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IFAC-PapersOnLine 49-1 (2016) 419–424

Motion Planning for a Fixed-Wing UAV in Urban Environments

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Abstract: This paper presents motion planning for fixed-wing unmanned aerial vehicles (UAVs) using Rapidly-exploring Random Trees (RRTs), given a starting location and destination in the presence of static obstacles. Generating a near optimal path in an obstacle rich environment within a very short time window was well addressed by the RRT approach and was tightly established for a 2-D domain. Adopting this solution, it is investigated for urban environments. In this paper, taking real time situations for a fixed-wing UAV into account, a practical solution for its path planning in an urban environment is proposed. The turn radius and climb rate constraints of a UAV are considered in this regard. A combination of pursuit guidance law and line of sight guidance law is used to track the UAV in 3-D domain. Simulations studies are presented to demonstrate the performance of the proposed algorithm along with the guidance law.

Keywords: Path planning, RRTs, autonomous vehicles, pursuit guidance and LOS guidance.

1. INTRODUCTION

In recent times, unmanned aerial vehicles (UAVs) have become a common sight. They are being used in a wide spectrum of tasks like search and surveillance, terrain mapping, rescue mission, patrolling, armed attacks and are entering into the spheres of cargo transport, crop spraying. They are also being used for leisure activities such as hobby projects and movie making. In the beginning, UAVs were controlled from a ground station which is still an existing practice in most of the cases. With the efforts to implement autonomy skyrocketing in the current scenario, researchers have been working on those lines and the two most important aspects being path planning and path following, jointly come under the domain of motion planning. In any mission, for a UAV to successfully accomplish the alloted tasks, it has to move along a path that is either prefixed or that is generated instantaneously on-board based on then existing constraints. In most cases, the mission criteria are preloaded into a UAV that consists of a starting point and a terminal point. An autonomous UAV has to fly along a path that is generated using a path planning algorithm based on the mission criteria given to the UAV. Designing a path planning algorithm for an obstacle rich environment is a demanding job.

For a path planning algorithm to be effective, it has to accommodate for the environmental constraints of the domain as well as the physical and kinematic constraints of a UAV. Moreover, it is of paramount importance that a path planning algorithm be robust and be able to achieve certain level of optimality for it to be applicable in real world. Additionally, the computational complexity is a critical condition in evaluating the performance of an algorithm. This is due to the fact that UAVs carry limited processing power and travel at relatively high speeds as demanded by a mission. In short, designing a path planning algorithm demands optimality, completeness, and minimal computational cost.

Some of the early path planning algorithms are based on cell decomposition (Hart et al. (1968)), potential field techniques (Barraquand et al. (1992)), elastic bands (Quinlan and Khatib (1993)), cost functions from optimal control theory (Stentz (1994)). A comprehensive study on the planning algorithms can be found in the works by Latombe (1996), LaValle (2006), Choset (2005), and Da Gu et al. (2006) those include schemes based on roadmaps, visibility line, potential fields, etc. Each of these techniques performs better than others depending on the constraints and nature of the mission. Also, few techniques tried to solve the problem in its full generality. However, it is a well known fact that the computational complexity of a deterministic and complete algorithm grows exponentially with the degrees of freedom in the chosen domain. Therefore, most of these algorithms cannot be applied on a real time UAV.

Sampling based approaches are the current phenomena in the field of motion planning that has grabbed the researchers' attention (Amato and Wu (1996), Kavraki et al. (1996)). Of those, rapidly exploring random trees (RRTs) are ideal for UAV applications for its quick response to the requirements (LaValle (1998), Kuffner and LaValle (2000)). The model and sensor inaccuracies, wind disturbances affect the performance of the algorithm to a great extent. The reason being it is a open loop path planner and desired performance is hard to achieve. Later, Saunders et al. (2005) have extended RRTs into the output space and a robust controller is employed to track the generated path under external disturbances. Being a computationally efficient algorithm, RRT suffers from lack of optimality as it randomly explores the search space. Recently, Kothari

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10.1016/j.ifacol.2016.03.090

et al. (2010) came up with a suboptimal algorithm using anytime approach (Ferguson and Stentz (2007)) to tackle static and pop-up obstacles. A heuristic approach to improve the algorithm's performance has also been explored by Kothari and Postlethwaite (2013) to account for the uncertainties within the formulation.

Designing a path planning algorithm for an obstacle rich environment is a challenging task and a lot of research is still being carried to achieve a near optimal solution in the constraints of computational cost, most of the which offer solutions in a 2D domain. This task becomes even more difficult when the domain is upgraded to 3D. Some of the recent works on path planning in a 3D environment include those of Gan et al. (2009), Yang and Sukkarieh (2008), Stoyanov et al. (2010), Teniente and Andrade-Cetto (2013). They have not taken the dynamics of a UAV into account, excluding the one by Gan et al. (2009) that has considered a rotary wing UAV performing vertical takeoff and landing (VTOL). Also, another work by Sujit and Beard (2009) includes a guidance algorithm to track a feasible path generated using anytime algorithm in 3D.

In this paper, we develop a path planning algorithm that serves a fixed-wing UAV to navigate between two fixed points in a three dimensional space with static obstacles. In an urban environment, these static obstacles can be taken as buildings of various height. The proposed algorithm is simple and intuitive that adapts partly the work done by Kothari et al. (2010). In order to track the generated path precisely, a combination of pursuit and line of sight guidance law is chosen.

The rest of the paper is organized as follows. Section 2 formally describes the problem statement that we have considered for path planning in urban areas. Section 3 presents the proposed path planning algorithm that is suitable for 3D urban environment along with a brief introduction to the existing algorithm for 2D environment. In Section 4, the kinematic model of a UAV and the guidance law that is used to track the generated path are discussed. The simulation results are presented and appropriate interferences are drawn in Section 5. The paper is concluded with Section 6.

2. PROBLEM STATEMENT

In this paper, a mission is considered in which a UAV has to fly from a starting point to a destination point through an obstacle rich urban environment. The obstacles are assumed to be static and can be thought of as buildings and skyscrapers in a realistic scenario. It is also assumed that the UAV has complete knowledge of the terrain before the mission starts. The problem statement is to quickly generate a path from the starting point to the target point, along which the UAV can fly. Therefore, it is necessary to consider the performance constraints of the UAV when flying in such an environment, such as its maximum climb rate and minimum turn radius. It is also vital to provide a buffer zone around the city structures for enhanced safety to ensure that the UAV does not come close beyond a certain limit. It is assumed that the UAV flies at a constant speed.

3. RRT PATH PLANNING ALGORITHM

In this section, a sub-optimal RRT algorithm and its variant used in this paper is discussed. The method used to implement buffers along the structures and its effects are also studied.

The modified RRT algorithm that has been used in the work by Kothari et al. (2010) is given in Algorithm 1. It has been shown that this scheme is efficient in generating paths for a 2D scenario. Hence, this scheme was considered as the basis for this paper.

Algorithm 1 A Sub-optimal RRT Path Planning Algorithm (Kothari et al. (2010))

1: Choose an initial node w_{init} and add to the tree τ.
2: Pick a random waypoint w_{rand} with little bias (set $w_{rand} = w_{goal}$ for the bias) in the search space S.
3: Using a metric ρ, determine the node w_{near} in the tree that is nearest w_{rand}.
4: Extend the branch toward w_{rand} by an incremental distance, resulting in node w_{extend}.
5: Check for collision.
6: If there is no collision add node w_{extend} to the tree τ, else return to step 2.
7: Keep extending the branch and adding new nodes w_{new} to the tree τ until an obstacle is encountered.
8: Repeat steps 2 to 7 until w_{goal} is included in the tree τ.
9: Find the complete path form w_{init} and w_{goal}.
10: Eliminate extraneous nodes from path while checking the turn radius constraint for actual trajectory.

This scheme has been modified to meet the requirements for a 3D scenario and is discussed below.

3.1 Implemented RRT algorithm

The objective of the path planning algorithm is to generate a path from the starting point (x_{start}) to the target point (x_{target}). It is important to note that a fixed-wing UAV is more effective during horizontal maneuvering in comparison to vertical maneuvering. Hence, in this work, a generated path is separated into different sections: climb/descent and horizontal cruise sections. In order to ensure that the UAV is able to dodge most of the obstacles in the cruise phase, an intermediary plane is considered. An intermediary plane between the starting and ending point can be chosen on the of basis mission constraints such as the locations of the starting and target points, the maximum climb rate of the UAV, etc. For example, if the both the starting and target points are on the ground level, then the intermediary plane is chosen on the basis of the maximum climb rate of the UAV and the cruise height. If the two points are at different altitudes with the starting point on the ground, then the intermediary plane can lie at half the height of the target point. Using the maximum climb and descent angles, from the starting and ending points, possible connection points on the intermediary plane are searched along an arc on it, achieved by changing the azimuth angle. However, the two points on the plane are selected such that the UAV can travel from the starting and target points to the plane

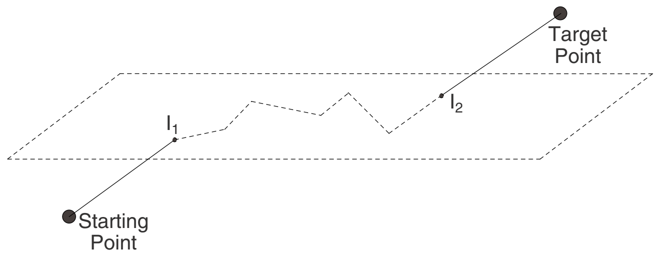

Fig. 1. The intermediate plane and the path generated by RRT

while satisfying its performance constraints. The prime constraint being the obstacle avoidance. A schematic of an intermediary plane with these connection points, I_1 and I_2, can be seen in Fig. 1. Once valid connection points are found, the path planning algorithm is then used to generated a path between the points I_1 and I_2, thereby simplifying the problem.

As the plane is considered to be at a different altitude to that of the starting point and the end point, not all obstacles will affect the path planning process. Hence, the obstacles that are lower than the intermediary plane and those that do not affect the path planning, are ignored. To take the deviation of UAV from the path into consideration, a buffer is added to the dimensions of the obstacles.

The proposed scheme for a UAV path planning in an urban environment is presented in Algorithm 2.

Algorithm 2 Implemented RRT Algorithm

1: Choose an intermediate plane between the altitudes of the initial and target points.
2: Based on the performance characteristics of the UAV, estimate two points on the plane from which the UAV can reach the starting and target points respectively.
3: Mark the two points w_{init} and w_{final} on the intermediate plane and add them to the tree τ along with the starting and ending points.
4: Ignore the irrelevant obstacles and add the buffer to the remaining obstacles.
5: Pick a random waypoint w_{rand} with little bias (set $w_{rand} = w_{final}$ for the bias) in the search space S.
6: Using a metric ρ, determine the node w_{near} in the tree that is nearest w_{rand}.
7: Extend the branch toward w_{rand} by an incremental distance, resulting in node w_{extend}.
8: Check for collision.
9: If there is no collision add node w_{extend} to the tree τ, else return to step 5.
10: Keep extending the branch and adding new nodes w_{new} to the tree τ until an obstacle is encountered.
11: Repeat steps 5 to 10 until w_{final} is included in the tree τ.
12: Find the complete path form w_{init} and w_{final}.
13: Eliminate extraneous nodes from path while checking the turn radius constraint for actual trajectory.

The following section presents kinematics of a UAV. Since the UAV mentioned in the problem statement is fixed-wing by design, the relevant performance parameters and constraints involved are also throughly analyzed and are considered in developing the guidance law.

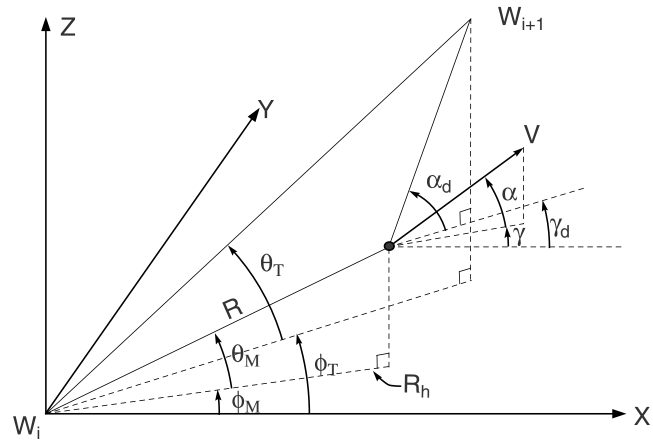

Fig. 2. UAV 3D guidance geometry

4. KINEMATICS OF A UAV

The main objective for a UAV is to reach a target point following the path generated by the RRT based algorithm presented in the former section. For the UAV to follow the path accurately, a guidance scheme is required. The guidance scheme implemented in this paper is developed from a combination of pursuit and line of sight guidance. This is done due to the simplicity and applicability of these guidance laws.

To design a guidance scheme, a constant speed point mass model is employed. The kinematics of the UAV is defined by the following relations:

$$\begin{aligned}
\dot{x} &= V\cos\phi\cos\theta \\
\dot{y} &= V\sin\phi\cos\theta \\
\dot{z} &= V\sin\theta \\
\dot{\phi} &= \frac{a_h}{V\cos\theta} \\
\dot{\theta} &= \frac{a_v}{V}
\end{aligned} \tag{1}$$

where, V is the velocity, a_h and a_v represent the lateral accelerations in horizontal and vertical planes respectively. θ and ϕ are the elevation and azimuth angles of the velocity vector of the UAV respectively.

Consider the engagement geometry in Fig. 2. Assume that the UAV has to travel from waypoint W_i to waypoint W_{i+1}; its current position is at (x, y, z). To reach the intended position on the assigned path, the UAV has to align itself with the line joining W_i and W_{i+1} and minimize its perpendicular distance to this line. The lateral accelerations generated by the guidance law for this purpose are defined as follows:

$$\begin{aligned}
a_h &= k_{1h}(\gamma_d - \gamma) + k_{2h}R_h\sin(\phi_M - \phi_T) \\
a_v &= k_{1v}(\alpha_d - \alpha) + k_{2v}R\sin(\theta_M - \theta_T)
\end{aligned} \tag{2}$$

where, k_{1h}, k_{1v}, k_{2h}, and k_{2v} are the gains. α and γ are the elevation and azimuth angles of the velocity vector, and, θ_M and ϕ_M are the elevation and azimuth angles of

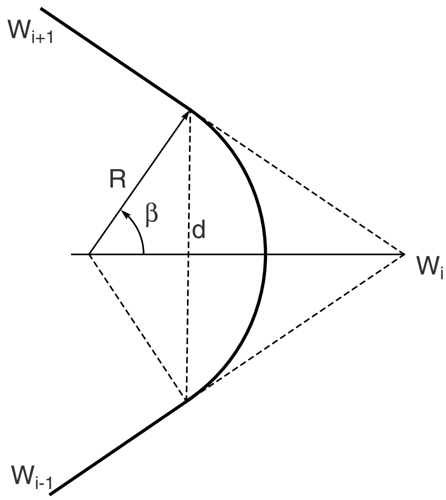

Fig. 3. Waypath geometry

the position vector of the UAV. The other terms in the equation are presented below:

$$
\begin{aligned}
R &= \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2 + (z_i - z_{i+1})^2} \\
R_h &= \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2} \\
\alpha_d &= \tan^{-1}\left(\frac{z_{i+1} - z}{\sqrt{(x_{i+1} - x)^2 + (y_{i+1} - y)^2}}\right) \\
\theta_T &= \tan^{-1}\left(\frac{z_{i+1} - z_i}{\sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}}\right) \\
\gamma_d &= \tan^{-1}\left(\frac{y_{i+1} - y}{x_{i+1} - x}\right) \\
\phi_t &= \tan^{-1}\left(\frac{y_{i+1} - y_i}{x_{i+1} - x_i}\right)
\end{aligned}
\tag{3}
$$

where, (x_i, y_i, z_i) and $(x_{i+1}, y_{i+1}, z_{i+1})$ are the Cartesian coordinates of the waypoints W_i and W_{i+1} respectively. The first and second terms on the right hand side of the expressions in 2, are pursuit and line of sight guidance law terms respectively. An appropriate guidance law can be developed by choosing the gains according to the requirements of the problem. The chosen gains in this case are

$$
\begin{aligned}
k_{1h} = k_{1v} = 10 \\
k_{2h} = k_{2v} = 1
\end{aligned}
$$

Now, consider the scenario shown in Fig. 3, where the UAV needs to travel from waypoint W_{i-1} to waypoint W_{i+1} via waypoint W_i. The UAV needs to travel along the path, while remaining within its performance constraints. Therefore, it travels along the arc seen in Fig. 3. The arc is governed by the following equation:

$$\frac{d}{2} = R \sin \beta \tag{4}$$

where, 2β is the angle made by the points where the UAV deviates from and returns to the path generated from the path planning algorithm at the center of the arc. When traveling along the arc, the line of sight guidance terms are ignored as traveling along an arc is different from traveling along a line. The maximum acceleration is the limiting factor in determining the arc of smallest possible deviation. The smallest possible radius of the arc can be determined using the following relation:

$$R_{min} = \frac{V^2}{a_{max}} \tag{5}$$

where, a_{max} is the maximum lateral acceleration that can be sustained by the UAV and V is the speed of the UAV. If the commanded acceleration is more than the maximum acceleration it is constrained by implementing the following equation:

$$
\begin{aligned}
a_h &= a_{max} \frac{a_{ch}}{\sqrt{a_{cv}^2 + a_{ch}^2}} \\
a_v &= a_{max} \frac{a_{cv}}{\sqrt{a_{cv}^2 + a_{ch}^2}}
\end{aligned}
\tag{6}
$$

where, a_{ch} and a_{cv} are the commanded horizontal and vertical acceleration respectively.

Now that the algorithm for the path generation and a guidance law that can be implemented on a UAV to follow the generated path are presented, the following section verifies the proposed scheme through simulations.

5. SIMULATION RESULTS

In this section, the proposed algorithm is demonstrated using one of the numerous simulations that we have carried out to analyze its efficacy. The simulations are carried out in a 3D domain of the size, $1km \times 1km \times 1km$, and random obstacles of various sizes are generated in it. As mentioned earlier, these obstacles are commensurable with skyscrapers and buildings in an urban environment. The objective of a UAV is to find a path in between its starting location to a target location and then traverse along the generated path in its kinematic constraints. The starting location and the target location are fixed at S $(10, 20, 30)$ m, and D $(750, 850, 950)$ m respectively. The speed of the UAV is chosen to be 13 m/s. The maximum climb rate is limited by its climb angle of 60°. The maximum acceleration is set by choosing the minimum turn radius to be 30 m. A buffer of 6 m is added to the height, and 2 m clearance is given along the perimeter of the obstacles.

A random selection of 150 obstacles are considered in the domain. The path is then generated using the path planning algorithm. These can be observed in Fig. 4. A mid-plane in between the horizontal planes containing the starting and target points is chosen as an intermediary plane for this simulation. The starting point and the target point are marked using a blue and red $'X'$ signs respectively. Note that the points connecting the plane to the starting and target points, the connections points on the intermediary plane, are also marked in red and blue respectively. The path in the figure validates the effectiveness of the path planning algorithm used in this paper.

As mentioned in the algorithm the obstacles not affecting the algorithm are removed. The top view of the domain with the remaining obstacles can be seen in Fig. 5. For further clarification, it can be compared with the domain

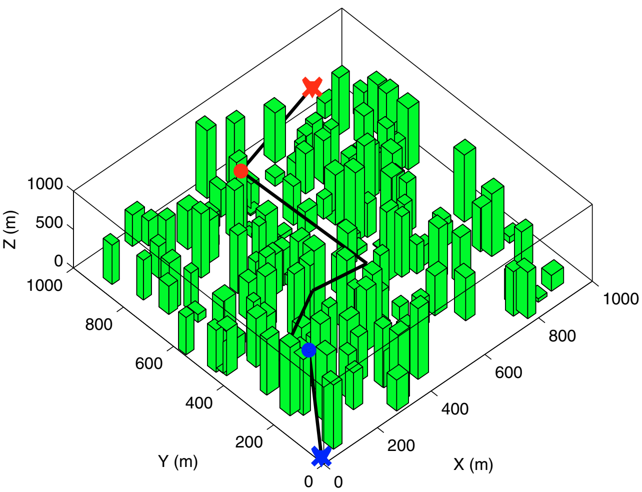

Fig. 4. Bird's eye view of the domain and the path generated

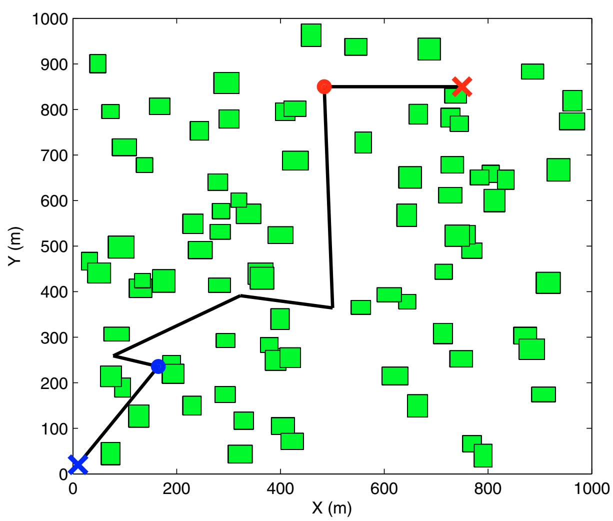

Fig. 5. Top view of the domain with only pertinent obstacles

in Fig. 6, which shows all the obstacles. It can be observed that the number of obstacles are greatly reduced, which will help with the speed of the path planning algorithm.

The UAV path tracking is simulated along the generated path, considering its performance constraints. The trajectory of the UAV superimposed over the path can be seen in Fig. 6. The path is represented by a set of black lines, whereas the trajectory of the UAV is represented by a red curve. It can be observed that the UAV more or less stays along the path and successfully reaches the target point. The effect of limiting the minimum radius can seen near the waypoints where the UAV deviates from the path, but eventually returns to it. This verifies the efficacy of the guidance scheme proposed and used in this paper. Also, from the path deviation that can be observed in Fig. 6, the need for the safety perimeter around and above the obstacles is emphasized.

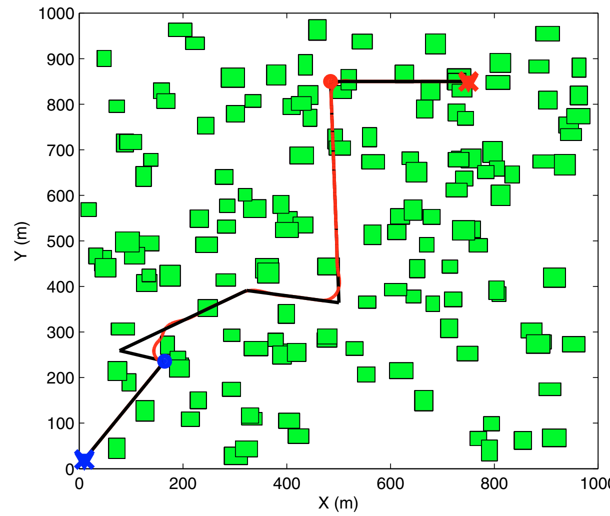

Fig. 6. Overlay of trajectory of the UAV and the path generated by RRT

Table 1. Computational time requirement for different scenarios

No. of obstacles	Minimum (sec)	Maximum (sec)	Average (sec)
100	0.258	11.035	2.909
125	0.102	10.547	3.803
150	0.277	10.426	5.0213

5.1 Computational Performance

The computational performance of the proposed path planning algorithm is presented in this subsection. The simulations are conducted in MATLAB using a machine powered by 3.1 GHz Intel Core i5 processor with 8 GB of RAM. For the scenario mentioned above, number of obstacles is taken to be 100, 125 and 150 for three different cases. For each of these cases, 20 simulations are carried out and the computational time to find a path is recorded. The results are summarized in Table 1. It can be observed that the required computational time is of the order of 10 sec even in the worst case scenario. During this time, the UAV is expected to head for the connection point, I_1, on the intermediary plane that is already generated. Hence, the proposed scheme is useful for real time UAV applications.

6. CONCLUSIONS

In this paper, a path planing strategy for a fixed-wing UAV in an urban domain with static obstacles is proposed. The algorithm is developed using a well-established 2D sub-optimal RRT strategy coupled with intuitive enhancements to accommodate for the disparities in a 3D environment. A 3D guidance law is also presented that is a combination of pursuit guidance law and line of sight guidance law for the UAV to successfully traverse on the path generated by the algorithm under external disturbances. Being an augmented version of a sub-optimal path planning algorithm, the path planner finds a path in a reasonably short time. The performance of the proposed motion planning scheme is demonstrated using a simulation. The scheme is found to be elegant and efficient.

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