

Roll No. []

KONGU ENGINEERING COLLEGE, PERUNDURAI 638 060

EVEN SEMESTER 2017-2018

CONTINUOUS ASSESSMENT TEST 1 - JANUARY 2018

(Regulations 2014)

Programme : BE	Date : 20.01.2018
Branch : CSE	Time : 9.15 am – 10.45 am
Semester : IV	
Course Code : 14CST43	Duration : 1 $\frac{1}{2}$ Hours
Course Name : Design and Analysis of Algorithms	Max. Marks : 50

PART - A (10 \times 2 = 20 Marks)

ANSWER ALL THE QUESTIONS

1.	Calculate gcd(60,22).	(CO1)	[K3]
2.	Define convex hull problem.	(CO1)	[K3]
3.	Compare interpolation and extrapolation.	(CO1)	[K2]
4.	How does dynamic visualization differ from static visualization?	(CO1)	[K2]
5.	Solve the following recurrence relation $x(n)=3x(n-1)$ for $n>1$; $x(1)=4$.	(CO2)	[K3]
6.	Find the order of growth of $\sum_{i=2}^{n-1} \lg i^2$	(CO1)	[K2]
7.	Arrange the following function according to their order $(n+1)! 2^{3n}, 2n^4 + 2n^3, n\log n, \log n, 6n, 8n^2$	(CO1)	[K3]
8.	Find the average case efficiency of sequential search.	(CO1)	[K4]
9.	Define θ notation.	(CO1)	[K1]
10.	Write the general plan for analyzing time efficiency of recursive algorithm.	(CO1)	[K1]

Part - B (3 \times 10 = 30 Marks)

ANSWER ANY THREE QUESTIONS

11.	Design a recursive algorithm for computing a^n for any non negative integer n that is based on the formula $a^n=a^{n-1}+a^{n-1}$. Set up a recurrence relation for the number of additions made by the algorithm and solve it.	(10)	(CO2)	[K4]
12.	Write the recursive algorithm to compute the n^{th} Fibonacci number and also compute the efficiency of the above algorithm using homogeneous and inhomogeneous recurrence equation.	(10)	(CO2)	[K3]
13. i)	Write the basic efficiency classes of algorithm analysis.	(5)	(CO1)	[K1]
ii)	Mention any five important problem types in computing.	(5)	(CO1)	[K1]

14.	Design a brute force algorithm for the given text, count the number of substrings that start with 'A' and end with 'B'. Also calculate the efficiency of the algorithm.	(10)	(CO2)	[K4]
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Bloom's Taxonomy Level	Remembering (K1)	Understanding (K2)	Applying (K3)	Analysing (K4)	Evaluating (K5)	Creating (K6)
Percentage	26.6	16.8	10	36.6	-	-

Module Test-1 Answer Key.

$$\textcircled{1} \quad \gcd(60, 22) : \gcd(22, 16) = \gcd(16, 6) : \gcd(6, 4) = \gcd(4, 2) : \gcd(2, 0) \\ = 2$$

$\textcircled{2}$ Convex Hull Problem: To find the smallest convex polygon that would include all the points of a given set.

$\textcircled{3}$ Interpolation: data points that fall within the range of the data you have

Extrapolation: data point from beyond the range of your data set.

$\textcircled{4}$ Dynamic visualizations vs static visualization
 Shows continuous movie like presentation of an algorithm's operations

Shows an algorithm's progress through a series of still images.

$$5. \quad x(n) = 3x(n-1) \quad \forall n > 1, \quad x(1) = 4$$

$$x(n) = 3[x(n-1)]$$

$$= 3^2 [x(n-2)]$$

$$x(n-2) = 3^3 [x(n-3)]$$

$$x(n-k) = 3^k (x(n-k))$$

$$n-k=1 \quad k=n-1$$

$$= 3^{n-1} (x(n-n+1))$$

$$= 3^{n-1} \cdot x(1)$$

$$= 4 \cdot 3^{n-1} //$$

11.

$$\textcircled{6} \quad \sum_{i=2}^{n-1} \lg i^2$$

$$\begin{aligned}
 &= 2 \sum_{i=1}^n \log_2 i - \log n \\
 &= 2 \left[\sum_{i=1}^n \log_2 i - \cancel{\log n} \right] \\
 &= 2 \mathcal{O}(n \log n) - 2 \log n \\
 &= \mathcal{O}(n \log n)
 \end{aligned}$$

$$\textcircled{7} \quad \log n, \ln n, n \log n, 8n^2, 2^{n^4+2n^3}, (n+1)! 2^{3n}$$

12. To

$$\frac{P(n+1)}{2} + n(1-P)$$

$$\textcircled{8} \quad C_2 \cdot g(n) \leq E(n) \leq C_1 \cdot g(n) \quad \forall n \geq n_0$$

\textcircled{9} O-notation:

\textcircled{10} (i) Decide on a parameter indicating an input's

\textcircled{ii} Identify the alg. basic operations

\textcircled{iii} check whether the no. of times the basic op. is executed can vary on different input of size. if it can, the WC, AC, BC must be given separately.

\textcircled{iv} Set up a recurrence relation with an appropriate initial condition

\textcircled{v} Solve the recurrence relation.

2. Homogeneous:

$$F(n) = F(n-1) + F(n-2)$$

$$= F(0) = 0 \quad F(1) = 1$$

$$\therefore F(n) - F(n-1) - F(n-2) = 0. \quad (1)$$

Apply linear second order linear recurrence with
const. coefficients.

$$ar^2 + br + c = 0.$$

After applying

$$F(n) = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^n$$

From (1)

$$r^2 - r - 1 = 0$$

$$= \frac{1}{\sqrt{5}} (\phi^n - \hat{\phi}^n)$$

$$r_{1,2} = \frac{1 \pm \sqrt{5}}{2}$$

2

$$P(n) = P(n-1) + P(n-2) + 1 \quad \text{for } n \geq 1$$

$$P(0) = 0 \quad P(1) = 0$$

$$\therefore [P(n)+1] - [P(n-1)+1] - [P(n-2)+1] = 0$$

$$P(n) = P(n+1)$$

$$\therefore P(n) - P(n-1) - P(n-2) = 0 \quad P(0) = 1 \quad P(1) = 0$$

$$\therefore P(n) - P(n-1) = F(n+1) - 1 = \frac{1}{\sqrt{5}} (\phi^{n+1} - \hat{\phi}^{n+1})$$

$$\therefore P(n) \in O(\phi^n)$$

(b) (i) Time efficiency

Space efficiency.

Measuring I/O size

Unit for measuring running time

Orders of growth

W, B, A^C

(ii) Important Pb types

Sorting, Searching, String Processing, Graph Problem

Combinatorial Problems, Geometric Problem,

Numerical Problem.

14. for $i \in 0 \rightarrow n-1$ do
 $j = i$

 for $j \in i+1 \rightarrow n-1$ if ($A[i] = 'A'$) then

 for $j \in i+1 \rightarrow n-1$

 { ~~j++~~

 if ($A[j] = 'B'$) then

 count++

}

 return count

$$= \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} (n-1-i-1)+1$$

$$\text{eff: } = \sum_{i=0}^{n-1} (n-1-i)$$

$$= \sum_{i=0}^{n-1} (n-1) - \sum_{i=0}^{n-1} i$$

$$= (n-1) \cdot n - \frac{(n-1)(n-2)}{2}$$

$$= \frac{(n-1)}{2} [2n - n + 2]$$

$$= \frac{(n-1)(n+2)}{2} = \frac{n^2 + O(n^2)}{2}$$

$$\frac{n(n-1)}{2}$$

KONGU ENGINEERING COLLEGE, PERUNDURAI, ERODE-638052
DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
MODULE TEST - I ANALYSIS

Year	II	Semester	IV	Section	B	Date of Exam		20.1.18								
COURSE CODE & NAME:		14CST43 - Design and Analysis of Algorithm				Expected Level %		71								
FACULTY NAME :		K. Kousalya														
TOTAL STRENGTH:		61	No.of Students appeared :				61									
S.No	ROLL NO	Part -A (10 X 2 = 20 Marks)								Total [Out of 50]						
		Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	
Expected Marks to attainment		2	1	1	2	1	1	2	2	2	1	6	6	6	0	7
1	16csr059	2	0	0	1	2	0	1	0	2	0	3	10	5		0 26
2	16csr060	0	0	0	0	0	0	0	0	2	0	1	7	1		0 11
3	16csr061	0	0	0	2	0	2	0	0	2	0	0	8	10		10 34
4	16csr062	0	0	0	2	0	0	0	0	0	0	0	0	0		1 8
5	16csr063	1	2	2	2	2	2	1	2	2	2	2	9.5	10		10 47.5
6	16csr064	2	2	2	2	2	2	2	2	2	2	0	10	10		10 50
7	16csr065	2	2	2	2	0	2	2	2	2	2	0	10	10		10 48
8	16csr067	0	0	0	1	0	0	1	0	0	0	0	0	3		3 9
9	16csr068	0	0	0	2	0	0	1	0	0	0	0	0	0		0 10
10	16csr069	2	2	2	0	2	0	1	1	1	2	0	10	7		7 37
11	16csr070	2	2	2	2	1	0	2	2	2	0	0	8	9		7 39
12	16csr071	2	2	2	0	2	0	2	0	2	2	0	10	7		7 38
13	16csr072	0	2	2	2	2	0	0	2	2	2	5	10	10		0 39
14	16csr073	2	0	2	2	0	0	2	2	2	0	8	10	7		0 37
15	16csr074	2	2	0	0	0	0	2	0	1	1	4	7	6		0 25
16	16csr075	2	2	2	2	2	0	0	2	2	2	6	10	10		0 42
17	16csr076	2	2	0	2	2	0	1	2	2	0	2	10	0		7 32
18	16csr077	2	0	2	0	0	1	2	2	1	0	0	0	6		1 17
19	16csr078	0	2	2	1	2	2	2	2	2	2	9	10	10		0 46
20	16csr079	2	2	2	2	2	0	1	2	2	2	0	10	8		4 39
21	16csr080	2	0	2	2	0	0	0	0	2	0	0	9	6		7 30
22	16csr081	0	0	0	1	0	0	2	0	2	0	0	0	0		5 10
23	16csr082	0	0	2	2	2	2	0	2	2	2	0	10	6		8 38
24	16csr083	2	0	1	0	0	0	1	0	0	0	0	10	10		1 25
25	16csr084	0	0	2	2	2	0	2	2	2	0	8.5	10	0		10 40.5
26	16csr085	0	0	0	1	0	0	1	0	1	0	7	9	6		0 25
27	16csr086	2	2	0	2	2	0	2	2	1	2	3	10	10		3 41
28	16csr087	1	0	0	0	0	0	1	0	2	0	0	0	6		0 10
29	16csr088	2	2	2	2	2	0	1	0	2	0	2	10	10		10 45
30	16csr089	0	0	0	2	0	0	0	0	0	0	0	0	0		0 2
31	16csr090	2	2	2	2	1	0	2	2	2	2	0	7	10		7 44
32	16csr091	0	0	0	2	0	0	2	0	2	2	0	7	10		0 25
33	16csr092	2	2	0.5	2	2	0	2	0	2	0	5	10	0		7 34.5
34	16csr093	0	2	2	2	2	0	2	2	2	1	0	7	6		7 35
35	16csr094	0	0	0	0	0	0	0	0	0	0	0	0	0		0 0
36	16csr095	2	2	2	2	2	0	0	2	2	2	0	10	9.5		7 42.5
37	16csr096	0	0	0	0	0	0	0	0	0	0	0	0	0		0 3
38	16csr097	2	2	2	2	0	0	1	2	2	2	6	10	0		7 38

39	16csr098	2	2	2	2	2	2	2	2	2	2	0	10	7	
40	16csr099	2	2	2	2	0	0	0	0	0	0	6	10	0	
41	16csr100	2	0	0	0	0	0	0	0	0	0	0	0	6	
42	16csr101	0	2	0	1	0	0	2	0	0	0	0	10	10	
43	16csr102	2	2	2	0	2	0	2	2	2	2	9	10	0	
44	16csr103	2	2	2	2	0	2	1	0	2	0	0	10	10	
45	16csr104	2	2	2	2	0	0	1	0	2	0	0	10	5	
46	16csr105	2	0	0	0	0	0	1	0	2	0	7	8	5	
47	16csr106	2	0	2	2	0	2	0	2	2	2	2	10	5	
48	16csr107	2	0	0	0	0	0	0	0	0	0	0	0	0	
49	16csr108	0	0	0	0	0	0	0	2	0	2	0	7	6	
50	16csr109	0	0	0	2	0	0	2	2	2	2	6	0	10	
51	16csr110	2	2	2	2	0	2	2	2	2	0	10	7	9	
52	16csr111	2	0	0	2	0	1	2	2	2	0	0	6	6	
53	16csr112	2	2	0	0	0	0	2	0	2	0	1	10	6	
54	16csr113	2	2	2	1	0	0	1	0	2	2	7	10	7	
55	16csr114	2	2	2	2	0	0	1	0	2	0	3	0	6	
56	16csr115	0	2	2	2	0	0	1	0	2	0	0	10	7	
57	16csr116	0	2	2	1	2	0	0	0	0	0	0	8		
58	16csr117	2	2	2	2	0	1	2	2	2	7	0	8		
59	16csl238	0	2	1	0	0	0	0	0	0	0	0	0	6	
60	16csl239	0	0	0	1	0	0	0	1	0	0	0	1	2	
61	16csl240	0	0	0	0	0	0	0	0	2	0	0	0	6	
No. of Students scores upto Expected Level	35	33	34	35	29	9	25	26	42	26	16	43	40	0	27
% of scoring above the Attainment	57.38	54.10	55.74	57.38	47.54	14.75	40.98	42.62	68.85	42.62	26.23	70.49	65.57	0.00	44.26

Course Outcome Attainment Level Indicator

Range of Attainment	5			4			3			2			1																							
	>			-	71	-	-	64	-	-	57	-	50	<																						
Expected attainment for each question	100	50	50	100	50	50	100	100	100	50	60	60	60	0	70																					
Satisfaction attainment level based on level indicator	5	1	1	5	1	1	5	5	5	1	3	3	3	0	4																					
Mapping with CO	CO1	CO1	CO1	CO1	CO2	CO1	CO1	CO1	CO1	CO1	CO2	CO2	CO1	CO1	CO2																					
Attainment level of All CO	2	2	2	2	1	1	1	1	4	1	1	4	4	0	1																					
Attainment Level of Each CO	CO1		2.00		CO2		1.75		CO3		-		CO4																							
Overall attainment of CO	CO5		-																																	
Mapping with PO	1.93																																			
Attainment of PO	a, b, c, f, l, pso1, pso2																																			
Staff Charge	1.93																																			
	Shivaji Patel HoD / CSE																																			

DEPT. OF COMPUTER SCIENCE & ENGG.
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PERUNDURAI (TK), ERODE - 638 060



SSK	<i>[Signature]</i>
Name and Signature of Hall Supdt. with Date	

KONGU ENGINEERING COLLEGE

PERUNDURAI ERODE - 638 060.

(Autonomous)



Name of the Student	K. KEERTHANA	Register No.	1 6 C S R 1 0 3
Programme	BE	Branch & Semester	CSE-B, IV
Course Code and Name	IHCST43 - Design and Analysis of algorithm.	Date	20.1.2018

No. of Pages Used
13.

MARKS TO BE FILLED IN BY THE EXAMINER

PART - A		PART - B		Grand Total Max. Marks : 50
Question No.	Max Marks : 2	Question No.	Max Marks : 10	
1	2	11	i)	
2	2	11	ii)	9
3	2	12	i)	
4	2	12	ii)	10
5	2	13	i)	
6	0	13	ii)	
7	2	14	i)	
8	2	14	ii)	
9	2			
10	2			
TOTAL	18	TOTAL	29	

Total Marks in Words : *fourty seven*

47

*Verified
K. Keerthana*

INSTRUCTION TO THE CANDIDATE

1. Check the Question Paper, Programme, Course Code, Branch Name etc., before answering the questions.
2. Use both sides of the paper for answering questions.
3. POSSESSION OF ANY INCRIMINATING MATERIAL AND MALPRACTICE OF ANY NATURE IS PUNISHABLE AS PER RULES.

JK
Name of the Examiner

K. Keerthana
Signature of the Examiner
with Date

part - A.

Answer all the questions :

$$1. \gcd(60, 22).$$

$$\gcd(m, n) = \gcd(n, m \bmod n)$$

$$= \gcd(22, 16)$$

$$= \gcd(16, 6)$$

$$= \gcd(6, 4)$$

$$= \gcd(4, 2)$$

$$\checkmark \quad \gcd(m, n) = \gcd(2, 0).$$

$\hookrightarrow x$ as output.

\therefore The result of $\gcd(60, 22)$ is 2. $1 - n = 0$ due

$$(1-1-n)x^8 = (1-n)x$$

$$(6-0)x^8 = (1-0)x$$

2) convex hull problem:

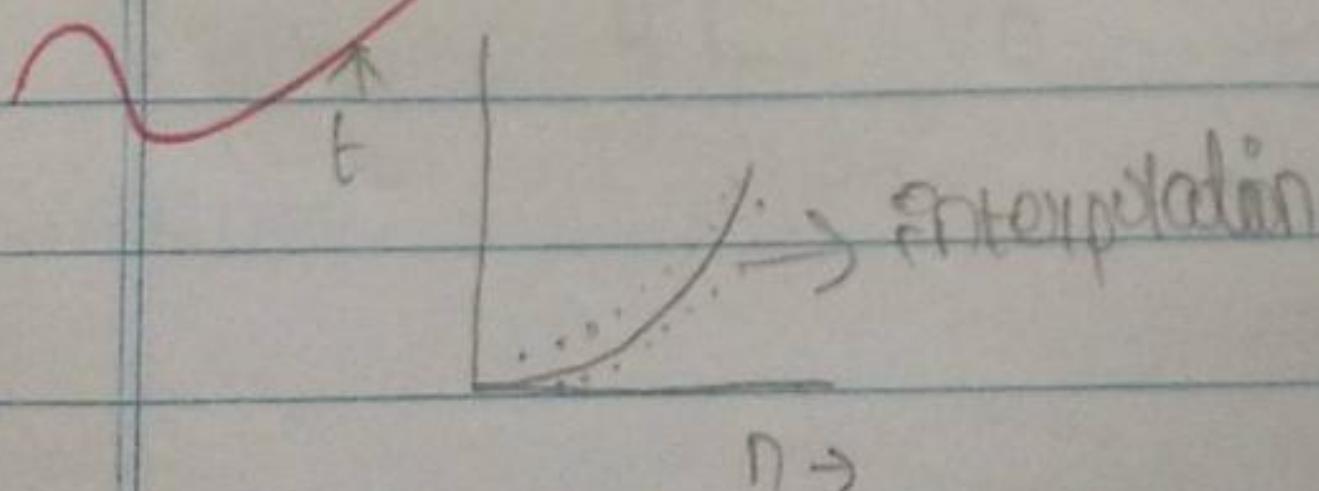
\checkmark To find a small object that covers all

the points in the given plane.

3.

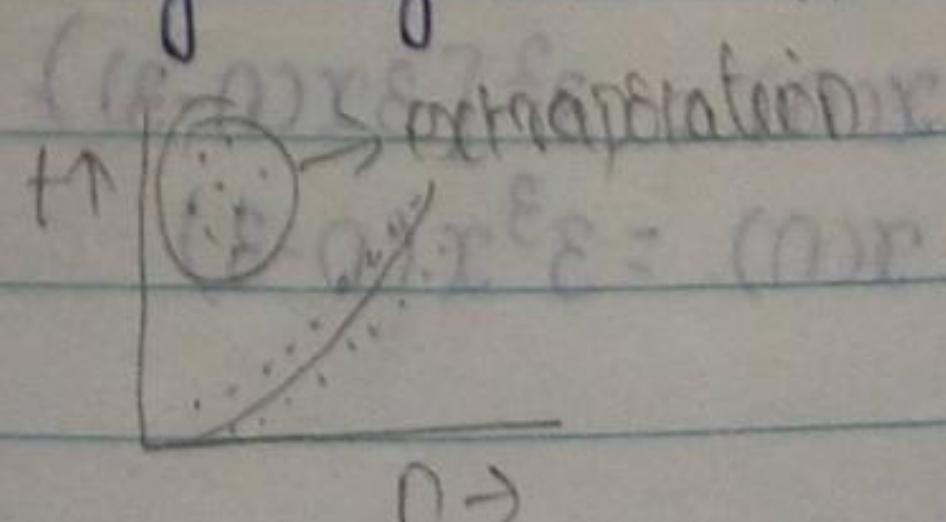
Interpolation

* Interpolation is the operation that the points are closer to the curve.



extrapolation.

* Extrapolation is the operation that the points are away from the curve.



4) dynamic visualization differ from static visualization.

→ in static visualization, the input can be given and it can displayed in form of graph, it doesn't have any movement.

✓ → in dynamic visualization, the input can be given and it can displayed in form of animations, it can have movements.

$$5) x(n) = 3x(n-1) \quad n > 1$$

$$x(1) = 4$$

$$x(n) = 3x(n-1)$$

$$(d, ec) \text{ bip} =$$

$$(d, d) \text{ bip} =$$

$$(d, d) \text{ bip} =$$

$$(e, d) \text{ bip} =$$

$$(0, e) \text{ bip} = (0, m) \text{ bip}$$

$$\text{Sub } n = n - 1$$

$$x(n-1) = 3x(n-1-1)$$

$$x(n-1) = 3x(n-2)$$

$$x(n) = 3[3x(n-2)]$$

$$x(n) = 3^2 x(n-2)$$

$$\text{Sub } n = n - 2$$

$$x(n-2) = 3x(n-2-1)$$

$$x(n-2) = 3x(n-3)$$

$$x(n) = 3^2 [3x(n-3)]$$

$$x(n) = 3^3 x(n-3)$$

In general equation

$$x(n) = 3^k x(n-k).$$

Sub $k=n-1$

$$= 3^{n-1} x(n-n+1)$$

$$= 3^{n-1} x(1)$$

$$= 3^{n-1} x(1)$$

$$= 3^{n-1} \cdot 1$$

$$= 1 \cdot 3^{n-1}$$

$$x(n) \approx 3^n$$

$$[x(n) \in O(3^n)]$$

b). $\sum_{i=2}^{n-1} \lg i^2$

$$\sum_{i=2}^{n-1} \lg i^2 = \lg \sum_{i=2}^{n-1} i^2.$$

$$\sum_{i=0}^{n-1} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$T(n) = \lg \left\lceil \frac{(n-1)n(n-1+1)(2(n-1)+1)}{6} \right\rceil$$

$$= \lg \left\lceil \frac{(n-1)n(2n-2+1)}{6} \right\rceil$$

$$= \lg \left\lceil \frac{(n-1)n(2n-1)}{6} \right\rceil = \lg \left\lceil \frac{(n^2-n)(2n-1)}{6} \right\rceil \approx n^2.$$

$$T(n) \in O(n^2),$$

7). Arranging order:

$\log n, 6n, n\log n, 8n^2, 8n^4 + 8n^3, (n+1)! 2^{3n}$

$C, \log n, n, \log n, \log \log n, n^{\frac{1}{2}}$

8). average case of sequential search:

Alg Linear Search

Alg linear [a[], n, key]

{ for $i=1$ to $n-1$.

{ if $[key == a[i]]$ basic operation.

return false

y

return true.

Average case = No. of comparison needed in 1st location \times probability of 1st location

No. of comparison needed in 2nd location \times probability of 2st location

No. of comparison needed in n^{th} location \times probability of n^{th} location

$$\text{average case } T(n) = 1 \times \frac{1}{n} + 2 \times \frac{1}{n} + \dots + n \times \frac{1}{n}$$

$$= \frac{1}{n} (1+2+\dots+n)$$

$$= \frac{1}{n} \left(\frac{n(n+1)}{2} \right)$$

$$= \frac{n+1}{2} \approx n$$

Removing constant values.

$$avg(T(n)) = O(n), \quad [avg(T(n)) \in O(n)]$$

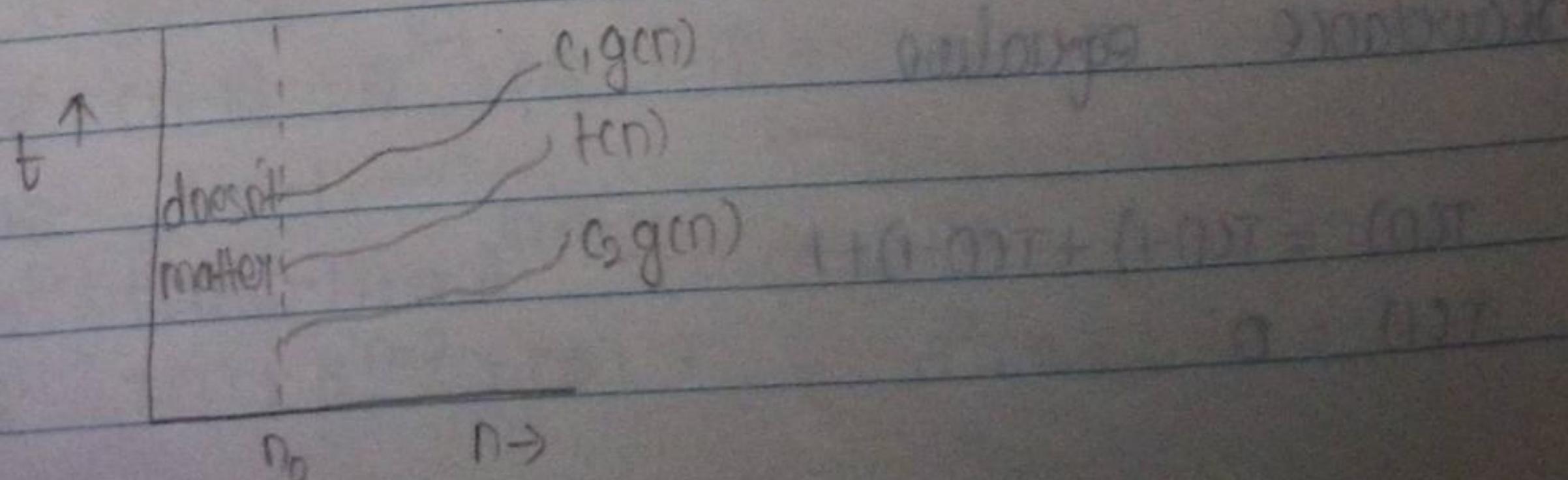
Q) Θ notation:-

* It is also called tight bound.

* If $T(n)$ is said to be $\Theta(g(n))$, then it is denoted by $T(n) \in \Theta(g(n))$.

e.g. If $T(n)$ is bounded both above and below by some constant multiples of $g(n)$. for all large n , there exists such as some positive constant c_1 and c_2 ($c_1 > c_2$) non-negative integer n_0

$$\text{i.e. } c_2 g(n) \leq T(n) \leq c_1 g(n) \quad \forall n \geq n_0.$$



10) general plan for analysing recursive algorithm.

* determine the input range.

* to identify the basic operations

* How long the basic operations can be executed?

* set the recurrence equation.

* solve and find the execution time

part-B.

answer any three:

11. Alg a pow (n,a).

{

if ($n = 1$)

return 0;

else

return (a pow (n-1) + a pow (n-1));

y.

recurrence equation

$$T(n) = T(n-1) + T(n-1) + 1$$

$$T(1) = 0.$$

Sub

$$T(n) = 2T(n-1) + 1$$

$$T(1) = 0$$

$$\text{Sub } T(n) = T(n-1)$$

$$T(n-1) =$$

$$T(n) = 2T(n-1) + 1$$

$$T(1) = 0$$

$$\text{Sub } n = n-1$$

$$T(n-1) = 2T(n-1-1) + 1$$

$$T(n-1) = 2T(n-2) + 1$$

$$T(n) = 2[2T(n-2) + 1] + 1$$

$$= 2^2 T(n-2) + 2 + 1$$

$$T(n) = 2^2 T(n-2) + 2 + 1$$

$$\text{Sub } n = n-2.$$

$$T(n-2) = 2T(n-2-1) + 1$$

$$T(n-2) = 2T(n-3) + 1$$

$$T(n) = 2^2 [2T(n-3) + 1] + 2 + 1$$

$$T(n) = 2^3 T(n-3) + 2^2 + 2 + 1$$

In general.

$$T(n) = 2^K T(n-K) + 2^{K-1} T(n-(K-1)) + \dots + 2^2 + 2 + 1$$

$$\text{Sub } K = n-1$$

$$= 2^{n-1} T(n-(n-1)) + 2^{(n-1-1)} T(n-(n-1-1)) + \dots + 2^2 + 2 + 1$$

$$= 2^{n-1} T(1) + 2^{n-2} T(2) + \dots + 2^2 + 2 + 1$$

$$= \alpha^{n-1}(0 + \alpha^{n-2} T(2) + \dots + \alpha^2 + \alpha + 1)$$

$$= \frac{\alpha^{n-1+1}-1}{\alpha-1}$$

$$= \frac{\alpha^n - 1}{1}$$

$$T(n) \approx \alpha^n$$

$$T(n) \in \Theta(\alpha^n)$$

$$\therefore T(n) \in \Theta(\alpha^n)$$

b). fibonacci numbers:

Alg fib(n).

{
if (n==1)

return 0;

else

return (fib(n-1) + fib(n-2));

y

homogeneous equation:

characteristics equation is

$$1 + \alpha + \alpha^2 + \dots + ((1-\lambda)-\alpha)T^{-\lambda} \alpha^0 + (\lambda-\alpha)T^\lambda \alpha = 0$$

$$ar^2 + br + c = 0$$

$$1 + \alpha + \alpha^2 + \dots + ((1-\lambda)-\alpha)T^{-\lambda} \alpha^0 + ((1-\lambda)-\alpha)T^\lambda \alpha = 0$$

The recurrence equation is

$$F(n) = F(n-1) + F(n-2)$$

$$F(n) - F(n-1) - F(n-2) = 0 \quad T(0) = 0$$

$$\text{det } F(n) = \gamma^2$$

$$\gamma^2 - \gamma - 1 = 0$$

$$a=1, b=-1, c=-1$$

$$\begin{aligned} \gamma &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{1 \pm \sqrt{1 - 4(-1)(-1)}}{2(-1)} \end{aligned}$$

The general form of eqn is $= \frac{1 \pm \sqrt{5}}{2}$

$$F(n) = \alpha \gamma_1^n + \beta \gamma_2^n$$

$$F(n) = \alpha \left(\frac{1+\sqrt{5}}{2}\right)^n + \beta \left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$F(0) = \alpha \left(\frac{1+\sqrt{5}}{2}\right)^0 + \beta \left(\frac{1-\sqrt{5}}{2}\right)^0 = 0$$

$$\alpha + \beta = 0 \rightarrow ①$$

$$F(1) = \alpha \left(\frac{1+\sqrt{5}}{2}\right)^1 + \beta \left(\frac{1-\sqrt{5}}{2}\right)^1 = 1$$

$$\Rightarrow \alpha \left(\frac{1+\sqrt{5}}{2}\right) + \beta \left(\frac{1-\sqrt{5}}{2}\right) = 1 \rightarrow ②$$

by solving ① & ②.

$$f(n) = \frac{1}{\sqrt{5}} (\phi^n - \hat{\phi}^n).$$

$$\phi^n = \frac{1+\sqrt{5}}{2} = 1.618$$

$$\hat{\phi}^n = \frac{1}{\phi} = -0.62$$

$$f(n) = \frac{1}{\sqrt{5}} \phi^n$$

$$\frac{a-1}{c} = r$$

$$\frac{a+1}{c} = r$$

non-homogeneous equation.

consider the recurrence equation

$$F(n) = F(n-1) + F(n-2) + 1 \quad F(0) = 0$$

$$F(1) = 0$$

$$F(n) - F(n-1) - F(n-2) - 1 = 0$$

To equating this we have to add ± 1 on LHS side

$$F(n) - F(n-1) - F(n-2) - 1 - 1 + 1 = 0$$

$$F(n) + 1 - [F(n-1) + 1] - [F(n-2) + 1] = 0$$

$$\text{Let } B(n) = F(n) + 1$$

$$B(n) - B(n-1) - B(n-2) = 0.$$

$$\text{Sub } B(0) = 0$$

$$\begin{aligned} B(0) &= F(0) + 1 \\ &= 0 + 1 \end{aligned}$$

$$B(0) = 1$$

$$\text{Sub } n=1$$

$$\begin{aligned} B(1) &= F(1) + 1 \\ &= 0 + 1 \end{aligned}$$

$$B(1) = 1$$

$$B(0) = F(0) + 1$$

$$F(n) = B(n) - 1$$

$$= F(n+1) - 1$$

$$= \frac{1}{\sqrt{5}} [g^{n+1} - g^{n-1} - 1]$$

$$\boxed{\therefore F(n) = \frac{1}{\sqrt{5}} g^n}$$

4) Brute force :-

```
big count[i] (T[0..n-1])  
{ count = 0  
for i=0 to n-1  
{  
if (T[i] == 'A')  
{  
for j=0 to n-1  
{  
if (T[j] == 'B')  
{  
count = count + 1;  
}  
}  
}  
return count;  
}.
```

efficiency of the algorithm :-

$$T(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} 1$$

$$= \sum_{i=0}^{n-1} (n-1-i+1).$$

$$= \sum_{i=0}^{n-1} (n).$$

$$= n \sum_{i=0}^{n-1} 1$$

$$= n(n-1-0+1)$$

$$= n(n)$$

$$T(n) = n^2$$

10

$$T(n) \in O(n^2)$$