Solutions Manual to Mechanics

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Contents

1 The Equations of Motion

1.1

Let the field be a uniform gravitational one with the acceleration g. Let the system be a coplanar double pendulum of the strings with the lengths l_1 and l_2 and the particles with masses m_1 and m_2 with the angles ϕ_1 and ϕ_2 from the y axis. Then,

$$\mathbf{r}_1 = \begin{bmatrix} l_1 \sin\phi_1 \\ l_1 \cos\phi_1 \end{bmatrix}, \mathbf{r}_2 = \begin{bmatrix} l_1 \sin\phi_1 + l_2 \sin\phi_2 \\ l_1 \cos\phi_1 + l_2 \cos\phi_2 \end{bmatrix}, \tag{1.1}$$

so that

$$\mathbf{v}_1 = \begin{bmatrix} l_1\dot{\phi}_1 \cos\phi_1 \\ -l_1\dot{\phi}_1 \sin\phi_1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} l_1\dot{\phi}_1 \cos\phi_1 + l_2\dot{\phi}_2 \cos\phi_2 \\ -l_1\dot{\phi}_1 \sin\phi_1 - l_2\dot{\phi}_2 \sin\phi_2 \end{bmatrix}. \tag{1.2}$$

Then, the Lagrangian is given by

$$L = \frac{1}{2}m_1\|\mathbf{v}_1\|^2 + mgr_{1y} + \frac{1}{2}m_2\|\mathbf{v}_2\|^2 + mgr_{2y}. \tag{1.3}$$

The right hand side can be written as

$$\begin{aligned} &\frac{1}{2}m_1 l_1^2 \dot{\phi}_1^2 + m_1 g l_1 \cos\phi_1 + \frac{1}{2}m_2 \left(l_1^2\dot{\phi}_1^2 + l_2^2\dot{\phi}_2^2 + 2l_1l_2\dot{\phi}_1\dot{\phi}_2\cos(\phi_1-\phi_2) \right) \\ &+ m_2 g \left(l_1\cos\phi_1 + l_2\cos\phi_2 \right). \end{aligned} \tag{1.4}$$

Therefore,

$$\begin{aligned} L =& \frac{1}{2}(m_1+m_2)l_1^2\dot{\phi}_1^2 + \frac{1}{2}m_2 l_2^2\dot{\phi}_2^2 + m_2 l_1 l_2 \dot{\phi}_1\dot{\phi}_2\cos(\phi_1-\phi_2) \\ &+ (m_1+m_2)g l_1\cos\phi_1 + m_2 g l_2 \cos\phi_2. \end{aligned} \tag{1.5}$$

1.2

Let the field be a uniform gravitational one with the acceleration g. Let the system be a pendulum of the string with the length l and the particles with the mass m_1 at the point of support which can move on the x axis and the mass m_2 at the end of the string with the angle ϕ from the y axis. Then,

$$\mathbf{r}_1 = \begin{bmatrix} x \\ 0 \end{bmatrix}, \mathbf{r}_2 = \begin{bmatrix} x + l\sin\phi \\ l\cos\phi \end{bmatrix}, \tag{1.6}$$

so that

$$\mathbf{v}_1 = \begin{bmatrix} \dot{x} \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} \dot{x} + l\dot{\phi}\cos\phi \\ -l\dot{\phi}\sin\phi \end{bmatrix}. \tag{1.7}$$

Then, the Lagrangian is given by

$$L = \frac{1}{2}m_1\|\mathbf{v}_1\|^2 + mgr_{1y} + \frac{1}{2}m_2\|\mathbf{v}_2\|^2 + mgr_{2y}. \tag{1.8}$$

The right hand side can be written as

$$\frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\left(\dot{x}^2 + l^2\dot{\phi}^2 + 2\dot{x}l\dot{\phi}\cos\phi\right) + m_2gl\cos\phi. \tag{1.9}$$

Therefore,

$$L = \frac{1}{2}(m_1 + m_2)\dot{x}^2 + \frac{1}{2}m_2l^2\dot{\phi}^2 + m_2\dot{x}l\dot{\phi}\cos\phi + m_2gl\cos\phi. \tag{1.10}$$

1.3

Let the field be a uniform gravitational one with the acceleration g. Let the system be a pendulum of the string with the length l and the particle with the mass m with the angle ϕ from the y axis.

(a)

If the point of support moves according to the law

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a\cos\gamma t \\ a\sin\gamma t \end{bmatrix}, \tag{1.11}$$

then

$$\mathbf{r} = \begin{bmatrix} a\cos\gamma t + l\sin\phi \\ a\sin\gamma t + l\cos\phi \end{bmatrix}, \tag{1.12}$$

so that

$$\mathbf{v} = \begin{bmatrix} -a\gamma\sin\gamma t + l\dot{\phi}\cos\phi \\ -a\gamma\cos\gamma t - l\dot{\phi}\sin\phi \end{bmatrix}. \tag{1.13}$$

Then, the Lagrangian is given by

$$L = \frac{1}{2}m\|\mathbf{v}\|^2 + mgr_y. \tag{1.14}$$

Therefore,

$$L = \frac{1}{2}ma^2\gamma^2 + \frac{1}{2}l^2\dot{\phi}^2 + al\gamma\dot{\phi}\sin(\phi - \gamma t) + mg(a\sin\gamma t + l\cos\phi). \tag{1.15}$$