Codetroopers ICPC Team Notebook 2019

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Sumário

1 Template

1.1 Macros

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i=(a); i<(b); i++)
#define pb push_back
#define mp make_pair
#define debug(x) cout<<__LINE__<<": "<<#x<<" = "<<x<<endl;
#define debug2(x, y) cout<<__LINE__<<": "<<#x<<" = "<<x<<\
                          "  "<<#y<<" = "<<y<<endl;
#define all(c) (c).begin(), (c).end()
#define F first
#define S second
#define UNIQUE(c) \
    sort(all(c)); \
    (c).resize(unique(all(c))-c.begin());

#define PI 3.1415926535897932384626433832795028841971

typedef long long ll;
typedef pair<int, int> ii;
typedef vector<int> vi;

const int INF = 0x3f3f3f3f;
const double EPS = 1e-9;

inline int cmp(double x, double y = 0, double tol = EPS){
  return ((x <= y+tol) ? (x+tol < y) ? -1:0:1);
}
```

2 Numerical algorithms

2.1 Triângulo de Pascal

```
// Calcula os numeros binomiais (N,K) = N!/(K!(N-K)!). (N,K)
// representa o numero de maneiras de criar um subconjunto de tamanho
// K dado um conjunto de tamanho N. A ordem dos elementos nao
// importa.
const int MAXN = 50;
long long C[MAXN][MAXN];
void calc_pascal() {
  memset(C, 0, sizeof(C));
  for (int i = 0; i < MAXN; ++i) {
    C[i][0] = C[i][i] = 1;
    for (int j = 1; j < i; ++j)
      C[i][j] = C[i - 1][j - 1] + C[i - 1][j];
  }
}
/*
Pascal triangle elements:
C(33, 16) = 1.166.803.110 [int limit]
C(34, 17) = 2.333.606.220 [unsigned int limit]
C(66, 33) = 7.219.428.434.016.265.740 [int64_t limit]
C(67, 33) = 14.226.520.737.620.288.370 [uint64_t limit]

Fatorial
12 ! = 479.001.600 [(unsigned)int limit]
20 ! = 2.432.902.008.176.640.000 [(unsigned)int64_t limit]
*/
```

2.2 GCD-LCM

```
// Calcula o maior divisor comum entre A e B
ll A, B;
cin >> A >> B;
cout << __gcd(A, B);

// Calcula o menor multiplo comum entre A e B
ll lcm(ll A, ll B) {
  if (A and B)
    return abs(A) / __gcd(A, B) * abs(B);
  else
    return abs(A | B);
}
```

2.3 Bezout Theorem

```
// Determina a solucao da equacao a*x+b*y = gcd(a, b), onde a e b sao
// dois numeros naturais. Como chamar: egcd(a, b), Retorna: a tupla
// {gcd(a, b), x, y}.  Determina tambem o Inverso Modular.
struct Triple {
  ll d, x, y;
  Triple(ll q, ll w, ll e) : d(q), x(w), y(e) {}
};
Triple egcd(ll a, ll b) {
  if (!b)
    return Triple(a, 1, 0);
  Triple q = egcd(b, a % b);
  return Triple(q.d, q.y, q.x - a / b * q.y);
}
// Retorna o inverso modular de A modulo N
// O inverso modular de um numero A em relacao a N eh um numero X tal
// que (A*X)%N = 1
ll invMod(ll a, ll n) {
  Triple t = egcd(a, n);
  if (t.d > 1)
    return 0;
  return (t.x % n + n) % n;
}
```

2.4 Teorema Chinês dos Restos

```
// crt() retorna um  X tal que X = a[i] (mod m[i]). Exemplo: Para a[]
// = {1, 2, 3} e m[] = {5, 6, 7} .: X = 206. Requer: Bezout Theorem
// para calcular o inverso modular
#define MAXN 1000
int n;
ll a[MAXN], m[MAXN];
ll crt() {
  ll M = 1, x = 0;
```

```
    for (int i = 0; i < n; ++i)
      M *= m[i];
    for (int i = 0; i < n; ++i)
      x += a[i] * invMod(M / m[i], m[i]) * (M / m[i]);
    return (((x % M) + M) % M);
}
```

2.5 Crivo de Eratóstenes

```
bitset<10000005> bs;
vector<int> primos;
void crivo(ll limite = 10000000LL) { // calcula primos ate limite
  primos.clear();
  bs.set();
  bs[0] = bs[1] = 0;
  for (ll i = 2; i <= limite; i++)
    if (bs[i]) {
      for (ll j = i * i; j <= limite; j += i)
        bs[j] = 0;
      primos.push_back(i);
    }
}
bool isPrime(ll N, ll limite) {
  if (N <= limite)
    return bs[N];
  for (int i = 0; i < (int)primos.size(); i++)
    if (N % primos[i] == 0)
      return false;
  return true;
}
```

2.6 Divisores de N

```
// Retorna todos os divisores naturais de N em O(sqrt(N)).
vector<ll> divisores(ll N) {
  vector<ll> divisors;
  for (ll div = 1, k; div * div <= N; ++div) {
    if (N % div == 0) {
      divisors.push_back(div);
      k = N / div;
      if (k != div)
        divisors.push_back(k);
    }
  }
  // caso precise ordenado
  sort(divisors.begin(), divisors.end());
  return divisors;
}
```

2.7 Funções com Números Primos (Crivo, Fatoração, PHI, etc)

```
// Encontra os fatores primos de N .: N = p1^e1 * ... *pi^ei
// factors armazena em first o fator primo e em segundo seu expoente
map<int, int> factors;
void primeFactors(ll N) {
  factors.clear();
  while (N % 2 == 0)
    ++factors[2], N >>= 1;
  for (ll PF = 3; PF * PF <= N; PF += 2) {
    while (N % PF == 0)
      N /= PF, factors[PF]++;
  }
  if (N > 1)
    factors[N] = 1;
}

// Funcoess derivadas dos numeros primos
void NumberTheory(ll N) {
  primeFactors(N);
  map<int, int>::iterator f; // iterador
  ll Totient = N;            // Totiente ou Euler-Phi de N
  // Totient(N) = qtos naturais x, tal que x < N && gcd(x,N) == 1
  ll numDiv = 1; // Quantidade de divisores de N
  ll sumDiv = 1; // Soma dos divisores de N
  ll sumPF = 0;  // Soma dos fatores primos de N (trivial)
  ll numDiffPF = factors.size(); // qtde de fatores distintos

  for (f = factors.begin(); f != factors.end(); f++) {
    ll PF = f->first, power = f->second;
    Totient -= Totient / PF;
    numDiv *= (power + 1);
    sumDiv *= ((ll)pow((double)PF, power + 1.0) - 1) / (PF - 1);
    sumPF += PF;
  }

  printf("Totiente/Euler-Phi de N = %lld\n", Totient);
  printf("qt de divisores de N = %lld\n", numDiv);
  printf("soma dos divisores de N = %lld\n", sumDiv);
  printf("qt de fatores primos distintos = %lld\n", numDiffPF);
  printf("soma dos fatores primos = %lld\n", sumPF);
}

// Calcula Euler Phi para cada valor do intervalo [1, N]
#define MM 1000010
int phi[MM];
void crivo_euler_phi(int N) {
  for (int i = 1; i <= N; i++)
    phi[i] = i;
  for (int i = 2; i <= N; i++)
    if (phi[i] == i) {
      for (int k = i; k <= N; k += i)
        phi[k] = (phi[k] / i) * (i - 1);
    }
}

// Qtde de fatores primos distintos de cada valor do range [2, MAX_N]
#define MAX_N 10000000
int NDPF[MAX_N]; //
void NumDiffPrimeFactors() {
  memset(NDPF, 0, sizeof NDPF);
  for (int i = 2; i < MAX_N; i++)
    if (NDPF[i] == 0)
      for (int j = i; j < MAX_N; j += i)
        NDPF[j]++;
}
```

```cpp
int main() { return 0; }
```

2.8 Exponenciação Modular Rápida

```cpp
/**
 * fastpow() realiza exponenciacao rapida de inteiros
 * #param ll b - base da exponenciacao
 * #param ll expo - expoente
 * #param ll mod - o resultado sera calculado modulo este valor
 * #return - o valor de (b ^ p) % mod
 * #complexidade - O(log(p))
**/

ll fastpow(ll b, ll expo, ll mod) {
  ll ret = 1, pot = b % mod;
  while (expo) {
    if (expo & 1) {
      ret = (ret * pot) % mod;
    }
    pot = (pot * pot) % mod;
    expo >>= 1;
  }
  return ret;
}
```

2.9 Exponenciação de Matriz

```cpp
/**
 * fastpow() realiza exponenciacao rapida de matrizes
 * #param matrix_t M - matriz a ser elevada
 * #param ll expo - expoente
 * #param ll mod - o resultado sera calculado modulo este valor
 * #return - o resultado de (M ^ expo) % mod
 * #complexidade - O(size^3 * log(expo))
**/

#define MAX (ChangeMe) // Max size of square matrix

struct matrix_t {
  ll m[MAX][MAX];
  int sz;
  matrix_t(int _sz) {
    memset(m, 0, sizeof(m));
    sz = _sz;
  }
  matrix_t multiply(const matrix_t other, ll mod) {
    matrix_t ret(other.sz);
    rep(i, 0, sz) rep(j, 0, sz) {
      ret.m[i][j] = 0;
      rep(k, 0, sz) ret.m[i][j] =
          (ret.m[i][j] + (m[i][k] * other.m[k][j]) % mod) % mod;
    }
    return ret;
  }
};

matrix_t fastpow(matrix_t M, ll expo, ll mod) {
  matrix_t ret(M.sz);
  // Identity matrix
  rep(i, 0, ret.sz) rep(j, 0, ret.sz) ret.m[i][j] = (i == j);

  while (expo) {
    if (expo & 1)
      ret = ret.multiply(M, mod);
    M = M.multiply(M, mod);
    expo >>= 1;
  }
  return ret;
}
```

2.10 Brent Cycle Detection

```cpp
// Dado uma sequencia formada por uma funcao f(.) e uma semente x0.
// f(x0), f(f(x0)), ..., f(f(...f(x0))), ela pode ser ciclica. Este
// algoritmo retorna o tamanho do ciclo e o valor xi que o inicia.
ii brent_cycle(int x) {
  int p = 1, length = 1, t = x, start = 0;
  int h = f(x);
  while (t != h) {
    if (p == length) {
      t = h;
      p *= 2;
      length = 0;
    }
    h = f(h);
    ++length;
  }
  t = h = x;
  for (int i = length; i != 0; --i)
    h = f(h);
  while (t != h) {
    t = f(t);
    h = f(h);
    ++start;
  }
  return ii(start, length);
}
```

2.11 Romberg's method - Calcula Integral (UFS2010)

```cpp
// Calcula a integral de f[a, b]
typedef long double ld;

ld f(double x) {
  // return f(x)
}

ld romberg(ld a, ld b) {
  ld R[16][16], div = (b - a) / 2;
  R[0][0] = div * (f(a) + f(b));
  for (int n = 1; n <= 15; n++, div /= 2) {
    R[n][0] = R[n - 1][0] / 2;
    for (ld sample = a + div; sample < b; sample += 2 * div)
      R[n][0] += div * f(a + sample);
  }
```

```
  for (int m = 1; m <= 15; m++)
    for (int n = m; n <= 15; n++)
      R[n][m] = R[n][m - 1] +
                1 / (pow(4, m) - 1) * (R[n][m - 1] - R[n - 1][m - 1]);
  return R[15][15];
}
```

2.12 Pollard's rho algorithm (UFS2010)

```
// Retorna um fator primo de N, util para fatorizacao quando N for
// grande.
ll pollard_r, pollard_n;
ll f(ll val) { return (val * val + pollard_r) % pollard_n; }
ll myabs(ll a) { return a >= 0 ? a : -a; }
ll pollard(ll n) {
  srand(unsigned(time(0)));
  pollard_n = n;
  long long d = 1;
  do {
    d = 1;
    pollard_r = rand() % n;
    long long x = 2, y = 2;
    while (d == 1)
      x = f(x), y = f(f(y)), d = __gcd(myabs(x - y), n);
  } while (d == n);
  return d;
}
```

2.13 Miller-Rabin's algorithm (UFS2010)

```
// Teste probabilistico de primalidade
bool miller_rabin(ll n, ll base) {
  if (n <= 1)
    return false;
  if (n % 2 == 0)
    return n == 2;
  ll s = 0, d = n - 1;
  while (d % 2 == 0)
    d /= 2, ++s;
  ll base_d = fastpow(base, d, n);
  if (base_d == 1)
    return true;
  ll base_2r = base_d;
  for (ll i = 0; i < s; ++i) {
    if (base_2r == 1)
      return false;
    if (base_2r == n - 1)
      return true;
    base_2r = base_2r * base_2r % n;
  }
  return false;
}
bool isprime(ll n) {
  if (n == 2 || n == 7 || n == 61)
    return true;
  return miller_rabin(n, 2) && miller_rabin(n, 7) &&
         miller_rabin(n, 61);
}
```

2.14 Quantidade de dígitos de N! na base B

```
int NumOfDigitsInFactorial(int N, int B) {
  double logFatN = 0;
  for (int i = 1; i <= N; i++)
    logFatN += log((double)i);
  int nd = floor(logFatN / log((double)B)) + 1;
  return nd;
}
```

2.15 Quantiade de zeros a direita de N! na base B

```
// Determina o numero de zeros a direita do fatorial de N na base B
// Ideia: Se a base for B for 10, e fatorarmos N! em fatores primos
// teremos algo como N! = 2^a * 3^b * 5^c ..., como cada par de primos
// 2 e 5 formam 10 que tem um zero, a quantidade seria min(a, c).
int NumOfTrailingZeros(int N, int B) {
  int nfact = fatora(B);
  int zeros = INF;
  // para cada fator de B, aux representa qtas vezes
  // fator[i]^expoente[i] aparece na representacao de N!
  for (int i = 0; i < nfact; i++) {
    int soma = 0;
    int NN = N;
    while (NN) {
      soma += NN / fator[i];
      NN /= fator[i];
    }
    int aux = soma / expoente[i];
    zeros = min(zeros, aux);
  }
  return zeros;
}
```

2.16 Baby Step Giant Step

```
// Determinar o menor E tal que B^E = N (mod P), -1 se for impossivel.
// Requer: Bezout Theorem para calcular o inverso modular
ll bsgs(ll b, ll n, ll p) {
  if (n == 1)
    return 0;
  map<ll, int> table;
  ll m = sqrt(p) + 1, pot = 1, pot2 = 1;
  for (int j = 0; j < m; ++j) {
    if (pot == n)
      return j;
    table[(n * invMod(pot, p)) % p] = j;
    pot = (pot * b) % p;
  }
  for (int i = 0; i < m; ++i) {
    if (table.find(pot2) != table.end())
      return i * m + table[pot2];
    pot2 = (pot * pot2) % p;
  }
  return -1;
}
```

2.17 Primos num intervalo

```
// Encontra os primos no intervalo [n,m]
vector<int> ret;
void primesBetween(int n, int m) {
  ret.clear();
  vector<int> primes(m - n + 1);
  for (int i = 0; i < m - n + 1; ++i)
    primes[i] = 0;
  for (int p = 2; p * p <= m; ++p) {
    int less = (n / p) * p;
    for (int j = less; j <= m; j += p)
      if (j != p && j >= n)
        primes[j - n] = 1;
  }
  for (int i = 0; i < m - n + 1; ++i) {
    if (primes[i] == 0 && n + i != 1) {
      ret.push_back(n + i);
    }
  }
}
```

2.18 FFT

```
typedef complex<double> comp;
const int MAX_N = 1 << 20;
int rev[MAX_N];
comp roots[MAX_N];

void preCalc(int N, int BASE) {
  for (int i = 1; i < N; ++i)
    rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (BASE - 1));
  int NN = N >> 1;
  roots[NN] = comp(1, 0);
  roots[NN + 1] = comp(cos(2 * PI / N), sin(2 * PI / N));
  for (int i = 2; i < NN; ++i)
    roots[NN + i] = roots[NN + i - 1] * roots[NN + 1];
  for (int i = NN - 1; i > 0; --i)
    roots[i] = roots[2 * i];
}

void fft(vector<comp> &a, bool invert) {
  int N = a.size();
  if (invert)
    rep(i, 0, N) a[i] = conj(a[i]);
  rep(i, 0, N) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < N; k *= 2) {
    for (int i = 0; i < N; i += 2 * k) {
      rep(j, 0, k) {
        comp B = a[i + j + k] * roots[k + j];
        a[i + j + k] = a[i + j] - B;
        a[i + j] = a[i + j] + B;
      }
    }
  }
  if (invert)
    rep(i, 0, a.size()) a[i] /= N;
}
```

```
vector<comp> multiply_real(vector<comp> a, vector<comp> b,
                           vector<comp> c) {
  int n = a.size();
  int m = b.size();

  int base = 0, N = 1;
  while (N < n + m - 1)
    base++, N <<= 1;
  preCalc(N, base);

  a.resize(N, comp(0, 0));
  c.resize(N);

  rep(i, 0, b.size()) a[i] = comp(real(a[i]), real(b[i]));
  fft(a, 0);
  rep(i, 0, N) {
    int j = (N - i) & (N - 1);
    c[i] = (a[i] * a[i] - conj(a[j] * a[j])) * comp(0, -0.25);
  }
  fft(c, 1);
  return c;
}
```

3 Geometria 2D

3.1 Geometria 2D Library

```
const double EPS = 1e-9;
inline int cmp(double x, double y = 0, double tol = EPS) {
  return ((x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1);
}

struct point {
  double x, y;
  point(double x = 0, double y = 0) : x(x), y(y) {}
  point operator+(point q) { return point(x + q.x, y + q.y); }
  point operator-(point q) { return point(x - q.x, y - q.y); }
  point operator*(double t) { return point(x * t, y * t); }
  point operator/(double t) { return point(x / t, y / t); }
  int cmp(point q) const {
    if (int t = ::cmp(x, q.x))
      return t;
    return ::cmp(y, q.y);
  }
  bool operator==(point q) const { return cmp(q) == 0; };
  bool operator!=(point q) const { return cmp(q) != 0; };
  bool operator<(point q) const { return cmp(q) < 0; };
};
ostream &operator<<(ostream &os, const point &p) {
  os << "(" << p.x << "," << p.y << ")";
}
#define vec(a, b) (b - a)
typedef vector<point> polygon;

double cross(point a, point b) { return a.x * b.y - a.y * b.x; }
double dot(point a, point b) { return a.x * b.x + a.y * b.y; }
double collinear(point a, point b, point c) {
```

```cpp
  return cmp(cross(b - a, c - a)) == 0;
}
// retorna 1 se R esta a esquerda do vetor P->Q, -1 se estiver a
// direita. 0 se P, Q e R forem colineares
int ccw(point p, point q, point r) { return cmp(cross(q - p, r - p)); }

// Rotaciona um ponto em relacao a origem em 90 graus sentido
// anti-horario
point RotateCCW90(point p) { return point(-p.y, p.x); }
// Rotaciona um ponto em relacao a origem em 90 graus sentido horario
point RotateCW90(point p) { return point(p.y, -p.x); }

// Rotaciona um ponto P em A graus no sentido anti-horario em relacao a
// origem; Para rotacionar no sentido horario, basta A ser negativo
point RotateCCW(point p, double a) {
  a = (a / 180.0) * acos(-1.0); // convertendo para radianos
  return point(p.x * cos(a) - p.y * sin(a),
               p.x * sin(a) + p.y * cos(a));
}
// Rotaciona P em A graus em relacao a Q.
point RotateCCW(point p, point q, double a) {
  return RotateCCW(p - q, a) + q;
}
// Tamanho ou norma de um vetor
double abs(point u) { return sqrt(dot(u, u)); }
// Projeta o vetor A sobre a direcao do vetor B
point project(point a, point b) { return b * (dot(a, b) / dot(b, b)); }
// Retorna a projecao do ponto P sobre reta definida por [A,B]
point projectPointLine(point p, point a, point b) {
  return p + project(p - a, b - a);
}
// Retorna o angulo que p faz com +x
double arg(point p) { return atan2(p.y, p.x); }
// Retorna o angulo entre os vetores AB e AC
double arg(point b, point a, point c) {
  point u = b - a, v = c - a;
  return atan2(cross(u, v), dot(u, v));
}

//////////////////////////////////////////////////////////////////////
/////////////////////////  Segmentos, Retas  /////////////////////////
//////////////////////////////////////////////////////////////////////

// Determina se P esta entre o segmento fechado [A,B], inclusive
bool between(point p, point a, point b) {
  return collinear(p, a, b) && dot(a - p, b - p) <= 0;
}

/* Distancia de ponto P para reta que passa por [A,B]. Armazena em C
 * (por ref) o ponto projecao de P na reta. */
double distancePointLine(point p, point a, point b, point &c) {
  c = projectPointLine(p, a, b);
  return fabs(cross(p - a, b - a)/abs(a - b); // or abs(p-c);
}

/* Distancia de ponto P ao segmento [A,B]. Armazena em C (por ref) o
 * ponto de projecao de P em [A,B]. Se este ponto estiver fora do
 * segmento, eh retornado o mais proximo. */
double distancePointSeg(point p, point a, point b, point &c) {
  if ((b - a) * (p - a) <= 0) {
    c = a;
    return abs(a - p);
  }
  if ((a - b) * (p - b) <= 0) {
    c = b;
    return abs(b - p);
  }

  c = projectPointLine(p, a, b);
  return fabs(cross(p - a, b - a)/abs(a - b); // or abs(p-c);
}

// Determina se os segmentos [A, B] e [C, D] se tocam
bool seg_intersect(point a, point b, point c, point d) {
  int d1, d2, d3, d4;
  d1 = ccw(c, a, d);
  d2 = ccw(c, b, d);
  d3 = ccw(a, c, b);
  d4 = ccw(a, d, b);
  if (d1 * d2 == -1 && d3 * d4 == -1)
    return true;
  if (d1 == 0 && between(c, a, d))
    return true;
  if (d2 == 0 && between(c, b, d))
    return true;
  if (d3 == 0 && between(a, c, b))
    return true;
  if (d4 == 0 && between(a, d, b))
    return true;
  return false;
}

// Encontra a interseccao das retas (p-q) e (r-s) assumindo que existe
// apenas 1 intereccao. Se for entre segmentos, verificar se
// interseptam primeiro.
point line_intersect(point p, point q, point r, point s) {
  point a = q - p, b = s - r, c = point(cross(p, q), cross(r, s));
  double x = cross(point(a.x, b.x), c);
  double y = cross(point(a.y, b.y), c);
  return point(x, y) / cross(a, b);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(point a, point b, point c, point d) { //! Nao
                                                         //! testado
  return fabs(cross(b - a, c - d)) < EPS;
}
bool LinesCollinear(point a, point b, point c,
                    point d) { //! Nao testado
  return LinesParallel(a, b, c, d) &&
         fabs(cross(a - b, a - c)) < EPS &&
         fabs(cross(c - d, c - a)) < EPS;
}

//////////////////////////////////////////////////////////////////////
////////////////////////////  Triangulos  ////////////////////////////
//////////////////////////////////////////////////////////////////////

bool pointInTriangle(point p, point a, point b, point c) {
```

```cpp
  // TODO
}

// Heron's formula - area do triangulo(a,b,c) -1 se nao existe
double area_heron(double a, double b, double c) {
  if (a < b)
    swap(a, b);
  if (a < c)
    swap(a, c);
  if (b < c)
    swap(b, c);
  if (a > b + c)
    return -1;
  return sqrt((a + (b + c)) * (c - (a - b)) * (c + (a - b)) *
              (a + (b - c)) / 16.0);
}

//////////////////////////////////////////////////////////////////////
////////////////////////////  Circulos  //////////////////////////////
//////////////////////////////////////////////////////////////////////

bool pointInCircle(point p, point c, double radius) {
  // Todo
}

/*Dado dois pontos (A, B) de uma circunferencia e seu raio R, eh
 * possivel obter seus possiveis centros (C1 e C2). Para obter o outro
 * centro, basta inverter os paramentros */
bool circle2PtsRad(point a, point b, double r, point &c) {
  point aux = a - b;
  double d = dot(aux, aux);
  double det = r * r / d - 0.25;
  if (det < 0.0)
    return false;
  double h = sqrt(det);
  c.x = (a.x + b.x) * 0.5 + (a.y - b.y) * h;
  c.y = (a.y + b.y) * 0.5 + (b.x - a.x) * h;
  return true;
}

//  Menor distancia entre dois pontos numa esfera de raio r
//  lat = [-90,90]; long = [-180,180]
double spherical_distance(double lt1, double lo1, double lt2,
                          double lo2, double r) {
  double pi = acos(-1);
  double a = pi * (lt1 / 180.0), b = pi * (lt2 / 180.0);
  double c = pi * ((lo2 - lo1) / 180.0);
  return r * acos(sin(a) * sin(b) + cos(a) * cos(b) * cos(c));
}

//////////////////////////////////////////////////////////////////////
/////////////////////////////  Planos  ///////////////////////////////
//////////////////////////////////////////////////////////////////////

// Distancia entre (x,y,z) e plano ax+by+cz=d
double distancePointPlane(double x, double y, double z, double a,
                          double b, double c, double d) {
  return fabs(a * x + b * y + c * z - d) / sqrt(a * a + b * b + c *
      c);
}
```

```cpp
//***[Inicio] Funcoes que usam numeros complexos para pontos***
typedef complex<double> cxpt;
struct circle {
  cxpt c;
  double r;
  circle(cxpt c, double r) : c(c), r(r) {}
  circle() {}
};
double cross(const cxpt &a, const cxpt &b) {
  return imag(conj(a) * b);
}
double dot(const cxpt &a, const cxpt &b) { return real(conj(a) * b); }

// Area da interseccao de dois circulos
double circ_inter_area(circle &a, circle &b) {
  double d = abs(b.c - a.c);
  if (d <= (b.r - a.r))
    return a.r * a.r * M_PI;
  if (d <= (a.r - b.r))
    return b.r * b.r * M_PI;
  if (d >= a.r + b.r)
    return 0;
  double A = acos((a.r * a.r + d * d - b.r * b.r) / (2 * a.r * d));
  double B = acos((b.r * b.r + d * d - a.r * a.r) / (2 * b.r * d));
  return a.r * a.r * (A - 0.5 * sin(2 * A)) +
         b.r * b.r * (B - 0.5 * sin(2 * B));
}

// Pontos de interseccao de dois circulos
// Intersects two circles and intersection points are in 'inter'
// -1-> outside, 0-> inside, 1-> tangent, 2-> 2 intersections
int circ_circ_inter(circle &a, circle &b, vector<cxpt> &inter) {
  double d2 = norm(b.c - a.c), rS = a.r + b.r, rD = a.r - b.r;
  if (d2 > rS * rS)
    return -1;
  if (d2 < rD * rD)
    return 0;
  double ca = 0.5 * (1 + rS * rD / d2);
  cxpt z = cxpt(ca, sqrt((a.r * a.r / d2) - ca * ca));
  inter.push_back(a.c + (b.c - a.c) * z);
  if (abs(z.imag()) > EPS)
    inter.push_back(a.c + (b.c - a.c) * conj(z));
  return inter.size();
}

// Line-circle intersection
// Intersects (infinite) line a-b with circle c
// Intersection points are in 'inter'
// 0 -> no intersection, 1 -> tangent, 2 -> two intersections
int line_circ_inter(cxpt a, cxpt b, circle c, vector<cxpt> &inter) {
  c.c -= a;
  b -= a;
  cxpt m = b * real(c.c / b);
  double d2 = norm(m - c.c);
  if (d2 > c.r * c.r)
    return 0;
  double l = sqrt((c.r * c.r - d2) / norm(b));
  inter.push_back(a + m + l * b);
  if (abs(l) > EPS)
    inter.push_back(a + m - l * b);
  return inter.size();
```

```
}
//***[FIM] Funcoes que usam numeros complexos para pontos***
```

4 Polígonos 2D

4.1 Polígono 2D Library

```
/*Poligono eh representado como um array de pontos T[i] sao os
    vertices
do poligono. Existe uma aresta que conecta T[i] com T[i+1], e
    T[size-1]
com T[0]. Logo assume-se que T[0] != T[size-1]
Poligono simples: Aquele em que as arestas nao se interceptam.
    Convexo:
O angulo interno de T[i] com T[i-1] e T[i+1] <= 180. Concavo: Existe
algum i que nao satisfaz a condicao anterior*/

/* Retorna a area com sinal de um poligono T. Se area > 0, T esta
 * listado na ordem CCW */
double signedArea(const polygon &T) {
  double area = 0;
  int n = T.size();
  if (n < 3)
    return 0;
  rep(i, 0, n) area += cross(T[i], T[(i + 1) % n]);
  return (area / 2.0);
}

/* Retorna a area de um poligono T. (pode ser concavo ou convexo) em
 * O(N)*/
double poly_area(const polygon &T) { return fabs(signedArea(T)); }

/* Retorna a centroide de um poligono T em O(N)*/
point centroide(const polygon &T) {
  int n = T.size();
  double sgnArea = signedArea(T);

  point c = point(0, 0);
  rep(i, 0, n) {
    int k = (i + 1) % n;
    c = c + (T[i] + T[k]) * cross(T[i], T[k]);
  }
  c = c / (sgnArea * 6.0);
  return c;
}

/* Retorna o perimetro do poligono T. (pode n funcionar como esperado
 * se o poligono for uma linha reta (caso degenerado))*/
double poly_perimeter(polygon &T) {
  double perimeter = 0;
  int n = T.size();
  if (n < 3)
    return 0;
  rep(i, 0, n) perimeter += abs(T[i] - T[(i + 1) % n]);
  return perimeter;
}

// tests whether or not a given polygon (in CW or CCW order) is simple
bool isSimple(const polygon &p) { // nao testado
  for (int i = 0; i < p.size(); i++) {
    for (int k = i + 1; k < p.size(); k++) {
      int j = (i + 1) % p.size();
      int l = (k + 1) % p.size();
      if (i == l || j == k)
        continue;
      if (seg_intersect(p[i], p[j], p[k], p[l]))
        return false;
    }
  }
  return true;
}

// Retorna True se T for convexo. O(N)
bool isConvex(polygon &T) {
  int n = T.size();
  if (n < 3)
    return false;

  int giro = 0;
  rep(i, 0, n) { // encontra um giro valido
    int t = ccw(T[i], T[(i + 1) % n], T[(i + 2) % n]);
    if (t != 0)
      giro = t;
  }
  if (giro == 0)
    return false; // todos pontos sao colineares

  rep(i, 0, n) {
    int t = ccw(T[i], T[(i + 1) % n], T[(i + 2) % n]);
    if (t != 0 && t != giro)
      return false;
  }
  return true;
}

//  Determina se P pertence a T, funciona para convexo ou concavo
// -1 borda, 0 fora, 1 dentro. O(N)
int in_poly(point p, polygon &T) {
  double a = 0;
  int N = T.size();
  rep(i, 0, N) {
    if (between(p, T[i], T[(i + 1) % N]))
      return -1;
    a += arg(T[i], p, T[(i + 1) % N]);
  }
  return cmp(a) != 0;
}

// determina se P pertence a B, funciona APENAS para convexo
bool PointInConvexPolygon(point P, const polygon &B) {
  int ini = 1, fim = B.size() - 2, mid, pos = -1;
  int giro = -1; // sentido horario
  while (ini <= fim) {
    mid = (ini + fim) / 2;
    int aux = ccw(B[0], B[mid], P);
    if (aux == giro) {
      pos = mid;
      ini = mid + 1;
    } else {
```

```
      fim = mid - 1;
    }
  }
  if (pos == -1)
    return false;
  if (ccw(B[0], B[pos], P) != giro * -1 &&
      ccw(B[0], B[pos + 1], P) != giro &&
      ccw(B[pos], B[pos + 1], P) == giro) // giro || 0 na borda
    return true;
  return false;
}

//  Determina o poligono interseccao de P e Q
//  P e Q devem estar orientados anti-horario.
polygon poly_intersect(polygon &P, polygon &Q) {
  int m = Q.size(), n = P.size();
  int a = 0, b = 0, aa = 0, ba = 0, inflag = 0;
  polygon R;
  while ((aa < n || ba < m) && aa < 2 * n && ba < 2 * m) {
    point p1 = P[a], p2 = P[(a + 1) % n], q1 = Q[b],
          q2 = Q[(b + 1) % m];
    point A = p2 - p1, B = q2 - q1;
    int cross = cmp(cross(A, B)), ha = ccw(p2, q2, p1),
        hb = ccw(q2, p2, q1);
    if (cross == 0 && ccw(p1, q1, p2) == 0 && cmp(dot(A, B)) < 0) {
      if (between(q1, p1, p2))
        R.push_back(q1);
      if (between(q2, p1, p2))
        R.push_back(q2);
      if (between(p1, q1, q2))
        R.push_back(p1);
      if (between(p2, q1, q2))
        R.push_back(p2);
      if (R.size() < 2)
        return polygon();
      inflag = 1;
      break;
    } else if (cross != 0 && seg_intersect(p1, p2, q1, q2)) {
      if (inflag == 0)
        aa = ba = 0;
      R.push_back(line_intersect(p1, p2, q1, q2));
      inflag = (hb > 0) ? 1 : -1;
    }
    if (cross == 0 && hb < 0 && ha < 0)
      return R;
    bool t = cross == 0 && hb == 0 && ha == 0;
    if (t ? (inflag == 1) : (cross >= 0) ? (ha <= 0) : (hb > 0)) {
      if (inflag == -1)
        R.push_back(q2);
      ba++;
      b++;
      b %= m;
    } else {
      if (inflag == 1)
        R.push_back(p2);
      aa++;
      a++;
      a %= n;
    }
  }
  if (inflag == 0) {
    if (in_poly(P[0], Q))
      return P;
    if (in_poly(Q[0], P))
      return Q;
  }
  R.erase(unique(all(R)), R.end());
  if (R.size() > 1 && R.front() == R.back())
    R.pop_back();
  return R;
}
```

4.2 Convex Hull

```
/*Encontra o convex hull de um conjunto de pontos em O(NlogN)
pivot: Ponto base para a criacao do convex hull;
radial_lt(): Ordena os pontos em sentido anti-horario (ccw).
Input: Conjunto de pontos 2D;
Output: Conjunto de pontos do convex hull, no sentido anti-horario;

(1)Se for preciso manter pontos colineares na borda do convex hull,
essa
parte evita que eles sejam removidos;
*/

point pivot;
bool radial_lt(point a, point b) {
  int R = ccw(pivot, a, b);
  if (R == 0) // sao colineares
    return (pivot - a) * (pivot - a) < (pivot - b) * (pivot - b);
  else
    return (R == 1); // 1 se A esta a direita de (pivot->B)
}
vector<point> convexhull(vector<point> &T) {
  // Se for necessario remover pontos duplicados
  sort(T.begin(), T.end()); // ordena por x e por y
  T.resize(unique(T.begin(), T.end()) - T.begin());

  int tam = 0, n = T.size();
  vector<point> U; // convex hull

  int idx = min_element(T.begin(), T.end()) - T.begin();
  // nesse caso, pivot = ponto com menor x, depois menor y
  pivot = T[idx];
  swap(T[0], T[idx]);
  sort(++T.begin(), T.end(), radial_lt);

  /*(1)*/ int k;
  for (k = n - 2; k >= 0 && ccw(T[0], T[n - 1], T[k]) == 0; k--)
    ;
  reverse((k + 1) + all(T)); /*(1)*/

  // troque <= por < para manter pontos colineares na borda
  for (int i = 0; i < T.size(); i++) {
    while (tam > 1 && ccw(U[tam - 2], U[tam - 1], T[i]) <= 0)
      U.pop_back(), tam--;
    U.pb(T[i]);
    tam++;
  }
  return U;
}
```

4.3 Minimum Enclosing Circle

```
// Finds a circle of the minimum area enclosing a 2D point set.
typedef pair<point, double> circle; // {ponto,raio}
bool in_circle(circle C, point p) { // ponto dentro de circulo?
  return cmp(abs(p - C.first), C.second) <= 0;
}
// menor circulo que engloba o triangulo (P,Q,R)
point circumcenter(point p, point q, point r) {
  point a = p - r, b = q - r, c, ret;
  c = point(dot(a, p + r), dot(b, q + r)) * 0.5;
  ret = point(cross(c, point(a.y, b.y)), cross(point(a.x, b.x), c)) /
        cross(a, b);
  return ret;
}
circle spanning_circle(const vector<point> &T) {
  int n = T.size();
  random_shuffle(all(T));
  circle C(point(), -INF);
  rep(i, 0, n) if (!in_circle(C, T[i])) {
    C = circle(T[i], 0);
    rep(j, 0, i) if (!in_circle(C, T[j])) {
      C = circle((T[i] + T[j]) / 2, abs(T[i] - T[j]) / 2);
      rep(k, 0, j) if (!in_circle(C, T[k])) {
        point O = circumcenter(T[i], T[j], T[k]);
        C = circle(O, abs(O - T[k]));
      }
    }
  }
  return C;
}
```

5 Geometria 3D

5.1 Geometria 3D Library

```
#define LINE 0
#define SEGMENT 1
#define RAY 2
int sgn(double x) { return (x > EPS) - (x < -EPS); }

#define vec(ini, fim) (fim - ini)
struct PT {
  double x, y, z;
  PT() { x = y = z = 0; }
  PT(double x, double y, double z) : x(x), y(y), z(z) {}
  PT operator+(PT q) { return PT(x + q.x, y + q.y, z + q.z); }
  PT operator-(PT q) { return PT(x - q.x, y - q.y, z - q.z); }
  PT operator*(double d) { return PT(x * d, y * d, z * d); }
  PT operator/(double d) { return PT(x / d, y / d, z / d); }
  double dist2() const { return x * x + y * y + z * z; }
  double dist() const { return sqrt(dist2()); }
  bool operator==(const PT &a) const {
    return fabs(x - a.x) < EPS && fabs(y - a.y) < EPS &&
           fabs(z - a.z) < EPS;
  }
};
double dot(PT A, PT B) { return A.x * B.x + A.y * B.y + A.z * B.z; }
PT cross(PT A, PT B) {
  return PT(A.y * B.z - A.z * B.y, A.z * B.x - A.x * B.z,
            A.x * B.y - A.y * B.x);
}
bool collinear(PT A, PT B, PT C) {
  return sgn(cross(B - A, C - A)) == 0;
}

inline double det(double a, double b, double c, double d) {
  return a * d - b * c;
}
inline double det(double a11, double a12, double a13, double a21,
                  double a22, double a23, double a31, double a32,
                  double a33) {

  return a11 * det(a22, a23, a32, a33) -
         a12 * det(a21, a23, a31, a33) + a13 * det(a21, a22, a31,
             a32);
}
inline double det(const PT &a, const PT &b, const PT &c) {
  return det(a.x, a.y, a.z, b.x, b.y, b.z, c.x, c.y, c.z);
}

// tamanho do vetor A
double norma(PT A) { return sqrt(dot(A, A)); }

// distancia^2 de (a->b)
double distSq(PT a, PT b) { return dot(a - b, a - b); }

// Projeta vetor A sobre o vetor B
PT project(PT A, PT B) { return B * dot(A, B) / dot(B, B); }

// Verifica se existe interseccao de segmentos
// (assumir que [A,B] e [C,D] sao coplanares)
bool seg_intersect(PT A, PT B, PT C, PT D) {
  return cmp(dot(cross(A - B, C - B), cross(A - B, D - B))) <= 0 &&
         cmp(dot(cross(C - D, A - D), cross(C - D, B - D))) <= 0;
}

// square distance between point and line, ray or segment
double ptLineDistSq(PT s1, PT s2, PT p, int type) {
  double pd2 = distSq(s1, s2);
  PT r;
  if (pd2 == 0)
    r = s1;
  else {
    double u = dot(p - s1, s2 - s1) / pd2;
    r = s1 + (s2 - s1) * u;
    if (type != LINE && u < 0.0)
      r = s1;
    if (type == SEGMENT && u > 1.0)
      r = s2;
  }
  return distSq(r, p);
}

//  Distancia de ponto P ao segmento [A,B]
double dist_point_seg(PT P, PT A, PT B) {
  PT PP = A + project(P - A, B - A);
```

```
  if (cmp(norma(A - PP) + norma(PP - B), norma(A - B)) == 0)
    return norma(P - PP); // distance point-line!
  else
    return min(norma(P - A), norma(P - B));
}

// Distance between lines ab and cd. TODO: Test this
double lineLineDistance(PT a, PT b, PT c, PT d) {
  PT v1 = b - a;
  PT v2 = d - c;
  PT cr = cross(v1, v2);
  if (dot(cr, cr) < EPS) {
    PT proj = v1 * (dot(v1, c - a) / dot(v1, v1));
    return sqrt(dot(c - a - proj, c - a - proj));
  } else {
    PT n = cr / sqrt(dot(cr, cr));
    PT p = dot(n, c - a);
    return sqrt(dot(p, p));
  }
}

//  Menor distancia do segmento [A,B] ao segmento [C,D] (lento*)
#define dps dist_point_seg
double dist_seg_seg(PT A, PT B, PT C, PT D) {
  PT E = project(A - D, cross(B - A, D - C));
  // distance between lines!
  if (seg_intersect(A, B, C + E, D + E)) {
    return norma(E);
  } else {
    double dA = dps(A, C, D), dB = dps(B, C, D);
    double dC = dps(C, A, B), dD = dps(D, A, B);
    return min(min(dA, dB), min(dC, dD));
  }
}

//  Menor distancia do segmento [A,B] ao segmento [C,D] (rapido*)
double dist_seg_seg2(PT A, PT B, PT C, PT D) {
  PT u(B - A), v(D - C), w(A - C);
  double a = dot(u, u), b = dot(u, v);
  double c = dot(v, v), d = dot(u, w), e = dot(v, w);

  double DD = a * c - b * b;
  double sc, sN, sD = DD;
  double tc, tN, tD = DD;
  if (DD < EPS) {
    sN = 0, sD = 1, tN = e, tD = c;
  } else {
    sN = (b * e - c * d);
    tN = (a * e - b * d);
    if (sN < 0) {
      sN = 0, tN = e, tD = c;
    } else if (sN > sD) {
      sN = sD, tN = e + b, tD = c;
    }
  }
  if (tN < 0) {
    tN = 0;
    if (-d < 0)
      sN = 0;
    else if (-d > a)
      sN = sD;
    else {
      sN = -d;
      sD = a;
    }
  } else if (tN > tD) {
    tN = tD;
    if ((-d + b) < 0)
      sN = 0;
    else if (-d + b > a)
      sN = sD;
    else {
      sN = -d + b;
      sD = a;
    }
  }
  sc = fabs(sN) < EPS ? 0 : sN / sD;
  tc = fabs(tN) < EPS ? 0 : tN / tD;
  PT dP = w + (u * sc) - (v * tc);
  return norma(dP);
}

//  Distancia de Ponto a Triangulo, dps = dist_point_seg
double dist_point_tri(PT P, PT A, PT B, PT C) {
  PT N = cross(B - A, C - A);
  PT PP = P - project(P - A, N);
  PT R1, R2, R3;
  R1 = cross(B - A, PP - A);
  R2 = cross(C - B, PP - B);
  R3 = cross(A - C, PP - C);

  if (cmp(dot(R1, R2)) >= 0 && cmp(dot(R2, R3)) >= 0 &&
      cmp(dot(R3, R1)) >= 0) {
    return norma(P - PP);
  } else {
    return min(dps(P, A, B), min(dps(P, B, C), dps(P, A, C)));
  }
}

// compute a, b, c, d such that all points lie on ax + by + cz = d.
// TODO: test this
void planeFromPts(PT p1, PT p2, PT p3, double &a, double &b, double
    &c,
                  double &d) {
  PT normal = cross(p2 - p1, p3 - p1);
  a = normal.x;
  b = normal.y;
  c = normal.z;
  d = -a * p1.x - b * p1.y - c * p1.z;
}

// project point onto plane. TODO: test this
PT ptPlaneProj(PT p, double a, double b, double c, double d) {
  double l =
      (a * p.x + b * p.y + c * p.z + d) / (a * a + b * b + c * c);
  return PT(p.x - a * l, p.y - b * l, p.z - c * l);
}

// distance from point p to plane aX + bY + cZ + d = 0
double ptPlaneDist(PT p, double a, double b, double c, double d) {
  return fabs(a * p.x + b * p.y + c * p.z + d) /
         sqrt(a * a + b * b + c * c);
```

```
}

// distance between parallel planes aX + bY + cZ + d1 = 0 and
// aX + bY + cZ + d2 = 0
double planePlaneDist(double a, double b, double c, double d1,
                      double d2) {
  return fabs(d1 - d2) / sqrt(a * a + b * b + c * c);
}

// Volume de Tetraedro
double signedTetrahedronVol(PT A, PT B, PT C, PT D) {
  double A11 = A.x - B.x;
  double A12 = A.x - C.x;
  double A13 = A.x - D.x;
  double A21 = A.y - B.y;
  double A22 = A.y - C.y;
  double A23 = A.y - D.y;
  double A31 = A.z - B.z;
  double A32 = A.z - C.z;
  double A33 = A.z - D.z;
  double det = A11 * A22 * A33 + A12 * A23 * A31 + A13 * A21 * A32 -
               A11 * A23 * A32 - A12 * A21 * A33 - A13 * A22 * A31;
  return det / 6;
}

// Parameter is a vector of vectors of points - each interior vector
// represents the 3 points that make up 1 face, in any order.
// Note: The polyhedron must be convex, with all faces given as
// triangles.
double polyhedronVol(vector<vector<PT>> poly) {
  int i, j;
  PT cent(0, 0, 0);
  for (i = 0; i < poly.size(); i++)
    for (j = 0; j < 3; j++)
      cent = cent + poly[i][j];
  cent = cent * (1.0 / (poly.size() * 3));
  double v = 0;
  for (i = 0; i < poly.size(); i++)
    v += fabs(signedTetrahedronVol(cent, poly[i][0], poly[i][1],
                                   poly[i][2]));
  return v;
}

// Outras implementacoes [Usa struct PT]

struct line { // reta definida por um ponto p e direcao v
  PT p, v;
  line(){};
  line(const PT &p, const PT &v) : p(p), v(v) { assert(!(v == PT()));
      }
  bool on(const PT &pt) const { return cross(pt - p, v) == PT(); }
};
struct plane {
  PT n;
  double d;
  plane() : d(0) {}
  plane(const PT &p1, const PT &p2, const PT &p3) {
    n = cross(p2 - p1, p3 - p1);
    d = -dot(n, p1);
    assert(side(p1) == 0);
    assert(side(p2) == 0);
    assert(side(p3) == 0);
  }
  int side(const PT &p) const { return sgn(dot(n, p) + d); }
};

// interesecao de retas
int intersec(const line &l1, const line &l2, PT &res) {
  assert(!(l1.v == PT()));
  assert(!(l2.v == PT()));
  if (cross(l1.v, l2.v) == PT()) {
    if (cross(l1.v, l1.p - l2.p) == PT())
      return 2; // same
    return 0;   // parallel
  }
  PT n = cross(l1.v, l2.v);
  PT p = l2.p - l1.p;
  if (sgn(dot(n, p)))
    return 0; // skew
  double t;
  if (sgn(n.x))
    t = (p.y * l2.v.z - p.z * l2.v.y) / n.x;
  else if (sgn(n.y))
    t = (p.z * l2.v.x - p.x * l2.v.z) / n.y;
  else if (sgn(n.z))
    t = (p.x * l2.v.y - p.y * l2.v.x) / n.z;
  else
    assert(false);
  res = l1.p + l1.v * t;
  assert(l1.on(res));
  assert(l2.on(res));
  return 1; // intersects
}

// distancia entre 2 retas
double dist(const line &l1, const line &l2) {
  PT ret = l1.p - l2.p;
  ret = ret - l1.v * (dot(l1.v, ret) / l1.v.dist2());
  PT tmp = l2.v - l1.v * (dot(l1.v, l2.v) / l1.v.dist2());
  if (sgn(tmp.dist2()))
    ret = ret - tmp * (dot(tmp, ret) / tmp.dist2());
  assert(fabs(dot(ret, l1.v)) < eps);
  assert(fabs(dot(ret, tmp)) < eps);
  assert(fabs(dot(ret, l2.v)) < eps);
  return ret.dist();
}

// Retorna os dois pontos mais proximos entre l1 e l2
void closest(const line &l1, const line &l2, PT &p1, PT &p2) {
  if (cross(l1.v, l2.v) == PT()) {
    p1 = l1.p;
    p2 = l2.p - l1.v * (dot(l1.v, l2.p - l1.p) / l1.v.dist2());
    return;
  }
  PT p = l2.p - l1.p;
  double t1 =
      (dot(l1.v, p) * l2.v.dist2() - dot(l1.v, l2.v) * dot(l2.v, p)) /
      cross(l1.v, l2.v).dist2();
  double t2 =
      (dot(l2.v, l1.v) * dot(l1.v, p) - dot(l2.v, p) * l1.v.dist2()) /
      cross(l2.v, l1.v).dist2();
  p1 = l1.p + l1.v * t1;
```

```
  p2 = l2.p + l2.v * t2;
  assert(l1.on(p1));
  assert(l2.on(p2));
}

// retorna a intersecao de reta com plano [retorna 1 se intersecao for
// pt]
int cross(const line &l, const plane &pl, PT &res) {
  double d = dot(pl.n, l.v);
  if (sgn(d) == 0) {
    return (pl.side(l.p) == 0) ? 2 : 0;
  }
  double t = (-dot(pl.n, l.p) - pl.d) / d;
  res = l.p + l.v * t;
#ifdef DEBUG
  assert(pl.side(res) == 0);
#endif
  return 1;
}

bool cross(const plane &p1, const plane &p2, const plane &p3,
           PT &res) {
  double d = det(p1.n, p2.n, p3.n);
  if (sgn(d) == 0) {
    return false;
  }
  PT px(p1.n.x, p2.n.x, p3.n.x);
  PT py(p1.n.y, p2.n.y, p3.n.y);
  PT pz(p1.n.z, p2.n.z, p3.n.z);
  PT p(-p1.d, -p2.d, -p3.d);
  res = PT(det(p, py, pz) / d, det(px, p, pz) / d, det(px, py, p) /
      d);
#ifdef DEBUG
  assert(p1.side(res) == 0);
  assert(p2.side(res) == 0);
  assert(p3.side(res) == 0);
#endif
  return true;
}

// retorna reta da intersecao de dois planos
int cross(const plane &p1, const plane &p2, line &res) {
  res.v = cross(p1.n, p2.n);
  if (res.v == PT()) {
    if ((p1.n * (p1.d / p1.n.dist2())) ==
        (p2.n * (p2.d / p2.n.dist2())))
      return 2;
    else
      return 0;
  }
  plane p3;
  p3.n = res.v;
  p3.d = 0;
  bool ret = cross(p1, p2, p3, res.p);
  assert(ret);
  assert(p1.side(res.p) == 0);
  assert(p2.side(res.p) == 0);
  return 1;
}

// testes
int main() {
  {
    line l;
    l.p = PT(1, 1, 1);
    l.v = PT(1, 0, -1);
    plane p(PT(10, 11, 12), PT(9, 8, 7), PT(1, 3, 2));
    PT res;
    assert(cross(l, p, res) == 1);
  }
  {
    plane p1(PT(1, 2, 3), PT(4, 5, 6), PT(-1, 5, -4));
    plane p2(PT(3, 2, 1), PT(6, 5, 4), PT(239, 17, -42));
    line l;
    assert(cross(p1, p2, l) == 1);
  }
  {
    plane p1(PT(1, 2, 3), PT(4, 5, 6), PT(-1, 5, -4));
    plane p2(PT(1, 2, 3), PT(7, 8, 9), PT(3, -1, 10));
    line l;
    assert(cross(p1, p2, l) == 2);
  }
  {
    plane p1(PT(1, 2, 3), PT(4, 5, 6), PT(-1, 5, -4));
    plane p2(PT(1, 2, 4), PT(4, 5, 7), PT(-1, 5, -3));
    line l;
    assert(cross(p1, p2, l) == 0);
  }

  line l1, l2;
  while (l1.p.load()) {
    l1.v.load();
    l1.v = l1.v - l1.p;
    l2.p.load();
    l2.v.load();
    l2.v = l2.v - l2.p;
    if (l1.v == PT() || l2.v == PT())
      continue;
    PT res;
    int cnt = intersec(l1, l2, res);
    double d = dist(l1, l2);
    if (fabs(d) < eps)
      assert(cnt >= 1);
    else
      assert(cnt == 0);
    PT p1, p2;
    closest(l1, l2, p1, p2);
    assert(fabs((p1 - p2).dist() - d) < eps);
  }
  plane a(PT(1, 0, 0), PT(0, 1, 0), PT(0, 0, 1));
  plane b(PT(-1, 0, 0), PT(0, -1, 0), PT(0, 0, -1));
  line l;
  assert((cross(a, b, l)) == 0);
  return 0;
}
```

6 Grafos

6.1 Topological Sort

```cpp
// Ordenacao topologia baseado em BFS. Ideia: Processar os vertices
// que nao tem aresta chegando neles. Apos processar, remover as
// arestas dele para seus vizinhos. Os vizinhos que nao tiverem mais
// arestas chegando sao inseridos na fila para serem processados
// depois.
#define MAXV 100001
vector<int> adj[MAXV];
vector<int> ordem;
void topo_sort(int N) {
  queue<int> q;
  // para mudar a ordem que os vertices sao processados pode-se se
  // usar uma priority_queue, outra estrutura para ordenar os vertices
  vector<int> in_degree(N, 0);

  rep(i, 0, N) rep(j, 0, adj[i].size()) in_degree[adj[i][j]]++;

  rep(i, 0, N) if (in_degree[i] == 0) q.push(i);
  while (!q.empty()) {
    int u = q.front();
    q.pop();
    ordem.push_back(u);
    rep(i, 0, adj[u].size()) {
      int v = adj[u][i];
      in_degree[v]--;
      if (in_degree[v] == 0)
        q.push(v);
    }
  }
  if (ordem.size() != N) {
    // grafo contem ciclos, nao eh um DAG
  }
}
int main() { return 0; }
```

6.2 Dijkstra

```cpp
/*
 * Encontra o custo do menor caminho uma origem para todos os outros
 * vertices do grafo.
 * So pode ser aplicado em grafos que nao possuem ciclos com peso
 * negativo.
 * Em dist[X] ficara armazenado o custo do menor caminho de src ate X;
 * e em pi[X] ficara o vertice anterior a X neste caminho.
 * Exemplo: src ->, ... , pi[ pi[X] ] -> pi[X] -> X
 * Complexidade O( (V+E)log(V) )
*/

#define MAXV 100000+10    // quantidade maxima de vertices
typedef long long cost_t; // tipo de variavel para o custo da aresta

int V, E;
vector<pair<int, cost_t> > adj[MAXV];
cost_t dist[MAXV];
int pi[MAXV];

void dijkstra(int src) {
  priority_queue< pair<cost_t, int> > PQ;

  memset(dist, INF, sizeof(dist));
  // nao utilize memset se cost_f for double, use um for
  dist[src] = 0;
  PQ.push( make_pair(dist[src], src));

  while (!PQ.empty()) {
    pair<cost_t, int> top = PQ.top();
    PQ.pop();

    int u = top.second;
    cost_t d = -top.first;

    if (d != dist[u]) continue;

    rep(i, 0, (int)adj[u].size()) {
      int v = adj[u][i].F;
      cost_t cost_uv = adj[u][i].S;

      if (dist[u] + cost_uv < dist[v]) {
        dist[v] = dist[u] + cost_uv;
        pi[v] = u;
        PQ.push( make_pair(-dist[v], v) );
      }
    }
  }
}
```

6.3 Floyd-Warshall

```cpp
#define MAXV 401
int adj[MAXV][MAXV], path[MAXV][MAXV];
int n, m; // #vertices, #arestas
// adj[u][v] = custo de {U->V}
// path[u][v] = k .: K vem logo apos U no caminho ate V
void read_graph() {
  memset(adj, INF, sizeof adj); // para menor caminho
  rep(i, 0, n) adj[i][i] = 0;   // para menor caminho
  int u, v, w;
  rep(i, 0, m) {
    cin >> u >> v >> w;
    adj[u][v] = w;
    path[u][v] = v;
  }
}
void floyd() {
  rep(k, 0, n) rep(i, 0, n)
      rep(j, 0, n) if (adj[i][k] + adj[k][j] < adj[i][j]) {
    adj[i][j] = adj[i][k] + adj[k][j];
    path[i][j] = path[i][k];
  }
}
vector<int> findPath(int s, int d) {
  vector<int> Path;
  Path.pb(s);
  while (s != d) {
    s = path[s][d];
    Path.pb(s);
  }
  return Path;
}
```

```
/*Aplicacoes:
1-Encontrar o fecho transitivo (saber se U consegue visitar V)
.: adj[u][v] |= (adj[u][k] & adj[k][v]);
   (inicializar adj com 0)

2-Minimizar a maior aresta do caminho entre U e V
.: adj[u][v] = min(adj[u][v], max(adj[u][k], adj[k][v]));
   (inicializar adj com INF)

3-Maximizar a menor aresta do caminho entre U e V
.: adj[u][v] = max(adj[u][v], min(adj[u][k], adj[k][u]));
   (inicializar adj com -INF)*/

int main() { return 0; }
```

6.4 Bellman-Ford

```
// Menor custo de uma origem s para todos vertices em O(V^3).
// bellman() retorna FALSE se o grafo tem ciclo com custo negativo.
// dist[v] contem o menor custo de s ate v.
#define MAXV 400

// Vertices indexados em 0.
int V, E; // #vertices, #arestas
vector<ii> adj[MAXV];
ll dist[MAXV];

bool bellman(int s) {
  rep(i, 0, V) dist[i] = INF;
  dist[s] = 0;
  rep(i, 0, V - 1) rep(u, 0, V) {
    rep(j, 0, adj[u].size()) {
      int v = adj[u][j].F, duv = adj[u][j].S;
      dist[v] = min(dist[v], dist[u] + duv);
    }
  }
  // verifica se tem ciclo com custo negativo
  rep(u, 0, V) rep(j, 0, adj[u].size()) {
    int v = adj[u][j].F, duv = adj[u][j].S;
    if (dist[v] > dist[u] + duv)
      return false;
  }
  return true;
}

int main() { return 0; }
```

6.5 Vértices de Articulação e Pontes

```
#define MAXV 100001
vector<int> adj[MAXV];
int dfs_num[MAXV], dfs_low[MAXV], dfs_parent[MAXV];
int dfscounter, V, dfsRoot, rootChildren, ans;
int articulation[MAXV], articulations;
vector<ii> bridges;

void articulationPointAndBridge(int u) {
  dfs_low[u] = dfs_num[u] = dfscounter++;
  rep(i, 0, adj[u].size()) {
    int v = adj[u][i];
    if (dfs_num[v] == -1) {
      dfs_parent[v] = u;
      if (u == dfsRoot)
        rootChildren++;

      articulationPointAndBridge(v);
      if (dfs_low[v] >= dfs_num[u])
        articulation[u] = true;
      if (dfs_low[v] > dfs_num[u])
        bridges.pb(mp(u, v));

      dfs_low[u] = min(dfs_low[u], dfs_low[v]);
    } else if (v != dfs_parent[u])
      dfs_low[u] = min(dfs_low[u], dfs_num[v]);
  }
}

int main() {
  // read graph
  dfscounter = 0;
  rep(i, 0, V) {
    dfs_low[i] = dfs_parent[i] = articulation[i] = 0;
    dfs_num[i] = -1;
  }
  articulations = 0;
  bridges.clear();
  rep(i, 0, V) if (dfs_num[i] == -1) {
    dfsRoot = i;
    rootChildren = 0;
    articulationPointAndBridge(i);
    articulation[dfsRoot] = (rootChildren > 1);
  }
  printf("#articulations = %d\n", articulations);
  rep(i, 0, V) if (articulation[i]) printf("Vertex %d\n", i);
  printf("#bridges = %d\n", bridges.size());
  rep(i, 0, bridges.size())
    printf("Bridge %d<->%d\n", bridges[i].F, bridges[i].S);
  return 0;
}
```

6.6 Tarjan

```
#define MAXV 100010
vector<int> adj[MAXV];
int V;
int dfs_num[MAXV], dfs_low[MAXV], vis[MAXV], SCC[MAXV];
int dfsCounter, numSCC;
vector<int> S; // global variables

void tarjanSCC(int u) {
  dfs_low[u] = dfs_num[u] = dfsCounter++; // dfs_low[u] <= dfs_num[u]
  S.push_back(u); // stores u in a vector based on order of
                  // visitation
  vis[u] = 1;
  rep(i, 0, adj[u].size()) {
    int v = adj[u][i];
    if (dfs_num[v] == -1)
      tarjanSCC(v);
```

```
    if (vis[v]) // condition for update
      dfs_low[u] = min(dfs_low[u], dfs_low[v]);
  }
  if (dfs_low[u] ==
      dfs_num[u]) { // if this is a root (start) of an SCC
    while (true) {
      int v = S.back();
      S.pop_back();
      vis[v] = 0;
      SCC[v] = numSCC; // wich SCC this vertex belong
      if (u == v)
        break;
    }
    numSCC++;
  }
}

int main() {
  // read graph
  rep(i, 0, V) {
    dfs_num[i] = -1;
    dfs_low[i] = vis[i] = 0;
    SCC[i] = -i;
  }
  dfsCounter = numSCC = 0;
  rep(i, 0, V) if (dfs_num[i] == -1) tarjanSCC(i);
  rep(i, 0, V) printf("vertice %d, componente %d\n", i, SCC[i]);
  return 0;
}
```

6.7 Kosaraju

```
// Encotra componentes conexos. Mesmo que Tarjan
#define MAXV 100000
#define DFS_WHITE 0
vector<int> adj[2][MAXV]; // adj[0][] original, adj[1][] transposto
vector<int> S, dfs_num;
int N, numSCC, SCC[MAXV];

void Kosaraju(int u, int t, int comp) {
  dfs_num[u] = 1;
  if (t == 1)
    SCC[u] = comp;
  for (int j = 0; j < (int)adj[t][u].size(); j++) {
    int v = adj[t][u][j];
    if (dfs_num[v] == DFS_WHITE)
      Kosaraju(v, t, comp);
  }
  S.push_back(u);
}
void doit() { // chamar na main
  S.clear();
  dfs_num.assign(N, DFS_WHITE);
  for (int i = 0; i < N; i++)
    if (dfs_num[i] == DFS_WHITE)
      Kosaraju(i, 0, -1);
  numSCC = 0;
  dfs_num.assign(N, DFS_WHITE);
  for (int i = N - 1; i >= 0; i--)
    if (dfs_num[S[i]] == DFS_WHITE) {
      Kosaraju(S[i], 1, numSCC);
      numSCC++;
    }
  printf("There are %d SCCs\n", numSCC);
}

int main() { return 0; }
```

6.8 2-Sat

```
#define MAXV 100001
// 2-sat - Codigo do problema X-Mart
// vertices indexado em 1
vector<int> adj[2 * MAXV];
vector<int> radj[2 * MAXV];
int seen[2 * MAXV], comp[2 * MAXV], order[2 * MAXV], ncomp, norder;
int N; // #variaveis
int n; // #vertices

#define NOT(x) ((x <= N) ? (x + N) : (x - N))
#define quero 1
void add_edge(int a, int b, int opcao) {
  if (a > b)
    swap(a, b);
  if (b == 0)
    return;
  if (a == 0) {
    if (opcao == quero)
      adj[NOT(b)].pb(b);
    else
      adj[b].pb(NOT(b));
  } else { // normal...
    if (opcao == quero) {
      adj[NOT(a)].pb(b);
      adj[NOT(b)].pb(a);
    } else {
      a = NOT(a);
      b = NOT(b);
      adj[NOT(a)].pb(b);
      adj[NOT(b)].pb(a);
    }
  }
}
void init() {
  rep(i, 0, n + 1) {
    adj[i].clear();
    radj[i].clear();
  }
}
void dfs1(int u) {
  seen[u] = 1;
  rep(i, 0, adj[u].size()) if (!seen[adj[u][i]]) dfs1(adj[u][i]);
  order[norder++] = u;
}
void dfs2(int u) {
  seen[u] = 1;
  rep(i, 0, radj[u].size()) if (!seen[radj[u][i]]) dfs2(radj[u][i]);
  comp[u] = ncomp;
}
void strongly_connected_components() {
```

```
  rep(v, 1, n + 1) rep(i, 0, (int)adj[v].size())
      radj[adj[v][i]].pb(v);

  norder = 0;
  memset(seen, 0, sizeof seen);
  rep(v, 1, n + 1) if (!seen[v]) dfs1(v);

  ncomp = 0;
  memset(seen, 0, sizeof seen);
  for (int i = n - 1, u = order[n - 1]; i >= 0; u = order[--i])
    if (!seen[u]) {
      dfs2(u);
      ncomp++;
    }
}
bool sat2() {
  strongly_connected_components();
  rep(i, 1, n + 1) if (comp[i] == comp[NOT(i)]) return false;
  return true;
}
int main() {
  int Clientes;
  while (cin >> Clientes >> N) {
    if (Clientes == 0 && N == 0)
      break;
    n = 2 * N;
    init();
    int u, v;
    rep(i, 0, Clientes) {
      scanf("%d %d", &u, &v);
      add_edge(u, v, quero);
      scanf("%d %d", &u, &v);
      add_edge(u, v, !quero);
    }
    sat2() ? printf("yes\n") : printf("no\n");
  }
  return 0;
}
```

6.9 LCA

```
/*Lowest Common Ancestor (LCA) entre dois vertices A, B de uma arvore.
LCA(A,B) = ancestral mais proximo de A adj B. O codigo abaixo tambem
calcula a menor aresta do caminho entre A adj B. Para saber quantas
arestas tem entre A adj B basta fazer:
level[A]+level[B]-2*level[lca(A,B)]
Pode-se modificar para retorna a
distancia entre A adj B. Como usar: (1) ler a arvore em adj[] adj W[],
chamar doit(raiz), passando a raiz da arvore. Indexar em 0 os vertices
(2) A funcao retorna o LCA adj a menor aresta entre A adj B.
*/

#define MAXV 101000
const int maxl = 20;      // profundidade maxima 2^(maxl) > MAXV
int pai[MAXV][maxl + 1];  // pai[v][i] = pai de v subindo 2^i arestas
int dist[MAXV][maxl + 1]; // dist[v][i] = menor aresta de v subindo
                          // 2^i arestas
int level[MAXV];          // level[v] = #arestas de v ate a raiz

int N, M;                         // numero de vertices adj arestas
vector<pair<int, int>> adj[MAXV]; // {v,custo}

void dfs(int v, int p, int peso) {
  level[v] = level[p] + 1;
  pai[v][0] = p;
  dist[v][0] = peso; // aresta de v--pai[v]
  for (int i = 1; i <= maxl; i++) {
    pai[v][i] = pai[pai[v][i - 1]][i - 1]; // subindo 2^i arestas
    dist[v][i] = min(dist[v][i - 1], dist[pai[v][i - 1]][i - 1]);
  }
  rep(i, 0, adj[v].size()) {
    int viz = adj[v][i].F;
    int cost = adj[v][i].S;
    if (viz == p)
      continue;
    dfs(viz, v, cost);
  }
}

void doit(int root) {
  level[root] = 0;
  for (int i = 0; i <= maxl; i++)
    pai[root][i] = root, dist[root][i] = INF;
  rep(i, 0, adj[root].size()) {
    int viz = adj[root][i].F;
    int cost = adj[root][i].S;
    dfs(viz, root, cost);
  }
}

pair<int, int> lca(int a, int b) {
  int menor = INF; // valor da menor aresta do caminho a->b
  if (level[a] < level[b])
    swap(a, b);

  for (int i = maxl; i >= 0; i--) {
    if (level[pai[a][i]] >= level[b]) {
      menor = min(menor, dist[a][i]);
      a = pai[a][i];
    }
  }
  if (a != b) {
    for (int i = maxl; i >= 0; i--) {
      if (pai[a][i] != pai[b][i]) {
        menor = min(menor, min(dist[a][i], dist[b][i]));
        a = pai[a][i];
        b = pai[b][i];
      }
    }
    // ultimo salto
    menor = min(menor, min(dist[a][0], dist[b][0]));
    a = pai[a][0];
    b = pai[b][0];
  }
  return make_pair(a, menor);
}

int main() { return 0; }
```

6.10 LCA (Sparse Table)

```
/*
Encontra o lca usando sparse table.
O(NlogN) de pre-processamento
O(1) em cada consulta

Como usar:
  main: level[root] = 0
      dfs(root, root);
  consulta:
      lca(u,v)
*/

typedef pair<int, int> ii;

#define MAXN (1e5 + 1);
#define LOGN (23)

vector<int> adj[MAXN];
int level[MAXN];

vector<int> num;
int f[MAXN];
ii st[4 * MAXN][LOGN];

void dfs(int u, int p) {
  level[u] = level[p] + 1;

  f[u] = num.size();
  num.pb(u);

  rep(i, 0, (int)adj[u].size()) {
    if (adj[u][i] == p)
      continue;
    dfs(adj[u][i], u);
    num.pb(u);
  }
}

ii comb(ii left, ii right) { return min(left, right); }

void SparseTable() {
  rep(i, 0, (int)num.size()) st[i][0] =
      make_pair(level[num[i]], num[i]);

  rep(k, 1, LOGN) for (int i = 0; (i + (1 << k) - 1) <
       (int)num.size();
                          i++) st[i][k] =
      comb(st[i][k - 1], st[i + (1 << (k - 1))][k - 1]);
}

int lca(int u, int v) {
  int l = f[u];
  int r = f[v];

  int k = log2(r - l + 1);
  return comb(st[l][k], st[r - (1 << k) + 1][k]).second;
}
```

6.11 Maximum Bipartite Matching

```
// Encontra o casamento bipartido maximo. Set de vertices X e Y.
// x = [0,X-1], y = [0,Y-1]. match[y] = x - contem quem esta casado
// com y. Teorema de Konig - Num grafo bipartido, o matching eh igual
// ao minimum vertex cover. Complexidade O(nm)
#define MAXV 1000
vector<int> adj[MAXV];
int match[MAXV], V, X, Y;
bool vis[MAXV];

int aug(int v) {
  if (vis[v])
    return 0;
  vis[v] = true;
  rep(i, 0, adj[v].size()) {
    int r = adj[v][i];
    if (match[r] == -1 || aug(match[r])) {
      match[r] = v; // augmenting path
      return 1;
    }
  }
  return 0;
}
int matching(int X, int Y) {
  int V = X + Y;
  rep(i, 0, V) match[i] = -1;
  int mcbm = 0;
  rep(i, 0, X) {
    rep(j, 0, X) vis[j] = false;
    mcbm += aug(i);
  }
  return mcbm;
}

int main() { return 0; }
```

6.12 Hopcroft Karp - Maximum Bipartite Matching (UNIFEI)

```
/*Encontra o casamento bipartido maximo em O(sqrt(V)*E)
1) Chamar init(L,R) #vertices da esquerda, #vertices da direita
2) Usar addEdge(Li,Ri) para adicionar a aresta Li -> Ri
3) maxMatching() retorna o casamento maximo.
matching[Rj] -> armazena Li */

#define MAXN1 3010
#define MAXN2 3010
#define MAXM 6020
int n1, n2, edges, last[MAXN1], pre[MAXM], head[MAXM];
int matching[MAXN2], dist[MAXN1], Q[MAXN1];
bool used[MAXN1], vis[MAXN1];

void init(int L, int R) {
  n1 = L, n2 = R;
  edges = 0;
  fill(last, last + n1, -1);
```

```
}
void addEdge(int u, int v) {
  head[edges] = v;
  pre[edges] = last[u];
  last[u] = edges++;
}
void bfs() {
  fill(dist, dist + n1, -1);
  int sizeQ = 0;
  for (int u = 0; u < n1; ++u) {
    if (!used[u]) {
      Q[sizeQ++] = u;
      dist[u] = 0;
    }
  }
  for (int i = 0; i < sizeQ; i++) {
    int u1 = Q[i];
    for (int e = last[u1]; e >= 0; e = pre[e]) {
      int u2 = matching[head[e]];
      if (u2 >= 0 && dist[u2] < 0) {
        dist[u2] = dist[u1] + 1;
        Q[sizeQ++] = u2;
      }
    }
  }
}
bool dfs(int u1) {
  vis[u1] = true;
  for (int e = last[u1]; e >= 0; e = pre[e]) {
    int v = head[e];
    int u2 = matching[v];
    if (u2 < 0 || !vis[u2] && dist[u2] == dist[u1] + 1 && dfs(u2)) {
      matching[v] = u1;
      used[u1] = true;
      return true;
    }
  }
  return false;
}
int maxMatching() {
  fill(used, used + n1, false);
  fill(matching, matching + n2, -1);
  for (int res = 0;;) {
    bfs();
    fill(vis, vis + n1, false);
    int f = 0;
    for (int u = 0; u < n1; ++u)
      if (!used[u] && dfs(u))
        ++f;
    if (!f)
      return res;
    res += f;
  }
}

int main() { return 0; }
```

6.13 Network Flow (lento)

```
// Ford-Fulkerson para fluxo maximo
#define MAXV 250
vector<int> edge[MAXV];
int cap[MAXV][MAXV];
bool vis[MAXV];

void init() {
  rep(i, 0, MAXV) edge[i].clear();
  memset(cap, 0, sizeof cap);
}

void add(int a, int b, int cap_ab, int cap_ba) {
  edge[a].pb(b), edge[b].pb(a);
  cap[a][b] += cap_ab, cap[b][a] += cap_ba;
}

int dfs(int src, int snk, int fl) {
  if (vis[src])
    return 0;
  if (snk == src)
    return fl;
  vis[src] = 1;
  rep(i, 0, edge[src].size()) {
    int v = edge[src][i];
    int x = min(fl, cap[src][v]);
    if (x > 0) {
      x = dfs(v, snk, x);
      if (!x)
        continue;
      cap[src][v] -= x;
      cap[v][src] += x;
      return x;
    }
  }
  return 0;
}

int flow(int src, int snk) {
  int ret = 0;
  while (42) {
    memset(vis, 0, sizeof vis);
    int delta = dfs(src, snk, 1 << 30);
    if (!delta)
      break;
    ret += delta;
  }
  return ret;
}
int main() { return 0; }
```

6.14 Network Flow - Dinic

```
// Dinic para fluxo maximo
// Grafo indexado em 1
// Inicializar maxN, maxE.
// Chamar init() com #nos, source e sink. Montar o grafo chamando
// add(a,b,c1,c2), sendo c1 cap. de a->b e c2 cap. de b->a
#define FOR(i, a, b) for (int i = a; i <= b; i++)
#define SET(c, v) memset(c, v, sizeof c)
const int maxN = 5000;
const int maxE = 70000;
```

```
const int inf = 1000000005;

int nnode, nedge, src, snk;
int Q[maxN], pro[maxN], fin[maxN], dist[maxN];
int flow[maxE], cap[maxE], to[maxE], prox[maxE];

void init(int _nnode, int _src, int _snk) {
  nnode = _nnode, nedge = 0, src = _src, snk = _snk;
  FOR(i, 1, nnode) fin[i] = -1;
}

void add(int a, int b, int c1, int c2) {
  to[nedge] = b, cap[nedge] = c1, flow[nedge] = 0,
  prox[nedge] = fin[a], fin[a] = nedge++;
  to[nedge] = a, cap[nedge] = c2, flow[nedge] = 0,
  prox[nedge] = fin[b], fin[b] = nedge++;
}

bool bfs() {
  SET(dist, -1);
  dist[src] = 0;
  int st = 0, en = 0;
  Q[en++] = src;
  while (st < en) {
    int u = Q[st++];
    for (int e = fin[u]; e >= 0; e = prox[e]) {
      int v = to[e];
      if (flow[e] < cap[e] && dist[v] == -1) {
        dist[v] = dist[u] + 1;
        Q[en++] = v;
      }
    }
  }
  return dist[snk] != -1;
}

int dfs(int u, int fl) {
  if (u == snk)
    return fl;
  for (int &e = pro[u]; e >= 0; e = prox[e]) {
    int v = to[e];
    if (flow[e] < cap[e] && dist[v] == dist[u] + 1) {
      int x = dfs(v, min(cap[e] - flow[e], fl));
      if (x > 0) {
        flow[e] += x, flow[e ^ 1] -= x;
        return x;
      }
    }
  }
  return 0;
}

ll dinic() {
  ll ret = 0;
  while (bfs()) {
    FOR(i, 1, nnode) pro[i] = fin[i];
    while (true) {
      int delta = dfs(src, inf);
      if (!delta)
        break;
      ret += delta;
    }
  }
  return ret;
}

int main() { return 0; }
```

6.15 Min Cost Max Flow

```
// Criar o grafo chamando MCMF g(V), onde g eh o grafo e V a qtde de
// vertices (indexado em 0). Chamar g.add(u,v,cap,cost) para add a
// aresta u->v, se for bidirecional, chamar tbm g.add(v,u,cap,cost)
struct MCMF {
  typedef int ctype;
  enum { MAXN = 550, INF = INT_MAX };
  struct Edge {
    int x, y;
    ctype cap, cost;
  };
  vector<Edge> E;
  vector<int> adj[MAXN];
  int N, prev[MAXN];
  ctype dist[MAXN], phi[MAXN];

  MCMF(int NN) : N(NN) {}

  void add(int x, int y, ctype cap, ctype cost) { // cost >= 0
    Edge e1 = {x, y, cap, cost}, e2 = {y, x, 0, -cost};
    adj[e1.x].push_back(E.size());
    E.push_back(e1);
    adj[e2.x].push_back(E.size());
    E.push_back(e2);
  }

  void mcmf(int s, int t, ctype &flowVal, ctype &flowCost) {
    int x;
    flowVal = flowCost = 0;
    memset(phi, 0, sizeof(phi));
    while (true) {
      for (x = 0; x < N; x++)
        prev[x] = -1;
      for (x = 0; x < N; x++)
        dist[x] = INF;
      dist[s] = prev[s] = 0;

      set<pair<ctype, int>> Q;
      Q.insert(make_pair(dist[s], s));
      while (!Q.empty()) {
        x = Q.begin()->second;
        Q.erase(Q.begin());
        for (vector<int>::iterator it = adj[x].begin();
             it != adj[x].end(); it++) {
          const Edge &e = E[*it];
          if (e.cap <= 0)
            continue;
          ctype cc = e.cost + phi[x] - phi[e.y];
          if (dist[x] + cc < dist[e.y]) {
            Q.erase(make_pair(dist[e.y], e.y));
            dist[e.y] = dist[x] + cc;
            prev[e.y] = *it;
```

```
          Q.insert(make_pair(dist[e.y], e.y));
        }
      }
    }
    if (prev[t] == -1)
      break;

    ctype z = INF;
    for (x = t; x != s; x = E[prev[x]].x)
      z = min(z, E[prev[x]].cap);
    for (x = t; x != s; x = E[prev[x]].x) {
      E[prev[x]].cap -= z;
      E[prev[x] ^ 1].cap += z;
    }
    flowVal += z;
    flowCost += z * (dist[t] - phi[s] + phi[t]);
    for (x = 0; x < N; x++)
      if (prev[x] != -1)
        phi[x] += dist[x];
  }
 }
};

int main() { return 0; }
```

6.16 Min Cost Max Flow (Stefano)

```
#define MAX_V 2003
#define MAX_E 2 * 3003
// Inicializar MAX_V e MAX_E corretamente. Chamar init(_V) com a qtde
// de vertices (indexado em 0) mesmo que seja bidirecional. Adicionar
// as arestas duas vezes no main(). Complexiade (rapido)

typedef int cap_type;
typedef long long cost_type;
const cost_type inf = LLONG_MAX;

int V, E, pre[MAX_V], last[MAX_V], to[MAX_E], nex[MAX_E];
bool visited[MAX_V];
cap_type flowVal, cap[MAX_E];
cost_type flowCost, cost[MAX_E], dist[MAX_V], pot[MAX_V];

void init(int _V) {
  memset(last, -1, sizeof(last));
  V = _V;
  E = 0;
}

void add_edge(int u, int v, cap_type _cap, cost_type _cost) {
  to[E] = v, cap[E] = _cap;
  cost[E] = _cost, nex[E] = last[u];
  last[u] = E++;
  to[E] = u, cap[E] = 0;
  cost[E] = -_cost, nex[E] = last[v];
  last[v] = E++;
}

// only if there is initial negative cycle
void BellmanFord(int s, int t) {
  bool stop = false;
  for (int i = 0; i < V; ++i)
    dist[i] = inf;
  dist[s] = 0;

  for (int i = 1; i <= V && !stop; ++i) {
    stop = true;

    for (int j = 0; j < E; ++j) {
      int u = to[j ^ 1], v = to[j];

      if (cap[j] > 0 && dist[u] != inf &&
          dist[u] + cost[j] < dist[v]) {
        stop = false;
        dist[v] = dist[u] + cost[j];
      }
    }
  }

  for (int i = 0; i < V; ++i)
    if (dist[i] != inf)
      pot[i] = dist[i];
}

void mcmf(int s, int t) {
  flowVal = flowCost = 0;
  memset(pot, 0, sizeof(pot));

  BellmanFord(s, t);

  while (true) {
    memset(pre, -1, sizeof(pre));
    memset(visited, false, sizeof(visited));
    for (int i = 0; i < V; ++i)
      dist[i] = inf;

    priority_queue<pair<cost_type, int>> Q;
    Q.push(make_pair(0, s));
    dist[s] = pre[s] = 0;

    while (!Q.empty()) {
      int aux = Q.top().second;
      Q.pop();

      if (visited[aux])
        continue;
      visited[aux] = true;

      for (int e = last[aux]; e != -1; e = nex[e]) {
        if (cap[e] <= 0)
          continue;
        cost_type new_dist =
            dist[aux] + cost[e] + pot[aux] - pot[to[e]];
        if (new_dist < dist[to[e]]) {
          dist[to[e]] = new_dist;
          pre[to[e]] = e;
          Q.push(make_pair(-new_dist, to[e]));
        }
      }
    }

    if (pre[t] == -1)
```

```
        break;

      cap_type f = cap[pre[t]];
      for (int i = t; i != s; i = to[pre[i] ^ 1])
        f = min(f, cap[pre[i]]);
      for (int i = t; i != s; i = to[pre[i] ^ 1]) {
        cap[pre[i]] -= f;
        cap[pre[i] ^ 1] += f;
      }

      flowVal += f;
      flowCost += f * (dist[t] - pot[s] + pot[t]);

      for (int i = 0; i < V; ++i)
        if (pre[i] != -1)
          pot[i] += dist[i];
    }
  }

int main() { return 0; }
```

6.17 Tree Isomorphism

```
// Verifica se dado duas arvores, desconsiderando o rotulo dos
// vertices, elas tem a mesma forma.
typedef vector<int> vi;
#define sz(a) (int)a.size()
#define fst first
#define snd second

struct tree {
  int n;
  vector<vi> adj;
  tree(int n) : n(n), adj(n) {}
  void add_edge(int src, int dst) {
    adj[src].pb(dst);
    adj[dst].pb(src);
  }
  vi centers() {
    vi prev;
    int u = 0;
    for (int k = 0; k < 2; ++k) {
      queue<int> q;
      prev.assign(n, -1);
      q.push(prev[u] = u);
      while (!q.empty()) {
        u = q.front();
        q.pop();
        for (auto i : adj[u]) {
          if (prev[i] >= 0)
            continue;
          q.push(i);
          prev[i] = u;
        }
      }
    }
    vi path = {u};
    while (u != prev[u])
      path.pb(u = prev[u]);
    int m = sz(path);
    if (m % 2 == 0)
      return {path[m / 2 - 1], path[m / 2]};
    else
      return {path[m / 2]};
  }
  vector<vi> layer;
  vi prev;
  int levelize(int r) {
    prev.assign(n, -1);
    prev[r] = n;
    layer = {{r}};
    while (true) {
      vi next;
      for (auto u : layer.back()) {
        for (int v : adj[u]) {
          if (prev[v] >= 0)
            continue;
          prev[v] = u;
          next.pb(v);
        }
      }
      if (next.empty())
        break;
      layer.pb(next);
    }
    return sz(layer);
  }
};

bool isomorphic(tree S, int s, tree T, int t) {
  if (S.n != T.n)
    return false;
  if (S.levelize(s) != T.levelize(t))
    return false;
  vector<vi> longcodeS(S.n + 1), longcodeT(T.n + 1);
  vi codeS(S.n), codeT(T.n);
  for (int h = S.layer.size() - 1; h >= 0; h--) {
    map<vi, int> bucket;
    for (int u : S.layer[h]) {
      sort(all(longcodeS[u]));
      bucket[longcodeS[u]] = 0;
    }
    for (int u : T.layer[h]) {
      sort(all(longcodeT[u]));
      bucket[longcodeT[u]] = 0;
    }
    int id = 0;
    for (auto &p : bucket)
      p.snd = id++;
    for (int u : S.layer[h]) {
      codeS[u] = bucket[longcodeS[u]];
      longcodeS[S.prev[u]].pb(codeS[u]);
    }
    for (int u : T.layer[h]) {
      codeT[u] = bucket[longcodeT[u]];
      longcodeT[T.prev[u]].pb(codeT[u]);
    }
  }
  return codeS[s] == codeT[t];
}
```

```cpp
bool isomorphic(tree S, tree T) {
  auto x = S.centers(), y = T.centers();
  if (sz(x) != sz(y))
    return false;
  if (isomorphic(S, x[0], T, y[0]))
    return true;
  return sz(x) > 1 and isomorphic(S, x[1], T, y[0]);
}

int main() {
  int N, u, v;
  cin >> N;
  tree A(N + 2), B(N + 2);
  rep(i, 0, N - 1) {
    scanf("%d %d", &u, &v);
    u--, v--;
    A.add_edge(u, v);
  }
  rep(i, 1, N) {
    scanf("%d %d", &u, &v);
    u--, v--;
    B.add_edge(u, v);
  }
  puts(isomorphic(A, B) ? "S" : "N");
}
```

6.18 Stoer Wagner-Minimum Cut (UNIFEI)

```cpp
/*
Retorna o corte minimo do grafo
(Conjunto de arestas que caso seja removido, desconecta o grafo)
Input: n = #vertices, g[i][j] = custo da aresta (i->j)
Output: Retorna o corte minimo
Complexidade: O(N^3)
*/

// Maximum number of vertices in the graph
#define NN 101
// Maximum edge weight (MAXW * NN * NN must fit into an int)
#define MAXW 110

// Adjacency matrix and some internal arrays
int g[NN][NN], v[NN], w[NN], na[NN], n;
bool a[NN];
int stoer_wagner() {
  // init the remaining vertex set
  for (int i = 0; i < n; i++)
    v[i] = i;
  // run Stoer-Wagner
  int best = MAXW * n * n;
  while (n > 1) {
    // initialize the set A and vertex weights
    a[v[0]] = true;
    for (int i = 1; i < n; i++) {
      a[v[i]] = false;
      na[i - 1] = i;
      w[i] = g[v[0]][v[i]];
    }
    // add the other vertices
    int prev = v[0];
    for (int i = 1; i < n; i++) {
      // find the most tightly connected non-A vertex
      int zj = -1;
      for (int j = 1; j < n; j++)
        if (!a[v[j]] && (zj < 0 || w[j] > w[zj]))
          zj = j;
      // add it to A
      a[v[zj]] = true;
      // last vertex?
      if (i == n - 1) {
        // remember the cut weight
        best = min(best, w[zj]);

        // merge prev and v[zj]
        for (int j = 0; j < n; j++)
          g[v[j]][prev] = g[prev][v[j]] += g[v[zj]][v[j]];
        v[zj] = v[--n];
        break;
      }
      prev = v[zj];
      // update the weights of its neighbours
      for (int j = 1; j < n; j++)
        if (!a[v[j]])
          w[j] += g[v[zj]][v[j]];
    }
  }
  return best;
}

int main() { return 0; }
```

6.19 Erdos Gallai (UNIFEI)

```cpp
// Determina se existe um grafo tal que b[i] eh o grau do i-esimo
// vertice. Vertices indexado em 1. Apenas armazenar em b[1...N] e
// chamar EGL()
long long b[100005], n;
long long dmax, dmin, dsum, num_degs[100005];

bool basic_graphical_tests() { // Sort and perform some simple tests
                               // on the sequence
  int p = n;
  memset(num_degs, 0, (n + 1) * sizeof(long long));

  dmax = dsum = n = 0;
  dmin = p;
  for (int d = 1; d <= p; d++) {
    if (b[d] < 0 || b[d] >= p)
      return false;
    else if (b[d] > 0) {
      if (dmax < b[d])
        dmax = b[d];
      if (dmin > b[d])
        dmin = b[d];
      dsum = dsum + b[d];
      n++;
      num_degs[b[d]]++;
    }
  }
  if (dsum % 2 || dsum > n * (n - 1))
```

```
    return false;
  return true;
}
bool EGL() {
  long long k, sum_deg, sum_nj, sum_jnj, run_size;

  if (!basic_graphical_tests())
    return false;
  if (n == 0 || 4 * dmin * n >= (dmax + dmin + 1) * (dmax + dmin + 1))
    return true;

  k = sum_deg = sum_nj = sum_jnj = 0;
  for (int dk = dmax; dk >= dmin; dk--) {
    if (dk < k + 1)
      return true;

    if (num_degs[dk] > 0) {
      run_size = num_degs[dk];
      if (dk < k + run_size)
        run_size = dk - k;
      sum_deg += run_size * dk;

      for (int v = 0; v < run_size; v++) {
        sum_nj += num_degs[k + v];
        sum_jnj += (k + v) * num_degs[k + v];
      }
      k += run_size;
      if (sum_deg > k * (n - 1) - k * sum_nj + sum_jnj)
        return false;
    }
  }
  return true;
}
int main() { return 0; }
```

6.20 Stable Marriage (UNIFEI)

```
/*Seja um conjunto de m homens e n mulheres, onde cada pessoa tem uma
preferencia por outra de sexo oposto. O algoritmo produz o casamento
estavel de cada homem com uma mulher. Estavel:
- Cada homem se casara com uma mulher diferente (n >= m)
- Dois casais H1M1 e H2M2 nao serao instaveis.
Dois casais H1M1 e H2M2 sao instaveis se:
- H1 prefere M2 ao inves de M1, e
- M1 prefere H2 ao inves de H1.
Entrada
(1) m = #homens, n = #mulheres
(2) R[x][y] = i, i: eh a ordem de preferencia do homem y pela mulher x
Obs.: Quanto maior o valor de i menor eh a preferencia do homem y pela
mulher x

(3) L[x][i] = y : A mulher y eh a i-esima preferencia do homem x
Obs.: 0 <= i <= n-1, quanto menor o valor de i maior eh a preferencia
do homem x pela mulher y
Saida
L2R[i]: a mulher do homem i (sempre entre 0 e n-1)
R2L[j]: o homem da mulher j (-1 se a mulher for solteira)

Complexidade O(m^2)
*/
#define MAXM 1000
#define MAXW 1000
int L[MAXM][MAXW];
int R[MAXW][MAXM];
int L2R[MAXM], R2L[MAXW];
int m, n;
int p[MAXM];

void stableMarriage() {
  static int p[MAXM];
  memset(R2L, -1, sizeof(R2L));
  memset(p, 0, sizeof(p));
  for (int i = 0; i < m; ++i) {
    int man = i;
    while (man >= 0) {
      int wom;
      while (42) {
        wom = L[man][p[man]++];
        if (R2L[wom] < 0 || R[wom][man] > R[wom][R2L[wom]])
          break;
      }
      int hubby = R2L[wom];
      R2L[L2R[man] = wom] = man;
      man = hubby;
    }
  }
}

int main() { return 0; }
```

6.21 Hungarian Max Bipartite Matching with Cost (UNIFEI)

```
/*Encontra o casamento bipartido maximo/minimo com peso nas arestas
Criar o grafo:
Hungarian G(L, R, ehMaximo)
L = #vertices a esquerda
R = #vertices a direita
ehMaximo = variavel booleana que indica se eh casamento maximo ou
minimo

Adicionar arestas:
G.add_edge(x,y,peso)
x = vertice da esquerda no intervalo [0,L-1]
y = vertice da direita no intervalo [0,R-1]
peso = custo da aresta
obs: tomar cuidado com multiplas arestas.

Resultado:
match_value = soma dos pesos dos casamentos
pairs = quantidade de pares (x-y) casados
xy[x] = vertice y casado com x
yx[y] = vertice x casado com y

Complexidade do algoritmo: O(V^3)
Problemas resolvidos: SCITIES (SPOJ)
 */

struct Hungarian {
```

```
enum { MAXN = 150, INF = 0x3f3f3f3f };
int cost[MAXN][MAXN];
int xy[MAXN], yx[MAXN];
bool S[MAXN], T[MAXN];
int lx[MAXN], ly[MAXN], slack[MAXN], slackx[MAXN], prev[MAXN];
int match_value, pairs;
bool ehMaximo;
int n;

Hungarian(int L, int R, bool _ehMaximo = true) {
  n = max(L, R);
  ehMaximo = _ehMaximo;
  if (ehMaximo)
    memset(cost, 0, sizeof cost);
  else
    memset(cost, INF, sizeof cost);
}

void add_edge(int x, int y, int peso) {
  if (!ehMaximo)
    peso *= (-1);
  cost[x][y] = peso;
}

int solve() {
  match_value = 0;
  pairs = 0;
  memset(xy, -1, sizeof(xy));
  memset(yx, -1, sizeof(yx));
  init_labels();
  augment();
  for (int x = 0; x < n; ++x)
    match_value += cost[x][xy[x]];
  return match_value;
}

void init_labels() {
  memset(lx, 0, sizeof(lx));
  memset(ly, 0, sizeof(ly));
  for (int x = 0; x < n; ++x)
    for (int y = 0; y < n; ++y)
      lx[x] = max(lx[x], cost[x][y]);
}

void augment() {
  if (pairs == n)
    return;
  int x, y, root;
  int q[MAXN], wr = 0, rd = 0;
  memset(S, false, sizeof(S));
  memset(T, false, sizeof(T));
  memset(prev, -1, sizeof(prev));
  for (x = 0; x < n; ++x)
    if (xy[x] == -1) {
      q[wr++] = root = x;
      prev[x] = -2;
      S[x] = true;
      break;
    }
  for (y = 0; y < n; ++y) {
    slack[y] = lx[root] + ly[y] - cost[root][y];
    slackx[y] = root;
  }
  while (true) {
    while (rd < wr) {
      x = q[rd++];
      for (y = 0; y < n; ++y)
        if (cost[x][y] == lx[x] + ly[y] && !T[y]) {
          if (yx[y] == -1)
            break;
          T[y] = true;
          q[wr++] = yx[y];
          add(yx[y], x);
        }
      if (y < n)
        break;
    }
    if (y < n)
      break;
    update_labels();
    wr = rd = 0;
    for (y = 0; y < n; ++y)
      if (!T[y] && slack[y] == 0) {
        if (yx[y] == -1) {
          x = slackx[y];
          break;
        } else {
          T[y] = true;
          if (!S[yx[y]]) {
            q[wr++] = yx[y];
            add(yx[y], slackx[y]);
          }
        }
      }
    if (y < n)
      break;
  }
  if (y < n) {
    ++pairs;
    for (int cx = x, cy = y, ty; cx != -2; cx = prev[cx], cy = ty) {
      ty = xy[cx];
      yx[cy] = cx;
      xy[cx] = cy;
    }
    augment();
  }
}

void add(int x, int prevx) {
  S[x] = true;
  prev[x] = prevx;
  for (int y = 0; y < n; ++y)
    if (lx[x] + ly[y] - cost[x][y] < slack[y]) {
      slack[y] = lx[x] + ly[y] - cost[x][y];
      slackx[y] = x;
    }
}

void update_labels() {
  int x, y, delta = INF;
  for (y = 0; y < n; ++y)
    if (!T[y])
```

```
        delta = min(delta, slack[y]);
    for (x = 0; x < n; ++x)
      if (S[x])
        lx[x] -= delta;
    for (y = 0; y < n; ++y)
      if (T[y])
        ly[y] += delta;
    for (y = 0; y < n; ++y)
      if (!T[y])
        slack[y] -= delta;
  }

  int casouComX(int x) { return xy[x]; }

  int casouComY(int y) { return yx[y]; }
};

// O codigo abaixo resolve o problema scities (Spoj)
int main() {
  int casos;
  cin >> casos;
  while (casos--) {
    int L, R;
    cin >> L >> R;
    Hungarian G(L, R, true);

    int x, y, w, aux[L][R];
    memset(aux, 0, sizeof aux);
    while (scanf("%d %d %d", &x, &y, &w) != EOF) {
      if (x == 0 && y == 0 && w == 0)
        break;
      aux[x - 1][y - 1] += w;
    }
    for (int x = 0; x < L; x++) {
      for (int y = 0; y < R; y++) {
        if (aux[x][y] != 0) {
          G.add_edge(x, y, aux[x][y]);
        }
      }
    }
    printf("%d\n", G.solve());
  }
  return 0;
}
```

6.22 Blossom

```
// Encontra o emparelhamento maximo em um grafo nao direcionado.
// Armazenar em n a quantidade de vertice e em mat[][] as adjacencias.
// edmond(n) retorna o emparelhamento maximo.
typedef vector<int> VI;
typedef vector<vector<int>> VVI;

int mat[205][205], n;

int lf[205];
VVI adj;
VI vis, inactive, match;
int N;

bool dfs(int x, VI &blossom) {
  if (inactive[x])
    return false;
  int i, y;
  vis[x] = 0;
  for (i = adj[x].size() - 1; i >= 0; i--) {
    y = adj[x][i];
    if (inactive[y])
      continue;
    if (vis[y] == -1) {
      vis[y] = 1;
      if (match[y] == -1 || dfs(match[y], blossom)) {
        match[y] = x;
        match[x] = y;
        return true;
      }
    }
    if (vis[y] == 0 || blossom.size()) {
      blossom.push_back(y);
      blossom.push_back(x);
      if (blossom[0] == x) {
        match[x] = -1;
        return true;
      }
      return false;
    }
  }
  return false;
}

bool augment() {
  VI blossom, mark;
  int i, j, k, s, x;
  for (i = 0; i < N; i++) {
    if (match[i] != -1)
      continue;
    blossom.clear();
    vis = VI(N + 1, -1);
    if (!dfs(i, blossom))
      continue;
    s = blossom.size();
    if (s == 0)
      return true;

    mark = VI(N + 1, -1);
    for (j = 0; j < s - 1; j++) {
      for (k = adj[blossom[j]].size() - 1; k >= 0; k--)
        mark[adj[blossom[j]][k]] = j;
    }

    for (j = 0; j < s - 1; j++) {
      mark[blossom[j]] = -1;
      inactive[blossom[j]] = 1;
    }

    adj[N].clear();
    for (j = 0; j < N; j++) {
      if (mark[j] != -1)
        adj[N].pb(j), adj[j].pb(N);
    }
```

```
      match[N] = -1;
      N++;
      if (!augment())
        return false;
      N--;

      for (j = 0; j < N; j++) {
        if (mark[j] != -1)
          adj[j].pop_back();
      }
      for (j = 0; j < s - 1; j++) {
        inactive[blossom[j]] = 0;
      }

      x = match[N];
      if (x != -1) {
        if (mark[x] != -1) {
          j = mark[x];
          match[blossom[j]] = x;
          match[x] = blossom[j];
          if (j & 1)
            for (k = j + 1; k < s; k += 2) {
              match[blossom[k]] = blossom[k + 1];
              match[blossom[k + 1]] = blossom[k];
            }
          else
            for (k = 0; k < j; k += 2) {
              match[blossom[k]] = blossom[k + 1];
              match[blossom[k + 1]] = blossom[k];
            }
        }
      }
      return true;
    }
  return false;
}

int edmond(int n) {
  int i, j, ret = 0;
  N = n;
  adj = VVI(2 * N + 1);
  for (i = 0; i < n; i++) {
    for (j = i + 1; j < n; j++) {
      if (mat[i][j]) {
        adj[i].pb(j);
        adj[j].pb(i);
      }
    }
  }
  match = VI(2 * N + 1, -1);
  inactive = VI(2 * N + 1);
  while (augment())
    ret++;
  return ret;
}
```

7 Estruturas de Dados

7.1 BIT

```
// Permite realizar operacoes de query e update em um vetor em O(logN)
// Obs: A[] deve ser indexado em 1, nao em 0.
#define MAXN 100001
ll ft[MAXN];
ll A[MAXN];
int N;

// ATUALIZA UM INDICE i, CONSULTA UM INTERVALO (i,j)
 /// update(i, valor) faz A[i] += valor em log(N)
void update(int i, ll valor) {
  for (; i <= N; i += i & -i)
    ft[i] += valor;
}

 /// query(i) retorna a soma A[1] + ... + A[i] em log(N)
ll query(int i) {
  ll sum = 0;
  for (; i > 0; i -= i & -i)
    sum += ft[i];
  return sum;
}

/// query(i,j) retorna a soma A[i] + A[i+1] + ... + A[j] em log(N)
ll query(int i, int j) { return query(j) - query(i - 1); }



// ATUALIZA UM INTERVALO (i,j), CONSULTA UM ELEMENTO i

/// range_update(i,j,valor) faz A[k] += valor, para i <= k <= j em
/// log(N) query(i): retorna o valor de A[i] em log(N)
void range_update(int i, int j, ll valor) {
  update(i, valor);
  update(j + 1, -valor);
}
/// Just a wrapper function... Returns the value at A[i] after
    range_update( ) calls
ll point_query(int i) {
  return query(i);
}

int main() { return 0; }
```

7.2 BIT 2D

```
#define MAXL 3001
#define MAXC 3001
ll ft[MAXL][MAXC];
int L, C;
// update(x,y,v) incrementa v na posicao (x,y)
// .: M[x][y] += v em O(log(N))
void update(int x, int y, int v) {
  for (; x <= L; x += x & -x)
```

```
    for (int yy = y; yy <= C; yy += yy & -yy)
      ft[x][yy] += v;
}

// query(x,y) retorna o somatorio da submatriz definida por
// (1,1)->(x,y) .: sum += M[i][j] para todo 1 <= i <= x e 1 <= j <= y,
// em O(log(N))
ll query(int x, int y) {
  if (x <= 0 || y <= 0)
    return 0;
  ll sum = 0;
  for (; x > 0; x -= x & -x)
    for (int yy = y; yy > 0; yy -= yy & -yy)
      sum += ft[x][yy];
  return sum;
}

// query(x1,y1,x2,y2) retorna o somatorio da submatriz definida por
// (x1,x1) -- (x2,y2) .: sum += M[i][j] para todo x1 <= i <= x2 e
// y1 <= j <= y2, em O(log(N))
ll query(int x1, int y1, int x2, int y2) {
  return query(x2, y2) - query(x2, y1 - 1) - query(x1 - 1, y2) +
         query(x1 - 1, y1 - 1);
}

// A ideia de atualizar um intervalo (submatriz) e consultar um
// elemento (i,j) tambem sao validos
```

7.3 Sparse Table

```
/*
Resolve problemas de consulta a intervalos (RSQ, RMQ etc) de um vetor
estatico, ou seja, os valores nao sofrem update.
Alterar a funcao comb() de acordo (min, max, soma etc)
Pre-processamento O(NlogN) e consulta em O(1).
N = tamanho do vetor a[]
a[] deve ser indexado em 0
*/
const int MAXN = (1e6 + 1);
#define LOGN (21)
int st[MAXN][LOGN];
int N, a[MAXN];

int comb(int left, int right) { return min(left, right); }

void SparseTable() {
  rep(k, 0, LOGN) for (int i = 0; (i + (1 << k) - 1) < N; i++)
      st[i][k] =
          k ? comb(st[i][k - 1], st[i + (1 << (k - 1))][k - 1]) :
              a[i];
}

int query(int l, int r) {
  int k = log2(r - l + 1);
  return comb(st[l][k], st[r - (1 << k) + 1][k]);
}
```

7.4 RMQ

```
// Range Minimum Query: idx do menor elemento num intervalo de um
// array. Permite consultas e updates no array em O(logN).  ATENCAO:
// Array A[] deve ser indexado em 0;
#define MAXN 500000
int A[MAXN], T[4 * MAXN];
int N; // #number of elements in A[]
int neutro = -1;

// combina o resultado de dois segmentos
int combine(int p1, int p2) {
  if (p1 == -1)
    return p2;
  if (p2 == -1)
    return p1;
  if (A[p1] <= A[p2])
    return p1;
  else
    return p2;
}

// chamar build() apos preencher o vetor A[]. O(N)
void build(int no = 1, int a = 0, int b = N - 1) {
  if (a == b) {
    T[no] = a;
  } else {
    int m = (a + b) / 2;
    int esq = 2 * no;
    int dir = esq + 1;
    build(esq, a, m);
    build(dir, m + 1, b);
    T[no] = combine(T[esq], T[dir]);
  }
}

// Modifica A[i] em O(logN), neste caso A[i] = v
void update(int i, int v, int no = 1, int a = 0, int b = N - 1) {
  if (a > i || b < i)
    return;
  if (a == i && b == i) {
    A[i] = v;
    T[no] = i; // desnecessario ;p
    return;
  }
  int m = (a + b) / 2;
  int esq = 2 * no;
  int dir = esq + 1;
  update(i, v, esq, a, m);
  update(i, v, dir, m + 1, b);
  T[no] = combine(T[esq], T[dir]);
}

// Retorna o idx k do menor valor A[k] no intervalo [i,j] em O(logN)
int query(int i, int j, int no = 1, int a = 0, int b = N - 1) {
  if (a > j || b < i)
    return neutro;
  if (a >= i && b <= j)
    return T[no];
  int m = (a + b) / 2;
```

```
    int esq = 2 * no;
    int dir = esq + 1;

    int p1 = query(i, j, esq, a, m);
    int p2 = query(i, j, dir, m + 1, b);
    return combine(p1, p2);
  }

  int main() { return 0; }
```

7.5 Persistent Seg Tree

```
const int MAX = 1e5 + 5;
const int MAXT = 80 * MAX;
int a[MAX], tree[MAXT];
int L[MAXT], R[MAXT];
vector<int> root;
int cnt;

void build(int no, int i, int j) {
  if (i == j) {
    tree[no] = a[i];
    return;
  }
  L[no] = ++cnt;
  R[no] = ++cnt;
  build(L[no], i, (i + j) / 2);
  build(R[no], (i + j) / 2 + 1, j);
  tree[no] = tree[L[no]] + tree[R[no]];
}

int update(int no, int i, int j, int p, int v) {
  int NO = ++cnt;
  if (i == j) {
    tree[NO] = v;
    return NO;
  }

  L[NO] = L[no];
  R[NO] = R[no];

  if (p <= (i + j)/2) L[NO] = update(L[NO], i, (i + j) / 2, p, v);
  else                R[NO] = update(R[NO], (i + j) / 2 + 1, j, p, v);

  tree[NO] = tree[L[NO]] + tree[R[NO]];
  return NO;
}

int query(int no, int i, int j, int l, int r) {
  if (i > r || j < l)   return 0;
  if (i >= l && j <= r) return tree[no];

  int left  = query(L[no], i, (i + j) / 2, l, r);
  int right = query(R[no], (i + j) / 2 + 1, j, l, r);
  return left + right;
}

void init() {
  memset(tree, 0, sizeof tree);
  root.clear();
  root.pb(0);
  cnt = 0;
}

int main(int argc, char **argv) {
  query(root[k], 1, N, l, r));  // consulta na versao k
  root.pb(update(root[sz], 1, N, l, v)); // update
  return 0;
}
```

7.6 Seg Tree com Lazy

```
// RSQ agora com queries e updates em intervalos. Precisa de Lazy
// Propagation. Array A[] deve ser indexado em 0. Nem sempre o array
// que sera modificado armazena apenas um valor, nesse caso usamos
// struct para representar cada no.
#define MAXN 500000
ll A[MAXN], tree[4 * MAXN], lazy[4 * MAXN];
int N;
int neutro = 0;

// funcao que realiza o merge de um intervalo, pode ser *, -, min,
// max, etc...
int combine(int segEsq, int segDir) { return segEsq + segDir; }

void build(int no = 1, int a = 0, int b = N - 1) {
  if (a == b) {
    tree[no] = A[a];
    return;
  }
  int m = (a + b) / 2;
  int esq = 2 * no;
  int dir = esq + 1;
  build(esq, a, m);
  build(dir, m + 1, b);
  tree[no] = combine(tree[esq], tree[dir]);
}

void propagate(int no, int a, int b) {
  if (lazy[no] != 0) {
    // esta parte depende do problema, neste caso queremos adicionar o
    // valor lazy[no] no intervalo [a,b], mas estamos atualizando
    // apenas o noh que representa este intervalo
    tree[no] += (b - a + 1) * lazy[no];
    if (a != b) {
      lazy[2 * no] += lazy[no];
      lazy[2 * no + 1] += lazy[no];
    }
    lazy[no] = 0;
  }
}

// update(i,j,v) faz A[k] += v, para i <= k <= j, em log(N)
void update(int i, int j, ll v, int no = 1, int a = 0, int b = N - 1)
    {
  if (lazy[no])
    propagate(no, a, b);
  if (a > j || b < i)
    return;
  if (a >= i && b <= j) {
```

```cpp
      lazy[no] += v; // atualiza apenas a flag da raiz da subarvore
      propagate(no, a, b);
      return;
    }
    int m = (a + b) / 2;
    int esq = 2 * no;
    int dir = esq + 1;
    update(i, j, v, esq, a, m);
    update(i, j, v, dir, m + 1, b);
    tree[no] = combine(tree[esq], tree[dir]);
  }

  // query(i,j) retorna o somatorio A[i] + A[i+1] + ... + A[j]
  ll query(int i, int j, int no = 1, int a = 0, int b = N - 1) {
    if (lazy[no])
      propagate(no, a, b);
    if (a > j || b < i)
      return neutro;
    if (a >= i && b <= j)
      return tree[no];
    int m = (a + b) / 2;
    int esq = 2 * no;
    int dir = esq + 1;
    ll q1 = query(i, j, esq, a, m);
    ll q2 = query(i, j, dir, m + 1, b);
    return combine(q1, q2);
  }

  int main() { return 0; }
```

7.7 Union-Find

```cpp
  // Conjuntos Disjuntos. Inicialmente cada elemento eh lider de seu
  // proprio conjunto. Operacoes de join(u,v) fazem com que os conjuntos
  // que u e v pertencem se unam. find(u) retorna o lider do conjunto
  // que u esta contido.
  #define MAXV 100000
  int V, pai[MAXV], rnk[MAXV], size[MAXV];

  void init() { rep(i, 0, V) pai[i] = i, rnk[i] = 0, size[i] = 1; }

  int find(int v) {
    if (v != pai[v])
      pai[v] = find(pai[v]);
    return pai[v];
  }

  void join(int u, int v) {
    u = find(u);
    v = find(v);
    if (u == v)
      return;

    if (rnk[u] < rnk[v])
      swap(u, v);
    pai[v] = u; // add v no conjunto de u
    size[u] += size[v];
    if (rnk[u] == rnk[v])
      rnk[u]++;
  }

  bool same_set(int u, int v) { return find(u) == find(v); }

  int main() { return 0; }
```

7.8 Treap

```cpp
  typedef struct node {
    int prior, size;
    int val;  // value stored in the array
    int sum;  // whatever info you want to maintain in segtree for each
              // node
    int lazy; // whatever lazy update you want to do
    int rev;
    struct node *l, *r;
  } node;
  typedef node *pnode;
  int sz(pnode t) { return t ? t->size : 0; }
  void upd_sz(pnode t) {
    if (t)
      t->size = sz(t->l) + 1 + sz(t->r);
  }
  void lazy(pnode t) {
    if (!t || t->lazy == -1)
      return;
    t->val = t->lazy; // operation of lazy
    t->sum = t->lazy * sz(t);
    if (t->l)
      t->l->lazy = t->lazy; // propagate lazy
    if (t->r)
      t->r->lazy = t->lazy;
    t->lazy = -1;
  }
  void reset(pnode t) {
    if (t)
      t->sum = t->val; // no need to reset lazy coz when we call this
                       // lazy would itself be propagated
  }

  // combining two ranges of segtree
  void combine(pnode &t, pnode l, pnode r) {
    if (!l || !r)
      return void(t = l ? l : r);
    t->sum = l->sum + r->sum;
  }
  void operation(pnode t) { // operation of segtree
    if (!t)
      return;
    reset(t); // reset the value of current node assuming it now
              // represents a single element of the array
    lazy(t->l);
    lazy(t->r); // imp:propagate lazy before combining t->l,t->r;
    combine(t, t->l, t);
    combine(t, t, t->r);
  }
  void push(pnode t) {
    if (!t || !t->rev)
      return;
    t->rev = false;
    swap(t->l, t->r);
```

```cpp
  if (t->l)
    t->l->rev ^= true;
  if (t->r)
    t->r->rev ^= true;
}
void split(pnode t, pnode &l, pnode &r, int pos, int add = 0) {
  if (!t)
    return void(l = r = NULL);
  push(t);
  lazy(t);
  int curr_pos = add + sz(t->l);
  if (curr_pos <= pos) // element at pos goes to left subtree(l)
    split(t->r, t->r, r, pos, curr_pos + 1), l = t;
  else
    split(t->l, l, t->l, pos, add), r = t;
  upd_sz(t);
  operation(t);
}

// l->leftarray,r->rightarray,t->resulting array
void merge(pnode &t, pnode l, pnode r) {
  push(l);
  push(r);
  lazy(l);
  lazy(r);
  if (!l || !r)
    t = l ? l : r;
  else if (l->prior > r->prior)
    merge(l->r, l->r, r), t = l;
  else
    merge(r->l, l, r->l), t = r;
  upd_sz(t);
  operation(t);
}
pnode init(int val) {
  pnode ret = new node;
  ret->prior = rand();
  ret->size = 1;
  ret->val = val;
  ret->sum = val;
  ret->lazy = -1;
  ret->rev = 0;
  ret->l = NULL, ret->r = NULL;
  return ret;
}
int range_query(pnode t, int l, int r) { //[l,r]
  pnode L, mid, R;
  split(t, L, mid, l - 1);
  split(mid, t, R, r - l); // note: r-l!!
  int ans = t->sum;
  merge(mid, L, t);
  merge(t, mid, R);
  return ans;
}
void range_update(pnode t, int l, int r, int val) { //[l,r]
  pnode L, mid, R;
  split(t, L, mid, l - 1);
  split(mid, t, R, r - l); // note: r-l!!
  t->lazy = val;           // lazy_update
  merge(mid, L, t);
  merge(t, mid, R);
}
void reverse(pnode t, int l, int r) {
  pnode L, mid, R;
  split(t, L, mid, l - 1);
  split(mid, mid, R, r - l);
  mid->rev ^= true;
  merge(t, L, mid);
  merge(t, t, R);
}
void output(pnode t) {
  if (!t)
    return;
  push(t);
  lazy(t);
  output(t->l);
  printf("%d ", t->val);
  output(t->r);
}

int valor(int val) { return val & 1 ? 0 : 1; }

int main() {
  int P, Q;
  while (scanf("%d %d", &P, &Q) != EOF) {
    pnode tree = NULL, T1 = NULL, T2 = NULL, T3 = NULL;
    int val;
    rep(i, 0, P) {
      scanf("%d", &val);
      split(tree, T1, T2, i);
      merge(T1, T1, init(valor(val)));
      merge(tree, T1, T2);
    }
    while (Q--) {
    }
  }
}
```

7.9 Seg Tree 2D

```cpp
struct node {
  int qt;
  int f1, f2, f3, f4;
};

node new_node() {
  node ret;
  ret.qt = ret.f1 = ret.f2 = ret.f3 = ret.f4 = 0;
  return ret;
}

vector<node> tree;
int cnt = 0;

bool inRange(int x1, int x2, int y1, int y2, int a1, int a2, int b1,
             int b2) {
  if (x2 < x1 || y2 < y1)
    return false;
  if (x2 < a1 || x1 > a2)
    return false;
  if (y2 < b1 || y1 > b2)
```

```
    return false;
  return true;
}

void update(int no, int x1, int x2, int y1, int y2, int a1, int a2,
            int b1, int b2, int val) {
  if (no == cnt)
    tree[cnt++] = new_node();

  if (x1 >= a1 && x2 <= a2 && y1 >= b1 && y2 <= b2) {
    tree[no].qt = val;
    return;
  }

  int f1 = 0, f2 = 0, f3 = 0, f4 = 0;
  if (inRange(x1, (x1 + x2) / 2, y1, (y1 + y2) / 2, a1, a2, b1, b2)) {
    if (!tree[no].f1)
      tree[no].f1 = cnt;
    update(tree[no].f1, x1, (x1 + x2) / 2, y1, (y1 + y2) / 2, a1, a2,
           b1, b2, val);
  }
  if (inRange(x1, (x1 + x2) / 2, (y1 + y2) / 2 + 1, y2, a1, a2, b1,
              b2)) {
    if (!tree[no].f2)
      tree[no].f2 = cnt;
    update(tree[no].f2, x1, (x1 + x2) / 2, (y1 + y2) / 2 + 1, y2, a1,
           a2, b1, b2, val);
  }
  if (inRange((x1 + x2) / 2 + 1, x2, y1, (y1 + y2) / 2, a1, a2, b1,
              b2)) {
    if (!tree[no].f3)
      tree[no].f3 = cnt;
    update(tree[no].f3, (x1 + x2) / 2 + 1, x2, y1, (y1 + y2) / 2, a1,
           a2, b1, b2, val);
  }
  if (inRange((x1 + x2) / 2 + 1, x2, (y1 + y2) / 2 + 1, y2, a1, a2,
     b1,
              b2)) {
    if (!tree[no].f4)
      tree[no].f4 = cnt;
    update(tree[no].f4, (x1 + x2) / 2 + 1, x2, (y1 + y2) / 2 + 1, y2,
           a1, a2, b1, b2, val);
  }

  if (tree[no].f1)
    f1 = tree[tree[no].f1].qt;
  if (tree[no].f2)
    f2 = tree[tree[no].f2].qt;
  if (tree[no].f3)
    f3 = tree[tree[no].f3].qt;
  if (tree[no].f4)
    f4 = tree[tree[no].f4].qt;

  tree[no].qt = f1 + f2 + f3 + f4;
}

int query(int no, int x1, int x2, int y1, int y2, int a1, int a2,
          int b1, int b2) {
  if (!inRange(x1, x2, y1, y2, a1, a2, b1, b2) || no >= cnt ||
      tree[no].qt == 0)
    return 0;

  if (x1 >= a1 && x2 <= a2 && y1 >= b1 && y2 <= b2)
    return tree[no].qt;

  int f1 = 0, f2 = 0, f3 = 0, f4 = 0;
  if (tree[no].f1)
    f1 = query(tree[no].f1, x1, (x1 + x2) / 2, y1, (y1 + y2) / 2, a1,
               a2, b1, b2);
  if (tree[no].f2)
    f2 = query(tree[no].f2, x1, (x1 + x2) / 2, (y1 + y2) / 2 + 1, y2,
               a1, a2, b1, b2);
  if (tree[no].f3)
    f3 = query(tree[no].f3, (x1 + x2) / 2 + 1, x2, y1, (y1 + y2) / 2,
               a1, a2, b1, b2);
  if (tree[no].f4)
    f4 = query(tree[no].f4, (x1 + x2) / 2 + 1, x2, (y1 + y2) / 2 + 1,
               y2, a1, a2, b1, b2);

  return f1 + f2 + f3 + f4;
}

void erase() {
  tree.clear();
  vector<node> xua;
  swap(tree, xua);
  tree.resize(1000010);
  cnt = 0;
}

int main() { return 0; }
```

7.10 Polyce

```
// https://codeforces.com/blog/entry/11080

#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace std;
using namespace __gnu_pbds;

typedef tree<int, // tipo da variavel
             null_type,
             less<int>, // funcao de comparacao(greater, less_equal,
             rb_tree_tag, tree_order_statistics_node_update>
    ordered_set;

void newSet() {
  // fuciona como um set normal, mas ha 2 funcoes especiais: log(n)
  ordered_set T;
  ordered_set::iterator it;
  int k = *T.find_by_order(0); // retorna o K-esimo elemento segundo
                               // a funcao de comparacao
  int kk = T.order_of_key(0);  // retorna a posicao que um elemento
                               // encaixaria segundo a funcao de
                               // comparacao
}

#include <ext/rope>
```

```
using namespace __gnu_cxx;

void newVector() {
  // funciona como um vector, mas consegue algo a mais: (log(n))
  rope<int> v;
  rope<int>::iterator it;
  int l, r; // segmento
  rope<int> cur =
      v.substr(l, r - l + 1);       // copia um segmento do vector
  v.erase(l, r - l + 1);            // apaga um segmento
  v.insert(v.mutable_begin(), cur); // insere um segmento
  for (it = cur.mutable_begin(); it != cur.mutable_end(); it++)
    cout << *it << " "; // percorre ele
}

int main() { return 0; }
```

7.11 KD2

```
struct point {
  int x, y, z;
  point(int x = 0, int y = 0, int z = 0) : x(x), y(y), z(z) {}
  point operator-(point q) { return point(x - q.x, y - q.y, z - q.z);
      }
  int operator*(point q) { return x * q.x + y * q.y + z * q.z; }
};

typedef vector<point> polygon;
priority_queue<double> vans;
int NN, CC, KK, DD;

struct KDTreeNode {
  point p;
  int level;
  KDTreeNode *below, *above;
  KDTreeNode(const point &q, int levl) {
    p = q;
    level = levl;
    below = above = 0;
  }
  ~KDTreeNode() { delete below, above; }
  int diff(const point &pt) {
    switch (level) {
    case 0:
      return pt.x - p.x;
    case 1:
      return pt.y - p.y;
    case 2:
      return pt.z - p.z;
    }
    return 0;
  }
  ll distSq(point &q) { return (p - q) * (p - q); }
  int rangeCount(point &pt, ll K) {
    int count = (distSq(pt) <= K * K) ? 1 : 0;
    if (count)
      vans.push(-sqrt(distSq(pt)));

    int d = diff(pt);
    if (~d <= K && above != 0)
      count += above->rangeCount(pt, K);
    if (d <= K && below != 0)
      count += below->rangeCount(pt, K);
    return count;
  }
};

class KDTree {
public:
  polygon P;
  KDTreeNode *root;
  int dimention;
  KDTree() {}
  KDTree(polygon &poly, int D) {
    P = poly;
    dimention = D;
    root = 0;
    build();
  }
  ~KDTree() { delete root; }
  // count the number of pairs that has a distance less than K
  ll countPairs(ll K) {
    ll count = 0;
    rep(i, 0, P.size()) count += root->rangeCount(P[i], K) - 1;
    return count;
  }

protected:
  void build() {
    // random_shuffle(all(P));
    rep(i, 0, P.size()) { root = insert(root, P[i], -1); }
  }
  KDTreeNode *insert(KDTreeNode *t, const point &pt, int parentLevel)
      {
    if (t == 0) {
      t = new KDTreeNode(pt, (parentLevel + 1) % dimention);
      return t;
    } else {
      int d = t->diff(pt);
      if (d <= 0)
        t->below = insert(t->below, pt, t->level);
      else
        t->above = insert(t->above, pt, t->level);
    }
    return t;
  }
};

int main() {
  point e;
  e.z = 0;
  polygon p;
  set<ii> st;

  while (scanf("%d %d %d %d", &NN, &CC, &KK, &DD) != EOF) {
    p.clear();
    KK = min(NN, KK);
    st.clear();

    rep(i, 0, NN) {
      scanf("%d %d", &e.x, &e.y);
```

```
      st.insert(mp(e.x, e.y));
      p.pb(e);
    }

    KDTree tree(p, 2);
    int ans = 0;
    rep(i, 0, CC) {
      scanf("%d %d", &e.x, &e.y);
      if (st.count(mp(e.x, e.y)))
        continue;

      ll at = 0;
      rep(i, 0, 30) {
        at = ll(1) << i;
        while (!vans.empty())
          vans.pop();
        int aux = tree.root->rangeCount(e, at);
        if (aux >= KK)
          break;
      }
      double sum = 0.0;
      rep(i, 0, KK) {
        sum += -vans.top();
        vans.pop();
      }
      if (sum >= DD)
        ans++;
    }
    printf("%d\n", ans);
  }
  return 0;
}
```

8 Strings

8.1 KMP

```
// obs: A funcao strstr (char* text, char* pattern) da biblioteca
// <cstring> implementa KMP (C-ANSI). A funcao retorna a primeira
// ocorrencia do padrao no texto, KMP retorna todas.  nres -> O numero
// de ocorrencias do padrao no texto res[] -> posicoes das nres
// ocorrencias do padrao no texto Complexidade do algoritmo: O(n+m)*/

#define MAXN 100001
int pi[MAXN], res[MAXN], nres;
void kmp(string text, string pattern) {
  nres = 0;
  pi[0] = -1;
  rep(i, 1, pattern.size()) {
    pi[i] = pi[i - 1];
    while (pi[i] >= 0 && pattern[pi[i] + 1] != pattern[i])
      pi[i] = pi[pi[i]];
    if (pattern[pi[i] + 1] == pattern[i])
      ++pi[i];
  }
  int k = -1; // k+1 eh o tamanho do match atual
  rep(i, 0, text.size()) {
    while (k >= 0 && pattern[k + 1] != text[i])
      k = pi[k];
    if (pattern[k + 1] == text[i])
      ++k;
    if (k + 1 == pattern.size()) {
      res[nres++] = i - k;
      k = pi[k];
    }
  }
}
```

8.2 Aho Corasick

```
const int cc = 26;
const int MAX = 100;

int cnt;
int sig[MAX][cc];
int term[MAX];
int T[MAX];
int v[MAX];

inline int C(char c) { return c - '0'; }

void add(string s, int id) {
  int x = 0;
  rep(i, 0, s.size()) {
    int c = C(s[i]);
    if (!sig[x][c]) {
      term[cnt] = 0;
      sig[x][c] = cnt++;
    }
    x = sig[x][c];
  }
  term[x] = 1;
  v[id] = x;
}

void aho() {
  queue<int> q;
  rep(i, 0, cc) {
    int x = sig[0][i];
    if (!x)
      continue;
    q.push(x);
    T[x] = 0;
  }
  while (!q.empty()) {
    int u = q.front();
    q.pop();
    rep(i, 0, cc) {
      int x = sig[u][i];
      if (!x)
        continue;
      int v = T[u];
      while (v && !sig[v][i])
        v = T[v];
      v = sig[v][i];
      T[x] = v;
      term[x] += term[v];
      q.push(x);
```

```
      }
    }
  }

  // Conta a quantidade de palavras de exatamente l caracteres que se
  // pode formar com um determinado alfabeto, dado que algumas palavras
  // sao "proibidas"

  int mod = 1e9 + 7;
  ll pd[100][MAX];

  ll solve(int pos, int no) {
    if (pos == 0)
      return 1;
    if (pd[pos][no] != -1)
      return pd[pos][no];
    ll ans = 0;
    rep(i, 0, cc) {
      int v = no;
      while (v && !sig[v][i])
        v = T[v];
      v = sig[v][i];
      if (term[v])
        continue;
      ans = (ans + solve(pos - 1, v)) % mod;
    }
    return pd[pos][no] = ans;
  }

  void Qttd_de_Palavras() {
    while (1) {
      memset(sig, 0, sizeof sig);
      memset(pd, -1, sizeof pd);
      cnt = 1;
      int l = readInt();
      if (!l)
        break;
      int n = readInt();
      string pattern;
      rep(i, 0, n) {
        cin >> pattern;
        add(pattern, i);
      }
      aho();
      ll ans = 0;
      rep(i, 1, l + 1) ans = (ans + solve(i, 0)) % mod;
      printf("%d\n", ans);
    }
  }

  // Verifica quais padroes ocorreram em um texto

  int alc[MAX];

  void busca(string s) {
    int x = 0;
    rep(i, 0, s.size()) {
      int c = C(s[i]);
      while (x && !sig[x][c])
        x = T[x];
      x = sig[x][c];
      alc[x] = 1;
    }
  }

  void Ql_Ocorreu() {
    string pattern, text;
    while (getline(cin, text)) {
      if (text == "*")
        break;
      memset(sig, 0, sizeof sig);
      memset(alc, 0, sizeof alc);
      cnt = 1;
      int n;
      cin >> n;
      rep(i, 0, n) {
        cin >> pattern;
        add(pattern, i);
      }
      aho();
      busca(text);
      for (int i = cnt - 1; i >= 0; i--) {
        if (alc[i])
          alc[T[i]] = 1;
      }
      rep(i, 0, n) {
        int u = v[i];
        if (alc[u])
          printf("Ocorreu\n");
        else
          printf("Nao ocorreu\n");
      }
    }
  }

  // Total de ocorrencias de cada padrao em uma string, mesmo com
  // sufixos iguais
  ll busca2(string s) {
    ll x = 0, cont = 0;
    rep(i, 0, s.size()) {
      int c = C(s[i]);
      while (x && !sig[x][c])
        x = T[x];
      x = sig[x][c];
      cont += term[x];
    }
    return cont;
  }

  void Qnts_vezes_Ocorreu() {
    string text, pattern;
    while (cin >> text) {
      if (text == "*")
        break;
      memset(sig, 0, sizeof sig);
      cnt = 1;
      int n = readInt();
      rep(i, 0, n) {
        cin >> pattern;
        add(pattern, i);
      }
      aho();
```

```
    rep(i, 1, 10) debug(T[i]) cout << busca2(text) << endl;
  }
}

// Encontra a primeira ocorrencia de cada padrao em uma string
void busca3(string s) {
  int x = 0;
  rep(i, 0, s.size()) {
    int c = C(s[i]);
    while (x && !sig[x][c])
      x = T[x];
    x = sig[x][c];
    if (!alc[x])
      alc[x] = i + 1;
  }
}

void Onde_Ocorreu() {
  string pattern, text;
  int tam[1000];
  while (cin >> text) {
    if (text == "*")
      break;
    memset(sig, 0, sizeof sig);
    memset(alc, 0, sizeof alc);
    cnt = 1;
    int n;
    cin >> n;
    rep(i, 0, n) {
      cin >> pattern;
      tam[i] = pattern.size();
      add(pattern, i);
    }

    aho();
    busca3(text);
    for (int i = cnt - 1; i >= 0; i--) {
      alc[T[i]] = min(alc[i], alc[T[i]]);
    }
    rep(i, 0, n) {
      int u = v[i];
      if (alc[u] != INF) {
        int k = alc[u] - tam[i] + 1;
        printf("De %d a %d\n", k, alc[u]);
      } else
        printf("Nao ocorreu\n");
    }
  }
}
```

8.3 Suffix Array

```
#define MAX 100010
#define MAX_N 100010
char T[MAX_N];
ll n;
int RA[MAX_N], tempRA[MAX_N];
int SA[MAX_N], tempSA[MAX_N];
int c[MAX_N];
int Phi[MAX_N], PLCP[MAX_N], LCP[MAX_N];

void countingSort(int k) {
  int i, sum, maxi = max((ll)300, n);
  memset(c, 0, sizeof c);
  for (i = 0; i < n; i++)
    c[i + k < n ? RA[i + k] : 0]++;
  for (i = sum = 0; i < maxi; i++) {
    int t = c[i];
    c[i] = sum;
    sum += t;
  }
  for (i = 0; i < n; i++)
    tempSA[c[SA[i] + k < n ? RA[SA[i] + k] : 0]++] = SA[i];
  for (i = 0; i < n; i++)
    SA[i] = tempSA[i];
}

void constructSA() {
  int i, k, r;
  for (i = 0; i < n; i++)
    RA[i] = T[i];
  for (i = 0; i < n; i++)
    SA[i] = i;

  for (k = 1; k < n; k <<= 1) {
    countingSort(k);
    countingSort(0);
    tempRA[SA[0]] = r = 0;
    for (i = 1; i < n; i++)
      tempRA[SA[i]] = (RA[SA[i]] == RA[SA[i - 1]] &&
                       RA[SA[i] + k] == RA[SA[i - 1] + k])
                         ? r
                         : ++r;
    for (i = 0; i < n; i++)
      RA[i] = tempRA[i];
    if (RA[SA[n - 1]] == n - 1)
      break;
  }
}

void computeLCP() {
  int i, L;
  Phi[SA[0]] = -1;
  for (i = 1; i < n; i++)
    Phi[SA[i]] = SA[i - 1];
  for (i = L = 0; i < n; i++) {
    if (Phi[i] == -1) {
      PLCP[i] = 0;
      continue;
    }
    while (T[i + L] == T[Phi[i] + L])
      L++;
    PLCP[i] = L;
    L = max(L - 1, 0);
  }
  for (i = 0; i < n; i++) {
    LCP[i] = PLCP[SA[i]];
  }
}
int main() {
  // concatenar $ no final
```

```
}
```

8.4 Suffix Array (Gugu)

```
const int MAX = 100010;
int gap, tam, sa[MAX], pos[MAX], lcp[MAX], tmp[MAX];

bool sufixCmp(int i, int j) {
  if (pos[i] != pos[j])
    return pos[i] < pos[j];
  i += gap, j += gap;
  return (i < tam && j < tam) ? pos[i] < pos[j] : i > j;
}
void buildSA(char s[]) {
  tam = strlen(s);
  for (int i = 0; i < tam; i++)
    sa[i] = i, pos[i] = s[i], tmp[i] = 0;

  for (gap = 1;; gap *= 2) {
    sort(sa, sa + tam, sufixCmp);
    tmp[0] = 0;
    for (int i = 0; i < tam - 1; i++)
      tmp[i + 1] = tmp[i] + sufixCmp(sa[i], sa[i + 1]);
    for (int i = 0; i < tam; i++)
      pos[sa[i]] = tmp[i];
    if (tmp[tam - 1] == tam - 1)
      break;
  }
}
ll buildLCP(char s[]) {
  ll sum = 0;
  for (int i = 0, k = 0; i < tam; i++) {
    if (pos[i] == tam - 1)
      continue;
    for (int j = sa[pos[i] + 1]; s[i + k] == s[j + k];)
      k++;
    lcp[pos[i] + 1] = k;
    sum += k;
    if (k > 0)
      k--;
  }
  return sum;
}
void PrintAll(char s[]) {
  printf("SA\ttam\tLCP\tSuffix\n");
  rep(i, 0, tam) printf("%2d\t%2d\t%2d\t%s\n", sa[i], tam - sa[i],
                      lcp[i], s + sa[i]);
}
ll num_subs(ll m) { return (ll)tam * (tam + 1) / 2 - m; }
ll num_subsrn() {
  ll ret = 0;
  rep(i, 1, tam) if (lcp[i] > lcp[i - 1]) ret += lcp[i] - lcp[i - 1];
  return ret;
}
void printans(char s[], int n) {
  int maior = 0, id = -1;
  rep(i, 0, tam) if (lcp[i] > n && lcp[i] > maior) maior = lcp[i],
                                                   id = i;
  if (id == -1)
    printf("*");
  else
    rep(i, sa[id], sa[id] + maior) printf("%c", s[i]);
  printf("\n");
}

char s[MAX];
int main() {
  while (1) {
    scanf("%s", s);
    if (s[0] == '*')
      break;

    buildSA(s);
    ll m = buildLCP(s);

    PrintAll(s); // printa sa, lcp, suffixs
    // printf("%lld\n", num_subs(m)); //numero de substrings nao
    // repetidas printf("%lld\n", num_subsrn()); //numero de
       substrings
    // que se repete printans(s, 2); //maior substring de tamanho
       maior
    // ou igual a n que se repete
  }
}
```

8.5 Rolling Hash

```
// Permite encontrar um hash de uma substring de S. precompute O(n),
// my_hash O(1)
#define NN 1000006
const ll mod = 1e9 + 7; // modulo do hash
const ll x = 33;        // num. primo > que o maior caracter de S.
ll H[NN], X[NN];
ll V(char c) { return c - 'A'; }
ll my_hash(int i, int j) {
  ll ret = H[j];
  if (!i)
    return ret;
  return ((ret - (H[i - 1] * X[j - i + 1]) % mod) + mod) % mod;
}
void precompute(string s) {
  X[0] = 1;
  rep(i, 1, NN) X[i] = (X[i - 1] * x) % mod;
  H[0] = V(s[0]);
  rep(i, 1, s.size()) H[i] = ((H[i - 1] * x) % mod + V(s[i])) % mod;
}
```

8.6 Longest Commom Prefix with Hash

```
// Longest Commom Prefix between S[i..] and S[j..]
int lcp(int i, int j, int tam) {
  int lo = 0, hi = tam, ans;
  while (lo <= hi) {
    int mid = (lo + hi) / 2;
    if (my_hash(i, i + mid - 1) == my_hash(j, j + mid - 1)) {
      ans = mid;
      lo = mid + 1;
    } else
```

```
    hi = mid - 1;
  }
  return ans;
}
```

8.7 Minimum Lexicographic Rotation

```
// Retorna a menor string lexicografica de s. Necessario my_hash() e
// lcp()
string min_lex_rot(string s) {
  int t = s.size();
  precompute(s); // hashing
  s += s;
  int idx = 0;
  for (int i = 1; i < t; i++) {
    // tam do prefix comum
    int len = lcp(i, idx, t);
    if (s[i + len] < s[idx + len])
      idx = i;
  }
  return s.substr(idx, t);
}
```

8.8 Longest Palindrome (Manacher algorithm)

```
string preProcess(string s) {
  int n = s.length();
  if (n == 0)
    return "^$";

  string ret = "^";
  for (int i = 0; i < n; i++)
    ret += "#" + s.substr(i, 1);
  ret += "#$";
  return ret;
}

string longestPalindrome(string s) {
  L = C = s.size();
  string T = preProcess(s);
  int n = T.length();
  int *P = new int[n];
  int C = 0, R = 0;

  for (int i = 1; i < n - 1; i++) {
    int i_mirror = 2 * C - i;
    P[i] = (R > i) ? min(R - i, P[i_mirror]) : 0;
    while (T[i + 1 + P[i]] == T[i - 1 - P[i]])
      P[i]++;
    if (i + P[i] > R) {
      C = i;
      R = i + P[i];
    }
  }

  int maxLen = 0;
  int centerIndex = 0;
  for (int i = 1; i < n - 1; i++) {
    if (!P[i])
      continue;
    if (P[i] > maxLen) {
      maxLen = P[i];
      centerIndex = i;
    }
  }
  delete[] P;
  return s.substr((centerIndex - 1 - maxLen) / 2, maxLen);
}
```

8.9 Autômato de Sufixos

```
struct state {
  int len, link;
  int next[26];
};

const int MAXN = 200020;
state st[2 * MAXN]; // vetor que armazena os estados
int sz;             // contador do numero de estados
int last;           // numero do estado que corresponde ao texto todo

void sa_init() {
  sz = 1;
  last = 0;
  st[0].len = 0;
  st[0].link = -1;
  rep(i, 0, 26) st[0].next[i] = 0;
  // limpa o mapeamento de transicoes
}

void sa_extend(int c, ll &ans) {
  int cur = sz++; // novo estado a ser criado
  st[cur].len = st[last].len + 1;
  rep(i, 0, 26) st[cur].next[i] = 0;
  int p; // variavel que itera sobre os estados terminais
  for (p = last; p != -1 && !st[p].next[c]; p = st[p].link) {
    st[p].next[c] = cur;
  }
  if (p == -1) { // nao ocorreu transicao c nos estados terminais
    st[cur].link = 0;
    ans += st[cur].len;
  } else { // ocorreu transicao c no estado p
    int q = st[p].next[c];
    if (st[p].len + 1 == st[q].len) {
      st[cur].link = q;
    } else {
      int clone = sz++; // criacao do vertice clone de q
      st[clone].len = st[p].len + 1;
      rep(i, 0, 26) st[clone].next[i] = st[q].next[i];
      st[clone].link = st[q].link;
      for (; p != -1 && st[p].next[c] == q;
           p = st[p].link) { // atualizacao das transicoes c
        st[p].next[c] = clone;
      }
      st[q].link = st[cur].link = clone;
    }
    ans += st[cur].len - st[st[cur].link].len;
```

```
  }
  // atualizacao do estado que corresponde ao texto
  last = cur;
}

bool busca_automato(int m, string p) {
  int i, pos = 0;
  for (i = 0; i < m; i++) {
    if (st[pos].next[p[i]] == 0) {
      return false;
    } else {
      pos = st[pos].next[p[i]];
    }
  }
  return true;
}

int maior_tamanho_em_comum(string s, string t) {
  ll nothing = 0;
  // Constroi o automato com o primeiro texto
  sa_init();
  for (int i = 0; i < (int)s.size(); i++)
    sa_extend(s[i] - 'a', nothing);
  int estado = 0, tamanho = 0, maior = 0;
  // Passando pelos caracteres do segundo texto
  for (int i = 0; i < (int)t.size(); ++i) {
    while (estado && !st[estado].next[t[i] - 'a']) {
      estado = st[estado].link;
      tamanho = st[estado].len;
    }
    if (st[estado].next[t[i] - 'a']) {
      estado = st[estado].next[t[i] - 'a'];
      tamanho++;
    }
    if (tamanho > maior) {
      maior = tamanho;
    }
  }
  return maior;
}

int main() {
  char s[MAXN];
  char p[MAXN];
  while (gets(s)) {
    sa_init();
    int tam = strlen(s);
    ll ans = 0;
    rep(i, 0, tam) { sa_extend(s[i] - 'a', ans); }
    gets(p);
    printf("%d\n", maior_tamanho_em_comum(s, p));
  }
  return 0;
}
```

8.10 Suffix Array

```
#define MAX 100010
#define MAX_N 100010
char T[MAX_N];
ll n;
int RA[MAX_N], tempRA[MAX_N];
int SA[MAX_N], tempSA[MAX_N];
int c[MAX_N];
int Phi[MAX_N], PLCP[MAX_N], LCP[MAX_N];

void countingSort(int k) {
  int i, sum, maxi = max((ll)300, n);
  memset(c, 0, sizeof c);
  for (i = 0; i < n; i++)
    c[i + k < n ? RA[i + k] : 0]++;
  for (i = sum = 0; i < maxi; i++) {
    int t = c[i];
    c[i] = sum;
    sum += t;
  }
  for (i = 0; i < n; i++)
    tempSA[c[SA[i] + k < n ? RA[SA[i] + k] : 0]++] = SA[i];
  for (i = 0; i < n; i++)
    SA[i] = tempSA[i];
}

void constructSA() {
  int i, k, r;
  for (i = 0; i < n; i++)
    RA[i] = T[i];
  for (i = 0; i < n; i++)
    SA[i] = i;

  for (k = 1; k < n; k <<= 1) {
    countingSort(k);
    countingSort(0);
    tempRA[SA[0]] = r = 0;
    for (i = 1; i < n; i++)
      tempRA[SA[i]] = (RA[SA[i]] == RA[SA[i - 1]] &&
                       RA[SA[i] + k] == RA[SA[i - 1] + k])
                         ? r
                         : ++r;
    for (i = 0; i < n; i++)
      RA[i] = tempRA[i];
    if (RA[SA[n - 1]] == n - 1)
      break;
  }
}

void computeLCP() {
  int i, L;
  Phi[SA[0]] = -1;
  for (i = 1; i < n; i++)
    Phi[SA[i]] = SA[i - 1];
  for (i = L = 0; i < n; i++) {
    if (Phi[i] == -1) {
      PLCP[i] = 0;
      continue;
    }
    while (T[i + L] == T[Phi[i] + L])
      L++;
    PLCP[i] = L;
    L = max(L - 1, 0);
  }
  for (i = 0; i < n; i++) {
```

```
      LCP[i] = PLCP[SA[i]];
    }
  }
  int main() {
    // concatenar $ no final
  }
```

8.11 Z Algorithm

```
// Algorithm produces an array Z where Z[i] is the length of the
// longest substring starting from S[i] which is also a prefix of S.
string s;
vector<int> z;

void Z() {
  int n = s.size(), L = 0, R = 0;
  z.assign(n, 0);

  for (int i = 1; i < n; i++) {
    if (i > R) {
      L = R = i;
      while (R < n && s[R - L] == s[R])
        R++;
      z[i] = R - L;
      R--;
    } else {
      int k = i - L;
      if (z[k] < R - i + 1)
        z[i] = z[k];
      else {
        L = i;
        while (R < n && s[R - L] == s[R])
          R++;
        z[i] = R - L;
        R--;
      }
    }
  }
}
```

9 PD

9.1 Soma acumulada 2D

```
/*Retorna o somatorio dos elementos de uma submatriz em O(1).
 * Submatriz definida por canto superior esquerdo (x1,y1) e  canto
 * inferior direito (x2,y2) .: x1 <= x2 && y1 <= y2 */
#define MAXN 3000
int N, M;                          // linhas colunas
long long V[MAXN + 2][MAXN + 2]; // matriz da entrada
long long S[MAXN + 2][MAXN + 2]; // matriz com as somas acumuladas

// precomputa as somas em O(N*M)
void precal() {
  rep(x, 0, N) rep(y, 0, M) {
    S[x][y] = V[x][y];
    if (x > 0)
      S[x][y] += S[x - 1][y];
    if (y > 0)
      S[x][y] += S[x][y - 1];
    if (x > 0 && y > 0)
      S[x][y] -= S[x - 1][y - 1];
  }
}
// retorna a soma da submatriz em O(1)
long long sum(int x1, int y1, int x2, int y2) {
  long long soma = S[x2][y2];
  if (x1 > 0)
    soma -= S[x1 - 1][y2];
  if (y1 > 0)
    soma -= S[x2][y1 - 1];
  if (x1 > 0 && y1 > 0)
    soma += S[x1 - 1][y1 - 1];
  return soma;
}
```

9.2 Knuth Optimization

```
int N, B, C, yep, save[MAXN][MAXN], sav[MAXN];
ll n[MAXN], mc[MAXN][MAXN], se[MAXN], sd[MAXN], pd[MAXN][MAXN];

ll solve(int i, int k) {
  if (i == N)
    return 0;
  if (k == 1)
    return pd[i][k] = mc[i][N - 1];
  if (pd[i][k] != -1)
    return pd[i][k];

  ll ret = LINF;
  int ini = i, fim = N - k + 1, best = -1;
  if (i && save[i - 1][k])
    ini = save[i - 1][k];
  if (save[i][k - 1])
    fim = save[i][k - 1] + 1;

  rep(l, ini, fim) {
    ll aux = solve(l + 1, k - 1) + mc[i][l];
    if (ret > aux) {
      best = l;
      ret = aux;
    }
  }
  save[i][k] = best;
  return pd[i][k] = ret;
}

int main() {
  rep(i, 0, N) scanf("%lld", &n[i]);

  se[0] = n[0];
  rep(i, 1, N) se[i] = se[i - 1] + n[i];

  sd[N - 1] = n[N - 1];
  for (int i = N - 2; i >= 0; i--)
    sd[i] = sd[i + 1] + n[i];
```

```
  rep(i, 1, N) pd[0][i] = pd[0][i - 1] + se[i - 1];
  for (int i = N - 2; i >= 0; i--)
    pd[N - 1][i] = pd[N - 1][i + 1] + sd[i + 1];

  rep(i, 1, N) {
    rep(j, i + 1, N) pd[i][j] = pd[i - 1][j] - n[i - 1] * (j - i + 1);
  }
  for (int i = N - 2; i >= 0; i--) {
    for (int j = i - 1; j >= 0; j--)
      pd[i][j] = pd[i + 1][j] - n[i + 1] * (i - j + 1);
  }

  rep(i, 0, N) {
    if (pd[i][i + 1] < pd[i + 1][i])
      mc[i][i + 1] = pd[i][i + 1], save[i][i + 1] = i + 1;
    else
      mc[i][i + 1] = pd[i + 1][i], save[i][i + 1] = i;
    rep(j, i + 2, N) {
      int ini = save[i][j - 1];
      mc[i][j] = pd[i][ini] + pd[j][ini], save[i][j] = ini;
      rep(k, ini + 1, j + 1) {
        ll a = pd[i][k] + pd[j][k];
        if (mc[i][j] <= a)
          break;
        mc[i][j] = a;
        save[i][j] = k;
      }
    }
    rep(j, 0, N + 1) { pd[i][j] = -1, save[i][j] = 0; }
  }

  rep(j, 0, N + 1) pd[N][j] = -1, save[N][j] = 0;

  solve();

  return 0;
}
```

9.3 Convex Hull Trick

```
bool bad(int l1, int l2, int l3) {
  return (B[l3] - B[l1]) * (M[l1] - M[l2]) <
         (B[l2] - B[l1]) * (M[l1] - M[l3]);
}
void add(long long m, long long b) {
  M.push_back(m);
  B.push_back(b);
  while (M.size() >= 3 &&
         bad(M.size() - 3, M.size() - 2, M.size() - 1)) {
    M.erase(M.end() - 2);
    B.erase(B.end() - 2);
  }
}
long long query(long long x) {
  if (pointer >= M.size())
    pointer = M.size() - 1;
  while (pointer < M.size() - 1 &&
         M[pointer + 1] * x + B[pointer + 1] <
             M[pointer] * x + B[pointer])
    pointer++;
  return M[pointer] * x + B[pointer];
}

struct hux {
  int a, b, id;
};
bool my_sort(hux a, hux b) {
  return a.b != b.b ? a.b > b.b : a.a > b.a;
}

const ll LINF = 1LL << 52;
const double EPS = 1e-9;
const int MAXV = 100010;

double intersept(hux a, hux b) {
  return double(b.b - a.b) / (a.a - b.a);
}

vector<pair<double, double>> convex_hux(const vector<hux> &v) {
  int p = 0, n = v.size(), bestai = v[0].a;
  double cross = 0.0;
  pair<double, int> aux;

  priority_queue<pair<double, int>> pq;
  vector<pair<double, double>> ret(n + 1, mp(-1, -1));

  pq.push(mp(cross, p));
  ret[v[p].id].F = cross, ret[v[p].id].S = LINF;

  rep(i, 1, n) {
    aux = pq.top();
    cross = aux.F, p = aux.S;

    if (v[i].a <= bestai)
      continue;
    bestai = v[i].a;

    double new_cross = intersept(v[i], v[p]);
    while (new_cross <= cross + EPS) {
      pq.pop();
      ret[v[p].id] = mp(-1.0, -1.0);

      aux = pq.top();
      cross = aux.F, p = aux.S;

      new_cross = intersept(v[i], v[p]);
    }

    pq.push(mp(new_cross, i));
    ret[v[p].id].S = new_cross;
    ret[v[i].id].F = new_cross;
    ret[v[i].id].S = LINF;
  }

  // rep(i, 0, n) cout << ret[i].F << " " << ret[i].S << "\n";

  return ret;
}
```

9.4 Longest Increasing Subsequence

```
// Maior subsequencia crescente
#define MAX_N 100
int vet[MAX_N], P[MAX_N], N;
void reconstruct_print(int end) {
  int x = end;
  stack<int> s;
  while (P[x] >= 0) {
    s.push(vet[x]);
    x = P[x];
  }
  printf("%d", vet[x]);
  while (!s.empty()) {
    printf(", %d", s.top());
    s.pop();
  }
}
int lis() {
  int L[MAX_N], L_id[MAX_N];
  int li = 0, lf = 0; // lis ini, lis end
  rep(i, 0, N) {
    int pos = lower_bound(L, L + li, vet[i]) - L;
    L[pos] = vet[i];
    L_id[pos] = i;
    P[i] = pos ? L_id[pos - 1] : -1;
    if (pos + 1 > li) {
      li = pos + 1;
      lf = i;
    }
  }
  reconstruct_print(lf);
  return li;
}
```

9.5 Kadane 1D

```
// Encontra maior soma contigua positiva num vetor em O(N). {s,f}
// contem o intervalo de maior soma.
int Kadane1D(int vet[], int N, int &s, int &f) {
  int ret = -INF, sum, saux;
  sum = s = f = saux = 0;
  rep(i, 0, N) {
    sum += vet[i];
    if (sum > ret) {
      ret = sum;
      s = saux;
      f = i;
    }
    if (sum < 0) {
      sum = 0;
      saux = i + 1;
    }
  }
  return ret;
}
```

9.6 Kadane 2D

```
/*Maior soma de uma sub-matriz a partir de valores positivos.
 * [x1,y1]=upper-left, [x2,y2]=bottom-right*/
int L, C, pd[MAX_L], mat[MAX_L][MAX_C];
int x1, y1, x2, y2;
int Kadane2D() {
  int ret = 0, aux;
  rep(left, 0, C) {
    rep(i, 0, L) pd[i] = 0;
    rep(right, left, C) {
      rep(i, 0, L) pd[i] += mat[i][right];
      int sum = aux = 0;
      rep(i, 0, L) { // Kadane1D
        sum += pd[i];
        if (sum > ret)
          ret = sum, x1 = aux, y1 = left, x2 = i, y2 = right;
        if (sum < 0)
          sum = 0, aux = i + 1;
      }
    }
  }
  return ret;
}
```

9.7 Knapsack0-1

```
//[IME] 0-1 Knapsack, v-valores, w-pesos, Cap-capacidade
int mochila01(vector<int> v, vector<int> w, int Cap) {
  int n = v.size();
  int dp[n + 1][Cap + 1];
  for (int i = 0; i <= n; i++)
    dp[i][0] = 0;
  for (int j = 0; j <= Cap; j++)
    dp[0][j] = 0;
  for (int i = 1; i <= n; i++)
    for (int j = 1; j <= Cap; j++) {
      if (w[i - 1] > j)
        dp[i][j] = dp[i - 1][j];
      else
        dp[i][j] =
            max(dp[i - 1][j], v[i - 1] + dp[i - 1][j - w[i - 1]]);
    }
  return dp[n][Cap];
}
```

9.8 Edit Distance

```
//[IME] menor custo para transformar a em b, dado as operacoes de
// inserir, remover e substituir caracteres de a
int editDistance(string a, string b) {
  int cost, insertCost = 1, deletCost = 1, substCost = 1;
  int m = a.size();
  int n = b.size();
  int d[m + 1][n + 1];
  for (int i = 0; i <= m; i++)
```

```
    d[i][0] = i * deletCost;
  for (int j = 0; j <= n; j++)
    d[0][j] = j * insertCost;
  for (int i = 1; i <= m; i++)
    for (int j = 1; j <= n; j++) {
      if (a[i - 1] == b[j - 1])
        cost = 0;
      else
        cost = substCost;
      d[i][j] =
          min(d[i - 1][j] + deletCost,
              min(d[i][j - 1] + insertCost, d[i - 1][j - 1] + cost));
    }
  return d[m][n];
}
```

10 Sorting

10.1 Merge Sort com num de Inversoes

```
// Ordena arr aplicando mergesort e conta o numero de inversoes
void merge(int *arr, int size1, int size2, ll &inversions) {
  int temp[size1 + size2 + 2];
  int ptr1 = 0, ptr2 = 0;

  while (ptr1 + ptr2 < size1 + size2) {
    if (ptr1 < size1 && arr[ptr1] <= arr[size1 + ptr2] ||
        ptr1 < size1 && ptr2 >= size2)
      temp[ptr1 + ptr2] = arr[ptr1++];

    if (ptr2 < size2 && arr[size1 + ptr2] < arr[ptr1] ||
        ptr2 < size2 && ptr1 >= size1) {
      temp[ptr1 + ptr2] = arr[size1 + ptr2++];
      inversions += size1 - ptr1;
    }
  }

  for (int i = 0; i < size1 + size2; i++)
    arr[i] = temp[i];
}

void mergeSort(int *arr, int size, ll &inversions) {
  if (size == 1)
    return;

  int size1 = size / 2, size2 = size - size1;
  mergeSort(arr, size1, inversions);
  mergeSort(arr + size1, size2, inversions);
  merge(arr, size1, size2, inversions);
}
```

10.2 Quick Sort

```
// No main, chamar quicksort(array, 0, tam-1);
int partition(int s[], int l, int h) {
  int i, p, firsthigh;
  p = h;
  firsthigh = l;
  for (i = l; i < h; i++)
    if (s[i] < s[p]) {
      swap(s[i], s[firsthigh]);
      firsthigh++;
    }
  swap(s[i], s[firsthigh]);
  return firsthigh;
}
void quicksort(int s[], int l, int h) {
  int p;
  if ((h - l) > 0) {
    p = partition(s, l, h);
    quicksort(s, l, p - 1);
    quicksort(s, p + 1, h);
  }
}
```

11 Miscelânia

11.1 Calendário

```
// Routines for performing computations on dates.  In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
// integers.
string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat",
    "Sun"};

// converts Gregorian date to integer (Julian day number)
int dateToInt(int m, int d, int y) {
  return 1461 * (y + 4800 + (m - 14) / 12) / 4 +
         367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
         3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 + d - 32075;
}
// converts integer (Julian day number) to Gregorian date:
// month/day/year
void intToDate(int jd, int &m, int &d, int &y) {
  int x, n, i, j;
  x = jd + 68569;
  n = 4 * x / 146097;
  x -= (146097 * n + 3) / 4;
  i = (4000 * (x + 1)) / 1461001;
  x -= 1461 * i / 4 - 31;
  j = 80 * x / 2447;
  d = x - 2447 * j / 80;
  x = j / 11;
  m = j + 2 - 12 * x;
  y = 100 * (n - 49) + i + x;
}
// converts integer (Julian day number) to day of week
string intToDay(int jd) { return dayOfWeek[jd % 7]; }
int main() {
  int jd = dateToInt(3, 24, 2004);
  int m, d, y;
  intToDate(jd, m, d, y);
  string day = intToDay(jd);
  // expected output:
```

```
//    2453089
//    3/24/2004
//    Wed
cout << jd << endl
     << m << "/" << d << "/" << y << endl
            << day << endl;
   }
```