Assignment 3

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Aim:-

Implement Gradient Descent algorithm to find the local minima of a function. For example, find the local minima of the function $y = (x+3)^2$, starting from the point $x = 2$.

Requirements:-

Jupyter notebook, python libraries - Numpy, Pandas, sklearn.

Theory:-

Gradient Descent algorithm-
Gradient descent is an optimization algorithm used to minimize a function by adjusting its parameters iteratively.
It is widely used in ML, particularly for training a ML model, including neural networks.
The primary idea behind Gradient Descent is to find the minimum or maximum of a function by taking steps in the direction of the steepest decrease (negative gradient) of the function.

Working of Gradient Descent algorithm-

1) Initialization-
Start with an initial guess for the model parameters. This can be random or set

to specific values.

2) Compute the gradient -
Calculate the gradient of the function with respect to the parameters. The gradient is a vector that points in the direction of the steepest increase in the function.

3) Update parameters -
Adjust the parameters by moving in the opposite direction of the gradient to minimize the function.

Local Minima :-
Local minima are the points within a function's domain, where the function reaches its lowest value but only within a limited, local neighbourhood.
A local minima is lower than the function's value at nearby points, but it may not be absolute lowest point in the entire function.

Local minima are significant in optimization problems as they can mislead optimization algorithm.

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Finding local minima of function $y = (x+3)^2$ –

In Gradient Descent, we update the value of x in the direction that decreases the function y most rapidly.
The update rule is as follows –

$$x_{new} = x_{old} - n \nabla y$$

Here,

x_{new} = updated value of x.
x_{old} = current value of x.
n = learning state, a small +ve value that controls step size.
∇y = gradient of the function y.

In our case,

$$\nabla y = 2(x+3)$$

Now,
we can perform Gradient Descent steps :

1) Initialize:
$x_{old} = 2$ & learning rule, $n = 0.1$

2) Compute the gradient :
$$\nabla y = 2(x_{old} + 3)$$

3) Update:
$$x_{new} = x_{old} + n . \nabla y.$$

Repeat steps 2 & 3 until convergence, where you might stop when change in x becomes very small.

First Iteration:

$$x_{new} = 2 - 0.1 \times 2(2+3)$$
$$= 2 - 0.1 \times 10$$
$$= 2 - 1$$
$$x_{new} = 1$$

$\therefore$ After first iteration, $x_{new} = 1$.

We continue these steps iteratively until convergence to find local minimum.

After several iteration, we find that local minimum is at $x \approx -3$.

Conclusion:-

Hence, we implemented Gradient Descent algorithm to find local minima of the function $y = (x+3)^2$ starting from the point $x = 2$.