Monotonic Abstraction for Programs with Muliply-Linked Structures

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Introduction

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- Verification of programs

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- Programs written in a subset of C

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- Operating on multiply linked data structures

Outline

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struct B {
    T1 t1;
    T2 t2;
    T3 t3;
    T4 t4;
}
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- Labels on nodes for variables

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- $\lambda : X \to \overline{M}$ is the *variable function*, where X is a set of variables

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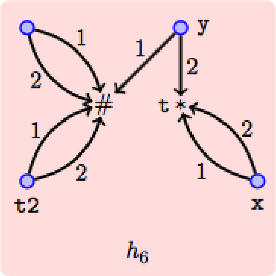

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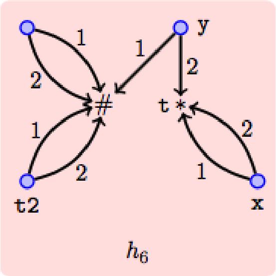

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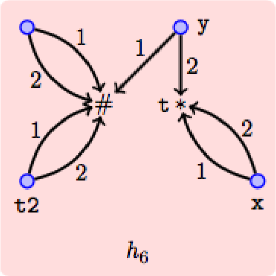

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- The heap invariant:
- $\forall c \in C\ \forall m \in M : |s^{-1}(m) \cap \tau^{-1}(c)| = 1$

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  - S is a set of states which are pairs of (pc, h) where pc is the Program Counter
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- Given states s, s', $s \rightarrow s'$ exists if an operation is defined which transforms h into h'

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A signature is a heap with some parts 'missing'. It represents the set of all heaps that have at least the property or structural information described by this signature.

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- We say that a heap h satisfies sig_1, written as $h \in [\![sig_1]\!]$, if $sig_1 \sqsubseteq h$

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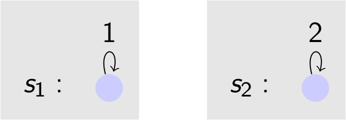

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A transition system T is monotonic if the following holds. For any states sig_1, sig_2 and sig_3 such that $sig_1 \sqsubseteq sig_2$ and $sig_1 \longrightarrow sig_3$, we can always find a state sig_4 such that $sig_2 \longrightarrow sig_4$ and $sig_3 \sqsubseteq sig_4$.

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Definition

We say $s \longrightarrow_A s'$ for two signatures s and s' iff there is an s" such that s" $\sqsubseteq s$ and $s \longrightarrow s'$.

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- If the set of signatures is disjoint from the initial set of states $\Rightarrow$ program execution is safe

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Table 1. Experimental results

Program	Struct	Time	#Sig.
Traverse	DLL	11.4 s	294
Insert	DLL	3.5 s	121
Ordered Insert	DLL	19.4 s	793
Merge	DLL	6 min 40 s	8171
Reverse	DLL	10.8 s	395
Search	Tree	1.2 s	51
Insert	Tree	6.8 s	241

Prototype Results [ACV13]

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Any questions?

Parosh Aziz Abdulla, Jonathan Cederberg, and Tomáš Vojnar, *Monotonic abstraction for programs with multiply-linked structures*, International Journal of Foundations of Computer Science **24** (2013), no. 02, 187–210.