Classical Mechanics: From Newton to Euler-Lagrange

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NOTE: These notes are a summary of Classical Mechanics: A Theoretical Minimum. They also contain some of David Morrin. Some of the notebooks are exercises I did through my own research.

NOTE: Note this "summary" is NOT a reproduction of the course materials nor is it copied from the corresponding courses. It was entirely written and typeset from scratch.

Hey, it is me, the author of these notes. I used to think reading physics was pointless practically speaking, so I did it a bit sparingly. But then.. I actually needed it in my own research! So I'm citing my work here [1]. Hopefully it encourages more academics to read physics at a moderate to advanced level, as you never know when you'll need it!

1 Lagrange

format

We use the Lagrangian method to derive the dynamics of the system [**?**, Chapter 6]. First, we define the position x_b, z_b of the mass m_b and its velocity $\dot{x}_b, \dot{z}_b$:

$$\begin{aligned} x_b &= x + l\ \sin(\theta) \\ z_b &= z + l\ \cos(\theta) \\ \dot{x}_b &= \dot{x} + \dot{l}\ \sin(\theta) + l\ cos(\theta)\ \dot{\theta} \\ \dot{z}_b &= \dot{z} + \dot{l}\ \cos(\theta) - l\ sin(\theta)\ \dot{\theta} \end{aligned} \tag{1}$$

Lagrange's method states that for a system with total kinetic energy T and potential energy U:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}_i} - \frac{\partial \mathcal{L}}{\partial \mathbf{q}_i} = \mathbf{f}_{ext}, \tag{2}$$

where $\mathcal{L} = T - U$ is the system's Lagrangian and $\mathbf{f}_{ext}$ are external forces applied to the system. We now need to compute the system's kinetic and potential energy.

In general, every link will have a rotational and translational kinetic energy component. For the wheel we include a rotational kinetic energy term $I_w\dot{\phi}^2$ where $I_w = m_w R_w^2$ is the moment of inertia of the wheel. Since the point mass has a zero moment of inertia, it only has a translational kinetic energy $m_b \mathbf{v}_b^T \mathbf{v}_b$, where $\mathbf{v}_b$ is the velocity of the point mass.

The only potential energy component is due to gravity g acting on the wheel and the point mass. This leads to:

$$T = \frac{1}{2}(I_w \ \dot{\phi}^2 + m_w \dot{x}^2 + m_w \dot{z}^2 + m_b \dot{x}_b^2 + m_b \dot{z}_b^2) \tag{3}$$

$$U = m_w g z + m_b g z_b \tag{4}$$

$$\mathcal{L} = T - U \tag{5}$$

References

[1] T. Dinev, S. Xin, W. Merkt, V. Ivan, and S. Vijayakumar, "Modeling and control of a hybrid wheeled jumping robot," *arXiv preprint arXiv:2003.01440*, 2020.