Solutions Manual to Pattern Recognition and Machine Learning

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1 Introduction

1.1

Setting the derivative of

$$E(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\left(y(x_n,\mathbf{w}) - t_n\right)^2 \tag{1.1}$$

as zero gives

$$\mathbf{0} = \sum_{n=1}^{N}\frac{\partial y(x_n,\mathbf{w})}{\partial \mathbf{w}}\left(y(x_n,\mathbf{w}) - t_n\right). \tag{1.2}$$

Substituting

$$y(x_n,\mathbf{w}) = \sum_{j=0}^{M} w_j x_n^j \tag{1.3}$$

gives

$$0 = \sum_{n=1}^{N} x_n^i \left(\sum_{j=0}^{M} w_j x_n^j - t_n\right). \tag{1.4}$$

Therefore, we have

$$\sum_{j=0}^{M} A_{ij} w_j = T_i \tag{1.5}$$

where

$$\begin{aligned} A_{ij} &= \sum_{n=1}^{N} x_n^{i+j}, \\ T_i &= \sum_{n=1}^{N} x_n^i t_n. \end{aligned} \tag{1.6}$$

1.2

Setting the derivative of

$$\tilde{E}(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\left(y(x_n,\mathbf{w}) - t_n\right)^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2 \tag{1.7}$$

as zero gives

$$\mathbf{0} = \sum_{n=1}^{N} \frac{\partial y(x_n, \mathbf{w})}{\partial \mathbf{w}} (y(x_n, \mathbf{w}) - t_n) + \lambda \mathbf{w}. \tag{1.8}$$

Substituting

$$y(x_n, \mathbf{w}) = \sum_{j=0}^{M} w_j x_n^j \tag{1.9}$$

gives

$$0 = \sum_{n=1}^{N} x_n^i \left(\sum_{j=0}^{M} w_j x_n^j - t_n \right) + \lambda w_i. \tag{1.10}$$

Therefore, we have

$$\sum_{j=0}^{M} \tilde{A}_{ij} w_j = T_i \tag{1.11}$$

where

$$\begin{aligned} \tilde{A}_{ij} &= \sum_{n=1}^{N} x_n^{i+j} + \lambda \delta_{ij}, \\ T_i &= \sum_{n=1}^{N} x_n^i t_n. \end{aligned} \tag{1.12}$$

1.3

Let a, o l be the events where an apple, orange and lime are selected respectively. The probability that an apple is selected is given by

$$p(a) = \frac{1}{5}\frac{3}{10} + \frac{1}{5}\frac{1}{2} + \frac{3}{5}\frac{3}{10} = \frac{17}{50}. \tag{1.13}$$

If an orange is selected, the probability that it came from the geen box is given by

$$p(g|o) = \frac{p(g, o)}{p(o)}. \tag{1.14}$$

Since

$$\begin{aligned} p(g, o) &= \frac{3}{5}\frac{3}{10} = \frac{9}{50}, \\ p(o) &= \frac{1}{5}\frac{2}{5} + \frac{1}{5}\frac{1}{2} + \frac{3}{5}\frac{3}{10} = \frac{9}{25}, \end{aligned} \tag{1.15}$$

we have

$$p(g|o) = \frac{1}{2}. \tag{1.16}$$