Rectangular Plate Bending Analysis

6” × 1.5” FCFC Steel Plate — Circular Patch Load

Plate Bending Solver (Rayleigh–Ritz Method)

February 8, 2026

1 Plate Geometry & Setup

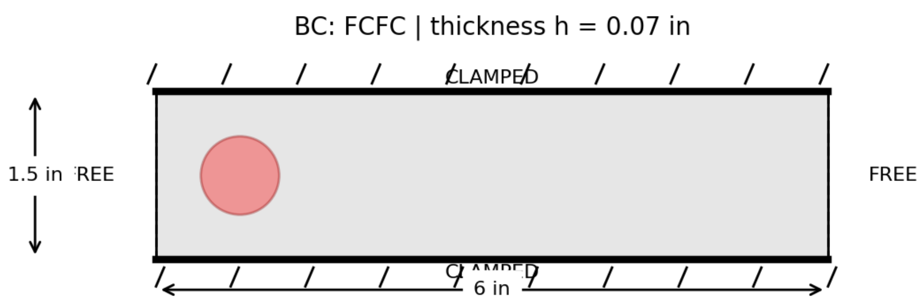

Figure 1: Plate geometry, boundary conditions, and applied load.

1.1 Geometry & Material

Parameter	Symbol	Value
Plate length (x-direction)	a	6.000 in
Plate width (y-direction)	b	1.500 in
Thickness	h	0.070 in
Young’s modulus	E	29×10^6 psi
Poisson’s ratio	ν	0.3

1.2 Flexural Rigidity

$$D = \frac{Eh^3}{12(1-\nu^2)} = \frac{29 \times 10^6 \times 0.070^3}{12(1-0.3^2)} = 910.9 \text{ lbf} \cdot \text{in} \tag{1}$$

1.3 Boundary Conditions

Configuration: **FCFC**

- $x = 0$ (short edge): **Free**
- $y = 0$ (long edge): **Clamped**
- $x = a$ (short edge): **Free**
- $y = b$ (long edge): **Clamped**

1.4 Loading

Concentrated force $P = 68$ lbf distributed over a $\varnothing 0.7''$ circular area ($R = 0.35''$), centered at $(x_0, y_0) = (0.75,\ 0.75)$ in.

$$q_0 = \frac{P}{\pi R^2} = \frac{68}{\pi \times 0.35^2} = 176.7 \text{ psi} \tag{2}$$

2 Governing Equation

$$D\nabla^4 w = D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = q(x, y) \tag{3}$$

3 Solution Method — Rayleigh–Ritz

FCFC is non-Levy (x-edges are free, not simply supported), so the Rayleigh–Ritz method is used:

$$w(x, y) = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn}\, \phi_m\left(\frac{x}{a}\right)\ \psi_n\left(\frac{y}{b}\right) \tag{4}$$

with Free–Free eigenfunctions ϕ_m in x and Clamped–Clamped eigenfunctions ψ_n in y. Final solution uses $M = N = 15$ (225 DOFs).

4 Results at Load Center

$(x, y) = (0.75,\ 0.75)$ in:

4.1 Deflection & Internal Moments

Quantity	Symbol	Value	Units
Deflection	w	0.000962	in (0.96 mil)
Bending moment (x)	M_x	218.6	lbf · in/in
Bending moment (y)	M_y	308.9	lbf · in/in

4.2 Bending Stresses

$$\sigma_x = \frac{6M_x}{h^2} = \frac{6 \times 218.6}{0.070^2} = 6800 \text{ psi } (6.80 \text{ ksi}) \tag{5}$$

$$\sigma_y = \frac{6M_y}{h^2} = \frac{6 \times 308.9}{0.070^2} = 9607 \text{ psi } (9.61 \text{ ksi}) \tag{6}$$

4.3 Global Maxima

Quantity		Value	Units
Max deflection	$\lvert w \rvert_{\max}$	0.000964	in
Max $\lvert\sigma_x\rvert$		6877	psi (6.88 ksi)
Max $\lvert\sigma_y\rvert$		11303	psi (11.30 ksi)

5 Deflection Contour

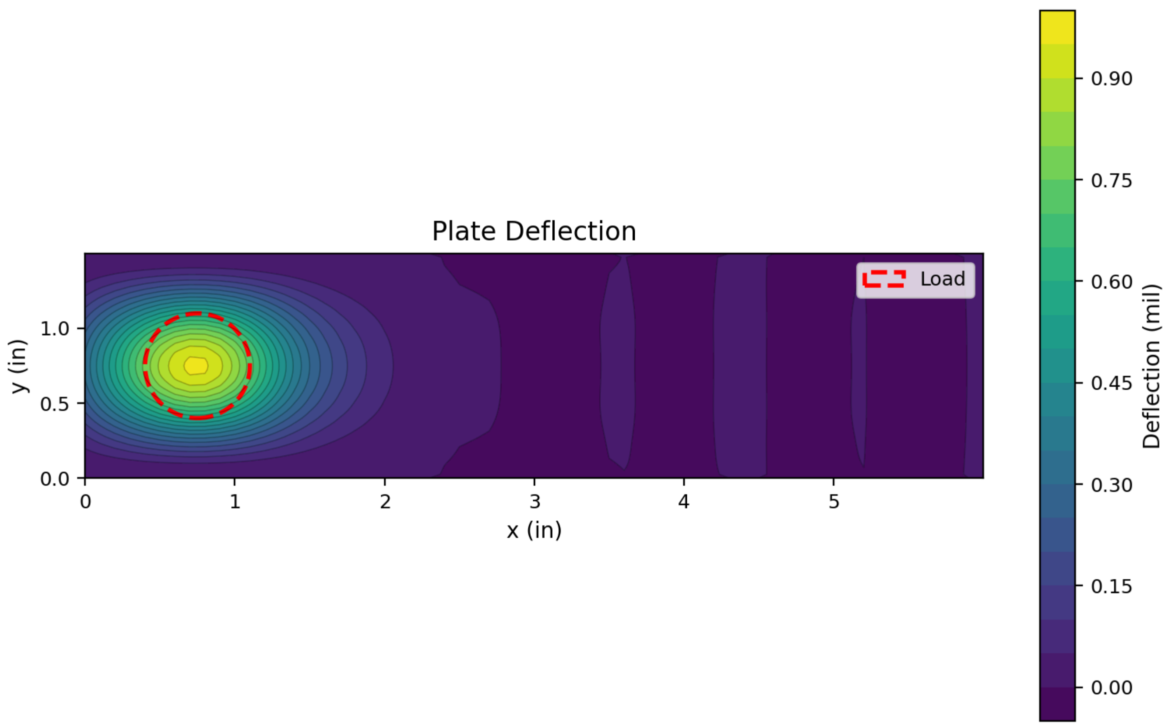

Figure 2: Deflection field (in mils). Dashed circle marks the loaded area.

6 Stress Contours

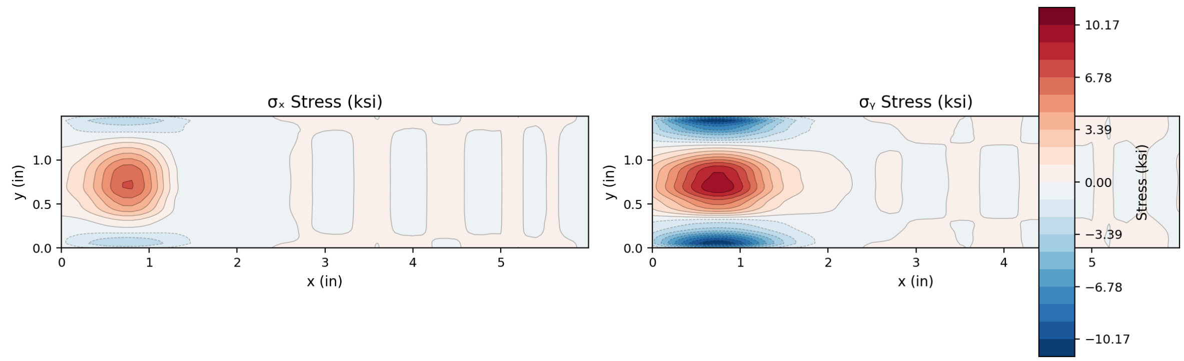

Figure 3: Bending stress fields σ_x and σ_y (ksi).

7 Deflection Profile

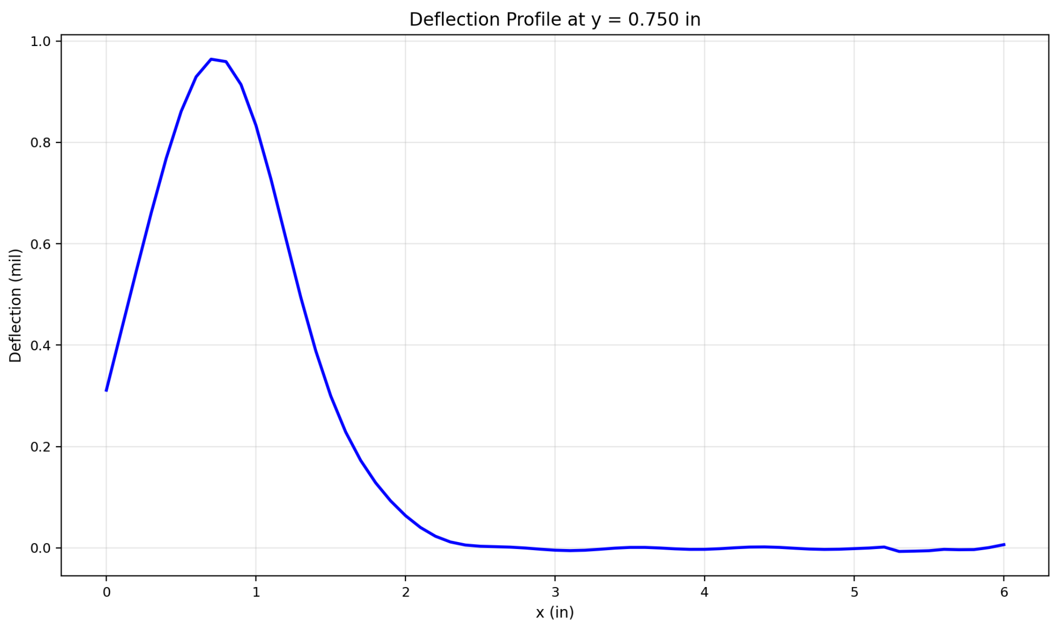

Figure 4: Deflection along the plate centerline ($y = 0.75''$).

Notes

- Material: structural steel ($E = 29$ Msi, $\nu = 0.3$).
- Kirchhoff thin plate theory ($h/b = 0.047 \ll 1$).
- Stresses are max surface values at $z = \pm h/2$.

A Step-by-Step Ritz Calculation

This appendix walks through the Ritz solution in detail, showing every number so the calculation can be followed or reproduced.

A.1 Setup

Quantity	Value	
a (plate length)	6	in
b (plate width)	1.5	in
h (thickness)	0.07	in
E (Young's modulus)	29×10^6	psi
ν (Poisson's ratio)	0.3	
q_0 (pressure)	176.7	psi

Flexural rigidity:

$$\begin{aligned} D &= \frac{Eh^3}{12(1-\nu^2)} \\ &= \frac{29 \times 10^6 \times (0.07)^3}{12(1-0.3^2)} \\ &= \frac{29 \times 10^6 \times 343 \times 10^{-6}}{10.92} \\ &= 910.9 \text{ lbf} \cdot \text{in} \end{aligned} \tag{7}$$

A.2 Trial Function

The deflection is approximated as a double sum of beam eigenfunctions:

$$w(x,y) = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn}\, \phi_m\left(\frac{x}{a}\right)\ \psi_n\left(\frac{y}{b}\right) \tag{8}$$

- $\phi_m(\xi)$: **FF** (Free–Free) beam eigenfunctions in x
- $\psi_n(\eta)$: **CC** (Clamped–Clamped) beam eigenfunctions in y

Minimizing the total potential energy $\Pi = U - W_{\text{ext}}$ gives:

$$\mathbf{K}\,\mathbf{A} = \mathbf{F} \tag{9}$$

where **K** is the stiffness matrix (from plate strain energy) and **F** is the load vector.

A.3 Convergence Study

The series is evaluated at increasing truncation levels. At each level, the deflection at the analysis point (0.75, 0.75) in is recorded.
$M = N = 2$: solve $\rightarrow w = 0.4987$ mil (first evaluation).
$M = N = 6$: $w = 0.744$ mil. $|\Delta w| = 0.245$ mil.
Continuing to $M = N = 15$: $w = 0.9618$ mil, $|\Delta w| = 0.0153$ mil (1.6%) — converged.

$M = N$	DOFs	w (mil)	$\lvert\Delta w\rvert$ (mil)
2	4	0.4987	—
6	36	0.744	0.245 (33%)
8	64	0.9029	0.159 (18%)
10	100	0.9431	0.0403 (4%)
12	144	0.9464	3.27×10^{-3} (0.3%)
15	**225**	**0.9618**	0.0153 (2%)

A.4 Physical Intuition

This plate has 2 clamped edges and 2 free edges. The clamped edges provide strong rotational restraint, limiting deflection but concentrating bending moments near the supports. The free edge(s) allow the plate to deflect without constraint, so maximum deflection typically occurs near or at the free edges. The high aspect ratio (4:1) means the plate behaves somewhat like a wide beam in the short direction.

References

1. Timoshenko, S. and Woinowsky-Krieger, S., *Theory of Plates and Shells*, 2nd ed., 1959.
2. Ventsel, E. and Krauthammer, T., *Thin Plates and Shells*, Marcel Dekker, 2001.