در تابع دو متغیره مشتق پذیر $z=f(x,y)$

چنانچه $f(x,2x)=1$ و $f_x(x,2x)=x$ مقدار

$f_y(1,2)$ کدام است؟

$$\begin{cases} f(x,2x)=1 \\ f_x(x,2x)=x \end{cases} \Rightarrow$$

تابعی را باید حدس بزنیم که این دو ویژگی را دارا باشد.

قطعا این تابع نمایی خواهد بود.

$$f(x,y)=e^{\frac{1}{2}x^2-\frac{1}{8}y^2}$$

$$\begin{cases} f(x,2x)=1 \\ f_x(x,2x)=x \end{cases}$$

$$f_y=\frac{-2}{8}y\,e^{\frac{1}{2}x^2-\frac{1}{8}y^2} \rightarrow f(1,2)=-\frac{1}{4}\times 2\times e^0=-\frac{1}{2}$$

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ODE معادله دیفرانسیل معمولی:

$$y = c_1 e^{2x} + c_2 e^{-x}$$

معادله دیفرانسیل حاکم چیست؟

$$y' = 2c_1 e^{2x} - c_2 e^{-x}$$

$$y'' = 4c_1 e^{2x} + c_2 e^{-x}$$

حذف c_1 و c_2 تشکیل می‌شود معادله دیفرانسیل

راه حل دیگر رونسکین را برابر با صفر قرار می‌دهیم:

$W = 0$

$$W = 0 \Rightarrow \begin{vmatrix} y & e^{2x} & e^{-x} \\ y' & 2e^{2x} & -e^{x} \\ y'' & 4e^{2x} & +e^{-x} \end{vmatrix} = 0$$

معادله دیفرانسیل تشکیل می‌شود.

معادله دیفرانسیل حاکم بر تابع $z = y f(x^2 - y^2)$ کدام است؟

$z = y f(x^2 - y^2)$ ‏z تابع و x و y متغیرهای مستقل

$$z = y f(\underbrace{x^2 - y^2}_{v}) \Rightarrow \begin{cases} u = f(v) \\ f(u, v) = 0 \end{cases}$$

u $\qquad$ v

$$\frac{z}{y} = f(x^2 - y^2)$$

$$\rightarrow J = 0 \rightarrow \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

$$\rightarrow \begin{vmatrix} \frac{z'_x \times y - 0 \times z}{y^2} & \frac{z'_y \times y - 1 \times z}{y^2} \\ 2x & 2y \end{vmatrix}$$

$$J = \frac{-2y(z'_x y)}{y^2} - \frac{2x(z'_y y - z)}{y^2}$$

$$= \frac{-2y^2 z'_x - 2xy z'_y + 2xz}{y^2}$$

$$J = 0 \quad \underbrace{(z-x)}_{u} = f(\underbrace{xy^2}_{v}) \longrightarrow$$

$$\rightarrow \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial x} \end{vmatrix}$$

$$\begin{vmatrix} z'_x - 1 & z'_y \\ y^2 & 2yx \end{vmatrix} = 0$$

$$2xy(z'_x - 1) - y^2 z'_y = 0$$

$$2xy\frac{\partial z}{\partial x} - 2xy - y^2\frac{\partial z}{\partial y} = 0$$

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$$\xrightarrow{\div y} \quad 2x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2x$$

$$z = x + f(xy^2)$$

مثال

$$\begin{cases} z = x + \sin(xy^2) \\ z = x + \ln(xy^2) \\ z = x + \operatorname{Arctan}(xy^2) \\ z = x + (xy^2)^3 \end{cases}$$

و خیلی دلیل که یک بسته بندی : xy^2 دارند

و x هم جمع شده است در معادله دیفرانسیل

بدست آورده قبلی صدق می‌کند.

مثال در تبدیلات حاصل

$$\begin{cases} u = x^2 + y^2 + z^2 \\ v = x + y + z \\ w = xy + xz + yz \end{cases}$$

$\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ کدام است؟

$$J = \frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

$$= \begin{vmatrix} 2x & 2x & 2x \\ 1 & 1 & 1 \\ y+z & x+z & x+y \end{vmatrix}$$

$$= 2x\big((x+y) - (x-z)\big)$$

$$- 2y\big((x+y) - (y+z)\big) + 2x\big((x+z) - (y+z)\big)$$

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$$J = \cancel{2xy} - \cancel{2xz} + \cancel{2yz} + \cancel{2zx} - \cancel{2zy} - \cancel{2xy} = 0$$

اگر $\int_{0}^{1} \frac{1}{\sqrt{1+t^2}}\, dt$ حاصل $\int_{0.03}^{0.98} \frac{1}{\sqrt{1+t^2}}\, dt$ کدام است؟

$\left(\frac{1}{\sqrt{2}} \simeq 0.7\right)$

$$\int_{0.03}^{0.98} \frac{1}{\sqrt{1+t^2}}\, dt \Rightarrow f(x,y) = \int_{x}^{y} \frac{1}{\sqrt{1+t^2}}\, dt$$

$$\begin{cases} x_0 = 0 \Rightarrow dx = 0.03 \\ y_0 = 1 \Rightarrow dy = -0.02 \end{cases}$$

$$\boxed{y_0 - y = -0.02}$$

مشتق از انتگرال گیری

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

$$df = \left(0 \times (\ ) - 1 \times \frac{1}{\sqrt{1+x^2}}\right) dx$$

$$\left(1 \times \frac{1}{\sqrt{1+y^2}} - 0 \times (\ )\right) dy$$

$$df = \left(-\frac{1}{\sqrt{1+(0)}}\right) \overset{0.03}{dx} + \left(\frac{1}{\sqrt{(1+1)}}\right) \overset{(-0.02)}{dy}$$

$$= -0.03 + (0.7 \times (-0.02)) =$$

$$df = -0.044$$ تغییرات تابع است

$$\int_{0.03}^{0.98} \frac{1}{\sqrt{1+t^2}} dt = \int_{0}^{1} \frac{1}{\sqrt{1+t^2}} dt + (-0.044) = 0.8 - 0.044 = 0.756$$

در تابع $z=\dfrac{2x-3y}{xy}$ در نقطه $(1,2)$ به ازای $dx=0.1$, $dy=0.2$ دیفرانسیل کامل مرتبه ۲ کدام است؟

$$d^2z=\frac{\partial^2 z}{\partial x^2}dx^2+\frac{\partial^2 z}{\partial y^2}dy^2+2\frac{\partial^2 z}{\partial x\partial y}dxdy$$

$$z=\frac{2x-3y}{xy}\Rightarrow z=\frac{2}{y}-\frac{3}{x}$$

$$\frac{\partial z}{\partial x}=0+\frac{3}{x^2}\qquad \frac{\partial^2 z}{\partial x^2}=-\frac{6x}{x^4}=-\frac{6}{x^3}$$

$$\frac{\partial^2 z}{\partial x\partial y}=0$$

$$\frac{\partial z}{\partial y}=-\frac{2}{y^2}\qquad \frac{\partial^2 z}{\partial y^2}=+\frac{4}{y^3}$$

$$d^2z = \left(\frac{-6}{x^3}\right)dx^2 + 2(0)\,dy\,dx + \left(\frac{4}{y^3}\right)dy^2 \Rightarrow @(1,2) \Rightarrow (-6)(0.1)^2 + \left(\frac{1}{2}\right)(0.2)^2$$

$dx = 0.1$

$dy = -0.2$

$$\boxed{dz = -0.04}$$

فرض $\begin{cases} z = 2x - y \\ f(x,y,z) = 0 \end{cases}$ در سه حالت

$\frac{dz}{dx}$ کدام است؟

$$\begin{cases} z = 2x - y \\ f(x,y,z) = 0 \end{cases} \Rightarrow \frac{dz}{dx} = 2$$

مشتق z نسبت به x

این غلط است.

$$\begin{cases} z = 2x - y \\ f(x,y,z) = 0 \end{cases}$$

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

$$dz = 2dx - dy$$

$$f(x,y,z) = 0 \longrightarrow \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz = 0$$

$$\begin{cases} dz = 2dx - dy \\ f'_x dx + f'_y dy + f'_z dz = 0 \end{cases}$$

$$\begin{cases} f_x dx + f_y (2dx - dz) + f_z dz = 0 \\ (f_x + 2f_y) dx + (f_z - f_y) dz = 0 \end{cases}$$

$$(f_y - f_z) dz = (f_x + 2f_y) dx$$

$$\frac{dz}{dx} = \frac{f_x + 2f_y}{f_y - f_z}$$

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مثال، مفروض است $z=f(x^2y, x\ln(2y-x^2), e^{y^2-x})$

$\frac{\partial z}{\partial x}$ کدام است؟

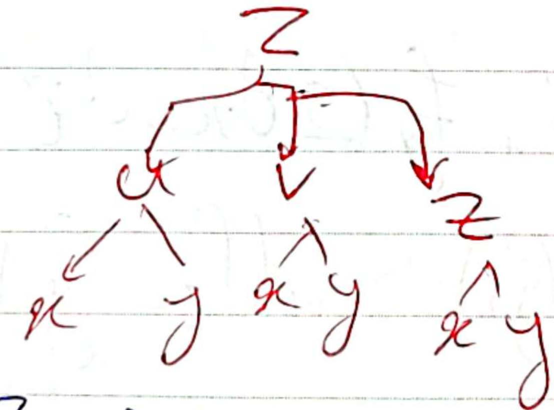
$$z=f(x^2y, x\ln(2y-x^2), e^{y^2-x})$$

$$u=x^2y$$

$$v=x\ln(2y-x^2)$$

$$w=e^{y^2-x}$$

$$\frac{\partial z}{\partial x}$$



$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$+\frac{\partial z}{\partial w}\cdot\frac{\partial w}{\partial x}$$

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$$\frac{\partial z}{\partial x} = f'_u \times 2xy + f'_v \left(\ln(2y-x^2) + x \frac{-2x}{2y-x^2} \right) + f'_w \times (-1 e^{y-x^2})$$

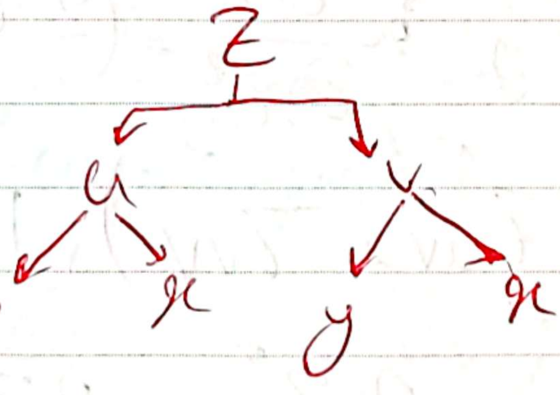
$$\frac{\partial z}{\partial x}(1,1) = 2f'_u - 2f'_v - f'_w$$

مثال معادله دیفرانسیل با مشتقات جزئی

$z_{xx}+z_{xy}=z_x$ با تغییر متغیرهای $u=2x+y$

$v=x+3y$ به معادله ای برحسب u , v ببرید

$$\begin{cases} u = 2x+y \\ v = x+3y \end{cases}$$

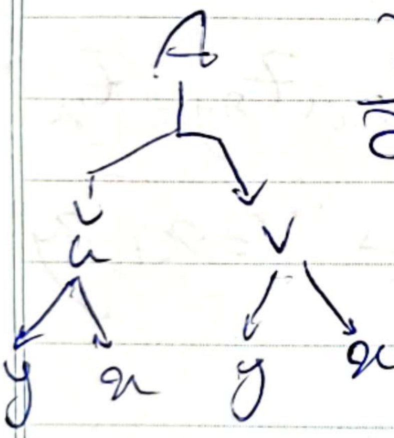


$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\frac{\partial z}{\partial x} = z'_u \cdot 2 + z'_v \times 1$$

$$\frac{\partial z}{\partial x} = 2z'_u + z'_v$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial x}\big(\overbrace{2z'_u + z'_v}^{A}\big)$$



$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial A}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial A}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\frac{\partial^2 z}{\partial x^2} = \left(2z''_{uu} + z''_{vu}\right) \times 2$$

$$+ \left(2z''_{uv} + z''_{vv}\right) \times 1$$

$$\frac{\partial^2 z}{\partial x^2} = 4z''_{uu} + 4z''_{uv} + z''_{vv}$$

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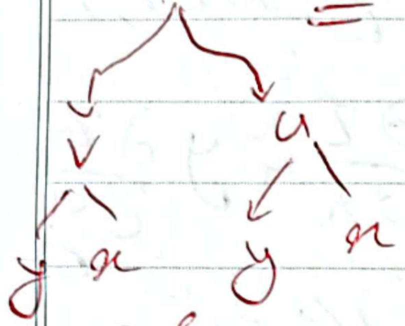
$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}\big(\underbrace{2z'_u + z'_v}_{A}\big)$$

$$A = \frac{\partial A}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial A}{\partial v} \cdot \frac{\partial v}{\partial y}$$



$$\frac{\partial^2 z}{\partial y \partial x} = (2z''_{uu} + z''_{vu}) \times 1 + (2z''_{uv} + z''_{vv}) \times 3$$

$$\frac{\partial^2 z}{\partial y \partial x} = \underbrace{2z''_{uu} + z''_{vu} + 6z''_{uv} + 3z''_{vv}}_{2z''_{uu} + 7z''_{uv} + 3z''_{vv}}$$

$$(4z_{uu} + 4z_{uv} + z_{vv}) + (2z_{uu} + 7z_{uv} + 3z_{vv})$$

$$= 2z_u + z_v$$

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$$6Z_{uu}+11Z_{uv}+9Z_{vv}-2Z_u-Z_v=0$$

در نقاط سطح $x^2+y^2+z^2=1$ حاصل

$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}$ کدام است؟

$x^2+y^2+z^2-1=0$ تابع ضمنی

$$\frac{\partial z}{\partial x}=-\frac{f'_x}{f'_z}=-\frac{2x}{2z}=-\frac{x}{z}$$

$$\frac{\partial z}{\partial y}=-\frac{f'_y}{f'_z}=-\frac{2y}{2z}=-\frac{y}{z}$$

$$-x^2-y^2=z^2-1$$

$$x\left(-\frac{x}{z}\right)+y\left(-\frac{y}{z}\right)=$$

$$\frac{-x^2}{z}+\frac{-y^2}{z}=\frac{-x^2-y^2}{z}=\frac{z^2-1}{z}$$

$$=z-\frac{1}{z}$$

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در معادله ضمنی $xyz^2-1=x^2-y^2$ در نقطه $(1,1,1)$ حاصل $\frac{\partial^2 z}{\partial x\partial y}$ کدام است؟

$$xyz^2-1=x^2-y^2$$

$$\xrightarrow{\frac{\partial}{\partial y}} x(z^2+y2zz'_y)=0-2y$$

$$= xz^2+2xyzz'_y=-2y$$

$$\xrightarrow{\frac{\partial}{\partial x}} 1z^2+x2zz'_x+2y(1\times zz'_y+$$

$$xz'_xz'_y+xzz''_{yx})=0$$

$$f:\ xyz^2-1+x^2+y^2=0$$

$$z'_x=\frac{\partial z}{\partial x}=-\frac{f'_x}{f'_z}=-\frac{yz^2-2x}{2xyz}\Bigg|_{(1,1,1)}$$

$$=\frac{1}{2}$$

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$$z'_y = \frac{\partial z}{\partial y} = -\frac{f'_y}{f'_z} = -\frac{xz^2+2y}{2xyz}\Bigg|_{(1,1,1)}$$

$$= \frac{-3}{2}$$

$$1\times z^2 + x\,2\,z\,z'_x + 2y\left(1\,z\,z'_y + x\,z'_x z'_y + x\,z\,z''_{yx}\right) = 0$$

$$@\,(1,1,1) = 1\times 2\times\frac{1}{2} + 2\left(\frac{-3}{2} + \frac{1}{2}\times-\frac{3}{2} + z''_{yx}\right) = 0$$

$$2z''_{yx} = \frac{5}{2} \Rightarrow z''_{yx} = \frac{5}{4}$$

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تجانس → تضیه اویلر

$$z = g(x,y) \rightarrow x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = ?$$

$$f(z) = g(x,y) \Rightarrow x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = ?$$

تابع شده z → صورت دیگر اویلر

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \alpha z$$ تضیه اویلر

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \alpha\frac{f(z)}{f'(z)}$$ صورت دیگر اویلر

اگر تابع همگن $z = \frac{\ln x - \ln y}{x^2 y + x y^2}$ در معادله (1)

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + (3m-1)z = 0$$ با مشتقات جزئی

صدق کند مقدار m چقدر است؟

$$\alpha\frac{f(z)}{f'(z)}$$

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}+(3m-1)z=0$$

تعمیم اویلر

$$z=\frac{\ln\frac{x}{y}}{x^2y+xy^2}\quad\begin{cases}x\to\lambda x\\ y\to\lambda y\end{cases}$$

$$z(x\lambda,\ x\lambda)$$

$$z=\frac{\ln\left(\frac{\cancel{\lambda}x}{\cancel{\lambda}y}\right)}{\lambda^3x^2y+\lambda^3xy^2}=\frac{\ln\left(\frac{x}{y}\right)}{\lambda^3(x^2y+xy^2)}$$

$$=\frac{1}{\lambda^3}\ \frac{\ln\left(\frac{x}{y}\right)}{(x^2y+xy^2)}$$

درجه همگن $\lambda=-3$

$$\alpha\frac{f(z)}{f'(z)}+(3m-1)z=0$$

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$$-3\,\frac{z}{1}+(3m-1)z=0$$

$$3z=(3m-1)z \Rightarrow m=\frac{4}{3}$$

در معادله‌ی $x^2+y^2+z^2=1$ حاصل $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}$ کدام است؟

$$f(z)$$

$$x^2+y^2+z^2=1 \rightarrow 1-z^2=x^2+y^2$$

درجه همگنی $\lambda=2$

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=\alpha\,\frac{f(z)}{f'(z)}$$

$$=2\,\frac{1-z^2}{-2z}=\frac{z^2-1}{z}=z-\frac{1}{z}$$

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