



Assignment -3: Modal Logic and Capabilities

1. Consider a system with three states: S1, S2, and S3. From S1, an action a can lead to S2 or S3. At S2, property P holds.

a) Express the system's behavior using Hennessy–Milner Logic (HML).

b) Determine if $[a]P$ holds at S1. Justify your answer.

2. A banking system maintains a balance B that should never be negative. If the system allows transactions of +100 or -50 at any step, prove that the invariant condition $B \geq 0$ holds for all reachable states, assuming the initial balance is 200.

3. Consider a CPU scheduler that assigns tasks in a round-robin fashion. If there are three processes P1, P2, and P3, formulate liveness and fairness properties using temporal logic (e.g., $G\Diamond P$ for fairness). Explain your reasoning.

4. Model a traffic light system using temporal logic where:

- The light always transitions from Green \rightarrow Yellow \rightarrow Red \rightarrow Green.
- A car must eventually see a green light after waiting at red.

Express these conditions using LTL (Linear Temporal Logic) operators.

5. Define a recursive formula in modal mu-calculus that ensures a process continues execution infinitely while allowing interruptions (e.g., a server that must eventually restart). Explain how the formula ensures termination or infinite execution.

6. A system has three states:

- S1 transitions to S2 with action a.
- S2 transitions to S3 with action b.
- S3 is a final state where property P holds.

Express the system behavior using modal logic and determine:

a) Whether $\Diamond P$ holds at S1.

b) Whether $\Box P$ holds at S1.

7. A traffic light controller cycles through states Green \rightarrow Yellow \rightarrow Red \rightarrow Green. The system must never transition from Green to Red directly.

Using state transition invariants, prove whether this safety property holds.



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8. Consider a printer queue where jobs are processed in First-In-First-Out (FIFO) order. A job J_n is submitted at time $t = 0$. Jobs are printed every 2 minutes.

- Express the liveness property ensuring J_n is eventually printed.
- Determine the maximum time J_n will wait if there are 4 jobs ahead.

9. A system has the following labeled transitions:

- $S1 \xrightarrow{a} S2$
- $S2 \xrightarrow{b} S3$
- $S3$ satisfies property P .

Formulate and verify the following statements in Hennessy–Milner Logic:

- Does $\langle a \rangle \langle b \rangle P$ hold at $S1$?
- Does $[a]P$ hold at $S1$?

10. A web server processes requests with the following conditions:

- If a request is received, it will be processed within 5 seconds.
- No request remains pending indefinitely.

Express these conditions using Linear Temporal Logic (LTL).

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