

Solutions Manual to Mechanics

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1 The Equations of Motion

1.1

Let the field be a uniform gravitational one with the acceleration g . Let the system be a coplanar double pendulum of the strings with lengths l_1 and l_2 and the particles with masses m_1 and m_2 and with the angles ϕ_1 and ϕ_2 from the y axis. Then,

$$\mathbf{r}_1 = \begin{bmatrix} l_1 \sin \phi_1 \\ l_1 \cos \phi_1 \end{bmatrix}, \mathbf{r}_2 = \begin{bmatrix} l_1 \sin \phi_1 + l_2 \sin \phi_2 \\ l_1 \cos \phi_1 + l_2 \cos \phi_2 \end{bmatrix}, \quad (1.1)$$

so that

$$\mathbf{v}_1 = \begin{bmatrix} l_1 \dot{\phi}_1 \cos \phi_1 \\ -l_1 \dot{\phi}_1 \sin \phi_1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} l_1 \dot{\phi}_1 \cos \phi_1 + l_2 \dot{\phi}_2 \cos \phi_2 \\ -l_1 \dot{\phi}_1 \sin \phi_1 - l_2 \dot{\phi}_2 \sin \phi_2 \end{bmatrix}. \quad (1.2)$$

Then, the Lagrangian is given by

$$L = \frac{1}{2}m_1\|\mathbf{v}_1\|^2 + mgr_{1y} + \frac{1}{2}m_2\|\mathbf{v}_2\|^2 + mgr_{2y}. \quad (1.3)$$

The right hand side can be written as

$$\begin{aligned} & \frac{1}{2}m_1 l_1^2 \dot{\phi}_1^2 + m_1 g l_1 \cos \phi_1 + \frac{1}{2}m_2 \left(l_1^2 \dot{\phi}_1^2 + l_2^2 \dot{\phi}_2^2 + 2l_1 l_2 \dot{\phi}_1 \dot{\phi}_2 \cos(\phi_1 - \phi_2) \right) \\ & + m_2 g (l_1 \cos \phi_1 + l_2 \cos \phi_2). \end{aligned} \quad (1.4)$$

Therefore,

$$\begin{aligned} L = & \frac{1}{2}(m_1 + m_2)l_1^2 \dot{\phi}_1^2 + \frac{1}{2}m_2 l_2^2 \dot{\phi}_2^2 + m_2 l_1 l_2 \dot{\phi}_1 \dot{\phi}_2 \cos(\phi_1 - \phi_2) \\ & + (m_1 + m_2)gl_1 \cos \phi_1 + m_2 gl_2 \cos \phi_2. \end{aligned} \quad (1.5)$$