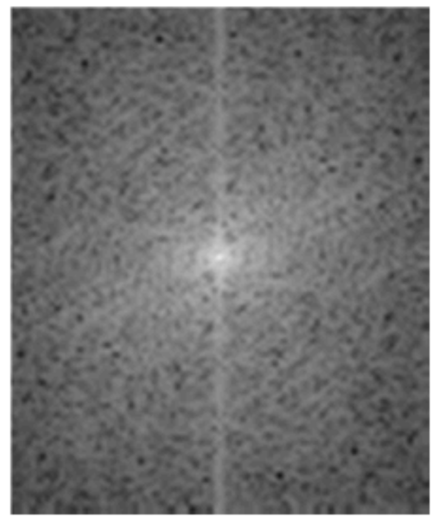
Kuliah 07: Fourier Transform (Teori Dasar)

Yeni Herdiyeni

Jean Baptiste Joseph Fourier

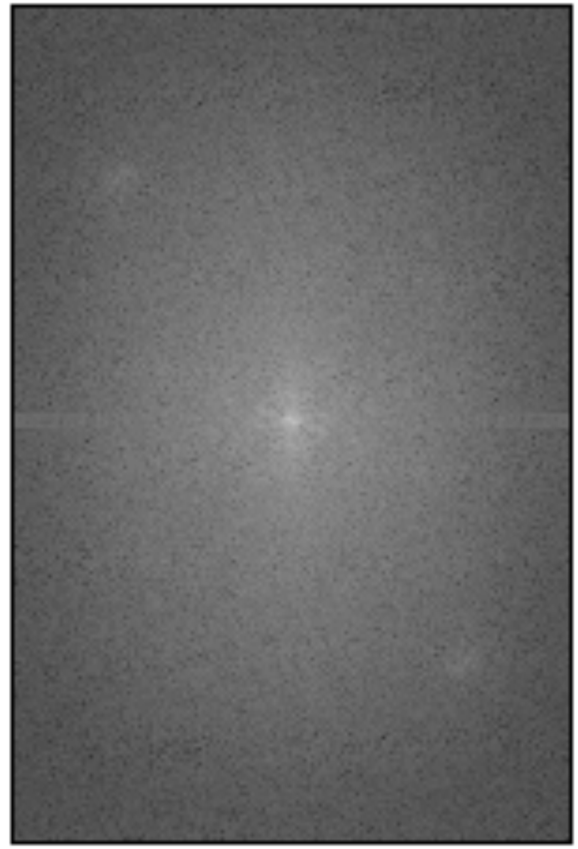
1768-1830

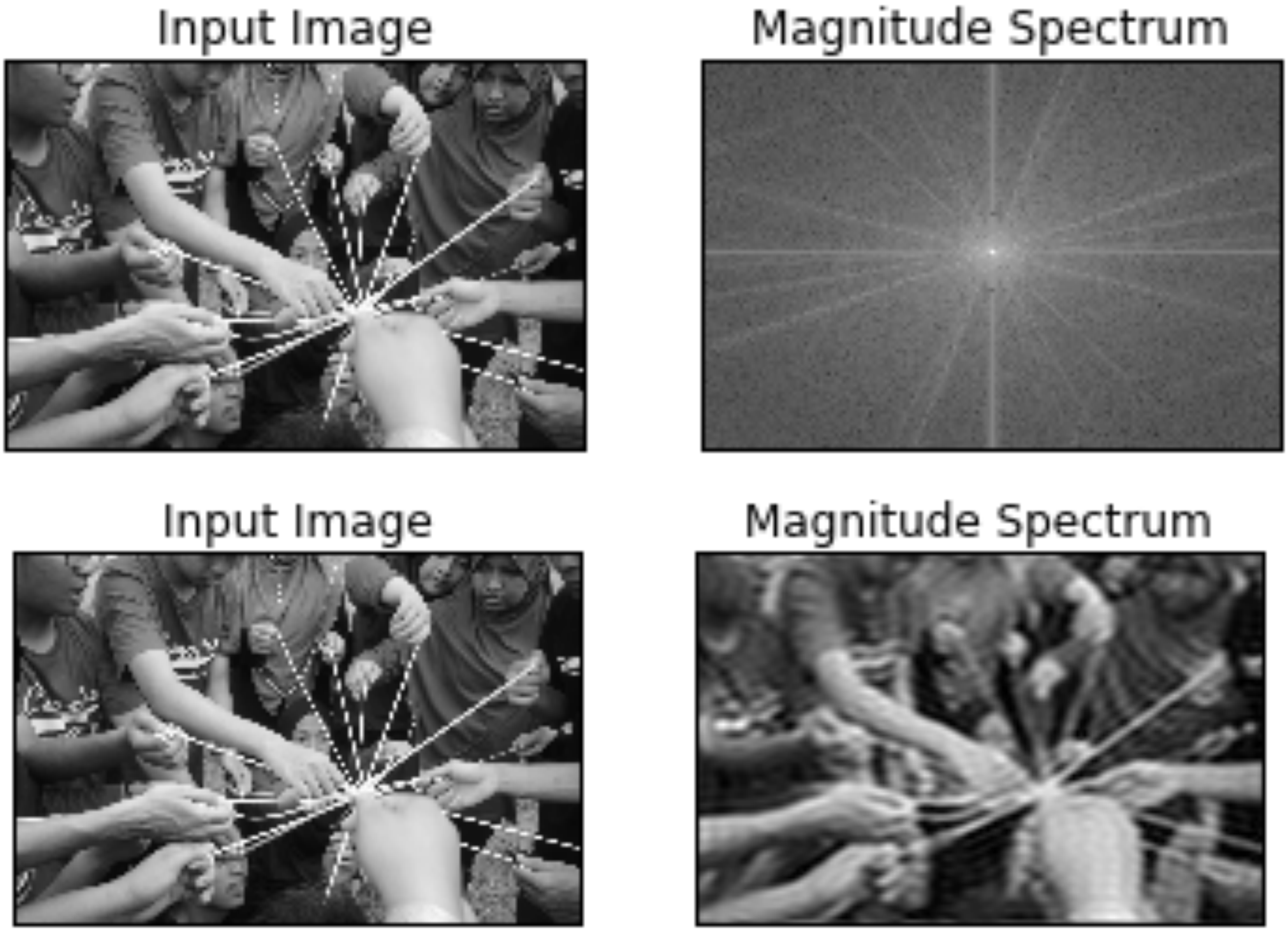
Jean Baptiste Joseph Fourier

- Fourier was born in Auxerre, France in 1768
  - Most famous for his work "*La Théorie Analitique de la Chaleur*" published in 1822
  - Translated into English in 1878: "*The Analytic Theory of Heat*"
- Nobody paid much attention when the work was first published
- One of the most important mathematical theories in modern engineering

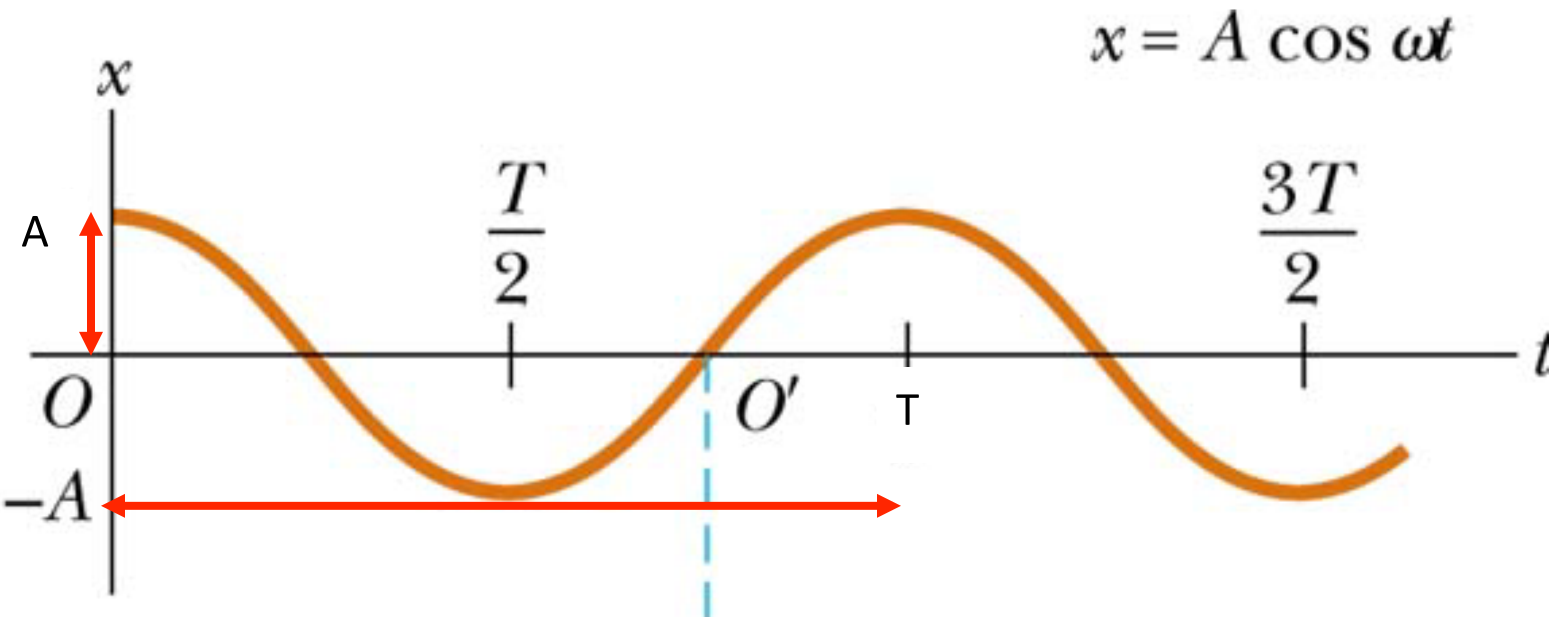
Input Image

Magnitude Spectrum



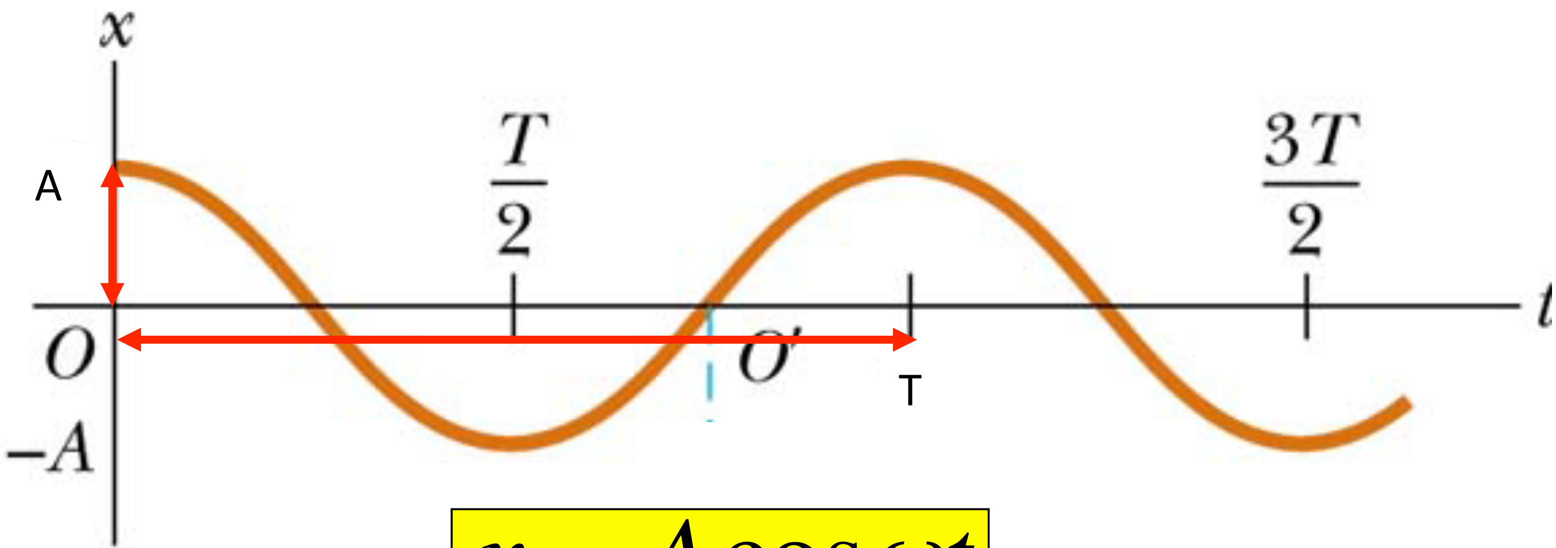

Input Image
Magnitude Spectrum
Input Image
Magnitude Spectrum

Gelombang



A : <u>amplitude</u> (length, m)   T : <u>period</u> (time, s)

Periode dan Frequency



$$\boxed{x = A\cos\omega t}$$

$$\omega T = 2\pi$$

$$\Downarrow$$

$$\boxed{T = \frac{2\pi}{\omega}, \quad f = \frac{\omega}{2\pi}}$$

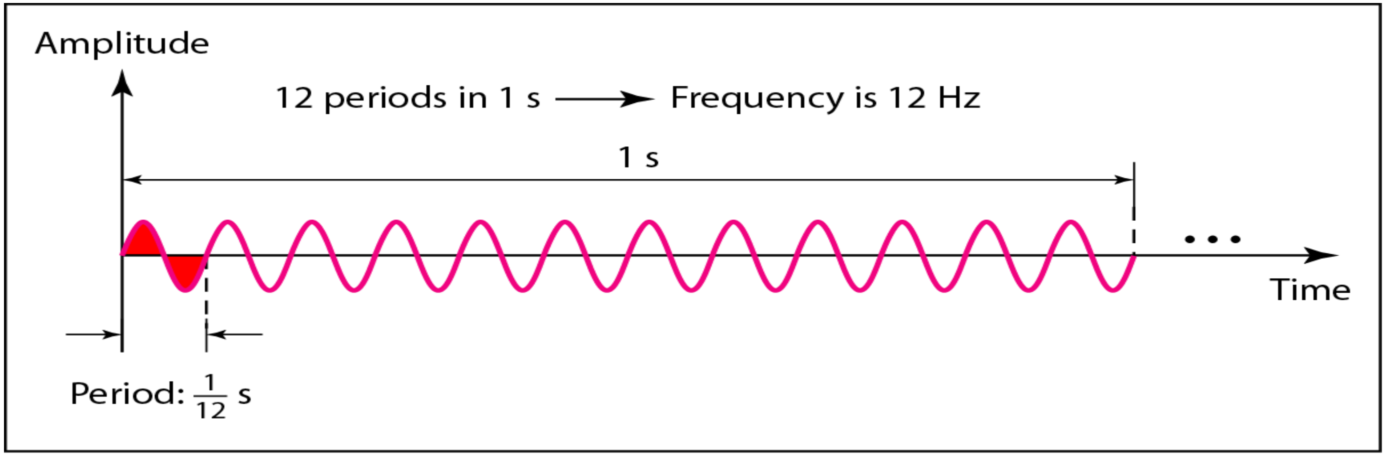
Amplitude: A

Period: T

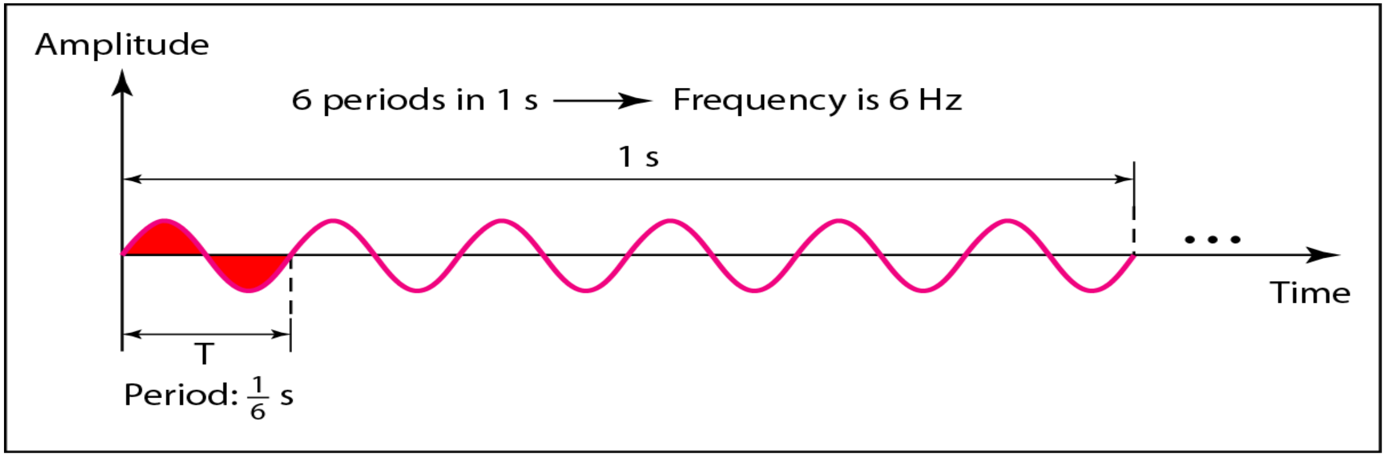
Frequency: f = 1/T

Angular frequency: ω

Gelombang



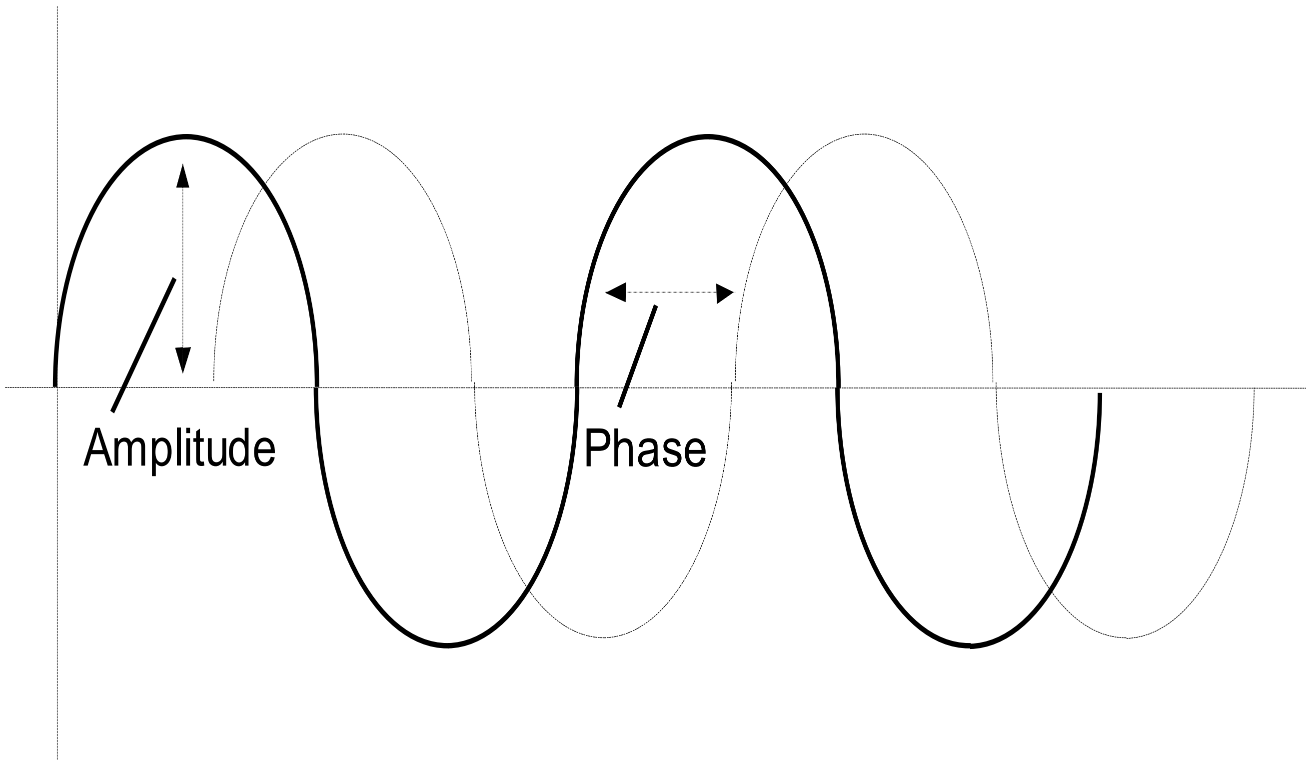
a. A signal with a frequency of 12 Hz



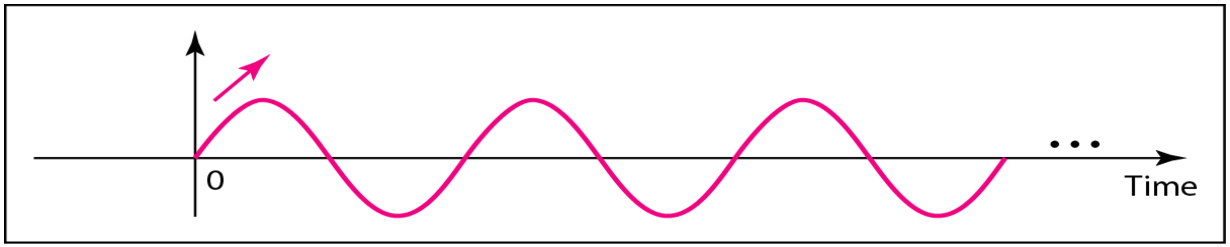
b. A signal with a frequency of 6 Hz

$$f = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{f}$$

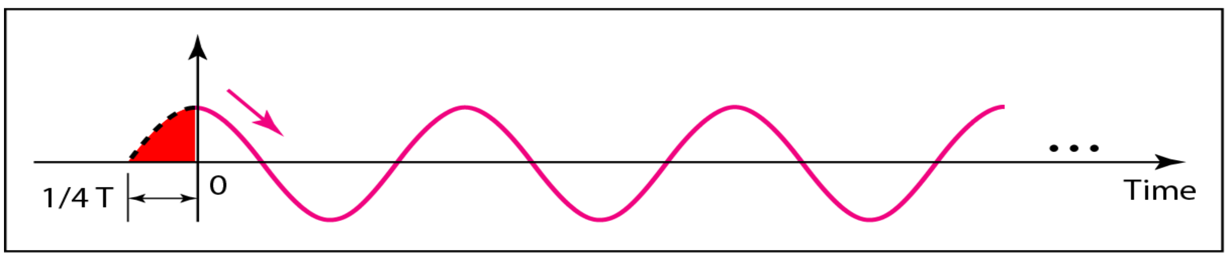
Amplitude dan Phase



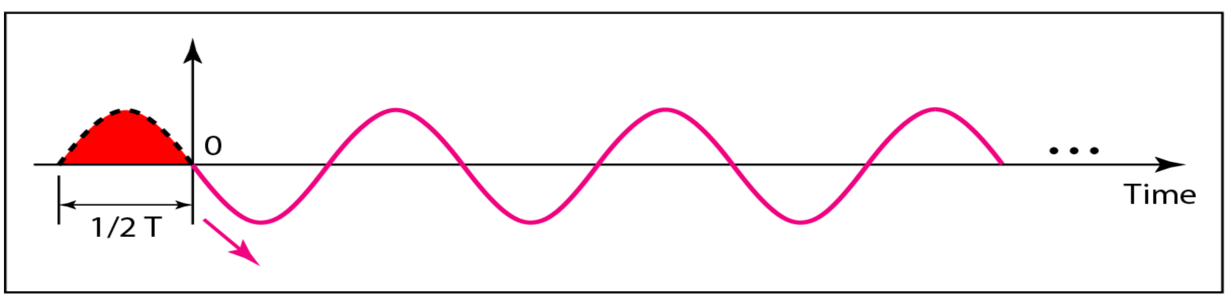
Phase



a. 0 degrees



b. 90 degrees



c. 180 degrees

Phases

Often a phase ϕ is included to shift the timing of the peak:

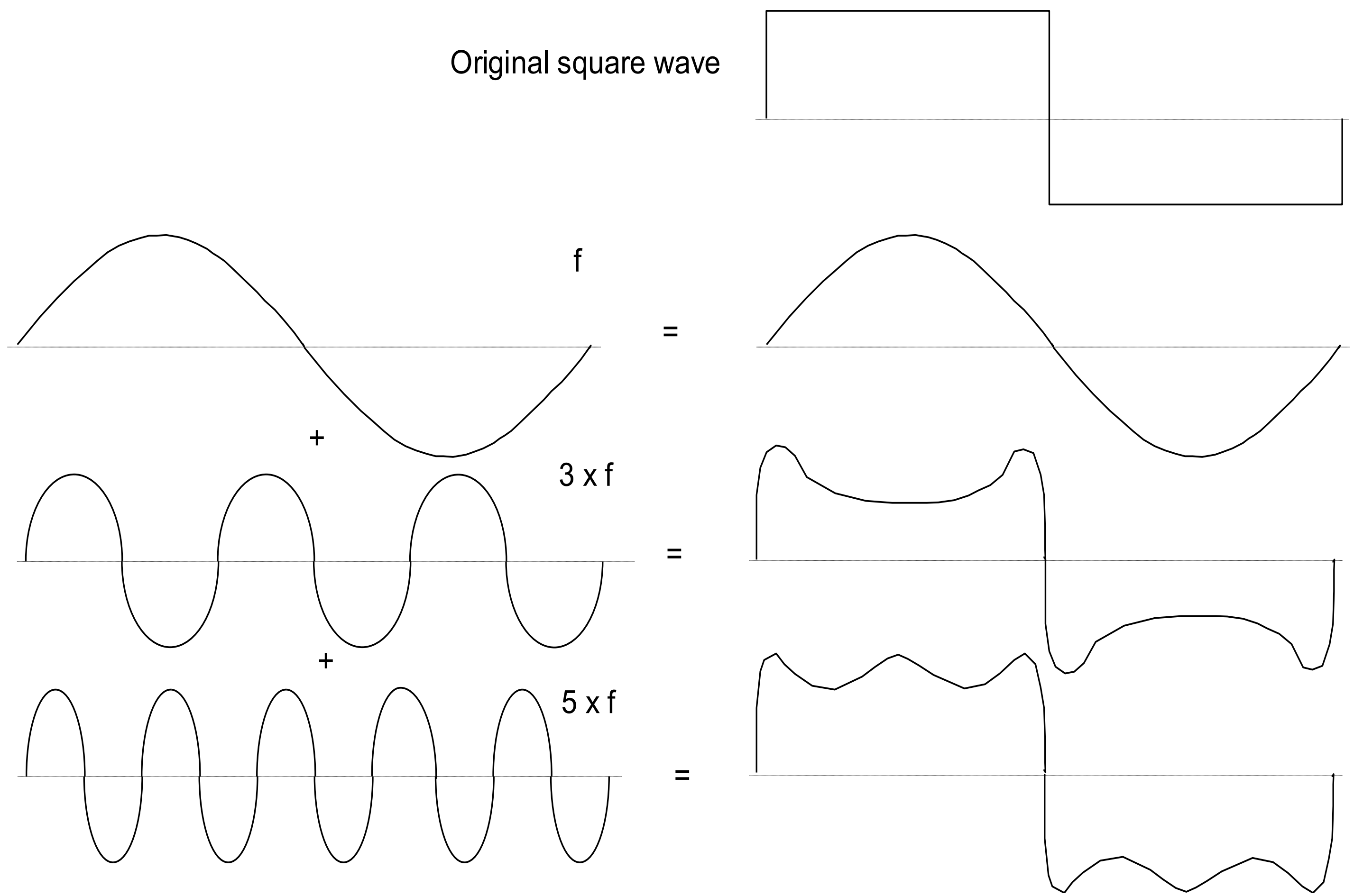
$$x = A\cos\left(\omega t - \phi\right)$$

$$= A\cos\left(\omega(t - t_0)\right)$$ for peak at $t = t_0$

Phase of 90-degrees changes cosine to sine

$$\cos\left(\omega t - \frac{\pi}{2}\right) = \sin\left(\omega t\right)$$

Fourier Transform



Frekuensi Domain

- ω**, angular frequency** in radians per unit distance,or
- ***f*, rotational frequency** in cycles per unit distance. $\boldsymbol{\omega=2\pi f}$
- The **period** of a signal, $\mathbf{T=1/f= 2\pi/ \omega}$

Examples:

- The signal [0 1 0 1 0 1…] has frequency f=.5 (.5 cycles per sample)

Transformasi Fourier 1D

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx \quad \text{where } j = \sqrt{-1}$$

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

Fungsi Basis

$$F(u)=\int_{-\infty}^{\infty} f(x)e^{-i2\pi ux}dx$$

Faktor Skala

Note: $e^{ik}=\cos k+i\sin k \quad i=\sqrt{-1}$

$$e^{\pm ix}=\cos(x)\pm i\sin(x)$$

$$F(u)=\int_{\infty}^{-\infty} f(x)(\cos 2\pi ux - i\sin 2\pi ux)\,dx$$

$$F(u)=\int_{-\infty}^{\infty} f(x)e^{-i2\pi ux}dx=\int_{-\infty}^{\infty} f(x)\{\cos(2\pi ux)-i\sin(2\pi ux)\}dx$$

$$f(x)=\int_{-\infty}^{\infty} F(u)e^{i2\pi ux}du=\int_{-\infty}^{\infty} F(u)\{\cos(2\pi ux)+i\sin(2\pi ux)\}du$$

Spektrum dan Phase

$$F(u) = R(u) + iI(u) = |F(u)|e^{i\phi(u)}$$

$$|F(u)| = \sqrt{R^2(u) + I^2(u)}$$

$$\Theta(u) = \tan^{-1}\left[\frac{I(u)}{R(u)}\right]$$

Fungsi Diskret

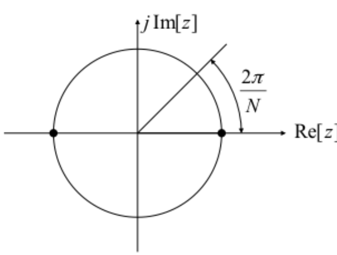
$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx \quad \text{where } j = \sqrt{-1}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

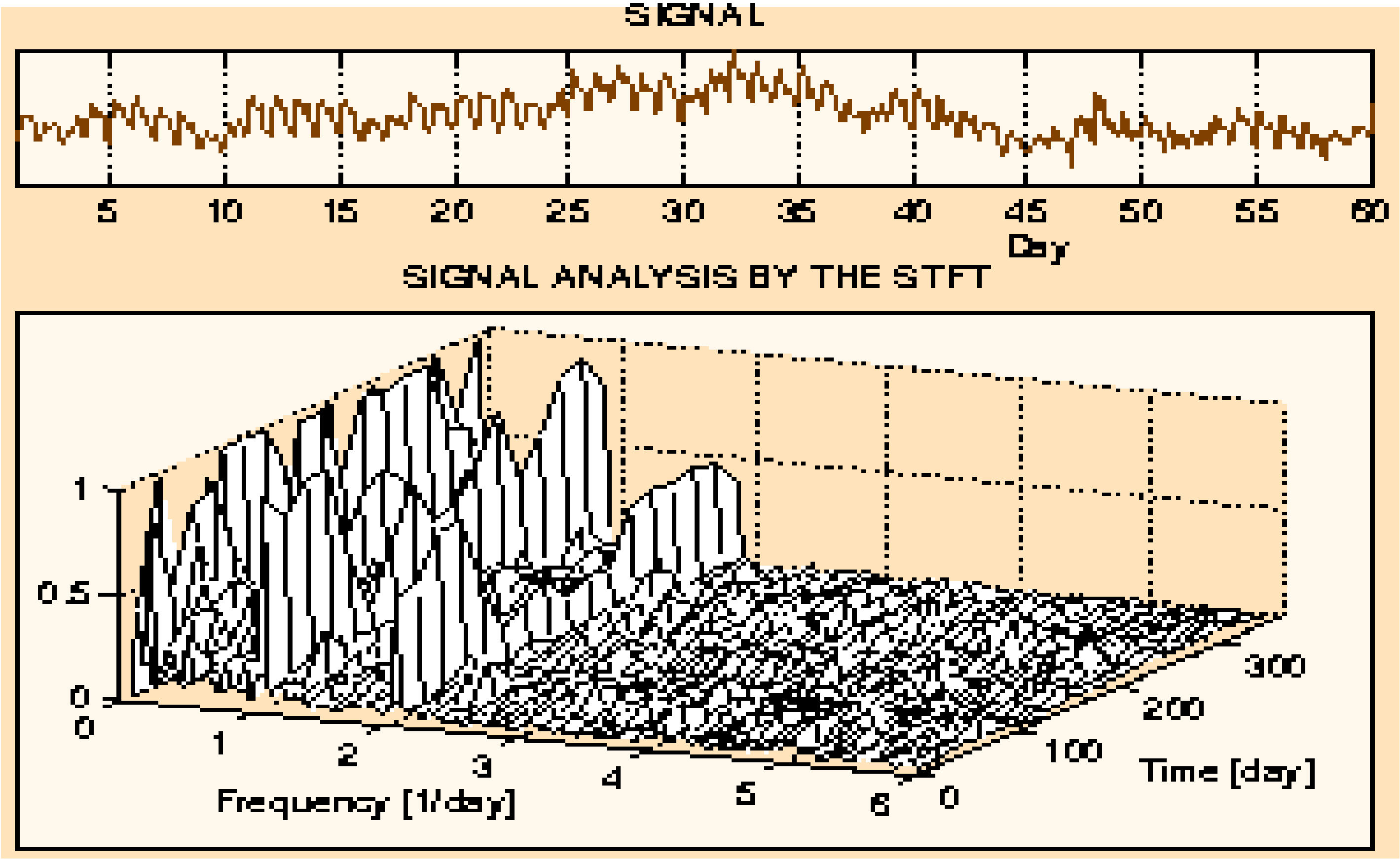
$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \quad \text{for } u = 0,1,2,\ldots,M-1$$

$$\cos(-\theta) = \cos\theta$$

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x)[\cos 2\pi ux/M - j\sin 2\pi ux/M]$$



Transformasi Fourier



Transformasi Fourier 2D

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{j2\pi(ux+vy)}dudv$$

Fungsi Diskret

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$$

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

$$\text{for } u = 0,1,2,\ldots,M-1, v = 0,1,2,\ldots,N-1$$

Spectrum dan Phase

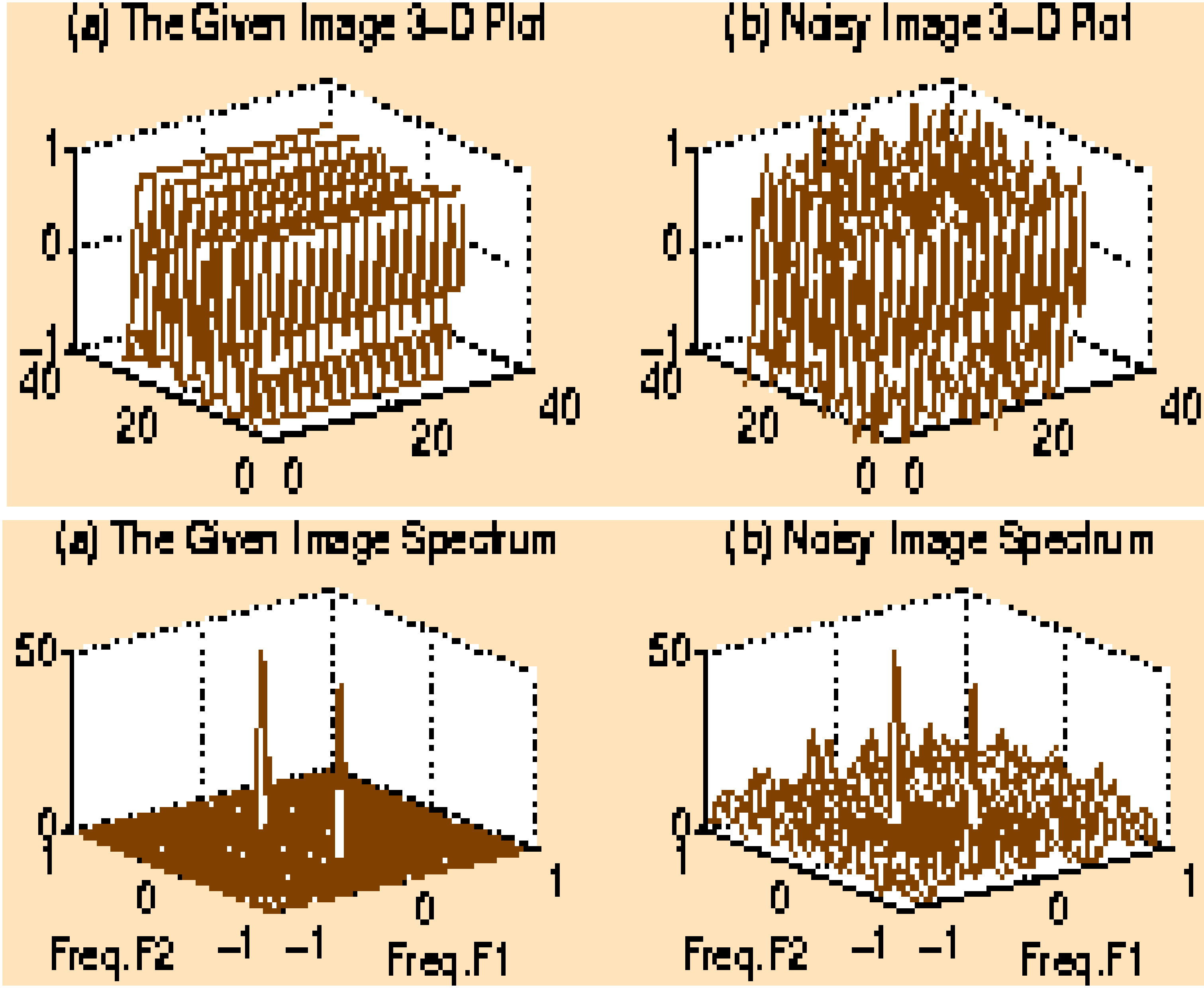
$$f(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi(ux/M+vy/N)}$$

$$\text{for } x=0,1,2,\ldots,M-1,\ y=0,1,2,\ldots,N-1$$

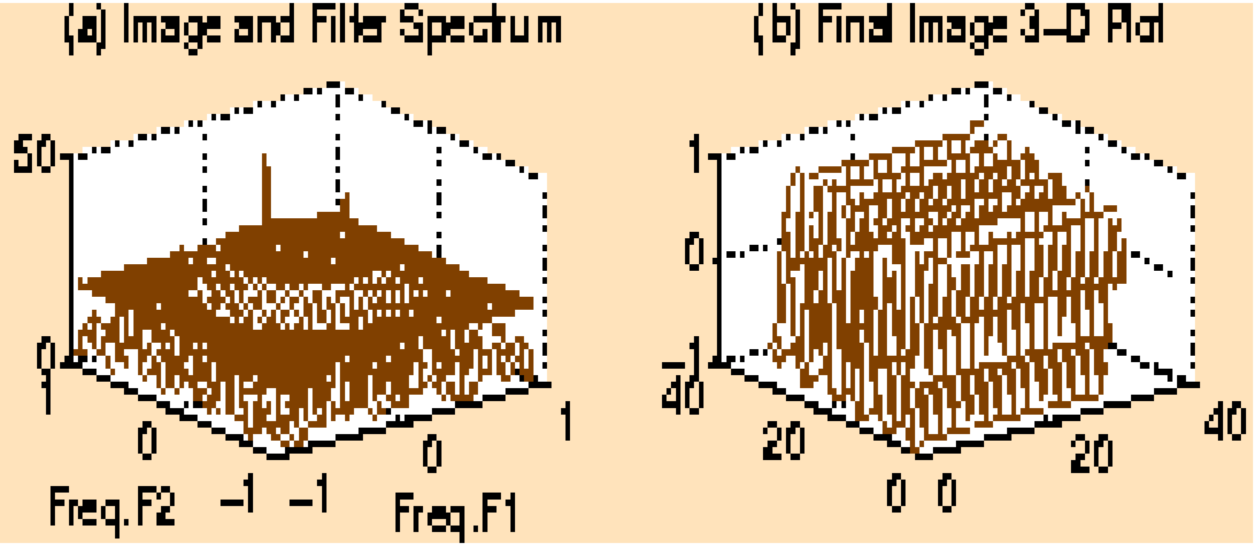
$$\left|F(u,v)\right|=\left[R^2(u,v)+I^2(u,v)\right]^{\frac{1}{2}} \quad (\text{ spectrum})$$

$$\phi(u,v)=\tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right] \quad (\text{phase angle})$$

$$P(u,v)=\left|F(u,v)\right|^2=R^2(u,v)+I^2(u,v) \quad (\text{power spectrum})$$



(a) The Given Image 3-D Plot
(b) Noisy Image 3-D Plot
1
0
-1
40
20
0 0
20
40
(a) The Given Image Spectrum
(b) Noisy Image Spectrum
50
0
1
0
-1 -1
0
1
Freq. F2
Freq. F1


(a) Image and Filter Spectrum
50
0
1
0
-1
-1
0
1
Freq.F2
Freq.F1
(b) Final Image 3-D Plot
1
0
-1
40
20
0
0
20
40

Berikut pseudocode untuk transformasi Fourier:

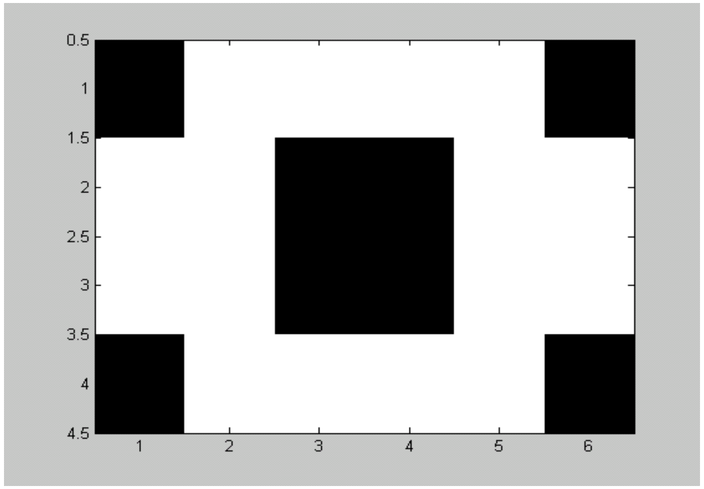
```
/* Data type for N set of complex number */
Double fx[N][2];
Double fu[N][2];
/*Fourier transform to get F(0)….F(N-1) */
For (u=0; u<N; u++) {
        For (k=0; k<N; k++){
                P=2*PI*u*k/N;
                /*real */
                Fu[u][0] += fx[k][0]*cos(p) + fx[k][1] * sin (p);
                /*imaginary */
                Fu[u][1] += fx[k][1]*cos(p) – fk[k][0]*sin(p)
        }
        /* multiply the result by 1/N */
        Fu[u][0] /= N;
        Fu[u][1] /= N;
                    }
```

Contoh

contoh :

Diketahui f(x,y) adalah sebagai berikut :

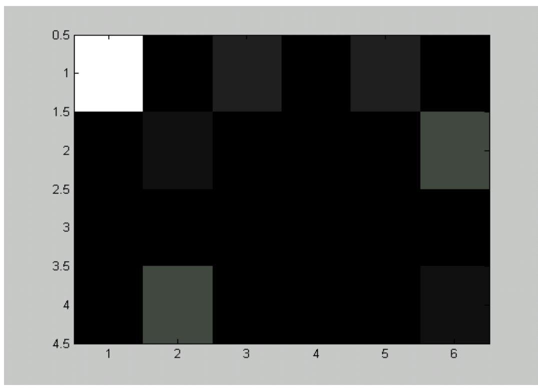
0	1	1	1	1	0
1	1	0	0	1	1
1	1	0	0	1	1
0	1	1	1	1	0

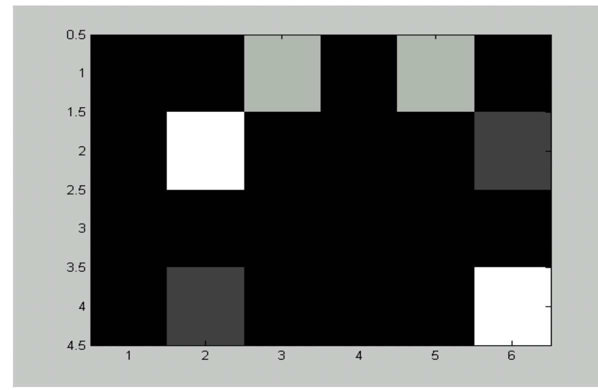


$$F(k_1,k_2)=\sum_{n_1=0}^{4}\sum_{n_2=0}^{6} f(n_1,n_2).e^{-j2\pi T(k_1n_1/4+k_2n_2/6)}$$

Hasil dari DFT adalah sebagai berikut :

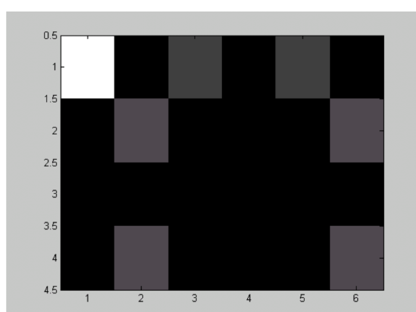
16	0	-2 - 3.46i	0	-2 + 3.46i	0
0	-1.27 - 4.73i	0	0	0	4.73 - 1.27i
0	0	0	0	0	0
0	-4.73+ 1.27i	0	0	0	1.27 + 4.73i

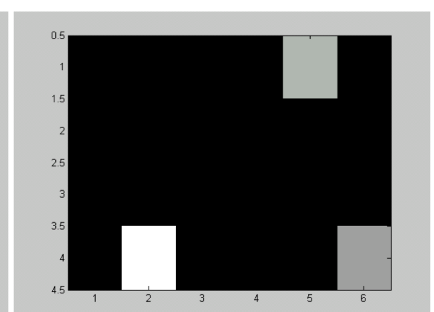




Bagian Real

Bagian Imaginer





Magnitude

Phase

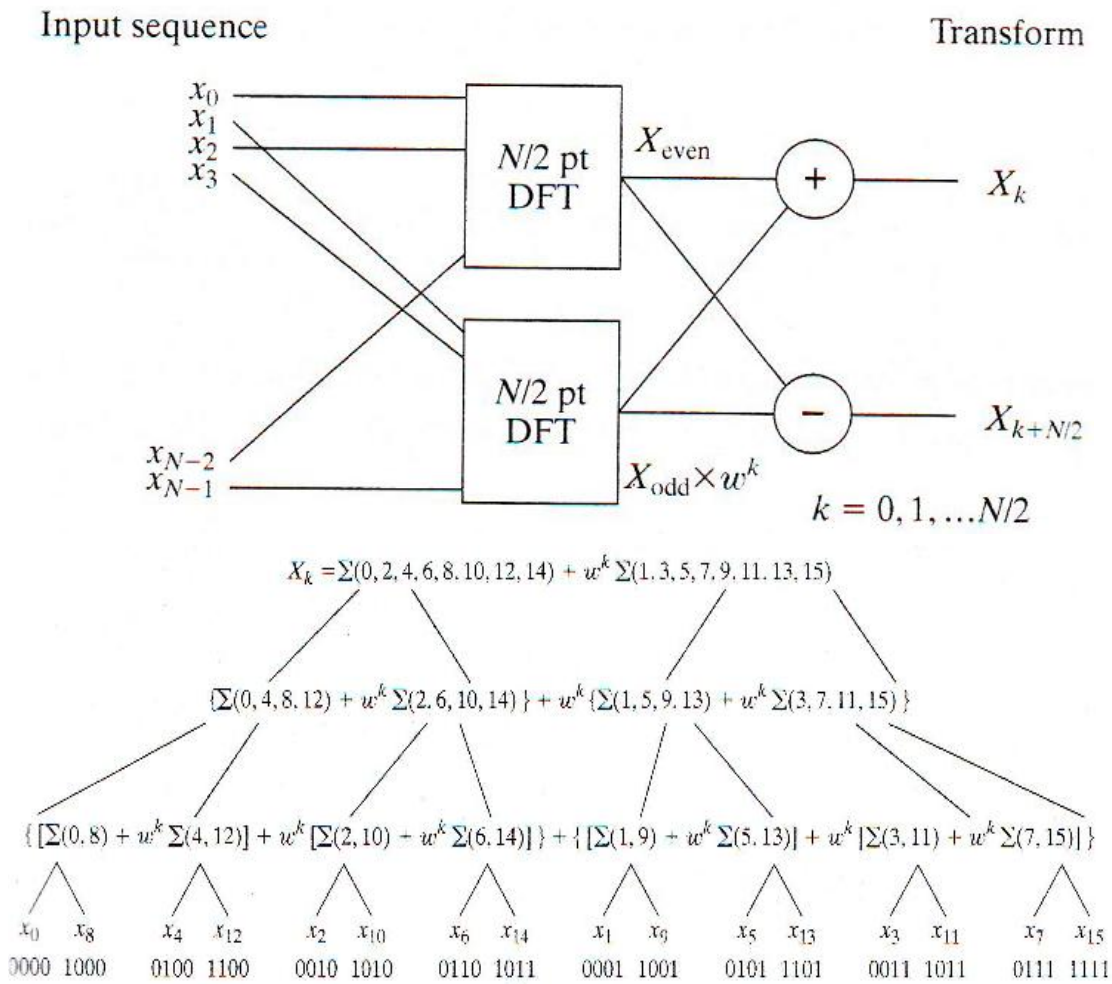
DFT

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N}kn} \qquad k = 0, \dots, N-1$$

$$X_k = x_0 w^0 + x_1 w^1 + x_2 w^2 + x_3 w^3 + \dots x_{N-1} w^{N-1}$$

$$\begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_k \\ \vdots \\ X_{N-1} \end{bmatrix} = \frac{1}{N} \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & w & w^2 & w^3 & \cdots & w^{N-1} \\ 1 & w^2 & w^4 & w^6 & \cdots & w^{2(N-1)} \\ 1 & w^3 & w^6 & w^9 & \cdots & w^{3(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & w^k & w^{2k} & w^{3k} & \cdots & w^{(N-1)k} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & w^{N-1} & w^{2(N-1)} & w^{3(N-1)} & \cdots & w^{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_k \\ \vdots \\ x_{N-1} \end{bmatrix}$$

Fast Fourier Transform



This method allows us to find the DFT in O(NLogN), in sequential time, instead of (N^2)