# iX-BSP: Incremental Belief Space Planning with Selective Resampling
# **Supplementary Material**

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This document provides supplementary material to the paper [1]. Therefore, it should not be considered a self-contained document, but instead regarded as an appendix of [1].

Throughout this report, all notations and definitions are with compliance to the ones presented in [1].

In order to address the more general and realistic scenario as presented in [2], the DA might require correction before proceeding to update the new acquired measurements. This report covers the possible scenarios of inconsistent data association and its graphical materialization - Appendix A, followed by a paradigm to update inconsistent DA from planning stage according to the actual DA attained in the consecutive inference stage - Appendix B. [omit or update]

## Appendix A: Multiple Importance Sampling Formulation

Let us assume we wish to express expectation over some function $f(x)$ with respect to distribution $p(x)$, by sampling $x$ from a different distribution $q(x)$,

$$\mathbb{E}_p f(x) = \int f(x) \cdot p(x) dx = \int \frac{f(x) \cdot p(x)}{q(x)} q(x) dx = \mathbb{E}_q \left( \frac{f(x) \cdot p(x)}{q(x)} \right). \tag{10}$$

Eq. (10) presents the basic importance sampling problem, where $\mathbb{E}_q$ denotes expectation for $x \sim q(x)$. The probability ratio between the nominal distribution $p$ and the importance sampling distribution $q$ is usually referred to as the likelihood ratio. Our problem is more complex, since our samples are potentially taken from $M$ different distributions while $M \in [1, (n_x \cdot n_z)^L]$, i.e. a multiple importance sampling problem

$$\mathbb{E}_p f(x) = \tilde{\mu}(x) \sim \sum_{m=1}^{M} \frac{1}{n_m} \sum_{i=1}^{n_m} w_m(x_{im}) \frac{f(x_{im}) p(x_{im})}{q_m(x_{im})}, \tag{11}$$

where $w_m(.)$ are weight functions satisfying $\Sigma_{m=1}^{M} w_m(x) = 1$. For $q_m(x) > 0$ whenever $w_m(x)p(x)f(x) \neq 0$, Eq. (11) forms an unbiased estimator

$$\mathbb{E}\left[\tilde{\mu}(x)\right] = \sum_{m=1}^{M} \mathbb{E}_{q_m} \left[ \frac{1}{n_m} \sum_{i=1}^{n_m} w_m(x_{im}) \frac{f(x_{im}) p(x_{im})}{q_m(x_{im})} \right] = \tilde{\mu}(x). \tag{12}$$

Although there are numerous options for weight functions satisfying $\Sigma_{m=1}^{M} w_m(x) = 1$, we chose to consider the Balance Heuristic [3], considered to be nearly optimal in the sense of estimation variance,

$$w_m(x) = w_m^{BH}(x) = \frac{n_m q_m(x)}{\Sigma_{s=1}^{M} n_s q_s(x)}. \tag{13}$$

Using (13) in (11) produces the multiple importance sampling with the balance heuristic

$$\mathbb{E}_p f(x) \sim \frac{1}{n} \sum_{m=1}^{M} \sum_{i=1}^{n_m} \frac{p(x_{im})}{\Sigma_{s=1}^{M} \frac{n_s}{n} q_s(x_{im})} f(x_{im}). \tag{14}$$

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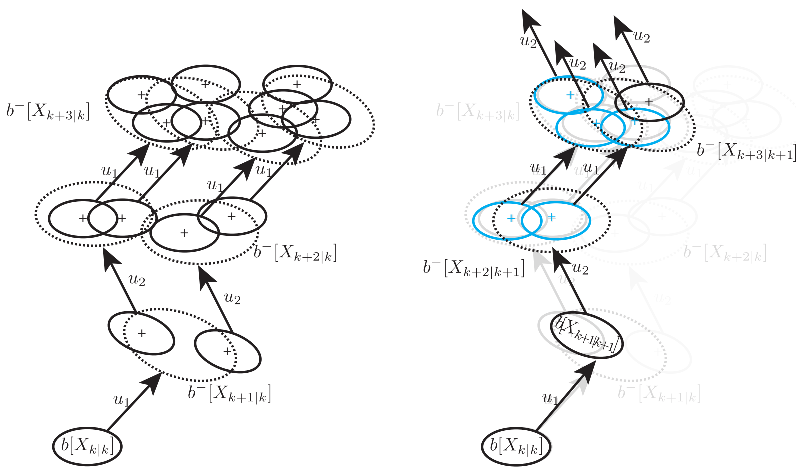


(a) (b)

Figure 4: (a) Horizon overlap between planning time k and planning time k+l, both with L steps horizon and same candidate actions: (i) The shared history of both planning sessions (ii) The possibly outdated information of planning time k, since in planning time k+l this time span is considered as known history (iii) Although in both it represents future prediction, it is conditioned over different history hence possibly different. (b) bla bla. (c) bla bla. [update]

## Appendix B: Toy example for iX-BSP

To better understand the problem let us consider a toy example. We have access to all calculations from planning time $k$, in-which we performed X-BSP (or iX-BSP) for a horizon of 3 steps, with $n_x = 2$ and $n_z = 1$, while considering 2 candidate actions $u_1$ and $u_2$. Figure 4a illustrates a specific action sequence, $u_1 \rightarrow u_2 \rightarrow u_1$, considered as part of planning at time k. We are currently at time k+1, performing planning using iX-BSP with the same horizon length and number of samples per action, for the action sequence $u_2 \rightarrow u_1 \rightarrow u_2$, see Figure 4b.

Following Alg. 1 line 1, out of the two available beliefs from planning time $k$, $\{b[X_{k+1|k}]\}_1^2$, the left one is closer to $b[X_{k+1|k+1}]$, so we consider all its descendants as the set $\mathcal{B}_{k+1|k}$, and denote the difference between $b[X_{k+1|k}]$ and $b[X_{k+1|k+1}]$ as $\{\delta\}$. We continue with re-using the beliefs in the set $\mathcal{B}_{k+1|k}$ (Alg. 1 line 3). First we check whether the two available samples from planning time $k$ constitute an adequate representation for $b^-[X_{k+2|k+1}]$ (Alg. 2 line 7); since they are, we flag them both for re-use, and update $\{b[X_{k+2|k}]\}_1^2$ into $\{b[X_{k+2|k+1}]\}_1^2$ (Alg. 2 line 11). Now for the next future time step, we propagate $\{b[X_{k+2|k+1}]\}_1^2$ with action $u_1$ to obtain $\{b^-[X_{k+3|k+1}]\}_1^2$ (Alg. 2 line 3), and check whether the four available samples from planning time k constitute an adequate representation for $\{b^-[X_{k+3|k+1}]\}_1^2$ (Alg. 2 line 7); since only three of them are, we flag them for re-use and sample the forth one (black colored belief at $k+3|k+1$ in Figure 4b) from $b^-[X_{k+3|k+1}]$. We then update $\{b[X_{k+3|k}]\}_1^3$ into $\{b[X_{k+3|k+1}]\}_1^3$, and $b^-[X_{k+3|k+1}]$ into $b^4[X_{k+3|k+1}]$ using the newly sampled measurement (Alg. 2 line 11). The last step of the horizon $k+4|k+1$ is calculated using X-BSP (Alg. 1 line 4).

At this point we have all inference results for all beliefs along the action sequence $u_2 \rightarrow u_1 \rightarrow u_2$, so we can calculate all reward(cost) values for this action sequence for planning at time k+1. For the look ahead at time $k+2$ of planning session at time $k+1$, i.e. $k+2|k+1$, we have two reward(cost) values, $\{r_{k+2|k+1}(b[X_{k+2|k+1}], u_2)\}_1^2$, each calculated with a different belief $b[X_{k+2|k+1}]$ considering a different sample $z_{k+2|k}$. Calculating the expected reward(cost) value for future time step $k+2|k+1$ would mean in this case, using measurements sampled from $\mathbb{P}(z_{k+2|k}|H_{k+1|k}, u_2)$ rather then from $\mathbb{P}(z_{k+2|k+1}|H_{k+1|k+1}, u_2)$. This problem, of performing estimation using forced samples is called importance sampling. Since for a single time step we might have samples from multiple different distributions, e.g. future time $k+3|k+1$ in Figure 4b, our problem falls within the special case of Multiple Importance Sampling (see Appendix A). Using the formulation of multiple importance sampling using the balance heuristic (14) we can write down the estimation for the expected reward value at planning time $k+2|k+1$,

$$\mathbb{E}\left[r_{k+2|k+1}(.)\right] \sim \frac{1}{2}\frac{p_1(z^1_{k+2|k})}{\frac{2}{2}q_1(z^1_{k+2|k})} \cdot r^1_{k+2|k+1}(.) + \frac{1}{2}\frac{p_1(z^2_{k+2|k})}{\frac{2}{2}q_1(z^2_{k+2|k})} \cdot r^2_{k+2|k+1}(.), \tag{15}$$

where $p_1(.) \doteq \mathbb{P}(z_{k+2|k+1}|H_{k+1|k+1}, u_2)$ and $q_1(.) \doteq \mathbb{P}(z_{k+2|k}|H_{k+1|k}, u_2)$. In the same manner, following (14), we can also write down the estimation for the expected reward(cost) value at look ahead step $k+3$ from planning session at time $k+1$, i.e. $k+3|k+1$,

$$\mathbb{E}\left[r_{k+3|k+1}(.)\right] \sim \frac{1}{4}\frac{p_2(z^1_{k+2:k+3|k})}{\frac{3}{4}q_2(z^1_{k+2:k+3|k})+\frac{1}{4}p_2(z^1_{k+2:k+3|k})}r^1_{k+3|k+1}(.)+\frac{1}{4}\frac{p_2(z^2_{k+2:k+3|k})}{\frac{3}{4}q_2(z^2_{k+2:k+3|k})+\frac{1}{4}p_2(z^2_{k+2:k+3|k})}r^2_{k+3|k+1}(.)+ \\ \frac{1}{4}\frac{p_2(z^3_{k+2:k+3|k})}{\frac{3}{4}q_2(z^3_{k+2:k+3|k})+\frac{1}{4}p_2(z^3_{k+2:k+3|k})}r^3_{k+3|k+1}(.)+\frac{1}{4}\frac{p_2(z^4_{k+2:k+3|k+1})}{\frac{3}{4}q_2(z^4_{k+2:k+3|k+1})+\frac{1}{4}p_2(z^4_{k+2:+3|k+1})}r^4_{k+3|k+1}(.), \quad (16)$$

where $p_2(.) \doteq \mathbb{P}(z_{k+2:k+3|k+1}|H_{k+1|k+1}, u_2, u_1)$ and $q_2(.) \doteq \mathbb{P}(z_{k+2:k+3|k}|H_{k+1|k}, u_2, u_1)$. When considering

$$\mathbb{P}(z_{k+1:k+L|k}|H_{k|k}, u_{k:k+L-1}) = \prod_{i=k+1}^{k+L} \mathbb{P}(z_{i|k}|H^-_{i|k}) \quad (17)$$

we can re-write the measurement likelihood from (16) into a product of measurement likelihoods per look ahead step, e.g. $p_2(z^1_{k+2:k+3|k}) = p_1(z^1_{k+2|k})\tilde{p}_2(z^1_{k+3|k})$, when $p_1(.)$ need not be calculated at look ahead step $k+3$, since it is given from (15).

# References


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