Solutions Manual to Mechanics

Hiromichi Inawashiro

May 12, 2025

Contents

1 The Equations of Motion

1.1

Let the field be a uniform gravitational one with the acceleration g. Let the system be a coplanar double pendulum of the strings with the lengths l_1 and l_2 and the particles with masses m_1 and m_2 with the angles ϕ_1 and ϕ_2 from the y axis. Then,

$$\mathbf{r}_1 = \begin{bmatrix} l_1 \sin\phi_1 \\ l_1 \cos\phi_1 \end{bmatrix}, \mathbf{r}_2 = \begin{bmatrix} l_1 \sin\phi_1 + l_2 \sin\phi_2 \\ l_1 \cos\phi_1 + l_2 \cos\phi_2 \end{bmatrix}, \tag{1.1}$$

so that

$$\mathbf{v}_1 = \begin{bmatrix} l_1\dot{\phi_1} \cos\phi_1 \\ -l_1\dot{\phi_1} \sin\phi_1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} l_1\dot{\phi_1} \cos\phi_1 + l_2\dot{\phi_2} \cos\phi_2 \\ -l_1\dot{\phi_1} \sin\phi_1 - l_2\dot{\phi_2} \sin\phi_2 \end{bmatrix}. \tag{1.2}$$

Then, the Lagrangian is given by

$$L = \frac{1}{2} m_1 \|\mathbf{v}_1\|^2 + mgr_{1y} + \frac{1}{2} m_2 \|\mathbf{v}_2\|^2 + mgr_{2y}. \tag{1.3}$$

The right hand side can be written as

$$\begin{aligned} &\frac{1}{2} m_1 l_1^2 \dot{\phi_1}^2 + m_1 g l_1 \cos\phi_1 + \frac{1}{2} m_2 \left(l_1^2 \dot{\phi_1}^2 + l_2^2 \dot{\phi_2}^2 + 2 l_1 l_2 \dot{\phi_1}\dot{\phi_2} \cos(\phi_1 - \phi_2) \right) \\ &+ m_2 g \left(l_1 \cos\phi_1 + l_2 \cos\phi_2 \right). \end{aligned} \tag{1.4}$$

Therefore,

$$\begin{aligned} L =& \frac{1}{2}(m_1 + m_2) l_1^2 \dot{\phi_1}^2 + \frac{1}{2} m_2 l_2^2 \dot{\phi_2}^2 + m_2 l_1 l_2 \dot{\phi_1}\dot{\phi_2} \cos(\phi_1 - \phi_2) \\ &+ (m_1 + m_2) g l_1 \cos\phi_1 + m_2 g l_2 \cos\phi_2. \end{aligned} \tag{1.5}$$

1.2 (Incomplete)

Let the field be a uniform gravitational one with the acceleration g. Let the system be a pendulum of the string with the length l and the particles with the mass m_1 at the point of support which can move on a horisontal line and the mass m_2 at the end of the string with the angle ϕ from the y axis.