Roll No. | | | | | | | | |

KONGU ENGINEERING COLLEGE, PERUNDURAI 638 060
EVEN SEMESTER 2017-2018
CONTINUOUS ASSESSMENT TEST 1 - JANUARY 2018
(Regulations 2014)

Programme : BE Branch : CSE Semester : IV	Date : 20.01.2018 Time : 9.15 am – 10.45 am
Course Code : 14CST43 Course Name : Design and Analysis of Algorithms	Duration : 1 ½ Hours Max. Marks : 50

	PART - A (10 × 2 = 20 Marks)		
	ANSWER ALL THE QUESTIONS		
1.	Calculate gcd(60,22).	(CO1)	[K3]
2.	Define convex hull problem.	(CO1)	[K1]
3.	Compare interpolation and extrapolation.	(CO1)	[K2]
4.	How does dynamic visualization differ from static visualization?	(CO1)	[K2]
5.	Solve the following recurrence relation $x(n)=3x(n-1)$ for $n>1$; $x(1)=4$.	(CO2)	[K3]
6.	Find the order of growth of $\sum_{i=2}^{n-1} \lg i^2$	(CO1)	[K2]
7.	Arrange the following function according to their order $(n+1)!\ 2^{3n}$, $2n^4 + 2n^3$, nlogn, log n, 6n, $8n^2$	(CO1)	[K3]
8.	Find the average case efficiency of sequential search.	(CO1)	[K4]
9.	Define θ notation.	(CO1)	[K1]
10.	Write the general plan for analyzing time efficiency of recursive algorithm.	(CO1)	[K1]

		Part – B (3× 10 = 30 Marks)			
		ANSWER ANY THREE QUESTIONS			
11.		Design a recursive algorithm for computing a^n for any non negative integer n that is based on the formula $a^n=a^{n-1}+a^{n-1}$. Set up a recurrence relation for the number of additions made by the algorithm and solve it.	(10)	(CO2)	[K4]
12.		Write the recursive algorithm to compute the n^{th} Fibonacci number and also compute the efficiency of the above algorithm using homogeneous and inhomogeneous recurrence equation.	(10)	(CO2)	[K3]
13.	i)	Write the basic efficiency classes of algorithm analysis.	(5)	(CO1)	[K1]
	ii)	Mention any five important problem types in computing.	(5)	(CO1)	[K1]
14.		Design a brute force algorithm for the given text, count the number of substrings that start with 'A' and end with 'B'. Also calculate the efficiency of the algorithm.	(10)	(CO2)	[K4]

Bloom's Taxonomy Level	Remembering (K1)	Understanding (K2)	Applying (K3)	Analysing (K4)	Evaluating (K5)	Creating (K6)
Percentage	26.6	16.8	10	36.6	-	-

Module Test-1 Answer key.

① $gcd(60,22) : gcd(22,16) = gcd(16,6) = gcd(6,4) = gcd(4,2)$
$gcd(2,0)$
$= 2$

② Convex Hull Problem: To find the smallest convex polygon that would include all the points of a given set.

③ Interpolation: data points that fall within the range of the data ~~you~~ you have

Extrapolation: data point from beyond the range of your data set.

④ Dynamic visualization vs static visualization

↳ Shows continuous movie like presentation of an algorithm's operations

Shows an algorithm's Progress through a series of still images.

5. $x(n) = 3x(n-1) \quad \forall n > 1, \quad x(1) = 4$

$x(n-1) = 3[3(x(n-2))]$

$= 3^2 [x(n-2)]$

$x(n-2) = 3^3 [x(n-3)]$

⋮

$x(n-k) = 3^k (x(n-k))$ $\qquad n-k=1 \qquad k=n-1$

$= 3^{n-1} (x(n-n+1))$

$= 3^{n-1} \cdot x(1)$

$= 4 \cdot 3^{n-1}$ //

(6) $\sum_{i=2}^{n-1} \lg i^2$

$= 2\sum_{i=1}^{n} \log_2 i - \log n$

$= 2\left[\sum_{i=1}^{n} \log_2 i - \sum \log n\right]$

$= 2\,\Theta(n \log n) - 2\log n$

$= \Theta(n \log n)$

(7) $\log n,\ 6n,\ n\log n,\ 8n^2,\ 2n^4+2n^3,\ (n+1)!\ 2^{3n}$

(8) $\frac{p(n+1)}{2} + n(1-p)$

(9) Θ-notation: $c_2 \cdot g(n) \le t(n) \le c_1 \cdot g(n) \quad \forall\ n \ge n_0$

(10) (i) Decide on a Parameter indicating an input's

(ii) Identify the alg. basic operation

(iii) check whether the no. of times the basic op is executed can vary on different input of the size. if it can, the WC, AC, BC must be in separately.

(iv) Set up a recurrence relation with an app initial condition

(v) Solve the recurrence relation.

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11.

12. Homogeneous :

$F(n) : F(n-1) + F(n-2)$

$= F(0) = 0 \quad F(1) = 1.$

$\therefore F(n) - F(n-1) - F(n-2) = 0.$ (1)

Apply linear second order linear recurrence with const. coefficients.

$ar^2 + br + c = 0.$

From (1)

$r^2 - r - 1 = 0$

$r_{1,2} = \frac{1 \pm \sqrt{5}}{2}$

After applying

$F(n) = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^n$

$= \frac{1}{\sqrt{5}} (\phi^n - \hat{\phi}^n)$

$A(n) = A(n-1) + A(n-2) + 1$ for $n > 1$

$A(0) = 0 \quad A(1) = 0$

$\therefore [A(n)+1] - [A(n-1)+1] - [A(n-2)+1] = 0$

$B(n) = A(n+1)$

$\therefore B(n) - B(n-1) - B(n-2) = 0 \quad B(0) = 1 \quad B(1)$

$\therefore A(n) = B(n) - 1 = F(n+1) - 1 = \frac{1}{\sqrt{5}}(\phi^{n+1} - \hat{\phi}^{n+1}) - 1$

$\therefore A(n) \in \Theta(\phi^n)$

13) (i) Time efficiency

Space Efficiency.

Measuring i/p size

Units for measuring Running time

Orders of growth

WC, BC, AC

(ii) Important Pb types

sorting, Searching, string Processing, Graph Problem

Combinatorial Problems, Geometric Problem,

Numerical Problem.

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for $i \leftarrow 0$ to $n-2$ do
{
$j := i$

~~for j←i+1~~ if (A[i] = 'A') then

for $j \leftarrow i+1$ to $n-1$

{ ~~j++~~

if (A[j] = 'B') then

count++

}

}

return count

}

$\frac{n(n-1)}{2}$

$(n-1)(n$

$$\text{Eff}: \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n-1-i-1)+1)$$

$$= \sum_{i=0}^{n-1} (n-1-i)$$

$$= \sum_{i=0}^{n-1} (n-1) - \sum_{i=0}^{n-1} i$$

$$= (n-1) \cdot n - \frac{(n-1)(n-2)}{2}$$

$$= \frac{(n-1)}{2} \left[2n - n + 2 \right]$$

$$= \frac{(n-1)(n+2)}{2} = \frac{n^2}{2} \in \Theta(n^2)$$

DEPT. OF COMPUTER SCIENCE & ENGG.
KONGU ENGINEERING COLLEGE,
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PERUNDURAI (TK), ERODE - 638 060

SSK	[signature] 20/01
Name and Signature of Hall Supdt. with Date	

Estd : 1984

KONGU ENGINEERING COLLEGE

PERUNDURAI ERODE - 638 060.

(Autonomous)

Name of the Student	K. KEERTHANA	Register No.	1	6	C	S	R	1	0	3
Programme	BE	Branch & Semester	CSE-B, IV							
Course Code and Name	14CST43 - Design and Analysis of algorithm.	Date	20.1.2018.	No. of Pages Used	13.					

MARKS TO BE FILLED IN BY THE EXAMINER

PART - A		PART - B			Grand Total Max. Marks : 50
Question No.	Max Marks : 2	Question No.		Max Marks : 10	
1	2	11	i)	9	47/50
2	2		ii)		
3	2				
4	2	12	i)		
5	2		ii)	10	
6	0	13	i)		
7	2		ii)		verified K. Keerthana
8	2				
9	2	14	i)	10	
10	2		ii)		
TOTAL	18	TOTAL		29	

Total Marks in Words : four seven

INSTRUCTION TO THE CANDIDATE

1. Check the Question Paper, Programme, Course Code, Branch Name etc., before answering the questions.
2. Use both sides of the paper for answering questions.
3. POSSESSION OF ANY INCRIMINATING MATERIAL AND MALPRACTICE OF ANY NATURE IS PUNISHABLE AS PER RULES.

Name of the Examiner: KK

Signature of the Examiner with Date: [signature] 20/1/18 part - A.

Answer all the Questions :-

1. gcd (60, 22).

$gcd(m,n) = gcd(n, m \bmod n)$

$= gcd(22, 16)$
$= gcd(16, 6)$
$= gcd(6, 4)$
$= gcd(4, 2)$

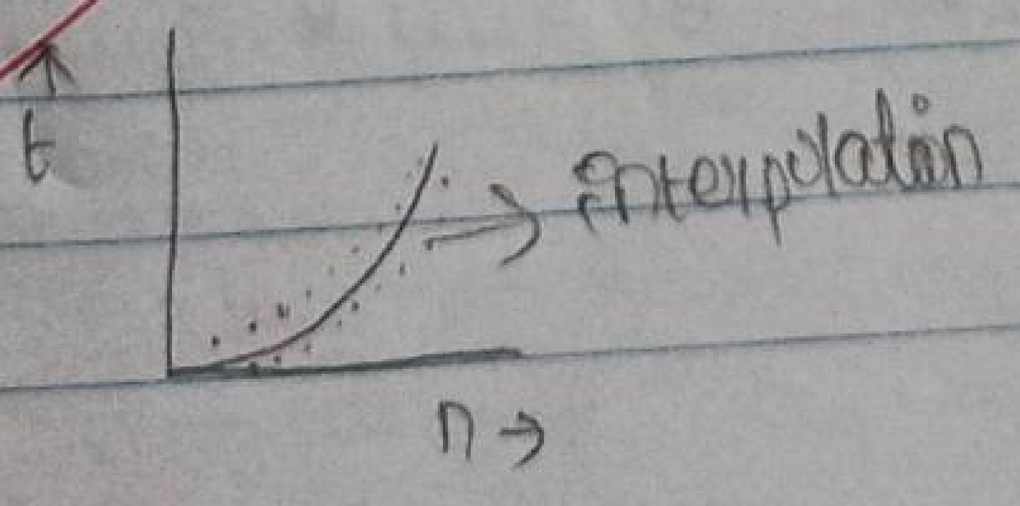
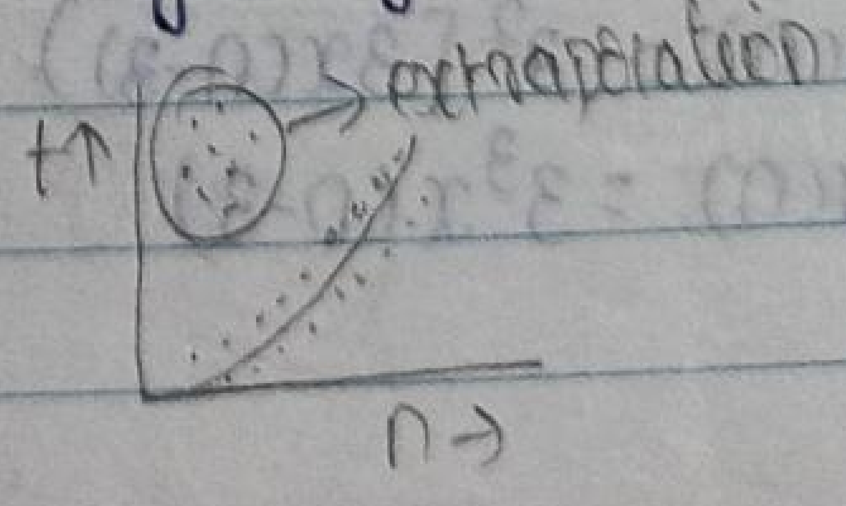
$gcd(m,n) = gcd(2, 0)$.
↳ x as output.

∴ The result of gcd (60, 22) is 2.

2) convex hull problem :-

to find a small of object that covers all the points in the given plane.

3.

Interpolation	extrapolation.
* Interpolation is the polation that the points are closer to the curve.	*. extrapolation is the polation that ~~the~~ points are away from the curve.
	

4) dynamic visualization differ from static visualization.

⇒ In static visualization, the input can be given and it can displayed inform of graph, it doesn't have any movement.

⇒ In dynamic visualization, the input can be given and it can displayed inform of animations, it can have movements.

5) $x(n) = 3x(n-1)$ $n>1$.

$x(1) = 4$.

$x(n) = 3x(n-1)$.

sub $n = n-1$

$x(n-1) = 3x(n-1-1)$

$x(n-1) = 3x(n-2)$

$x(n) = 3[3x(n-2)]$

$x(n) = 3^2 x(n-2)$

sub $n = n-2$

$x(n-2) = 3x(n-2-1)$

$x(n-2) = 3x(n-3)$

$x(n) = 3^2 [3x(n-3))$

$x(n) = 3^3 x(n-3)$

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In general equation

$$x(n) = 3^k x(n+k).$$

Sub $k = n-1$

$$= 3^{n-1} x(n-n+1)$$

$$= 3^{n-1} x(1)$$

$$= 3^{n-1} x(1)$$

$$= 3^{n-1} \cdot 4$$

$$= 4 \cdot 3^{n-1}$$

$$x(n) \simeq 3^n.$$

$$\boxed{x(n) \in O(3^n).}$$

6). $\sum_{i=2}^{n-1} \lg i^2$

$$\sum_{i=2}^{n-1} \lg i^2 = \lg \sum_{i=2}^{n-1} i^2.$$

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$T(n) = \lg\left[\frac{(n-1)(n-1+1)((2(n-1)+1)}{6}\right]$$

$$= \lg\left[\frac{n-1(n)(2n-2+1)}{6}\right]$$

$$= \lg\left\{\frac{n-1(n)(2n-1)}{6}\right\} = \lg\left(\frac{(n^2-n)(2n-1)}{6}\right) \simeq n^2.$$

$$T(n) \in O(n^2)$$

1.

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7). Arranging order:

$C, \log n, n, n\log n, n^2, n^3, n!, n^n$

$\log n$, $6n$, $n\log n$, $8n^2$, $2n^4+2n^3$, $(n+1)!$, 2^{3n}.

8). average case of sequential search:

Alg: linear search

```
Alg linear [a[], n, key)
{
 for i=1 to n-1.
  {
   if [key == a[i])    basic operation.
     return false
  }
 return true.
}.
```

average case = No. of comparison needed in 1st location × probability of location
+
no. of comparison needed in 2nd location × probability of location
+
no. of comparison needed in n^{th} Location × probability location

$$\text{average case}(n) = 1\times\frac{1}{n} + 2\times\frac{1}{n} + \cdots n\times\frac{1}{n}$$

$$= \frac{1}{n}(1+2+\cdots n)$$

$$= \frac{1}{\cancel{n}}\left(\frac{\cancel{n}(n+1)}{2}\right)$$

$$= \frac{n+1}{2} \simeq n$$

Removing constant values.

$avg(n) = O(n)$ //. $\boxed{avg(n) \in O(n)}$ //.

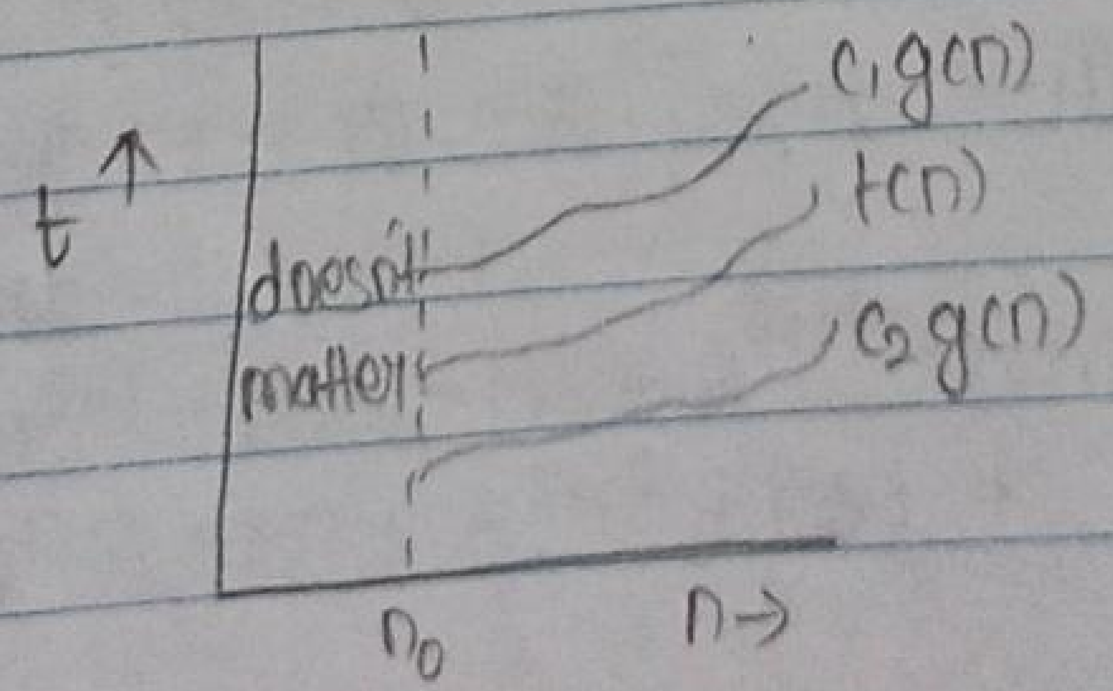
9) Θ notation :-

* It is also called tight bound.

* If $T(n)$ is said to be $\Theta(g(n))$, then it is denoted by $T(n) \in \Theta(g(n))$.

* If $T(n)$ is bounded both above and below by some constant multiplies of $g(n)$ for all large n, there exists such as some positive constant c_1 and c_2 and non-negative integer n_0.

ie) $c_2 g(n) \leq t(n) \leq c_1 g(n) \quad \forall n \geq n_0$.



10) general plan for analysing recursive algorithm.

* Determine the input range.

* To identify the basic operations

* How long the basic operations can be executed?

* set the recurrance equation.

* solve and find the execution time

part - B.

Answer any three:

11. Alg apow (n, a).

```
{.
  if (n==1)
    return 0;
  else
    return (apow(n-1) + apow(n-1));
}.
```

recurrance equation

$T(n) = T(n-1) + T(n-1) + 1$

$T(1) = 0.$

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sub

$T(n) = 2T(n-1)+1$

$T(1) = 0$

Sub $T(n) = T(n-1)$

$T(n-1) =$

$T(n) = 2T(n-1)+1$

$T(1) = 0.$

Sub $n = n-1$

$T(n-1) = 2T(n-1-1)+1$

$T(n-1) = 2T(n-2)+1$

$T(n) = 2[2T(n-2)+1]+1$

$= 2^2 T(n-2)+2+1$

$T(n) = 2^2 T(n-2)+2+1$

Sub $n = n-2.$

$T(n-2) = 2T(n-2-1)+1$

$T(n-2) = 2T(n-3)+1$

$T(n) = 2^2[2T(n-3)+1]+2+1$

$T(n) = 2^3 T(n-3)+2^2+2+1.$

In general

$T(n) = 2^k T(n-k) + 2^{k-1} T(n-(k-1)) + \cdots\ 2^2+2+1$

sub $k = n-1.$

$= 2^{n-1} T(n-(n-1)) + 2^{(n-1-1)} T(n-(n-1-1)) + \cdots\ 2^2+2+1$

$= 2^{n-1} T(1) + 2^{n-2} T(2) + \cdots\ 2^2+2+1.$

$= 2^{n-1}(0) + 2^{n-2}\, T(2) + \dots\ 2^2 + 2 + 1.$

$1 + a + a^2 + \dots a^n = \frac{a^{n+1} - 1}{a - 1}$

$= \frac{2^{n-1+1} - 1}{2 - 1}$

$= \frac{2^n - 1}{1}$

$T(n) \simeq 2^n.$

$T(n) \in \Theta(2^n).$

$\therefore \boxed{T(n) \in \Theta(2^n)}$

12). fibonacci number:

```
Alg fib(n).
{
 if (n==1)
    return 0;
 else
 return (fib(n-1) + fib(n-2));
}
```

homogeneous equation:

characteristics equation is

$ar^2 + br + c = 0.$

The reccurance equation is

$F(n) = F(n-1) + F(n-2)$

$F(n) - F(n-1) - F(n-2) = 0.$ $T(\phi) = 0.$

Let $F(n) = r^2$

$r^2 - r - 1 = 0.$

$a=1,\ b=-1,\ c=-1$

$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$r_1 = \frac{1+\sqrt{5}}{2}$ $r_2 = \frac{1-\sqrt{5}}{2}.$

$= \frac{1 \pm \sqrt{1-4(1)(-1)}}{2(1)}$

The general form of eqn is $= \frac{1 \pm \sqrt{5}}{2}.$

$F(n) = \alpha r_1^n + \beta r_2^n$

$F(n) = \alpha\left(\frac{1+\sqrt{5}}{2}\right)^n + \beta\left(\frac{1-\sqrt{5}}{2}\right)^n$

$F(0) = \alpha\left(\frac{1+\sqrt{5}}{2}\right)^0 + \beta\left(\frac{1-\sqrt{5}}{2}\right)^0 = 0.$

$\alpha + \beta = 0. \rightarrow (1)$

$F(1) = \alpha\left(\frac{1+\sqrt{5}}{2}\right)^1 + \beta\left(\frac{1-\sqrt{5}}{2}\right)^1 = 1$

$\Rightarrow \alpha\left(\frac{1+\sqrt{5}}{2}\right) + \beta\left(\frac{1-\sqrt{5}}{2}\right) = 1. \rightarrow (2)$

by solving ① & ②:

$$F(n) = \frac{1}{\sqrt{5}}\left(\phi^n - \hat{\phi}^n\right).$$

$$\phi^n = \frac{1+\sqrt{5}}{2} = 1.618.$$

$$\hat{\phi} = \frac{1}{\phi} = -0.62$$

$$\boxed{F(n) = \frac{1}{\sqrt{5}}\phi^n.}$$

non- homogeneous equation.

consider the recurrance equation

$$F(n) = F(n-1) + F(n-2) + 1 \qquad F(0) = 0$$
$$F(1) = 0$$

$$F(n) - F(n-1) + F(n-2) - 1 = 0.$$

To equating this we have to add ± 1 on LHS

$$F(n) - F(n-1) - F(n-2) - 1 - 1 + 1 = 0$$

$$F(n)+1 - [F(n-1)+1] - (F(n-2)+1) = 0.$$

Let $B(n) = F(n)+1.$

$B(n) - B(n-1) - B(n-2) = 0.$

sub $n = 0$

$B(0) = F(0) + 1$

$= 0 + 1$

$B(0) = 1$

sub $n = 1$

$B(1) = F(1) + 1$

$= 0 + 1$

$B(1) = 1$

$B(n) = F(n) + 1$

$F(n) = B(n) - 1$

$= F(n+1) - 1$

$= \frac{1}{\sqrt{5}} [\phi^{n+1} - \phi^{n-1} - 1]$

$$\boxed{\therefore F(n) = \frac{1}{\sqrt{5}} \phi^{n}.}$$

10

14) Brute force:-

```
Alg countstr (T[0....n-1])
{ count=0
 for i=0 to n-1
 {
  if (T[i] == 'A')
  {
   for j=0 to n-1
   {
    if (T[j] == 'B')
    {
     count = count+1 ;
    }
   }
  }
 }
return count ;
}.
```

efficiency of the algorithm:-

$$T(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} 1$$

$$= \sum_{i=0}^{n-1} (n-1-0+1).$$

$$= \sum_{i=0}^{n-1} (n).$$

$$= n \sum_{i=0}^{n-1} 1$$

$$= n \, (n-1-0+1)$$

$$= n(n)$$

$$T(n) = n^2$$

10

$$\boxed{T(n) \in O(n^2)}$$