Planned Contrasts

Following a significant one-way analysis of variance (ANOVA), the researcher may be interested in following up the analysis with some specific comparisons. In the case of the *planned contrast* or *planned comparison*, only a few predicted or *a priori* hypotheses are of interest, and familywise error is not likely to be a serious concern. *Post hoc* tests that adjust for familywise error typically follow a significant one-way ANOVA when many or all possible comparisons are of interest. Philosophically, the distinction between an *a priori* and *post hoc* test has to do with whether or not the group means compared were predicted to be different in advance or are decided after looking at the results. Statistically, the distinction also concerns whether there are a few or many contrasts conducted. Statisticians will cite either the philosophical or the statistical reason for deciding between the two approaches. We know that with many contrasts, familywise error becomes a problem, but if not very many contrasts are performed (e.g., 2, 3, 4?) than it is likely to be safe to use a planned contrast approach.

Planned contrasts typically involve the comparison of just two means. More complicated tests can be conducted (e.g., in a three-group design, the average of two groups might be compared to the third group), but I will not get into demonstrating more complicated comparisons in this handout (see Keppel & Wickens, 2004, for more detail). The approach is to develop a set of weights that eliminate any group means that are not involved in the comparison by giving them a zero weight and to specify the group means to be compared by giving them opposite values, usually -1 and +1. Thus, the first step is to obtain the weighted sum, $\hat{\psi}$, that gives the appropriate difference between the two means that one wishes to compare.

$$\hat{\psi} = \sum w_j \bar{Y}_j$$

In the formula, $\bar{Y}_j$ represents the group mean for each cell and w_i represents the contrast weights or "coefficients." In a three-group design, a comparison between the first and second group means uses the weights of -1, +1, and 0. The third mean drops out because it is multiplied by 0. The next step is to compute the standard error: [1]

$$s_{\hat{\psi}} = \sqrt{MS_{s/A} \sum \frac{w_j^2}{n_j}}$$

The mean-square error, $MS_{s/A}$, is then obtained from the full one-way (omnibus) ANOVA. Here, we use an estimate of error derived from using within-group variability of <u>all cases</u> in the study. This approach gives a more stable estimate of error and a more powerful statistical test than if we simply conducted a standard t test. The next step is simply to compute the t value using the familiar ratio of the difference to the standard error.

$$t_{contrast} = \frac{\hat{\psi}}{s_{\hat{\psi}}}$$

Although still fairly common practice, it is not advisable to conduct a standard t test after a significant ANOVA. Statistical power is lower with the standard t test compared than it is with the planned contrast version for two reasons: a) the sample size is smaller with the t test, because only the cases in the two groups are selected; and b) in the planned contrast the error term is smaller than it is with the standard t test because it is based on all the cases from the ANOVA. As with a standard t test, we can use a Welch's robust approach if there are concerns about equal variances (see Myers, Well, & Lorch, p. 245, for details).

---

[1] I've followed the text book (Myers, Well, & Lorch, 2010) by computing a planned t-test, but many books and some software packages us an F-test. In the F-test version, the contrast weights are used to compute the numerator sum of squares. The mean square for the contrast is then divided by the means square error in a familiar F ratio.   The two tests are statistically equivalent, however, because $t_\psi^2 = F_\psi$.

I will use the teacher satisfaction example from the one-way ANOVA handout to illustrate the computation of the planned contrast: public ($M = 6$) , charter ($M = 9$), and private ($M = 6$). The omnibus ANOVA was significant, and it might be desirable to follow the test with a comparison of the means for public and charter schools for theoretical or policy reasons. Thus, the comparison involves the first two groups, and the contrast weights should be -1, +1, and 0.

$$\hat{\psi} = \sum w_j \bar{Y}_j$$
$$= -1(6) + 1(9) + 0(6)$$
$$= -6 + 9$$
$$= 3$$

$$s_{\hat{\psi}} = \sqrt{MS_{s/A} \sum \frac{w_j^2}{n_j}}$$
$$= \sqrt{1.833 \left[\frac{(-1)^2}{5} + \frac{(1)^2}{5} + \frac{(0)^2}{5} \right]}$$
$$= \sqrt{1.83 \left[\frac{2}{5} \right]}$$
$$= \sqrt{.732}$$
$$= .856$$

$$t_{contrast} = \hat{\psi} / s_{\hat{\psi}}$$
$$= \frac{3}{.856}$$
$$= 3.50$$

The critical value for a t test with $df = N - a = 15 - 3 = 12$ is 2.179. Because our calculated value of 3.50 is greater the critical value, the difference is significant.

To obtain this contrast in SPSS, a contrast subcommand can be added to the one-way ANOVA using the following syntax and this produces a t test version of the planned comparison.[2]

```
ONEWAY satisfaction BY school
    /CONTRAST= -1 1 0.
```

ANOVA

satisfaction

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	30.000	2	15.000	8.182	.006
Within Groups	22.000	12	1.833		
Total	52.000	14			

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
satisfaction	Assume equal variances	1	3.000000000	.8563488386	3.503	12	.004
	Does not assume equal variances	1	3.000000000	...	3.000	5.882	.025

R (uses F-test version, but $t^2 = F$)

```
#compare the first and second means
#(each = n specifies number of cases in each group)
mydata$c1 <- rep(c(-1, 1, 0), each = 5)
#compare the first and third means
mydata$c3 <- rep(c(-1, 0, 1), each = 5)

#request the first contrast, c1, and second contrast, c3
anova(lm(satisfaction ~ c1 + c3, mydata))

Analysis of Variance Table

Response: satisfaction
          Df Sum Sq Mean Sq F value   Pr(>F)
c1         1   22.5 22.5000 12.2727 0.004356
c3         1    7.5  7.5000  4.0909 0.065982
Residuals 12   22.0  1.8333
```
Notice that the square of the t-value from SPSS (assumed equal variances) equals the c1 F from R.

---

[2] The MANOVA command can also be used, but the subcommand requires that you specify a -1 comparisons, where a is the number of levels (see the simple effects analysis handout for more details).