

# Aperiodic control of automobiles

Subjecting theory to practice

Indeevar Shyam Lanka

Literature Survey



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September 29, 2020



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# Abstract

This report is a literature survey on the emerging field of aperiodic sampling and control. The focus is limited to a subset of cyber-physical systems (CPS) with sensing, computing and actuation components distributed over a network, also referred to as networked control systems (NCS). The report begins with an introduction to aperiodic control. Advantages over the conventional periodic implementation are discussed and applications are provided, where such a paradigm can outperform periodic control. Event-triggered control (ETC) and self-triggered control (STC) are the two design options studied in detail.

Modern automobiles are an example of CPS with multiple electronic control units (ECUs) connected over a network that requires coordination in real-time. Due to the widespread usage of automobiles and their impact on our society, it becomes a natural choice to test aperiodic (or event-based) control. By choosing some control loops within the vehicular network, their components, controller design and connection architecture are studied.

Based on the above survey, the last chapter is dedicated to elaborating on gaps in the literature that assist in formulating a new thesis. Further, an outline of the methodology is devised and possible validation methods are discussed.



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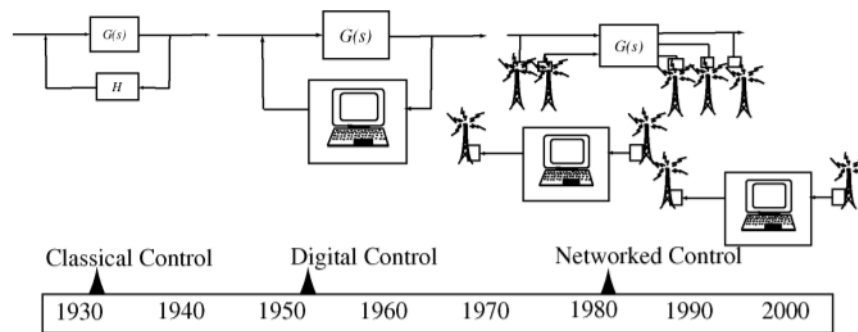
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# Chapter 1

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## Introduction

Feedback control systems that are implemented on digital platforms implicitly assume the notions of periodic sensing, control and zero-order hold actuation, as seen in textbooks [1] and [2] for sampled-data implementations. Real-time software is guided by clocks, time-triggered models and periodicity [3] and they account for a widespread adoption of periodic control in manufacturing, automotive, aerospace and defence systems. In periodic control, the sample time is chosen using thumb rules that consider the worst-case scenario and the sample period remains fixed, irrespective of the current state of the system. This makes periodic control conservative for many cases, e.g. when the system is at a stable equilibrium point or in a neighbourhood around it.



**Figure 1-1:** Timeline of certain developments in control, from [4]

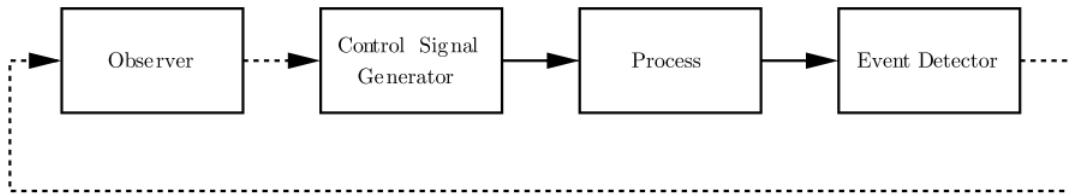
With an increase in affordable and high-speed communication hardware, the controllers were no longer required to be physically collocated with sensing and actuation units. With communication networks came time-varying delays, which poses a significant challenge to performance but sufficiently addressed in the past [5]. However, other challenges that remain persistent to periodic control are related to limited resources, asynchronous communication and multi-rate systems that often arise in large-scale networked systems. It established that

an aperiodic or event-based paradigm is capable of addressing all the issues mentioned above and also outperform traditional sampled-data control in some cases [6].

Aperiodic control refers to sampling and/or control only when the states of the system violate a trigger condition or measured output crosses a threshold. An event is a violation of some triggering condition and this could occur at any point of time, hence arises aperiodicity. This is also referred to as event-based control. Since the idea is intuitive and simple, it emerged by the early 1960s ([7], [8]) but failed to gain the attention of the research community. In the late 1990s, [9] compared Riemann sampling (periodic) and Lebesgue sampling (threshold-based) to show that Lebesgue sampling showed superior performance both in terms of communication efficiency and variance of the controlled variable for simple stochastic systems. This idea was carried further when event-based control PID (proportional-integral-derivative) control showed a significant reduction in resource utilization for a small performance trade-off [10]. These studies were sufficient to attract the attention of the research community, who was then working on large-scale networked systems with shared computation and communication resources.

Since the early 2000s, event-based control has gained much traction for the reasons mentioned above and research was driven by the need to improve resource utilization while guaranteeing stability and desired performance levels. The developments in the first decade after resurgence are summarized in the subsequent sections.

## 1-1 Architecture and hardware requirements



**Figure 1-2:** Process monitored by event-based control [6] - an early attempt to systematize design

An early attempt to structure the design can be seen in [6], which describes the primary components as an event detector, control signal generator and possibly an observer as shown in Figure 1-2. The event detector in conventional periodic control is a clock, but in the aperiodic paradigm uses a triggering condition. Until the triggering condition, based on states, input or output, is not violated, the control action remains unchanged and the communication loop is not closed. In Figure 1-2, the dashed lines represent selective communication and the solid lines indicate uninterrupted communication. The control signal generator uses the last received information to act on the plant until a new event is triggered. As aptly described in [6], control generator design is an open loop problem of regulation to the desired state because the behaviour of the system between samples is completely open loop. Therefore, many variations of hold circuits like zero-order, exponential and impulse holds were studied for designing the control signal generator in [6]. This idea was not carried further as the rest of the literature only exploited zero-order hold actuation.

The earliest survey by Heemels, Johansson & Tabuada [11] presents a good starting point to get acquainted with developments in ETC and STC design. Later, another survey by Hetel, Fiter, Omran, Seuret, Fridman, Richard & Niculescu [12] emerged that discusses a broad range of stability aspects related to aperiodic sampling, most of which are beyond the scope of this survey. Aperiodic control can be classified as event-triggered control (ETC) and self-triggered control (STC), which were first elaborated in Anta and Tabuada [13] based on initial ideas of Velasco et al. [14]. ETC requires continuous monitoring of the triggering condition and when the condition is violated, the communication loop is closed and the controller/actuator is updated with new information. This continuous monitoring requires special hardware in many cases and if dedicated hardware for such monitoring is not possible, then one can resort to STC. In STC, the controller also calculates the current control input and also estimates the next triggering instant [11]. STC combines the best of time and event-based paradigms, as sensing the inter-sample behaviour is absent. Both these schemes will be discussed independently in the following chapters.

## 1-2 Design objectives

As the requirements and constraints of each control system are different, hence designing a suitable triggering condition for every closed loop process is not straight forward. From the literature, it is clear that addressing the following issues while ensuring a "good" aperiodic control scheme.

- Inter-sample times: Non-zero inter-sample times or Zeno behaviour needs to be avoided as this is impossible over digital implementations. Additionally, STC schemes need to minimize the conservativeness in estimating the next event.
- Closed loop stability: The closed loop system is expected to be stable with respect to (w.r.t.) measurement errors, delays in the network, external noise or model uncertainties.
- Schedulability: This is important when the processor is shared with other tasks. Scheduling strategies are not addressed in this work.
- Performance: In all applications, the possibility to outperform periodic implementations may not exist. A tuning variable is required for adjusting performance, such that some trade-off between control and communication cost can be obtained.

## 1-3 Applications

A wide range of applications already uses some or the other form of aperiodic or event-based control, which can be hidden in plain sight. Wheel encoders are event-based sensors found in motors and axles, where the rotation of a toothed wheel generates pulses for calculating wheel speed. Relay control systems [15] are inherently aperiodic in nature as they are driven by events. Examples of relay control systems include thermostats, voltage regulators, overload protection and simple servo systems [15], which are all based on continuous-time event detection. event-based control also happens to be dominant in biological systems where feedback is

involved. Aperiodic control also exists in automotive systems, such as fuel injection in engine control, also in attitude control systems employed in satellites and for an extensive collection of existing examples one can refer to Aström [6].

Moving away from existing examples of aperiodic control, it is necessary to gauge the potential impact this field could have shortly. Therefore, the rest of this section is dedicated to systems, applications and use-cases proposed in the recent literature.

*Resource-limited NCS:* First and foremost application discussed is related to control of the networked system. Some application-oriented works have emerged that focus on implementing existing ETC design methods. These include climate control for greenhouse [16], vehicle platooning [17] and wireless irrigation network [18]. These either address the challenges of limited bandwidth (*shared medium*), computation (*shared processor*) or energy resources over a wired or wireless network while maintaining sufficient control performance.

*Shared processors* is a common phenomenon faced while meeting the increasing demands of the user interacting with the computer meant for controlling a process. These demands are usually other software (s/w) tasks, whose deadlines should also be met while performing control operations. Tabuada's [19] work addresses the scheduling of stabilizing control tasks over a shared processor and it applies to general nonlinear systems, as detailed in the next chapter.

*Shared medium* is another phenomenon where the communications are limited by the finite capacity of the network, e.g. a sensor-actuator network (SAN). Mazo Jr. and Tabuada [20] proposed distributed ETC and decentralized STC for reducing the number of samplings and energy consumption. Decentralized STC broadcasts next sampling time information to all sensor nodes once the actuator node receives current measurement information from the sensor nodes. In contrast, distributed ETC is based on a *token* system to indicate threshold violation. When all children nodes issue the tokens, the actuator requests new measurements to broadcast the updated control action. In both methods, the sensors are set to sleep during the inter-sample period. Another work that deals with STC for wireless SANs is presented by Araujo et al. [21] that is compatible with the IEEE 802.15.4 standard.

*Safety-critical systems:* In addition to reducing energy consumption, network congestion and processor usage, ETC can also be designed for safety. Resilience and security are of prime importance in safety-critical systems like power-grids, connected and automated vehicles (CAVs) and automated factory floors, where cybersecurity and control strategies require integration. Denial-of-service (DoS) attack is an example where security is compromised by an attacker intending to disrupt services over a network through some network jams. With control loops communicating over the network, such attacks could lead to degraded performance or instability. In Dolk et al. [22], an ETC scheme is proposed that tolerates a class of DoS attacks without jeopardizing the system stability. The design allows tradeoffs between performance, resource utilization and robustness to DoS attacks. No assumptions are made on the strategy of the attacker and it addresses output-based feedback control of a class of nonlinear systems, hence making it applicable to a wide range of applications. A case study is also provided, where cooperative adaptive cruise control (CACC) during vehicle platooning is analyzed.

## 1-4 Attention to nonlinear systems

The literature available on aperiodic control for linear systems far outweighs the focus on nonlinear systems, possibly due to wide-scale applicability of linear (or linearizable) systems for control or a deeper understanding of linear systems theory compared to nonlinear systems.

This is evident from the survey paper Hemeels et al. [11] that emphasised only on linear systems and specifically to output-based control over a wireless network. In a later survey, Hetel et al. [12] places limited focus on ETC and STC altogether.

As the dynamics of a general nonlinear system are highly dependent on the operating point compared to linear systems, aperiodicity (state or event dependent sampling) is expected to be more benefiting for nonlinear systems [23]. For these reasons, this literature survey considers only nonlinear systems.

## 1-5 Notation and preliminaries

For the sake of convenience and consistency, the following notations and definitions are adhered to build up rest of the report. The p-norm of vector  $x$  is denoted by  $\|x\|_p$  and for specifically representing 2-norm the subscript  $p$  is removed  $\|x\|$ . A scalar continuous function  $\alpha$  is of class  $\mathcal{K}$  if  $\alpha(0) = 0$ ,  $\alpha(r) > 0 \forall r > 0$  and strictly increasing. Additionally, if  $\alpha(r) \rightarrow \infty$  when  $r \rightarrow \infty$ , then  $\alpha(\cdot)$  belongs to class  $\mathcal{K}_\infty$ . A scalar continuous function  $\beta(r, s)$  is of class  $\mathcal{KL}$ , if the mapping  $\beta(r, s)$  belongs to class  $\mathcal{K}$  for a fixed  $s$  and the mapping  $\beta(r, s)$  is decreasing with  $\beta(r, s) \rightarrow 0$  as  $s \rightarrow \infty$  for a fixed  $r$ , defined over  $r \in [0, a)$  and  $s \in [0, \infty)$ . The space  $\mathcal{L}_p^m$  refers to all piecewise continuous functions/signals  $u : [0, \infty) \rightarrow \mathbb{R}^m$  that satisfy the norm,

$$\|u\|_{\mathcal{L}_p} = \left( \int_0^\infty \|u(t)\|^p \right)^{1/p} < \infty.$$

Further,  $\mathcal{L}_e^m$  refers to the extended space of  $\mathcal{L}^m$  where the signal  $u_\tau(t)$  refers to the truncated signal  $u(t)$  such that after time  $t > \tau$  the function value is zero.

**Definition 1.1** (Input-to-State Stable [24]). A system  $\dot{x} = f(x, u)$  with  $f : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$  is said to be input-to-state stable (ISS) if there exist a class  $\mathcal{KL}$  function  $\beta$  and class  $\mathcal{K}$  function  $\gamma$  that satisfy,

$$\|x(t)\| \leq \beta(\|x_0\|, t - t_0) + \gamma\left(\sup_{t_0 \leq \tau \leq t} \|u(\tau)\|\right), \quad \forall t \geq t_0 \quad (1-1)$$

where  $x_0 = x(t_0)$  is the initial state and the origin of the unforced system  $\dot{x} = f(x, 0)$  is asymptotically stable.

**Definition 1.2** (ISS-Lyapunov function [19]). A smooth function  $V : \mathbb{R}^n \rightarrow \mathbb{R}_0^+$  is an ISS-Lyapunov function for a system  $\dot{x} = f(x, u)$  with feedback law  $u = k(x + e)$  subject to error  $e(t)$ , if there exist class  $\mathcal{K}_\infty$  functions  $\underline{\alpha}, \bar{\alpha}, \alpha$  and  $\gamma$  satisfying,

$$\underline{\alpha}(|x|) \leq V(x) \leq \bar{\alpha}(|x|) \quad (1-2)$$

$$\frac{\partial V}{\partial x} f(x, k(x + e)) \leq -\alpha(|x|) + \gamma(|e|). \quad (1-3)$$

**Remark.** Existence of ISS-Lyapunov function for a closed loop system implies that closed loop is ISS [24].

**Definition 1.3** (Finite-gain  $\mathcal{L}$  stable [25]). Consider a system with input-output relation represented by  $y = Hu$ , where  $y \in \mathbb{R}^q$ ,  $u \in \mathbb{R}^m$  and  $H : \mathcal{L}_e^m \rightarrow \mathcal{L}_e^q$ . The system is finite-gain  $\mathcal{L}$  stable if there exist nonnegative  $\gamma$  and  $\beta$  such that,

$$\|y\| \leq \gamma \|u_\tau\| + \beta, \quad (1-4)$$

for all  $u \in \mathcal{L}^m$  and  $\tau \in [0, \infty)$ .

**Theorem 1.1** (Small gain theorem [25]). *Consider two finite-gain  $\mathcal{L}$  stable systems  $y_1 = H_1 u$  and  $y_2 = H_2 u$  with gains  $\gamma_1$  and  $\gamma_2$ . The closed loop formed by interconnecting the two systems is also  $\mathcal{L}$  stable if,*

$$\gamma_1 \gamma_2 < 1. \quad (1-5)$$

In literature, inter-sample times is also referred to as inter-execution times or inter-event times. To emphasis state or input values at a particular sampling instant  $x(t_k)$  or  $x_k$  is utilized. The triggering condition of an event-based controller is denoted by  $\mathcal{C}(x, x_k)$  and the condition is violated when  $\mathcal{C}(x, x_k) > 0$ , which closes the loop to update control input with new measurement  $x_{k+1}$ .

# Event-triggered Control

Consider a continuous-time general nonlinear system in closed loop,

$$\dot{x} = f(x, u), \quad (\text{plant dynamics}) \quad (2-1)$$

$$u = k(x), \quad (\text{controller}) \quad (2-2)$$

where  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^m$  with regularity assumptions such as  $f(0, 0) = 0$  and  $f$  is locally Lipschitz in a compact set. The controller, in most cases, is a static feedback law that renders the continuous-time system stable in some sense. Various assumptions on the type of stability are made in the literature to design ETC. Each section describes the assumptions made and methods developed in the literature, along with some discussion on results.

### 2-1 Perturbation approach

This method employs perturbed systems analysis (see [25]) to study the stability of the ETC systems and construct a triggering condition. First studied by Tabuada [19], the simplicity of this method has appealed other authors to provide extensions, see e.g. [13] and [23].

#### 2-1-1 Stabilization

In Tabuada [19], a feedback controller  $u = k(x)$  is assumed to exist that renders the continuous-time system (2-1) ISS w.r.t. measurement errors ( $e$ ). The error arises due to the digital implementation of the controller, where the inter-sample information is not available for control. When implemented in event-triggered paradigm, continuous monitoring of the state is required to check  $\mathcal{C}(x, x_k)$ . The sampling error is described as,

$$e(t) = x(t_k) - x(t), \quad (2-3)$$

$$\implies \dot{x} = f(x, k(x + e)), \quad (2-4)$$

with  $t_k$  representing the sample instants. The ISS property of the closed loop system is established by assuming the existence of ISS-Lyapunov function ( $V : \mathbb{R}^n \rightarrow \mathbb{R}^+$ ) that satisfies (1-3). Assuming no delay ( $\Delta = 0$ ) in communication, restricting the error with this condition

$$\begin{aligned} -\alpha(|x|) + \gamma(|e|) &\leq 0, \\ \implies \gamma(|e|) &\leq \sigma\alpha(|x|), \end{aligned} \quad (2-5)$$

$$\implies \frac{\partial V}{\partial x} f(x, k(x+e)) \leq (1-\sigma)\alpha(|x|) < 0, \quad \forall \sigma \in (0, 1) \quad (2-6)$$

provides an asymptotic decrease of Lyapunov function. The sampling policy can be designed to satisfy the above inequality by updating control action when  $\sigma\alpha(\|x\|) - \gamma(\|e\|) := \mathcal{C}(x, e) < 0$ . This policy renders the event-triggered system with global asymptotic stability. By satisfying additional Lipschitz conditions on  $f$  and  $k$  over compacts, a minimum bound on inter-sampling time can be established. Even in the presence of bounded delays ( $0 \leq \Delta \leq \epsilon$ ), semi-global asymptotic stability can be guaranteed. In the presence of sufficiently small delay  $\Delta > 0$ , a lower bound on inter-sampling time is ensured. Tabuada [19] proposed this for scheduling of *shared processors*, which follow preemptive scheduling with the highest priority assigned to control task. A sufficient condition for co-schedulability is also provided therein.

### 2-1-2 Trajectory tracking

Tallapragada and Chopra [26] extended the idea of Tabuada [19] for trajectory tracking with some improvements. The continuous-time system (2-1) assumes the existence of a feedback controller that provides global uniform asymptotic (GUA) tracking of the reference trajectory defined by a dynamical system,

$$\dot{x}_d = f_r(x_d, v) \quad (\text{reference system}) \quad (2-7)$$

$$u = \gamma(\xi), \quad \xi = [\tilde{x}; x_d; v] \quad (\text{controller}) \quad (2-8)$$

$$\frac{\partial V}{\partial x} \left[ f(\tilde{x} + x_d, \gamma(\xi)) - \dot{x}_d \right] \leq -\alpha_3(\|\tilde{x}\|) \quad (\text{Lyapunov rate of } \tilde{x}) \quad (2-9)$$

where  $x_d$  is the desired trajectory,  $v$  is the input to reference system,  $\tilde{x} = x - x_d$  represents tracking error and  $\alpha_3 \in \text{class } \mathcal{K}_\infty$ . Since the measurements are not continuously updated on a digital platform, the error in measurement is defined as,  $e \triangleq \xi(t_i) - \xi$ .

The event-based tracking controller guarantees uniform ultimate bound (UUB) on tracking error and non-Zeno sampling behaviour under certain assumptions of Lipschitz over compacts, admissibility of inputs and stability of continuous-time closed loop system. The error dynamics are re-written in perturbed systems form,

$$\dot{\tilde{x}} = \underbrace{[f(\tilde{x} + x_d, \gamma(\xi)) - \dot{x}_d]}_{\text{nominal}} + \underbrace{[f(\tilde{x} + x_d, \gamma(\xi + e)) - f(\tilde{x} + x_d, \gamma(\xi))]}_{\text{perturbation}} \quad (2-10)$$

and the Lyapunov rate of the error dynamics is given by,

$$\dot{V} \leq -\alpha_3(\|\tilde{x}\|) + \beta(\|\tilde{x}\|)L(R)^T|e| \quad (2-11)$$



where  $\alpha_3(\cdot)$  is a class  $\mathcal{K}_\infty$  function that upper bounds  $\dot{V}$  of continuous-time closed and  $\beta(R) \geq \max_{\|w\| \leq R} \left| \frac{\partial V(w)}{\partial w} \right|$ ,  $\forall R \geq 0$ . Therefore the triggering condition is formulated as,

$$t_0 = \min\{t \geq 0 : \|\tilde{x}\| \geq r, r > 0\} \quad (2-12)$$

$$t_{i+1} = \min \left\{ t \geq t_i : L(\|\tilde{x}_i\|)|e| - \frac{\sigma \alpha_3(\|\tilde{x}\|)}{\beta(\|\tilde{x}\|)} \geq 0, \|\tilde{x}\| \geq r \right\} \quad (2-13)$$

with design parameters  $r$  (ultimate bound of the tracking error) and  $\sigma \in (0, 1)$ . A preliminary version of this work was presented in Tallapragada and Chopra [27], which is specifically for control-affine nonlinear systems.

## 2-2 Hybrid systems approach

Consider a continuous-time closed loop nonlinear system,

$$\dot{x}_p = f_p(x_p, u), \quad (2-14)$$

$$\dot{x}_c = f_c(x_c, x_p), \quad u = g_c(x_c, x_p). \quad (2-15)$$

In sampled data implementation, the plant and controller do not have access to  $u(t)$  and  $x_p(t)$  during the inter-sample period  $[t_k, t_{k+1})$ , therefore only  $u_k$  and  $x_k = [x_p(t_k); x_c(t_k)]$  are used for control. At sample times  $(t_i, \forall i \in \mathbb{N})$  the information is updated as  $x_p^+ = x_p(t)$  and  $u^+ = u(t)$  or the measurement error  $e^+ = 0$ . The combination of continuous dynamics with jumps makes it hybrid, therefore the formalisms of hybrid systems theory [28] can be applied to solve the control problem. One of the many ways to rewrite the dynamics is,

$$\begin{aligned} \dot{q} = f_q(q) &\triangleq \begin{cases} \dot{x} = f_x(x, e), \\ \dot{e} = f_e(x, e), \\ \dot{\eta} = f_\eta(x, e, \eta) \end{cases} & q \in C \\ q^+ = h_q(q) &\triangleq \begin{cases} x^+ = x, \\ e^+ = h_e(x, e), \\ \eta^+ = h_\eta(x, e, \eta) \end{cases} & q \in D \end{aligned} \quad (2-16)$$

where the closed sets  $C, D$  represent continuous and jump set, respectively and  $\eta$  represents auxiliary variables that can be used by the triggering condition [29].

### 2-2-1 Stabilization

Wang & Lemmon [30] designed an exponentially stable ETC using switched system representation. The closed loop is assumed to be ISS w.r.t. measurement error like in Tabuada [19]. One major difference is that by using piecewise continuous bounding function a non-monotonous decrease of Lyapunov function is allowed, unlike in Tabuada [19], therefore resulting in larger inter-event times at all times.

Postoyan, Tabuada & Nesic [31] used hybrid system strategies to capture a unified version of various ETC schemes. This work also proposes two techniques, but in contrary to Seuret et

al. [32], there exists a uniform minimum dwell time and it is semi-global. The closed loop can be ensured to have global uniform asymptotic stability under some assumptions. The first strategy employs a threshold variable ( $\eta$ ) for the triggering condition,

$$\dot{\eta} = -\delta(\eta), \quad \eta^+ = W(e), \quad (\text{threshold variable dynamics}) \quad (2-17)$$

$$W(e) \geq \max\{V(x), \eta\} \quad (\text{triggering condition}). \quad (2-18)$$

The transmission intervals are longer than Wang & Lemmon [30] and Tabuada [19]. The second strategy modifies an existing idea on periodic sampling for nonlinear systems, to derive a clock that is state-dependent.

### 2-2-2 Trajectory tracking

Tracking is an important control problem that requires to follow a time varying reference trajectory. Vamvoudakis, Mojoodi & Ferraz [33] designed a trajectory tracking controller using impulsive systems formulation. The main difference from the previous tracking controller Tallapragada & Chopra [26] is that solutions are optimal with respect to the cost function. An actor-critic structure is employed, where the actor approximates the optimal ETC and critic estimates the optimal cost.

## 2-3 Periodic ETC

Periodic ETC (PETC) offers a smooth transition between the conventional periodic to event-based sampling. In PETC, the digital platform continues sense the system periodically. At each time period the triggering condition is checked to decide if control action requires any update. Postoyan et al. [34] introduced periodic ETC (PETC) for nonlinear systems. Starting with a known continuous-time ETC, a systematic procedure is provided for sample period selection and approximately emulate the continuously evaluated ETC. The approach similar to hybrid systems, where the continuous-time closed loop system is modelled as an impulsive system with continuous and jump dynamics,

$$\dot{z}(t) = g(z(t)), \quad \mathcal{C}(z) < 0 \quad (2-19)$$

$$z(t_k^+) = b(z(t_k)), \quad b(z) = [x; 0; c(x, e, \chi)], \quad \mathcal{C}(z) \geq 0, \quad (2-20)$$

$$\mathcal{C}(x, e) = \gamma(|e|) - \sigma\alpha(|x|) \quad (\text{triggering condition})$$

where  $z = [x; e; \chi]$ ,  $\chi$  denotes additional variables that may be introduced and  $g, c, b$  are locally Lipschitz functions. First, a reasonable assumption regarding the continuous-time ETC implementation is made that the closed loop has a minimum inter-sample time ( $T$ ) within a compact set  $z \in \Omega$ , e.g. a Lyapunov level set. The sampling period ( $h$ ) of PETC is chosen such that,

$$0 < h < T, \quad (2-21)$$

Zeno-behaviour is avoided by design. The sampling policy is given by,

$$t_{k+1} = t_k + \min\{t > 0 : (\tilde{\mathcal{C}}(z) \geq 0) \wedge (t = nh, n \in \mathbb{Z}_{\geq 0})\}. \quad (2-22)$$

This new triggering condition  $\tilde{\mathcal{C}}(z)$  is designed such the time period  $h$  is split into  $N$  horizons to guarantee that  $\mathcal{C}(z(nh + i\frac{h}{N})) \leq 0$ . This is done by assuming that both  $\mathcal{C}(z)$  and  $g(z)$  are  $p$ -times continuously differentiable and that for every initial state  $z(0) \in \Omega$  the future states  $z(t, z_0)$  also remain in  $\Omega$ . In this method,  $h$  and  $N$  can be chosen to approximately meet the specifications of the original closed loop systems. This method can be extended to dynamic feedback systems and the actuation need not be zero-order hold. For an output feedback version of PETC for nonlinear systems, the reader can refer to Wang et al. [35].

## 2-4 Small gain approach

Liu and Jiang [36] applied the small-gain theorem for stabilization under disturbance and uncertainty, starting with an assumption that continuous-time closed loop is ISS w.r.t. measurement errors,

$$\begin{aligned} \dot{x} &= f(x, u) \triangleq \bar{f}(x, x + e), \\ |x| &\leq \max\{\beta(|x(0)|, t), \gamma(\|e\|_\infty)\} \quad (\text{another form of ISS}), \end{aligned}$$

where  $\beta \in \text{class } \mathcal{KL}$  and the ISS gain  $\gamma \in \text{class } \mathcal{K}$ . From the ISS assumption, if the sampling error satisfies  $|e| \leq \rho(|x|)$ , then the small-gain condition that provides asymptotic convergence of  $x(t)$  is given as

$$\rho \circ \gamma < I_d, \quad (2-23)$$

where  $\rho$  is a class  $\mathcal{K}$  function,  $I_d$  represents identity matrix (or 1, if scalar) and  $\circ$  indicates function composition. The right choice of  $\rho$  satisfying the above inequality leads to the triggering rule,

$$t_{k+1} = \inf\{t > t_k : \rho(|x|) - |x - x_k| = 0\}.$$

In the presence of disturbance, the system is can be described as

$$\dot{x} = f(x, u, d) \triangleq \tilde{f}(x, x + e, d),$$

where the disturbance may not allow system to converge to the origin and possibly result in infinitely fast sampling. To overcome this, Liu and Jiang [36] assumed ISS w.r.t. sampling error and disturbance,

$$|x| \leq \max\{\beta(|x(0)|, t), \gamma(\|e\|_\infty), \gamma^d(\|d\|_\infty)\}, \quad (\gamma^d \in \text{class } \mathcal{K}) \quad (2-24)$$

to propose an event-triggering condition with  $\epsilon$  modification, wherein

$$t_{k+1} = \inf\{t > t_k : \max\{\rho(|x|), \epsilon\} - |x - x_k| = 0\}, \quad \epsilon > 0. \quad (2-25)$$

Unlike the approach in Tabuada [19], this work doesn't require an ISS-Lyapunov function or locally Lipschitz  $\alpha^{-1}$  in (2-5), as such a condition may not be easy to check even for linear systems. In small-gain approach, locally Lipschitz ISS gain  $\gamma$  can avoid Zeno-behaviour. Further, Liu and Jiang [36] also addressed ETC design for uncertain systems in the presence of disturbance, which are represented as

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \Delta_i(\bar{x}_i, d), \quad i \in \{1, 2, \dots, n-1\} \\ \dot{x}_n &= u + \Delta_n(\bar{x}_n, d), \end{aligned} \quad (2-26)$$

where  $\Delta_i$ 's are locally Lipschitz uncertain functions and  $d$  is measurable and bounded external disturbance. The closed loop system is transformed to a large-scale system of ISS-subsystems. Accurate knowledge of the system is not required but ensuring that system is locally Lipschitz can avoid infinitely fast sampling. By using the following cyclic-small-gain theorem, the closed loop is guaranteed to be ISS so that influence of sampling can be analysed,

$$\gamma_{i_1 i_2} \circ \gamma_{i_2 i_3} \cdots \gamma_{i_N i_1} < I_d, \quad (2-27)$$

where  $i_1$  to  $i_N$  are the ISS-subsystems and  $\gamma_{i_p i_q}$  is the gain from component  $i_p$  to  $i_q$ .

## 2-5 Dynamic triggering

Dynamic triggering uses a varying triggering condition, which depends on an internal dynamic variable. In contrary, the ETC scheme (2-5) by Tabuada [19] is referred as static triggering since the triggering condition depends only on the current value of  $x$  and  $e$ . Girard [37] proposed the idea of dynamic triggering mechanism for ETC. The closed loop system (2-1) and (2-2) is assumed to be ISS w.r.t. measurement errors and an ISS-Lyapunov function is known. The internal dynamic variable  $\eta$  satisfies the following equation,

$$\dot{\eta} = -\beta(\eta) + \sigma(\|x\|) - \gamma(\|e\|), \quad \eta(0) = \eta_0 \quad (2-28)$$

where  $\beta$  is a locally Lipschitz class  $\mathcal{K}_\infty$  function and  $\sigma, \eta_0$  are the design parameters. The advantage that is exploited by dynamic triggering is that  $\mathcal{C}(x, e) = \sigma(\|x\|) - \gamma(\|e\|)$  is non-negative on average, but static ETC requires  $\mathcal{C}(x, e)$  to always be non-negative. The sampling time instants follow the rule,

$$t_{i+1} = \inf \left\{ t : (t > t_i) \wedge \left( \eta + \theta(\sigma(\|x\|) - \gamma(\|e\|)) \leq 0 \right) \right\}, \quad (2-29)$$

where  $\theta \in \mathbb{R}_0^+$  is another design parameter. The dynamic triggering mechanism in (2-29) with design variables  $\beta, \sigma, \eta_0$  and  $\theta$ , under the assumption that  $f, k, \alpha^{-1}$  and  $\gamma$  are Lipschitz on compacts, ensure that there exists a minimum and uniform inter-sample time  $\tau > 0$  and provides asymptotic convergence of  $x(t)$  and  $\eta(t)$  to the origin. Girard [37] also extends the analysis to linear systems to study the influence of the design parameters.

## 2-6 Extension of conventional methods

Some efforts were made to derive a universal formula for event-based stabilization, based on Sontag's general formula for feedback stabilization. This idea was explored in Marchand, Durand & Castellanos [38], which guarantees global asymptotic stability by assuming the existence of a smooth CLF. Also, a minimum uniform inter-event time is ensured in a given compact set. But this analysis is restricted to only control affine systems, in the absence of any disturbance or uncertainty.

Behra & Bandyopadhyay [39] provided an event-triggered version of sliding mode control (SMC), which provides robust stabilization for the following class of nonlinear systems that

is subject to matched external disturbances (entering through input channel):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1(x_1) + B_1 x_2 \\ f_2(x_1, x_2) + B_2 u + B_2 d \end{bmatrix} \quad (2-30)$$

where the states  $x_1 \in \mathbb{R}^{n-1}$ ,  $x_2 \in \mathbb{R}$ , input  $u \in \mathbb{R}$ , disturbance  $d \in \mathbb{R}$ ,  $B_2 \neq 0$  and  $f_1, f_2$  satisfy some Lipschitz properties.

Once the system trajectories arrive at the sliding manifold in finite time, the triggering condition retains the trajectories in a predefined band. In the sliding manifold, the closed loop is ISS, even in the presence of bounded execution delays. The ultimate bound on steady-state is dependent on the delays, but independent of the magnitude of the matched disturbances. This is also referred as *practical sliding mode*, as the ideal sliding surface cannot be achieved through discontinuous updates. In practical sliding mode, the band size remains constant irrespective of inter-sample periods. The control law that enforces practical sliding mode for the system (2-30) in an event-triggered fashion is given by *theorem 3.1* of [39],

$$u(t) = -B_2^{-1}(c^T f(x(t_i)) + K \text{sign}(s(t_i))), \quad (2-31)$$

where the gain  $K$  is selected as  $K > |B_2|\Delta_d + \alpha$  for  $\alpha > L\|c\|\|e(t)\|$ . On the sliding surface, the dynamics is represented by a reduced order system and the trajectories in the sliding phase is ISS with respect to measurement error of the sliding surface. This method retains most properties of conventional periodic control, but limited to a small class of nonlinear systems.



# Self-triggered Control

In the absence of monitoring hardware, one can resort to STC. But the lack of information to check the triggering condition necessitates the prediction of inter-sampling time. Unlike linear systems, nonlinear systems can have multiple equilibria and predicting the state trajectories as a closed-form expression is not always possible. Therefore extending the ideas of linear STC, such as discretization of plant or computation of state transition matrix, can be infeasible and computationally expensive [13]. This has necessitated addressing a class of systems in which the structure can be exploited and later resort to generalizations. In essence, the triggering condition of ETC and STC remain similar but the difference is that ETC checks the triggering condition, but STC emulates ETC's sampling times.

### 3-1 Stabilization of structured systems

In this section, all the papers discussed address state feedback systems without any external disturbances or uncertainty. Additionally, the continuous-time system (2-1) assumes a controller (2-2) that renders the closed loop ISS w.r.t. measurement error.

#### 3-1-1 Homogeneous systems

Earliest work on STC addressing nonlinear systems was done on homogeneous systems, as they can represent local approximations of general nonlinear systems. A homogeneous function  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$  satisfies the following property:

$$f_i(\lambda x) = \lambda^{d+r_i} f_i(x),$$

where  $x \in \mathbb{R}^n$ ,  $\lambda \in \mathbb{R}$  and  $d$  is the degree of homogeneity with respect to standard dilations (all  $r_i = 1$ ) [13]. A system is linear if  $d = 0$  w.r.t standard dilation.

Anta and Tabuada [13] modified the ETC triggering condition (2-5) of Tabuada [19] to

$$\tau(x_k) := \min \left\{ t > t_k : |e(t, x_k)| = \sigma \frac{a}{b} |x(t, x_k)| \right\},$$

where  $a, b$  and  $\sigma$  are scalars and  $\tau(x_k)$  represents the inter-sample time, which is implicitly defined in the above equation. First, it is established that linear systems follow the property:

$$\tau(\lambda x) = \tau(x), \quad (\lambda > 0) \quad (3-1)$$

which indicates that the inter-execution times coincide for all values of  $x(t_i)$  along a ray [13, Proposition 3.1]. Next, this idea is extended to nonlinear homogeneous case, where the inter-sample times are found to follow this scaling law [13]:

$$\tau(x) = \lambda^{-d} \tau(\tilde{x}), \quad (3-2)$$

where  $d$  represents the order of homogeneity and  $\tilde{x} \in$  boundary of  $\Gamma$  and  $x$  lies on the homogeneous ray of  $\tilde{x}$  at relative distance of  $\lambda = |x|/|\tilde{x}|$ . The above equation means that inter-sample period becomes shorter as the current state is radially away from  $\Gamma$ . To explicitly calculate  $\tau$  for every state, the 4-step algorithm is followed in Anta and Tabuada [13]:

- An invariant set  $\Omega$  is defined around the origin. e.g. a Lyapunov level set.
- In the set  $\Omega$ , an upper bounding linear system description is attained as follows:

$$\begin{aligned} \dot{x} &= f(x, k(x+e)) \triangleq \tilde{f}(x, e) \\ |\tilde{f}(x, e)| &= |H(x, e)x + G(x, e)e| \end{aligned} \quad (3-3)$$

$$|\tilde{f}(x, e)| \leq H^*|x| + G^*|e| \quad (3-4)$$

where  $H^*, G^*$  are the maximized values of weighted Jacobians  $H(x, e), G(x, e)$ . Refer Anta and Tabuada [13] for expressions and further details on  $H(x, e)$  and  $G(x, e)$ .

- Using  $H^*$  and  $G^*$  the stabilizing inter-sampling time  $\tau^*$  is calculated in the same way as done for a linear system, that is the time taken to for  $\frac{|e|}{|x|}$  to reach from 0 to  $\sigma \frac{a}{b}$  [19]. The inter-sample time is explicitly computed as,

$$\tau = -\Psi - \frac{2}{i\Theta} \left( \arctan \left( \frac{\alpha_1 + 2a\sigma\alpha_2/b}{-i\Theta} \right) \right) \quad (3-5)$$

with

$$\begin{aligned} \Psi &= -\frac{2}{i\Theta} \left( \arctan \left( \frac{\alpha_1 + 2\phi_0\alpha_2}{-i\Theta} \right) \right) \\ \Theta &= \sqrt{4\alpha_2\alpha_0 - \alpha_1^2} \\ \alpha_0 &= |H(x, e)|, \quad \alpha_1 = |H(x, e)| + |G(x, e)|, \quad \alpha_2 = |G(x, e)|. \end{aligned}$$

A minimum inter-sample time  $\tau^*$  can be obtained by using the values of  $H^*$  and  $G^*$  in all  $\alpha_i$ . Such  $\tau^*$  can stabilize any point  $x \in \Omega$ .



- By defining the largest ball  $\Gamma$  inside  $\Omega$  with radius  $p$ , the inter-sample period of any point in the state-space can be linked to a point on the boundary of the ball  $\Gamma$ . Using homogeneity, the inter-sample period of a point on the homogeneous ray can be calculated as,

$$\tau(x(t_{i+1})) = \left(\frac{|x|}{p}\right)^{-d} \tau^*, \quad (3-6)$$

where  $\tau^*$  can be precomputed offline for a set of points on the boundary of the ball  $\Gamma$ .

An example simulation for 40 seconds showed that STC executed 74% lesser than periodic control. Due to conservativeness in the estimate of  $\tau^*$ , STC showed 67% more executions compared to ETC [13]. Subsequently, these results were extended to non-homogeneous systems [23]. Any polynomial system can be converted into a first-order homogeneous system by increasing the dimension of the state-space by one through the use of auxiliary variables as described in detail in Anta and Tabuada [23].

As [13] dealt with execution times only *along* homogeneous rays and [23] assumed spherical manifolds to calculate a set of lower bounds on the surface, together they could not effectively capture the entire state space due to the conservativeness associated with  $\tau^*$  for points across the sphere. The authors in [40] addresses inter-execution times *across* homogeneous rays by introducing the idea of *isochronous manifolds*, i.e points in the state space where the inter-execution time remains a constant.

Later in [41], the authors summarize a combined STC for evaluating event times *along and across* homogeneous rays using isochrony. Approximate manifolds are shown to provide tighter bounds than spherical manifolds in estimating inter-event times. This work is also applicable when the triggering condition is different from [19], as [41] provides formal method to address non-homogeneous triggering condition as well. Exploiting isochrony has resulted in comparable number of executions of ETC [19] and STC [41] strategy [41], which was not observed in earlier STC strategies. The potential applicability of [41] is beyond the control problem being addressed, as it shown to predict temporal aspects of nonlinear system trajectories.

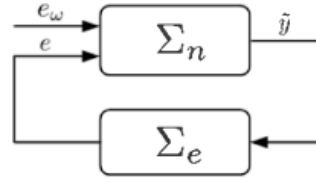
A recent contribution to homogeneous systems is by Delimpaltadakis and Mazo Jr. [42], in which a region-based STC is introduced for nonlinear systems. The state space is partitioned using isochronous manifolds as discussed in [41]. But, [42] points out many shortcomings in [41] and addresses them in detail. Firstly, the Lemma V.2 [41] is corrected by providing a new bounding lemma based on higher order trajectory derivatives. Secondly, since [41] uses zero-level sets of triggering function instead of actual isochronous manifolds, [42] becomes the first to actually implement approximations of isochronous manifolds. Finally, to compute this approximation [42] suggests using SMT solvers. The state-space partitioning method was earlier available to only linear systems in [43], where the next sample time was dependent on the current region  $\mathcal{R}_i$ , where the set of regions  $\mathcal{R}$  was generated by an algorithm. Alternatively in [42], one can choose the number of state-space regions to be generated by a-priori selection of inter-event times. In this was, dimensionality of the system doesn't affect the number of partitions.

## 3-2 Stability under external influence

This section is dedicated to the previous works that deal with external disturbances, uncertainties and delays that are inherent in the real world. Although ETC may seem more reliable for such applications, some attempts have been made using STC schemes as discussed below.

### 3-2-1 Disturbances

The work by Tolić et al. [44] studies a networked closed loop system affected by input and external disturbance using the hybrid systems approach. All the measurements of the interconnected system ( $\Sigma_n$  - nominal system,  $\Sigma_e$  - output error system) in Figure 3-1 are not affected by delays.



**Figure 3-1:** Interconnected system (source: [44])

Since the standard  $\mathcal{L}_p$  gain is independent of time ( $\forall t \geq 0$ , or infinite-horizon), it does not allow the study of temporal aspects. Therefore, for predicting the next event, [44] uses finite-horizon  $\mathcal{L}_p$  gains defined for  $t \in [t_0, t_0 + \tau]$ . If the error dynamics can be expressed as,

$$\dot{e} = g(t, x, e, w) \leq Ae + \tilde{y}(t, x, w, e) \quad (3-7)$$

the stabilizing sampling policy is given by,

$$\tau_i^* = \frac{1}{\|A\|} \ln \left( k \frac{\|A\|}{\gamma_n} + 1 \right) \quad (3-8)$$

where  $\gamma_n$  is the gain of  $\Sigma_n$  from  $e$  to  $\tilde{y}$ .

Here,  $\tau$  represents the minimum inter-sample time, which is to be maximized ( $\tau^*$ ) in the interest of reducing conservativeness. The sampling policy aims to satisfy the small-gain condition for all times, as it is a sufficient condition for stability. Therefore, triggering occurs when  $\gamma_1(\tau_i)\gamma_2(\tau_i) \geq 1$ . The stability of the closed loop system is based on assumptions made on the external inputs, therefore one could obtain stable, asymptotically stable or  $\mathcal{L}_p$  stable system depending on whether the measurements ( $u, y, \omega$ ) are affected by noise.

Liu and Jiang [36] provided a self-triggering extension of ETC based on small-gain theorem for a class of nonlinear systems subject to external disturbances. The system is described as,

$$\begin{aligned} \dot{x} &= f(x, u, d) \triangleq \bar{f}(x, x + e, d), \\ |x(t)| &\leq \max\{\beta(|x_0, t|), \gamma(\|e\|_\infty), \gamma^d(\|d\|_\infty)\}, \end{aligned} \quad (3-9)$$

and assumed to be ISS w.r.t measurement error and disturbance. If sampling error satisfies  $|e| \leq \rho(|x|)$ ,  $\rho \in \mathcal{K}_\infty$ ,  $\rho^{-1}$  is locally Lipschitz and  $\bar{f}(0, 0, 0) = 0$ , then  $\rho$  can be identified such

that the small-gain condition (2-23) is satisfied. Given that the disturbance is bounded and decays to zero, by choosing  $\mathcal{X}, \mathcal{X}^d \in \mathcal{K}_\infty$  such that  $\mathcal{X}^{-1}, (\mathcal{X}^d)^{-1}$  are locally Lipschitz, the following self-triggered sampler,

$$t_{k+1} = \frac{1}{L_{\bar{f}}(\max\{\bar{\mathcal{X}}, \bar{\mathcal{X}}^d(B^d)\})} + t_k \quad (3-10)$$

achieves global asymptotic stability. In (3-10)  $\bar{\mathcal{X}}(s) = \max\{\mathcal{X}(s), s\}$ ,  $\bar{\mathcal{X}}^d(s) = \max\{\mathcal{X}^d(s), s\}$ ,  $L_{\bar{f}}$  is the Lipschitz constant of  $\bar{f}$  and  $\mathcal{X} = \rho \circ (I_d + \rho)^{-1}$ . If the disturbance doesn't attenuate, the trajectories can be steered to a neighbourhood around the origin depending on the size of  $\|d\|_\infty$ .

### 3-2-2 Actuation delays

Theodosios and Dimarogonas [45] addressed STC design for globally asymptotically stabilizable systems with actuator delays. The assumptions include that state-feedback  $u = h(x)$  is possible, the actuator delays ( $\Delta_k$ ) are smaller than inter-sample times and the continuous-time system is globally asymptotically stable,

$$\Delta V(x)f(x, h(x)) \leq -\alpha(x).$$

The design objectives are Zeno-free behaviour and practical stability, i.e convergence of states to a small neighbourhood around an equilibrium. This neighborhood is selected a priori to derive an upper bound on the actuation delays ( $\Delta$ ). The sampling strategy for the proposed STC is,

$$t_{k+1} = t_k + \frac{1}{L_f} \ln \left( \frac{\frac{\tau}{L_{f,h}\beta} + L_h|x_k|}{2L_{f,h}\gamma\Delta_k + L_h|x_k|} \right) \quad (3-11)$$

where  $\tau, \beta, \gamma$  are positive scalars,  $L_f, L_h, L_{f,h} = L_f L_h$  are Lipschitz constants of  $f$  and  $h$  over a Lyapunov level set ( $V(x) \leq \xi, \xi > 0$ ) and  $\Delta_k \in [0, \Delta)$ . The lower bound of inter-sample times is the maximum delay, which is expressed as

$$\Delta = \frac{L_h^2 \tau}{L_{f,h}(L_{f,h} + 2(1 + \epsilon)L_{f,h})\beta\gamma + L_f\tau} \quad (\epsilon > 0) \quad (3-12)$$

If the delays are longer than expected, the practical stability region also grows larger and then the states converge to a larger neighbourhood around the equilibrium point. According to the initial assumption, larger delays might imply longer inter-sample times, but this is achieved at the cost of system stability and hence detrimental. From an example simulation, it is established that this method is more conservative than Anta and Tabuada [13] in the absence of any delays.

### 3-2-3 Perturbation

The first work in literature that addressed uncertainties is [46], where a locally stabilizable system is perturbed by norm-bounded parametric uncertainty and disturbances in the presence of bounded actuation delay. The STC scheme relies on a polynomial approximation

of the Lyapunov function to predict the subsequent event times. This method guarantees convergence to an arbitrarily small neighbourhood around the origin.

Further, Tiberi & Johansson [47] developed a simple self-triggered sampler for perturbed non-linear systems, using perturbed systems analysis. The STC scheme ensures global uniform ultimate boundedness (GUUB) of the closed loop trajectories along with some robustness to disturbance and time delays. It assumes a state feedback law that ensures global asymptotic stability of the continuous-time unperturbed closed loop. The sampler developed for STC is utilized to design disturbance observers, which further reduce conservativeness of any ETC implementations. A preliminary version of this work [48] shows that the sampler is more conservative than Anta and Tabuada [23] since the structure of homogeneity is not exploited therein.

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## Chapter 4

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# Automotive systems

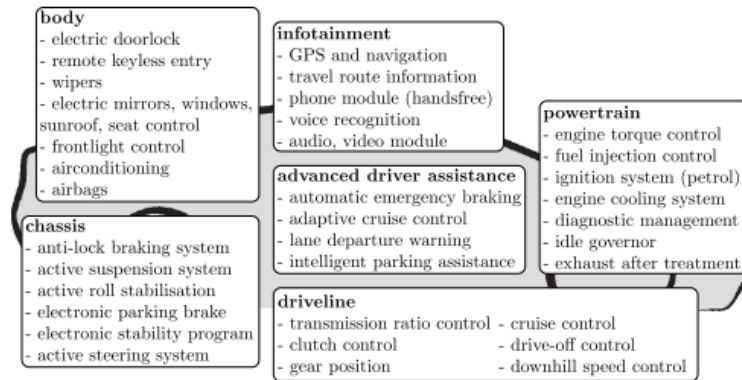
Automobiles provide a means for transporting humans and goods and hence an integral part of human society. In this chapter, the notion of automobile or vehicle is restricted to a four-wheeled passenger-only vehicle that corresponds to category M vehicles in the European Commission.

### 4-1 Automobile: A cyber-physical system

Today's vehicle is a computer on wheels with a large number of electronic components to govern many vehicle functions. A modern automobile has at least 70 independent electronic control units (ECUs) [49]. These ECUs are connected to actuators and sensors via data buses that consists of 1900 wires, weighs 40 kilograms and spans 4 kilometres of length [50]. The ECUs host limited capacity microprocessor that is dedicated to performing a single task. This complete system can be viewed as a CPS, as the components of the control system are distributed over a network with limited computing and communication resources. Only through the integration of communication and computation with control, it is possible to ensure efficient use of limited resources.

#### 4-1-1 Subsystems

Based on the functionalities, vehicle subsystems can be grouped as engine, driveline, chassis, body, infotainment and advanced driver assistance systems (ADAS) as shown in 4-1. Based on the type of fuel used and the architecture employed, powertrain and engine controls can vary significantly. With the trend of vehicle electrification [51], research has shifted focus to hybrid powertrain control [52] and headed towards independent in-wheel motors. Due to the transitioning in technologies, these subsystems are avoided from further study. ADAS and chassis control systems are independent of energy generation and transmission architecture in a vehicle. But the high cost associated with driving automation hinders widespread adoption of ADAS into every vehicle. Chassis control systems, on the other hand, are almost two



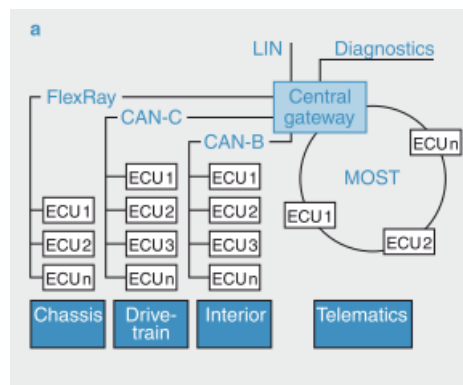
**Figure 4-1:** Vehicular subsystems (source: [50])

decades old and have been available as a standard on every modern vehicle in the form of ESP, ABS, traction control, suspension control and steering control.

#### 4-1-2 In-vehicle networks

To capture the needs of various subsystems, more than one type of network is employed, namely, LIN, MOST, Controller area network (CAN) and FlexRay [53]. Among these network types, LIN and Bluetooth are low cost and small bandwidth networks used for non-critical applications such as body control or switches. MOST is exclusively used for infotainment purposes. Finally, for near real-time or hard real-time control applications of body, driveline and chassis, both CAN and FlexRay are used. Therefore, addressing chassis control systems is more rewarding due to its availability in almost all vehicles.

The CAN system was developed by Robert Bosch GmbH in 1986 as an automotive-specific network that provides a low cost, robust and real-time communication. It is the most dominant network type among vehicles and the success of this serial communication bus extends to other domains such as automation, medical and manufacturing. CAN is a multi-master bus that connects all nodes over a common bus and performs arbitration to deliver messages based on a fixed priority [54].



**Figure 4-2:** A schematic of in-vehicle network as per Bosch (source: [53])

The CAN network is subdivided as low-speed CAN (or CAN-B at 125 kbps) for body interiors and comfort and high-speed CAN (or CAN-C at 500 kbps) for engine management, transmission and ABS/ESP network [53]. The standard CAN is event-triggered and there also exists time-triggered CAN (TT-CAN) and Flexible data rate CAN (FD-CAN) [55]. Standard CAN provides good response times to asynchronous events and it offers the flexibility to append the network nodes, but the worst-case response times (WCRT) is not upper-bounded. Whereas, the TT-CAN provides more determinism over message latency and reliability, but requires more planning and hence restrictive to later changes [53].

FlexRay [56] was developed in 2006 by a consortium of major manufacturers that include Robert Bosch, NXP, BMW, GM, Daimler and Volkswagen and currently available as an ISO standard 17458-1 to 17458-5. It provides the best of event and time-triggered communication for addressing the future needs of automotive systems such as drive-by-wire applications with high bandwidth of 10 to 20 Mbps. For these reasons, Flexray is significantly costlier than using CAN.

### 4-1-3 Architecture and trends

The in-vehicle electronic and electrical (e/e) architecture started as a point-to-point communication system and evolved into a distributed system over the CAN bus. With the increase system integration for better safety, stability and assistance, the current distributed e/e architecture is a hindrance, as more and more ECUs across domains are required to communicate in real-time.

Higher levels of coordination in subsystems and performance are observed with integrated vehicle dynamics controllers (IVDCs) that use hierarchical and multi-layered structures [57]. These structures can be viewed as near-to-decentralization, as only a handful of supervisory controllers would be required to coordinate the vehicle. Although there is no consensus about having a standardized e/e structure in the future, the current direction is certainly towards more centralization [58], [59].

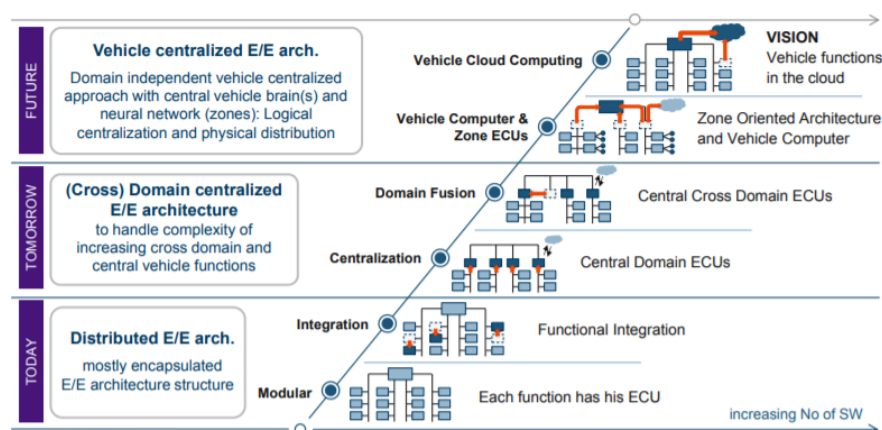
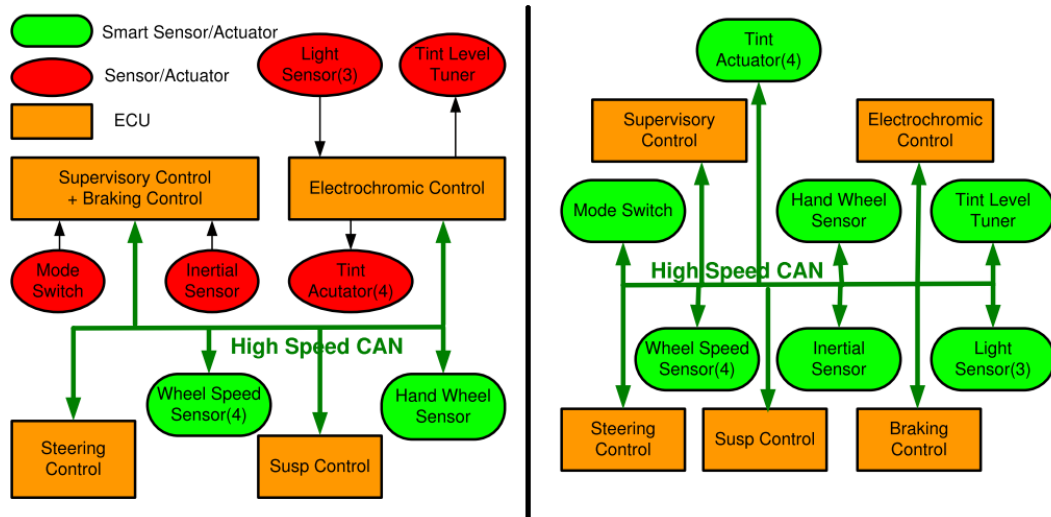


Figure 4-3: Future of e/e architectures (source: [59])

Finally, a completely centralized computer is another possibility that was disclosed by Audi [60] as their plans towards a central vehicle dynamics computer. To achieve this solution, it

would be necessary to build a real-time operating system and address the scheduling aspects of a *shared processor*, which computes all the control functions of the vehicle. Kanajan et



**Figure 4-4:** MDICC vs DIDC architecture (source:[61], edited)

al. [61] performed an exploratory study to compare various topologies of centralized and decentralized e/e architectures. Using a virtual simulation environment Metropolis, which based on formal verification tools, Kanajan et al. [61] arrived at quantitative results for latencies, geometric attributes (like wiring length, nodes), bus utilization and flexibility. The topologies are characterized based on how inputs/outputs (i/o) and computation elements (ECUs) are arranged. Figure 4-4 shows distributed i/o distributed computing (DIDC) and Mixed distributed i/o and centralized computing (MDICC). Similarly, other possible types like CICC and CIDC also taken into account. The results show that the most suitable topology is MDICC. On the other hand, DIDC provides excellent flexibility and geometric aspects with comparable latencies with MDICC, but the bus usage and traffic is significantly higher than the rest of the architectures.

## 4-2 Chassis control systems

Automotive chassis primarily consist of brakes, transmission, steering and suspensions. Among these, the transmission system that comprises of gearbox, axles and differentials are affected by current trends of hybridization and/or electrification. Additionally, the transmission systems are not standardized and vary across models and variants of cars. Contrarily, systems such as brakes, steering and suspensions have minimal variations across vehicles. Active safety systems that include ESC and ABS are standardized control systems that need to qualify homologation tests during vehicle release. Therefore, any developments achieved in such standardized systems readily affects large number of vehicles. A list of all the variables and parameters used in this section is found in Table 4-1.



$l_f, l_r$	longitudinal distance from vehicle center of gravity (CoG) to front and rear axle
$L$	Total vehicle length from front to rear axle
$M$	Vehicle total mass
$u, v, r$	Longitudinal, lateral and yaw velocity at CoG
$I_z$	Vehicle moment of inertia about CG along z-axis
$\delta$	Angle of the steered front wheel
$\alpha, \lambda$	Slip angle and slip ratio of a tire
$\beta$	Body slip angle
$R_w$	Radius of wheel
$C_\alpha, C_\lambda$	Lateral and longitudinal tire stiffness
$F_{1,2}$	Tire forces. Subscript 1 $\in \{x, y\}$ represents longitudinal or lateral forces. Subscript 2 $\in \{f, r\}$ indicates front or rear wheel position.

Table 4-1: Vehicle parameters and variables

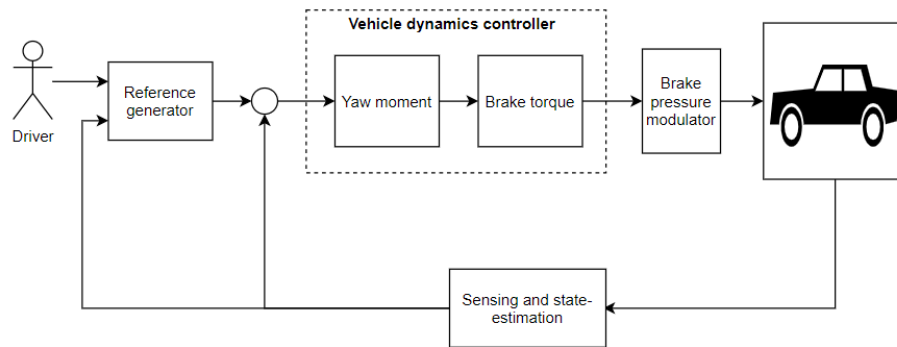


Figure 4-5: A typical VDC based on differential braking

### 4-2-1 Electronic Stability Control

Electronic stability control (ESC), also referred to as vehicle dynamics control (VDC), electronic stability program (ESP), yaw moment control (YMC) or vehicle stability control (VSC), was first developed and introduced by Bosch in 1995 [62] and it is an essential feature in modern automobiles for improved handling and safety. ESC assists the driver in case of understeer or oversteer situations, where the vehicle doesn't respond as desired by the driver. Such situations arise in high-speed cornering, evasive-maneuvring and driving on low or mixed friction surfaces. The components of ESC include wheel speed sensors, steering wheel sensor, brake pressure sensor, inertial measurement unit (for lateral and yaw acceleration), an ECU and brake modulator, which communicate via CAN network [63].

To design vehicle stability controllers, many design methods exist that can be classified on the basis of type of vehicle model (linear/nonlinear), complexity of model (bicycle/planar/-planar+roll), dimensionality of the vehicle model (two/three/four degrees-of-freedom) and structure of controller (hierarchical two-step/ slip-based single-step). To keep the design simple but capture the necessary dynamics one can resort to bicycle model, which was used by OEMs such as Ford [64] and Toyota [65]. But unlike both, the dynamics here is described

using three degrees-of-freedom (3-DoF) and nonlinearity is preserved during controller synthesis. To design a nonlinear controller often a sliding mode control is used due to its inherent robustness to inaccuracies in plant description [66]. This has been exploited for designing ESC systems in Yi et al. [67], Rajamani [68], and Li et al. [69]. Since a bicycle model can not be directly used for differential braking, a hierarchical SMC design is employed, which is inspired from Rajamani [68]. This allows to first calculate the yaw moment required for stabilizing the vehicle and then decide on how to allocate the control effort among the four wheels. The 3-DoF nonlinear dynamical bicycle model is given as,

$$\dot{u} = \frac{1}{M} (F_{xf} \cos \delta - F_{yf} \sin \delta + F_{xr}) + vr, \quad (4-1)$$

$$\dot{v} = \frac{1}{M} (F_{xf} \sin \delta + F_{yf} \cos \delta + F_{yr}) - ur, \quad (4-2)$$

$$\dot{r} = \frac{1}{I_z} (l_f [F_{xf} \sin \delta + F_{yf} \cos \delta] - l_r F_{yr}) + \frac{1}{I_z} \mathcal{M}_z, \quad (4-3)$$

where  $\mathcal{M}_z$  is the control input that represents the external yaw moment that can act on vehicle. The forces at the tires can be represented with a simple linear model similar to [67] and [68],

$$F_x(t) = C_\kappa \kappa_i(t) = C_\kappa \left( \frac{u - R_w \omega_i}{u} \right), \quad (4-4)$$

$$F_{yf}(t) = C_{\alpha f} \alpha_f(t) = C_{\alpha f} \left( \delta - \frac{v + l_f r}{u} \right), \quad (4-5)$$

$$F_{yr}(t) = C_{\alpha r} \alpha_r(t) = C_{\alpha r} \left( -\frac{v - l_r r}{u} \right). \quad (4-6)$$

The reference generator, as the name suggests, provides desired trajectory for certain state variables that need to be controlled. The desired trajectory based on the driver inputs is the steady-state response of a 2-DoF linear bicycle model to the same inputs,

$$r_{des}(t) = \frac{u}{L + K_{us} u^2 / g} \delta(t) \quad (4-7)$$

where the understeer gradient ( $K_{us}$ ) is calculated as  $K_{us} = \frac{Mg}{L} \left( \frac{l_r}{C_{\alpha f}} - \frac{l_f}{C_{\alpha r}} \right)$ .

### Sliding mode controller design

A brief introduction to sliding mode control can be found in A-1. A sliding surface is defined such that yaw rate error or slip angle error or a combination of both is achieved. Rajamani [68] suggests using the following sliding surface,

$$s = (r - r_{des}) + \xi(\beta - \beta_{des}), \quad (4-8)$$

where  $\xi$  is a positive constant and  $\beta$  represents the body slip angle of the vehicle. By enforcing convergence of sliding variable to zero, the desired dynamics is achieved. The evolution of this sliding variable is given by,

$$\dot{s} = (\dot{r} - \dot{r}_{des}) + \xi(\dot{\beta} - \dot{\beta}_{des}) \quad (4-9)$$

To achieve asymptotic stability, a positive constant  $\eta$  can be chosen such that,

$$\dot{s} = -\eta s, \quad (4-10)$$

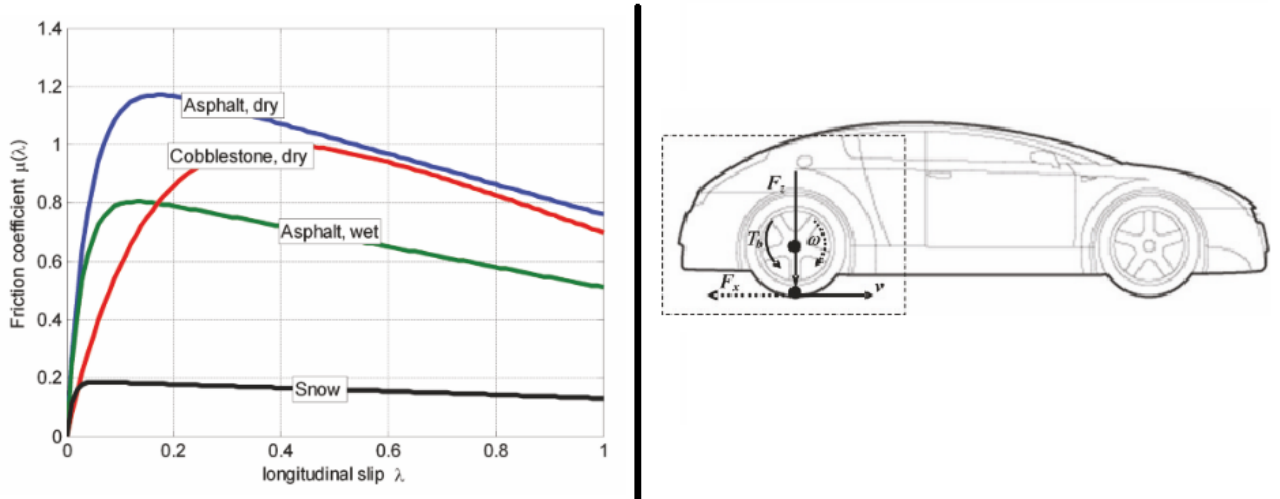
which upon substitution of (4-3) of yields a control law similar to Rajamani [68],

$$\mathcal{M}_z = -l_f [F_{xf} \sin \delta + F_{yf} \cos \delta] + l_r F_{yr} + I_z [-\eta s + \dot{r}_{des} - \xi(\dot{\beta} - \dot{\beta}_{des})] \quad (4-11)$$

This yaw moment is realized as through the brakes system by through some control allocation technique. The target wheel's brake pressure will be varied to aid the vehicle in cornering.

### 4-2-2 Anti-lock Braking System

The tire-road interaction generates traction (or grip), which is the only interface for support during braking, cornering and acceleration. Friction coefficient  $\mu$  is a quantitative measure of the grip with the highest grip available on a dry surface and lowest grip available on snow. The tire has to slip to generate grip, but too much slip of tire can lead to loss of traction or wheel lock. In this case, the vehicle is assumed to be pure longitudinal motion and a single corner model is used to study the tire dynamics [70].



**Figure 4-6:** Effect of wheel slip and road surface on friction & single corner model for ABS (source: [70], edited)

During longitudinal motion over dry asphalt surface consider that driver demands maximum braking action. The slip ratio of a tire varies over a fixed range  $\lambda \in [0, 1]$  where  $\lambda = 0$  refers to no forces at the tire-road interface and  $\lambda = 1$  indicates a wheel lock. The wheel lock is undesirable as the driver can lose steerability due to loss of traction. Wheel slip and friction steadily increase when the braking demand is small, but once the wheel slip crosses the  $\mu - \lambda$  peak at  $\lambda = 0.15$ , the wheel quickly loses traction and tends to a wheel lock. To generate maximum force and retain steerability, it is required to control the slip ratio near the  $\mu - \lambda$  peak and especially not allow unstable behaviour such as wheel lock. Anti-lock braking system (ABS) regulates the brake pressure using an electro-hydraulic valve and therefore controls the braking torque and indirectly the slip ratio of the tire [70]. As the surface of the road changes,

the maximum available traction also varies. Consider the green curve in Figure 4-6 where the wet surface can only provide a maximum of  $\mu = 0.8$  friction coefficient, as compared to asphalt dry that can provide about  $\mu = 1.2$ . With a higher friction coefficient, a larger amount of force can be sustained at the tire, which leads to improved braking performance.

Similar to ESC, sliding mode control is widely used for wheel slip control and the work of Unsal & Kachroo [71] is followed to obtain the following dynamics and the control law. The system dynamics is based on the evolution of wheel speed ( $x_1 = \omega$ ) and vehicle speed ( $x_2 = V$ ) as,

$$\dot{x}_1 = -f_1(x_1) + b_{1,N} \cdot \mu(\lambda), \quad (4-12)$$

$$\dot{x}_2 = -f_2(x_2) - b_{2,N} \cdot \mu(\lambda) + b_3 \cdot T, \quad (4-13)$$

where,  $f_1, f_2$  are nonlinear functions based on geometry of vehicle,  $b_{1N}, b_{2N}, b_3$  are scalar constants depending on vehicle,  $\mu(\lambda)$  represents the nonlinear relationship between  $\mu - \lambda$  (see figure 4-6) and  $T = T_{drive} - T_{brake}$  is the net torque acting on a wheel. The control law is derived by using a sliding variable,

$$s(\lambda, t) = \lambda - \lambda_d, \quad (4-14)$$

which denotes the error in slip ratio. The state trajectories converge to the sliding surface ( $s = 0$ ) in finite-time. On the sliding surface, the trajectories exponentially converge to the origin using the control law,

$$u = b_3^{-1}[-\hat{f} - \dot{\lambda}_d - k \operatorname{sgn}(s)] \quad (4-15)$$

$$k \geq \alpha \cdot (F + \eta) + (\alpha - 1)|\hat{f} + \dot{\lambda}_d|, \quad k \in \mathbb{R} \quad (4-16)$$

where,  $\frac{1}{2} \frac{d}{dt} s^2 \leq \eta |s|$ ,  $\alpha \in (0, 1)$ ,  $\hat{f}$  is an estimate of

$$f = \frac{(1 + \lambda)f_1(x_1) - f_2(x_2) - [b_{2N} + (1 + \lambda)b_{1N}] \cdot \mu(\lambda)}{x_1}. \quad (4-17)$$

and the estimation error is bounded by  $|\hat{f} - f| \leq F$ .

# Thesis proposal

### 5-1 Problem formulation

Research on aperiodic control has recently gained attention from the control community as it shows good promise for many problems that CPS needs to address, such as limited computation, energy resources and communication bandwidth. This field is less than two decades old and far from maturity in comparison with periodic control systems, which have been dominant since the 1960s. Aperiodic control approaches are generally classified as ETC and STC. Only one survey [11] encompasses these methods, but the focus is on output-feedback linear control systems. Since the control in ETC (or STC) is usually state-dependent, this can be found to be more advantageous to nonlinear systems as their behaviour is highly dependent on the operating point.

As of now, studies on aperiodic control for nonlinear systems has reached a point where one can begin to classify the major design approaches. For example, ETC approaches are mostly perturbation-based as in chapter two. One can resort to STC schemes in the absence of triggering condition monitoring hardware. STC methods usually exploit the mathematical structure of the nonlinear system description, such as homogeneous or smooth polynomial systems, as seen in chapter three. There also exist variants in ETC and STC schemes that are capable of handling disturbances, noise, uncertainties and delays, as seen in the earlier chapters.

Most applications that have been mentioned in the first chapter are modelled as linear systems. The results of these studies are based on academic examples that may not exist in the real world. Although the simplicity of the examples might be aimed to benefit the reader for better understanding of theory and replication of results, there is also a matter of concern whether ETC/STC can stand the test of real-world scenarios as claimed in theory. Some examples of ETC implemented on real-world systems do exist, which are addressed here. In the earliest work, greenhouse climate control by Powlowski et al. [16], uses gain scheduling to address linearizable nonlinear dynamics. An error function and time intervals are explicitly checked in reference to an upper bound and triggered if at least one condition is violated. There is no

formal proof provided for stability of the sampled-data closed loop or minimum inter-event time. The results appear to be stable due to the only reason that the dynamics of the system is slow and the possibility that it is open loop stable. More serious and formal works can be found in [18], [17] and [72]. Hashish [17] addresses the problem of vehicle platooning using ETC, Lont [18] uses ETC for irrigation systems and Hop [72] abstracts the in-vehicle NCS for scheduling purpose. In all these works, the dynamics of the system is only linear or linearizable and the verification methods are limited to only software simulations. Only one study, carried out in Kartakis et al. [73], uses an experimental setup to implement aperiodic sampling and control for the application of smart water network. But similar to the previous examples, Kartakis et al. [73] models the plant as a linear time-invariant system.

The literature survey and the above discussion points out the following gaps in the study of event-based control -

- Most literature on event-based control is inclined to address linear systems than non-linear systems, despite the notion that the nonlinear system can benefit more from aperiodic control [23].
- A majority of validation for design methods in literature is done through simulation studies, which consider academic examples that fail to represent complexities of real world
- Some practical studies that implement event-triggered control have emerged, but they are limited to linear systems and the system is tested mostly on virtual simulation environments.

**Problem.** Aperiodic control paradigm is often motivated by present-day practical issues of cyber-physical systems. But, the design techniques developed for this cause often rely on virtual simulation of academic examples that lack complexities of the real world. This questions the applicability and efficacy of the developments made in theory, thereby motivating the need for experimental corroboration.

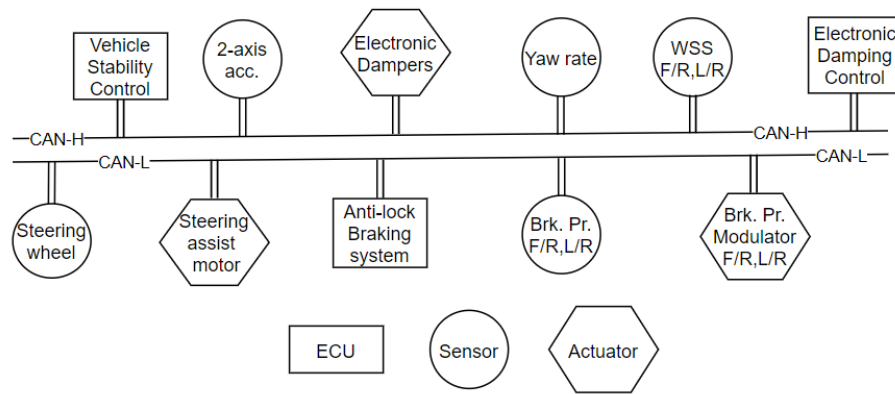
To tackle this problem, the automotive domain is chosen as the CPS to be studied. Apart from subjecting theory to practise, it is intuitive to expect that event based control might offer its unique advantages to the domain of automotive engineering that would allow one or many of the following points to manifest:

- easing network congestion
- possibilities to explore new e/e architectures, for e.g. distributed and mixed topology
- increased actuator life with lesser actuation than periodic control
- lowering computation burden on shared processors, for e.g. in centralized computing
- higher design flexibility, lesser wiring and increased reliability through distributed sensing, computation and actuation

## 5-2 Proposed methodology

### 1. Identify and study conventional control loops

An automobile has many functional domains of which the chassis systems are chosen as they are widely on every automobile and some are necessitated through standardized. Moreover, their functionality is to coordinate the vehicle motion through non-collocated sensors, actuators and ECUs that are connected over the CAN network. Since vehicle dynamics is defined along three principal directions, we consider the control loops: ABS (longitudinal dynamics/safety), ESC (lateral dynamics/stability) and Electronic damping control (vertical dynamics/comfort), covers the major functions of a modern vehicle. Figure 5-1 shows a chassis subnetwork of how these control loops can be connected.

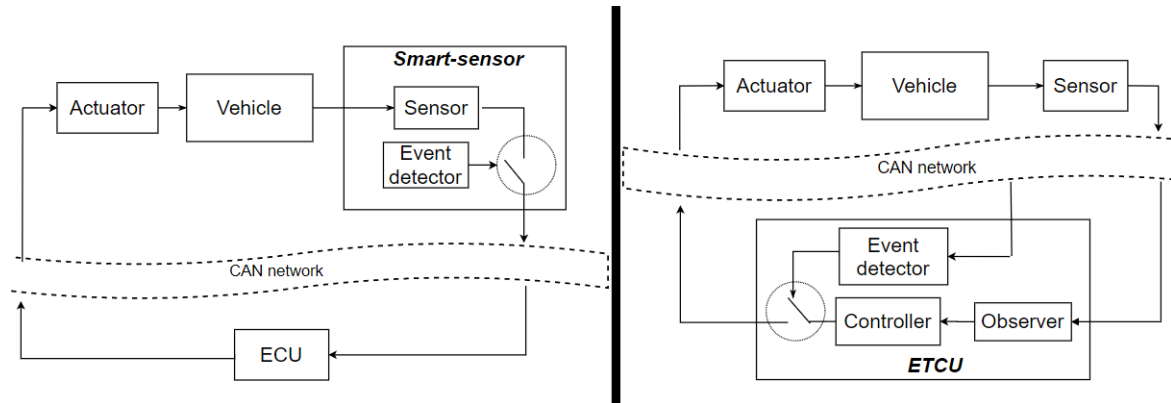


**Figure 5-1:** An example of chassis subnetwork with DIDC architecture

### 2. Aperiodic control design

The triggering condition in most literature is usually checked at the sensor and sometimes also after the controller if the network has to be accessed at both instances. A sensor that checks the triggering condition or calculates the next sensing event is commonly referred to as *smart-sensor* in the literature. In automobiles, multiple sensors share information with multiple ECUs, so sensors are not exclusive to a single control loop. Therefore, the idea of smart-sensor that satisfies all control loops is withheld and an intermediate option for event-based control is considered that retains the existing periodic sensing policy but triggering is checked at the controller, as shown in figure 5-1. The event-triggered ECUs (ETCU) can continue receiving sensor information over CAN, but updating the actuators via CAN takes place according to the designed triggering condition. This notion of periodic sensing but trigger-based control is close to PETC framework. Since PETC requires a continuous-time event-triggered controller a priori, it is first necessary to design an event-triggered version of the conventional chassis controllers.

For example, to design an aperiodic ESC system it would be necessary to convert the SMC control law to an equivalent event-triggered SMC (ET-SMC), similar to Behera and Bandyopadhyay [39]. This ET-SMC should satisfy the earlier design objectives of Zeno-freeness, stability in some sense and certain performance guarantees in continuous-time. Using the ET-SMC control law, the PETC framework [34] can be applied to



**Figure 5-2:** Smart-sensor vs. ETCU based aperiodic control

achieve a digitally implementable ET-SMC.

### 3. Simulation and evaluation

There exist various categories of validating automotive controllers and these include model-in-loop (MiL), software-in-loop (SiL), hardware-in-loop (HiL) and ultimately vehicle-in-loop (ViL). The simulation of HiL is considered near to real-world in a laboratory and a combination of SiL and HiL is a common practice. Due to reasons of cost ViL testing is limited to specific cases.

To factor the effects of the vehicle CAN bus in real-time control, one can implement the controller on real-time machines with actual CAN communication among the nodes, e.g. National Instruments (NI-XNET) [74]. On the other hand, the CAN bus can be emulated by network simulators such as OMNET++ [75], [76].

The effectiveness of the controllers can be studied using vehicle simulation environments, which achieve close to real-world scenarios through detailed multi-body dynamical simulations, e.g. Dynacar [77]. It is also possible to simulate Dynacar in real-time the NI machines [78] for evaluating event-triggered controllers on HiL setup.

In this way this thesis can be validated over SiL or HiL experimental setups.

To conclude, the thesis proposal to solve the above-identified problem is as follows:

**Proposal.** Design ETC implementations of ABS, ESC and EDC to provide stability and performance guarantees for their implementation over a vehicle CAN bus. The effectiveness of aperiodic control is tested through HiL simulations using some vehicle dynamic scenarios.



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# Appendix A

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## Conventional control design

### A-1 Sliding mode control

For a detailed introduction to sliding mode control (SMC) and its applications to electro-mechanical systems, an interested reader can refer to [66]. Sliding mode occurs due to the existence of discontinuous right-hand sides of the ordinary differential equations. The design can be divided into two-stages, as explained using an example from [25, ch. 10].

Consider a control-affine single-input system nonlinear system,

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= h(x) + g(x)u,\end{aligned}\tag{A-1}$$

where  $h, g$  are locally Lipschitz functions,  $g(x) > 0$  and  $f, g$  can be unknown. First define a sliding surface,

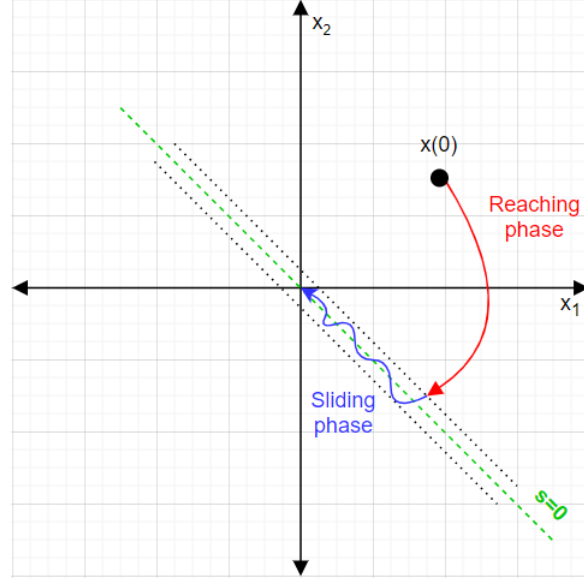
$$s := ax_1 + x_2 = 0\tag{A-2}$$

to which the trajectories remain confined, resulting in a reduced-order system description. On the sliding surface ( $s = 0$ ) the system dynamics can be represented as

$$\dot{x}_1 = -ax_1,\tag{A-3}$$

which is independent of  $f, g$  as  $s = 0$  is satisfied. Therefore, by choosing  $a > 0$  we arrive at exponential convergence of the states to the origin. At the end of the first step, the convergence to the origin on the sliding surface is ensured by choosing appropriate parameters (like  $a$ ) that define the sliding surface. Secondly, it is necessary to ensure that trajectories starting outside  $s = 0$  will reach the sliding surface and continue to stay thereafter. Consider the dynamics of the sliding variable,

$$\begin{aligned}\dot{s} &= a\dot{x}_1 + \dot{x}_2 \\ &= ax_2 + h(x) + g(x)u\end{aligned}\tag{A-4}$$



**Figure A-1:** An illustration of SMC in action

and by defining a Lyapunov function candidate  $V(s)$ , it is possible to analyse the behaviour of  $s$ . Define,

$$V(s) = \frac{1}{2}s^2 \quad (\text{A-5})$$

$$\Rightarrow \dot{V} = s\dot{s} \quad (\text{A-6})$$

$$= sax_2 + sh(x) + sg(x)u \quad (\text{A-7})$$

$$\leq |s|g(x)\rho(x) + sg(x)u \quad (\text{A-8})$$

$$\leq |s|g(x)\rho(x) + sg(x)(-\text{sgn}(s)\tilde{\rho}) \quad (\text{A-9})$$

$$\leq |s|g(x)(\rho(x) - \tilde{\rho}(x)) \leq 0 \iff \rho < \tilde{\rho} \quad (\text{A-10})$$

where  $\rho(x) \geq \left| \frac{ax_2 + h(x)}{g(x)} \right|$  and  $\tilde{\rho} = \rho + k$ ,  $k > 0$ . By ensuring a negative (semi) definite Lyapunov rate, the desired behaviour of the reaching phase is achieved using the control law,

$$u = -\tilde{\rho}(x)\text{sgn}(s). \quad (\text{A-11})$$

From Figure A-1, it is easy to visualize how SMC would achieve the desired behaviour using reaching and sliding phases in control. Chattering is one of the challenges in SMC where the control signal switches between two dynamics (here,  $\text{sgn}$  function at zero). There exist many methods in literature to alleviate chattering [66]. Robustness to matched disturbances is a key feature of SMC [25, pg. 255].

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