Shaheed Zulfikar Ali Bhutto Institute of Science & Technology University

DEPARTMENT OF ROBOTICS & ARTIFICIAL INTELLIGENCE

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Total Marks: 04

Obtained Marks: _____

Differential Equation

Assignment # 01

Last Date of Submission: 01 March 2025

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DEPARTMENT OF ROBOTICS & ARTIFICIAL INTELLIGENCE

***Instructions**: Copied or shown assignments will be marked zero. Late submissions are not entertained in any case.*

CLO 1 – CX – PLO X

Q1: *(Marks 4)*

Solve the Separable differential equation

$$(xy + 2x + y + 2)dx + (x^2 + 2x)dy = 0$$

Solutions

Solution: (xy+2x+y+2)dx+(x2+2x)dy = 0

(xy+2x+y+2)dx = -(x2+2x)dy

(xy+2x+y+2)dx = -x(x+2)dy

(x+1)(y+2)dx = -x(x+2)dy

(y+2)dx / [x(x+2)] = -dy / (x+1) (y+2)

[A/x + B/(x+2)]dx = -dy / (x+1) 1 = A(x+2) + Bx

When x = 0:

1 = 2A → A = 1/2

When x = -2:

1 = -2B

B = -1/2

$(y+2)[1/(2x) - 1/(2(x+2))]dx = -dy / (x+1)$

Apply Integration

$\int(y+2)[1/(2x) - 1/(2(x+2))]dx = -\int dy / (x+1)$

$(y+2)[(1/2)\ln|x| - (1/2)\ln|x+2|] = -\ln|x+1| + C$

$(y+2)[(1/2)\ln|x/(x+2)|] = -\ln|x+1| + C$

$(y+2)[(1/2)\ln|x/(x+2)|] + \ln|x+1| = C$

Therefore, $(y+2)[(1/2)\ln|x/(x+2)|] + \ln|x+1| = C$ is the solution.

Q2: Solve the following differential equation

(a) $y(1+x)dx + x(1+y)dy = 0$

Solution

Solution:

$y(1+x)dx+x(1+y)dy=0$

$y(1+x)dx + x(1+y)dy = 0$

$y(1+x)dx = -x(1+y)dy$

$dx/x + dx/(x(1+y)) = -dy/y - dy/(y(1+x))$

Apply Integration

$\int dx/x + \int dx/(x(1+y)) = -\int dy/y - \int dy/(y(1+x))$

$\ln|x| + \ln|1+y| = -\ln|y| - \ln|1+x| + C$

$\ln|x| + \ln|1+y| + \ln|y| + \ln|1+x| = C$

$\ln|xy(1+x)(1+y)| = C$

xy(1+x)(1+y) = K, where K = e^C

(b) $\frac{dy}{dx} = \frac{y \sin x}{(1+2y^2)}$

Solution

Solution: dy/dx=(y sin x)/((1+2y^2))

dy/dx = (y sin x)/(1+2y^2)

(1+2y^2)dy = y sin x dx

(1/y + 2y)dy = sin x dx

Apply Integration

∫(1/y + 2y)dy = ∫sin x dx

ln|y| + y^2 = -cos x + C

ln|y| + y^2 = -cos x + C

Therefore, ln|y| + y^2 = -cos x + C is the solution.

(c) $\mathbf{2(y-1)dy = (3x^2 + 4x + 2)dx}$

Solution

2(y-1)dy=(3x^2+4x+2)dx

2(y-1)dy = (3x^2+4x+2)dx

Apply Integration

∫2(y-1)dy = ∫(3x^2+4x+2)dx

2(y^2/2 - y) = x^3 + 2x^2 + 2x + C

y^2 - 2y = x^3 + 2x^2 + 2x + C

(y-1)^2 = x^3 + 2x^2 + 2x + C

(y-1)^2 = x^3 + 2x^2 + 2x + K, where K = C + 1

Therefore, (y-1)^2 = x^3 + 2x^2 + 2x + K is the solution.

End