Solutions Manual to Pattern Recognition and Machine Learning

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1 Introduction

1.1

To minimise

$$E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^{N} \left(y(x_n, \mathbf{w}) - t_n\right)^2, \tag{1.1}$$

setting its derivative as zero gives

$$\mathbf{0} = \sum_{n=1}^{N} \frac{\partial y(x_n, \mathbf{w})}{\partial \mathbf{w}} \left(y(x_n, \mathbf{w}) - t_n\right). \tag{1.2}$$

Substituting

$$y(x_n, \mathbf{w}) = \sum_{j=0}^{M} w_j x_n^j \tag{1.3}$$

gives

$$0 = \sum_{n=1}^{N} x_n^i \left(\sum_{j=0}^{M} w_j x_n^j - t_n \right). \tag{1.4}$$

Therefore, we have

$$\sum_{j=0}^{M} A_{ij} w_j = T_i \tag{1.5}$$

where

$$\begin{aligned} A_{ij} &= \sum_{n=1}^{N} x_n^{i+j}, \\ T_i &= \sum_{n=1}^{N} x_n^i t_n. \end{aligned} \tag{1.6}$$

1.2

To minimise

$$\tilde{E}(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^{N} \left(y(x_n, \mathbf{w}) - t_n\right)^2 + \frac{\lambda}{2} \|\mathbf{w}\|^2, \tag{1.7}$$

setting its derivative as zero gives

$$\mathbf{0} = \sum_{n=1}^{N} \frac{\partial y(x_n, \mathbf{w})}{\partial \mathbf{w}} (y(x_n, \mathbf{w}) - t_n) + \lambda \mathbf{w}. \tag{1.8}$$

Substituting

$$y(x_n, \mathbf{w}) = \sum_{j=0}^{M} w_j x_n^j \tag{1.9}$$

gives

$$0 = \sum_{n=1}^{N} x_n^i \left(\sum_{j=0}^{M} w_j x_n^j - t_n \right) + \lambda w_i. \tag{1.10}$$

Therefore, we have

$$\sum_{j=0}^{M} \tilde{A}_{ij} w_j = T_i \tag{1.11}$$

where

$$\begin{aligned} \tilde{A}_{ij} &= \sum_{n=1}^{N} x_n^{i+j} + \lambda \delta_{ij}, \\ T_i &= \sum_{n=1}^{N} x_n^i t_n. \end{aligned} \tag{1.12}$$

1.3

Let a, o and l be the events where an apple, orange and lime are selected respectively. The probability that an apple is selected is given by

$$p(a) = p(a|r)p(r) + p(a|b)p(b) + p(a|g)p(g). \tag{1.13}$$

Substituting $p(a|r) = \frac{3}{10}$, $p(r) = \frac{1}{5}$, $p(a|g) = \frac{1}{2}$, $p(r) = \frac{1}{5}$, $p(a|g) = \frac{3}{10}$ and $p(g) = \frac{3}{5}$ gives

$$p(a) = \frac{17}{50}. \tag{1.14}$$

If an orange is selected, the probability that it came from the geen box is given by

$$p(g|o) = \frac{p(g, o)}{p(o)}. \tag{1.15}$$

Here,

$$\begin{aligned} p(g,o) &= p(o|g)p(g), \\ p(o) &= p(o|r)p(r) + p(o|b)p(b) + p(o|g)p(g). \end{aligned} \tag{1.16}$$

Substituting $p(o|r) = \frac{2}{5}$, $p(r) = \frac{1}{5}$, $p(o|b) = \frac{1}{2}$, $p(b) = \frac{1}{5}$, $p(o|g) = \frac{3}{10}$ and $p(g) = \frac{3}{5}$ gives $p(g,o) = \frac{9}{50}$ and $p(o) = \frac{9}{25}$. Therefore,

$$p(g|o) = \frac{1}{2}. \tag{1.17}$$

1.5

By the definition,

$$\mathrm{var} f(x) = \mathrm{E}\left(f(x) - \mathrm{E}f(x)\right)^2. \tag{1.18}$$

The right hand side can be written as

$$\mathrm{E}\left((f(x))^2 - 2f(x)\mathrm{E}f(x) + (\mathrm{E}f(x))^2\right) = \mathrm{E}\left(f(x)\right)^2 - \left(\mathrm{E}f(x)\right)^2. \tag{1.19}$$

Therefore,

$$\mathrm{var} f(x) = \mathrm{E}\left(f(x)\right)^2 - \left(\mathrm{E}f(x)\right)^2. \tag{1.20}$$

1.6

By the definition,

$$\mathrm{cov}(x,y) = \mathrm{E}xy - \mathrm{E}x\mathrm{E}y. \tag{1.21}$$

Here,

$$\mathrm{E}xy = \int xyf(x,y)dxdy \tag{1.22}$$

If x and y are independent, by the definition

$$f(x,y) = f(x)f(y), \tag{1.23}$$

then

$$\int xyf(x,y)dxdy = \int f(x)dx \int f(y)dy. \tag{1.24}$$

Therefore,

$$\mathrm{E}xy = \mathrm{E}x\mathrm{E}y, \tag{1.25}$$

and thus

$$\mathrm{cov}(x,y) = 0. \tag{1.26}$$