1. 개요

● 목차

1) 트리 정의 및 표현
   - Definitions
   - 3 Reresentation: Adjacency Matrix, Adjacency List, Adjacency M-list

2) 그래프 기본 연산 (Elementary Graph Operation)) LAB
   - Depth First search, Breadth First search

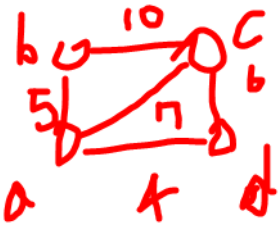
3) Minimum Cost Spanning Tree (MST)) HW
   - Kruskal, Prim, Sollin

4) Shortest Path (single source all destination)
   ex) NAVI

v = n
E = n-1

gragh

v = n
E = n



여기에 gragh 알고리즘 적용

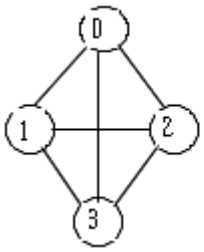
가장 최단거리 찾는 알고리즘

1.1 정의

A graph, G, consists of two sets, a finite set of vertices and a finite set of edges.

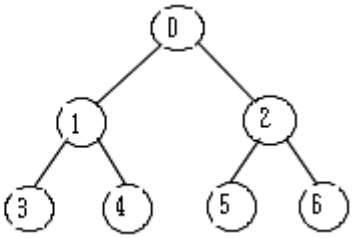
. G = (V, E) V: set of vertex E: Set of edges

. Undirected Graph (무방향): (v1, v2) = (v2, v1) G1, G2
. Directed Graph (방향) : <v1, v2> ≠ <v2, v1> G3



G1 not tree

정점 2의 차수 : 3
전체 차수 : 12, 선의개수 :6



G2 tree

정점 0의 차수 2
전체 차수 : 12, 선의 개수 : 6

화살표 -> 방향성이 있따.

G3 not tree

전체 차수 : 6, 선의 개수 : 3

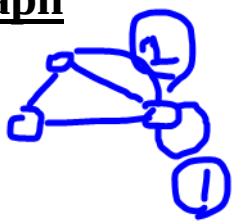
V(G1) = {0,1,2,3}
V(G2) = {0,1,2,3,4,5,6}
V(G3) ={0,1,2,}

E(G1) = {(0,1)(0,2)(0,3)(1,2)(1,3)(2,3)}
E(G2)={(0,1)(0,2)(1,3)(1,4)(2,5)(2,6)}
E(G3) = {<0,1><1,0><1,2>}

무방향은 () , 방향은 <>

- Restriction on a graph: **no self-loop, and no Multigraph**

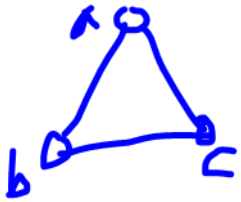
self-loop : 자기 자신을 가리키는 간선
multigraph: 두 정점간에 여러 간선있는 graph

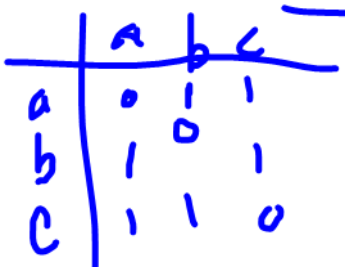


- Complete Graph 란? →

the maximum number of edges 를 갖는 그래프
(ex. G1 is a complete Graph, G2, G3 is not complete graph)

⇨ For undirected graph max number of edges => **n(n-1)/2**
⇨ For directed graph, max number of edges => **n(n-1)**



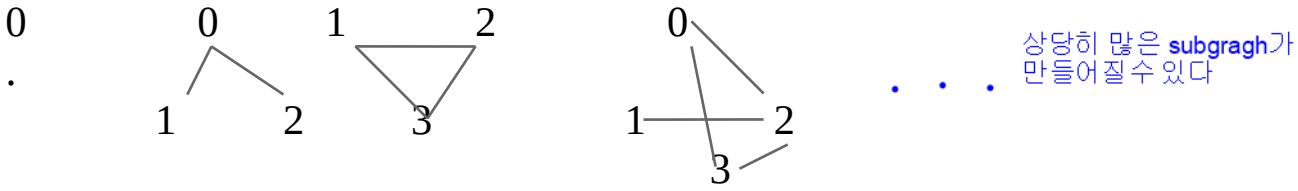


- If (0,1) is an edge in undirected graph, then vertices 0 and 1 are **ADJACENT**,
  and the edge (0,1) is **INCIDENT** on vertices 0 and 1

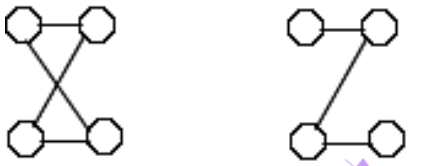
  ex) In Graph G2, vertices 3,4,0 are adjacent to vertex 1 and edges (0,1), (1,3), (1,4) are incident on vertex 1

- Subgraph: A subgraph of graph G is G', such that
  $V(G') \subseteq V(G)$ & $E(G') \subseteq E(G)$

ex) in Graph G1, many **subgraphs**, such as



- Cycle and Path



Cycle path =3

- Path(경로) : 정점(간선)들의 연속
  - 경로의 길이 : 경로상의 간선 수
  - Simple PATH(단순경로) : 서로 다른 정점으로 구성된 경로

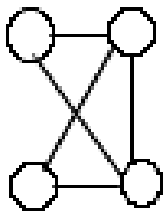
- CYCLE: (처음과 끝 정점이 같은 단순경로)

Cycle is a simple path in which the FIRST and LAST vertices are the same vertex

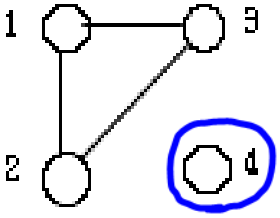
- CONNECTED: 연결(*connected*) 그래프, G

  vi, *vj* ∈ V(G) ⇒ *vi* 에서 *vj* 로의 경로 존재

모든 정점들이 연결되어 있기 때문에



Connected



Disconnected

- Connected Component: number of subgraphs

  ex) G4 has two components H1 and H2

  * Diff with Tree and Graph (트리와 그래프의 차이점)

  1) Tree is special case of graph
  2) Tree is a graph that is connected
  3) Tree is a graph that has no cycle

  (ex. G1, G3 is not Tree, G2 is tree)

차수

- DEGREE: number of edges incident to that vertex

  . Undirected Graph :

  . Directed Graph : (indegree, outdegree)

  In G3, vertex 1: indegree 1, outdegree 2 ➔ degree 3.

  If d_i is degree of vertex ***i*** in G, with n vertices and ***e*** edges:

  ⇨ number of edges : $e = \left(\sum_{0}^{n-1} d_i \right) / 2$

1.2 Graph Representation (3 가지 표현방법)

1)인접행렬 (Adjacent matrix)
2)인접리스트 (Adjacent list)
3)인접다중리스트 (Adjacent multilist)

1) Adjacency Matrix

G1

	1	2	3	4
1	0	1	1	1
2	1	0	1	1
3	1	1	0	1
4	1	1	1	0

. Undirected Graph: **Symmetric**

. can save space by storing only upper/lower triangle of matrix

. Degree of any vertex = ~~low sum~~ row sum

G3

	1	2	3
1	0	1	0
2	1	0	1
3	0	0	0

. Directed Graph : **Not symmetric**

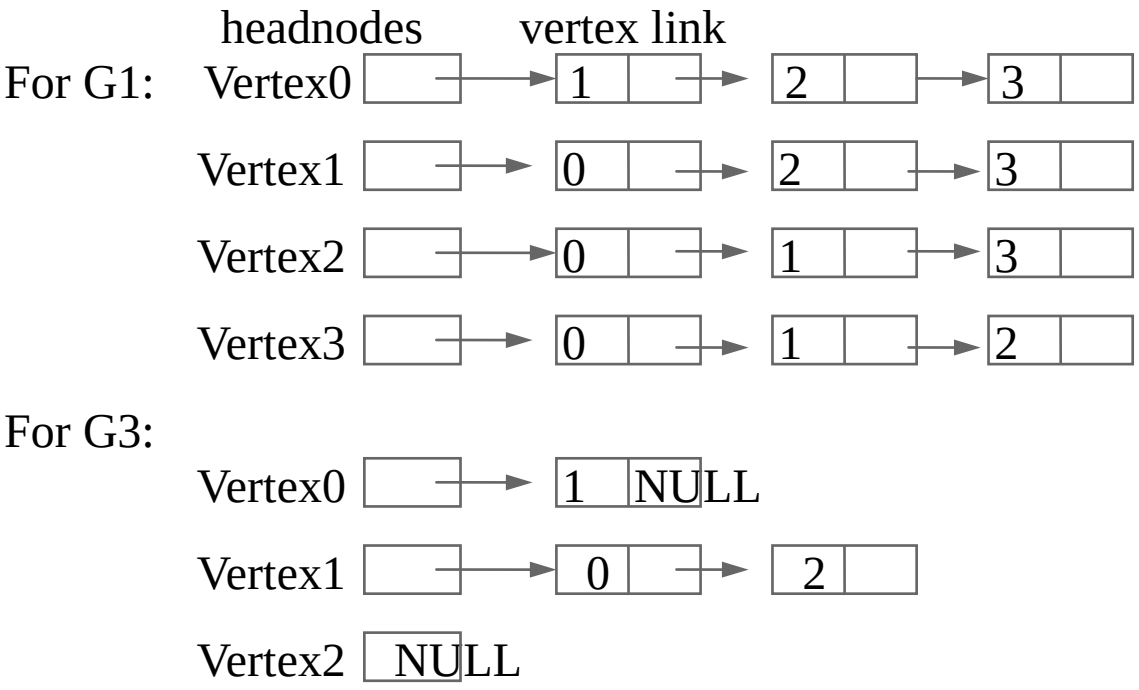
. Degree of vertex: row sum -> outdegree
col sum -> indegree

- **Space complexity = $O(n^2)$**
- Sparse graphs 란?: 간선의 개수가 적은 그래프를 뜻함
- sparse graph 를 adjacency matrix 로 표현하면 memory waste 임. adjacency list 가 적합함.

2) Adjacent List

space complexity = O(n+e)// e=edges

Replace n rows of adjacency matrix with n linked list, one for each vertex in G (각 정점에 대해 1 개의 리스트 존재)



3) Adjacent Multilist

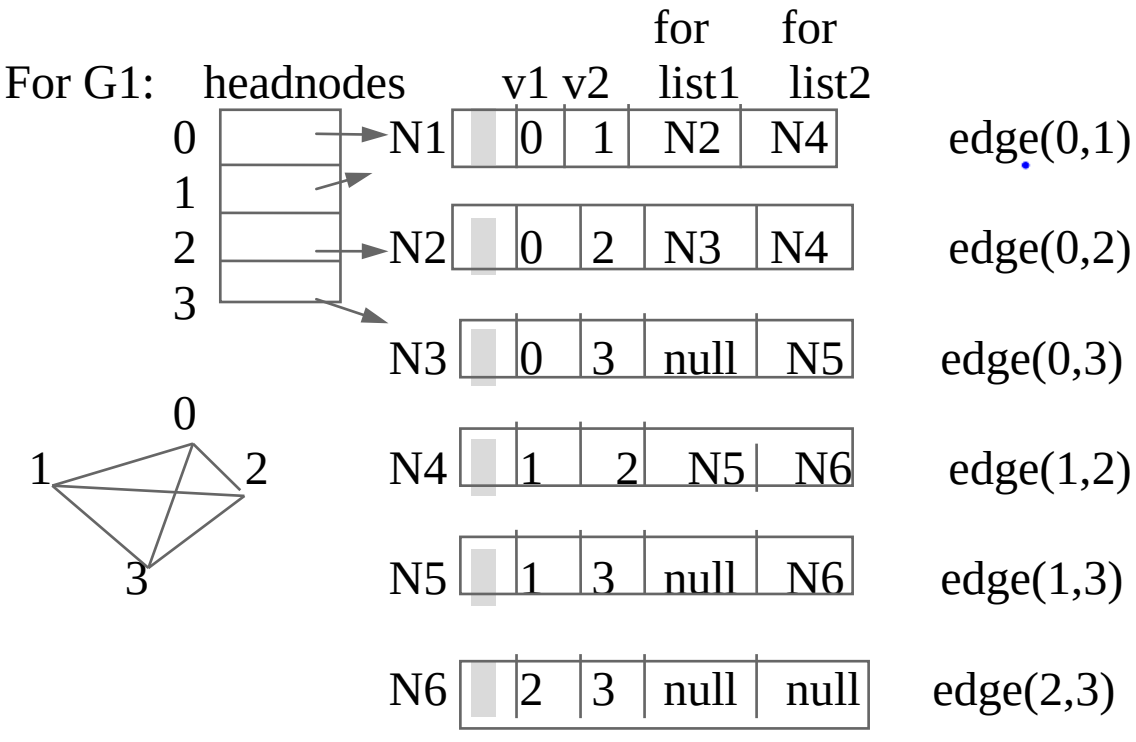
인접리스트에서는 각 간선이 두 번 표현되었음 (예: (1,0))

=> Multilist 로 해결 가능

- Node structure

marked	vertex1	vertex2	path1	path2

```
typedef  struct  edge *edge-ptr;
typedef  struct  edge {
            shortint marked;
            int vertex1;
            int vertex2;
            edge-ptr path1;
            edge-ptr path2;    };
edge-ptr  graph[max];
```



The lists are: vertex0: N1->N2->N3
vertex1: N1->N4->N5
vertex2: N2->N4->N6
vertex3: N3->N5->N6

2. Elementary Graph Operations

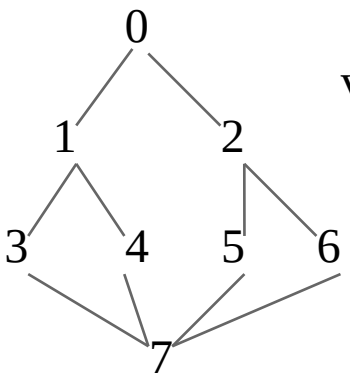
1) DFS (Depth First Search): 깊이 우선탬색

. use Adjacency linked list

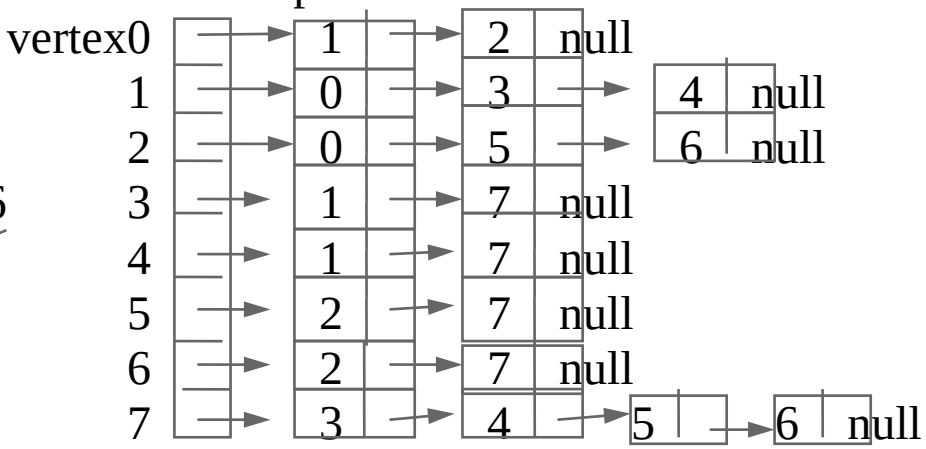
. *visited*[*MAX_VERTICES*] : 배열 (초기치 = FALSE)

. *visited*[*i*] = TRUE : 정점 *i* 방문

```
void DFS(int v)              . 시작정점 v 방문 (visited[v]=true)
  {                          . For each vertex W adjacent to v do
    Node   *w;                     if not visited[W] then DFS(W);
    visited[v] = true;       . 더이상 없으면 dfs 끝
    cout << v;
   for (w= graph[v]; w!=NULL; w=w->link)
      if (!visited[w->link])   DFS(w->link);

   }
  }
```



A. list representation:



Start from V_0,: 0, 1, 3,7,4,5,2,6

visited

0	1	2	3	4	5	6	7
T	T		T	T	T		T

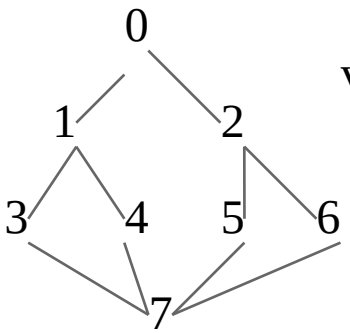
- Analysis: total time of DFS by A. List: O(e),
  by A. matrix: $O(n^2)$

2) BFS (Breadth First Search) . Implement with Linked Queue

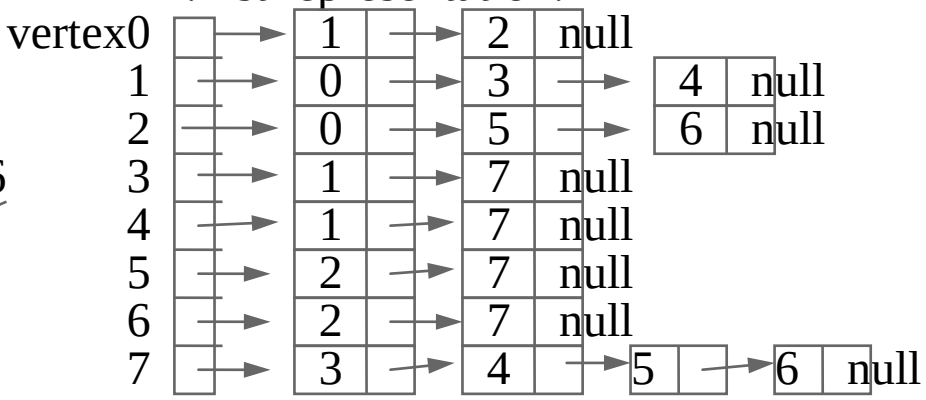
. **void Graph::BFS(int v) {**

Node p;

```
visited[v] = true;          addq(Q, v);
cout<< v;

while (front) {
   v = dequeue();
   for (p=graph[v]; p; p=p->next;   //인접된 모든 노드에 대해
      if (!visited[p->vertex]) {        //if not visited
            eneuque(p->vertext);
            visited[w] = true;  마킹
            cout<<p->vertex;
      }}}}
```



A. list representation:

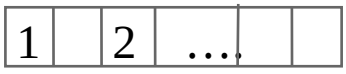


Start from V_0,: 0, 1, 2,3,4,5,6,7

visited

0	1	2	3	4	5	6	7
T	T	T	T	T			T

Q



- Analysis of BFS: same as DFS

- Sample Code

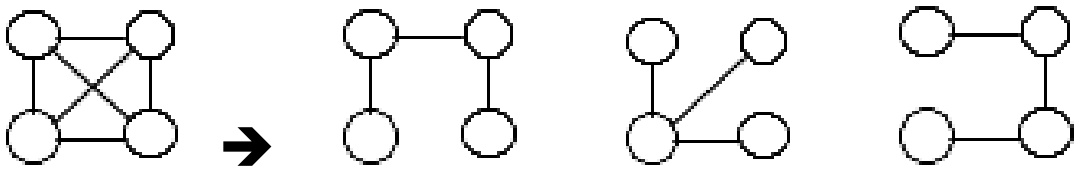
```
class node {
        int vertex;          node *next;
        node(int num)    { vertex = num;      next = 0;}
    friend class Graph;
};

class Graph {
        private:
            node *graph[MaxVertices];
            bool    visited[MaxVertices];
            node    *front;      node *rear;
        public:
            Graph() {front = 0; rear = 0;}
            void initGraph();
            void insertGraph(int num1, int num2);
            void displayGraph();
            void enqueue(int v);
            int dequeue();
            void bfs(int v);
    };

void Graph::initGraph()
{    for (int i = 0; i < MaxVertices; i++) {
                graph[i] = 0;      visited[i] = false;      }      }

void Graph::enqueue(int v)
{      node *temp = new node(v);      ……..
    }

int Graph::dequeue()
{      node *p;      int vertex;      .....
    }
```

3) SPANNING TREES (신장트리)

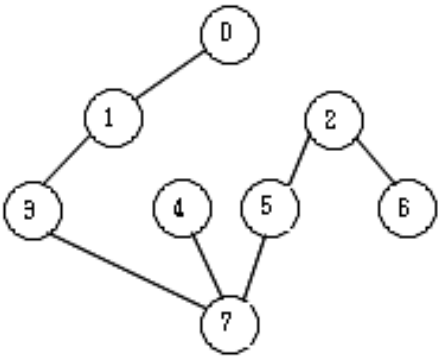
상당히 중요
응용분야가 상당함

Definition: any tree that includes all the vertices in G

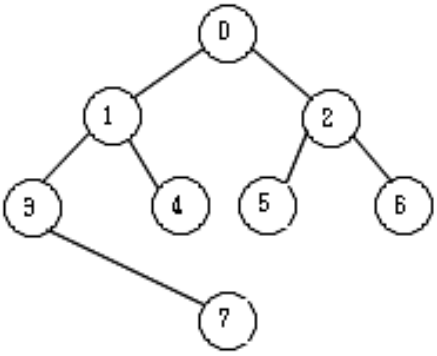
예) 하나의 연결 G 는 출발점이나 검색방법에 따라 각기 다른 신장 트리가 만들어진다.



- We can use dfs or bfs to create a spanning tree
  - when dfs is used => the result is dfs spanning tree
  - when bfs is used => the result is bfs spanning tree



(DFS Spanning Tree)



(BFS Spanning Tree)

- **Greedy Method**

문제를 해결하는 각 단계에서 가장 최선의 방법을 결정하는 것으로 optimal solution 을 구할 수 있다. (best solution 이 아닐 수도 있다)

예) 현재, 11¢, 5¢, 1¢ 가 있는데 15¢ 만들기
➔ 1-11¢, 4-1¢ 로 만들 수 있다.
그러나 실제로는 3-5¢ 로 만들 것이다.

3. Minimum cost spanning trees(MST)

▶ cost가 제일 적은 신장트리 최소비용 알고리즘 반드시 알고있어야됨

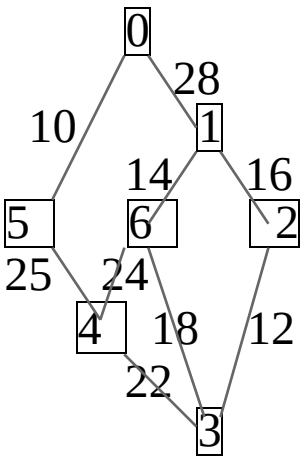
▶ Greedy Method 의 응용 예 ▶ 총 (n-1) edges 이다

- 대표적인 MST algorithms
  - ⇨ Kruskal's, Prim's, Sollin's

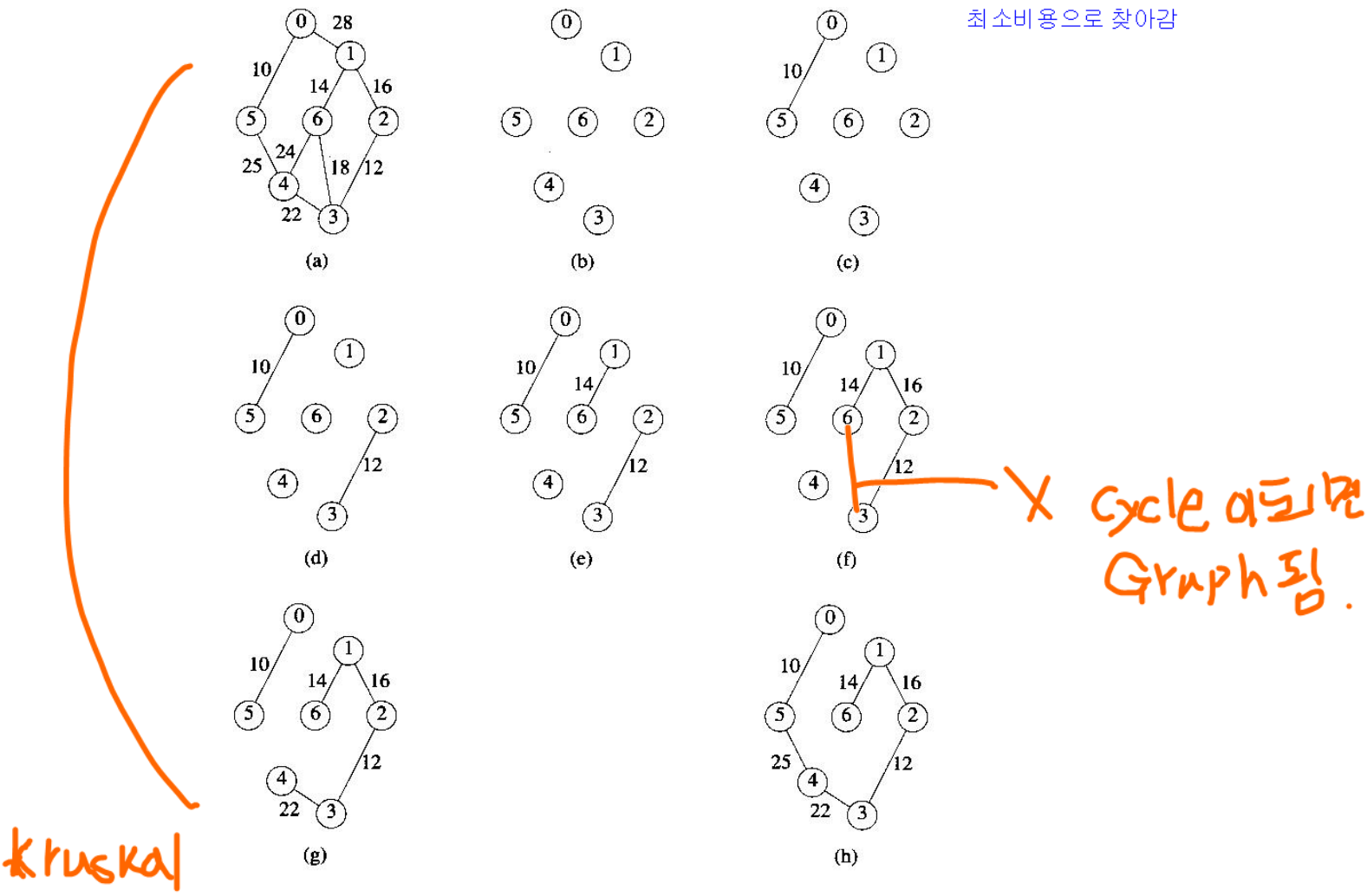
1) Kruskal's Algorithm (Greedy method)

tree = n & n-1

```
T = { };
while (T contains <n-1 edges) & (E is not empty) {
    choose a least cost edge (v,w) from E;
    delete (v,w) from E;
    if ((v,w) does not create a CYCLE in T)
            add(v,w) to T      => ACCEPT
    else    discard(v,w);      => REJECT
}
If T contains fewer than n-1 edges than
     printf("No spanning tree");
```



cost	edge	action
10	(0,5)	accept
12	(2,3)	accept
14	(1,6)	accept
16	(1,2)	accept
18	(3,6)	reject => cycle
22	(3,4)	accept
24	(4,6)	reject => cycle
25	(4,5)	accept
28	stop	already (n-1) edges added



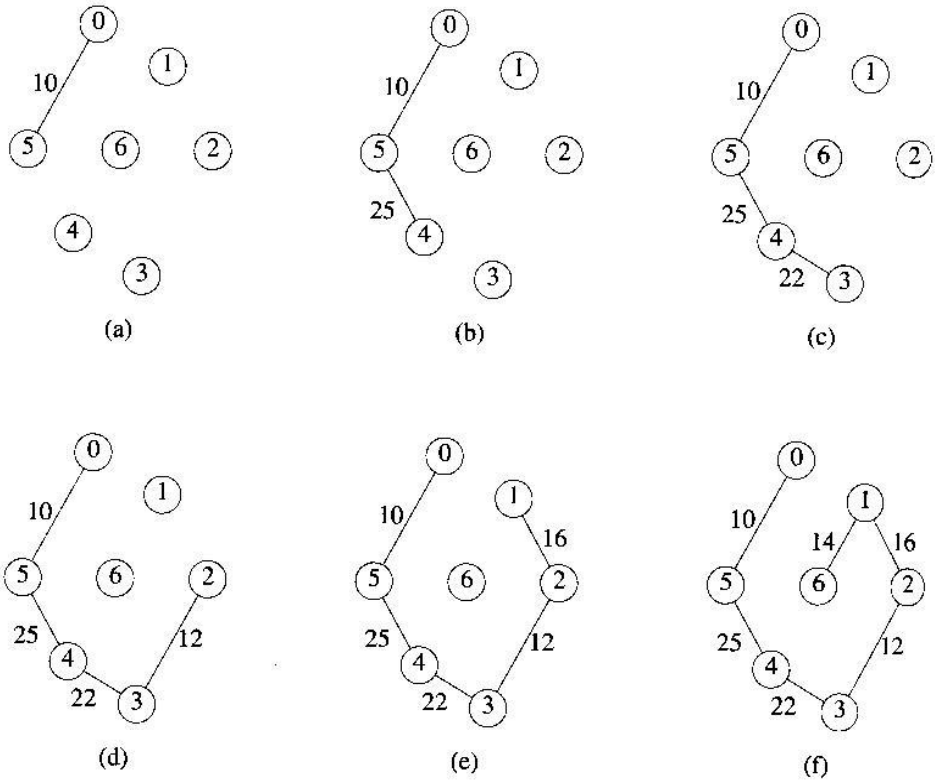
2) Prim's Algorithm- 각 단계에서 선택된 간선의 집합 = 트리

Prim's algorithm form a tree at each stage, but Kruskal's form a forest

Algorithm : {can start with any vertex}

```
T = { };
TV={0} //start with vertex 0//
while (T contains fewer than n-1 edges) {
    let (u,v) be a least cost edge such that u∈TV & v∉TV;
    if (there is no such edge) break;
    else add v to TV;
         add (u,v) to T;
}
if (T contains fewer than n-1 edges)
         printf ("No spanning Tree");
```

ex) starting vertex '0'



3) Sollin's Algorithm 각 단계별, T에 포함될 간선을 여러개 선택

(i) 그래프의 모든 n 정점을 포함하는 신장트리 구성

(ii) forest 내의 각 트리에 대해 하나의 간선 선택, (최소비용선택)

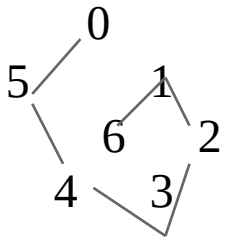
ex) 최소 0 / 1 / 5 6 2 / 4 / 3 => 0 : (0,5) 1 : (1,6) 2: (2,3) 3: (3,2) 4: (4,3) 5: (5,0) 6: (6,1) => 0 / 5 1 / 6 2 / 4 3 forest => (0,5) (1,6) (2,3) (4,3)

. Tree{0,5}:=> (1,0),(4,5), will select (4,5) since cost is 25 (minimum)

. Tree{1,6}: => (1,2), (6,3), will select (1,2) since cost is 16 (min)

. Tree{2,3,4}:=> (2,1),(3,6)(4,6), will select (2,1) since cost is 16 (min)

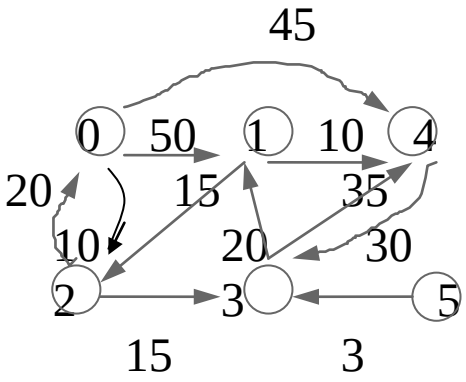
⇨ 결과



4. Shortest Path (최단경로)

1) Single Source All Destination (단일 출발점-> 모든 도착지)

- *v*0(*source*)에서 G 의 다른 모든 정점(도착지)까지의 최단경로



path	length
1) v0 v2	10
2) v0 v2 v3	25
3) v0 v2 v3 v1	45
4) v0 v4	45

< shortest path from v0 to v1,v2,v3,v4>
<no path from v0 to v5>

* *found*[*i*] : if *found*[*i*]=TRUE *v*i 까지의 최단경로 발견
distance[*i*] : *v*0 에서 S 내의 정점만을 거친 *v*i 까지의 최단거리
(S=최단경로가 발견된 정점의 집합)

- 초기치 : *distance*[*i*] = *cost*[0][*i*] — 그래프 전체의 초기값
- *cost*[*i*][*j*] : <*i*, *j*>의 가중치

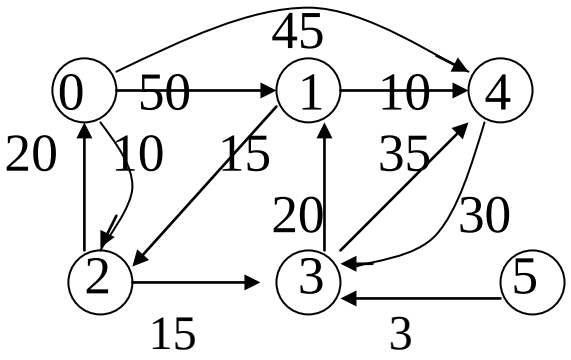
* 그래프 : 비용 인접 행렬(*cost adjacency matrix*)로 표현

```
void initCostMatrix(int cost[][8])   {
int I, j;


for (i = 0; i < 8; i++)
   for (j = 0; j < 8; j++)
         if (i == j)   cost[i][j] = 0;
         else          cost[i][j] = 10000;
}
```

- Algo (Shortest path) by Dijkstra's algorithm

```
Void Shortestpath (int v, int cost[][], int dist[], int n, bool found[]) {
      int I,u,w;
      for (I=0; I<n; I++)    {                    . O(n)
           found[I] = false;                      . found all FALSE
           distance[I] = cost[v,I]; }            . initial value assign

      found[v]=true;                  // start vertex mark
      distance[v]=0;                  // start vertex 0

      for (I=0; I<n-2; I++)    {
         u = choose(distance, n, found);          // find min value node
         found[u]= true;                           // mark that node

         for (w =0; w<n; w++)              // and replace if revised value
핵심       if (!found[w])                  // if not marked
            if (distance[u]+cost[u,w]<distance[w])    //is smaller than org
             distance[w] = distance[u] + cost[u,w];      // value
} }

int choose(int dist[], int n, bool found[])
{     int i, min, minpos;

      min = INT_MAX;
      minpos = -1;

    for (i = 0; i < n; i++)
          if (dist[i] < min && !found[i]) {
                min = dist[i];
                minpos = i;
          }
    return minpos;
}
```

시험문제 ★★★★



0→2→3→1→4

	0	1	2	3	4	5
0		50	10		45	
1			15		10	
2	20			15		
3		20			35	
4				30		
5				3		

비용 인접 행렬 - *cost*

Vertex	0	1	2	3	4	5
Distance	0	50	10	999	45	999
S	1	0	0	0	0	0

1. S = {*v*0} : 초기는 공백

distance(1) = 50
distance(2) = 10 <= *min*
distance(3) = 999 임의로 큰수 넣어놓음
distance(4) = 45
distance(5) = 999

Vertex	0	1	2	3	4	5
distance	0	50	10	999	45	999
S	1	0	1	0	0	0

2. S = S ∪ {*v*2} = {*v*0, *v*2}

distance(1)<- min{distance(1), distance(2)+(*v*2,*v*1,999)} 50
distance(3)<-min{distance(3), distance(2)+(*v*2,*v*3,15)} 25 <= *min*
distance(4)<- min{distance(4), distance(2)+(*v*2,*v*4,999)} 45
distance(5)<- min{distance(5), distance(2)+(*v*2,*v*5,999)} 999

vertex	0	1	2	3	4	5
distance	0	50	10	25	45	999
S	1	0	1	1	0	0

3. S = S ∪ {v3} = {v0, v2, v3}

25

distance(1)<- min{distance(1), distance(3)+*v*3,*v*1,20}} 45 <= *min*

distance(4)<- min{distance(4), distance(3)+(*v*3,*v*4,35)} 45

앞에꺼 먼저선택해도 무관

distance(5)<- min{distance(5), distance(3)+(*v*3,*v*5,999)} 999

Vertex	0	1	2	3	4	5
Distance	0	45	10	25	45	999
S	1	1	1	1	0	0

4. S = S ∪ {*v*1} = {*v*0, *v*1, *v*2, *v*3}

distance(4)<-min{distance(4), distance(1)+(*v*1,*v*4,10)} 45<= *min*

distance(5)<- min{distance(5), distance(1)+(*v*1,*v*5,999)} 999

Vertex	0	1	2	3	4	5
distance	0	45	10	25	45	999
S	1	1	1	1	1	0

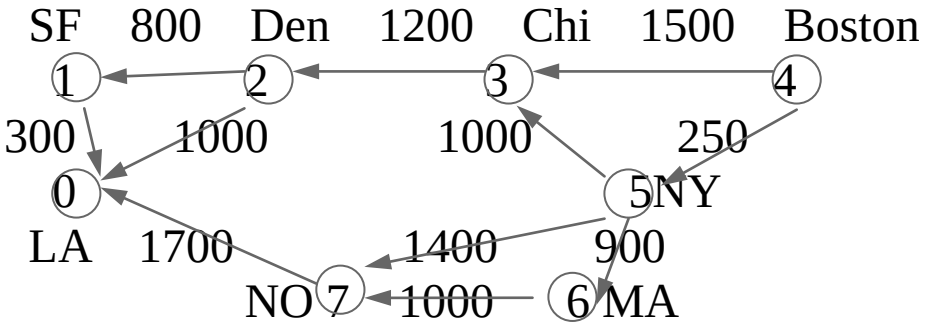
5. S = S ∪ {*v*4}

distance(5) <-min{distance(5), distance(4)+(*v*4,*v*5,999)} 999 <= *min*

Vertex	0	1	2	3	4	5
Distance	0	45	10	25	45	999
S	1	1	1	1	1	1

6. S = S ∪ {*v*5}

Ex2)



Matrix (cost[v,I])		0	1	2	3	4	5	6	7
	0	0	∞	∞	∞	∞	∞	∞	∞
	1	300	0						
	2	1000	800	0					
	3			1200	0				
	4				1500	0	250		
	5				1000		0	900	1400
	6							0	1000
	7	1700							0

- Found: | F | F | F | F | F | F | F | F | false initially

- Distance: | 0 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 1st iteration

Iteration	visited	vertex selected	Distance LA [0]	SF [1]	DEN [2]	CHI [3]	BOS [4]	NY [5]	MA [6]	NO [7]
Initial	**-**	**-**	∞	∞	∞	**1500**	**0**	**250**	∞	∞
1	**4**	**5**	∞	∞	∞	**1250**	**0**	**250**	**1150**	**1650**
2	**4,5**	**6**	∞	∞	∞	**1250**	**0**	**250**	**1150**	**1650**
3	**4,5,6**	**3**	∞	∞	**2450**	**1250**	**0**	**250**	**1150**	**1650**
4	**4,5,6,3**	**7**	**3350**	∞	**2450**	**1250**	**0**	**250**	**1150**	**1650**
5	**4,5,6,3,7**	**2**	**3350**	**3250**	**2450**	**1250**	**0**	**250**	**1150**	**1650**
6	**4,5,6,3,7,2**	**1**	**3350**	**3250**	**2450**	**1250**	**0**	**250**	**1150**	**1650**
	{4,5,6,3,7,2,1}									

* 250 + 1000 < 1500

if (distance[u] + cost[u,w] <distance[w]) // is smaller than org

distance[w] = distance[u] + cost[u,w]; // value

1250 250 + 1000

therefore CHI has been changed to 1250 thereafter

but this do not change