DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

ASSIGNMENT QUESTIONS

Subject Code & Name : 14CST52 – Theory of Computation

Year / Sem : III / V

BATCH 5 – RollNo.15CSR174-181, 14CSR133,15CSL254,15CSL255,15CSL256

1. Convert the equivalent DFA for the given NFA

ð	a	b
→ p	{p,q}	{p}
q	{r}	{r}
r	{s}	Ø
* s	{s}	{s}

2. Construct a NFA for the given Regular Expression 011(0+1)*
3. Obtain a regular expression for finite automata using both R_{ij} and State Elimination Method.

ð	0	1	€
→ q0	Ø	Ø	{q1,q2}
q1	{q4}	Ø	Ø
q2	{q2}	{q3}	Ø
* q3	Ø	Ø	Ø
* q4	Ø	{q4)	Ø

4. Construct the minimum state equivalent DFA for the given FA

ð	0	1
→ q0	{q1,q2}	{q0}
q1	{q0,q1}	Ø
* q2	{q1}	{q0,q1}

5.Check whether the given grammar is ambiguous or not.

S→0B/1A

A→0S/1AA

B→1/1S/0BB

6.Consider the CFG. Find Leftmost and Rightmost derivation for the grammar

i. S→aB/bA

A→a/aS/bAA

B→b/bS/aBB for the input strings w=aabbabba, aabbabaa

ii. S→XaaX

X→aX/bX/£ for the input string w=baaaabab

7.Design a CFG for the language $L=\{a^n b^m a^m b^n / n,m>=1\}$

8.Simplify the following grammar and find its equivalent in GNF form.

S→AB/aB, A→aab/ , B→bbA

9.Convert the PDA P=({p,q},{0,1}, {X,Z},ð,q, Z_0,p) to a CFG, if transition function is given by,

ð(q,1, Z)={(q,XZ)}

ð(q,1,X)={(q,XX)}

ð(q, £,X)={(q, £)}

ð(q, 0,X)={(p, X)}

ð(p, 1,X)={(p, £)}

ð(p,0, Z)={(q, Z)}

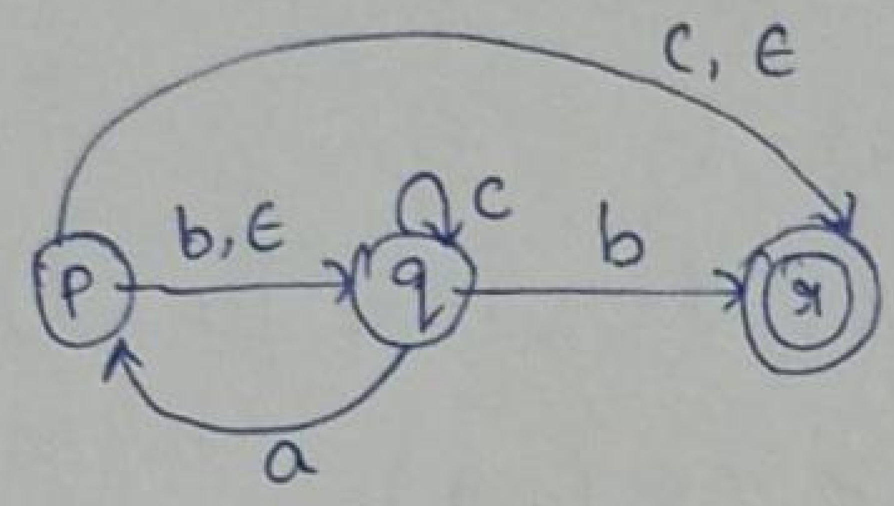
10. Design a Turing Machine to implement proper subtraction.

11. Prove by mathematical induction: 5^n-2^n is divisible by 3 for n>0.

ASSIGNMENT - SCHEME OF VALUATION

1) Convert the given ENFA to DFA

δ	a	b	c	ϵ
$\rightarrow p$	ϕ	$\{q\}$	$\{r\}$	$\{q, r\}$
q	$\{p\}$	$\{r\}$	$\{p, q\}$	ϕ
$*r$	ϕ	ϕ	ϕ	ϕ



Let ENFA, $E = (Q_E, \Sigma, \delta_E, q_0, F_E)$

then equivalent DFA, $D = (Q_D, \Sigma, \delta_D, q_D, F_D)$

① Finding Starting state of DFA

$q_D = ECLOSE(p) = \{p, q, r\}$

② Find Eclose of q & r

$ECLOSE(q) = \{q\}$

$ECLOSE(r) = \{r\}$

③ Apply all input symbols on start state

$\delta_D(q_D, a) = \delta_D(\{p, q, r\}, a)$

$= ECLOSE(\delta_E(p, a) \cup \delta_E(q, a) \cup \delta_E(r, a))$

$= ECLOSE(\phi \cup p \cup \phi)$

$= \{p, q, r\}$

$\delta_D(q_D, b) = \delta_D(\{p, q, r\}, b)$

$= ECLOSE(\delta_E(p, b) \cup \delta_E(q, b) \cup \delta_E(r, b))$

$= ECLOSE(\{q\} \cup \{r\} \cup \phi)$

$= \{q, r\}$

$$\delta_D(q_D, c) = \delta_D(\{p,q,r\}, c)$$
$$= \text{ECLOSE}(\delta_E(p,c) \cup \delta_E(q,c) \cup \delta_E(r,c))$$
$$= \text{ECLOSE}(\{q\} \cup \{r\})$$
$$= \{q,r\}$$

$$\delta_D(\{q,r\}, a) = \text{ECLOSE}(\delta_E(q,a) \cup \delta_E(r,a))$$
$$= \text{ECLOSE}(\{p\} \cup \phi)$$
$$= \{p,q,r\}$$

$$\delta_D(\{q,r\}, b) = \text{ECLOSE}(\delta_E(q,b) \cup \delta_E(r,b))$$
$$= \text{ECLOSE}(\{r\} \cup \phi)$$
$$= \{r\}$$

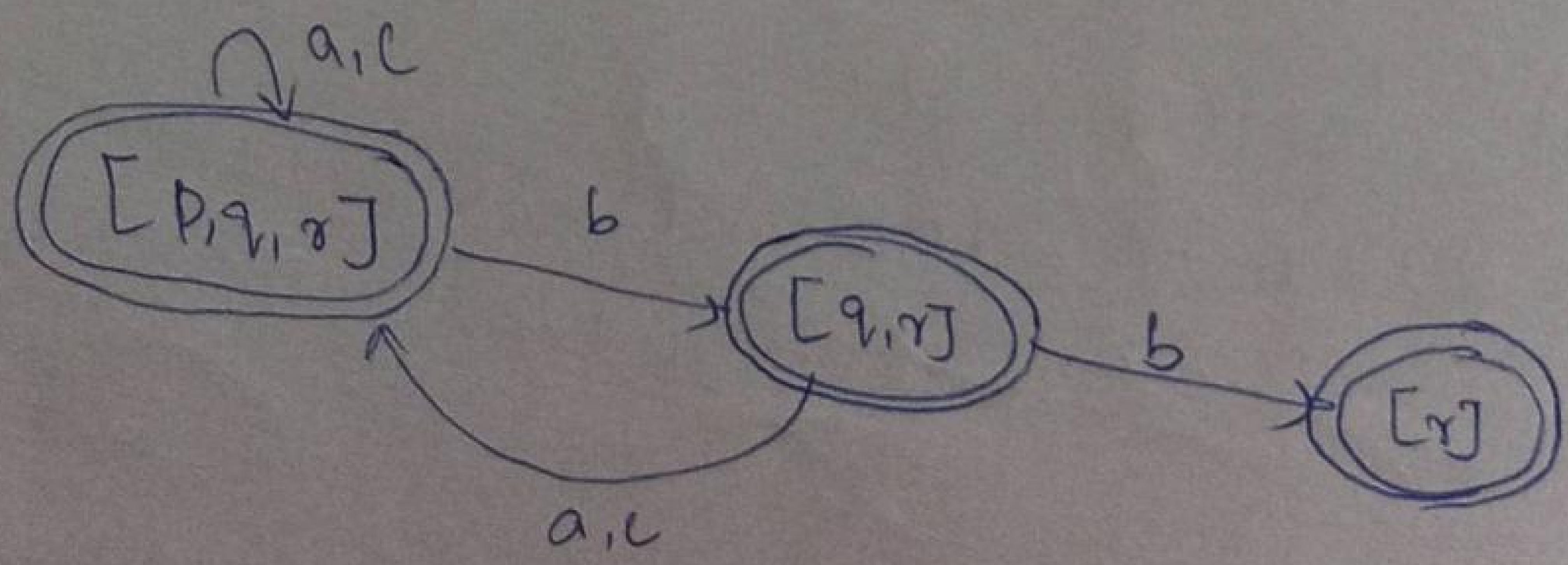
$$\delta_D(\{q,r\}, c) = \text{ECLOSE}(\delta_E(q,c) \cup \delta_E(r,c))$$
$$= \text{ECLOSE}(\{p,q\} \cup \phi) = \{p,q,r\}$$

$$\delta_D(\{r\}, a) = \text{ECLOSE}(\delta_E(r,a)) = \phi$$

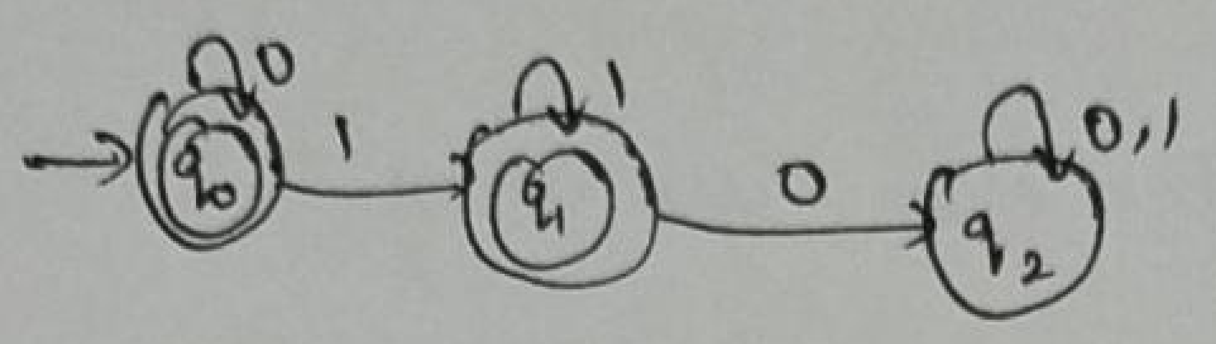
$$\delta_D(\{r\}, b) = \text{ECLOSE}(\delta_E(r,b)) = \phi$$

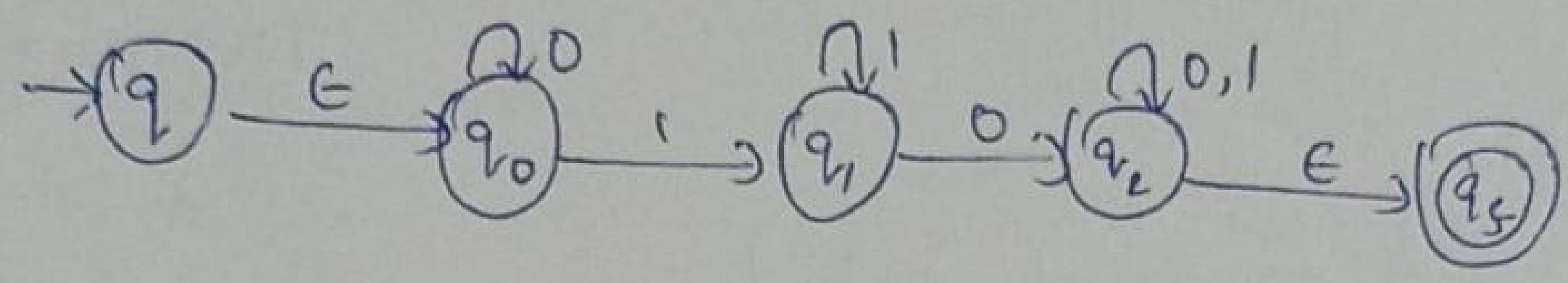
$$\delta_D(\{r\}, c) = \text{ECLOSE}(\delta_E(r,c)) = \phi$$

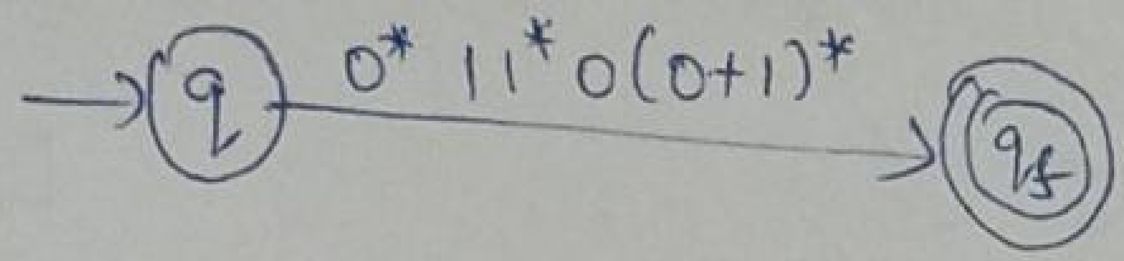
	a	b	c
p	[p,q,r]	[q,r]	[q,r]
q	[p,q,r]	[r]	[p,q,r]
r	ϕ	ϕ	ϕ



2. Obtain a regular expression for finite automata using State elimination method.







3) Construct a minimum state equivalent DFA for the given DFA

δ	0	1
$\to q_1$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_3\}$	$\{q_5\}$
$* q_3$	$\{q_4\}$	$\{q_3\}$
q_4	$\{q_3\}$	$\{q_5\}$
$* q_5$	$\{q_2\}$	$\{q_5\}$

Step 1: Find 0^{th} equivalent class

$\Pi_0 (1,2) = \{\{q_3, q_5\}, \{q_1, q_2, q_4\}\}$

	q_1	q_2	q_4
0	2	1	1
1	1	1	1

Step 2:

$\Pi_1 (1,3,4) = \{\{q_3, q_5\}, \{q_2, q_4\}, \{q_1\}\}$

	q_1	q_2	q_4
0	3	1	1
1	1	1	1

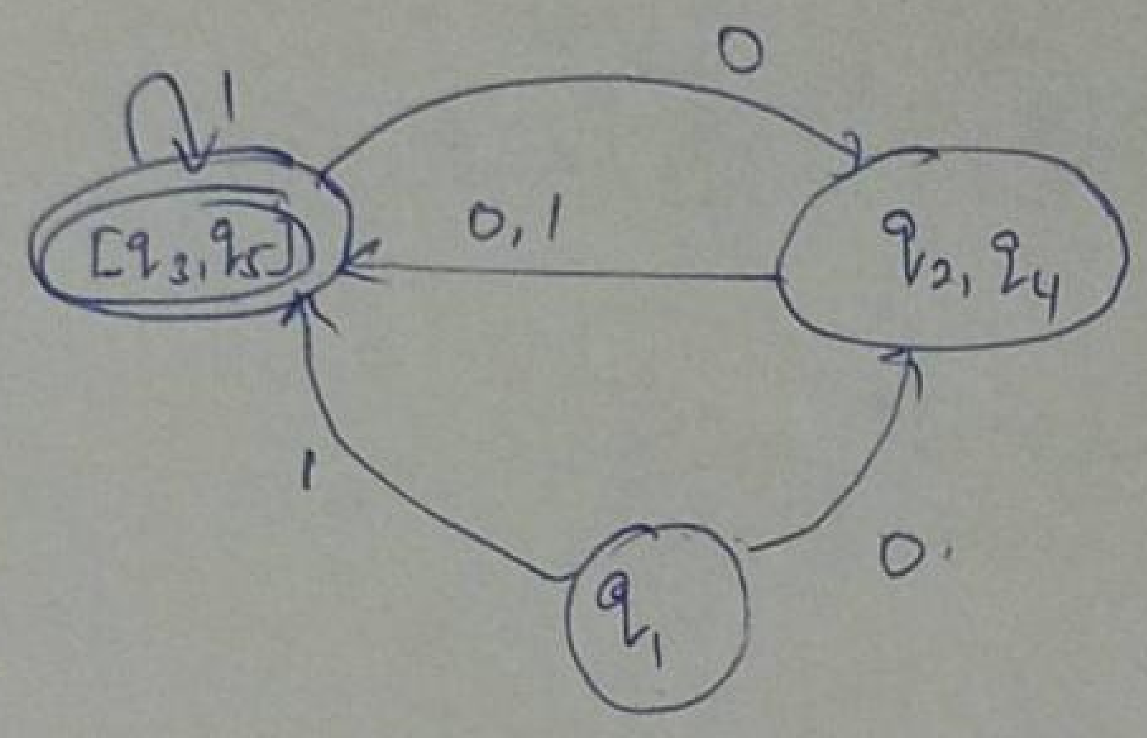
steps:

$\Pi_2(1,3,4) = \{\{q_3, q_5\}, \{q_2, q_4\}, \{q_1\}\}$

From the above

1^{st} & 3^{rd} Equivalent class are Same

q_2 & q_4 are equivalent class



Transistion Table:

δ	0	1
$[q_1]$	$\{q_2\}$	[illegible]
$[q_2, q_4]$	$\{q_3\}$	[illegible]
* $[q_3, q_5]$	$\{q_4, q_2\}$	[illegible]

Table Filling Algorithm:

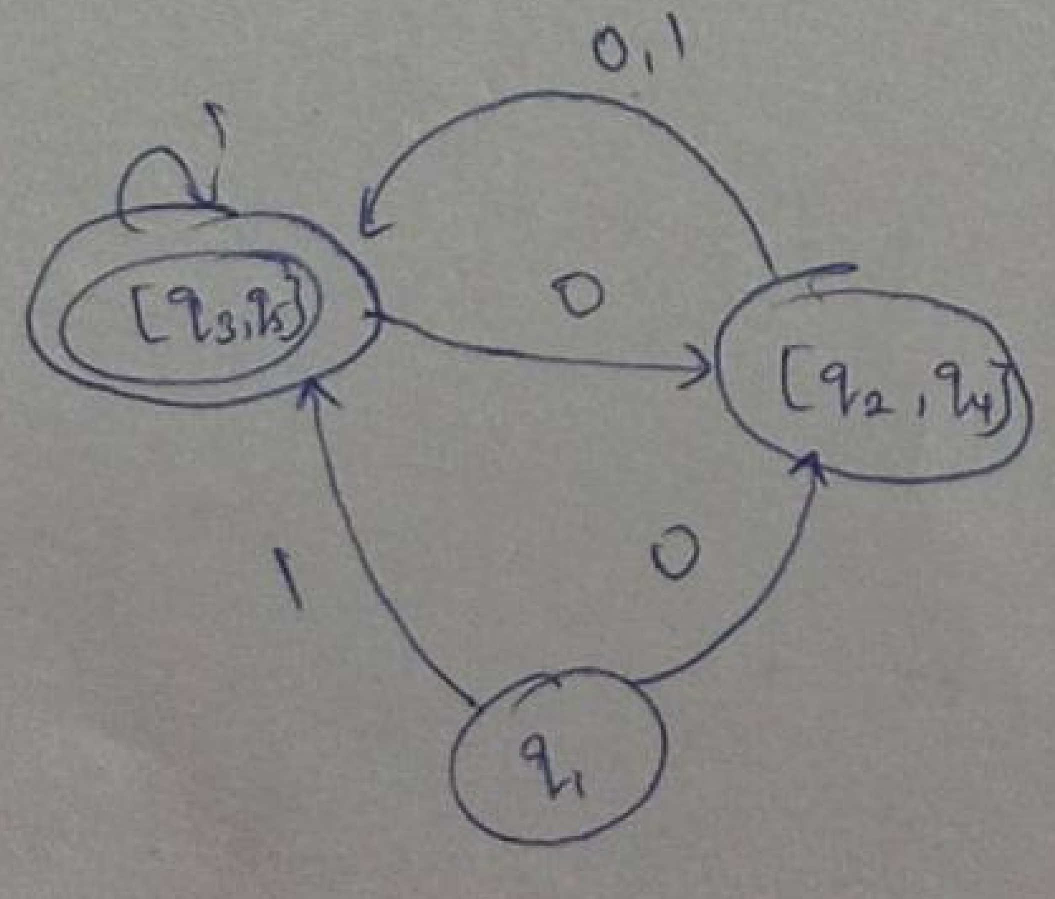
q_2	X			
q_3	X	X		
q_4	X	•	X	
q_5	X	X	X	X
	q_1	q_2	q_3	q_4

Distinguishable States.

$\therefore q_3 = q_1$

$\delta(q_3, 0) = \{q_4\}$ $\qquad$ $\delta(q_3, 1) = \{q_3\}$

$\delta(q_1, 0) = \{q_2\}$ $\qquad$ $\delta(q_1, 1) = \{q_3\}$



Transistion Table:

δ	0	1
→ $[q_1]$	$\{q_2\}$	$\{q_3\}$
$[q_2, q_4]$	$\{q_3\}$	$\{q_5\}$
* $[q_3, q_5]$	$\{q_4, q_2\}$	[illegible]

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4) $L = \{0^n 1^m 2^n \mid$ where m & n are +ve integers$\}$ find out whether the language is regular or not:

Let $w = 0^n 1^m 2^n$

Let n be a pumping lemma constant

$|w| = 2n+m$

Break w into 3 parts such that $|xy| \leq n$ & $|y| \neq 0$

$x = 0^{n-i}$

$y = 0^i$

$z = 1^m . 2^n$

$w = 0^{n-i} . 0^i . 1^m . 2^n$

$xy^k z = (0^{n-i})(0^i)^2 . (1^m 2^n)$

$= 0^{n-i} . 0^i . 0^i . 1^m . 2^n$

$= 0^{n+i} . 1^m 2^n \notin L$

Hence L is not regular language.

5) Design CFG for language $L = \{ww^R \mid w$ is formed over the string $\{a,b\}^*\}$

P: $S \rightarrow aSa$

$S \rightarrow bSb$

$S \rightarrow \epsilon$.

The Grammar: $G = \{\{S\}, \{a,b\}, P, S\}$

6. Convert the PDA $P = (\{q_0, q_1\}, \{a, b\}, \{Z_0\}, \delta, q_0, Z_0, q_1)$ to a CFG, if transition function is given by,

$\delta(q_0, \epsilon, Z_0) = \{(q_1, \epsilon)\}$

$\delta(q_0, 0, Z_0) = \{(q_0, 0Z_0)\}$

$\delta(q_0, 0, 0) = \{(q_0, 00)\}$

$\delta(q_0, 1, 0) = \{(q_0, 10)\}$

$\delta(q_0, 1, 1) = \{(q_0, 11)\}$

$\delta(q_0, 0, 1) = \{(q_1, \epsilon)\}$

$\delta(q_1, 0, 1) = \{(q_1, \epsilon)\}$

$\delta(q_0, 0, 0) = \{(q_1, \epsilon)\}$

$\delta(q_1, \epsilon, Z_0) = \{(q_1, \epsilon)\}$

We have to define CFG, $G = (V, T, P, S)$

$T = \Sigma = \{0, 1\}$

Set of terminals:

$V = \{S, [q_0 Z_0 q_0], [q_0 Z_0 q_1], [q_1 Z_0 q_0] [q_1 Z_0 q_1]\}$

Production of G:

1) For Start Symbol: $S \rightarrow [q_0 Z_0 q_0] / [q_0 Z_0 q_1]$

2) Find production for $\delta(q_0, 0, 1) = \{(q_1, \epsilon)\}$

$[q_0 1 q_1] \rightarrow 0$

Find production for $\delta(q_0, 0, 0) = \{(q_1, \epsilon)\}$

$[q_0 0 q_1] \rightarrow 0$

Find production for $\delta(q_1, \epsilon, Z_0) = \{(q_1, \epsilon)\}$

$[q_1 Z_0 q_1] = \epsilon$

Find production for $\delta(q_0, \epsilon, Z_0) = \{(q_1, \epsilon)\}$

$[q_0 Z_0 q_1] = \epsilon$

For $\delta(q_0, 0, Z_0) = \{(q_0, 0Z_0)\}$

$[q_0 Z_0 q_0] \rightarrow 0[q_0 0 q_0][q_0 Z_0 q_0] / 0[q_0 0 q_1][q_1 Z_0 q_0]$

$[q_0 Z_0 q_1] \rightarrow 0[q_0 0 q_0][q_0 Z_0 q_1] / 0[q_0 0 q_1][q_1 Z_0 q_1]$

For $\delta(q_0, 0, 0) = \{(q_0, 00)\}$

$[q_0 0 q_0] \rightarrow 0[q_0 0 q_0][q_0 0 q_0] / 0[q_0 0 q_1][q_1 0 q_0]$

$[q_0 0 q_1] \rightarrow 0[q_0 0 q_0][q_0 0 q_1] / 0[q_0 0 q_1][q_1 0 q_1]$

For $\delta(q_0, 1, 0) = \{(q_0, 10)\}$

$[q_0 0 q_0] \Rightarrow 0[q_0 1 q_0][q_0 0 q_0] / 0[q_0 1 q_1][q_1 0 q_0]$

$[q_0 0 q_1] \rightarrow 0[q_0 1 q_0][q_0 0 q_1] / 0[q_0 1 q_1][q_1 0 q_1]$

For $\delta(q_0, 1, 1) = \{(q_0, 11)\}$

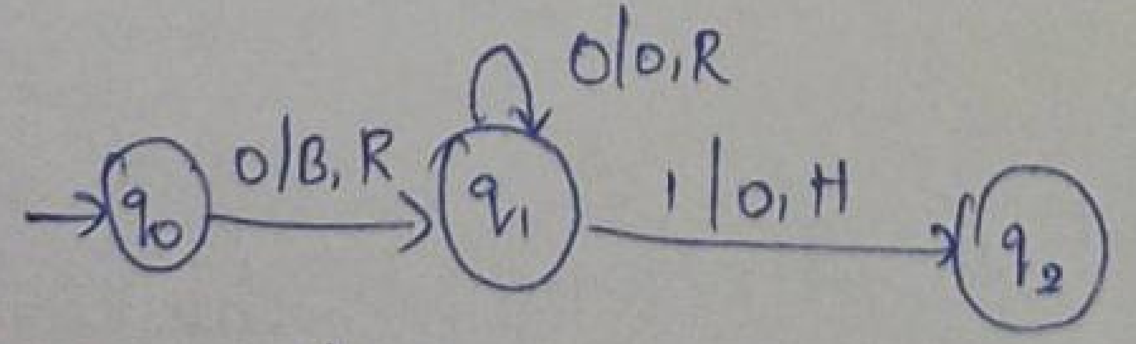
$[q_0 1 q_0] \Rightarrow 0[q_0 1 q_0][q_0 1 q_0] / 0[q_0 1 q_1][q_1 0 q_0]$

7. Design Turing Machine to perform addition of 2 integer.

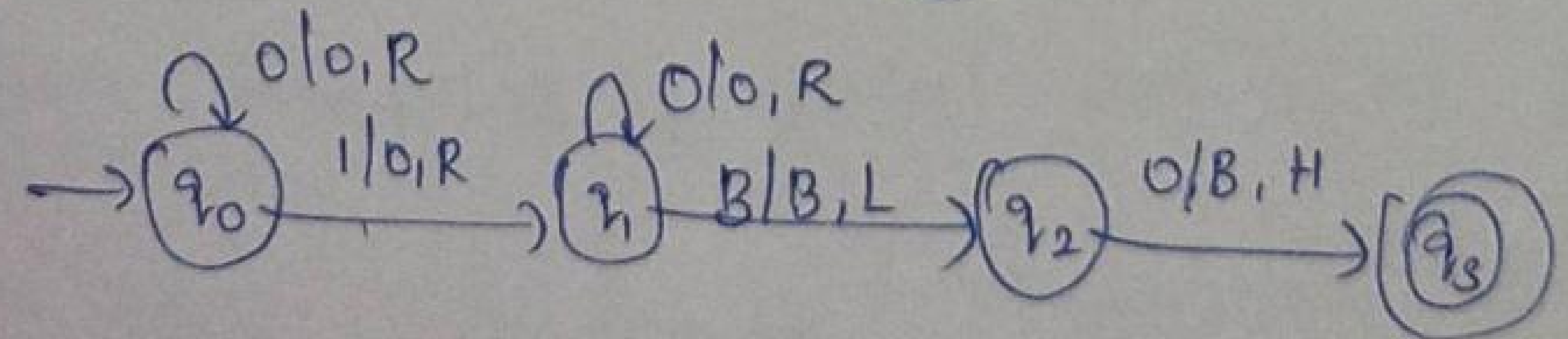
$f(m, n) = m+n$

$f(4, 3) = 7$

B	0	0	0	0	1	0	0	0	B	B



(or)



8. Prove the following Statement by proof by Counter example. There is no pair of integers a and b such that a mod b = b mod a.

i.e. There are no pair of integers such that $a\%b = b\%a$

Case (i) : $a<b$

L.H.S : $a\%b = a \rightarrow (1)$

Rhs $b\%a \rightarrow (2)$

$b\%a = 0$ to $a-1 \rightarrow (3)$

From (1) & (3)

$a\%b \neq b\%a$

Case (ii) : $a>b$

Lhs : $a\%b = 0$ to $b-1 \rightarrow (4)$

R.h.s $a\%b = b \rightarrow (5)$

From (4) & (5)

$a\%b \neq b\%a$

Case (iii) : $a=b$

Lhs = $a\%b = a\%a = 0 \rightarrow (6)$

Rhs = $b\%a = b\%b = 0 \rightarrow (7)$

From (6) & (7)

$a\%b = b\%a$

The theorem Statement is disproved in Case (iii) and is rested as for some pairs of integers.

a & b, $a\%b = b\%a$ iff $a=b$

From the hypothesis

$A \rightarrow a\%b = b\%a$

$B \rightarrow a=b$

Only if Part : If A then B

If $a \% b = b \% a$ then $a = b$

Statement	Hypothesis
1. $a \neq b$	
2) i) $a < b$	
$a \% b = a$	From case (i)
$b \% a = 0$ to $a-1$	
ii) $a > b$	From case (ii)
$a \% b = 0$ to $b-1$	
$b \% a = b$	
From ② $a \% b = b \% a$	False of the given statement.
$\therefore a \% b = b \% a$ then $a = b$	

If Part:

If $a = b$ then $a \% b = b \% a$

Statement	Justification
1) $a = b$	Given statement
2) $a \% b = b \% b = 0$	From ①.
3) $b \% a = b \% b = 0$	From ①
4) ② & ③ $a \% b = b \% a$	From ② & ③.

Hence statement & theorem is proved.

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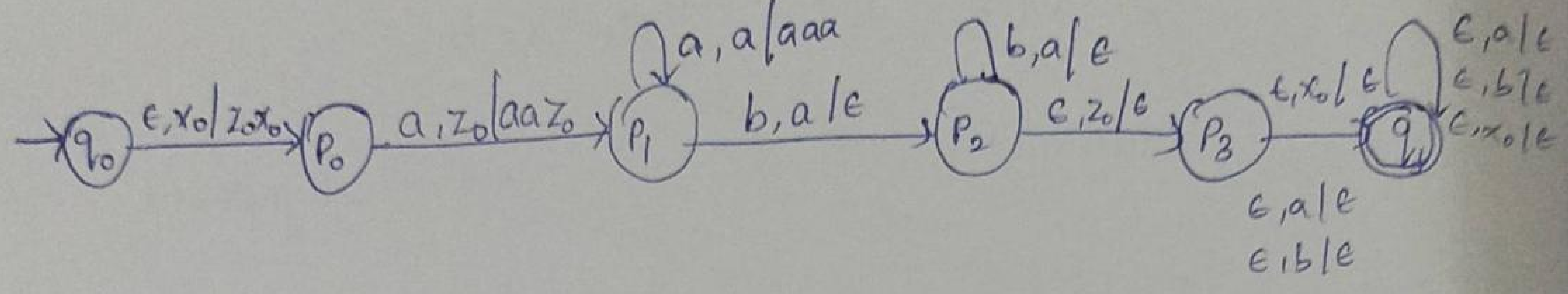
10. Design a PDA that accept the language $L = \{a^{3n} b^n / n \geq 0\}$



11. Simplify the grammar and find its equivalent in GNF form.

$S \to ASA \mid aB$

$A \to B \mid S$

$B \to b \mid \epsilon$

i) Step i: Convert to CNF.

Remove ϵ production : $B \to \epsilon$

$S \to ASA \mid aB \mid a$

$A \to B \mid S \mid \epsilon$

$B \to b$

Remove ϵ production : $A \to \epsilon$

$S \to ASA \mid aB \mid a \mid AS \mid SA \mid S$

$A \to B \mid S$

$B \to b$

Remove $S \to S$ because it is recursive.

$S \to ASA \mid aB \mid a \mid AS \mid SA$

$A \to B \mid S$

$B \to b$

Remove unit production: $A \to B$.

$S \to ASA \mid aB \mid a \mid AS \mid SA$

$A \to b \mid S$

$B \to b$

Useless Symbol:

Reacheable Symbol: $\{S, A, B, a, b\}$

Generating Symbol: $\{S, A, B, a, b\}$

No useless Symbol.

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CNF form:

$S \to ASA \mid aB \mid a \mid AS \mid SA$

$A \to b$

$B \to b.$

$S \to ASA$

$C_1 \to A$

$C_2 \to C_1 S$

$S \to C_2 C_1$

$S \to aB$

$C_3 \to a$

$S \to C_3 B$

$S \to a$

$S \to AS$

$S \to C_1 S$

$S \to SA$

$S \to SC_1.$

$A \to b$

$B \to b$

Grammar:

$S \to C_2 C_1 \mid C_3 B \mid a \mid C_1 S \mid SC_1$

$A \to b$

$B \to b$, $C_1 \to A$, $C_2 \to C_1 S$, $C_3 \to a$

Relabel the NTs:

$S \to A_1$, $A \to A_2$, $B \to A_3$, $C_1 \to A_4$, $C_2 \to A_5$, $C_3 \to A_6$.

$A_1 \to A_5 A_4 \mid A_6 A_3 \mid a \mid A_4 A_1 \mid A_1 A_4$

$A_2 \to b$ → It is in GNF

$A_3 \to b$ → It is in GNF.

$A_1 \to A_1 A_4.$, $i = j$. Introduce new Variable.

$B_1 \to A_4 \mid A_4 B_1$

Sub other rule of A_1 in A_1.

$A_1 \rightarrow A_1 B_1 \mid A_5 A_4 \mid A_6 A_3 \mid a \mid A_4 A_1$

$A_1 \rightarrow A_5 A_4 B_1 \mid A_6 A_3 B_1 \mid a B_1 \mid A_4 A_1 B_1$

Sub Rule of A_1 in B_1.

$B_1 \rightarrow a A_4 \mid a A_4 B_1 \mid a A_5 A_4 \mid a A_3 B_1 \mid a B_1 \mid a A_1 B_1$

$A_2 \rightarrow b$

$A_3 \rightarrow b$.

Toc Assignment

Name: M. Prem Kumar

Roll.NO: 15CSR145

Date: 21.10.17

Class: CSE-'C'

10/10

21/10/17

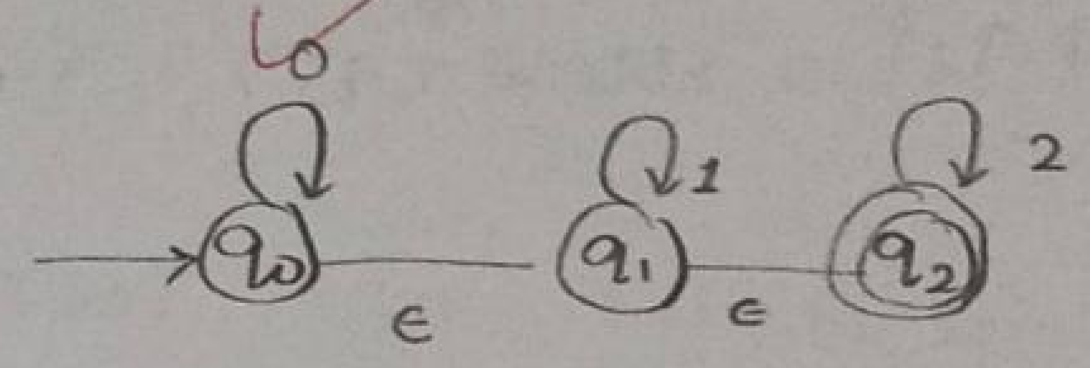
Find equivalent NFA without ϵ moves $M = (\{q_0, q_1, q_2\}, \{0,1,2\}, \delta, \{q_0\}, \{q_2\})$

(i) $\delta(q_0, \epsilon) = \{q_1\}$ (ii) $\delta(q_0, 0) = \{q_0\}$ (iii) $\delta(q_0, 1) = \{\phi\}$

(iv) $\delta(q_0, 2) = \{\phi\}$ (v) $\delta(q_1, \epsilon) = \{q_2\}$ (vi) $\delta(q_1, 0) = \{\phi\}$

(vii) $\delta(q_1, 1) = \{q_1\}$ (viii) $\delta(q_1, 2) = \{\phi\}$ (ix) $\delta(q_2, \epsilon) = \{\phi\}$

(x) $\delta(q_2, 0) = \{\phi\}$, xi) $\delta(q_2, 1) = \{\phi\}$ xii) $\delta(q_2, 2) = \{q_2\}$.



Sol: Let ENFA $N = (Q, \Sigma, \delta, q_0, F)$, Equivalent NFA (without ϵ)

$N' = (Q', \Sigma', \delta', q_0', F')$

$Q' = Q = \{q_0, q_1, q_2\}$

$\Sigma = \Sigma' = \{0, 1, 2\}$

$q_0' = q_0$

Find δ'

Step 1: Eclose on all states:

ECLOSE $(q_0) = \{q_0, q_1, q_2\}$

ECLOSE $(q_1) = \{q_1, q_2\}$

Eclose $(q_2) = \{q_2\}$

Step 2: For state q_0, apply all input symbol & find $\hat{\delta}$.

$\hat{\delta}(q_0, 0) = \text{ECLOSE}(\delta(\hat{\delta}(q_0, \epsilon), 0))$

$= \text{ECLOSE}(\delta(\{q_0, q_1, q_2\}, 0))$

$= \text{ECLOSE}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$

$= \text{ECLOSE}(q_0 \cup \phi \cup \phi)$

$= \text{ECLOSE}(q_0) = \{q_0, q_1, q_2\}$.

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$\hat{\delta}(q_0, 1) = ECLOSE(\delta(\hat{\delta}(q_0, E), 1)) = ECLOSE(\delta(\{q_0, q_1, q_2\}, 1))$

$= ECLOSE(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)\}$

$= ECLOSE(\phi \cup q_1 \cup \phi)$

$= ECLOSE(q_1) = \{q_1, q_2\}$

$\hat{\delta}(q_0, 2) = ECLOSE(\delta(\hat{\delta}(q_0, \epsilon), 2)) = ECLOSE(\delta(q_0, q_1, q_2\}, 2\})$

$= ECLOSE(\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2))$

$= ECLOSE(\phi \cup \phi\ q_2) = ECLOSE(q_2) = \{q_2\}.$

Step 3:-

For state q_1

$\hat{\delta}(q_1, 0) = ECLOSE(\delta(\hat{\delta}(q_1, \epsilon), 0))$

$= ECLOSE(\delta(\{q_1, q_2\}, 0))$

$= ECLOSE(\phi \cup \phi) = \phi$

$\hat{\delta}(q_1, 1) = ECLOSE(\delta(\hat{\delta}(q_1, \epsilon), 1)) = ECLOSE(\delta(\{q_1, q_2\}, 1))$

$= ECLOSE(q_1 \cup \phi) = \{q_1, q_2\}$

$\hat{\delta}(q_1, 2) = ECLOSE(\delta(\hat{\delta}(q_1, \epsilon), 2)) = ECLOSE(\delta(q_1, q_2), 2)$

$= ECLOSE(\phi \cup q_2) = ECLOSE(\{q_2\}) = \{q_2\}$

Step 4:-

$\hat{\delta}(q_2\ 0) = ECLOSE(\delta(\hat{\delta}(q_2, \epsilon), 0) = ECLOSE(\delta(q_2\}, 0) = ECLOSE($

$= \{\phi\}$

$\hat{\delta}(q_2, 1) = ECLOSE(\delta(\hat{\delta}(q_2, \epsilon), 1) = ECLOSE(\delta(\{q_2\}, 1) = ECl$

$= \{\phi\}$

$\hat{\delta}(q_2, 2) = ECLOSE(\delta(\hat{\delta}(q_2, \epsilon), 2) = ECLOSE(\delta(q_2), 2)$

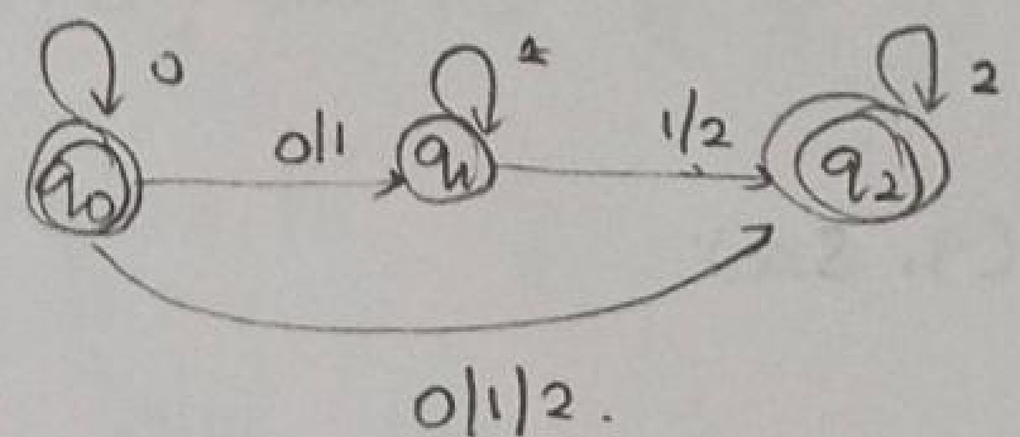
$= \{q_2\}.$

Transition Table:

δ	0	1	2
q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
q_2	ϕ	ϕ	$\{q_2\}$

$F' = \{q_2\} \cup \{q_0\}$ since ECLOSE(q_0) contains F.

Transition diagram:-



Find whether the language L is regular or not, where

$L = \{ww^R \mid$ where w from $(a, b)\}$.

$W = ab^n$, $W^R = a^n b$

$W = a^n b . ab^n$

Let n - Pumping lemma constant

$|W| = 2n + 2 \geq n$

break w into 3 parts

such that $|xy| \leq n$ & $|y| \neq 0$

$x = a^{n-i}$; $y = a^i$; $z = b b^n a$

(i) $|a^n| \Rightarrow n \leq n$ (ii) $|a^i| \neq 0 \Rightarrow i \neq 0$,

(iii) $xy^k z$

$= (a^{n-i}) (a^i)^n (b^n ba)$.

$k = 0$ $xy^0 z = a^{n-i} b^n ba$ $\in L$

$k=2$, xy^2z = $a^{n-i}\, a^i \times a^i\, b^n\, ba$

= $a^{n+i}\, b^n\, ba \in L$

hence, L is not Regular Language.

6) Convert the following CFG to PDA:

$S \rightarrow aAA$

$A \rightarrow aS \mid bS \mid a.$

Let PDA, $P = (Q, \Sigma, \Gamma, \delta, q, Z_0)$

$\Sigma = \{a, b\}$. $Q = \{q\}$, $\Gamma = \{a, b, A, S\}$

① $\delta(q, \epsilon, Z_0) = (q, SZ_0)$

② $S \rightarrow aAA$.

$\delta(q, \epsilon, S) = (q, aAA)$

③ For $A \rightarrow aS$

$\delta(q, \epsilon, A) = (q, aS)$

④ For $A \rightarrow bS$

$\delta(q, \epsilon, A) = (q, a)$

⑤ For $A \rightarrow a$

$\delta(q, \epsilon, A) = (q, a)$

⑥

$\delta(q, \epsilon, Z_0) = (q, \epsilon)$

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8] simplify the following grammer & find its equivalent in GNF,

$S \to AA|0$, $A \to SS|1$:-

sol :-

The given grammer is in CNF,

Relabel the Non-terminals,

sub $S = A_1$, $A = A_2$

$A_1 \to A_2 A_2 | 0$ $\quad$ $A_2 \to A_1 A_1 | 1$.

$A_2 \to A_1 A_1 | 1$. $i > j$ sub rule of A_1

$A_2 \to A_1 A_1 A_1 | 0 A_1 | 1$

introduce new variable since there is a left recursion

$B_2 \to A_2 A_1 | A_2 A_1 B_2$

sub other rule of A_2 in A_2

$A_2 \to A_2 B_2 | 0 A_1 | 1$

$A_2 \to 0 A_1 B_2 | 1 B_2 | 0 A_1 | 1$

sub other rule of A_2 in B_2

$B_2 \to 0 A_1 B_2 A_1 | 1 B_2 A_1 | 0 A_1 A_1 | 1 A_1 | 0 A_1 B_2 A_1 B_2 | 1 B_2 A_1 B_2 |$
$0 A_1 A_1 B_2 | 1 A_1 B_2$

sub

$A_1 \to 0 A_1 B_2 A_2 | 1 B_2 A_2 | 0 A_1 A_2 | 1 A_2 | 0$

The resultant grammer in GNF is

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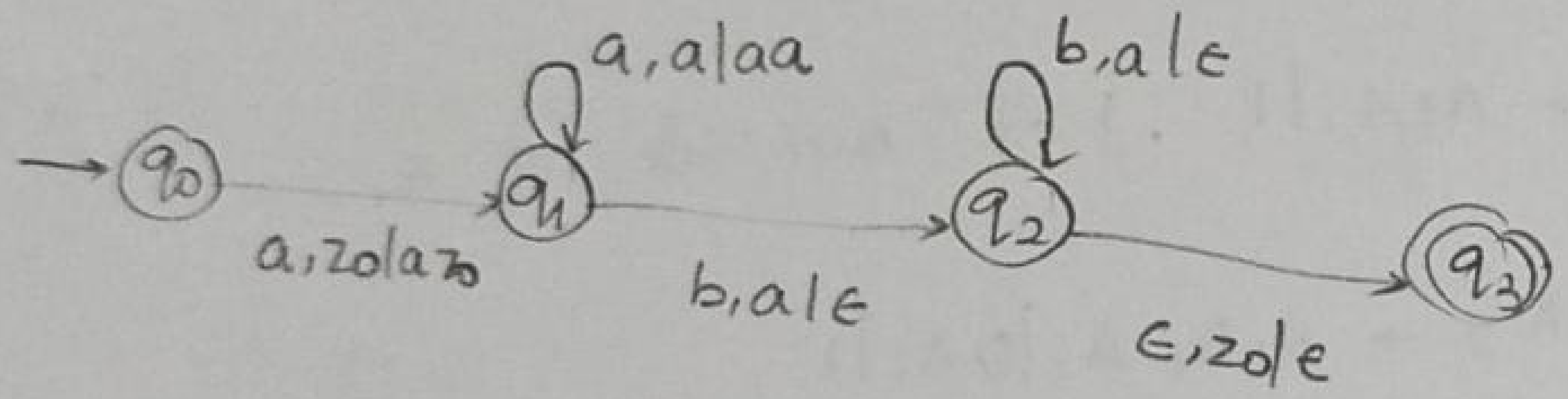
$A_1 \to 0A_1B_2A_2 \mid 1B_2A_2 \mid 0A_1A_2 \mid 1A_2 \mid 0$

$A_2 \to 0A_1B_2 \mid 1B_2 \mid 0A_1 \mid 1$

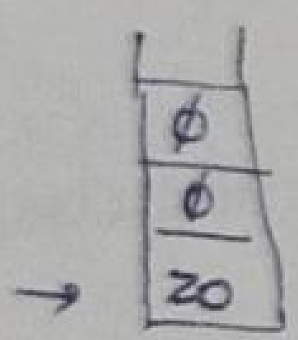
$B_2 \to 0A_1B_2A_1 \mid 1B_2A_1 \mid 0A_1A_1 \mid 1A_1 \mid 0A_1B_2A_1B_2 \mid$
$1B_2A_1B_2 \mid 0A_1A_1B_2 \mid 1A_1B_2.$

a) Design a PDA that accept the language $L = \{0^{2n}1^{2n} \mid n \geq 1\}$

if $n = 1 \Rightarrow 0011.$

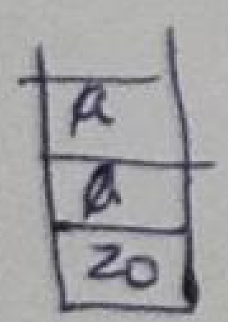


⇒ 0011



If ends with q_3, so accepting state

⇒ 00111



∴ b,z0 is not in PDA

∴ It is not accepting state.

5) convert the PDA $P = \{ \{p,q\}, \{0,1\}, \{X, z_0\}, \delta, q, z_0 P)$ to CFG if transition function is given by

$\delta(q, 0, z_0) = \{(q, x z_0)\}$ | $\delta(p, \epsilon, x) = \{(p, \epsilon)\}$

$\delta(q, 0, x) = \{(q, xx)\}$ $\delta(p, 1, z_0) = \{(p, xx)\}$.

$\delta(q, 1, x) = \{(q, x)\}$

$\delta(q, \epsilon, x) = \{(p, \epsilon)\}$

We have CFG, $G = (V, T, P, S)$

$\Sigma = \{b, 1\}$

set of non-Terminals:

$V = \{ S, [q z_0 q], [q x q], [q z_0 p], [q x p], [p z_0 q],$
$[p x q], [p z p], [p x p] \}$

Production of S:-

(i) for start symbol:-

$S \to [q z_0 q] / [q z_0 p]$

(ii) find production for (4)

$[q x p] \to \epsilon$

(iii) find production for (5)

$[p x p] \to \epsilon$.

for $\delta(q, 0, z_0) = (q, x z_0)$

$[q z_0 q] \to 0 [q x q] [q z_0 q] / 0 [q x p] [p z_0 q]$

$[q z_0 p] \to 0 [q x q] [q z_0 p] / 0 [q x p] [p z_0 p]$

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for $\delta(q, 0, x) = (q, xx)$

$[qxq] \rightarrow 0[qxq][qxq] \,/\, 0[qxp][pxq]$

$[qxp] \rightarrow 0[qxq][qxp] \,/\, 0[qxp][pxp]$

for $\delta(q, 1, x) = (q, x)$

$[qxq] \rightarrow 1[qxq]$

$[qxp] \Rightarrow 1[qxp]$

for $[p, 1, z_0] = (p, xx)$

$[pxp] \rightarrow 1[pxp][pxp] \mid 1[pxq][qxp]$

$[pxq] \rightarrow 1[pxp][pxq] \mid 1[pxq][qxq]$

Remove unnecessary production, we get

$S \rightarrow [qz_0p]$

$[qz_0p] \rightarrow 0[qxq][qz_0p]$

$[qxp] \rightarrow 0[qxq][qxp] \mid 0[qxp][pxp]$

$[qxq] \rightarrow 1[qxq]$

$[qxp] \rightarrow 1[qxp].$

$[pxp] \rightarrow 1[pxq][qxp]$

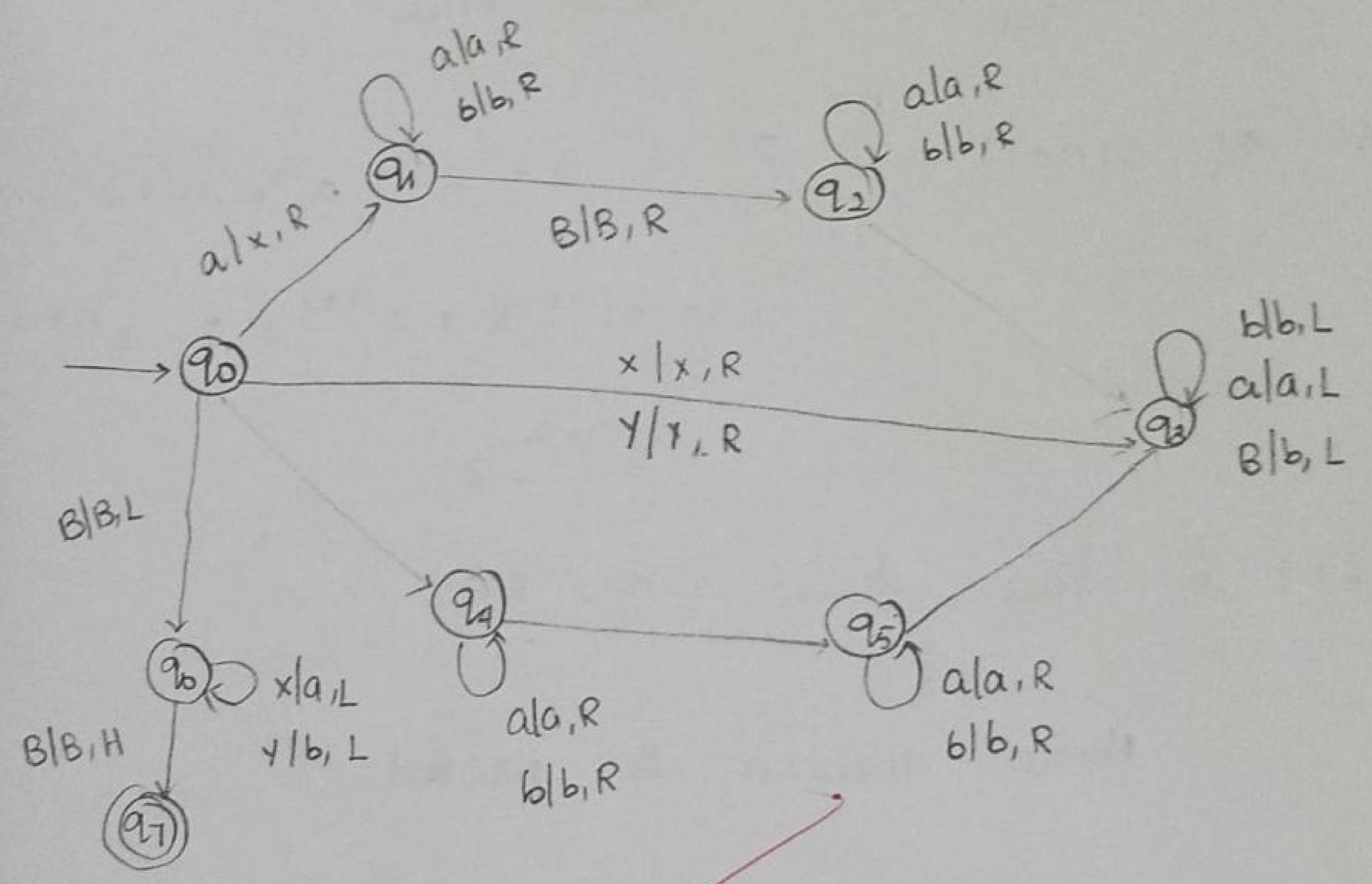
$[pxq] \rightarrow 1[pxq][qxq]$

10) Design a Turning Machine to copy string:.

I/P Baba BBB

after string copy

O/P Baba Baba B.



11) Prove the following statement by indication For $n \geq 1$,

$$2 + 2^2 + 2^3 + 2^4 + \ldots + 2^n = 2^{n+1} - 2.$$

Basis :

$n = 1$

we have to proof

$$\sum_{i=1}^{n} 2^i = 2^{n+1} - 2$$

L.H.S : $\sum_{i=1}^{1} 2^i = 2$

R.H.S. $2^{1+1} - 2 = 2^n - 2 = 4 - 2 = 2$

$\therefore$ Basis is true, since LHS = RHS.

Inductive step:

assume $S(n)$ $\sum_{i=1}^{n} 2^i = 2^{n+1} - 2$ is true

L.HS of $S(n+1) = \sum_{i=1}^{n+1} 2^i = 2 + 2^2 + 2^3 + \dots + 2^n +$

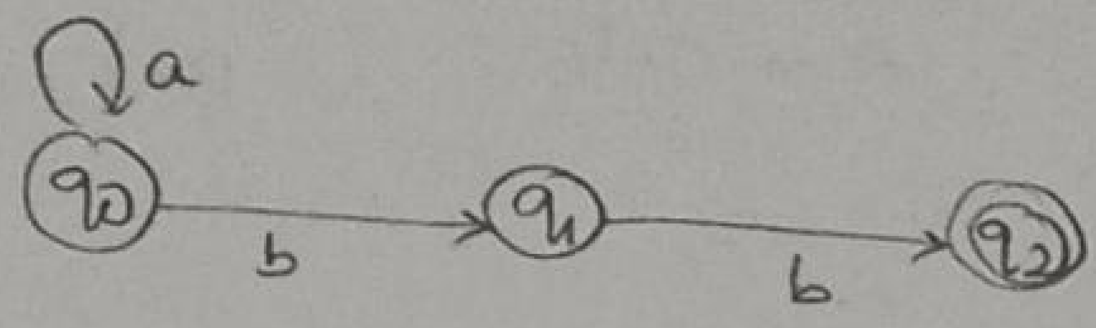
$= 2^{n+1} - 2 + 2^{n+1} = 2 \cdot 2^{n+1}$

$= 2^{n+2} - 2.$

$S(n+1)$ is true since LHS = RHS

Hence theorem is proved.

7) construct a minimum state DFA for the regular expression a^*bb.



Step 1 : Find 0^{th} equivalent class:

$$\pi_0(a,b) = \{ \{q_2\}, \{q_0, q_1\} \}$$

	q_0	q_1	q_2
a	2	ϕ	ϕ
b	2	1	ϕ

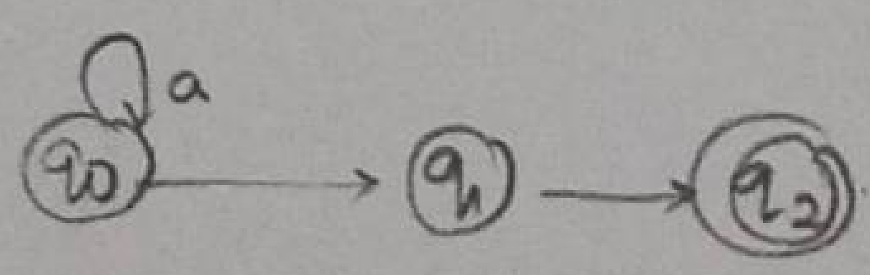
Step 2:

$$\pi(1,2,3) = \{ \{q_2\}, \{q_0\}, \{q_2\} \}$$

	q_0	q_1	q_2
a	2	ϕ	ϕ
b	3	1	ϕ

Step 3:

$$\pi(1,2,3) = \{ \{q_1\}, \{q_2\}, \{q_3\} \}$$



Transition table:

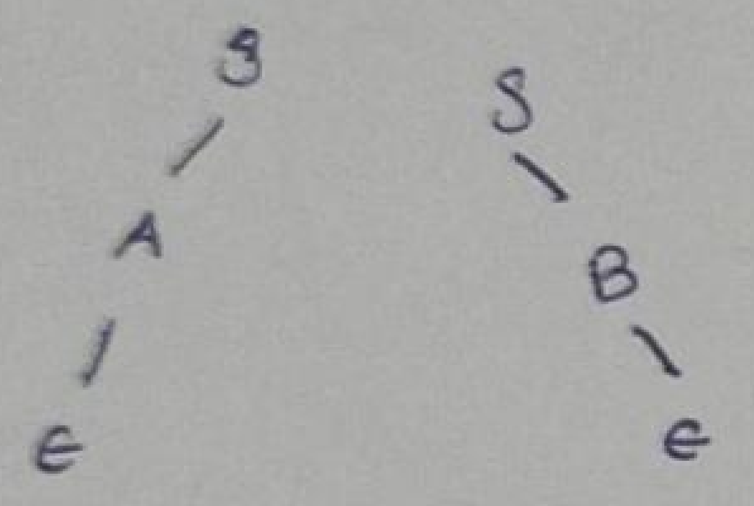
δ	a	b
q_0	q_0	q_1
q_1	ϕ	q_2
q_2	ϕ	ϕ.

3) Check whether the given grammar is ambiguous or not

$S \to A|B \quad A \to 0A|\epsilon \quad , \quad B \to 1B|0B|\epsilon$.

$S \to A|B$

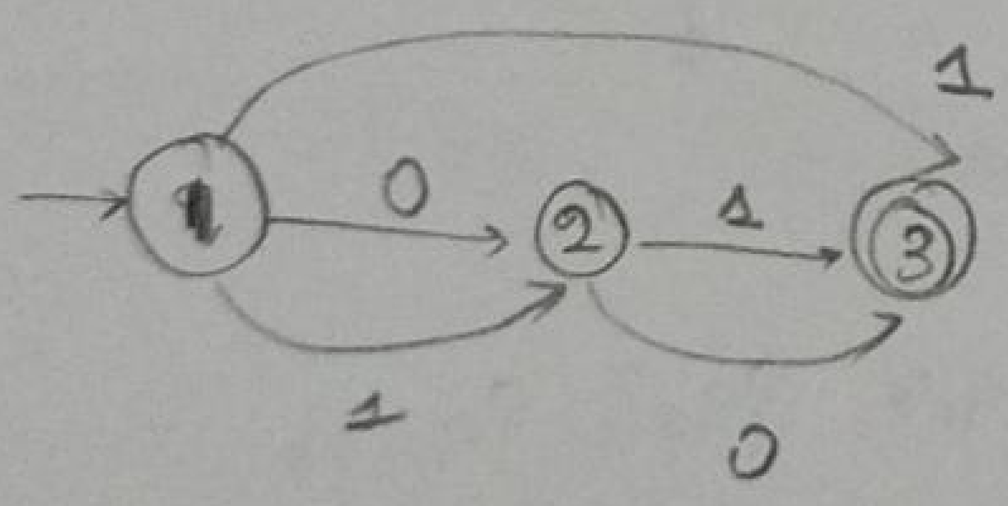
$A \to 0A|\epsilon \qquad B \to 1B|0B|\epsilon$



, The string obtain same gram

∴ the grammar is ambig

2) Obtain a regular expression for finite automate using both
and state elimination method.



Rij method:

Step 1: $K=0$

$R_{11}^{(0)} = \epsilon$, $\quad R_{21}^{(0)} = \phi \quad R_{31}^{(0)} = \phi$

$R_{12}^{(0)} = 0+1. \quad R_{22}^{(0)} = \epsilon \quad R_{32}^{(0)} = \phi$

$R_{13}^{(0)} = 1 \quad R_{23}^{(0)} = 1+0 \quad R_{33}^{(0)} = \epsilon$.

Step 2. $k=1$

$R_{11}^{(1)} = R_{11}^{(0)} + R_{11}^{(0)} \cdot (R_{11}^{(0)})^* \cdot R_{11}^{(0)}$

$= \epsilon + [\epsilon \cdot \epsilon^* \cdot \epsilon) = \epsilon$.

$R_{12}^{(1)} = R_{12}(0) + R_{11}(0) \cdot (R_{11}(0))^* \cdot R_{12}$

$= 0+1 + [\epsilon \cdot \epsilon^* \cdot (0+1)] = 0+1+0+1 = 0+1$

$R_{13}^{(1)} = R_{13}(0) + R_{11}(0) \, (R_{11}(0))^* \quad R_{13}(0)$.

$= 1 + [\epsilon \cdot \epsilon^* \cdot 1] = 1 + [\epsilon^* \cdot 1] = 1+1 = 1$.

$R_{21}^{(1)} = R_{21}^{(0)} + R_{11}(0) \cdot (R_{11}(0))^* \cdot R_{21}^{(0)}$

$= 0 + (\epsilon^* \cdot 0) = \phi$,

$R_{22}^{(1)} = R_{22}(0) + R_{21}(0) \cdot (R_{11}(0))^* \cdot R_{12}(0)$

$= \epsilon + [\phi \cdot \epsilon^* \cdot (0+1)] = \epsilon + \phi = \epsilon$.

$R_{23}^{(1)} = R_{23}(0) + R_{21}(0) \, [R_{11}(0)]^* \quad R_{13}(0)$.

$R_{13}^{(1)} = R_{13}(1) + R_{13}^{(1)} (R_{22}^{(1)})^* \quad R_{23}^{(1)}$

$= 1 + [(0+1) \cdot \epsilon^* \cdot (1+0)]$.

$= 1+1+0 = 0+1$.

$R_{21}(2) = R_{21}^{(1)} + R_{22}^{(1)} \cdot (R_{22}^{(1)})^* \cdot R_{21}^{(1)}$

$= \phi + [\epsilon \cdot \epsilon^* \cdot \phi]. \quad = \phi + \phi = \phi$.

$R_{22}^{(2)} = R_{22}^{(1)} + R_{22}^{(1)} \, (R_{22}^{(1)})^* \cdot R_{22}^{(1)}$

$= \epsilon + \epsilon^* = \epsilon^*$.

$R_{23}^{(2)} = R_{23}^{(1)} + R_{22}^{(1)} \cdot (R_{22}^{(1)})^* \cdot R_{22}^{(1)}$

$= 1+0+[\epsilon \cdot \epsilon^* \cdot \epsilon] = 1+0+\epsilon^* = 1+0.$

$R_{31}^{(2)} = R_{31}^{(1)} + R_{32}^{(1)} (R_{02}^{(1)})^* R_{21}^{(1)}$

$= \phi + \phi\,[\epsilon^* \cdot \phi] = \phi.$

$R_{32}^{(2)} : R_{32}^{(1)} + R_{32}^{(1)} \cdot (R_{32}^{(1)})^* \cdot R_{22}^{(1)}$

$= \phi + [\phi \cdot \epsilon^* \cdot \epsilon] = \phi + \phi = \phi.$

$R_{33}^{(2)} : R_{33}^{(1)} + R_{32}^{(1)} \cdot (R_{22}^{(1)})^* R_{22}^{(1)} = \epsilon + [\phi \cdot \epsilon^* \cdot \epsilon]$

$= \phi + \epsilon = \epsilon.$

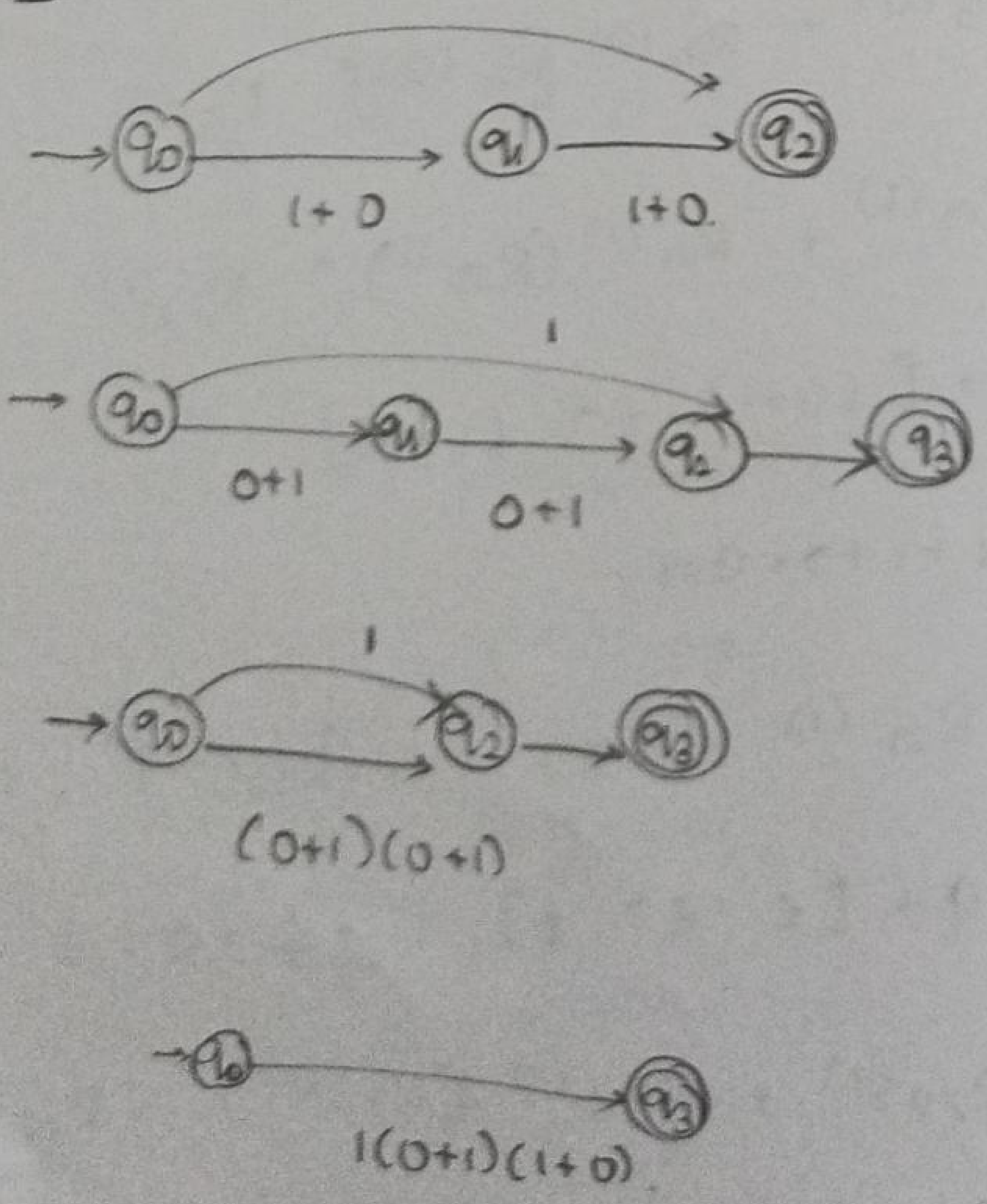
Step 4: $K=3$

$R_{13}^{(3)} = R_{13}^{(2)} + R_{13}^{(2)} \cdot (R_{33}^{(2)})^* \cdot R_{33}^{(2)}$

$= 1+0 + [(1+0) \cdot \epsilon^* \cdot \epsilon) = 1+0+[(1+0$

$= (1+0) \cdot \epsilon^*.$

State Elimination Method:-



ASSIGNMENT

NAME : A. RAJA RAJESWARI

CLASS : CSE - 'C'

SUBJECT : THEORY OF COMPUTATION

ROLL NO : 15CSR161

10/10

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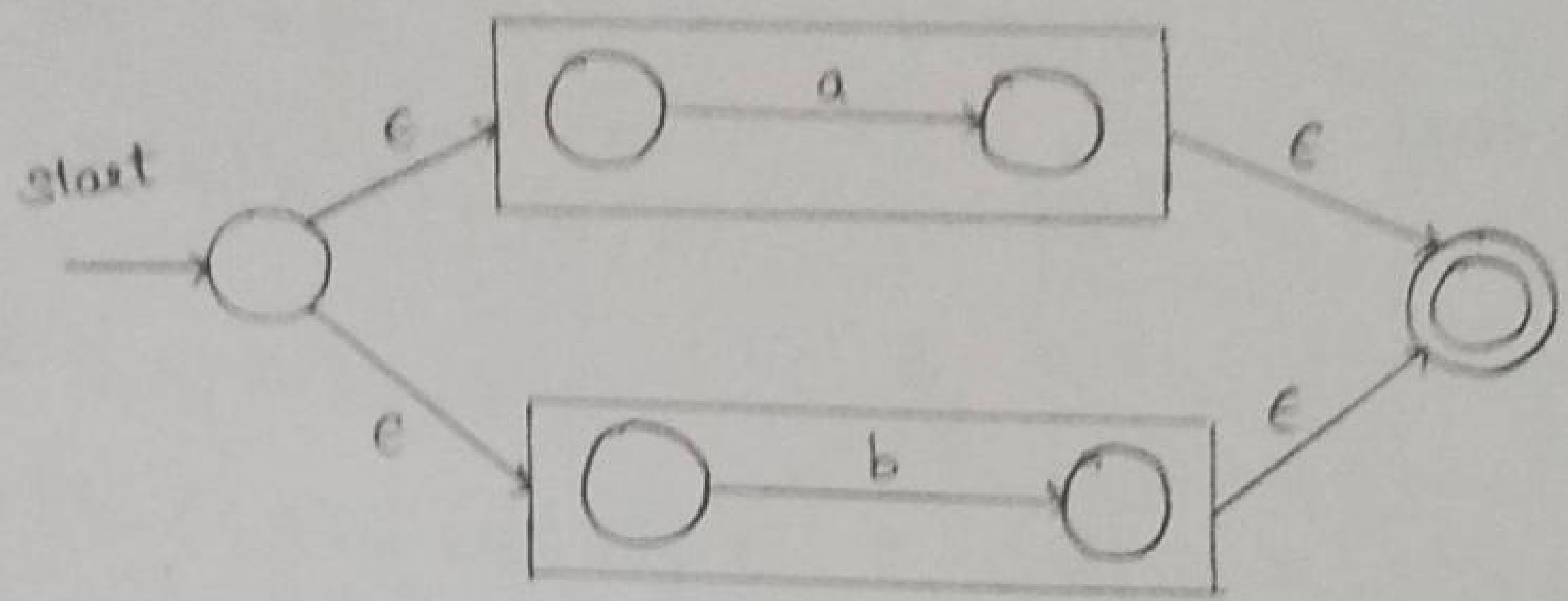
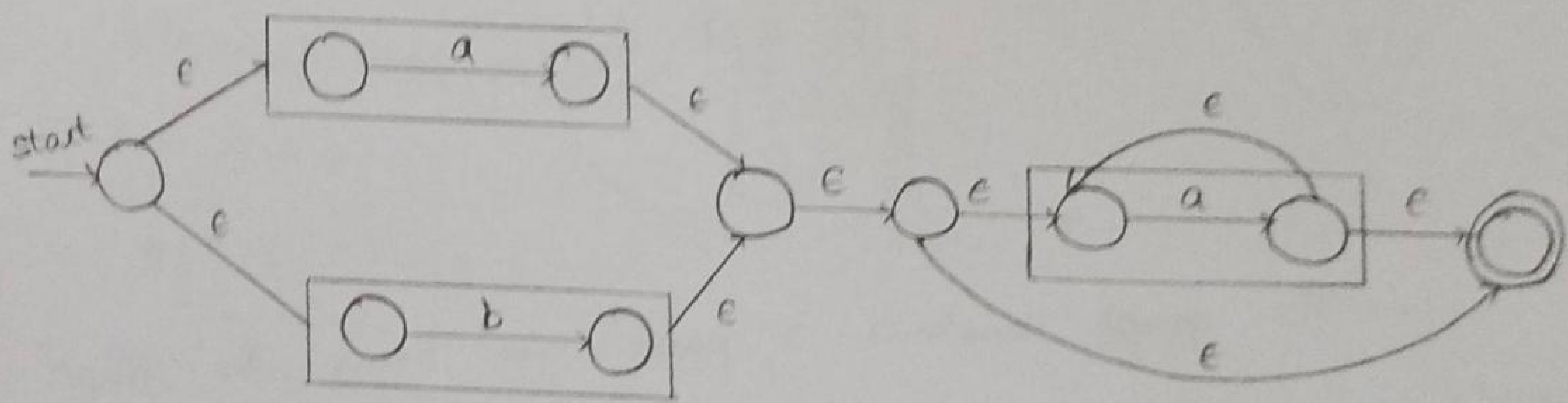
3. Construct a minimum state DFA for the regular expression $(a/b)a^*b$.

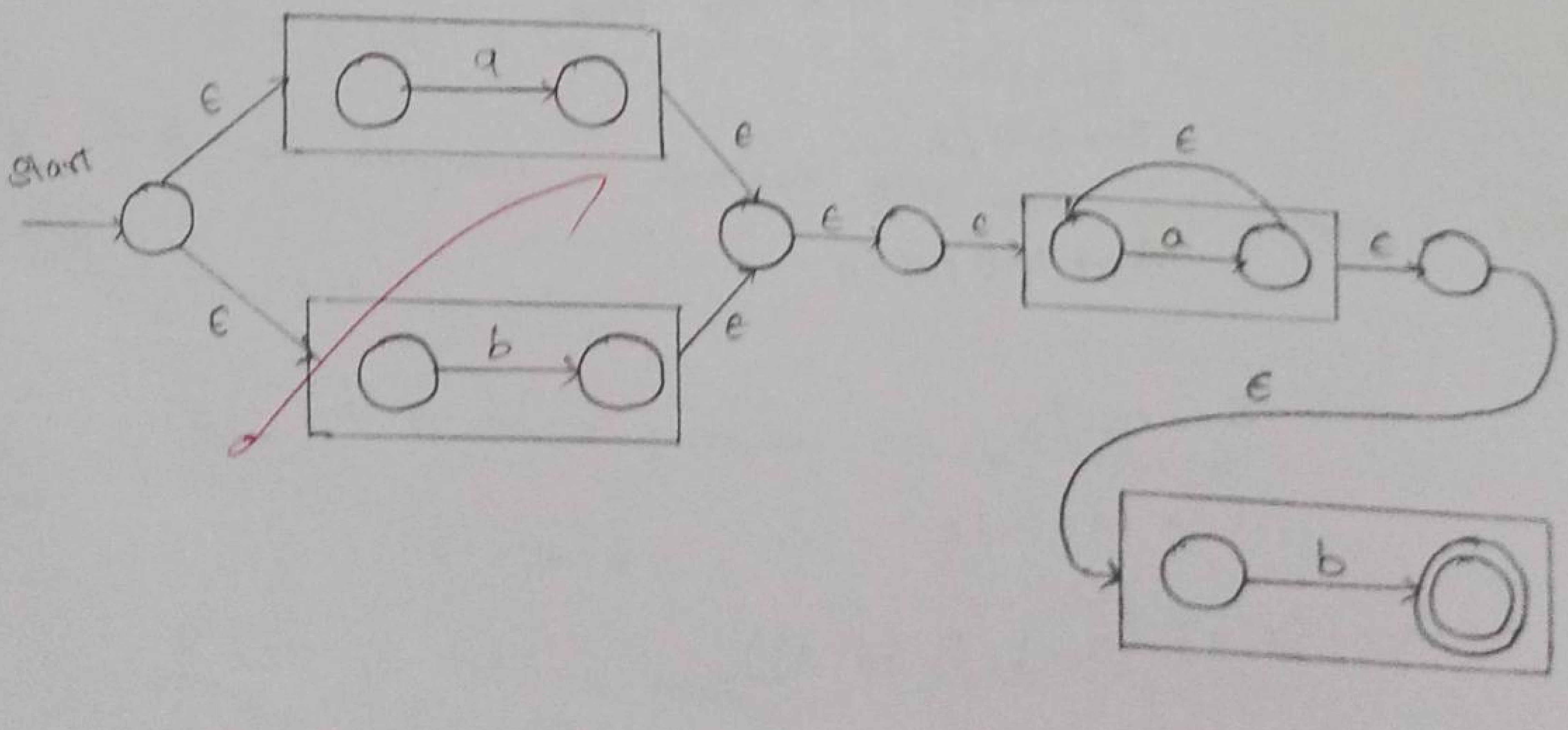
a/b :



$(a/b)a^*$:



$(a/b)a^*b$:

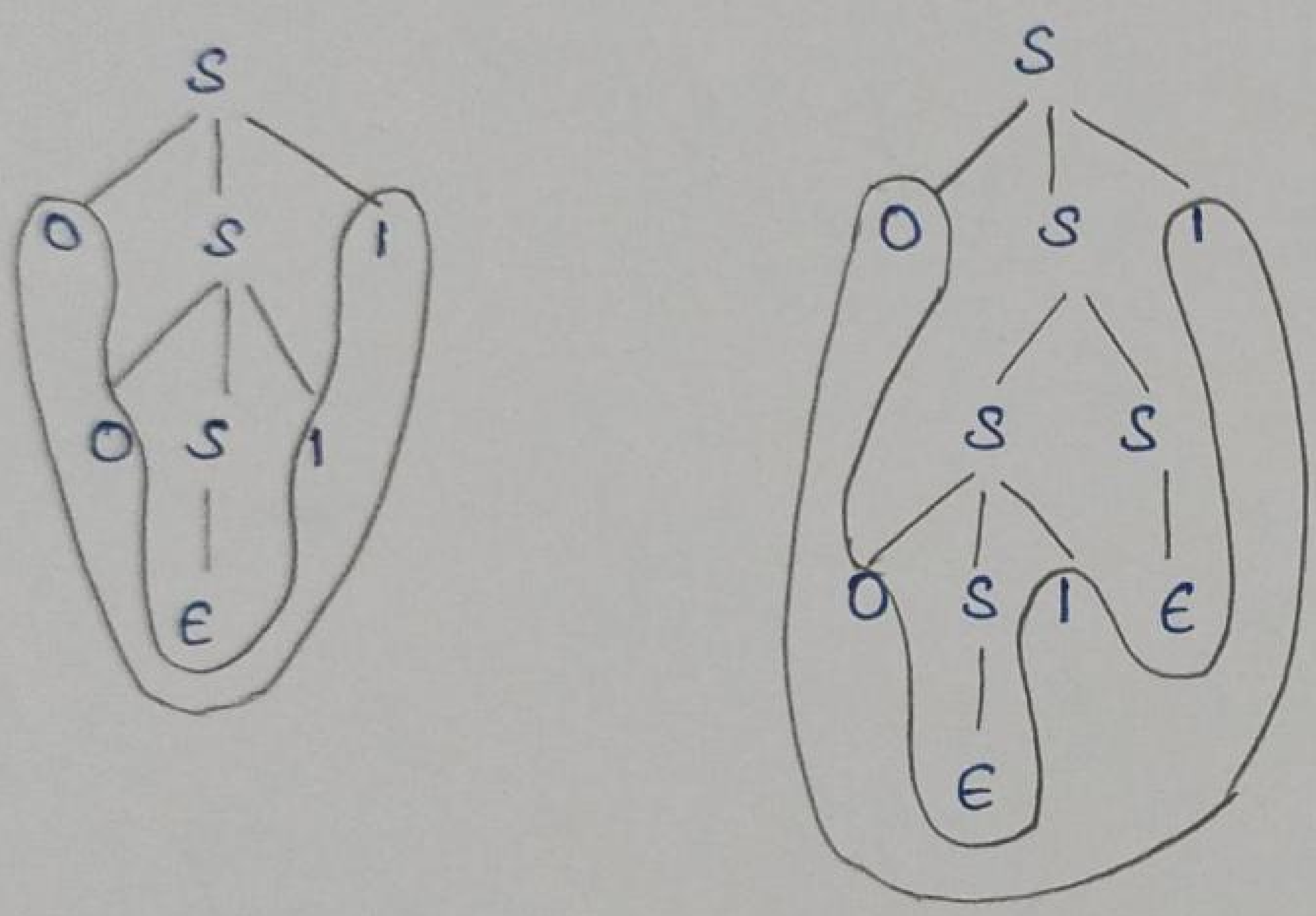


4. check whether the given grammar is ambiguous or not.

$S \rightarrow 0S1 / SS / e.$

$S \rightarrow 0S1 / SS / \epsilon$

$\omega = 0011$



∴ The graph construct 2 parse tree. Hence, the graph is ambiguous.

6. Convert the following CFG to PDA.

$S \rightarrow 0S1 / A$

$A \rightarrow 1A0 / S / e.$

$S \rightarrow 0S1 / A$

$A \rightarrow 1A0 / S / \epsilon$

$G = (\{S, A\}, \{0, 1\}, P, \{S, A\})$

Let PDA $\Rightarrow P = (Q, \Sigma, \Gamma, \delta, q, Z_0)$

$\Sigma = \{0, 1\}$

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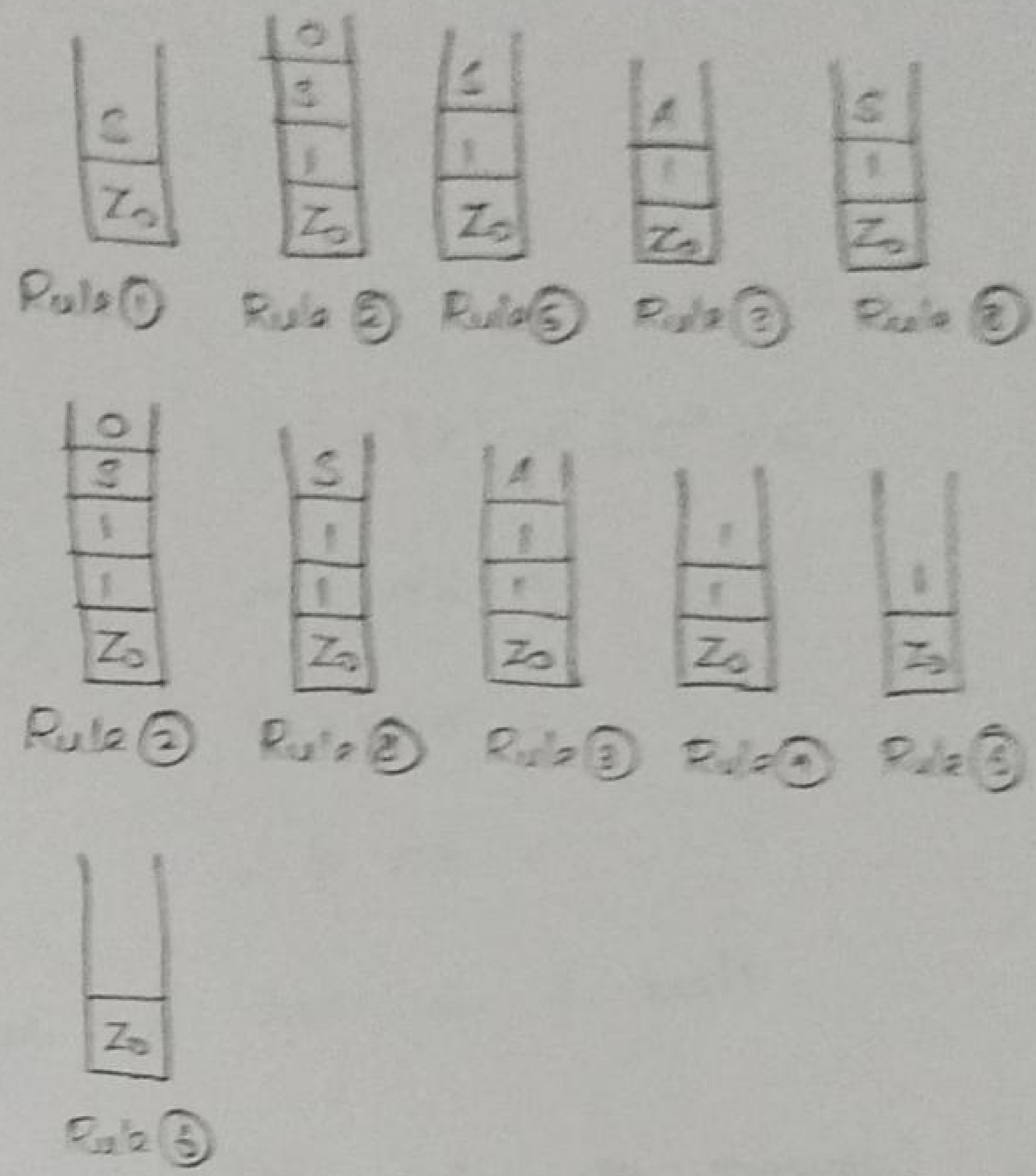
1. $\delta(q, \epsilon, Z_0) = (q, S Z_0)$

$\omega = 0011$

2. For $S \rightarrow 0S1$

$\delta(q, \epsilon, S) = (q, 0S1)$



3. For $S \rightarrow A$

$\delta(q, \epsilon, S) = (q, A)$

4. $\delta(q, \epsilon, Z_0) = (q, \epsilon)$

5. $\delta(q, 0, 0) = (q, \epsilon)$

6. $\delta(q, 1, 1) = (q, \epsilon)$

7. For $A \rightarrow 1A0$

$\delta(q, \epsilon, A) = (q, 1A0)$

8. For $A \rightarrow S$

$\delta(q, \epsilon, A) = (q, S)$

9. For $A \rightarrow \epsilon$

$\delta(q, \epsilon, A) = (q, \epsilon)$

$\omega = 0011$, The string is accepted.

8. Find out language is context free grammer / language $L = \{a^{2n} / n \geq 1\}$.

Let $\omega = a^{2n}$

Let n be a pumping lemma constant.

$|\omega| = 2n \geq n$

Break ω into 3 parts,

$x = a^n, \; y = a^{2n}, \; z = \epsilon$

$|xy| = 2n \leq n$

$|y| = n \neq 0$

$xy^k z \notin L$

$k = 2$

$|xy^2z| = |x| + |y| + |z| + |y|$

$= 2n + n$

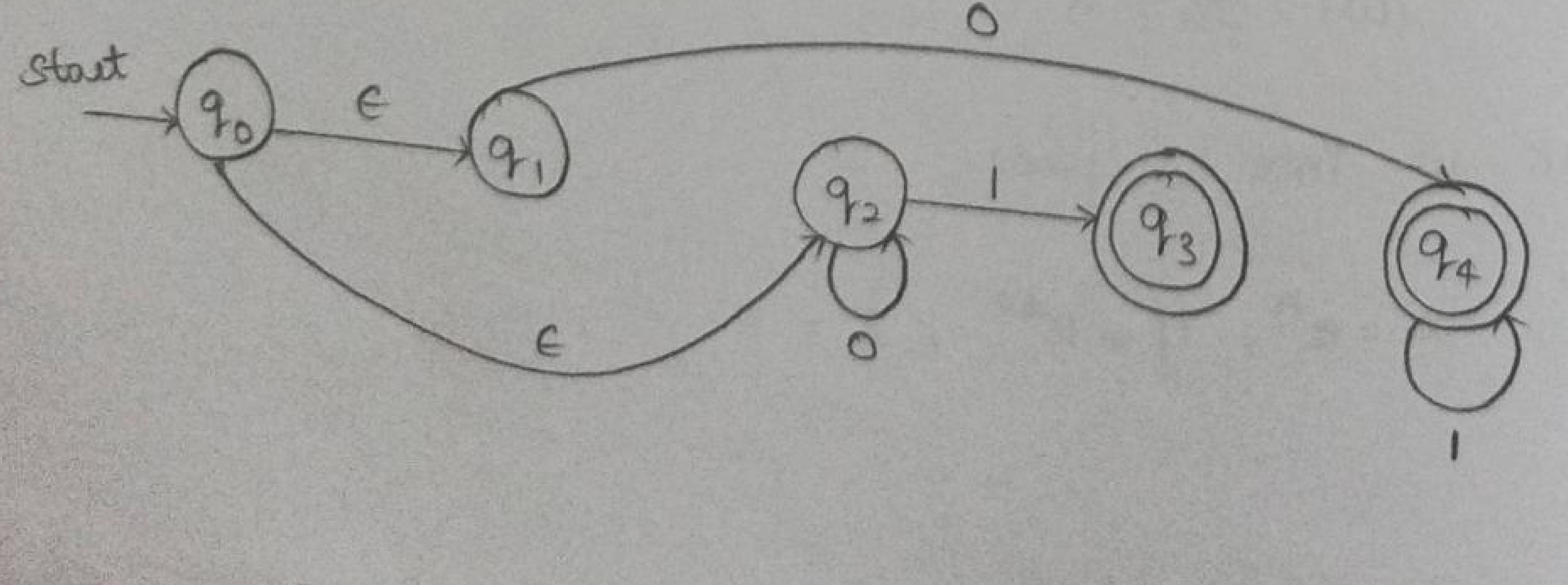
$= n(2+1)$

$= 3n \notin L$

Here L is not a regular language

1. Convert the given ε-NFA to DFA.

δ	0	1	ε
$\rightarrow q_0$	ϕ	ϕ	$\{q_1, q_2\}$
q_1	$\{q_4\}$	ϕ	ϕ
q_2	$\{q_2\}$	$\{q_3\}$	ϕ
$* q_3$	ϕ	ϕ	ϕ
$* q_4$	ϕ	$\{q_4\}$	ϕ



ϵ-NFA $E = (Q_E, \Sigma, \delta_E, q_E, F_E)$ then the equivalent DFA $D = (Q_D, \Sigma, \delta_D, q_D, F_D)$.

1. Find starting state of DFA

$$q_D = ECLOSE(q_D) = \{q_0, q_1, q_2, q_3, q_4\}$$

2. Find ECLOSE of q_1, q_2

$$ECLOSE(q_1) = \{q_1, q_4\}$$
$$ECLOSE(q_2) = \{q_2, q_3\}$$
$$ECLOSE(q_3) = \{q_3\}$$
$$ECLOSE(q_4) = \{q_4\}$$

3. Apply all input symbol on start symbol / state.

$$\delta_D(q_D, 0) = \delta_D(\{q_0, q_1, q_2, q_3, q_4\}, 0) = ECLOSE(\delta_E(q_0, 0) \cup \delta_E(q_1, 0) \cup \delta_E(q_2, 0) \cup \delta_E(q_3, 0) \cup \delta_E(q_4, 0))$$

$$= ECLOSE(q_2, q_4) = \{q_2, q_3, q_4\}$$

$$\delta_D(q_D, 1) = \delta_D(\{q_0, q_1, q_2, q_3, q_4\}, 1) = ECLOSE(\delta_E(q_0, 1) \cup \delta_E(q_1, 1) \cup \delta_E(q_2, 1) \cup \delta_E(q_3, 1) \cup \delta_E(q_4, 1))$$

$$= ECLOSE(q_3, q_4) = \{q_3, q_4\}.$$

$$\delta_D(q_D, \epsilon) = \delta_D(\{q_0, q_1, q_2, q_3, q_4\}, \epsilon) = ECLOSE(\delta_E(q_0, \epsilon) \cup \delta_E(q_1, \epsilon) \cup \delta_E(q_2, \epsilon) \cup \delta_E(q_3, \epsilon) \cup \delta_E(q_4, \epsilon))$$

$$= ECLOSE(q_1, q_2) = \{q_1, q_4\}.$$

$$= \{q_1, q_2, q_3, q_4\}$$

4. $\delta_D(\{q_2, q_3, q_4\}, 0) = ECLOSE(\delta_E(q_2, 0) \cup \delta_E(q_3, 0) \cup \delta_E(q_4, 0))$

$= ECLOSE(q_2) = \{q_2, q_3\}$.

$\delta_D(\{q_2, q_3, q_4\}, 1) = ECLOSE(\delta_E(q_2, 0) \cup \delta_E(q_3, 0) \cup \delta_E(q_4, 0))$

$= ECLOSE(q_3, q_4) = \{q_3, q_4\}$

$\delta_D(\{q_2, q_3, q_4\}, \epsilon) = ECLOSE(\delta_E(q_2, \epsilon) \cup \delta_E(q_3, \epsilon) \cup \delta_E(q_4, \epsilon))$

$= ECLOSE(\phi) = \phi$.

5. $\delta_D((\{q_3\}, \{q_4\}), 0) = ECLOSE(\delta_E(q_3, 0) \cup \delta_E(q_4, 0))$

$= ECLOSE(\phi) = \phi$

$\delta_D(\{q_3, q_4\}, 1) = ECLOSE(\delta_E(q_3, 1) \cup \delta_E(q_4, 1))$

$= ECLOSE(\phi) = \phi$

$\delta_D(\{q_3, q_4\}, \epsilon) = ECLOSE(\delta_E(q_3, \epsilon) \cup \delta_E(q_4, \epsilon))$

$= ECLOSE(\phi) = \phi$.

6. $\delta_D(\{q_1, q_2, q_3, q_4\}, 0) = ECLOSE(\delta_E(q_1, 0) \cup \delta_E(q_2, 0) \cup \delta_E(q_3, 0) \cup \delta_E(q_4, 0))$

$= ECLOSE(q_4, q_2) = \{q_2, q_3, q_4\}$

$\delta_D(\{q_1, q_2, q_3, q_4\}, 1) = ECLOSE(\delta_E(q_1, 1) \cup \delta_E(q_2, 1) \cup \delta_E(q_3, 1) \cup \delta_E(q_4, 1))$

$= ECLOSE(q_3, q_4) = \{q_3, q_4\}$

$\delta_D(\{q_1, q_2, q_3, q_4\}, \epsilon) = ECLOSE(\delta_E(q_1, \epsilon) \cup \delta_E(q_2, \epsilon) \cup \delta_E(q_3, \epsilon) \cup \delta_E(q_4, \epsilon))$

$= ECLOSE(\phi) = \phi$

7. $\delta_D(\{q_2, q_3\}, 0) = ECLOSE(\delta_E(q_2, 0) \cup \delta_E(q_3, 0)) = ECLOSE(q_2)$

$= \{q_2\}$

$\delta_D(\{q_2, q_3\}, 1) = ECLOSE(\delta_E(q_2, 1) \cup \delta_E(q_3, 1)) = ECLOSE(q_3)$

$= \{q_3\}$

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$\delta_D(\{q_2,q_3\},\epsilon) = ECLOSE(\delta_E(q_2,\epsilon) \cup \delta_E(q_3,\epsilon)) = ECLOSE(\phi)$

$= \phi$

8. $\delta_D(\{q_2\},0) = ECLOSE(\delta_E(q_2),0) = \{q_2,q_3\}$

$\delta_D(\{q_2\},1) = ECLOSE(\delta_E(q_2),1) = \{q_3\}$

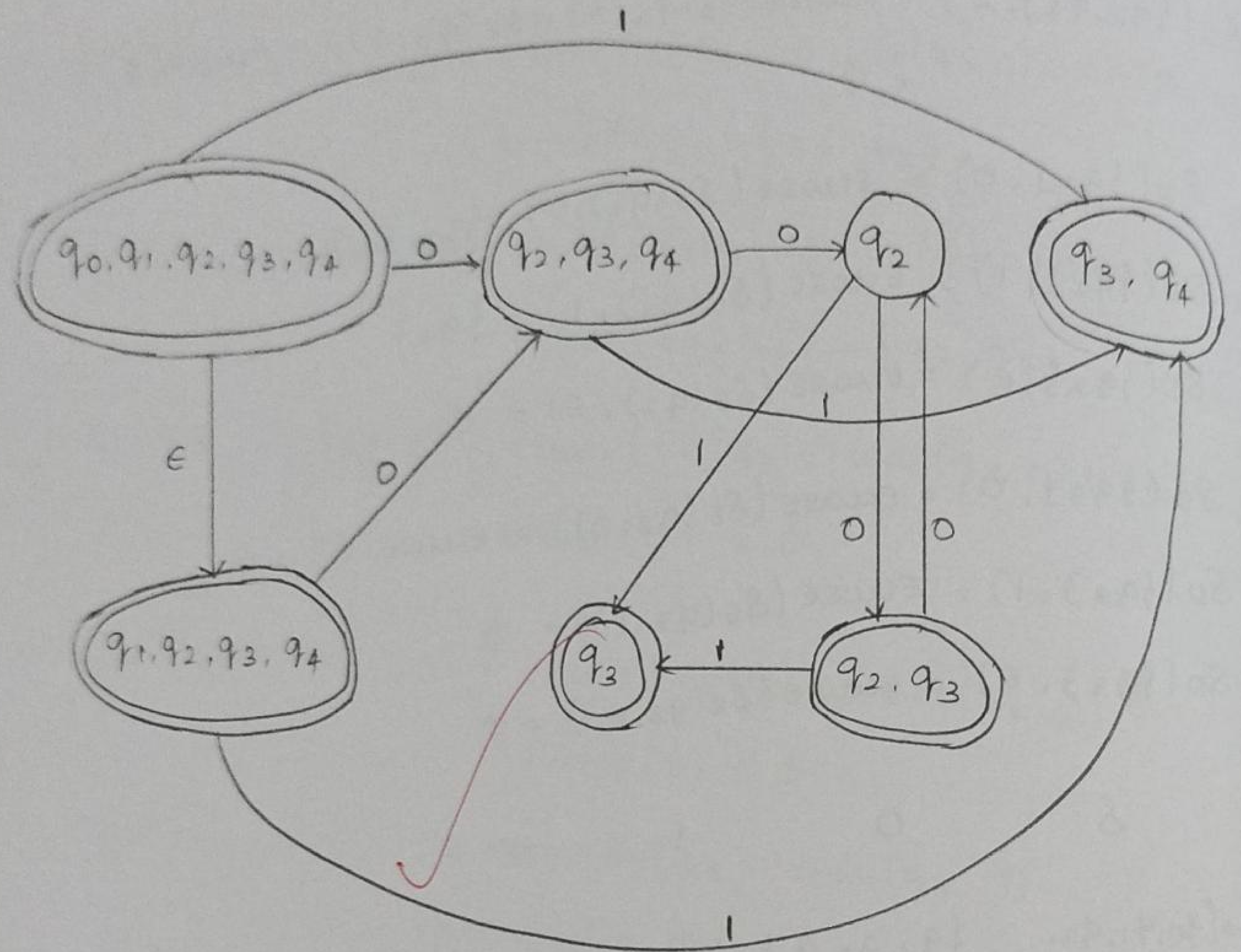
$\delta_D(\{q_2\},\epsilon) = ECLOSE(\delta_E(q_2),\epsilon) = \phi$

9. $\delta_D(\{q_3\},0) = ECLOSE(\delta_E(q_3,0)) = ECLOSE(\phi) = \phi$

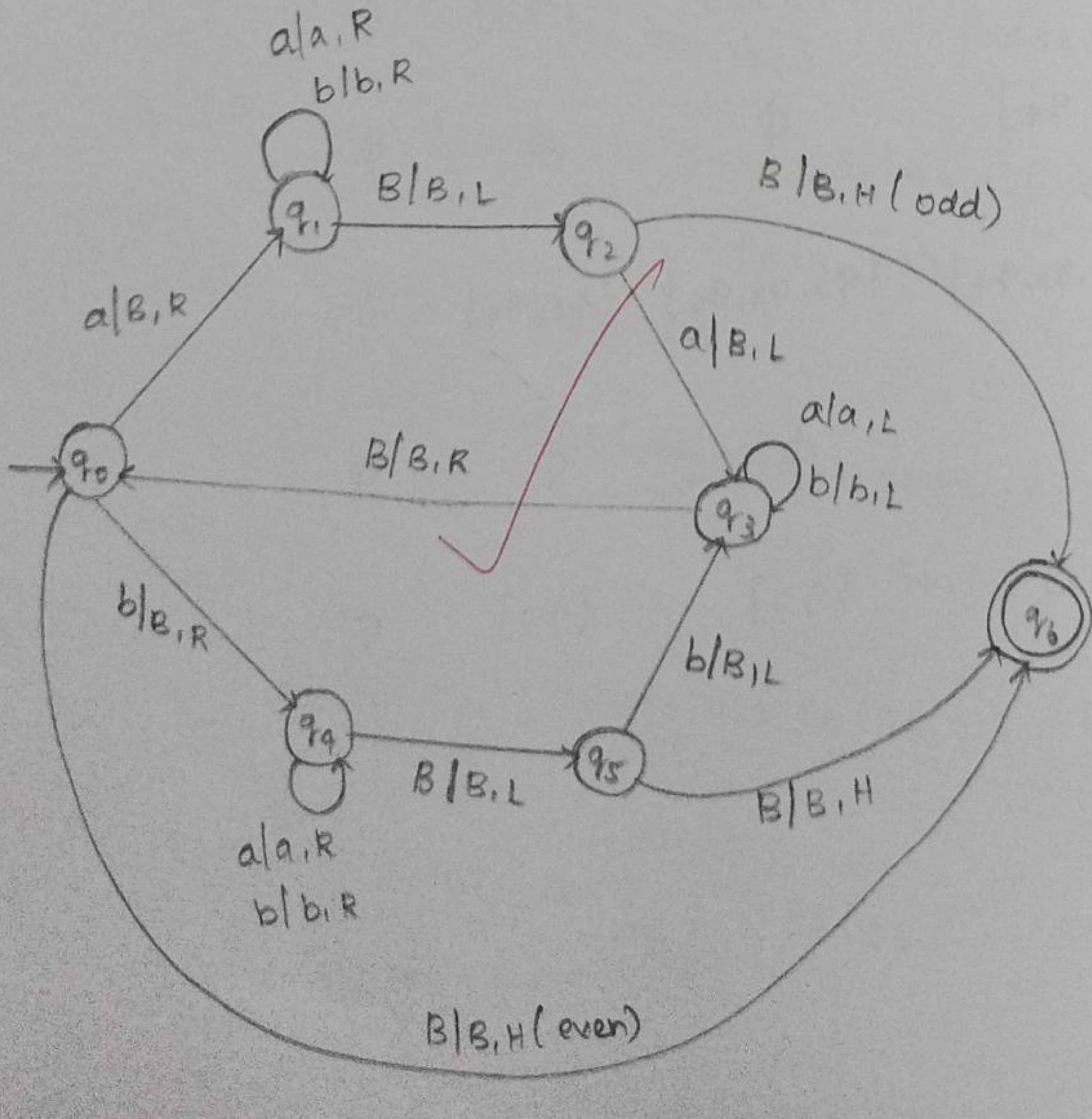
$\delta_D(\{q_3\},1) = ECLOSE(\delta_E(q_3,1)) = \phi$

$\delta_D(\{q_3\},\epsilon) = ECLOSE(\delta_E(q_3,\epsilon)) = \phi$.

δ	0	1	ϵ
* $\rightarrow[q_0,q_1,q_2,q_3,q_4]$	$\{q_2,q_3,q_4\}$	$\{q_3,q_4\}$	$\{q_1,q_2,q_3,q_4\}$
* $[q_2,q_3,q_4]$	$\{q_2\}$	$\{q_3,q_4\}$	ϕ
$[q_2]$	$\{q_2,q_3\}$	$\{q_3\}$	ϕ
* $[q_3,q_4]$	ϕ	ϕ	ϕ
* $[q_1,q_2,q_3,q_4]$	$\{q_2,q_3,q_4\}$	$\{q_3,q_4\}$	ϕ
* q_3	ϕ	ϕ	ϕ
* $[q_2,q_3]$	$\{q_2\}$	$\{q_3\}$	ϕ



10. Construct turing machine to check whether the given string is palindrome or not.



$q_0 abbaB \mapsto Bq_1bbaB$

$\vdash Bbq_1baB$

$\vdash Bbbq_1aB$

$\vdash Bbbaq_1B$

$\vdash Bbbq_2aB$

$\vdash Bbq_3bBB$

$\vdash Bq_3bbBB$

$\vdash q_3BbbB$

$\vdash Bq_0bbB$

$\vdash BBq_4bB$

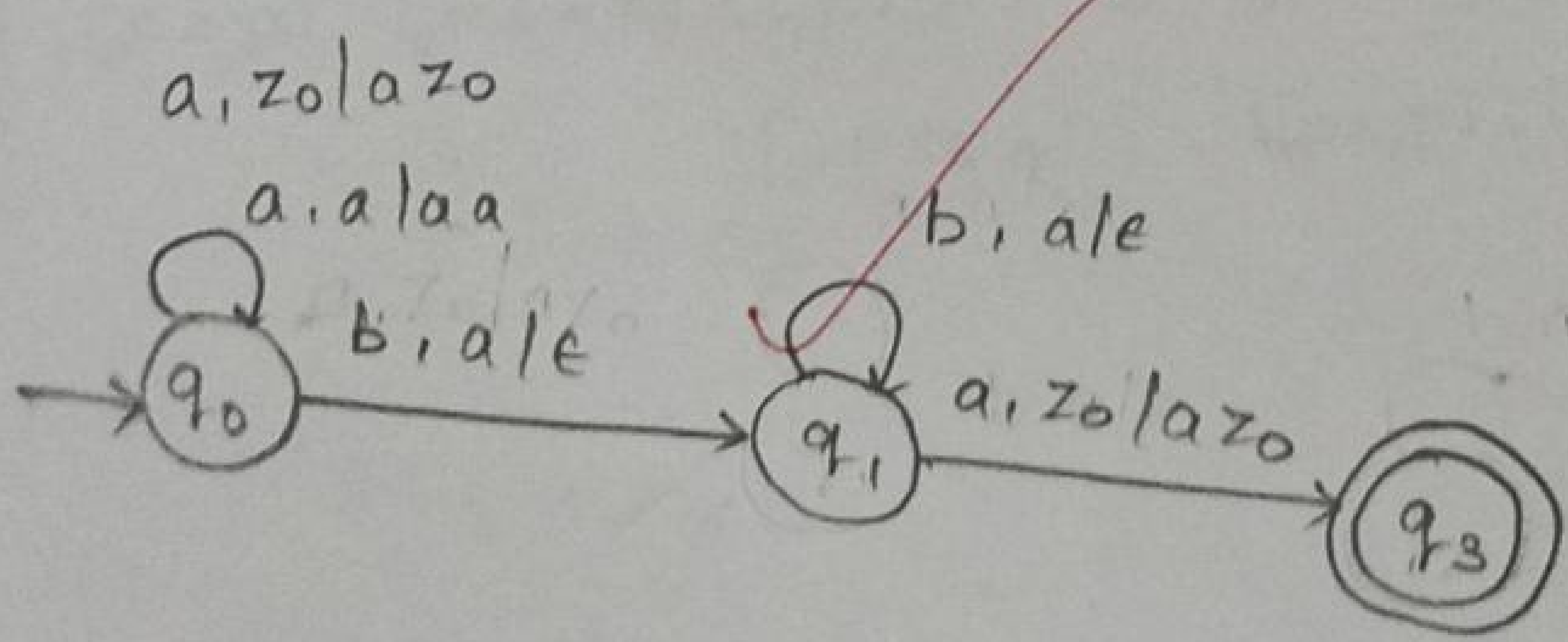
$\vdash BBq_5bB$

$\vdash BBq_3BB$

$\vdash BBBq_0B$

$\vdash Bq_6B$

9. Construct an PDA that accepts the language $L = \{a^n b^n a \mid n \geq 1\}$.



5. Convert the PDA $P = (\{q_0, q_1\}, \{a, b\}, \{x, Z_0\}, \delta, q_0, Z_0, q_1\})$ to a CFG, if transition function is given by

$$\delta(q_0, 0, Z_0) = \{(q_0, xZ_0)\}$$

$$\delta(q_0, 0, x) = \{(q_0, xx)\}$$

$$\delta(q_0, 1, x) = \{(q_1, \epsilon)\}$$

$$\delta(q_1, 1, x) = \{(q_1, \epsilon)\}$$

$$\delta(q_1, \epsilon, Z_0) = \{(q_1, \epsilon)\}.$$

SOLUTION :

we have to define CFG $G = (V, T, P, S)$

$$T = \Sigma = \{0, 1\}$$

Set of ~~the~~ non-terminals :

$$V = \left\{ \begin{array}{l} S, [q_0 Z_0 q_0], [q_0 x q_0], [q_0 x q_1], [q_1 Z_0 q_0], [q_1 x q_0], \\ [q_1 Z_0 q_1], [q_1 x q_1], [q_0 Z q_1] \end{array} \right\}$$

Production of G,

1. For start symbol

$$S \rightarrow [q_0 Z_0 q_0] \mid [q_0 Z_0 q_1]$$

2. Find production for ③, $[q_0 x q_1] \rightarrow 1$

Find production for ④, $\delta [q_1 x q_1] \rightarrow 1$

For ⑤ transition function, $[q_1 Z q_1] \rightarrow e$

For $\delta(q_0, 0, Z_0) = (q_0, xZ_0)$

$[q_0 Z_0 q_0] \rightarrow 0[q_0 x q_0][q_0 Z_0 q_0] \,/\, 0[q_0 x q_1][q_1 Z_0 q_0]$

$[q_0 Z_0 q_1] \rightarrow 0[q_0 x q_0][q_0 Z_0 q_1] \,/\, 0[q_0 x q_1][q_1 Z_0 q_1]$

For $\delta(q_0, 0, x) \rightarrow (q_0, xx)$

$[q_0 x q_0] \rightarrow 0[q_0 x q_0][q_0 x q_0] \,/\, 0[q_0 x q_1][q_1 x q_0]$

$[q_0 x q_1] \rightarrow 0[q_0 x q_0][q_0 x q_1] \,/\, 0[q_0 x q_1][q_1 x q_1]$

Resultant production :

$[q_0 Z_0 q_1] \rightarrow 0[q_0 x q_1][q_1 Z_0 q_1]$

$[q_0 x q_1] \rightarrow 0[q_0 x q_1][q_1 x q_1]$.

$[q_0 Z_0 q_0] \rightarrow 0[q_0, x, q_1][q_1, Z, q_1]$

$[q_0 x q_0] \rightarrow 0[q_0 x q_0][q_0 x q_0] \,/\, 0[q_0 x q_1][q_1 x_0$

$S = [q_0 Z_0 q_1], [q_0 Z_0 q_0]$, $A = [q_0 Z_0 q_0]$, $B = [q_0, Z_0, q_1]$

$C = [q_1, x, q_1]$, $D = [q_1, Z, q_1]$.

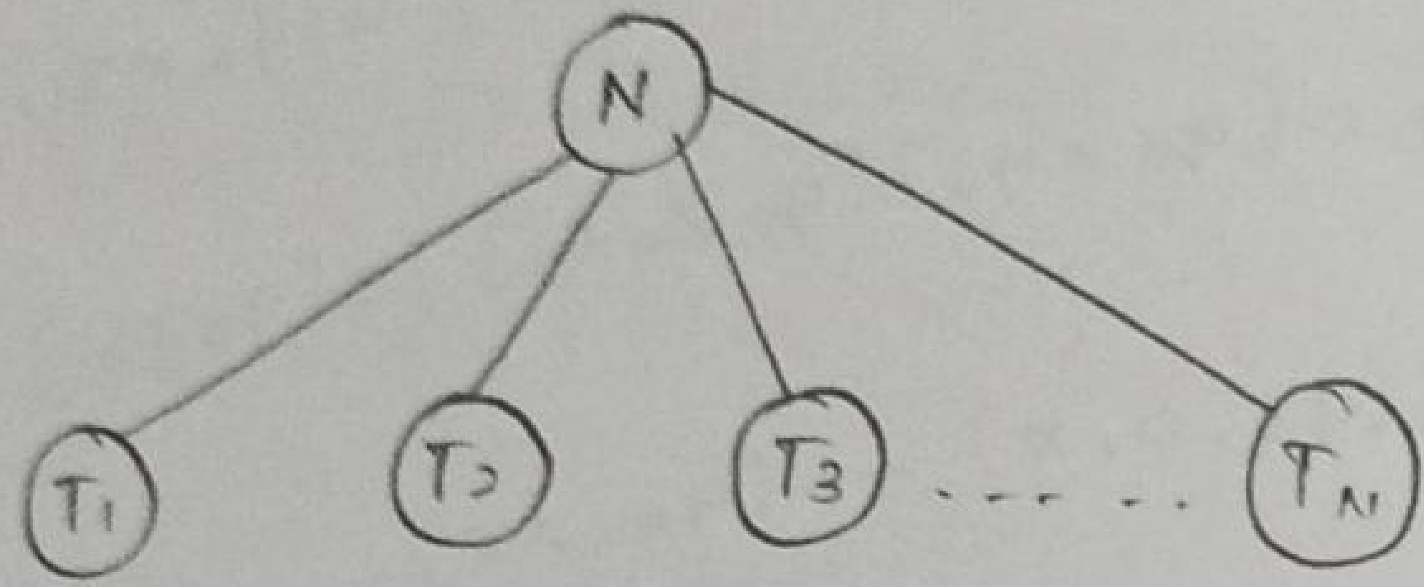
$S \rightarrow A / B$ $\qquad$ $B \rightarrow 0FD / 0EB$

$C \rightarrow 1$ $\qquad$ $E \rightarrow 0EE / \epsilon$

$D \rightarrow \epsilon$ $\qquad$ $F \rightarrow 0FC / 0EF / 1$

$A \rightarrow 0\epsilon A$

11. Prove that "Every tree has one more node than its edges"



$n_1 = e_1 + 1$

$n_2 = e_2 + 1$

No. of nodes in new tree $n(T) = 1 + n_1 + n_2 + \dots + n_K$

No. of edges in new tree $E(T) = K + e_1 + e_2 + \dots + e_K$

$n(T) = 1 + (e_1+1) + (e_2+1) + (e_3+1) + \dots + (e_K+1)$

$= 1 + K + e_1 + e_2 + \dots + e_K$

$= 1 + E(K)$

If T is a tree and T has n nodes and E edges then $n = e+1$.

BASIS :

When T is a single node, then $n=1$, $e=0$, $n=e+1$ holds/satisfied is true.

INDUCTIVE :

Let T be a tree build by inductive state from root node n and K smaller trees $t_1, t_2, \dots, t_K$. Assume $s(t_i)$ holds for $i = 1$ to K, t_i have n_i nodes and e_i edges then $n_i = e_i + 1$.

Nodes of tree T are node n and all nodes of T_i i.e. no. of. nodes in tree T is equal to

The edge of T are K edges which connects nodes n and t.

No. of. edges $= K + e_1 + e_2 + e_3 + \ldots + e_k$ sub, $n_i = e_i + 1$ for all $i = 1, 2, 3, \ldots K$

No. of. nodes in $T = 1 + n_1 + n_2 + \ldots + n_k$.

$= 1 + (e_1 + 1) + (e_2 + 1) + (e_3 + 1) + \ldots + (e_k + 1)$

$= 1 + K + e_1 + e_2 + \ldots + e_k$

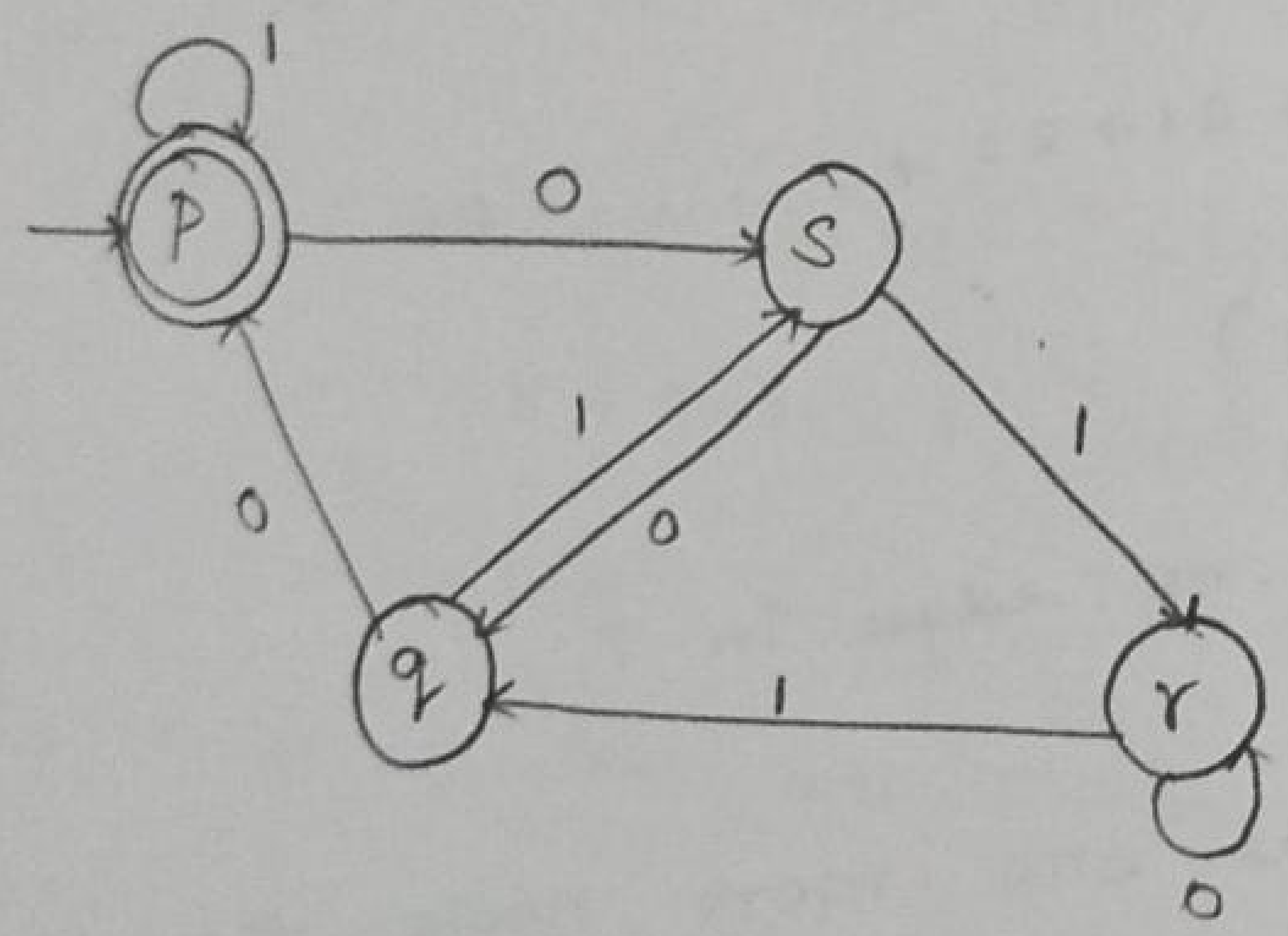
$= 1 + E(T)$

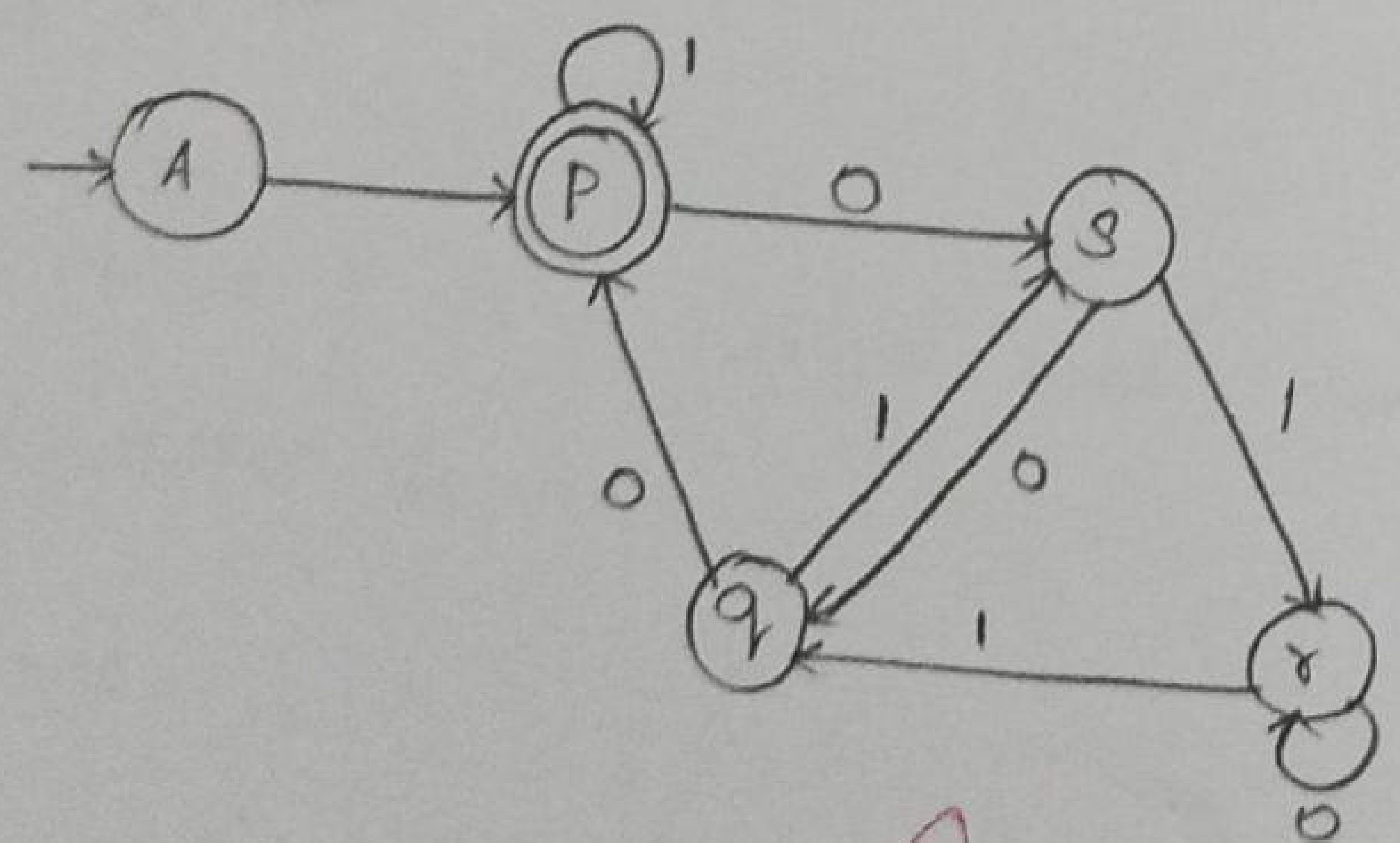
$= 1 +$ no. of. edges in T

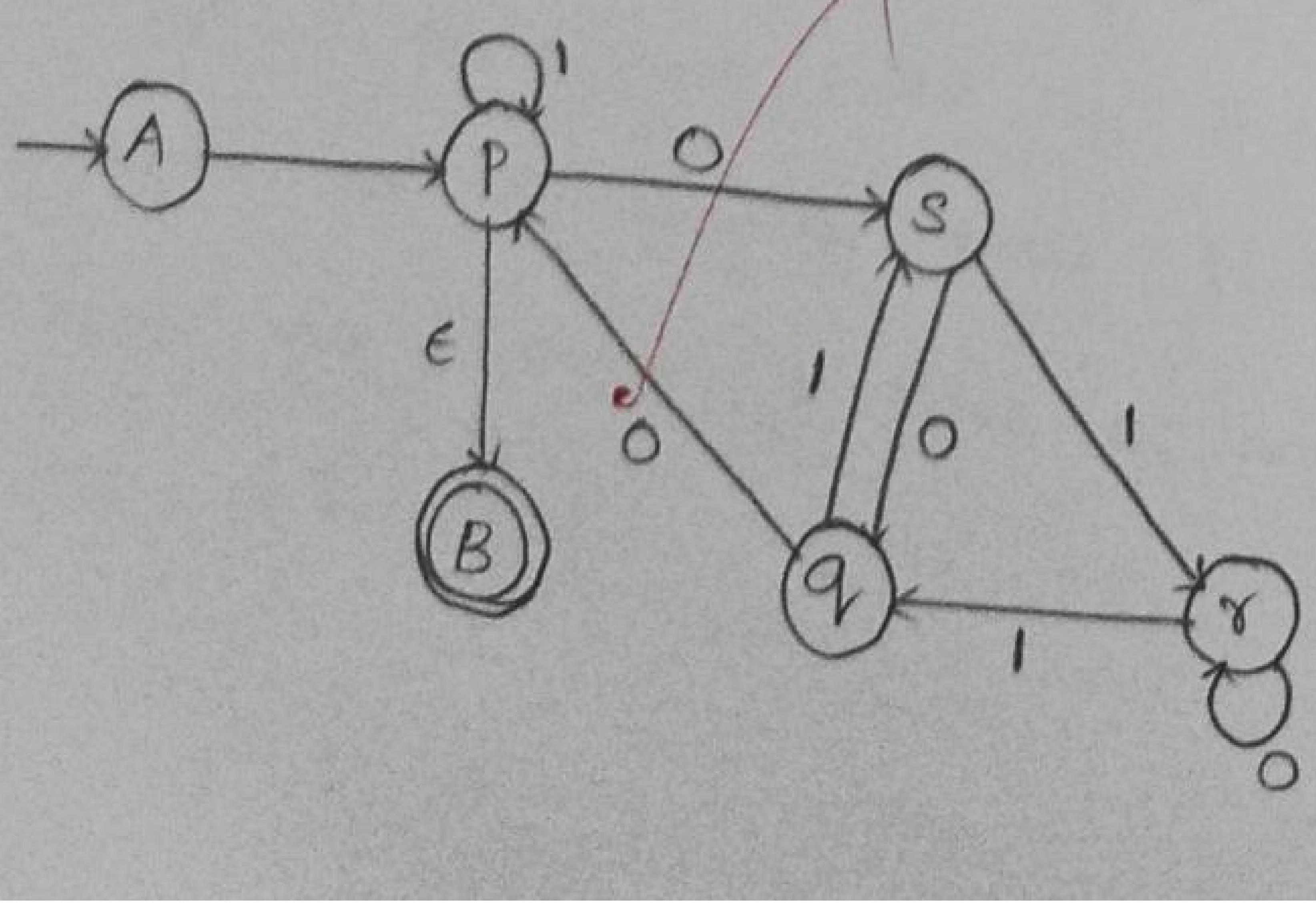
Thus, T has one more node than its edge.

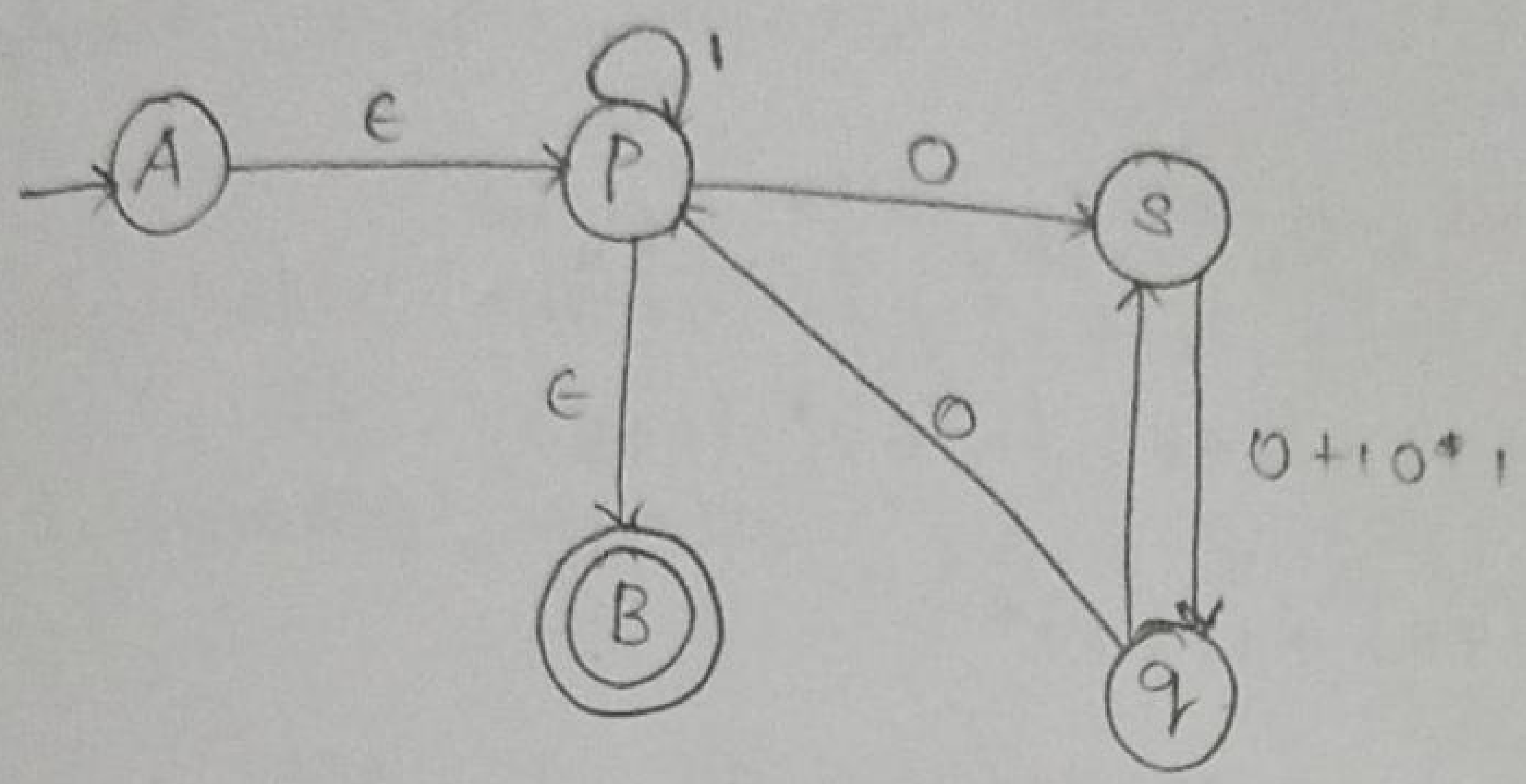
2. Obtain a regular expression for finite automata using both R_{ij} and state elimination method.

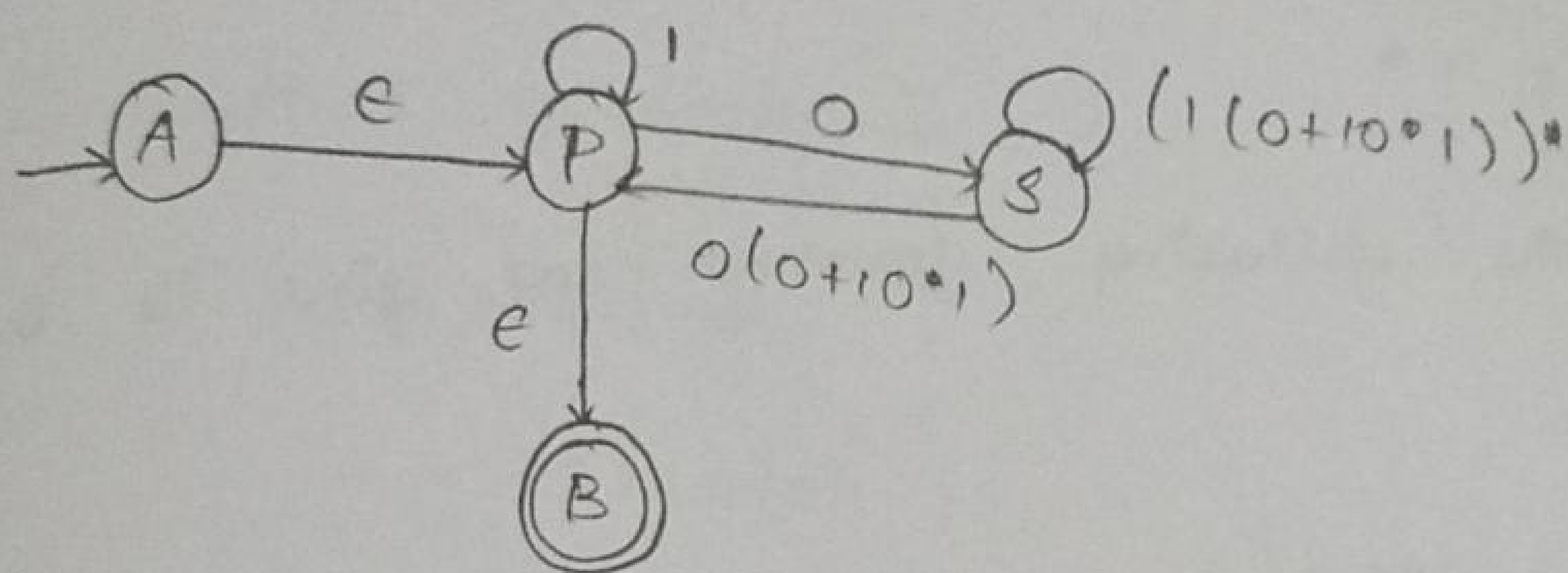
δ	0	1
$* \rightarrow p$	$\{s\}$	$\{p\}$
q	$\{p\}$	$\{s\}$
r	$\{r\}$	$\{q\}$
s	$\{q\}$	$\{r\}$

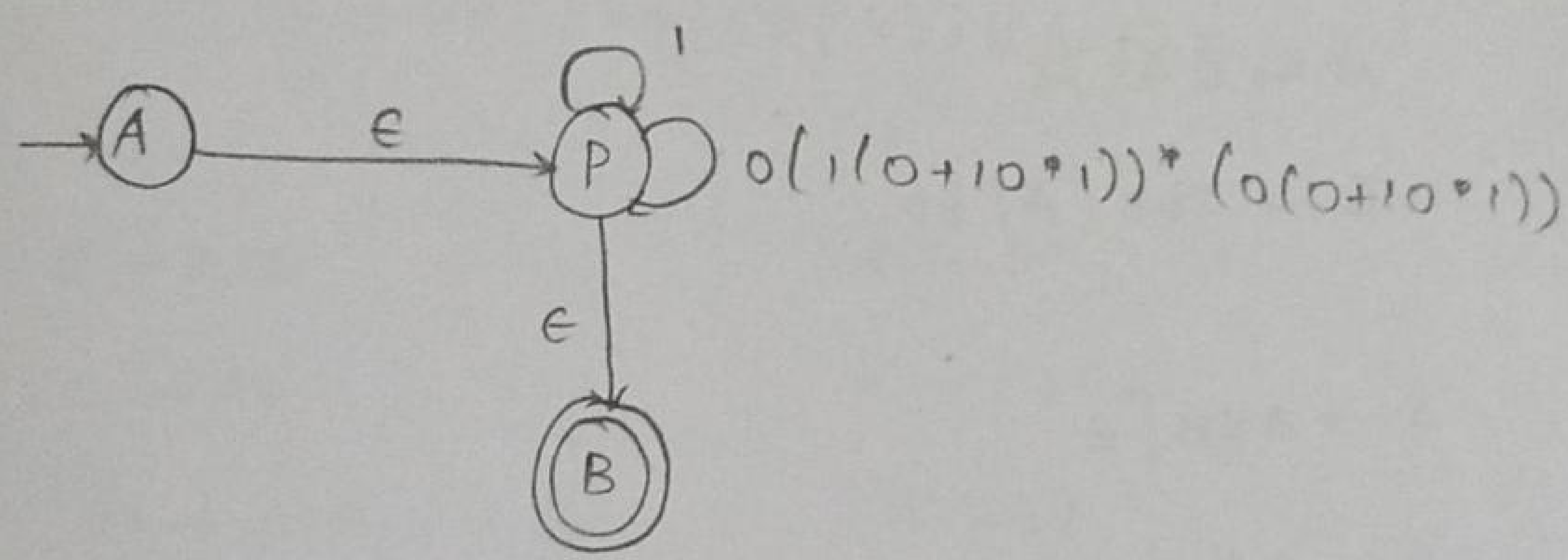


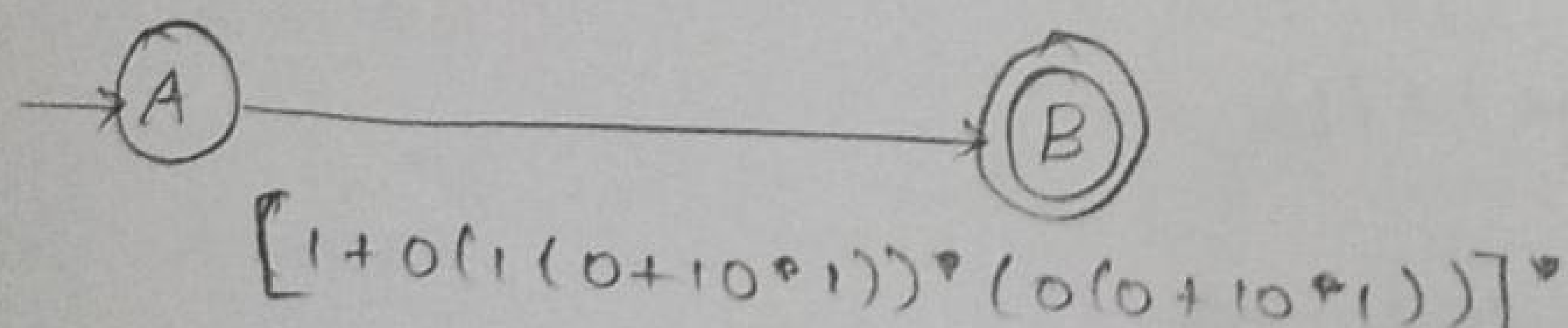












R_{ij} method :

STEP 1 : $k = 0$

$R_{11}^{(0)} = 1 + \epsilon$ $\quad R_{21}^{(0)} = \phi$ $\quad R_{31}^{(0)} = 0$ $\quad R_{41}^{(0)} = \phi$

$R_{12}^{(0)} = 0$ $\quad R_{22}^{(0)} = \epsilon$ $\quad R_{32}^{(0)} = 1$ $\quad R_{42}^{(0)} = \phi$

$R_{13}^{(0)} = \phi$ $\quad R_{23}^{(0)} = 1$ $\quad R_{33}^{(0)} = \epsilon$ $\quad R_{43}^{(0)} = 1$

$R_{14}^{(0)} = \phi$ $\quad R_{24}^{(0)} = 0$ $\quad R_{34}^{(0)} = \phi$ $\quad R_{44}^{(0)} = 0 + \epsilon$

STEP 2 : K = 1

$$R_{11}^{(1)} = R_{11}^{(0)} + R_{11}^{(0)} \cdot (R_{11}^{(0)})^* \ R_{11}^{(0)}$$

$$= (1+\epsilon) + (1+\epsilon)(1+\epsilon)^*(1+\epsilon)$$

$$= (1+\epsilon) + (1+\epsilon)^*$$

$$= (1+\epsilon)^*$$

$$R_{11}^{(1)} = 1^*$$

7. Simplify the following grammar and find its equivalent GNF form

$S \rightarrow ASB \mid \epsilon$

$A \rightarrow aAs \mid a$

$B \rightarrow sbs \mid A \mid bb.$

$S \rightarrow ASB \mid \epsilon$

$A \rightarrow aAs \mid a$

$B \rightarrow Sbs \mid A \mid bb$

1. Eliminate ϵ - production

$S \rightarrow ASB \mid AB$

$A \rightarrow aAS \mid a \mid aA$

$B \rightarrow Sbs \mid A \mid bb \mid sb \mid bs \mid b$

2. Eliminate unit production

$S \rightarrow ASB \mid AB$

$A \rightarrow aAs \mid a \mid aA$

$B \rightarrow Sbs \mid bb \mid Sb \mid bs \mid b \mid aAs \mid a \mid aA$

3. Eliminate useless symbol

Reachable symbol = {S, A, B, a, b}

Generating symbol = {S, A, B, a, b}

∴ There is no useless symbol.

The resultant grammer is

$S \to ASB \mid AB$

$A \to aAS \mid a \mid aA$

$B \to SbS \mid bb \mid Sb \mid bS \mid b \mid aAS \mid a \mid aA$

1. $S \to ASB$

$C_1 \to AS$

$S \to C_1B$

$S \to AB$

2. $A \to aAS$

$C_2 \to a$

$A \to C_2AS$

$A \to C_2C_1$

$A \to a$

$A \to C_2A$

$B \to SbS$

$C_3 \to SS$

$C_4 \to b$

$B \to C_4C_3$

$B \to bb$

$B \to C_4C_4$

$B \to b$

$B \to aAS$

$B \to C_2C_1$

$B \to a$

$B \to aA$

$B \to C_2A$

CNF :

$S \to C_1B \mid AB$

$A \to C_2C_1 \mid a \mid C_2A$

$B \to C_4C_3 \mid C_4C_4 \mid b \mid C_2C_1 \mid a \mid C_2A$.