n;
}
```

## 2.4 Teorema Chinês dos Restos

```
// crt() retorna um X tal que X = a[i] (mod m[i]). Exemplo: Para a[]
// = {1, 2, 3} e m[] = {5, 6, 7} .: X = 206. Requer: Bezout Theorem
// para calcular o inverso modular
#define MAXN 1000
int n;
ll a[MAXN], m[MAXN];
ll crt() {
    ll M = 1, x = 0;
```

```

for (int i = 0; i < n; ++i)
    M *= m[i];
for (int i = 0; i < n; ++i)
    x += a[i] * invMod(M / m[i], m[i]) * (M / m[i]);
return ((x % M) + M) % M;
}

```

## 2.5 Crivo de Eratóstenes

```

bitset<10000005> bs;
vector<int> primos;
void crivo(ll limite = 10000000LL) { // calcula primos ate limite
    primos.clear();
    bs.set();
    bs[0] = bs[1] = 0;
    for (ll i = 2; i <= limite; i++)
        if (bs[i]) {
            for (ll j = i * i; j <= limite; j += i)
                bs[j] = 0;
            primos.push_back(i);
        }
}
bool isPrime(ll N, ll limite) {
    if (N <= limite)
        return bs[N];
    for (int i = 0; i < (int)primos.size(); i++)
        if (N % primos[i] == 0)
            return false;
    return true;
}

```

## 2.6 Divisores de N

```

// Retorna todos os divisores naturais de N em O(sqrt(N)).
vector<ll> divisores(ll N) {
    vector<ll> divisors;
    for (ll div = 1, k; div * div <= N; ++div) {
        if (N % div == 0) {
            divisors.push_back(div);
            k = N / div;
            if (k != div)
                divisors.push_back(k);
        }
    }
    // caso precise ordenado
    sort(divisors.begin(), divisors.end());
    return divisors;
}

```

## 2.7 Funções com Números Primos (Crivo, Fatoração, PHI, etc)

```

// Encontra os fatores primos de N :: N = p1^e1 * ... * pi^ei
// factors armazena em first o fator primo e em segundo seu expoente
map<int, int> factors;

```

```

void primeFactors(ll N) {
    factors.clear();
    while (N % 2 == 0)
        ++factors[2], N >>= 1;
    for (ll PF = 3; PF * PF <= N; PF += 2) {
        while (N % PF == 0)
            N /= PF, factors[PF]++;
    }
    if (N > 1)
        factors[N] = 1;
}

// Funcoess derivadas dos numeros primos
void NumberTheory(ll N) {
    primeFactors(N);
    map<int, int>::iterator f; // iterador
    ll Totient = N; // Totiente ou Euler-Phi de N
    // Totient(N) = qtos naturais x, tal que x < N && gcd(x,N) == 1
    ll numDiv = 1; // Quantidade de divisores de N
    ll sumDiv = 1; // Soma dos divisores de N
    ll sumPF = 0; // Soma dos fatores primos de N (trivial)
    ll numDiffPF = factors.size(); // qtde de fatores distintos

    for (f = factors.begin(); f != factors.end(); f++) {
        ll PF = f->first, power = f->second;
        Totient -= Totient / PF;
        numDiv *= (power + 1);
        sumDiv *= ((ll)pow((double)PF, power + 1.0) - 1) / (PF - 1);
        sumPF += PF;
    }

    printf("Totiente/Euler-Phi de N = %lld\n", Totient);
    printf("qt de divisores de N = %lld\n", numDiv);
    printf("soma dos divisores de N = %lld\n", sumDiv);
    printf("qt de fatores primos distintos = %lld\n", numDiffPF);
    printf("soma dos fatores primos = %lld\n", sumPF);
}

```

```

// Calcula Euler Phi para cada valor do intervalo [1, N]
#define MM 1000010
int phi[MM];
void crivo_euler_phi(int N) {
    for (int i = 1; i <= N; i++)
        phi[i] = i;
    for (int i = 2; i <= N; i++)
        if (phi[i] == i) {
            for (int k = i; k <= N; k += i)
                phi[k] = (phi[k] / i) * (i - 1);
        }
}

// Qtde de fatores primos distintos de cada valor do range [2, MAX_N]
#define MAX_N 1000000
int NDPF[MAX_N]; //
void NumDiffPrimeFactors() {
    memset(NDPF, 0, sizeof NDPF);
    for (int i = 2; i < MAX_N; i++)
        if (NDPF[i] == 0)
            for (int j = i; j < MAX_N; j += i)
                NDPF[j]++;
}

```

```
int main() { return 0; }
```

## 2.8 Exponenciação Modular Rápida

```
/**
 * fastpow() realiza exponenciacao rapida de inteiros
 * #param ll b - base da exponenciacao
 * #param ll expo - expoente
 * #param ll mod - o resultado sera calculado modulo este valor
 * #return - o valor de (b ^ p) % mod
 * #complexidade - O(log(p))
 */

ll fastpow(ll b, ll expo, ll mod) {
    ll ret = 1, pot = b % mod;
    while (expo) {
        if (expo & 1) {
            ret = (ret * pot) % mod;
        }
        pot = (pot * pot) % mod;
        expo >>= 1;
    }
    return ret;
}
```

## 2.9 Exponenciação de Matriz

```
/**
 * fastpow() realiza exponenciacao rapida de matrizes
 * #param matrix_t M - matriz a ser elevada
 * #param ll expo - expoente
 * #param ll mod - o resultado sera calculado modulo este valor
 * #return - o resultado de (M ^ expo) % mod
 * #complexidade - O(size^3 * log(expo))
 */

#define MAX (ChangeMe) // Max size of square matrix

struct matrix_t {
    ll m[MAX][MAX];
    int sz;
    matrix_t(int _sz) {
        memset(m, 0, sizeof(m));
        sz = _sz;
    }
    matrix_t multiply(const matrix_t other, ll mod) {
        matrix_t ret(other.sz);
        rep(i, 0, sz) rep(j, 0, sz) {
            ret.m[i][j] = 0;
            rep(k, 0, sz) ret.m[i][j] =
                (ret.m[i][j] + (m[i][k] * other.m[k][j]) % mod) % mod;
        }
        return ret;
    }
};

matrix_t fastpow(matrix_t M, ll expo, ll mod) {
```

```
matrix_t ret(M.sz);
// Identity matrix
rep(i, 0, ret.sz) rep(j, 0, ret.sz) ret.m[i][j] = (i == j);

while (expo) {
    if (expo & 1)
        ret = ret.multiply(M, mod);
    M = M.multiply(M, mod);
    expo >>= 1;
}
return ret;
}
```

## 2.10 Brent Cycle Detection

```
// Dado uma sequencia formada por uma funcao f(.) e uma semente x0.
// f(x0), f(f(x0)), ..., f(f(...f(x0))), ela pode ser ciclica. Este
// algoritmo retorna o tamanho do ciclo e o valor xi que o inicia.
ii brent_cycle(int x) {
    int p = 1, length = 1, t = x, start = 0;
    int h = f(x);
    while (t != h) {
        if (p == length) {
            t = h;
            p *= 2;
            length = 0;
        }
        h = f(h);
        ++length;
    }
    t = h = x;
    for (int i = length; i != 0; --i)
        h = f(h);
    while (t != h) {
        t = f(t);
        h = f(h);
        ++start;
    }
    return ii(start, length);
}
```

## 2.11 Romberg's method - Calcula Integral (UFS2010)

```
// Calcula a integral de f[a, b]
typedef long double ld;

ld f(double x) {
    // return f(x)
}

ld romberg(ld a, ld b) {
    ld R[16][16], div = (b - a) / 2;
    R[0][0] = div * (f(a) + f(b));
    for (int n = 1; n <= 15; n++, div /= 2) {
        R[n][0] = R[n - 1][0] / 2;
        for (ld sample = a + div; sample < b; sample += 2 * div)
            R[n][0] += div * f(a + sample);
    }
}
```

```

for (int m = 1; m <= 15; m++)
    for (int n = m; n <= 15; n++)
        R[n][m] = R[n][m - 1] +
            1 / (pow(4, m) - 1) * (R[n][m - 1] - R[n - 1][m - 1]);
return R[15][15];
}

```

## 2.12 Pollard's rho algorithm (UFS2010)

```

// Retorna um fator primo de N, util para fatorizacao quando N for
// grande.
ll pollard_rho, pollard_n;
ll f(ll val) { return (val * val + pollard_rho) % pollard_n; }
ll myabs(ll a) { return a >= 0 ? a : -a; }
ll pollard(ll n) {
    srand(unsigned(time(0)));
    pollard_n = n;
    long long d = 1;
    do {
        d = 1;
        pollard_rho = rand() % n;
        long long x = 2, y = 2;
        while (d == 1)
            x = f(x), y = f(f(y)), d = __gcd(myabs(x - y), n);
    } while (d == n);
    return d;
}

```

## 2.13 Miller-Rabin's algorithm (UFS2010)

```

// Teste probabilistico de primalidade
bool miller_rabin(ll n, ll base) {
    if (n <= 1)
        return false;
    if (n % 2 == 0)
        return n == 2;
    ll s = 0, d = n - 1;
    while (d % 2 == 0)
        d /= 2, ++s;
    ll base_d = fastpow(base, d, n);
    if (base_d == 1)
        return true;
    ll base_2r = base_d;
    for (ll i = 0; i < s; ++i) {
        if (base_2r == 1)
            return false;
        if (base_2r == n - 1)
            return true;
        base_2r = base_2r * base_2r % n;
    }
    return false;
}
bool isprime(ll n) {
    if (n == 2 || n == 7 || n == 61)
        return true;
    return miller_rabin(n, 2) && miller_rabin(n, 7) &&
        miller_rabin(n, 61);
}

```

## 2.14 Quantidade de dígitos de N! na base B

```

int NumOfDigitsInFactorial(int N, int B) {
    double logFatN = 0;
    for (int i = 1; i <= N; i++)
        logFatN += log((double)i);
    int nd = floor(logFatN / log((double)B)) + 1;
    return nd;
}

```

## 2.15 Quantiade de zeros a direita de N! na base B

```

// Determina o numero de zeros a direita do fatorial de N na base B
// Ideia: Se a base for B for 10, e fatorarmos N! em fatores primos
// teremos algo como N! = 2^a * 3^b * 5^c ..., como cada par de primos
// 2 e 5 formam 10 que tem um zero, a quantidade seria min(a, c).
int NumOfTrailingZeros(int N, int B) {
    int nfact = fatora(B);
    int zeros = INF;
    // para cada fator de B, aux representa qtas vezes
    // fator[i]^expoente[i] aparece na representacao de N!
    for (int i = 0; i < nfact; i++) {
        int soma = 0;
        int NN = N;
        while (NN) {
            soma += NN / fator[i];
            NN /= fator[i];
        }
        int aux = soma / expoente[i];
        zeros = min(zeros, aux);
    }
    return zeros;
}

```

## 2.16 Baby Step Giant Step

```

// Determinar o menor E tal que B^E = N (mod P), -1 se for impossivel.
// Requer: Bezout Theorem para calcular o inverso modular
ll bsgs(ll b, ll n, ll p) {
    if (n == 1)
        return 0;
    map<ll, int> table;
    ll m = sqrt(p) + 1, pot = 1, pot2 = 1;
    for (int j = 0; j < m; ++j) {
        if (pot == n)
            return j;
        table[(n * invMod(pot, p)) % p] = j;
        pot = (pot * b) % p;
    }
    for (int i = 0; i < m; ++i) {
        if (table.find(pot2) != table.end())
            return i * m + table[pot2];
        pot2 = (pot2 * pot2) % p;
    }
    return -1;
}

```

## 2.17 Primos num intervalo

```
// Encontra os primos no intervalo [n,m]
vector<int> ret;
void primesBetween(int n, int m) {
    ret.clear();
    vector<int> primes(m - n + 1);
    for (int i = 0; i < m - n + 1; ++i)
        primes[i] = 0;
    for (int p = 2; p * p <= m; ++p) {
        int less = (n / p) * p;
        for (int j = less; j <= m; j += p)
            if (j != p && j >= n)
                primes[j - n] = 1;
    }
    for (int i = 0; i < m - n + 1; ++i) {
        if (primes[i] == 0 && n + i != 1) {
            ret.push_back(n + i);
        }
    }
}
```

## 2.18 FFT

```
typedef complex<double> comp;
const int MAX_N = 1 << 20;
int rev[MAX_N];
comp roots[MAX_N];

void preCalc(int N, int BASE) {
    for (int i = 1; i < N; ++i)
        rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (BASE - 1));
    int NN = N >> 1;
    roots[NN] = comp(1, 0);
    roots[NN + 1] = comp(cos(2 * PI / N), sin(2 * PI / N));
    for (int i = 2; i < NN; ++i)
        roots[NN + i] = roots[NN + i - 1] * roots[NN + 1];
    for (int i = NN - 1; i > 0; --i)
        roots[i] = roots[2 * i];
}

void fft(vector<comp> &a, bool invert) {
    int N = a.size();
    if (invert)
        rep(i, 0, N) a[i] = conj(a[i]);
    rep(i, 0, N) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < N; k *= 2) {
        for (int i = 0; i < N; i += 2 * k) {
            rep(j, 0, k) {
                comp B = a[i + j + k] * roots[k + j];
                a[i + j + k] = a[i + j] - B;
                a[i + j] = a[i + j] + B;
            }
        }
    }
    if (invert)
        rep(i, 0, a.size()) a[i] /= N;
}
```

```
vector<comp> multiply_real(vector<comp> a, vector<comp> b,
                           vector<comp> c) {
    int n = a.size();
    int m = b.size();

    int base = 0, N = 1;
    while (N < n + m - 1)
        base++, N <= 1;
    preCalc(N, base);

    a.resize(N, comp(0, 0));
    c.resize(N);

    rep(i, 0, b.size()) a[i] = comp(real(a[i]), real(b[i]));
    fft(a, 0);
    rep(i, 0, N) {
        int j = (N - i) & (N - 1);
        c[i] = (a[i] * a[i] - conj(a[j] * a[j])) * comp(0, -0.25);
    }
    fft(c, 1);
    return c;
}
```

## 3 Geometria 2D

### 3.1 Geometria 2D Library

```
const double EPS = 1e-9;
inline int cmp(double x, double y = 0, double tol = EPS) {
    return ((x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1);
}

struct point {
    double x, y;
    point(double x = 0, double y = 0) : x(x), y(y) {}
    point operator+(point q) const { return point(x + q.x, y + q.y); }
    point operator-(point q) const { return point(x - q.x, y - q.y); }
    point operator*(double t) const { return point(x * t, y * t); }
    point operator/(double t) const { return point(x / t, y / t); }
    int cmp(point q) const {
        if (int t = ::cmp(x, q.x))
            return t;
        return ::cmp(y, q.y);
    }
    bool operator==(point q) const { return cmp(q) == 0; }
    bool operator!=(point q) const { return cmp(q) != 0; }
    bool operator<(point q) const { return cmp(q) < 0; }
};

ostream &operator<<(ostream &os, const point &p) {
    os << "(" << p.x << "," << p.y << ")";
}

#define vec(a, b) (b - a)
typedef vector<point> polygon;

double cross(point a, point b) { return a.x * b.y - a.y * b.x; }
double dot(point a, point b) { return a.x * b.x + a.y * b.y; }
double collinear(point a, point b, point c) {
```

```

    return cmp(cross(b - a, c - a)) == 0;
}
// retorna 1 se R esta a esquerda do vetor P->Q, -1 se estiver a
// direita. 0 se P, Q e R forem colineares
int ccw(point p, point q, point r) { return cmp(cross(q - p, r - p));
}

// Rotaciona um ponto em relacao a origem em 90 graus sentido
// anti-horario
point RotateCCW90(point p) { return point(-p.y, p.x); }
// Rotaciona um ponto em relacao a origem em 90 graus sentido horario
point RotateCW90(point p) { return point(p.y, -p.x); }

// Rotaciona um ponto P em A graus no sentido anti-horario em relacao
// a
// origem; Para rotacionar no sentido horario, basta A ser negativo
point RotateCCW(point p, double a) {
    a = (a / 180.0) * acos(-1.0); // convertendo para radianos
    return point(p.x * cos(a) - p.y * sin(a),
                p.x * sin(a) + p.y * cos(a));
}
// Rotaciona P em A graus em relacao a Q.
point RotateCCW(point p, point q, double a) {
    return RotateCCW(p - q, a) + q;
}
// Tamanho ou norma de um vetor
double abs(point u) { return sqrt(dot(u, u)); }
// Projeta o vetor A sobre a direcao do vetor B
point project(point a, point b) { return b * (dot(a, b) / dot(b, b));
}
// Retorna a projecao do ponto P sobre reta definida por [A,B]
point projectPointLine(point p, point a, point b) {
    return p + project(p - a, b - a);
}
// Retorna o angulo que p faz com +x
double arg(point p) { return atan2(p.y, p.x); }
// Retorna o angulo entre os vetores AB e AC
double arg(point b, point a, point c) {
    point u = b - a, v = c - a;
    return atan2(cross(u, v), dot(u, v));
}

////////////////////////////////////
//////////////////////////////////// Segmentos, Retas //////////////////////////////////////
////////////////////////////////////

// Determina se P esta entre o segmento fechado [A,B], inclusive
bool between(point p, point a, point b) {
    return collinear(p, a, b) && dot(a - p, b - p) <= 0;
}

/* Distancia de ponto P para reta que passa por [A,B]. Armazena em C
 * (por ref) o ponto projecao de P na reta. */
double distancePointLine(point p, point a, point b, point &c) {
    c = projectPointLine(p, a, b);
    return fabs(cross(p - a, b - a)/abs(a - b); // or abs(p-c);
}

/* Distancia de ponto P ao segmento [A,B]. Armazena em C (por ref) o
 * ponto de projecao de P em [A,B]. Se este ponto estiver fora do
 * segmento, eh retornado o mais proximo. */

```

```

double distancePointSeg(point p, point a, point b, point &c) {
    if ((b - a) * (p - a) <= 0) {
        c = a;
        return abs(a - p);
    }
    if ((a - b) * (p - b) <= 0) {
        c = b;
        return abs(b - p);
    }

    c = projectPointLine(p, a, b);
    return fabs(cross(p - a, b - a)/abs(a - b); // or abs(p-c);
}

// Determina se os segmentos [A, B] e [C, D] se tocam
bool seg_intersect(point a, point b, point c, point d) {
    int d1, d2, d3, d4;
    d1 = ccw(c, a, d);
    d2 = ccw(c, b, d);
    d3 = ccw(a, c, b);
    d4 = ccw(a, d, b);
    if (d1 * d2 == -1 && d3 * d4 == -1)
        return true;
    if (d1 == 0 && between(c, a, d))
        return true;
    if (d2 == 0 && between(c, b, d))
        return true;
    if (d3 == 0 && between(a, c, b))
        return true;
    if (d4 == 0 && between(a, d, b))
        return true;
    return false;
}

// Encontra a interseccao das retas (p-q) e (r-s) assumindo que existe
// apenas 1 interseccao. Se for entre segmentos, verificar se
// interseptam primeiro.
point line_intersect(point p, point q, point r, point s) {
    point a = q - p, b = s - r, c = point(cross(p, q), cross(r, s));
    double x = cross(point(a.x, b.x), c);
    double y = cross(point(a.y, b.y), c);
    return point(x, y) / cross(a, b);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(point a, point b, point c, point d) { //!! Nao
                                                         //!! testado
    return fabs(cross(b - a, c - d)) < EPS;
}
bool LinesCollinear(point a, point b, point c,
                    point d) { //!! Nao testado
    return LinesParallel(a, b, c, d) &&
        fabs(cross(a - b, a - c)) < EPS &&
        fabs(cross(c - d, c - a)) < EPS;
}

////////////////////////////////////
//////////////////////////////////// Triangulos //////////////////////////////////////
////////////////////////////////////

bool pointInTriangle(point p, point a, point b, point c) {

```

```

// TODO
}

// Heron's formula - area do triangulo(a,b,c) -1 se nao existe
double area_heron(double a, double b, double c) {
    if (a < b)
        swap(a, b);
    if (a < c)
        swap(a, c);
    if (b < c)
        swap(b, c);
    if (a > b + c)
        return -1;
    return sqrt((a + (b + c)) * (c - (a - b)) * (c + (a - b)) *
                (a + (b - c)) / 16.0);
}

////////////////////////////////////
////////////////////////////////////  CIRCULOS  //////////////////////////////////////
////////////////////////////////////

bool pointInCircle(point p, point c, double radius) {
    // Todo
}

/*Dado dois pontos (A, B) de uma circunferencia e seu raio R, eh
 * possivel obter seus possiveis centros (C1 e C2). Para obter o outro
 * centro, basta inverter os paramentros */
bool circle2PtsRad(point a, point b, double r, point &c) {
    point aux = a - b;
    double d = dot(aux, aux);
    double det = r * r / d - 0.25;
    if (det < 0.0)
        return false;
    double h = sqrt(det);
    c.x = (a.x + b.x) * 0.5 + (a.y - b.y) * h;
    c.y = (a.y + b.y) * 0.5 + (b.x - a.x) * h;
    return true;
}

// Menor distancia entre dois pontos numa esfera de raio r
// lat = [-90,90]; long = [-180,180]
double spherical_distance(double lt1, double lo1, double lt2,
                          double lo2, double r) {
    double pi = acos(-1);
    double a = pi * (lt1 / 180.0), b = pi * (lt2 / 180.0);
    double c = pi * ((lo2 - lo1) / 180.0);
    return r * acos(sin(a) * sin(b) + cos(a) * cos(b) * cos(c));
}

////////////////////////////////////
////////////////////////////////////  PLANOS  //////////////////////////////////////
////////////////////////////////////

// Distancia entre (x,y,z) e plano ax+by+cz=d
double distancePointPlane(double x, double y, double z, double a,
                           double b, double c, double d) {
    return fabs(a * x + b * y + c * z - d) / sqrt(a * a + b * b + c * c);
}

```

```

/**[Inicio] Funcoes que usam numeros complexos para pontos**
typedef complex<double> cxpt;
struct circle {
    cxpt c;
    double r;
    circle(cxpt c, double r) : c(c), r(r) {}
    circle() {}
};
double cross(const cxpt &a, const cxpt &b) {
    return imag(conj(a) * b);
}
double dot(const cxpt &a, const cxpt &b) { return real(conj(a) * b); }

// Area da interseccao de dois circulos
double circ_inter_area(circle &a, circle &b) {
    double d = abs(b.c - a.c);
    if (d <= (b.r - a.r))
        return a.r * a.r * M_PI;
    if (d <= (a.r - b.r))
        return b.r * b.r * M_PI;
    if (d >= a.r + b.r)
        return 0;
    double A = acos((a.r * a.r + d * d - b.r * b.r) / (2 * a.r * d));
    double B = acos((b.r * b.r + d * d - a.r * a.r) / (2 * b.r * d));
    return a.r * a.r * (A - 0.5 * sin(2 * A)) +
        b.r * b.r * (B - 0.5 * sin(2 * B));
}

// Pontos de interseccao de dois circulos
// Intersects two circles and intersection points are in 'inter'
// -1-> outside, 0-> inside, 1-> tangent, 2-> 2 intersections
int circ_circ_inter(circle &a, circle &b, vector<cxpt> &inter) {
    double d2 = norm(b.c - a.c), rS = a.r + b.r, rD = a.r - b.r;
    if (d2 > rS * rS)
        return -1;
    if (d2 < rD * rD)
        return 0;
    double ca = 0.5 * (1 + rS * rD / d2);
    cxpt z = cxpt(ca, sqrt((a.r * a.r / d2) - ca * ca));
    inter.push_back(a.c + (b.c - a.c) * z);
    if (abs(z.imag()) > EPS)
        inter.push_back(a.c + (b.c - a.c) * conj(z));
    return inter.size();
}

// Line-circle intersection
// Intersects (infinite) line a-b with circle c
// Intersection points are in 'inter'
// 0 -> no intersection, 1 -> tangent, 2 -> two intersections
int line_circ_inter(cxpt a, cxpt b, circle c, vector<cxpt> &inter) {
    c.c -= a;
    b -= a;
    cxpt m = b * real(c.c / b);
    double d2 = norm(m - c.c);
    if (d2 > c.r * c.r)
        return 0;
    double l = sqrt((c.r * c.r - d2) / norm(b));
    inter.push_back(a + m + l * b);
    if (abs(l) > EPS)
        inter.push_back(a + m - l * b);
    return inter.size();
}

```



```

}
/**[FIM] Funcoes que usam numeros complexos para pontos**/

```

## 4 Polígonos 2D

### 4.1 Polígono 2D Library

```

/*Poligono eh representado como um array de pontos T[i] sao os
vertices
do poligono. Existe uma aresta que conecta T[i] com T[i+1], e
T[size-1]
com T[0]. Logo assume-se que T[0] != T[size-1]
Poligono simples: Aquele em que as arestas nao se interceptam.
Convexo:
O angulo interno de T[i] com T[i-1] e T[i+1] <= 180. Concavo: Existe
algum i que nao satisfaz a condicao anterior*/

```

```

/* Retorna a area com sinal de um poligono T. Se area > 0, T esta
* listado na ordem CCW */
double signedArea(const polygon &T) {
    double area = 0;
    int n = T.size();
    if (n < 3)
        return 0;
    rep(i, 0, n) area += cross(T[i], T[(i + 1) % n]);
    return (area / 2.0);
}

```

```

/* Retorna a area de um poligono T. (pode ser concavo ou convexo) em
* O(N)*/
double poly_area(const polygon &T) { return fabs(signedArea(T)); }

```

```

/* Retorna a centroide de um poligono T em O(N)*/

```

```

point centroide(const polygon &T) {
    int n = T.size();
    double sgnArea = signedArea(T);

    point c = point(0, 0);
    rep(i, 0, n) {
        int k = (i + 1) % n;
        c = c + (T[i] + T[k]) * cross(T[i], T[k]);
    }
    c = c / (sgnArea * 6.0);
    return c;
}

```

```

/* Retorna o perimetro do poligono T. (pode n funcionar como esperado
* se o poligono for uma linha reta (caso degenerado))*/

```

```

double poly_perimeter(polygon &T) {
    double perimeter = 0;
    int n = T.size();
    if (n < 3)
        return 0;
    rep(i, 0, n) perimeter += abs(T[i] - T[(i + 1) % n]);
    return perimeter;
}

```

```

// tests whether or not a given polygon (in CW or CCW order) is simple

```

```

bool isSimple(const polygon &p) { // nao testado
    for (int i = 0; i < p.size(); i++) {
        for (int k = i + 1; k < p.size(); k++) {
            int j = (i + 1) % p.size();
            int l = (k + 1) % p.size();
            if (i == l || j == k)
                continue;
            if (seg_intersect(p[i], p[j], p[k], p[l]))
                return false;
        }
    }
    return true;
}

```

```

// Retorna True se T for convexo. O(N)

```

```

bool isConvex(polygon &T) {
    int n = T.size();
    if (n < 3)
        return false;

    int giro = 0;
    rep(i, 0, n) { // encontra um giro valido
        int t = ccw(T[i], T[(i + 1) % n], T[(i + 2) % n]);
        if (t != 0)
            giro = t;
    }
    if (giro == 0)
        return false; // todos pontos sao colineares

    rep(i, 0, n) {
        int t = ccw(T[i], T[(i + 1) % n], T[(i + 2) % n]);
        if (t != 0 && t != giro)
            return false;
    }
    return true;
}

```

```

// Determina se P pertence a T, funciona para convexo ou concavo
// -1 borda, 0 fora, 1 dentro. O(N)

```

```

int in_poly(point p, polygon &T) {
    double a = 0;
    int N = T.size();
    rep(i, 0, N) {
        if (between(p, T[i], T[(i + 1) % N]))
            return -1;
        a += arg(T[i], p, T[(i + 1) % N]);
    }
    return cmp(a) != 0;
}

```

```

// determina se P pertence a B, funciona APENAS para convexo

```

```

bool PointInConvexPolygon(point P, const polygon &B) {
    int ini = 1, fim = B.size() - 2, mid, pos = -1;
    int giro = -1; // sentido horario
    while (ini <= fim) {
        mid = (ini + fim) / 2;
        int aux = ccw(B[0], B[mid], P);
        if (aux == giro) {
            pos = mid;
            ini = mid + 1;
        } else {

```

```

    fim = mid - 1;
}
}
if (pos == -1)
    return false;
if (ccw(B[0], B[pos], P) != giro * -1 &&
    ccw(B[0], B[pos + 1], P) != giro &&
    ccw(B[pos], B[pos + 1], P) == giro) // giro || 0 na borda
    return true;
return false;
}

// Determina o poligono interseccao de P e Q
// P e Q devem estar orientados anti-horario.
polygon poly_intersect(polygon &P, polygon &Q) {
    int m = Q.size(), n = P.size();
    int a = 0, b = 0, aa = 0, ba = 0, inflag = 0;
    polygon R;
    while ((aa < n || ba < m) && aa < 2 * n && ba < 2 * m) {
        point p1 = P[a], p2 = P[(a + 1) % n], q1 = Q[b],
            q2 = Q[(b + 1) % m];
        point A = p2 - p1, B = q2 - q1;
        int cross = cmp(cross(A, B)), ha = ccw(p2, q2, p1),
            hb = ccw(q2, p2, q1);
        if (cross == 0 && ccw(p1, q1, p2) == 0 && cmp(dot(A, B)) < 0) {
            if (between(q1, p1, p2))
                R.push_back(q1);
            if (between(q2, p1, p2))
                R.push_back(q2);
            if (between(p1, q1, q2))
                R.push_back(p1);
            if (between(p2, q1, q2))
                R.push_back(p2);
            if (R.size() < 2)
                return polygon();
            inflag = 1;
            break;
        } else if (cross != 0 && seg_intersect(p1, p2, q1, q2)) {
            if (inflag == 0)
                aa = ba = 0;
            R.push_back(line_intersect(p1, p2, q1, q2));
            inflag = (hb > 0) ? 1 : -1;
        }
        if (cross == 0 && hb < 0 && ha < 0)
            return R;
        bool t = cross == 0 && hb == 0 && ha == 0;
        if (t ? (inflag == 1) : (cross >= 0) ? (ha <= 0) : (hb > 0)) {
            if (inflag == -1)
                R.push_back(q2);
            ba++;
            b++;
            b %= m;
        } else {
            if (inflag == 1)
                R.push_back(p2);
            aa++;
            a++;
            a %= n;
        }
    }
    if (inflag == 0) {

```

```

        if (in_poly(P[0], Q))
            return P;
        if (in_poly(Q[0], P))
            return Q;
    }
    R.erase(unique(all(R), R.end()));
    if (R.size() > 1 && R.front() == R.back())
        R.pop_back();
    return R;
}

```

## 4.2 Convex Hull

*/\*Encontra o convex hull de um conjunto de pontos em  $O(N \log N)$   
 pivot: Ponto base para a criacao do convex hull;  
 radial\_lt(): Ordena os pontos em sentido anti-horario (ccw).  
 Input: Conjunto de pontos 2D;  
 Output: Conjunto de pontos do convex hull, no sentido anti-horario;*

*(1) Se for preciso manter pontos colineares na borda do convex hull,  
 essa  
 parte evita que eles sejam removidos;  
 \*/*

```

point pivot;
bool radial_lt(point a, point b) {
    int R = ccw(pivot, a, b);
    if (R == 0) // sao colineares
        return (pivot - a) * (pivot - a) < (pivot - b) * (pivot - b);
    else
        return (R == 1); // 1 se A esta a direita de (pivot->B)
}
vector<point> convexhull(vector<point> &T) {
    // Se for necessario remover pontos duplicados
    sort(T.begin(), T.end()); // ordena por x e por y
    T.resize(unique(T.begin(), T.end()) - T.begin());

    int tam = 0, n = T.size();
    vector<point> U; // convex hull

    int idx = min_element(T.begin(), T.end()) - T.begin();
    // nesse caso, pivot = ponto com menor x, depois menor y
    pivot = T[idx];
    swap(T[0], T[idx]);
    sort(++T.begin(), T.end(), radial_lt);

    /*(1)*/ int k;
    for (k = n - 2; k >= 0 && ccw(T[0], T[n - 1], T[k]) == 0; k--);
    reverse((k + 1) + all(T)); /*(1)*/

    // troque <= por < para manter pontos colineares na borda
    for (int i = 0; i < T.size(); i++) {
        while (tam > 1 && ccw(U[tam - 2], U[tam - 1], T[i]) <= 0)
            U.pop_back(), tam--;
        U.pb(T[i]);
        tam++;
    }
    return U;
}

```

## 4.3 Minimum Enclosing Circle

```
// Finds a circle of the minimum area enclosing a 2D point set.
typedef pair<point, double> circle; // {ponto,raio}
bool in_circle(circle C, point p) { // ponto dentro de circulo?
    return cmp(abs(p - C.first), C.second) <= 0;
}
// menor circulo que engloba o triangulo (P,Q,R)
point circumcenter(point p, point q, point r) {
    point a = p - r, b = q - r, c, ret;
    c = point(dot(a, p + r), dot(b, q + r)) * 0.5;
    ret = point(cross(c, point(a.y, b.y)), cross(point(a.x, b.x), c)) /
        cross(a, b);
    return ret;
}
circle spanning_circle(const vector<point> &T) {
    int n = T.size();
    random_shuffle(all(T));
    circle C(point(), -INF);
    rep(i, 0, n) if (!in_circle(C, T[i])) {
        C = circle(T[i], 0);
        rep(j, 0, i) if (!in_circle(C, T[j])) {
            C = circle((T[i] + T[j]) / 2, abs(T[i] - T[j]) / 2);
            rep(k, 0, j) if (!in_circle(C, T[k])) {
                point O = circumcenter(T[i], T[j], T[k]);
                C = circle(O, abs(O - T[k]));
            }
        }
    }
    return C;
}
```

## 5 Geometria 3D

### 5.1 Geometria 3D Library

```
#define LINE 0
#define SEGMENT 1
#define RAY 2
int sgn(double x) { return (x > EPS) - (x < -EPS); }

#define vec(ini, fim) (fim - ini)
struct PT {
    double x, y, z;
    PT() { x = y = z = 0; }
    PT(double x, double y, double z) : x(x), y(y), z(z) {}
    PT operator+(PT q) { return PT(x + q.x, y + q.y, z + q.z); }
    PT operator-(PT q) { return PT(x - q.x, y - q.y, z - q.z); }
    PT operator*(double d) { return PT(x * d, y * d, z * d); }
    PT operator/(double d) { return PT(x / d, y / d, z / d); }
    double dist2() const { return x * x + y * y + z * z; }
    double dist() const { return sqrt(dist2()); }
    bool operator==(const PT &a) const {
        return fabs(x - a.x) < EPS && fabs(y - a.y) < EPS &&
            fabs(z - a.z) < EPS;
    }
};
```

```
double dot(PT A, PT B) { return A.x * B.x + A.y * B.y + A.z * B.z; }
PT cross(PT A, PT B) {
    return PT(A.y * B.z - A.z * B.y, A.z * B.x - A.x * B.z,
        A.x * B.y - A.y * B.x);
}
bool collinear(PT A, PT B, PT C) {
    return sgn(cross(B - A, C - A)) == 0;
}

inline double det(double a, double b, double c, double d) {
    return a * d - b * c;
}
inline double det(double a11, double a12, double a13, double a21,
    double a22, double a23, double a31, double a32,
    double a33) {
    return a11 * det(a22, a23, a32, a33) -
        a12 * det(a21, a23, a31, a33) + a13 * det(a21, a22, a31,
        a32);
}
inline double det(const PT &a, const PT &b, const PT &c) {
    return det(a.x, a.y, a.z, b.x, b.y, b.z, c.x, c.y, c.z);
}

// tamanho do vetor A
double norma(PT A) { return sqrt(dot(A, A)); }

// distancia^2 de (a->b)
double distSq(PT a, PT b) { return dot(a - b, a - b); }

// Projeta vetor A sobre o vetor B
PT project(PT A, PT B) { return B * dot(A, B) / dot(B, B); }

// Verifica se existe interseccao de segmentos
// (assumir que [A,B] e [C,D] sao coplanares)
bool seg_intersect(PT A, PT B, PT C, PT D) {
    return cmp(dot(cross(A - B, C - B), cross(A - B, D - B))) <= 0 &&
        cmp(dot(cross(C - D, A - D), cross(C - D, B - D))) <= 0;
}

// square distance between point and line, ray or segment
double ptLineDistSq(PT s1, PT s2, PT p, int type) {
    double pd2 = distSq(s1, s2);
    PT r;
    if (pd2 == 0)
        r = s1;
    else {
        double u = dot(p - s1, s2 - s1) / pd2;
        r = s1 + (s2 - s1) * u;
        if (type != LINE && u < 0.0)
            r = s1;
        if (type == SEGMENT && u > 1.0)
            r = s2;
    }
    return distSq(r, p);
}

// Distancia de ponto P ao segmento [A,B]
double dist_point_seg(PT P, PT A, PT B) {
    PT PP = A + project(P - A, B - A);
```

```

    if (cmp(norma(A - PP) + norma(PP - B), norma(A - B)) == 0)
        return norma(P - PP); // distance point-line!
    else
        return min(norma(P - A), norma(P - B));
}

// Distance between lines ab and cd. TODO: Test this
double lineLineDistance(PT a, PT b, PT c, PT d) {
    PT v1 = b - a;
    PT v2 = d - c;
    PT cr = cross(v1, v2);
    if (dot(cr, cr) < EPS) {
        PT proj = v1 * (dot(v1, c - a) / dot(v1, v1));
        return sqrt(dot(c - a - proj, c - a - proj));
    } else {
        PT n = cr / sqrt(dot(cr, cr));
        PT p = dot(n, c - a);
        return sqrt(dot(p, p));
    }
}

// Menor distancia do segmento [A,B] ao segmento [C,D] (lento*)
#define dps dist_point_seg
double dist_seg_seg(PT A, PT B, PT C, PT D) {
    PT E = project(A - D, cross(B - A, D - C));
    // distance between lines!
    if (seg_intersect(A, B, C + E, D + E)) {
        return norma(E);
    } else {
        double dA = dps(A, C, D), dB = dps(B, C, D);
        double dC = dps(C, A, B), dD = dps(D, A, B);
        return min(min(dA, dB), min(dC, dD));
    }
}

// Menor distancia do segmento [A,B] ao segmento [C,D] (rapido*)
double dist_seg_seg2(PT A, PT B, PT C, PT D) {
    PT u(B - A), v(D - C), w(A - C);
    double a = dot(u, u), b = dot(u, v);
    double c = dot(v, v), d = dot(u, w), e = dot(v, w);

    double DD = a * c - b * b;
    double sc, sN, sD = DD;
    double tc, tN, tD = DD;
    if (DD < EPS) {
        sN = 0, sD = 1, tN = e, tD = c;
    } else {
        sN = (b * e - c * d);
        tN = (a * e - b * d);
        if (sN < 0) {
            sN = 0, tN = e, tD = c;
        } else if (sN > sD) {
            sN = sD, tN = e + b, tD = c;
        }
    }
    if (tN < 0) {
        tN = 0;
        if (-d < 0)
            sN = 0;
        else if (-d > a)
            sN = sD;
    }
}

```

```

    else {
        sN = -d;
        sD = a;
    }
} else if (tN > tD) {
    tN = tD;
    if ((-d + b) < 0)
        sN = 0;
    else if (-d + b > a)
        sN = sD;
    else {
        sN = -d + b;
        sD = a;
    }
}
sc = fabs(sN) < EPS ? 0 : sN / sD;
tc = fabs(tN) < EPS ? 0 : tN / tD;
PT dP = w + (u * sc) - (v * tc);
return norma(dP);
}

// Distancia de Ponto a Triangulo, dps = dist_point_seg
double dist_point_tri(PT P, PT A, PT B, PT C) {
    PT N = cross(B - A, C - A);
    PT PP = P - project(P - A, N);
    PT R1, R2, R3;
    R1 = cross(B - A, PP - A);
    R2 = cross(C - B, PP - B);
    R3 = cross(A - C, PP - C);

    if (cmp(dot(R1, R2)) >= 0 && cmp(dot(R2, R3)) >= 0 &&
        cmp(dot(R3, R1)) >= 0) {
        return norma(P - PP);
    } else {
        return min(dps(P, A), min(dps(P, B), min(dps(P, C), dps(P, A, C))));
    }
}

// compute a, b, c, d such that all points lie on ax + by + cz = d.
// TODO: test this
void planeFromPts(PT p1, PT p2, PT p3, double &a, double &b, double
    &c,
    double &d) {
    PT normal = cross(p2 - p1, p3 - p1);
    a = normal.x;
    b = normal.y;
    c = normal.z;
    d = -a * p1.x - b * p1.y - c * p1.z;
}

// project point onto plane. TODO: test this
PT ptPlaneProj(PT p, double a, double b, double c, double d) {
    double l =
        (a * p.x + b * p.y + c * p.z + d) / (a * a + b * b + c * c);
    return PT(p.x - a * l, p.y - b * l, p.z - c * l);
}

// distance from point p to plane ax + by + cz + d = 0
double ptPlaneDist(PT p, double a, double b, double c, double d) {
    return fabs(a * p.x + b * p.y + c * p.z + d) /
        sqrt(a * a + b * b + c * c);
}

```

```

}

// distance between parallel planes  $ax + by + cz + d1 = 0$  and
//  $ax + by + cz + d2 = 0$ 
double planePlaneDist(double a, double b, double c, double d1,
                      double d2) {
    return fabs(d1 - d2) / sqrt(a * a + b * b + c * c);
}

// Volume de Tetraedro
double signedTetrahedronVol(PT A, PT B, PT C, PT D) {
    double A11 = A.x - B.x;
    double A12 = A.x - C.x;
    double A13 = A.x - D.x;
    double A21 = A.y - B.y;
    double A22 = A.y - C.y;
    double A23 = A.y - D.y;
    double A31 = A.z - B.z;
    double A32 = A.z - C.z;
    double A33 = A.z - D.z;
    double det = A11 * A22 * A33 + A12 * A23 * A31 + A13 * A21 * A32 -
                 A11 * A23 * A32 - A12 * A21 * A33 - A13 * A22 * A31;
    return det / 6;
}

// Parameter is a vector of vectors of points - each interior vector
// represents the 3 points that make up 1 face, in any order.
// Note: The polyhedron must be convex, with all faces given as
// triangles.
double polyhedronVol(vector<vector<PT>> poly) {
    int i, j;
    PT cent(0, 0, 0);
    for (i = 0; i < poly.size(); i++)
        for (j = 0; j < 3; j++)
            cent = cent + poly[i][j];
    cent = cent * (1.0 / (poly.size() * 3));
    double v = 0;
    for (i = 0; i < poly.size(); i++)
        v += fabs(signedTetrahedronVol(cent, poly[i][0], poly[i][1],
                                       poly[i][2]));
    return v;
}

// Outras implementacoes [Usa struct PT]

struct line { // reta definida por um ponto p e direcao v
    PT p, v;
    line();
    line(const PT &p, const PT &v) : p(p), v(v) { assert(!(v == PT())); }
    bool on(const PT &pt) const { return cross(pt - p, v) == PT(); }
};

struct plane {
    PT n;
    double d;
    plane() : d(0) {}
    plane(const PT &p1, const PT &p2, const PT &p3) {
        n = cross(p2 - p1, p3 - p1);
        d = -dot(n, p1);
        assert(side(p1) == 0);
        assert(side(p2) == 0);
    }

```

```

    assert(side(p3) == 0);
}
int side(const PT &p) const { return sgn(dot(n, p) + d); }
};

```

// intersecao de retas

```

int intersec(const line &l1, const line &l2, PT &res) {
    assert(!(l1.v == PT()));
    assert(!(l2.v == PT()));
    if (cross(l1.v, l2.v) == PT()) {
        if (cross(l1.v, l1.p - l2.p) == PT())
            return 2; // same
        return 0; // parallel
    }
    PT n = cross(l1.v, l2.v);
    PT p = l2.p - l1.p;
    if (sgn(dot(n, p)))
        return 0; // skew
    double t;
    if (sgn(n.x))
        t = (p.y * l2.v.z - p.z * l2.v.y) / n.x;
    else if (sgn(n.y))
        t = (p.z * l2.v.x - p.x * l2.v.z) / n.y;
    else if (sgn(n.z))
        t = (p.x * l2.v.y - p.y * l2.v.x) / n.z;
    else
        assert(false);
    res = l1.p + l1.v * t;
    assert(l1.on(res));
    assert(l2.on(res));
    return 1; // intersects
}

```

// distancia entre 2 retas

```

double dist(const line &l1, const line &l2) {
    PT ret = l1.p - l2.p;
    ret = ret - l1.v * (dot(l1.v, ret) / l1.v.dist2());
    PT tmp = l2.v - l1.v * (dot(l1.v, l2.v) / l1.v.dist2());
    if (sgn(tmp.dist2()))
        ret = ret - tmp * (dot(tmp, ret) / tmp.dist2());
    assert(fabs(dot(ret, l1.v)) < eps);
    assert(fabs(dot(ret, tmp)) < eps);
    assert(fabs(dot(ret, l2.v)) < eps);
    return ret.dist();
}

```

// Retorna os dois pontos mais proximos entre l1 e l2

```

void closest(const line &l1, const line &l2, PT &p1, PT &p2) {
    if (cross(l1.v, l2.v) == PT()) {
        p1 = l1.p;
        p2 = l2.p - l1.v * (dot(l1.v, l2.p - l1.p) / l1.v.dist2());
        return;
    }
    PT p = l2.p - l1.p;
    double t1 =
        (dot(l1.v, p) * l2.v.dist2() - dot(l1.v, l2.v) * dot(l2.v, p)) /
        cross(l1.v, l2.v).dist2();
    double t2 =
        (dot(l2.v, l1.v) * dot(l1.v, p) - dot(l2.v, p) * l1.v.dist2()) /
        cross(l2.v, l1.v).dist2();
    p1 = l1.p + l1.v * t1;

```

```

    p2 = l2.p + l2.v * t2;
    assert(l1.on(p1));
    assert(l2.on(p2));
}

// retorna a intersecao de reta com plano [retorna 1 se intersecao for
// pt]
int cross(const line &l, const plane &pl, PT &res) {
    double d = dot(pl.n, l.v);
    if (sgn(d) == 0) {
        return (pl.side(l.p) == 0) ? 2 : 0;
    }
    double t = (-dot(pl.n, l.p) - pl.d) / d;
    res = l.p + l.v * t;
#ifdef DEBUG
    assert(pl.side(res) == 0);
#endif
    return 1;
}

bool cross(const plane &p1, const plane &p2, const plane &p3,
           PT &res) {
    double d = det(p1.n, p2.n, p3.n);
    if (sgn(d) == 0) {
        return false;
    }
    PT px(p1.n.x, p2.n.x, p3.n.x);
    PT py(p1.n.y, p2.n.y, p3.n.y);
    PT pz(p1.n.z, p2.n.z, p3.n.z);
    PT p(-p1.d, -p2.d, -p3.d);
    res = PT(det(p, py, pz) / d, det(px, p, pz) / d, det(px, py, p) /
            d);
#ifdef DEBUG
    assert(p1.side(res) == 0);
    assert(p2.side(res) == 0);
    assert(p3.side(res) == 0);
#endif
    return true;
}

// retorna reta da intersecao de dois planos
int cross(const plane &p1, const plane &p2, line &res) {
    res.v = cross(p1.n, p2.n);
    if (res.v == PT()) {
        if ((p1.n * (p1.d / p1.n.dist2())) ==
            (p2.n * (p2.d / p2.n.dist2())))
            return 2;
        else
            return 0;
    }
    plane p3;
    p3.n = res.v;
    p3.d = 0;
    bool ret = cross(p1, p2, p3, res.p);
    assert(ret);
    assert(p1.side(res.p) == 0);
    assert(p2.side(res.p) == 0);
    return 1;
}

// testes

```

```

int main() {
    {
        line l;
        l.p = PT(1, 1, 1);
        l.v = PT(1, 0, -1);
        plane p(PT(10, 11, 12), PT(9, 8, 7), PT(1, 3, 2));
        PT res;
        assert(cross(l, p, res) == 1);
    }
    {
        plane p1(PT(1, 2, 3), PT(4, 5, 6), PT(-1, 5, -4));
        plane p2(PT(3, 2, 1), PT(6, 5, 4), PT(239, 17, -42));
        line l;
        assert(cross(p1, p2, l) == 1);
    }
    {
        plane p1(PT(1, 2, 3), PT(4, 5, 6), PT(-1, 5, -4));
        plane p2(PT(1, 2, 3), PT(7, 8, 9), PT(3, -1, 10));
        line l;
        assert(cross(p1, p2, l) == 2);
    }
    {
        plane p1(PT(1, 2, 3), PT(4, 5, 6), PT(-1, 5, -4));
        plane p2(PT(1, 2, 4), PT(4, 5, 7), PT(-1, 5, -3));
        line l;
        assert(cross(p1, p2, l) == 0);
    }

    line l1, l2;
    while (l1.p.load()) {
        l1.v.load();
        l1.v = l1.v - l1.p;
        l2.p.load();
        l2.v.load();
        l2.v = l2.v - l2.p;
        if (l1.v == PT() || l2.v == PT())
            continue;
        PT res;
        int cnt = intersec(l1, l2, res);
        double d = dist(l1, l2);
        if (fabs(d) < eps)
            assert(cnt >= 1);
        else
            assert(cnt == 0);
        PT p1, p2;
        closest(l1, l2, p1, p2);
        assert(fabs((p1 - p2).dist() - d) < eps);
    }
    plane a(PT(1, 0, 0), PT(0, 1, 0), PT(0, 0, 1));
    plane b(PT(-1, 0, 0), PT(0, -1, 0), PT(0, 0, -1));
    line l;
    assert((cross(a, b, l)) == 0);
    return 0;
}

```

## 6 Grafos

### 6.1 Topological Sort

```
// Ordenacao topologia baseado em BFS. Ideia: Processar os vertices
// que nao tem aresta chegando neles. Apos processar, remover as
// arestas dele para seus vizinhos. Os vizinhos que nao tiverem mais
// arestas chegando sao inseridos na fila para serem processados
// depois.
#define MAXV 100001
vector<int> adj[MAXV];
vector<int> ordem;
void topo_sort(int N) {
    queue<int> q;
    // para mudar a ordem que os vertices sao processados pode-se se
    // usar uma priority_queue, outra estrutura para ordenar os vertices
    vector<int> in_degree(N, 0);

    rep(i, 0, N) rep(j, 0, adj[i].size()) in_degree[adj[i][j]]++;

    rep(i, 0, N) if (in_degree[i] == 0) q.push(i);
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        ordem.push_back(u);
        rep(i, 0, adj[u].size()) {
            int v = adj[u][i];
            in_degree[v]--;
            if (in_degree[v] == 0)
                q.push(v);
        }
    }
    if (ordem.size() != N) {
        // grafo contem ciclos, nao eh um DAG
    }
}
int main() { return 0; }
```

## 6.2 Dijkstra

```
#define MAXV 100000
int dist[MAXV], pi[MAXV]; // dist from s and pointer to parent
vector<ii> adj[MAXV]; // edge = {v, dist}
int dijkstra(int s, int t, int n) {
    priority_queue<ii> pq;
    memset(pi, -1, sizeof pi);
    memset(dist, INF, sizeof dist);
    pq.push(ii(dist[s] = 0, s));
    while (!pq.empty()) {
        ii top = pq.top();
        pq.pop();
        int u = top.second, d = -top.first;
        if (d != dist[u])
            continue;
        if (u == t)
            break; // terminou antes
        rep(i, 0, (int)adj[u].size()) {
            int v = adj[u][i].F;
            int cost = adj[u][i].S;
            if (dist[v] > dist[u] + cost) {
                dist[v] = dist[u] + cost;
                pi[v] = u;
                pq.push(ii(-dist[v], v));
            }
        }
    }
}
```

```
}
}
return dist[t];
}

int main() { return 0; }
```

## 6.3 Floyd-Warshall

```
#define MAXV 401
int adj[MAXV][MAXV], path[MAXV][MAXV];
int n, m; // #vertices, #arestas
// adj[u][v] = custo de {U->V}
// path[u][v] = k .: K vem logo apos U no caminho ate V
void read_graph() {
    memset(adj, INF, sizeof adj); // para menor caminho
    rep(i, 0, n) adj[i][i] = 0; // para menor caminho
    int u, v, w;
    rep(i, 0, m) {
        cin >> u >> v >> w;
        adj[u][v] = w;
        path[u][v] = v;
    }
}
void floyd() {
    rep(k, 0, n) rep(i, 0, n)
        rep(j, 0, n) if (adj[i][k] + adj[k][j] < adj[i][j]) {
            adj[i][j] = adj[i][k] + adj[k][j];
            path[i][j] = path[i][k];
        }
}
vector<int> findPath(int s, int d) {
    vector<int> Path;
    Path.pb(s);
    while (s != d) {
        s = path[s][d];
        Path.pb(s);
    }
    return Path;
}

/*Aplicacoes:
1-Encontrar o fecho transitivo (saber se U consegue visitar V)
.: adj[u][v] != (adj[u][k] & adj[k][v]);
   (inicializar adj com 0)

2-Minimizar a maior aresta do caminho entre U e V
.: adj[u][v] = min(adj[u][v], max(adj[u][k], adj[k][v]));
   (inicializar adj com INF)

3-Maximizar a menor aresta do caminho entre U e V
.: adj[u][v] = max(adj[u][v], min(adj[u][k], adj[k][u]));
   (inicializar adj com -INF)*/

int main() { return 0; }
```

## 6.4 Bellman-Ford

```
// Menor custo de uma origem s para todos vertices em O(V^3).
// bellman() retorna FALSE se o grafo tem ciclo com custo negativo.
// dist[v] contem o menor custo de s ate v.
#define MAXV 400

// Vertices indexados em 0.
int V, E; // #vertices, #arestas
vector<ii> adj[MAXV];
ll dist[MAXV];

bool bellman(int s) {
    rep(i, 0, V) dist[i] = INF;
    dist[s] = 0;
    rep(i, 0, V - 1) rep(u, 0, V) {
        rep(j, 0, adj[u].size()) {
            int v = adj[u][j].F, duv = adj[u][j].S;
            dist[v] = min(dist[v], dist[u] + duv);
        }
    }
    // verifica se tem ciclo com custo negativo
    rep(u, 0, V) rep(j, 0, adj[u].size()) {
        int v = adj[u][j].F, duv = adj[u][j].S;
        if (dist[v] > dist[u] + duv)
            return false;
    }
    return true;
}

int main() { return 0; }
```

## 6.5 Vértices de Articulação e Pontes

```
#define MAXV 100001
vector<int> adj[MAXV];
int dfs_num[MAXV], dfs_low[MAXV], dfs_parent[MAXV];
int dfscounter, V, dfsRoot, rootChildren, ans;
int articulation[MAXV], articulations;
vector<ii> bridges;

void articulationPointAndBridge(int u) {
    dfs_low[u] = dfs_num[u] = dfscounter++;
    rep(i, 0, adj[u].size()) {
        int v = adj[u][i];
        if (dfs_num[v] == -1) {
            dfs_parent[v] = u;
            if (u == dfsRoot)
                rootChildren++;

            articulationPointAndBridge(v);
            if (dfs_low[v] >= dfs_num[u])
                articulation[u] = true;
            if (dfs_low[v] > dfs_num[u])
                bridges.pb(mp(u, v));

            dfs_low[u] = min(dfs_low[u], dfs_low[v]);
        } else if (v != dfs_parent[u])
            dfs_low[u] = min(dfs_low[u], dfs_num[v]);
    }
}
```

```
int main() {
    // read graph
    dfscounter = 0;
    rep(i, 0, V) {
        dfs_low[i] = dfs_parent[i] = articulation[i] = 0;
        dfs_num[i] = -1;
    }
    articulations = 0;
    bridges.clear();
    rep(i, 0, V) if (dfs_num[i] == -1) {
        dfsRoot = i;
        rootChildren = 0;
        articulationPointAndBridge(i);
        articulation[dfsRoot] = (rootChildren > 1);
    }
    printf("#articulations = %d\n", articulations);
    rep(i, 0, V) if (articulation[i]) printf("Vertex %d\n", i);
    printf("#bridges = %d\n", bridges.size());
    rep(i, 0, bridges.size())
        printf("Bridge %d<->%d\n", bridges[i].F, bridges[i].S);
    return 0;
}
```

## 6.6 Tarjan

```
#define MAXV 100010
vector<int> adj[MAXV];
int V;
int dfs_num[MAXV], dfs_low[MAXV], vis[MAXV], SCC[MAXV];
int dfsCounter, numSCC;
vector<int> S; // global variables

void tarjanSCC(int u) {
    dfs_low[u] = dfs_num[u] = dfsCounter++; // dfs_low[u] <= dfs_num[u]
    S.push_back(u); // stores u in a vector based on order of
    // visitation
    vis[u] = 1;
    rep(i, 0, adj[u].size()) {
        int v = adj[u][i];
        if (dfs_num[v] == -1)
            tarjanSCC(v);
        if (vis[v]) // condition for update
            dfs_low[u] = min(dfs_low[u], dfs_low[v]);
    }
    if (dfs_low[u] ==
        dfs_num[u]) { // if this is a root (start) of an SCC
        while (true) {
            int v = S.back();
            S.pop_back();
            vis[v] = 0;
            SCC[v] = numSCC; // wich SCC this vertex belong
            if (u == v)
                break;
        }
        numSCC++;
    }
}

int main() {
    // read graph
```



```

rep(i, 0, V) {
    dfs_num[i] = -1;
    dfs_low[i] = vis[i] = 0;
    SCC[i] = -i;
}
dfsCounter = numSCC = 0;
rep(i, 0, V) if (dfs_num[i] == -1) tarjanSCC(i);
rep(i, 0, V) printf("vertice %d, componente %d\n", i, SCC[i]);
return 0;
}

```

## 6.7 Kosaraju

```

// Encotra componentes conexos. Mesmo que Tarjan
#define MAXV 100000
#define DFS_WHITE 0
vector<int> adj[2][MAXV]; // adj[0][] original, adj[1][] transposto
vector<int> S, dfs_num;
int N, numSCC, SCC[MAXV];

void Kosaraju(int u, int t, int comp) {
    dfs_num[u] = 1;
    if (t == 1)
        SCC[u] = comp;
    for (int j = 0; j < (int)adj[t][u].size(); j++) {
        int v = adj[t][u][j];
        if (dfs_num[v] == DFS_WHITE)
            Kosaraju(v, t, comp);
    }
    S.push_back(u);
}

void doit() { // chamar na main
    S.clear();
    dfs_num.assign(N, DFS_WHITE);
    for (int i = 0; i < N; i++)
        if (dfs_num[i] == DFS_WHITE)
            Kosaraju(i, 0, -1);
    numSCC = 0;
    dfs_num.assign(N, DFS_WHITE);
    for (int i = N - 1; i >= 0; i--)
        if (dfs_num[S[i]] == DFS_WHITE) {
            Kosaraju(S[i], 1, numSCC);
            numSCC++;
        }
    printf("There are %d SCCs\n", numSCC);
}

int main() { return 0; }

```

## 6.8 2-Sat

```

#define MAXV 100001
// 2-sat - Código do problema X-Mart
// vertices indexado em 1
vector<int> adj[2 * MAXV];
vector<int> radj[2 * MAXV];
int seen[2 * MAXV], comp[2 * MAXV], order[2 * MAXV], ncomp, norder;
int N; // #variaveis

```

```

int n; // #vertices

#define NOT(x) ((x <= N) ? (x + N) : (x - N))
#define quero 1
void add_edge(int a, int b, int opcao) {
    if (a > b)
        swap(a, b);
    if (b == 0)
        return;
    if (a == 0) {
        if (opcao == quero)
            adj[NOT(b)].pb(b);
        else
            adj[b].pb(NOT(b));
    } else { // normal...
        if (opcao == quero) {
            adj[NOT(a)].pb(b);
            adj[NOT(b)].pb(a);
        } else {
            a = NOT(a);
            b = NOT(b);
            adj[NOT(a)].pb(b);
            adj[NOT(b)].pb(a);
        }
    }
}

void init() {
    rep(i, 0, n + 1) {
        adj[i].clear();
        radj[i].clear();
    }
}

void dfs1(int u) {
    seen[u] = 1;
    rep(i, 0, adj[u].size()) if (!seen[adj[u][i]]) dfs1(adj[u][i]);
    order[norder++] = u;
}

void dfs2(int u) {
    seen[u] = 1;
    rep(i, 0, radj[u].size()) if (!seen[radj[u][i]]) dfs2(radj[u][i]);
    comp[u] = ncomp;
}

void strongly_connected_components() {
    rep(v, 1, n + 1) rep(i, 0, (int)adj[v].size())
        radj[adj[v][i]].pb(v);

    norder = 0;
    memset(seen, 0, sizeof seen);
    rep(v, 1, n + 1) if (!seen[v]) dfs1(v);

    ncomp = 0;
    memset(seen, 0, sizeof seen);
    for (int i = n - 1, u = order[n - 1]; i >= 0; u = order[--i])
        if (!seen[u]) {
            dfs2(u);
            ncomp++;
        }
}

bool sat2() {
    strongly_connected_components();
    rep(i, 1, n + 1) if (comp[i] == comp[NOT(i)]) return false;
}

```

```

    return true;
}
int main() {
    int Clientes;
    while (cin >> Clientes >> N) {
        if (Clientes == 0 && N == 0)
            break;
        n = 2 * N;
        init();
        int u, v;
        rep(i, 0, Clientes) {
            scanf("%d %d", &u, &v);
            add_edge(u, v, quero);
            scanf("%d %d", &u, &v);
            add_edge(u, v, !quero);
        }
        sat2() ? printf("yes\n") : printf("no\n");
    }
    return 0;
}

```

## 6.9 LCA

*/\*Lowest Common Ancestor (LCA) entre dois vertices A, B de uma arvore. LCA(A,B) = ancestral mais proximo de A adj B. O codigo abaixo tambem calcula a menor aresta do caminho entre A adj B. Para saber quantas arestas tem entre A adj B basta fazer:  
 $level[A] + level[B] - 2 * level[lca(A,B)]$   
 Pode-se modificar para retorna a distancia entre A adj B. Como usar: (1) ler a arvore em adj[] adj W[], chamar doit(raiz), passando a raiz da arvore. Indexar em 0 os vertices (2) A funcao retorna o LCA adj a menor aresta entre A adj B.  
 \*/*

```

#define MAXV 101000
const int maxl = 20; // profundidade maxima 2^(maxl) > MAXV
int pai[MAXV][maxl + 1]; // pai[v][i] = pai de v subindo 2^i arestas
int dist[MAXV][maxl + 1]; // dist[v][i] = menor aresta de v subindo 2^i arestas
int level[MAXV]; // level[v] = #arestas de v ate a raiz

int N, M; // numero de vertices adj arestas
vector<pair<int, int>> adj[MAXV]; // {v,custo}

void dfs(int v, int p, int peso) {
    level[v] = level[p] + 1;
    pai[v][0] = p;
    dist[v][0] = peso; // aresta de v--pai[v]
    for (int i = 1; i <= maxl; i++) {
        pai[v][i] = pai[pai[v][i - 1]][i - 1]; // subindo 2^i arestas
        dist[v][i] = min(dist[v][i - 1], dist[pai[v][i - 1]][i - 1]);
    }
    rep(i, 0, adj[v].size()) {
        int viz = adj[v][i].F;
        int cost = adj[v][i].S;
        if (viz == p)
            continue;
        dfs(viz, v, cost);
    }
}

```

```

void doit(int root) {
    level[root] = 0;
    for (int i = 0; i <= maxl; i++)
        pai[root][i] = root, dist[root][i] = INF;
    rep(i, 0, adj[root].size()) {
        int viz = adj[root][i].F;
        int cost = adj[root][i].S;
        dfs(viz, root, cost);
    }
}

pair<int, int> lca(int a, int b) {
    int menor = INF; // valor da menor aresta do caminho a->b
    if (level[a] < level[b])
        swap(a, b);

    for (int i = maxl; i >= 0; i--) {
        if (level[pai[a][i]] >= level[b]) {
            menor = min(menor, dist[a][i]);
            a = pai[a][i];
        }
    }
    if (a != b) {
        for (int i = maxl; i >= 0; i--) {
            if (pai[a][i] != pai[b][i]) {
                menor = min(menor, min(dist[a][i], dist[b][i]));
                a = pai[a][i];
                b = pai[b][i];
            }
        }
        // ultimo salto
        menor = min(menor, min(dist[a][0], dist[b][0]));
        a = pai[a][0];
        b = pai[b][0];
    }
    return make_pair(a, menor);
}

int main() { return 0; }

```

## 6.10 LCA (Sparse Table)

```

/*
Encontra o lca usando sparse table.
O(NlogN) de pre-processamento
O(1) em cada consulta

Como usar:
main: level[root] = 0
      dfs(root, root);
consulta:
      lca(u,v)
*/

typedef pair<int, int> ii;

#define MAXN (1e5 + 1);
#define LOGN (23)

```

```

vector<int> adj[MAXN];
int level[MAXN];

vector<int> num;
int f[MAXN];
ii st[4 * MAXN][LOGN];

void dfs(int u, int p) {
    level[u] = level[p] + 1;

    f[u] = num.size();
    num.pb(u);

    rep(i, 0, (int)adj[u].size()) {
        if (adj[u][i] == p)
            continue;
        dfs(adj[u][i], u);
        num.pb(u);
    }

    ii comb(ii left, ii right) { return min(left, right); }

    void SparseTable() {
        rep(i, 0, (int)num.size()) st[i][0] =
            make_pair(level[num[i]], num[i]);

        rep(k, 1, LOGN) for (int i = 0; (i + (1 << k) - 1) <
            (int)num.size(); i++) st[i][k] =
            comb(st[i][k - 1], st[i + (1 << (k - 1))][k - 1]);
    }

    int lca(int u, int v) {
        int l = f[u];
        int r = f[v];

        int k = log2(r - l + 1);
        return comb(st[l][k], st[r - (1 << k) + 1][k]).second;
    }
}

```

## 6.11 Maximum Bipartite Matching

```

// Encontra o casamento bipartido maximo. Set de vertices X e Y.
// x = [0,X-1], y = [0,Y-1]. match[y] = x - contem quem esta casado
// com y. Teorema de Konig - Num grafo bipartido, o matching eh igual
// ao minimum vertex cover. Complexidade O(nm)
#define MAXV 1000
vector<int> adj[MAXV];
int match[MAXV], V, X, Y;
bool vis[MAXV];

int aug(int v) {
    if (vis[v])
        return 0;
    vis[v] = true;
    rep(i, 0, adj[v].size()) {
        int r = adj[v][i];
        if (match[r] == -1 || aug(match[r])) {
            match[r] = v; // augmenting path
        }
    }
}

```

```

        return 1;
    }
}
return 0;
}

int matching(int X, int Y) {
    int V = X + Y;
    rep(i, 0, V) match[i] = -1;
    int mcbm = 0;
    rep(i, 0, X) {
        rep(j, 0, X) vis[j] = false;
        mcbm += aug(i);
    }
    return mcbm;
}

int main() { return 0; }

```

## 6.12 Hopcroft Karp - Maximum Bipartite Matching (UNI-FEI)

*/\*Encontra o casamento bipartido maximo em  $O(\sqrt{V}) * E$*   
*1) Chamar init(L,R) #vertices da esquerda, #vertices da direita*  
*2) Usar addEdge(Li,Ri) para adicionar a aresta Li -> Ri*  
*3) maxMatching() retorna o casamento maximo.*  
*matching[Rj] -> armazena Li \*/*

```

#define MAXN1 3010
#define MAXN2 3010
#define MAXM 6020
int n1, n2, edges, last[MAXN1], pre[MAXM], head[MAXM];
int matching[MAXN2], dist[MAXN1], Q[MAXN1];
bool used[MAXN1], vis[MAXN1];

void init(int L, int R) {
    n1 = L, n2 = R;
    edges = 0;
    fill(last, last + n1, -1);
}

void addEdge(int u, int v) {
    head[edges] = v;
    pre[edges] = last[u];
    last[u] = edges++;
}

void bfs() {
    fill(dist, dist + n1, -1);
    int sizeQ = 0;
    for (int u = 0; u < n1; ++u) {
        if (!used[u]) {
            Q[sizeQ++] = u;
            dist[u] = 0;
        }
    }

    for (int i = 0; i < sizeQ; i++) {
        int u1 = Q[i];
        for (int e = last[u1]; e >= 0; e = pre[e]) {
            int u2 = matching[head[e]];
            if (u2 >= 0 && dist[u2] < 0) {
                dist[u2] = dist[u1] + 1;
            }
        }
    }
}

```

```

        Q[sizeQ++] = u2;
    }
}
}
bool dfs(int u1) {
    vis[u1] = true;
    for (int e = last[u1]; e >= 0; e = pre[e]) {
        int v = head[e];
        int u2 = matching[v];
        if (u2 < 0 || !vis[u2] && dist[u2] == dist[u1] + 1 && dfs(u2)) {
            matching[v] = u1;
            used[u1] = true;
            return true;
        }
    }
    return false;
}
int maxMatching() {
    fill(used, used + n1, false);
    fill(matching, matching + n2, -1);
    for (int res = 0;;) {
        bfs();
        fill(vis, vis + n1, false);
        int f = 0;
        for (int u = 0; u < n1; ++u)
            if (!used[u] && dfs(u))
                ++f;
        if (!f)
            return res;
        res += f;
    }
}
int main() { return 0; }

```

## 6.13 Network Flow (lento)

```

// Ford-Fulkerson para fluxo maximo
#define MAXV 250
vector<int> edge[MAXV];
int cap[MAXV][MAXV];
bool vis[MAXV];

void init() {
    rep(i, 0, MAXV) edge[i].clear();
    memset(cap, 0, sizeof cap);
}

void add(int a, int b, int cap_ab, int cap_ba) {
    edge[a].pb(b), edge[b].pb(a);
    cap[a][b] += cap_ab, cap[b][a] += cap_ba;
}

int dfs(int src, int snk, int fl) {
    if (vis[src])
        return 0;
    if (snk == src)
        return fl;
    vis[src] = 1;

```

```

    rep(i, 0, edge[src].size()) {
        int v = edge[src][i];
        int x = min(fl, cap[src][v]);
        if (x > 0) {
            x = dfs(v, snk, x);
            if (!x)
                continue;
            cap[src][v] -= x;
            cap[v][src] += x;
            return x;
        }
    }
    return 0;
}

int flow(int src, int snk) {
    int ret = 0;
    while (42) {
        memset(vis, 0, sizeof vis);
        int delta = dfs(src, snk, 1 << 30);
        if (!delta)
            break;
        ret += delta;
    }
    return ret;
}
int main() { return 0; }

```

## 6.14 Network Flow - Dinic

```

// Dinic para fluxo maximo
// Grafo indexado em 1
// Inicializar maxN, maxE.
// Chamar init() com #nos, source e sink. Montar o grafo chamando
// add(a,b,c1,c2), sendo c1 cap. de a->b e c2 cap. de b->a
#define FOR(i, a, b) for (int i = a; i <= b; i++)
#define SET(c, v) memset(c, v, sizeof c)
const int maxN = 5000;
const int maxE = 70000;
const int inf = 1000000005;

int nnode, nedge, src, snk;
int Q[maxN], pro[maxN], fin[maxN], dist[maxN];
int flow[maxE], cap[maxE], to[maxE], prox[maxE];

void init(int _nnode, int _src, int _snk) {
    nnode = _nnode, nedge = 0, src = _src, snk = _snk;
    FOR(i, 1, nnode) fin[i] = -1;
}

void add(int a, int b, int c1, int c2) {
    to[nedge] = b, cap[nedge] = c1, flow[nedge] = 0,
    prox[nedge] = fin[a], fin[a] = nedge++;
    to[nedge] = a, cap[nedge] = c2, flow[nedge] = 0,
    prox[nedge] = fin[b], fin[b] = nedge++;
}

bool bfs() {
    SET(dist, -1);
    dist[src] = 0;

```

```

int st = 0, en = 0;
Q[en++] = src;
while (st < en) {
    int u = Q[st++];
    for (int e = fin[u]; e >= 0; e = prox[e]) {
        int v = to[e];
        if (flow[e] < cap[e] && dist[v] == -1) {
            dist[v] = dist[u] + 1;
            Q[en++] = v;
        }
    }
}
return dist[snk] != -1;
}

int dfs(int u, int fl) {
    if (u == snk)
        return fl;
    for (int &e = pro[u]; e >= 0; e = prox[e]) {
        int v = to[e];
        if (flow[e] < cap[e] && dist[v] == dist[u] + 1) {
            int x = dfs(v, min(cap[e] - flow[e], fl));
            if (x > 0) {
                flow[e] += x, flow[e ^ 1] -= x;
                return x;
            }
        }
    }
    return 0;
}

ll dinic() {
    ll ret = 0;
    while (bfs()) {
        FOR(i, 1, nnode) pro[i] = fin[i];
        while (true) {
            int delta = dfs(src, inf);
            if (!delta)
                break;
            ret += delta;
        }
    }
    return ret;
}

int main() { return 0; }

```

## 6.15 Min Cost Max Flow

// Criar o grafo chamando MCMF g(V), onde g eh o grafo e V a qtde de  
// vertices (indexado em 0). Chamar g.add(u,v,cap,cost) para add a  
// aresta u->v, se for bidirecional, chamar tbm g.add(v,u,cap,cost)

```

struct MCMF {
    typedef int ctype;
    enum { MAXN = 550, INF = INT_MAX };
    struct Edge {
        int x, y;
        ctype cap, cost;
    };
    vector<Edge> E;

```

```

    vector<int> adj[MAXN];
    int N, prev[MAXN];
    ctype dist[MAXN], phi[MAXN];

    MCMF(int NN) : N(NN) {}

    void add(int x, int y, ctype cap, ctype cost) { // cost >= 0
        Edge e1 = {x, y, cap, cost}, e2 = {y, x, 0, -cost};
        adj[e1.x].push_back(E.size());
        E.push_back(e1);
        adj[e2.x].push_back(E.size());
        E.push_back(e2);
    }

    void mcmf(int s, int t, ctype &flowVal, ctype &flowCost) {
        int x;
        flowVal = flowCost = 0;
        memset(phi, 0, sizeof(phi));
        while (true) {
            for (x = 0; x < N; x++)
                prev[x] = -1;
            for (x = 0; x < N; x++)
                dist[x] = INF;
            dist[s] = prev[s] = 0;

            set<pair<ctype, int>> Q;
            Q.insert(make_pair(dist[s], s));
            while (!Q.empty()) {
                x = Q.begin()->second;
                Q.erase(Q.begin());
                for (vector<int>::iterator it = adj[x].begin();
                    it != adj[x].end(); it++) {
                    const Edge &e = E[*it];
                    if (e.cap <= 0)
                        continue;
                    ctype cc = e.cost + phi[x] - phi[e.y];
                    if (dist[x] + cc < dist[e.y]) {
                        Q.erase(make_pair(dist[e.y], e.y));
                        dist[e.y] = dist[x] + cc;
                        prev[e.y] = *it;
                        Q.insert(make_pair(dist[e.y], e.y));
                    }
                }
            }
            if (prev[t] == -1)
                break;

            ctype z = INF;
            for (x = t; x != s; x = E[prev[x]].x)
                z = min(z, E[prev[x]].cap);
            for (x = t; x != s; x = E[prev[x]].x) {
                E[prev[x]].cap -= z;
                E[prev[x] ^ 1].cap += z;
            }
            flowVal += z;
            flowCost += z * (dist[t] - phi[s] + phi[t]);
            for (x = 0; x < N; x++)
                if (prev[x] != -1)
                    phi[x] += dist[x];
        }
    }
}

```

```
};

int main() { return 0; }
```

## 6.16 Min Cost Max Flow (Stefano)

```
#define MAX_V 2003
#define MAX_E 2 * 3003
// Inicializar MAX_V e MAX_E corretamente. Chamar init(_V) com a qtd
// de vertices (indexado em 0) mesmo que seja bidirecional. Adicionar
// as arestas duas vezes no main(). Complexiade (rapido)

typedef int cap_type;
typedef long long cost_type;
const cost_type inf = LLONG_MAX;

int V, E, pre[MAX_V], last[MAX_V], to[MAX_E], nex[MAX_E];
bool visited[MAX_V];
cap_type flowVal, cap[MAX_E];
cost_type flowCost, cost[MAX_E], dist[MAX_V], pot[MAX_V];

void init(int _V) {
    memset(last, -1, sizeof(last));
    V = _V;
    E = 0;
}

void add_edge(int u, int v, cap_type _cap, cost_type _cost) {
    to[E] = v, cap[E] = _cap;
    cost[E] = _cost, nex[E] = last[u];
    last[u] = E++;
    to[E] = u, cap[E] = 0;
    cost[E] = -_cost, nex[E] = last[v];
    last[v] = E++;
}

// only if there is initial negative cycle
void BellmanFord(int s, int t) {
    bool stop = false;
    for (int i = 0; i < V; ++i)
        dist[i] = inf;
    dist[s] = 0;

    for (int i = 1; i <= V && !stop; ++i) {
        stop = true;

        for (int j = 0; j < E; ++j) {
            int u = to[j ^ 1], v = to[j];

            if (cap[j] > 0 && dist[u] != inf &&
                dist[u] + cost[j] < dist[v]) {
                stop = false;
                dist[v] = dist[u] + cost[j];
            }
        }
    }

    for (int i = 0; i < V; ++i)
        if (dist[i] != inf)
            pot[i] = dist[i];
}
```

```
}

void mcmf(int s, int t) {
    flowVal = flowCost = 0;
    memset(pot, 0, sizeof(pot));

    BellmanFord(s, t);

    while (true) {
        memset(pre, -1, sizeof(pre));
        memset(visited, false, sizeof(visited));
        for (int i = 0; i < V; ++i)
            dist[i] = inf;

        priority_queue<pair<cost_type, int>> Q;
        Q.push(make_pair(0, s));
        dist[s] = pre[s] = 0;

        while (!Q.empty()) {
            int aux = Q.top().second;
            Q.pop();

            if (visited[aux])
                continue;
            visited[aux] = true;

            for (int e = last[aux]; e != -1; e = nex[e]) {
                if (cap[e] <= 0)
                    continue;
                cost_type new_dist =
                    dist[aux] + cost[e] + pot[aux] - pot[to[e]];
                if (new_dist < dist[to[e]]) {
                    dist[to[e]] = new_dist;
                    pre[to[e]] = e;
                    Q.push(make_pair(-new_dist, to[e]));
                }
            }
        }

        if (pre[t] == -1)
            break;

        cap_type f = cap[pre[t]];
        for (int i = t; i != s; i = to[pre[i] ^ 1])
            f = min(f, cap[pre[i]]);
        for (int i = t; i != s; i = to[pre[i] ^ 1]) {
            cap[pre[i]] -= f;
            cap[pre[i] ^ 1] += f;
        }

        flowVal += f;
        flowCost += f * (dist[t] - pot[s] + pot[t]);

        for (int i = 0; i < V; ++i)
            if (pre[i] != -1)
                pot[i] += dist[i];
    }
}

int main() { return 0; }
```

## 6.17 Tree Isomorphism

*// Verifica se dado duas arvores, desconsiderando o rotulo dos  
// vertices, elas tem a mesma forma.*

```
typedef vector<int> vi;
#define sz(a) (int)a.size()
#define fst first
#define snd second

struct tree {
    int n;
    vector<vi> adj;
    tree(int n) : n(n), adj(n) {}
    void add_edge(int src, int dst) {
        adj[src].pb(dst);
        adj[dst].pb(src);
    }
    vi centers() {
        vi prev;
        int u = 0;
        for (int k = 0; k < 2; ++k) {
            queue<int> q;
            prev.assign(n, -1);
            q.push(prev[u] = u);
            while (!q.empty()) {
                u = q.front();
                q.pop();
                for (auto i : adj[u]) {
                    if (prev[i] >= 0)
                        continue;
                    q.push(i);
                    prev[i] = u;
                }
            }
        }
        vi path = {u};
        while (u != prev[u])
            path.pb(u = prev[u]);
        int m = sz(path);
        if (m % 2 == 0)
            return {path[m / 2 - 1], path[m / 2]};
        else
            return {path[m / 2]};
    }
    vector<vi> layer;
    vi prev;
    int levelize(int r) {
        prev.assign(n, -1);
        prev[r] = n;
        layer = {{r}};
        while (true) {
            vi next;
            for (auto u : layer.back()) {
                for (int v : adj[u]) {
                    if (prev[v] >= 0)
                        continue;
                    prev[v] = u;
                    next.pb(v);
                }
            }
        }
    }
};
```

```
        if (next.empty())
            break;
        layer.pb(next);
    }
    return sz(layer);
};

bool isomorphic(tree S, int s, tree T, int t) {
    if (S.n != T.n)
        return false;
    if (S.levelize(s) != T.levelize(t))
        return false;
    vector<vi> longcodes(S.n + 1, longcodeT(T.n + 1));
    vi codeS(S.n), codeT(T.n);
    for (int h = S.layer.size() - 1; h >= 0; h--) {
        map<vi, int> bucket;
        for (int u : S.layer[h]) {
            sort(all(longcodes[u]));
            bucket[longcodes[u]] = 0;
        }
        for (int u : T.layer[h]) {
            sort(all(longcodeT[u]));
            bucket[longcodeT[u]] = 0;
        }
        int id = 0;
        for (auto &p : bucket)
            p.snd = id++;
        for (int u : S.layer[h]) {
            codeS[u] = bucket[longcodes[u]];
            longcodes[S.prev[u]].pb(codeS[u]);
        }
        for (int u : T.layer[h]) {
            codeT[u] = bucket[longcodeT[u]];
            longcodeT[T.prev[u]].pb(codeT[u]);
        }
    }
    return codeS[s] == codeT[t];
}

bool isomorphic(tree S, tree T) {
    auto x = S.centers(), y = T.centers();
    if (sz(x) != sz(y))
        return false;
    if (isomorphic(S, x[0], T, y[0]))
        return true;
    return sz(x) > 1 and isomorphic(S, x[1], T, y[0]);
}

int main() {
    int N, u, v;
    cin >> N;
    tree A(N + 2), B(N + 2);
    rep(i, 0, N - 1) {
        scanf("%d %d", &u, &v);
        u--, v--;
        A.add_edge(u, v);
    }
    rep(i, 1, N) {
        scanf("%d %d", &u, &v);
        u--, v--;
    }
}
```

```

    B.add_edge(u, v);
}
puts(isomorphic(A, B) ? "S" : "N");
}

```

## 6.18 Stoer Wagner-Minimum Cut (UNIFEI)

```

/*
Retorna o corte minimo do grafo
(Conjunto de arestas que caso seja removido, desconecta o grafo)
Input: n = #vertices, g[i][j] = custo da aresta (i->j)
Output: Retorna o corte minimo
Complexidade: O(N^3)
*/

// Maximum number of vertices in the graph
#define NN 101
// Maximum edge weight (MAXW * NN * NN must fit into an int)
#define MAXW 110

// Adjacency matrix and some internal arrays
int g[NN][NN], v[NN], w[NN], na[NN], n;
bool a[NN];
int stoer_wagner() {
    // init the remaining vertex set
    for (int i = 0; i < n; i++)
        v[i] = i;
    // run Stoer-Wagner
    int best = MAXW * n * n;
    while (n > 1) {
        // initialize the set A and vertex weights
        a[v[0]] = true;
        for (int i = 1; i < n; i++) {
            a[v[i]] = false;
            na[i - 1] = i;
            w[i] = g[v[0]][v[i]];
        }
        // add the other vertices
        int prev = v[0];
        for (int i = 1; i < n; i++) {
            // find the most tightly connected non-A vertex
            int zj = -1;
            for (int j = 1; j < n; j++)
                if (!a[v[j]] && (zj < 0 || w[j] > w[zj]))
                    zj = j;
            // add it to A
            a[v[zj]] = true;
            // last vertex?
            if (i == n - 1) {
                // remember the cut weight
                best = min(best, w[zj]);

                // merge prev and v[zj]
                for (int j = 0; j < n; j++)
                    g[v[j]][prev] = g[prev][v[j]] += g[v[zj]][v[j]];
                v[zj] = v[--n];
                break;
            }
        }
        prev = v[zj];
        // update the weights of its neighbours

```

```

        for (int j = 1; j < n; j++)
            if (!a[v[j]])
                w[j] += g[v[zj]][v[j]];
    }
    return best;
}

int main() { return 0; }

```

## 6.19 Erdos Gallai (UNIFEI)

```

// Determina se existe um grafo tal que b[i] eh o grau do i-esimo
// vertice. Vertices indexado em 1. Apenas armazenar em b[1..N] e
// chamar EGL()
long long b[100005], n;
long long dmax, dmin, dsum, num_degs[100005];

bool basic_graphical_tests() { // Sort and perform some simple tests
    // on the sequence

    int p = n;
    memset(num_degs, 0, (n + 1) * sizeof(long long));

    dmax = dsum = n = 0;
    dmin = p;
    for (int d = 1; d <= p; d++) {
        if (b[d] < 0 || b[d] >= p)
            return false;
        else if (b[d] > 0) {
            if (dmax < b[d])
                dmax = b[d];
            if (dmin > b[d])
                dmin = b[d];
            dsum = dsum + b[d];
            n++;
            num_degs[b[d]]++;
        }
    }
    if (dsum % 2 || dsum > n * (n - 1))
        return false;
    return true;
}

bool EGL() {
    long long k, sum_deg, sum_nj, sum_jnj, run_size;

    if (!basic_graphical_tests())
        return false;
    if (n == 0 || 4 * dmin * n >= (dmax + dmin + 1) * (dmax + dmin + 1))
        return true;

    k = sum_deg = sum_nj = sum_jnj = 0;
    for (int dk = dmax; dk >= dmin; dk--) {
        if (dk < k + 1)
            return true;

        if (num_degs[dk] > 0) {
            run_size = num_degs[dk];
            if (dk < k + run_size)
                run_size = dk - k;
            sum_deg += run_size * dk;

```



```

    for (int v = 0; v < run_size; v++) {
        sum_nj += num_degs[k + v];
        sum_jnj += (k + v) * num_degs[k + v];
    }
    k += run_size;
    if (sum_deg > k * (n - 1) - k * sum_nj + sum_jnj)
        return false;
}
return true;
}
int main() { return 0; }

```

## 6.20 Stable Marriage (UNIFEI)

*/\*Seja um conjunto de m homens e n mulheres, onde cada pessoa tem uma preferencia por outra de sexo oposto. O algoritmo produz o casamento estavel de cada homem com uma mulher. Estavel:*

- Cada homem se casara com uma mulher diferente ( $n \geq m$ )
- Dois casais H1M1 e H2M2 nao serao instaveis.

*Dois casais H1M1 e H2M2 sao instaveis se:*

- H1 prefere M2 ao inves de M1, e
- M1 prefere H2 ao inves de H1.

*Entrada*

(1)  $m = \#homens$ ,  $n = \#mulheres$

(2)  $R[x][y] = i$ ,  $i$ : eh a ordem de preferencia do homem  $y$  pela mulher  $x$

*Obs.: Quanto maior o valor de  $i$  menor eh a preferencia do homem  $y$  pela mulher  $x$*

(3)  $L[x][i] = y$ : A mulher  $y$  eh a  $i$ -esima preferencia do homem  $x$

*Obs.:  $0 \leq i \leq n-1$ , quanto menor o valor de  $i$  maior eh a preferencia do homem  $x$  pela mulher  $y$*

*Saida*

$L2R[i]$ : a mulher do homem  $i$  (sempre entre 0 e  $n-1$ )

$R2L[j]$ : o homem da mulher  $j$  ( $-1$  se a mulher for solteira)

*Complexidade  $O(m^2)$*

```

*/
#define MAXM 1000
#define MAXW 1000
int L[MAXM][MAXW];
int R[MAXW][MAXM];
int L2R[MAXM], R2L[MAXW];
int m, n;
int p[MAXM];

void stableMarriage() {
    static int p[MAXM];
    memset(R2L, -1, sizeof(R2L));
    memset(p, 0, sizeof(p));
    for (int i = 0; i < m; ++i) {
        int man = i;
        while (man >= 0) {
            int wom;
            while (42) {
                wom = L[man][p[man]++];
                if (R2L[wom] < 0 || R[wom][man] > R[wom][R2L[wom]])
                    break;
            }
        }
    }
}

```

```

    int hubby = R2L[wom];
    R2L[L2R[man] = wom] = man;
    man = hubby;
}
}
int main() { return 0; }

```

## 6.21 Hungarian Max Bipartite Matching with Cost (UNIFEI)

*/\*Encontra o casamento bipartido maximo/minimo com peso nas arestas  
Criar o grafo:  
Hungarian G(L, R, ehMaximo)  
L = #vertices a esquerda  
R = #vertices a direita  
ehMaximo = variavel booleana que indica se eh casamento maximo ou minimo*

*Adicionar arestas:  
G.add\_edge(x,y,peso)  
x = vertice da esquerda no intervalo [0,L-1]  
y = vertice da direita no intervalo [0,R-1]  
peso = custo da aresta  
obs: tomar cuidado com multiplas arestas.*

*Resultado:  
match\_value = soma dos pesos dos casamentos  
pairs = quantidade de pares (x-y) casados  
xy[x] = vertice y casado com x  
yx[y] = vertice x casado com y*

*Complexidade do algoritmo:  $O(V^3)$   
Problemas resolvidos: SCITIES (SPOJ)  
\*/*

```

struct Hungarian {
    enum { MAXN = 150, INF = 0x3f3f3f3f };
    int cost[MAXN][MAXN];
    int xy[MAXN], yx[MAXN];
    bool S[MAXN], T[MAXN];
    int lx[MAXN], ly[MAXN], slack[MAXN], slackx[MAXN], prev[MAXN];
    int match_value, pairs;
    bool ehMaximo;
    int n;
}

```

```

Hungarian(int L, int R, bool _ehMaximo = true) {
    n = max(L, R);
    ehMaximo = _ehMaximo;
    if (ehMaximo)
        memset(cost, 0, sizeof cost);
    else
        memset(cost, INF, sizeof cost);
}

```

```

void add_edge(int x, int y, int peso) {
    if (!ehMaximo)
        peso *= (-1);
}

```

```

    cost[x][y] = peso;
}

int solve() {
    match_value = 0;
    pairs = 0;
    memset(xy, -1, sizeof(xy));
    memset(yx, -1, sizeof(yx));
    init_labels();
    augment();
    for (int x = 0; x < n; ++x)
        match_value += cost[x][xy[x]];
    return match_value;
}

void init_labels() {
    memset(lx, 0, sizeof(lx));
    memset(ly, 0, sizeof(ly));
    for (int x = 0; x < n; ++x)
        for (int y = 0; y < n; ++y)
            lx[x] = max(lx[x], cost[x][y]);
}

void augment() {
    if (pairs == n)
        return;
    int x, y, root;
    int q[MXN], wr = 0, rd = 0;
    memset(S, false, sizeof(S));
    memset(T, false, sizeof(T));
    memset(prev, -1, sizeof(prev));
    for (x = 0; x < n; ++x)
        if (xy[x] == -1) {
            q[wr++] = root = x;
            prev[x] = -2;
            S[x] = true;
            break;
        }
    for (y = 0; y < n; ++y) {
        slack[y] = lx[root] + ly[y] - cost[root][y];
        slackx[y] = root;
    }
    while (true) {
        while (rd < wr) {
            x = q[rd++];
            for (y = 0; y < n; ++y)
                if (cost[x][y] == lx[x] + ly[y] && !T[y]) {
                    if (yx[y] == -1)
                        break;
                    T[y] = true;
                    q[wr++] = yx[y];
                    add(yx[y], x);
                }
            if (y < n)
                break;
        }
        if (y < n)
            break;
        update_labels();
        wr = rd = 0;
        for (y = 0; y < n; ++y)

```

```

        if (!T[y] && slack[y] == 0) {
            if (yx[y] == -1) {
                x = slackx[y];
                break;
            } else {
                T[y] = true;
                if (!S[yx[y]]) {
                    q[wr++] = yx[y];
                    add(yx[y], slackx[y]);
                }
            }
        }
        if (y < n)
            break;
    }
    if (y < n) {
        ++pairs;
        for (int cx = x, cy = y, ty; cx != -2; cx = prev[cx], cy = ty) {
            ty = xy[cx];
            yx[cy] = cx;
            xy[cx] = cy;
        }
        augment();
    }
}

void add(int x, int prevx) {
    S[x] = true;
    prev[x] = prevx;
    for (int y = 0; y < n; ++y)
        if (lx[x] + ly[y] - cost[x][y] < slack[y]) {
            slack[y] = lx[x] + ly[y] - cost[x][y];
            slackx[y] = x;
        }
}

void update_labels() {
    int x, y, delta = INF;
    for (y = 0; y < n; ++y)
        if (!T[y])
            delta = min(delta, slack[y]);
    for (x = 0; x < n; ++x)
        if (S[x])
            lx[x] -= delta;
    for (y = 0; y < n; ++y)
        if (T[y])
            ly[y] += delta;
    for (y = 0; y < n; ++y)
        if (!T[y])
            slack[y] -= delta;
}

int casouComX(int x) { return xy[x]; }

int casouComY(int y) { return yx[y]; }
};

// 0 código abaixo resolve o problema scities (Spoj)
int main() {
    int casos;
    cin >> casos;

```

```

while (casos--) {
    int L, R;
    cin >> L >> R;
    Hungarian G(L, R, true);

    int x, y, w, aux[L][R];
    memset(aux, 0, sizeof aux);
    while (scanf("%d %d %d", &x, &y, &w) != EOF) {
        if (x == 0 && y == 0 && w == 0)
            break;
        aux[x - 1][y - 1] += w;
    }
    for (int x = 0; x < L; x++) {
        for (int y = 0; y < R; y++) {
            if (aux[x][y] != 0) {
                G.add_edge(x, y, aux[x][y]);
            }
        }
    }
    printf("%d\n", G.solve());
}
return 0;
}

```

## 6.22 Blossom

*// Encontra o emparelhamento maximo em um grafo nao direcionado.  
// Armazenar em n a quantidade de vertice e em mat[][] as adjacencias.  
// edmond(n) retorna o emparelhamento maximo.*

```

typedef vector<int> VI;
typedef vector<vector<int>> VVI;

```

```
int mat[205][205], n;
```

```
int lf[205];
```

```
VVI adj;
```

```
VI vis, inactive, match;
```

```
int N;
```

```

bool dfs(int x, VI &blossom) {
    if (inactive[x])
        return false;
    int i, y;
    vis[x] = 0;
    for (i = adj[x].size() - 1; i >= 0; i--) {
        y = adj[x][i];
        if (inactive[y])
            continue;
        if (vis[y] == -1) {
            vis[y] = 1;
            if (match[y] == -1 || dfs(match[y], blossom)) {
                match[y] = x;
                match[x] = y;
                return true;
            }
        }
        if (vis[y] == 0 || blossom.size()) {
            blossom.push_back(y);
            blossom.push_back(x);
            if (blossom[0] == x) {

```

```

                match[x] = -1;
                return true;
            }
        }
        return false;
    }
}

```

```

bool augment() {
    VI blossom, mark;
    int i, j, k, s, x;
    for (i = 0; i < N; i++) {
        if (match[i] != -1)
            continue;
        blossom.clear();
        vis = VI(N + 1, -1);
        if (!dfs(i, blossom))
            continue;
        s = blossom.size();
        if (s == 0)
            return true;

```

```

        mark = VI(N + 1, -1);
        for (j = 0; j < s - 1; j++) {
            for (k = adj[blossom[j]].size() - 1; k >= 0; k--)
                mark[adj[blossom[j]][k]] = j;
        }

```

```

        for (j = 0; j < s - 1; j++) {
            mark[blossom[j]] = -1;
            inactive[blossom[j]] = 1;
        }

```

```

        adj[N].clear();
        for (j = 0; j < N; j++) {
            if (mark[j] != -1)
                adj[N].pb(j), adj[j].pb(N);
        }

```

```

        match[N] = -1;
        N++;
        if (!augment())
            return false;
        N--;

```

```

        for (j = 0; j < N; j++) {
            if (mark[j] != -1)
                adj[j].pop_back();
        }
        for (j = 0; j < s - 1; j++) {
            inactive[blossom[j]] = 0;
        }

```

```

        x = match[N];
        if (x != -1) {
            if (mark[x] != -1) {
                j = mark[x];
                match[blossom[j]] = x;
                match[x] = blossom[j];
                if (j & 1)

```

```

    for (k = j + 1; k < s; k += 2) {
        match[blossom[k]] = blossom[k + 1];
        match[blossom[k + 1]] = blossom[k];
    }
    else
        for (k = 0; k < j; k += 2) {
            match[blossom[k]] = blossom[k + 1];
            match[blossom[k + 1]] = blossom[k];
        }
    }
    return true;
}
return false;
}

int edmond(int n) {
    int i, j, ret = 0;
    N = n;
    adj = VVI(2 * N + 1);
    for (i = 0; i < n; i++) {
        for (j = i + 1; j < n; j++) {
            if (mat[i][j]) {
                adj[i].pb(j);
                adj[j].pb(i);
            }
        }
    }
    match = VI(2 * N + 1, -1);
    inactive = VI(2 * N + 1);
    while (augment())
        ret++;
    return ret;
}

```

## 7 Estruturas de Dados

### 7.1 BIT

```

// Permite realizar operacoes de query e update em um vetor em O(logN)
// Obs: A[] deve ser indexado em 1, nao em 0.
#define MAXN 100001
ll ft[MAXN];
ll A[MAXN];
int N;

// ATUALIZA UM INDICE i, CONSULTA UM INTERVALO (i,j)
// update(i, valor) faz A[i] += valor em log(N)
void update(int i, ll valor) {
    for (; i <= N; i += i & -i)
        ft[i] += valor;
}

// query(i) retorna a soma A[1] + ... + A[i] em log(N)
ll query(int i) {
    ll sum = 0;
    for (; i > 0; i -= i & -i)
        sum += ft[i];
}

```

```

    return sum;
}

// query(i,j) retorna a soma A[i] + A[i+1] + ... + A[j] em log(N)
ll query(int i, int j) { return query(j) - query(i - 1); }
// ATUALIZA UM INTERVALO (i,j), CONSULTA UM ELEMENTO i
// range_update(i,j,valor) faz A[k] += valor, para i <= k <= j em
// log(N) query(i): retorna o valor de A[i] em log(N)
void range_update(int i, int j, ll valor) {
    update(i, valor);
    update(j + 1, -valor);
}

int main() { return 0; }

```

### 7.2 BIT 2D

```

#define MAXL 3001
#define MAXC 3001
ll ft[MAXL][MAXC];
int L, C;
// update(x,y,v) incrementa v na posicao (x,y)
// .: M[x][y] += v em O(log(N))
void update(int x, int y, int v) {
    for (; x <= L; x += x & -x)
        for (int yy = y; yy <= C; yy += yy & -yy)
            ft[x][yy] += v;
}

// query(x,y) retorna o somatorio da submatriz definida por
// (1,1)-(x,y) .: sum += M[i][j] para todo 1 <= i <= x e 1 <= j <= y,
// em O(log(N))
ll query(int x, int y) {
    if (x <= 0 || y <= 0)
        return 0;
    ll sum = 0;
    for (; x > 0; x -= x & -x)
        for (int yy = y; yy > 0; yy -= yy & -yy)
            sum += ft[x][yy];
    return sum;
}

// query(x1,y1,x2,y2) retorna o somatorio da submatriz definida por
// (x1,x1) -- (x2,y2) .: sum += M[i][j] para todo x1 <= i <= x2 e
// y1 <= j <= y2, em O(log(N))
ll query(int x1, int y1, int x2, int y2) {
    return query(x2, y2) - query(x2, y1 - 1) - query(x1 - 1, y2) +
        query(x1 - 1, y1 - 1);
}

// A ideia de atualizar um intervalo (submatriz) e consultar um
// elemento (i,j) tambem sao validos

```

### 7.3 Sparse Table

```

/*
Resolve problemas de consulta a intervalos (RSQ, RMQ etc) de um vetor
estatico, ou seja, os valores nao sofrem update.

```

```

Alterar a funcao comb() de acordo (min, max, soma etc)
Pre-processamento  $O(N \log N)$  e consulta em  $O(1)$ .
N = tamanho do vetor a[]
a[] deve ser indexado em 0
*/
const int MAXN = (1e6 + 1);
#define LOGN (21)
int st[MAXN][LOGN];
int N, a[MAXN];

int comb(int left, int right) { return min(left, right); }

void SparseTable() {
    rep(k, 0, LOGN) for (int i = 0; (i + (1 << k) - 1) < N; i++)
        st[i][k] =
            k ? comb(st[i][k - 1], st[i + (1 << (k - 1))][k - 1]) :
                a[i];
}

int query(int l, int r) {
    int k = log2(r - l + 1);
    return comb(st[l][k], st[r - (1 << k) + 1][k]);
}

```

## 7.4 RMQ

```

// Range Minimum Query: idx do menor elemento num intervalo de um
// array. Permite consultas e updates no array em  $O(\log N)$ . ATENCAO:
// Array A[] deve ser indexado em 0;
#define MAXN 500000
int A[MAXN], T[4 * MAXN];
int N; // #number of elements in A[]
int neutro = -1;

// combina o resultado de dois segmentos
int combine(int p1, int p2) {
    if (p1 == -1)
        return p2;
    if (p2 == -1)
        return p1;
    if (A[p1] <= A[p2])
        return p1;
    else
        return p2;
}

// chamar build() apos preencher o vetor A[].  $O(N)$ 
void build(int no = 1, int a = 0, int b = N - 1) {
    if (a == b) {
        T[no] = a;
    } else {
        int m = (a + b) / 2;
        int esq = 2 * no;
        int dir = esq + 1;
        build(esq, a, m);
        build(dir, m + 1, b);
        T[no] = combine(T[esq], T[dir]);
    }
}

```

```

// Modifica A[i] em  $O(\log N)$ , neste caso A[i] = v
void update(int i, int v, int no = 1, int a = 0, int b = N - 1) {
    if (a > i || b < i)
        return;
    if (a == i && b == i) {
        A[i] = v;
        T[no] = i; // desnecessario ;p
        return;
    }
    int m = (a + b) / 2;
    int esq = 2 * no;
    int dir = esq + 1;
    update(i, v, esq, a, m);
    update(i, v, dir, m + 1, b);
    T[no] = combine(T[esq], T[dir]);
}

// Retorna o idx k do menor valor A[k] no intervalo [i, j] em  $O(\log N)$ 
int query(int i, int j, int no = 1, int a = 0, int b = N - 1) {
    if (a > j || b < i)
        return neutro;
    if (a >= i && b <= j)
        return T[no];
    int m = (a + b) / 2;
    int esq = 2 * no;
    int dir = esq + 1;

    int p1 = query(i, j, esq, a, m);
    int p2 = query(i, j, dir, m + 1, b);
    return combine(p1, p2);
}

int main() { return 0; }

```

## 7.5 Persistent Seg Tree

```

const int MAX = 1e5 + 5;
const int MAXT = 80 * MAX;
int a[MAX], tree[MAXT];
int L[MAXT], R[MAXT];
vector<int> root;
int cnt;

void build(int no, int i, int j) {
    if (i == j) {
        tree[no] = a[i];
        return;
    }
    L[no] = ++cnt;
    R[no] = ++cnt;
    build(L[no], i, (i + j) / 2);
    build(R[no], (i + j) / 2 + 1, j);
    tree[no] = tree[L[no]] + tree[R[no]];
}

int update(int no, int i, int j, int p, int v) {
    int NO = ++cnt;
    if (i == j) {
        tree[NO] = v;
        return NO;
    }

```

```

}

L[NO] = L[no];
R[NO] = R[no];

if (p <= (i + j)/2) L[NO] = update(L[NO], i, (i + j) / 2, p, v);
else R[NO] = update(R[NO], (i + j) / 2 + 1, j, p, v);

tree[NO] = tree[L[NO]] + tree[R[NO]];
return NO;
}

int query(int no, int i, int j, int l, int r) {
    if (i > r || j < l) return 0;
    if (i >= l && j <= r) return tree[no];

    int left = query(L[no], i, (i + j) / 2, l, r);
    int right = query(R[no], (i + j) / 2 + 1, j, l, r);
    return left + right;
}

void init() {
    memset(tree, 0, sizeof tree);
    root.clear();
    root.pb(0);
    cnt = 0;
}

int main(int argc, char **argv) {
    query(root[k], 1, N, l, r); // consulta na versao k
    root.pb(update(root[sz], 1, N, l, v)); // update
    return 0;
}

```

## 7.6 Seg Tree com Lazy

```

// RSQ agora com queries e updates em intervalos. Precisa de Lazy
// Propagation. Array A[] deve ser indexado em 0. Nem sempre o array
// que sera modificado armazena apenas um valor, nesse caso usamos
// struct para representar cada no.
#define MAXN 500000
ll A[MAXN], tree[4 * MAXN], lazy[4 * MAXN];
int N;
int neutro = 0;

// funcao que realiza o merge de um intervalo, pode ser *, -, min,
// max, etc...
int combine(int segEsq, int segDir) { return segEsq + segDir; }

void build(int no = 1, int a = 0, int b = N - 1) {
    if (a == b) {
        tree[no] = A[a];
        return;
    }
    int m = (a + b) / 2;
    int esq = 2 * no;
    int dir = esq + 1;
    build(esq, a, m);
    build(dir, m + 1, b);
    tree[no] = combine(tree[esq], tree[dir]);
}

```

```

}

void propagate(int no, int a, int b) {
    if (lazy[no] != 0) {
        // esta parte depende do problema, neste caso queremos adicionar o
        // valor lazy[no] no intervalo [a,b], mas estamos atualizando
        // apenas o noh que representa este intervalo
        tree[no] += (b - a + 1) * lazy[no];
        if (a != b) {
            lazy[2 * no] += lazy[no];
            lazy[2 * no + 1] += lazy[no];
        }
        lazy[no] = 0;
    }
}

// update(i,j,v) faz A[k] += v, para i <= k <= j, em log(N)
void update(int i, int j, ll v, int no = 1, int a = 0, int b = N - 1)
{
    if (lazy[no])
        propagate(no, a, b);
    if (a > j || b < i)
        return;
    if (a >= i && b <= j) {
        lazy[no] += v; // atualiza apenas a flag da raiz da subarvore
        propagate(no, a, b);
        return;
    }
    int m = (a + b) / 2;
    int esq = 2 * no;
    int dir = esq + 1;
    update(i, j, v, esq, a, m);
    update(i, j, v, dir, m + 1, b);
    tree[no] = combine(tree[esq], tree[dir]);
}

// query(i,j) retorna o somatorio A[i] + A[i+1] + ... + A[j]
ll query(int i, int j, int no = 1, int a = 0, int b = N - 1) {
    if (lazy[no])
        propagate(no, a, b);
    if (a > j || b < i)
        return neutro;
    if (a >= i && b <= j)
        return tree[no];
    int m = (a + b) / 2;
    int esq = 2 * no;
    int dir = esq + 1;
    ll q1 = query(i, j, esq, a, m);
    ll q2 = query(i, j, dir, m + 1, b);
    return combine(q1, q2);
}

int main() { return 0; }

```

## 7.7 Union-Find

```

// Conjuntos Disjuntos. Inicialmente cada elemento eh lider de seu
// proprio conjunto. Operacoes de join(u,v) fazem com que os conjuntos
// que u e v pertencem se unam. find(u) retorna o lider do conjunto
// que u esta contido.

```

```

#define MAXV 100000
int V, pai[MAXV], rnk[MAXV], size[MAXV];

void init() { rep(i, 0, V) pai[i] = i, rnk[i] = 0, size[i] = 1; }

int find(int v) {
    if (v != pai[v])
        pai[v] = find(pai[v]);
    return pai[v];
}

void join(int u, int v) {
    u = find(u);
    v = find(v);
    if (u == v)
        return;

    if (rnk[u] < rnk[v])
        swap(u, v);
    pai[v] = u; // add v no conjunto de u
    size[u] += size[v];
    if (rnk[u] == rnk[v])
        rnk[u]++;
}

bool same_set(int u, int v) { return find(u) == find(v); }

int main() { return 0; }

```

## 7.8 Treap

```

typedef struct node {
    int prior, size;
    int val; // value stored in the array
    int sum; // whatever info you want to maintain in segtree for each
             // node
    int lazy; // whatever lazy update you want to do
    int rev;
    struct node *l, *r;
} node;

typedef node *pnode;
int sz(pnode t) { return t ? t->size : 0; }
void upd_sz(pnode t) {
    if (t)
        t->size = sz(t->l) + 1 + sz(t->r);
}

void lazy(pnode t) {
    if (!t || t->lazy == -1)
        return;
    t->val = t->lazy; // operation of lazy
    t->sum = t->lazy * sz(t);
    if (t->l)
        t->l->lazy = t->lazy; // propagate lazy
    if (t->r)
        t->r->lazy = t->lazy;
    t->lazy = -1;
}

void reset(pnode t) {
    if (t)
        t->sum = t->val; // no need to reset lazy coz when we call this

```

```

// lazy would itself be propagated
}

// combining two ranges of segtree
void combine(pnode &t, pnode l, pnode r) {
    if (!l || !r)
        return void(t = l ? l : r);
    t->sum = l->sum + r->sum;
}

void operation(pnode t) { // operation of segtree
    if (!t)
        return;
    reset(t); // reset the value of current node assuming it now
              // represents a single element of the array
    lazy(t->l);
    lazy(t->r); // imp:propagate lazy before combining t->l,t->r;
    combine(t, t->l, t);
    combine(t, t, t->r);
}

void push(pnode t) {
    if (!t || !t->rev)
        return;
    t->rev = false;
    swap(t->l, t->r);
    if (t->l)
        t->l->rev ^= true;
    if (t->r)
        t->r->rev ^= true;
}

void split(pnode t, pnode &l, pnode &r, int pos, int add = 0) {
    if (!t)
        return void(l = r = NULL);
    push(t);
    lazy(t);
    int curr_pos = add + sz(t->l);
    if (curr_pos <= pos) // element at pos goes to left subtree(l)
        split(t->r, t->r, r, pos, curr_pos + 1), l = t;
    else
        split(t->l, l, t->l, pos, add), r = t;
    upd_sz(t);
    operation(t);
}

// l->leftarray,r->rightarray,t->resulting array
void merge(pnode &t, pnode l, pnode r) {
    push(l);
    push(r);
    lazy(l);
    lazy(r);
    if (!l || !r)
        t = l ? l : r;
    else if (l->prior > r->prior)
        merge(l->r, l->r, r), t = l;
    else
        merge(r->l, l, r->l), t = r;
    upd_sz(t);
    operation(t);
}

pnode init(int val) {
    pnode ret = new node;
    ret->prior = rand();

```

## 7.9 Seg Tree 2D

```

ret->size = 1;
ret->val = val;
ret->sum = val;
ret->lazy = -1;
ret->rev = 0;
ret->l = NULL, ret->r = NULL;
return ret;
}

int range_query(pnode t, int l, int r) { //[l,r]
    pnode L, mid, R;
    split(t, L, mid, l - 1);
    split(mid, t, R, r - 1); // note: r-1!!
    int ans = t->sum;
    merge(mid, L, t);
    merge(t, mid, R);
    return ans;
}

void range_update(pnode t, int l, int r, int val) { //[l,r]
    pnode L, mid, R;
    split(t, L, mid, l - 1);
    split(mid, t, R, r - 1); // note: r-1!!
    t->lazy = val; // lazy_update
    merge(mid, L, t);
    merge(t, mid, R);
}

void reverse(pnode t, int l, int r) {
    pnode L, mid, R;
    split(t, L, mid, l - 1);
    split(mid, mid, R, r - 1);
    mid->rev ^= true;
    merge(t, L, mid);
    merge(t, t, R);
}

void output(pnode t) {
    if (!t)
        return;
    push(t);
    lazy(t);
    output(t->l);
    printf("%d ", t->val);
    output(t->r);
}

int valor(int val) { return val & 1 ? 0 : 1; }

int main() {
    int P, Q;
    while (scanf("%d %d", &P, &Q) != EOF) {
        pnode tree = NULL, T1 = NULL, T2 = NULL, T3 = NULL;
        int val;
        rep(i, 0, P) {
            scanf("%d", &val);
            split(tree, T1, T2, i);
            merge(T1, T1, init(valor(val)));
            merge(tree, T1, T2);
        }
        while (Q--) {
        }
    }
}

```

```

struct node {
    int qt;
    int f1, f2, f3, f4;
};

node new_node() {
    node ret;
    ret.qt = ret.f1 = ret.f2 = ret.f3 = ret.f4 = 0;
    return ret;
}

vector<node> tree;
int cnt = 0;

bool inRange(int x1, int x2, int y1, int y2, int a1, int a2, int b1,
             int b2) {
    if (x2 < x1 || y2 < y1)
        return false;
    if (x2 < a1 || x1 > a2)
        return false;
    if (y2 < b1 || y1 > b2)
        return false;
    return true;
}

void update(int no, int x1, int x2, int y1, int y2, int a1, int a2,
           int b1, int b2, int val) {
    if (no == cnt)
        tree[cnt++] = new_node();

    if (x1 >= a1 && x2 <= a2 && y1 >= b1 && y2 <= b2) {
        tree[no].qt = val;
        return;
    }

    int f1 = 0, f2 = 0, f3 = 0, f4 = 0;
    if (inRange(x1, (x1 + x2) / 2, y1, (y1 + y2) / 2, a1, a2, b1, b2)) {
        if (!tree[no].f1)
            tree[no].f1 = cnt;
        update(tree[no].f1, x1, (x1 + x2) / 2, y1, (y1 + y2) / 2, a1, a2,
              b1, b2, val);
    }
    if (inRange(x1, (x1 + x2) / 2, (y1 + y2) / 2 + 1, y2, a1, a2, b1,
              b2)) {
        if (!tree[no].f2)
            tree[no].f2 = cnt;
        update(tree[no].f2, x1, (x1 + x2) / 2, (y1 + y2) / 2 + 1, y2, a1,
              a2, b1, b2, val);
    }
    if (inRange((x1 + x2) / 2 + 1, x2, y1, (y1 + y2) / 2, a1, a2, b1,
              b2)) {
        if (!tree[no].f3)
            tree[no].f3 = cnt;
        update(tree[no].f3, (x1 + x2) / 2 + 1, x2, y1, (y1 + y2) / 2, a1,
              a2, b1, b2, val);
    }
    if (inRange((x1 + x2) / 2 + 1, x2, (y1 + y2) / 2 + 1, y2, a1, a2,
              b1,

```



```

        b2)) {
    if (!tree[no].f4)
        tree[no].f4 = cnt;
    update(tree[no].f4, (x1 + x2) / 2 + 1, x2, (y1 + y2) / 2 + 1, y2,
        a1, a2, b1, b2, val);
}

if (tree[no].f1)
    f1 = tree[tree[no].f1].qt;
if (tree[no].f2)
    f2 = tree[tree[no].f2].qt;
if (tree[no].f3)
    f3 = tree[tree[no].f3].qt;
if (tree[no].f4)
    f4 = tree[tree[no].f4].qt;

tree[no].qt = f1 + f2 + f3 + f4;
}

int query(int no, int x1, int x2, int y1, int y2, int a1, int a2,
    int b1, int b2) {
    if (!inRange(x1, x2, y1, y2, a1, a2, b1, b2) || no >= cnt ||
        tree[no].qt == 0)
        return 0;

    if (x1 >= a1 && x2 <= a2 && y1 >= b1 && y2 <= b2)
        return tree[no].qt;

    int f1 = 0, f2 = 0, f3 = 0, f4 = 0;
    if (tree[no].f1)
        f1 = query(tree[no].f1, x1, (x1 + x2) / 2, y1, (y1 + y2) / 2, a1,
            a2, b1, b2);
    if (tree[no].f2)
        f2 = query(tree[no].f2, x1, (x1 + x2) / 2, (y1 + y2) / 2 + 1, y2,
            a1, a2, b1, b2);
    if (tree[no].f3)
        f3 = query(tree[no].f3, (x1 + x2) / 2 + 1, x2, y1, (y1 + y2) / 2,
            a1, a2, b1, b2);
    if (tree[no].f4)
        f4 = query(tree[no].f4, (x1 + x2) / 2 + 1, x2, (y1 + y2) / 2 + 1,
            y2, a1, a2, b1, b2);

    return f1 + f2 + f3 + f4;
}

void erase() {
    tree.clear();
    vector<node> xua;
    swap(tree, xua);
    tree.resize(1000010);
    cnt = 0;
}

int main() { return 0; }

```

## 7.10 Polyce

// <https://codeforces.com/blog/entry/11080>

```
#include <bits/stdc++.h>
```

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace std;
using namespace __gnu_pbds;

typedef tree<int, // tipo da variavel
    null_type,
    less<int>, // funcao de comparacao(greater, less_equal,
    rb_tree_tag, tree_order_statistics_node_update>
    ordered_set;

void newSet() {
    // funciona como um set normal, mas ha 2 funcoes especiais: log(n)
    ordered_set T;
    ordered_set::iterator it;
    int k = *T.find_by_order(0); // retorna o K-esimo elemento segundo
    // a funcao de comparacao
    int kk = T.order_of_key(0); // retorna a posicao que um elemento
    // encaixaria segundo a funcao de
    // comparacao
}

#include <ext/rope>
using namespace __gnu_cxx;

void newVector() {
    // funciona como um vector, mas consegue algo a mais: (log(n))
    rope<int> v;
    rope<int>::iterator it;
    int l, r; // segmento
    rope<int> cur =
        v.substr(l, r - l + 1); // copia um segmento do vector
    v.erase(l, r - l + 1); // apaga um segmento
    v.insert(v.mutable_begin(), cur); // insere um segmento
    for (it = cur.mutable_begin(); it != cur.mutable_end(); it++)
        cout << *it << " "; // percorre ele
}

int main() { return 0; }

```

## 7.11 KD2

```

struct point {
    int x, y, z;
    point(int x = 0, int y = 0, int z = 0) : x(x), y(y), z(z) {}
    point operator-(point q) { return point(x - q.x, y - q.y, z - q.z); }
    int operator*(point q) { return x * q.x + y * q.y + z * q.z; }
};

typedef vector<point> polygon;
priority_queue<double> vans;
int NN, CC, KK, DD;

struct KDTreeNode {
    point p;
    int level;
    KDTreeNode *below, *above;
    KDTreeNode(const point &q, int lvl) {

```

```

    p = q;
    level = lev1;
    below = above = 0;
}
~KDTreeNode() { delete below, above; }
int diff(const point &pt) {
    switch (level) {
        case 0:
            return pt.x - p.x;
        case 1:
            return pt.y - p.y;
        case 2:
            return pt.z - p.z;
    }
    return 0;
}
ll distSq(point &q) { return (p - q) * (p - q); }
int rangeCount(point &pt, ll K) {
    int count = (distSq(pt) <= K * K) ? 1 : 0;
    if (count)
        vans.push(-sqrt(distSq(pt)));

    int d = diff(pt);
    if (~d <= K && above != 0)
        count += above->rangeCount(pt, K);
    if (d <= K && below != 0)
        count += below->rangeCount(pt, K);
    return count;
}
};

class KDTree {
public:
    polygon P;
    KDTreeNode *root;
    int dimation;
    KDTree() {}
    KDTree(polygon &poly, int D) {
        P = poly;
        dimation = D;
        root = 0;
        build();
    }
    ~KDTree() { delete root; }
    // count the number of pairs that has a distance less than K
    ll countPairs(ll K) {
        ll count = 0;
        rep(i, 0, P.size()) count += root->rangeCount(P[i], K) - 1;
        return count;
    }
protected:
    void build() {
        // random_shuffle(all(P));
        rep(i, 0, P.size()) { root = insert(root, P[i], -1); }
    }
    KDTreeNode *insert(KDTreeNode *t, const point &pt, int parentLevel)
    {
        if (t == 0) {
            t = new KDTreeNode(pt, (parentLevel + 1) % dimation);
            return t;
        }

```

```

        } else {
            int d = t->diff(pt);
            if (d <= 0)
                t->below = insert(t->below, pt, t->level);
            else
                t->above = insert(t->above, pt, t->level);
        }
        return t;
    }
};

int main() {
    point e;
    e.z = 0;
    polygon p;
    set<ii> st;

    while (scanf("%d %d %d %d", &NN, &CC, &KK, &DD) != EOF) {
        p.clear();
        KK = min(NN, KK);
        st.clear();

        rep(i, 0, NN) {
            scanf("%d %d", &e.x, &e.y);
            st.insert(mp(e.x, e.y));
            p.pb(e);
        }

        KDTree tree(p, 2);
        int ans = 0;
        rep(i, 0, CC) {
            scanf("%d %d", &e.x, &e.y);
            if (st.count(mp(e.x, e.y)))
                continue;

            ll at = 0;
            rep(i, 0, 30) {
                at = ll(1) << i;
                while (!vans.empty())
                    vans.pop();
                int aux = tree.root->rangeCount(e, at);
                if (aux >= KK)
                    break;
            }
            double sum = 0.0;
            rep(i, 0, KK) {
                sum += -vans.top();
                vans.pop();
            }
            if (sum >= DD)
                ans++;
        }
        printf("%d\n", ans);
    }
    return 0;
}

```

## 8 Strings

### 8.1 KMP

```
// obs: A funcao strstr (char* text, char* pattern) da biblioteca
// <cstring> implementa KMP (C-ANSI). A funcao retorna a primeira
// ocorrencia do padrao no texto, KMP retorna todas. nres -> O numero
// de ocorrencias do padrao no texto res[] -> posicoes das nres
// ocorrencias do padrao no texto Complexidade do algoritmo: O(n+m)*

#define MAXN 100001
int pi[MAXN], res[MAXN], nres;
void kmp(string text, string pattern) {
    nres = 0;
    pi[0] = -1;
    rep(i, 1, pattern.size()) {
        pi[i] = pi[i - 1];
        while (pi[i] >= 0 && pattern[pi[i] + 1] != pattern[i])
            pi[i] = pi[pi[i]];
        if (pattern[pi[i] + 1] == pattern[i])
            ++pi[i];
    }
    int k = -1; // k+1 eh o tamanho do match atual
    rep(i, 0, text.size()) {
        while (k >= 0 && pattern[k + 1] != text[i])
            k = pi[k];
        if (pattern[k + 1] == text[i])
            ++k;
        if (k + 1 == pattern.size()) {
            res[nres++] = i - k;
            k = pi[k];
        }
    }
}
```

### 8.2 Aho Corasick

```
const int cc = 26;
const int MAX = 100;

int cnt;
int sig[MAX][cc];
int term[MAX];
int T[MAX];
int v[MAX];

inline int C(char c) { return c - '0'; }

void add(string s, int id) {
    int x = 0;
    rep(i, 0, s.size()) {
        int c = C(s[i]);
        if (!sig[x][c]) {
            term[cnt] = 0;
            sig[x][c] = cnt++;
        }
        x = sig[x][c];
    }
}
```

```

    }
    term[x] = 1;
    v[id] = x;
}

void aho() {
    queue<int> q;
    rep(i, 0, cc) {
        int x = sig[0][i];
        if (!x)
            continue;
        q.push(x);
        T[x] = 0;
    }
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        rep(i, 0, cc) {
            int x = sig[u][i];
            if (!x)
                continue;
            int v = T[u];
            while (v && !sig[v][i])
                v = T[v];
            v = sig[v][i];
            T[x] = v;
            term[x] += term[v];
            q.push(x);
        }
    }
}
```

// Conta a quantidade de palavras de exatamente 1 caracteres que se  
// pode formar com um determinado alfabeto, dado que algumas palavras  
// sao "proibidas"

```
int mod = 1e9 + 7;
ll pd[100][MAX];

ll solve(int pos, int no) {
    if (pos == 0)
        return 1;
    if (pd[pos][no] != -1)
        return pd[pos][no];
    ll ans = 0;
    rep(i, 0, cc) {
        int v = no;
        while (v && !sig[v][i])
            v = T[v];
        v = sig[v][i];
        if (term[v])
            continue;
        ans = (ans + solve(pos - 1, v)) % mod;
    }
    return pd[pos][no] = ans;
}

void Qttd_de_Palavras() {
    while (1) {
        memset(sig, 0, sizeof sig);
        memset(pd, -1, sizeof pd);
    }
}
```

```

cnt = 1;
int l = readInt();
if (!l)
    break;
int n = readInt();
string pattern;
rep(i, 0, n) {
    cin >> pattern;
    add(pattern, i);
}
aho();
ll ans = 0;
rep(i, 1, l + 1) ans = (ans + solve(i, 0)) % mod;
printf("%d\n", ans);
}

```

*// Verifica quais padroes ocorreram em um texto*

```

int alc[MAX];

void busca(string s) {
    int x = 0;
    rep(i, 0, s.size()) {
        int c = C(s[i]);
        while (x && !sig[x][c])
            x = T[x];
        x = sig[x][c];
        alc[x] = 1;
    }
}

void Ql_Ocorreu() {
    string pattern, text;
    while (getline(cin, text)) {
        if (text == "*")
            break;
        memset(sig, 0, sizeof sig);
        memset(alc, 0, sizeof alc);
        cnt = 1;
        int n;
        cin >> n;
        rep(i, 0, n) {
            cin >> pattern;
            add(pattern, i);
        }
        aho();
        busca(text);
        for (int i = cnt - 1; i >= 0; i--) {
            if (alc[i])
                alc[T[i]] = 1;
        }
        rep(i, 0, n) {
            int u = v[i];
            if (alc[u])
                printf("Ocorreu\n");
            else
                printf("Nao ocorreu\n");
        }
    }
}

```

*// Total de ocorrencias de cada padrao em uma string, mesmo com sufixos iguais*

```

ll busca2(string s) {
    ll x = 0, cont = 0;
    rep(i, 0, s.size()) {
        int c = C(s[i]);
        while (x && !sig[x][c])
            x = T[x];
        x = sig[x][c];
        cont += term[x];
    }
    return cont;
}

void Qnts_vezes_Ocorreu() {
    string text, pattern;
    while (cin >> text) {
        if (text == "*")
            break;
        memset(sig, 0, sizeof sig);
        cnt = 1;
        int n = readInt();
        rep(i, 0, n) {
            cin >> pattern;
            add(pattern, i);
        }
        aho();
        rep(i, 1, 10) debug(T[i]) cout << busca2(text) << endl;
    }
}

// Encontra a primeira ocorrencia de cada padrao em uma string
void busca3(string s) {
    int x = 0;
    rep(i, 0, s.size()) {
        int c = C(s[i]);
        while (x && !sig[x][c])
            x = T[x];
        x = sig[x][c];
        if (!alc[x])
            alc[x] = i + 1;
    }
}

void Onde_Ocorreu() {
    string pattern, text;
    int tam[1000];
    while (cin >> text) {
        if (text == "*")
            break;
        memset(sig, 0, sizeof sig);
        memset(alc, 0, sizeof alc);
        cnt = 1;
        int n;
        cin >> n;
        rep(i, 0, n) {
            cin >> pattern;
            tam[i] = pattern.size();
            add(pattern, i);
        }
    }
}

```

```

aho();
busca3(text);
for (int i = cnt - 1; i >= 0; i--) {
    alc[T[i]] = min(alc[i], alc[T[i]]);
}
rep(i, 0, n) {
    int u = v[i];
    if (alc[u] != INF) {
        int k = alc[u] - tam[i] + 1;
        printf("De %d a %d\n", k, alc[u]);
    } else
        printf("Nao ocorreu\n");
}
}
}

```

### 8.3 Suffix Array

```

#define MAX 100010
#define MAX_N 100010
char T[MAX_N];
ll n;
int RA[MAX_N], tempRA[MAX_N];
int SA[MAX_N], tempSA[MAX_N];
int c[MAX_N];
int Phi[MAX_N], PLCP[MAX_N], LCP[MAX_N];

void countingSort(int k) {
    int i, sum, maxi = max((ll)300, n);
    memset(c, 0, sizeof c);
    for (i = 0; i < n; i++)
        c[i + k < n ? RA[i + k] : 0]++;
    for (i = sum = 0; i < maxi; i++) {
        int t = c[i];
        c[i] = sum;
        sum += t;
    }
    for (i = 0; i < n; i++)
        tempSA[c[SA[i] + k < n ? RA[SA[i] + k] : 0]++] = SA[i];
    for (i = 0; i < n; i++)
        SA[i] = tempSA[i];
}

void constructSA() {
    int i, k, r;
    for (i = 0; i < n; i++)
        RA[i] = T[i];
    for (i = 0; i < n; i++)
        SA[i] = i;

    for (k = 1; k < n; k <= 1) {
        countingSort(k);
        countingSort(0);
        tempRA[SA[0]] = r = 0;
        for (i = 1; i < n; i++)
            tempRA[SA[i]] = (RA[SA[i]] == RA[SA[i - 1]]) &&
                RA[SA[i] + k] == RA[SA[i - 1] + k])
                ? r
                : ++r;
    }
}

```

```

    for (i = 0; i < n; i++)
        RA[i] = tempRA[i];
    if (RA[SA[n - 1]] == n - 1)
        break;
}

void computeLCP() {
    int i, L;
    Phi[SA[0]] = -1;
    for (i = 1; i < n; i++)
        Phi[SA[i]] = SA[i - 1];
    for (i = L = 0; i < n; i++) {
        if (Phi[i] == -1) {
            PLCP[i] = 0;
            continue;
        }
        while (T[i + L] == T[Phi[i] + L])
            L++;
        PLCP[i] = L;
        L = max(L - 1, 0);
    }
    for (i = 0; i < n; i++) {
        LCP[i] = PLCP[SA[i]];
    }
}

int main() {
    // concatenar $ no final
}

```

### 8.4 Suffix Array (Gugu)

```

const int MAX = 100010;
int gap, tam, sa[MAX], pos[MAX], lcp[MAX], tmp[MAX];

bool suffixCmp(int i, int j) {
    if (pos[i] != pos[j])
        return pos[i] < pos[j];
    i += gap, j += gap;
    return (i < tam && j < tam) ? pos[i] < pos[j] : i > j;
}

void buildSA(char s[]) {
    tam = strlen(s);
    for (int i = 0; i < tam; i++)
        sa[i] = i, pos[i] = s[i], tmp[i] = 0;

    for (gap = 1; gap * 2 < tam; gap *= 2) {
        sort(sa, sa + tam, suffixCmp);
        tmp[0] = 0;
        for (int i = 0; i < tam - 1; i++)
            tmp[i + 1] = tmp[i] + suffixCmp(sa[i], sa[i + 1]);
        for (int i = 0; i < tam; i++)
            pos[sa[i]] = tmp[i];
        if (tmp[tam - 1] == tam - 1)
            break;
    }
}

ll buildLCP(char s[]) {
    ll sum = 0;
    for (int i = 0, k = 0; i < tam; i++) {

```

```

    if (pos[i] == tam - 1)
        continue;
    for (int j = sa[pos[i] + 1]; s[i + k] == s[j + k];)
        k++;
    lcp[pos[i] + 1] = k;
    sum += k;
    if (k > 0)
        k--;
}
return sum;
}

void PrintAll(char s[]) {
    printf("SA\ttam\tLCP\tSuffix\n");
    rep(i, 0, tam) printf("%2d\t%2d\t%2d\t%s\n", sa[i], tam - sa[i],
        lcp[i], s + sa[i]);
}

ll num_subs(ll m) { return (ll)tam * (tam + 1) / 2 - m; }
ll num_subsrn() {
    ll ret = 0;
    rep(i, 1, tam) if (lcp[i] > lcp[i - 1]) ret += lcp[i] - lcp[i - 1];
    return ret;
}

void printans(char s[], int n) {
    int maior = 0, id = -1;
    rep(i, 0, tam) if (lcp[i] > n && lcp[i] > maior) maior = lcp[i],
        id = i;

    if (id == -1)
        printf("*");
    else
        rep(i, sa[id], sa[id] + maior) printf("%c", s[i]);
    printf("\n");
}

}

char s[MAX];
int main() {
    while (1) {
        scanf("%s", s);
        if (s[0] == '*')
            break;

        buildSA(s);
        ll m = buildLCP(s);

        PrintAll(s); // printa sa, lcp, suffixs
        // printf("%lld\n", num_subs(m)); //numero de substrings nao
        // repetidas printf("%lld\n", num_subsrn()); //numero de
        // substrings
        // que se repete printans(s, 2); //maior substring de tamanho
        // maior
        // ou igual a n que se repete
    }
}

```

## 8.5 Rolling Hash

```

// Permite encontrar um hash de uma substring de S. precompute O(n),
// my_hash O(1)
#define NN 1000006
const ll mod = 1e9 + 7; // modulo do hash
const ll x = 33;        // num. primo > que o maior caracter de S.

```

```

ll H[NN], X[NN];
ll V(char c) { return c - 'A'; }
ll my_hash(int i, int j) {
    ll ret = H[j];
    if (!i)
        return ret;
    return ((ret - (H[i - 1] * X[j - i + 1]) % mod) + mod) % mod;
}

void precompute(string s) {
    X[0] = 1;
    rep(i, 1, NN) X[i] = (X[i - 1] * x) % mod;
    H[0] = V(s[0]);
    rep(i, 1, s.size()) H[i] = ((H[i - 1] * x) % mod + V(s[i])) % mod;
}

```

## 8.6 Longest Common Prefix with Hash

```

// Longest Common Prefix between S[i..] and S[j..]
int lcp(int i, int j, int tam) {
    int lo = 0, hi = tam, ans;
    while (lo <= hi) {
        int mid = (lo + hi) / 2;
        if (my_hash(i, i + mid - 1) == my_hash(j, j + mid - 1)) {
            ans = mid;
            lo = mid + 1;
        } else
            hi = mid - 1;
    }
    return ans;
}

```

## 8.7 Minimum Lexicographic Rotation

```

// Retorna a menor string lexicografica de s. Necessario my_hash() e
// lcp()
string min_lex_rot(string s) {
    int t = s.size();
    precompute(s); // hashing
    s += s;
    int idx = 0;
    for (int i = 1; i < t; i++) {
        // tam do prefixo comum
        int len = lcp(i, idx, t);
        if (s[i + len] < s[idx + len])
            idx = i;
    }
    return s.substr(idx, t);
}

```

## 8.8 Longest Palindrome (Manacher algorithm)

```

string preProcess(string s) {
    int n = s.length();
    if (n == 0)
        return "^$";

    string ret = "^";

```

```

for (int i = 0; i < n; i++)
    ret += "#" + s.substr(i, 1);
ret += "$";
return ret;
}

string longestPalindrome(string s) {
    L = C = s.size();
    string T = preprocess(s);
    int n = T.length();
    int *P = new int[n];
    int C = 0, R = 0;

    for (int i = 1; i < n - 1; i++) {
        int i_mirror = 2 * C - i;
        P[i] = (R > i) ? min(R - i, P[i_mirror]) : 0;
        while (T[i + 1 + P[i]] == T[i - 1 - P[i]])
            P[i]++;
        if (i + P[i] > R) {
            C = i;
            R = i + P[i];
        }
    }

    int maxLen = 0;
    int centerIndex = 0;

    for (int i = 1; i < n - 1; i++) {
        if (!P[i])
            continue;
        if (P[i] > maxLen) {
            maxLen = P[i];
            centerIndex = i;
        }
    }
    delete[] P;
    return s.substr((centerIndex - 1 - maxLen) / 2, maxLen);
}

```

## 8.9 Autômato de Sufixos

```

struct state {
    int len, link;
    int next[26];
};

const int MAXN = 200020;
state st[2 * MAXN]; // vetor que armazena os estados
int sz; // contador do numero de estados
int last; // numero do estado que corresponde ao texto todo

void sa_init() {
    sz = 1;
    last = 0;
    st[0].len = 0;
    st[0].link = -1;
    rep(i, 0, 26) st[0].next[i] = 0;
    // limpa o mapeamento de transicoes
}

```

```

void sa_extend(int c, ll &ans) {
    int cur = sz++; // novo estado a ser criado
    st[cur].len = st[last].len + 1;
    rep(i, 0, 26) st[cur].next[i] = 0;
    int p; // variavel que itera sobre os estados terminais
    for (p = last; p != -1 && !st[p].next[c]; p = st[p].link) {
        st[p].next[c] = cur;
    }
    if (p == -1) { // nao ocorreu transicao c nos estados terminais
        st[cur].link = 0;
        ans += st[cur].len;
    } else { // ocorreu transicao c no estado p
        int q = st[p].next[c];
        if (st[p].len + 1 == st[q].len) {
            st[cur].link = q;
        } else {
            int clone = sz++; // criacao do vertice clone de q
            st[clone].len = st[p].len + 1;
            rep(i, 0, 26) st[clone].next[i] = st[q].next[i];
            st[clone].link = st[q].link;
            for (; p != -1 && st[p].next[c] == q; p = st[p].link) { // atualizacao das transicoes c
                st[p].next[c] = clone;
            }
            st[q].link = st[cur].link = clone;
        }
        ans += st[cur].len - st[st[cur].link].len;
    }
    // atualizacao do estado que corresponde ao texto
    last = cur;
}

bool busca_automato(int m, string p) {
    int i, pos = 0;
    for (i = 0; i < m; i++) {
        if (st[pos].next[p[i]] == 0) {
            return false;
        } else {
            pos = st[pos].next[p[i]];
        }
    }
    return true;
}

int maior_tamanho_em_comum(string s, string t) {
    ll nothing = 0;
    // Constroi o automato com o primeiro texto
    sa_init();
    for (int i = 0; i < (int)s.size(); i++)
        sa_extend(s[i] - 'a', nothing);
    int estado = 0, tamanho = 0, maior = 0;
    // Passando pelos caracteres do segundo texto
    for (int i = 0; i < (int)t.size(); ++i) {
        while (estado && !st[estado].next[t[i] - 'a']) {
            estado = st[estado].link;
            tamanho = st[estado].len;
        }
        if (st[estado].next[t[i] - 'a']) {
            estado = st[estado].next[t[i] - 'a'];
            tamanho++;
        }
    }
}

```

```

    if (tamanho > maior) {
        maior = tamanho;
    }
}
return maior;
}

int main() {
    char s[MAXN];
    char p[MAXN];
    while (gets(s)) {
        sa_init();
        int tam = strlen(s);
        ll ans = 0;
        rep(i, 0, tam) { sa_extend(s[i] - 'a', ans); }
        gets(p);
        printf("%d\n", maior_tamanho_em_comum(s, p));
    }
    return 0;
}

```

## 8.10 Z Algorithm

*// Algorithm produces an array Z where Z[i] is the length of the  
// longest substring starting from S[i] which is also a prefix of S.*

```

string s;
vector<int> z;

```

```

void Z() {
    int n = s.size(), L = 0, R = 0;
    z.assign(n, 0);

    for (int i = 1; i < n; i++) {
        if (i > R) {
            L = R = i;
            while (R < n && s[R - L] == s[R])
                R++;
            z[i] = R - L;
            R--;
        } else {
            int k = i - L;
            if (z[k] < R - i + 1)
                z[i] = z[k];
            else {
                L = i;
                while (R < n && s[R - L] == s[R])
                    R++;
                z[i] = R - L;
                R--;
            }
        }
    }
}

```

## 9 PD

### 9.1 Soma acumulada 2D

```

/*Retorna o somatorio dos elementos de uma submatriz em O(1).
 * Submatriz definida por canto superior esquerdo (x1,y1) e canto
 * inferior direito (x2,y2) .: x1 <= x2 && y1 <= y2 */
#define MAXN 3000
int N, M; // linhas colunas
long long V[MAXN + 2][MAXN + 2]; // matriz da entrada
long long S[MAXN + 2][MAXN + 2]; // matriz com as somas acumuladas

// precomputa as somas em O(N*M)
void precal() {
    rep(x, 0, N) rep(y, 0, M) {
        S[x][y] = V[x][y];
        if (x > 0)
            S[x][y] += S[x - 1][y];
        if (y > 0)
            S[x][y] += S[x][y - 1];
        if (x > 0 && y > 0)
            S[x][y] -= S[x - 1][y - 1];
    }
}

// retorna a soma da submatriz em O(1)
long long sum(int x1, int y1, int x2, int y2) {
    long long soma = S[x2][y2];
    if (x1 > 0)
        soma -= S[x1 - 1][y2];
    if (y1 > 0)
        soma -= S[x2][y1 - 1];
    if (x1 > 0 && y1 > 0)
        soma += S[x1 - 1][y1 - 1];
    return soma;
}

```

## 9.2 Knuth Optimization

```

int N, B, C, yep, save[MAXN][MAXN], sav[MAXN];
ll n[MAXN], mc[MAXN][MAXN], se[MAXN], sd[MAXN], pd[MAXN][MAXN];

ll solve(int i, int k) {
    if (i == N)
        return 0;
    if (k == 1)
        return pd[i][k] = mc[i][N - 1];
    if (pd[i][k] != -1)
        return pd[i][k];

    ll ret = LINF;
    int ini = i, fim = N - k + 1, best = -1;
    if (i && save[i - 1][k])
        ini = save[i - 1][k];
    if (save[i][k - 1])
        fim = save[i][k - 1] + 1;

    rep(l, ini, fim) {
        ll aux = solve(l + 1, k - 1) + mc[i][l];
        if (ret > aux) {
            best = l;
            ret = aux;
        }
    }
    save[i][k] = best;
}

```



```

    return pd[i][k] = ret;
}

int main() {
    rep(i, 0, N) scanf("%lld", &n[i]);

    se[0] = n[0];
    rep(i, 1, N) se[i] = se[i - 1] + n[i];

    sd[N - 1] = n[N - 1];
    for (int i = N - 2; i >= 0; i--)
        sd[i] = sd[i + 1] + n[i];

    rep(i, 1, N) pd[0][i] = pd[0][i - 1] + se[i - 1];
    for (int i = N - 2; i >= 0; i--)
        pd[N - 1][i] = pd[N - 1][i + 1] + sd[i + 1];

    rep(i, 1, N) {
        rep(j, i + 1, N) pd[i][j] = pd[i - 1][j] - n[i - 1] * (j - i + 1);
    }
    for (int i = N - 2; i >= 0; i--) {
        for (int j = i - 1; j >= 0; j--)
            pd[i][j] = pd[i + 1][j] - n[i + 1] * (i - j + 1);
    }

    rep(i, 0, N) {
        if (pd[i][i + 1] < pd[i + 1][i])
            mc[i][i + 1] = pd[i][i + 1], save[i][i + 1] = i + 1;
        else
            mc[i][i + 1] = pd[i + 1][i], save[i][i + 1] = i;
        rep(j, i + 2, N) {
            int ini = save[i][j - 1];
            mc[i][j] = pd[i][ini] + pd[j][ini], save[i][j] = ini;
            rep(k, ini + 1, j + 1) {
                ll a = pd[i][k] + pd[j][k];
                if (mc[i][j] <= a)
                    break;
            }
            mc[i][j] = a;
            save[i][j] = k;
        }
        rep(j, 0, N + 1) { pd[i][j] = -1, save[i][j] = 0; }
    }

    rep(j, 0, N + 1) pd[N][j] = -1, save[N][j] = 0;

    solve();

    return 0;
}

```

### 9.3 Convex Hull Trick

```

bool bad(int l1, int l2, int l3) {
    return (B[l3] - B[l1]) * (M[l1] - M[l2]) <
           (B[l2] - B[l1]) * (M[l1] - M[l3]);
}

void add(long long m, long long b) {
    M.push_back(m);
    B.push_back(b);
}

```

```

while (M.size() >= 3 &&
       bad(M.size() - 3, M.size() - 2, M.size() - 1)) {
    M.erase(M.end() - 2);
    B.erase(B.end() - 2);
}

long long query(long long x) {
    if (pointer >= M.size())
        pointer = M.size() - 1;
    while (pointer < M.size() - 1 &&
           M[pointer + 1] * x + B[pointer + 1] <
           M[pointer] * x + B[pointer])
        pointer++;
    return M[pointer] * x + B[pointer];
}

struct hux {
    int a, b, id;
};

bool my_sort(hux a, hux b) {
    return a.b != b.b ? a.b > b.b : a.a > b.a;
}

const ll LINF = 1LL << 52;
const double EPS = 1e-9;
const int MAXV = 100010;

double intersept(hux a, hux b) {
    return double(b.b - a.b) / (a.a - b.a);
}

vector<pair<double, double>> convex_hux(const vector<hux> &v) {
    int p = 0, n = v.size(), bestai = v[0].a;
    double cross = 0.0;
    pair<double, int> aux;

    priority_queue<pair<double, int>> pq;
    vector<pair<double, double>> ret(n + 1, mp(-1, -1));

    pq.push(mp(cross, p));
    ret[v[p].id].F = cross, ret[v[p].id].S = LINF;

    rep(i, 1, n) {
        aux = pq.top();
        cross = aux.F, p = aux.S;

        if (v[i].a <= bestai)
            continue;
        bestai = v[i].a;

        double new_cross = intersept(v[i], v[p]);
        while (new_cross <= cross + EPS) {
            pq.pop();
            ret[v[p].id] = mp(-1.0, -1.0);

            aux = pq.top();
            cross = aux.F, p = aux.S;

            new_cross = intersept(v[i], v[p]);
        }
    }
}

```

```

    pq.push(mp(new_cross, i));
    ret[v[p].id].S = new_cross;
    ret[v[i].id].F = new_cross;
    ret[v[i].id].S = LINF;
}

// rep(i, 0, n) cout << ret[i].F << " " << ret[i].S << "\n";

return ret;
}

```

## 9.4 Longest Increasing Subsequence

```

// Maior subsequencia crescente
#define MAX_N 100
int vet[MAX_N], P[MAX_N], N;
void reconstruct_print(int end) {
    int x = end;
    stack<int> s;
    while (P[x] >= 0) {
        s.push(vet[x]);
        x = P[x];
    }
    printf("%d", vet[x]);
    while (!s.empty()) {
        printf(" ", s.top());
        s.pop();
    }
}

int lis() {
    int L[MAX_N], L_id[MAX_N];
    int li = 0, lf = 0; // lis ini, lis end
    rep(i, 0, N) {
        int pos = lower_bound(L, L + li, vet[i]) - L;
        L[pos] = vet[i];
        L_id[pos] = i;
        P[i] = pos ? L_id[pos - 1] : -1;
        if (pos + 1 > li) {
            li = pos + 1;
            lf = i;
        }
    }
    reconstruct_print(lf);
    return li;
}

```

## 9.5 Kadane 1D

```

// Encontra maior soma contigua positiva num vetor em O(N). {s,f}
// contem o intervalo de maior soma.
int Kadane1D(int vet[], int N, int &s, int &f) {
    int ret = -INF, sum, saux;
    sum = s = f = saux = 0;
    rep(i, 0, N) {
        sum += vet[i];
        if (sum > ret) {
            ret = sum;
            s = saux;

```

```

        f = i;
    }
    if (sum < 0) {
        sum = 0;
        saux = i + 1;
    }
}
return ret;
}

```

## 9.6 Kadane 2D

```

/*Maior soma de uma sub-matriz a partir de valores positivos.
 * [x1,y1]=upper-left, [x2,y2]=bottom-right*/
int L, C, pd[MAX_L], mat[MAX_L][MAX_C];
int x1, y1, x2, y2;
int Kadane2D() {
    int ret = 0, aux;
    rep(left, 0, C) {
        rep(i, 0, L) pd[i] = 0;
        rep(right, left, C) {
            rep(i, 0, L) pd[i] += mat[i][right];
            int sum = aux = 0;
            rep(i, 0, L) { // Kadane1D
                sum += pd[i];
                if (sum > ret)
                    ret = sum, x1 = aux, y1 = left, x2 = i, y2 = right;
                if (sum < 0)
                    sum = 0, aux = i + 1;
            }
        }
    }
    return ret;
}

```

## 9.7 Knapsack0-1

```

//[IME] 0-1 Knapsack, v-valores, w-pesos, Cap-capacidade
int mochila01(vector<int> v, vector<int> w, int Cap) {
    int n = v.size();
    int dp[n + 1][Cap + 1];
    for (int i = 0; i <= n; i++)
        dp[i][0] = 0;
    for (int j = 0; j <= Cap; j++)
        dp[0][j] = 0;
    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= Cap; j++) {
            if (w[i - 1] > j)
                dp[i][j] = dp[i - 1][j];
            else
                dp[i][j] =
                    max(dp[i - 1][j], v[i - 1] + dp[i - 1][j - w[i - 1]]);
        }
    return dp[n][Cap];
}

```

## 9.8 Edit Distance

```
//[IME] menor custo para transformar a em b, dado as operacoes de
// inserir, remover e substituir caracteres de a
int editDistance(string a, string b) {
    int cost, insertCost = 1, deletCost = 1, substCost = 1;
    int m = a.size();
    int n = b.size();
    int d[m + 1][n + 1];
    for (int i = 0; i <= m; i++)
        d[i][0] = i * deletCost;
    for (int j = 0; j <= n; j++)
        d[0][j] = j * insertCost;
    for (int i = 1; i <= m; i++)
        for (int j = 1; j <= n; j++) {
            if (a[i - 1] == b[j - 1])
                cost = 0;
            else
                cost = substCost;
            d[i][j] =
                min(d[i - 1][j] + deletCost,
                    min(d[i][j - 1] + insertCost, d[i - 1][j - 1] + cost));
        }
    return d[m][n];
}
```

## 10 Sorting

### 10.1 Merge Sort com num de Inversoes

```
// Ordena arr aplicando mergesort e conta o numero de inversoes
void merge(int *arr, int size1, int size2, ll &inversions) {
    int temp[size1 + size2 + 2];
    int ptr1 = 0, ptr2 = 0;

    while (ptr1 + ptr2 < size1 + size2) {
        if (ptr1 < size1 && arr[ptr1] <= arr[size1 + ptr2] ||
            ptr1 < size1 && ptr2 >= size2)
            temp[ptr1 + ptr2] = arr[ptr1++];

        if (ptr2 < size2 && arr[size1 + ptr2] < arr[ptr1] ||
            ptr2 < size2 && ptr1 >= size1) {
            temp[ptr1 + ptr2] = arr[size1 + ptr2++];
            inversions += size1 - ptr1;
        }
    }

    for (int i = 0; i < size1 + size2; i++)
        arr[i] = temp[i];
}

void mergeSort(int *arr, int size, ll &inversions) {
    if (size == 1)
        return;

    int size1 = size / 2, size2 = size - size1;
    mergeSort(arr, size1, inversions);
```

```
mergeSort(arr + size1, size2, inversions);
merge(arr, size1, size2, inversions);
}
```

### 10.2 Quick Sort

```
// No main, chamar quicksort(array, 0, tam-1);
int partition(int s[], int l, int h) {
    int i, p, firsthigh;
    p = h;
    firsthigh = l;
    for (i = l; i < h; i++)
        if (s[i] < s[p]) {
            swap(s[i], s[firsthigh]);
            firsthigh++;
        }
    swap(s[i], s[firsthigh]);
    return firsthigh;
}

void quicksort(int s[], int l, int h) {
    int p;
    if ((h - l) > 0) {
        p = partition(s, l, h);
        quicksort(s, l, p - 1);
        quicksort(s, p + 1, h);
    }
}
```

## 11 Miscelânea

### 11.1 Calendário

```
// Routines for performing computations on dates. In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
// integers.
string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat",
    "Sun"};

// converts Gregorian date to integer (Julian day number)
int dateToInt(int m, int d, int y) {
    return 1461 * (y + 4800 + (m - 14) / 12) / 4 +
        367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
        3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 + d - 32075;
}

// converts integer (Julian day number) to Gregorian date:
// month/day/year
void intToDate(int jd, int &m, int &d, int &y) {
    int x, n, i, j;
    x = jd + 68569;
    n = 4 * x / 146097;
    x -= (146097 * n + 3) / 4;
    i = (4000 * (x + 1)) / 1461001;
    x -= 1461 * i / 4 - 31;
    j = 80 * x / 2447;
    d = x - 2447 * j / 80;
    x = j / 11;
```

```
    m = j + 2 - 12 * x;  
    y = 100 * (n - 49) + i + x;  
}  
// converts integer (Julian day number) to day of week  
string intToDay(int jd) { return dayOfWeek[jd % 7]; }  
int main() {  
    int jd = dateToInt(3, 24, 2004);  
    int m, d, y;  
    intToDate(jd, m, d, y);  
    string day = intToDay(jd);
```

```
// expected output:  
//      2453089  
//      3/24/2004  
//      Wed  
cout << jd << endl  
      << m << "/" << d << "/" << y << endl  
      << day << endl;  
}
```

---