MAE 250H, Spring 2019
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Final Project, Due Friday, June 14

The final project requires the solution for the viscous external flow past a bluff body, as shown in Figure 1. The body's shape is arbitrary (though a cylinder is a natural target to start with). The Reynolds number is on the order of 100, for which we should expect that the wake will be a von Kármán vortex street. You are to solve this using the immersed boundary projection method (developed by Taira and Colonius), which we have already used in Homework 5 for potential flow.

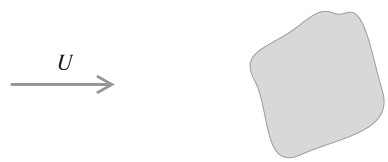

Figure 1: Bluff body

The most effective way to express the governing equations are for the velocity disturbance, $q' = q - U$. The advantage of using this, instead of the velocity q, is that it is expected to be negligible at large distances from the body. The semi-discrete governing equations are thus

$$\frac{\mathrm{d}q'}{\mathrm{d}t} = \frac{1}{\Delta x}N(q) + \frac{1}{Re\,\Delta x^2}Lq' - \frac{1}{\Delta x}Gp + E^T f_b, \tag{1}$$

where $N(q)$ is the usual non-linear term (with q, not q'). The associated no-slip condition on the body is

$$Eq' = u_b - U, \tag{2}$$

which ensures that the velocity equals u_b on the body's Lagrange points.

You are welcome to choose either of two approaches for solving the Navier–Stokes system: the velocity–pressure formulation (e.g., in delta formulation), like we used for the midterm project, but now in a fractional step formulation that must consider the solution for f_b (see Taira lecture notes); or a vorticity formulation,

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = \frac{1}{Re\,\Delta x^2}L\gamma + \frac{C^T}{\Delta x}\mathcal{N}(q) + C^T E^T f_b, \tag{3}$$

where $\gamma = C^T q$. This system would be combined with solving

$$Ls = -\gamma \tag{4}$$

$$q = Cs + U \tag{5}$$

$$-ECL^{-1}\gamma = u_b - U \tag{6}$$

where s is the streamfunction. The overall system would be solved similarly to the velocity–pressure form, using a fractional step method.

What you'll need to do:

1. Extend the immersed boundary projection method so that it can be used for Navier–Stokes problems. This requires regularizing vector force data on Lagrange points to the grid, and interpolating grid velocity data to the Lagrange points.

2. Implement a means of solving the Poisson equation in infinite domains (i.e., domains that are not meant to have external boundaries). We will discuss this in class, and I will present the means of doing it — the lattice Green's function. I will likely provide some functions to do this, as well as a means of carrying out viscous diffusion with an integrating factor method.

3. Formulate a fractional-step algorithm that can be used to solve the overall system.

4. Solve the problem for flow past a circular cylinder (of radius 1) with uniform flow of velocity 1 (i.e., non-dimensionalized by radius R and velocity U, at Reynolds number 100. Note that this flow will remain symmetric unless you explicitly perturb it; the perturbed flow will exhibit alternating vortex shedding. Compute the time-varying force on the body, from a sum over the Lagrange forces:

$$F = \sum_{p=1}^{N_b} f_{b,p} \tag{7}$$

   From this, compute the time-varying lift and drag coefficients, $C_L = 2F_y/\rho U^2 D$ and $C_D = 2F_x/\rho U^2 D$, and compare them with the results reported in the provided paper by Taira & Colonius (JCP 2007). You should provide plots of these coefficients versus time, along with some snapshots of your vorticity field (the best to visualize your flow).

5. Write a report that details your results, including your convergence analysis and your comparison with the results of previous work. Comment on the physics of the problem, including the behaviors as Reynolds number increases.

6. For the M.S. comprehensive exam: compute the flow past an interesting shape (e.g., square, flat plate, airfoil, or something more exotic), at around the same Reynolds number, and report your results.