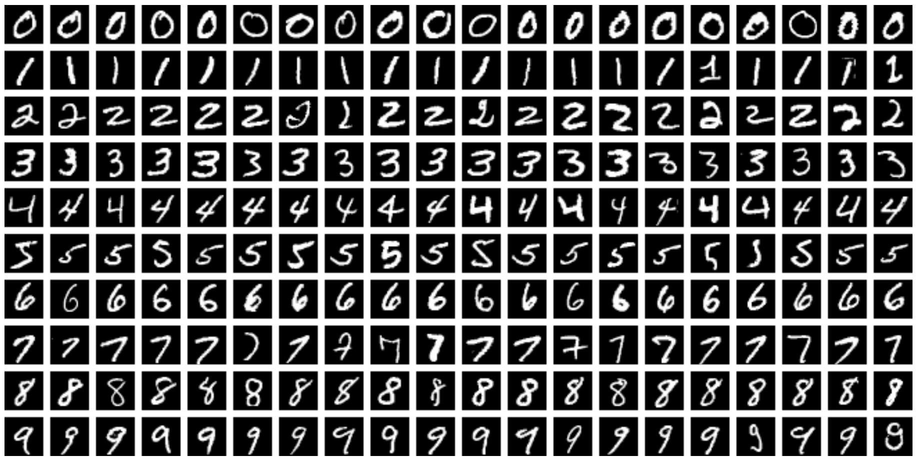
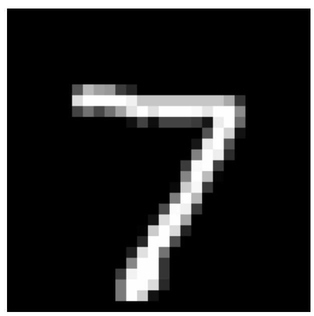
MNIST





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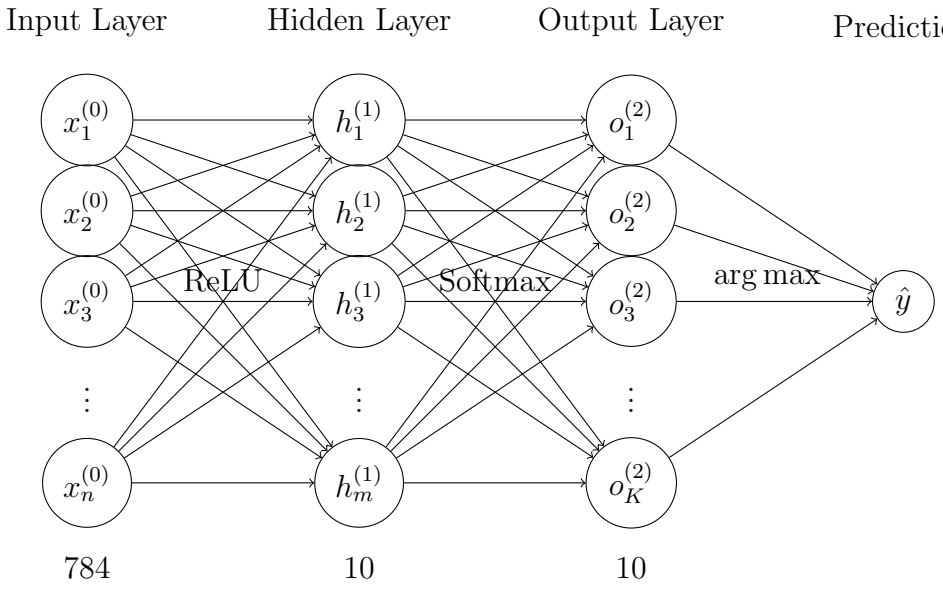
Neural Network



- **Neuron** - Basic computational unit that processes inputs
- **Weight** (w) - Connection strength between neurons
- **Bias** (b) - Offset value added to weighted sum
- **Forward propagation** - Computing outputs from inputs through the network
- **Logits** (z) - Pre-activation output
- **Activation** (a) - Non-linear function applied to neuron outputs
- **Backward propagation** - Computing gradients from outputs back to inputs
- **One-hot** - Binary vector representation of categorical labels
- **Loss** (l) - Measure of prediction error to be minimized
- **Gradient** (d) - Derivative of loss with respect to a parameter
- **Cross-entropy** - Loss function that measures prediction accuracy
- **Gradient descent** - Optimization algorithm that updates weights and biases
- **Learning rate** (α) - Step size for parameter updates in gradient descent

1. Forward propagation produces predictions

2. Loss measures how wrong we are
3. Backward propagation computes gradients from the loss
4. Gradient descent uses those gradients to update parameters

Data Structure

$$X = \begin{bmatrix} \cdots & x^{(1)} & \cdots \\ \cdots & x^{(2)} & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & x^{(m)} & \cdots \end{bmatrix}^{\top} = \begin{bmatrix} x_1^{(1)} & x_1^{(2)} & \cdots & x_1^{(m)} \\ x_2^{(1)} & x_2^{(2)} & \cdots & x_2^{(m)} \\ \vdots & \vdots & \ddots & \vdots \\ x_n^{(1)} & x_n^{(2)} & \cdots & x_n^{(m)} \end{bmatrix}$$

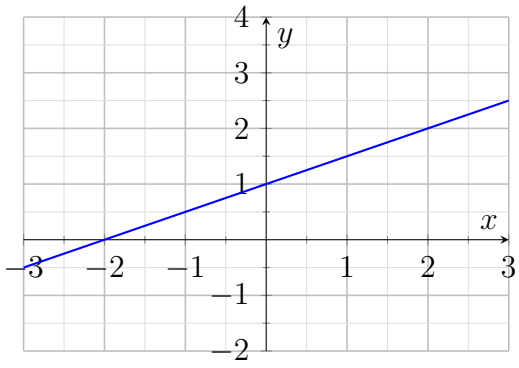
Matrix Multiplication

$$C_{ij} = \sum_{p=1}^{k} A_{ip} \cdot B_{pj}$$

$$\begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} (0.1 \times 1) + (0.2 \times 2) + (0.3 \times 3) \\ (0.4 \times 1) + (0.5 \times 2) + (0.6 \times 3) \end{bmatrix} = \begin{bmatrix} 1.4 \\ 3.2 \end{bmatrix}$$

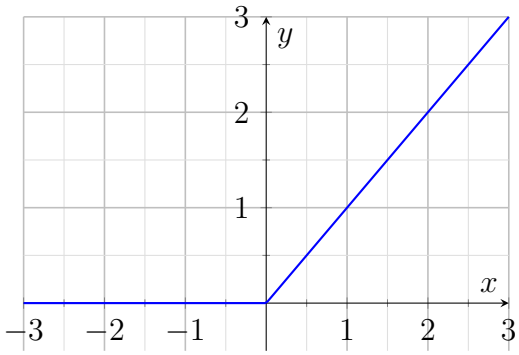
Linear Equation

$$y = ax + b$$



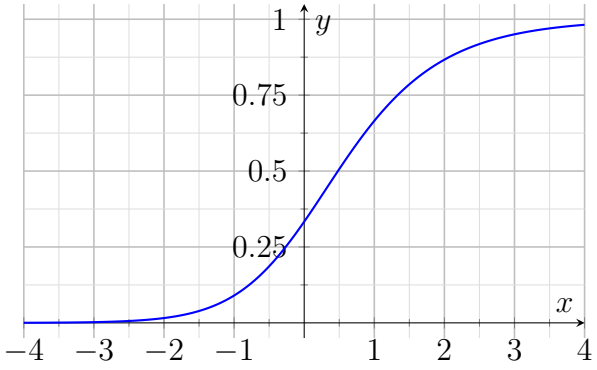
ReLU

$$\text{ReLU}(x) = \max(0, x)$$



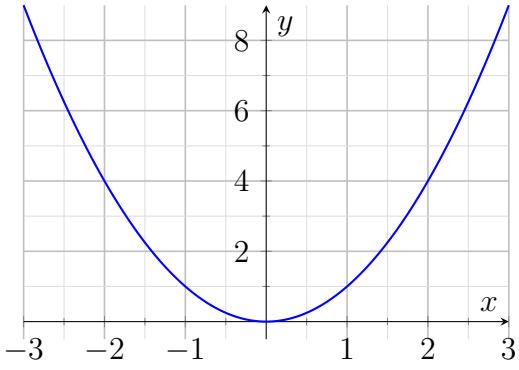
Softmax

$$\text{softmax}(z_i) = \frac{e^{z_i}}{\sum_{j=1}^{K} e^{z_j}} \quad \text{for } i = 1, \ldots, K.$$

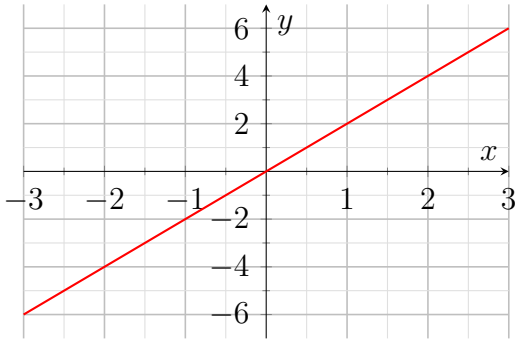


Derivative

$$f(x) = x^2$$



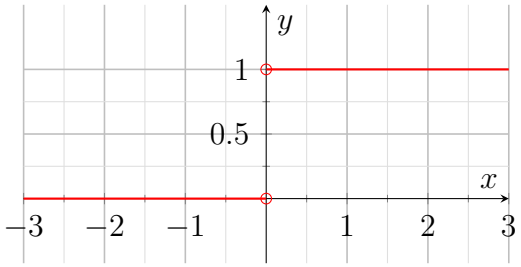
$$f'(x) = 2x$$



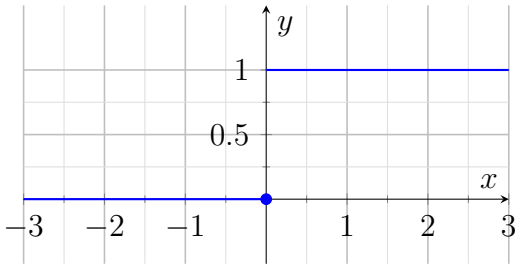
The derivative tells us the **rate of change**: how much the output changes when we make a change to the input.

ReLU Derivative

$$\text{ReLU}'(x) = \begin{cases} 0, & x < 0, \\ 1, & x > 0, \\ \text{undefined}, & x = 0 \end{cases}$$



$$\text{ReLU}'(x) = \begin{cases} 0, & x \leq 0, \\ 1, & x > 0 \end{cases}$$



Cross-Entropy Loss

$$L = -\sum_{i=0}^{C-1} y_i \log(\hat{y}_i)$$

Where:

- C = number of classes
- y_i = true label (one-hot encoded): 1 if class i is correct, 0 otherwise
- $\hat{y}_i$ = predicted probability for class i (output from softmax)

Key Properties:

- If prediction is perfect ($\hat{y} = 1$ for correct class): $L = -\log(1) = 0$
- If prediction is terrible ($\hat{y} \to 0$ for correct class): $L \to \infty$
- Only the correct class contributes to the loss (all other $y_i = 0$)