



Unit: V – Numerical Assignment

1. Let $\phi = E[p \cup (q \wedge EX r)]$. Show how this formula can be broken down using fixed point expressions. Explain how nested fixed points can be evaluated in model checking.
2. Assume you have a finite Kripke structure with 5 states and transitions forming a DAG (no cycles). Design an algorithm to compute EF ϕ using fixpoint iteration. Now suppose ϕ holds in exactly one sink state. How many iterations would it take (in the worst case) for the fixpoint to converge? Justify.
3. Prove using the fixed point formulation that: $E[p \cup q] \equiv q \vee (p \wedge EX E[p \cup q])$
Clearly define the fixpoint function involved. Demonstrate how the recursive formulation corresponds to a least fixpoint computation.
4. Consider a CTL formula EG p. Let the following transition system be defined:
 - States: s0, s1, s2
 - Transitions: $s0 \rightarrow s1, s1 \rightarrow s2, s2 \rightarrow s0$
 - Labeling: All states satisfy p

Using the greatest fixpoint characterization of EG p, show how the fixpoint is computed iteratively. Does EG p hold in s0? Justify each step.

5. Given the following Kripke structure with atomic propositions {p, q}:

States and transitions:

- $S0 \rightarrow S1, S0 \rightarrow S2$
- $S1 \rightarrow S1$
- $S2 \rightarrow S3$
- $S3 \rightarrow S3$

Labeling:

- $L(S0) = \{p\}, L(S1) = \{p\}, L(S2) = \{q\}, L(S3) = \{q\}$

Apply the CTL model checking algorithm to determine whether the formula $E[p \cup q]$ holds in state S0. Explain each step of the fixpoint iteration and show the evolution of the set of states satisfying the formula.

*Note: Also complete the theory questions given in the lecture slides of 5th unit.