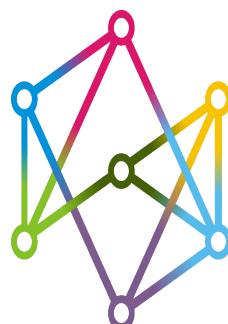


Probabilistic Qualitative Geometry SLAM

Roe Mor

Supervised by assoc. prof. Vadim Indelman



ANPL
Autonomous Navigation and
Perception Lab

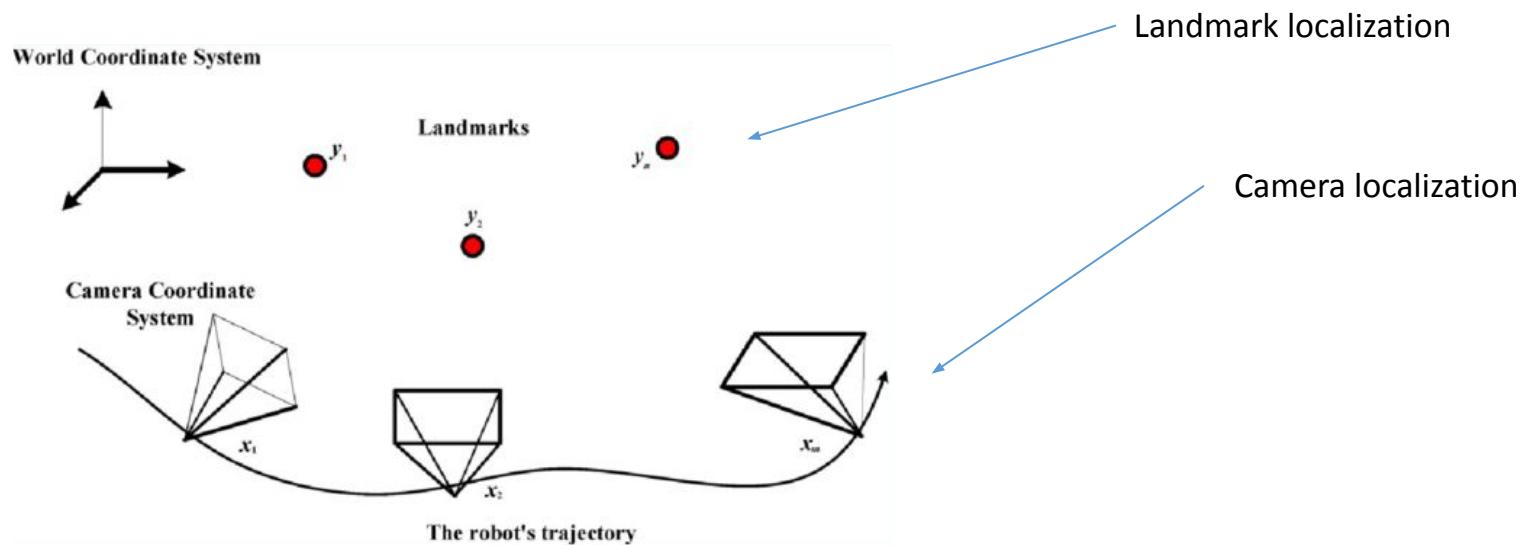
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Introduction

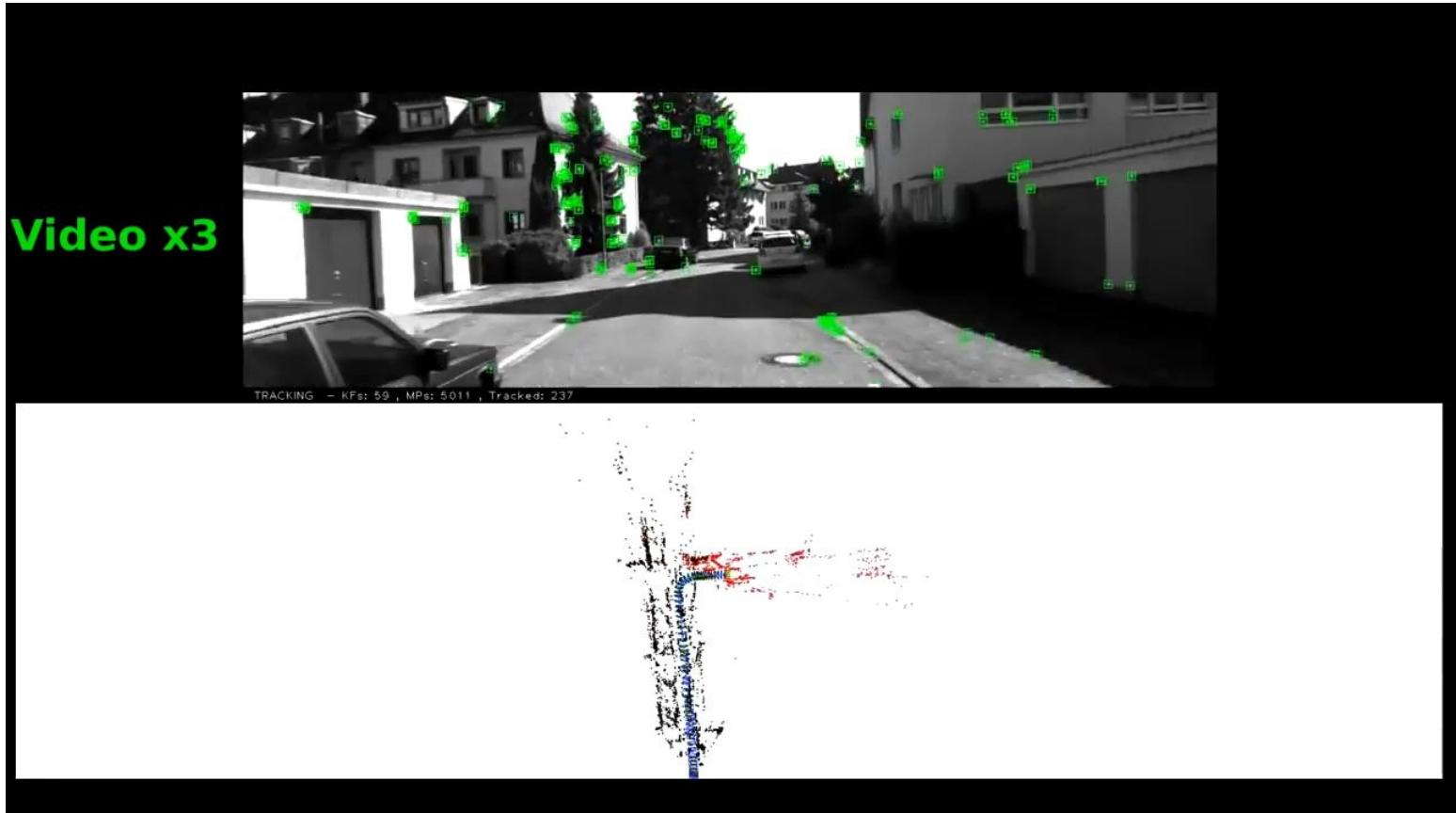
Introduction

- SLAM – simultaneous localization and mapping:



Introduction

- SLAM – simultaneous localization and mapping:



Introduction

- SLAM – simultaneous localization and mapping:

Status:

- Well researched (also today), many open-source libraries
- Partial success in real world autonomous systems
- Online performance

Challenges:

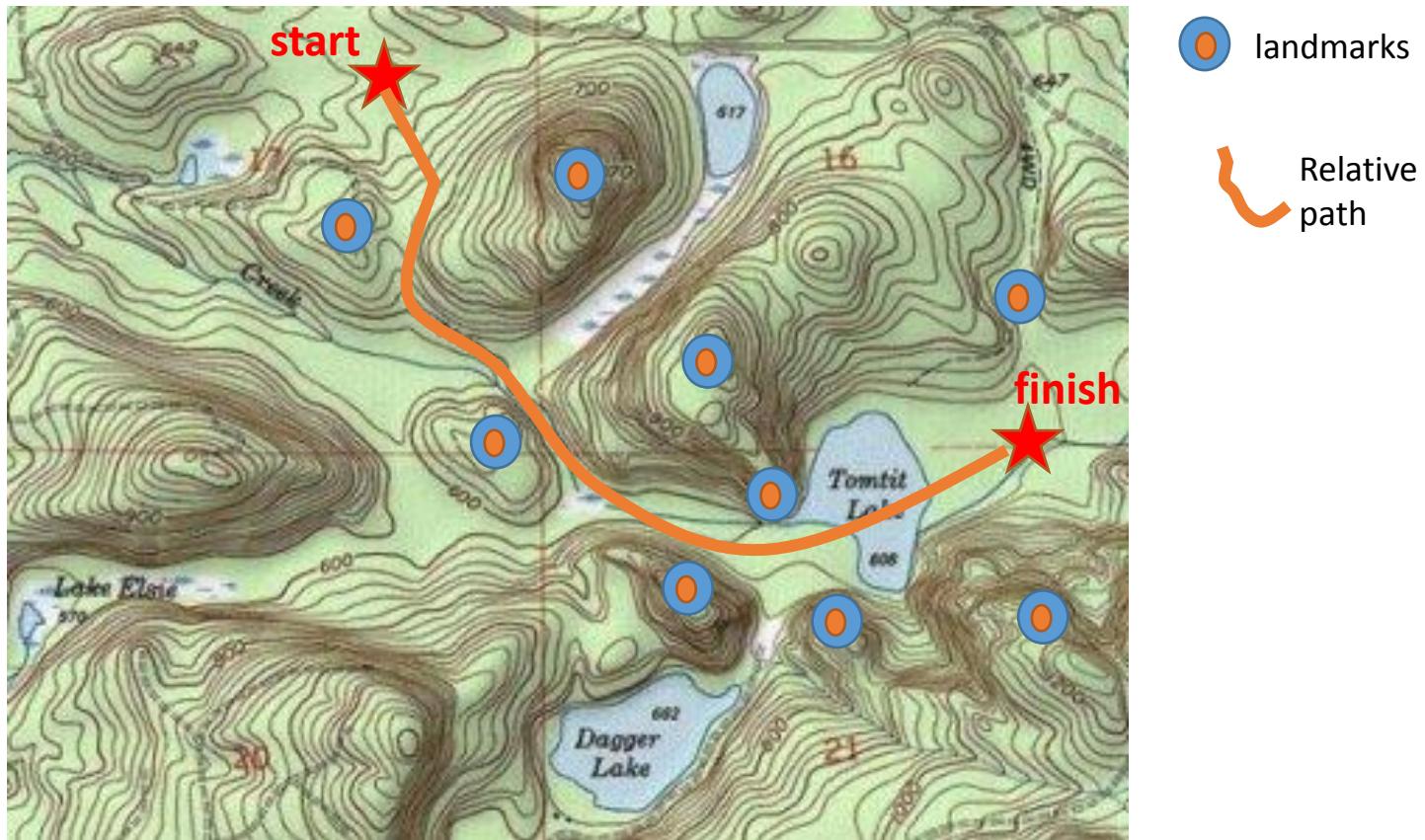
- Accumulated error (Linearization, Measurement noise, miss identification)
- High complexity – not real-time. Uses much power.

Motivation

Qualitative spatial reasoning – easier, and good enough

Human navigation:

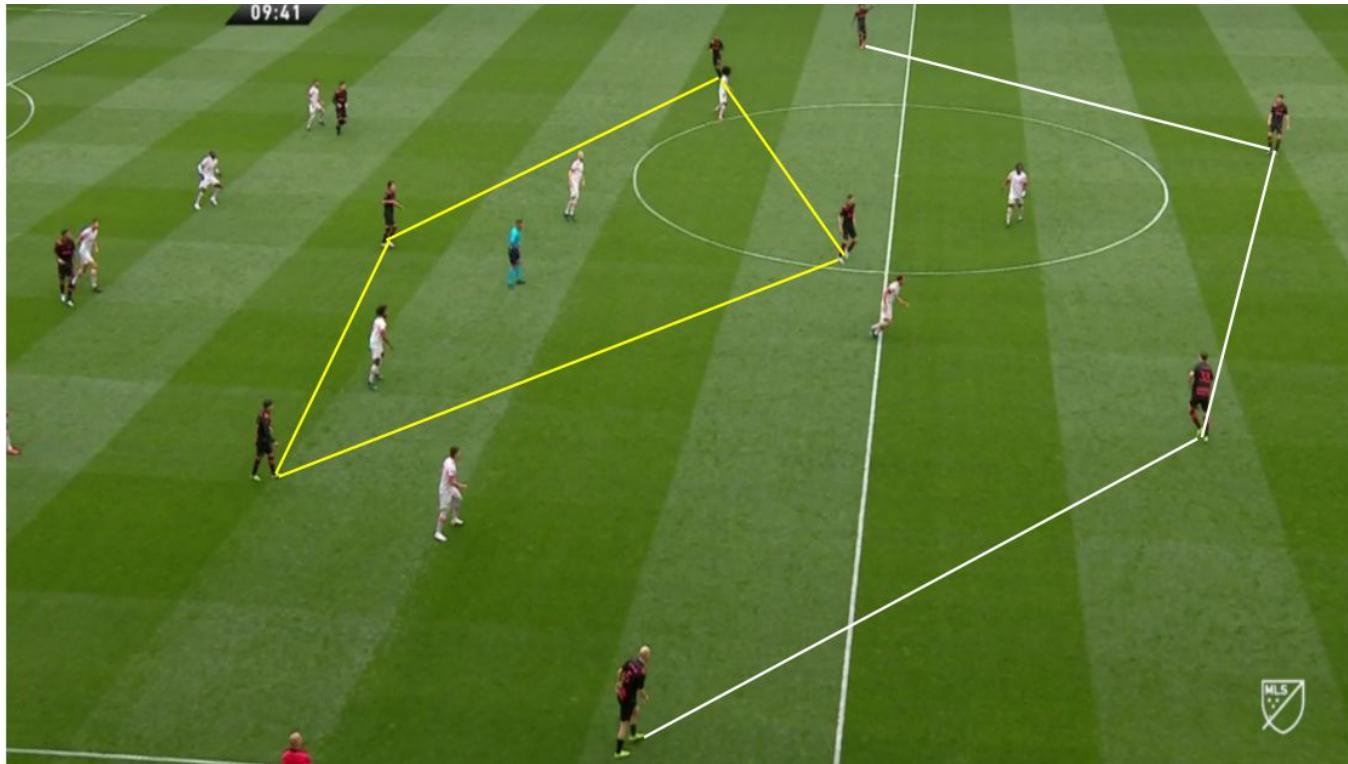
- Landmark Relative path
- Qualitative geometry
- Local accurate navigation for minimal effort



Qualitative path Vs metric path

Motivation

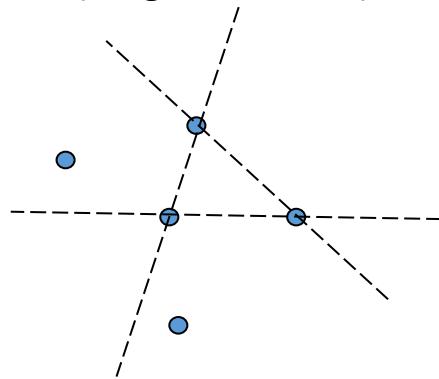
Qualitative spatial reasoning – easier, and good enough



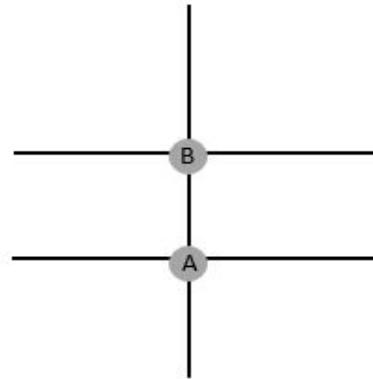
Motivation

Qualitative spatial reasoning – easier, and good enough

relative location
(no global frame)



qualitative localization
(qualitative geometric relations)

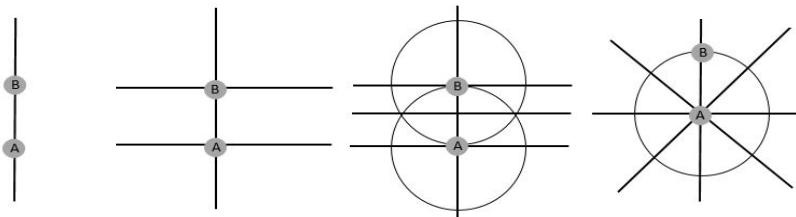


- ✓ less sensitive to noise
- ✓ No Long term error accumulation
- ✓ Low complexity

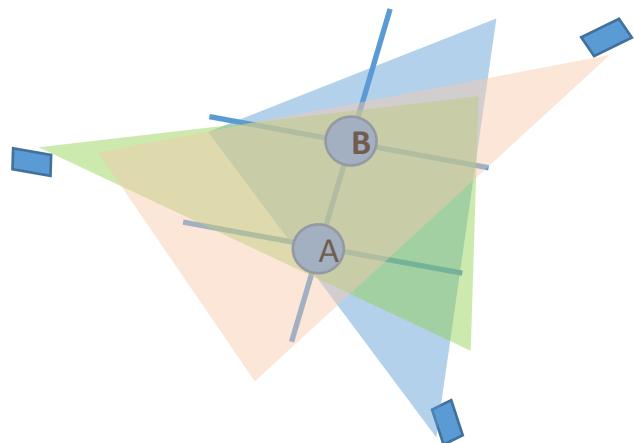
Concept Overview

Concept - Intuition

- Many small two-landmark relative frames of reference – no global frame
- Qualitative spatial partition instead of metric location



- Estimate state from landmark relative measurements



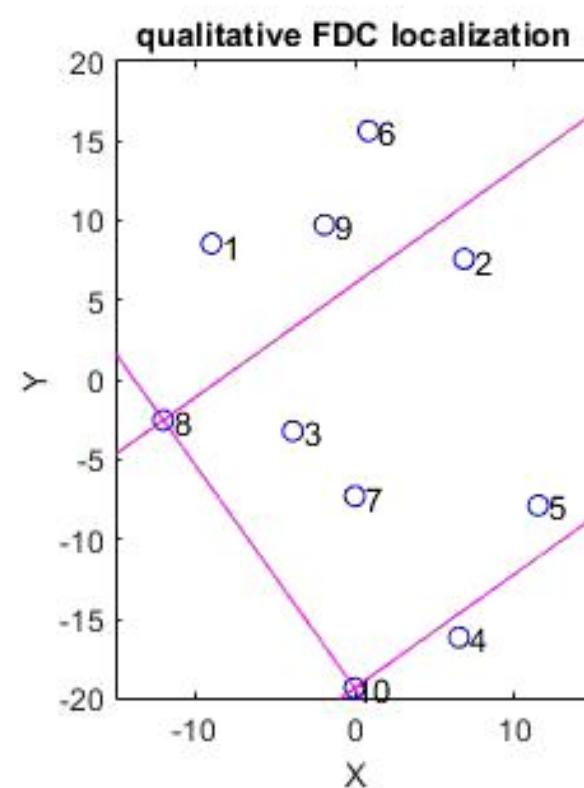
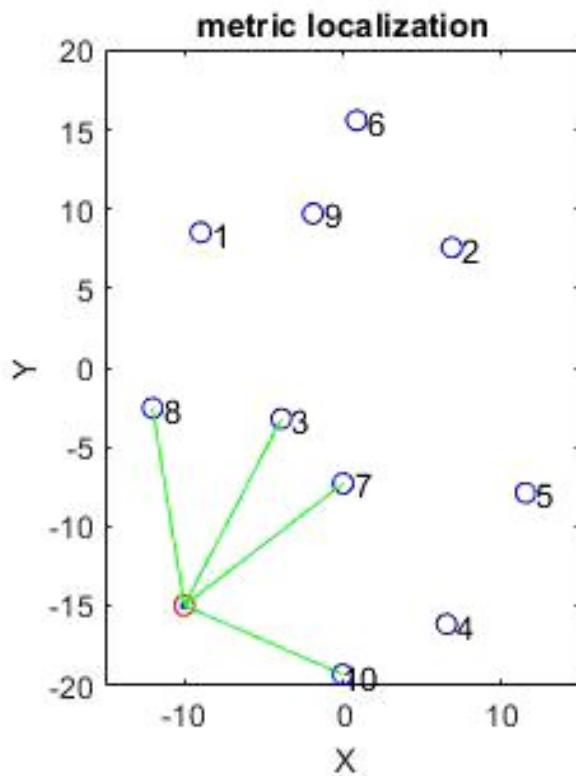
[1] Freksa 1992 . On the utilization of spatial structures for cognitively plausible and efficient reasoning.

[2] Schlieder 1993 Representing visible locations for qualitative navigation.

[3] Scivos 2004 The finest of its class: The natural pointbased ternary calculus lr for qualitative spatial reasoning.

Concept - Intuition

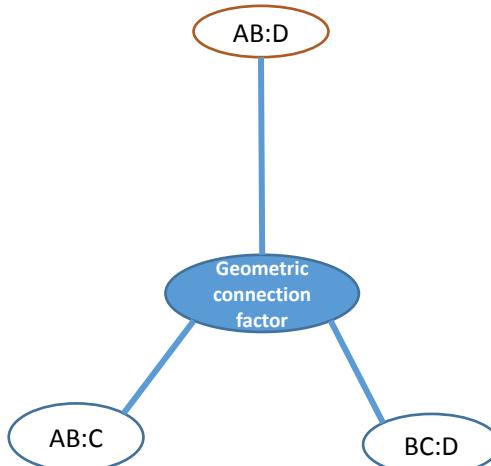
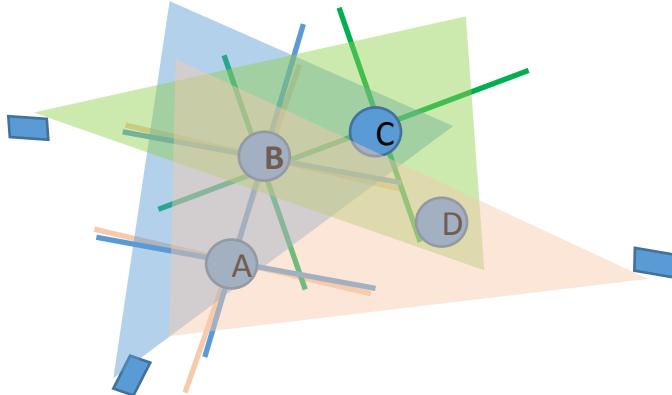
- Qualitative relational localization



Concept - Intuition

- Qualitative relational mapping

Map \rightarrow connected graph of landmark triplets

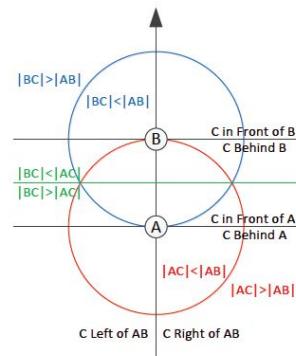


QSR related work

- Spatial Qualitative Reasoning (QSR) approaches:

McClelland,2013

- Typically assume data association is given
- Address mainly mapping, less localization
- Not probabilistic
- Extended double cross



(a) Region Boundaries

McClelland,2013, Qualitative relational mapping for planetary rovers

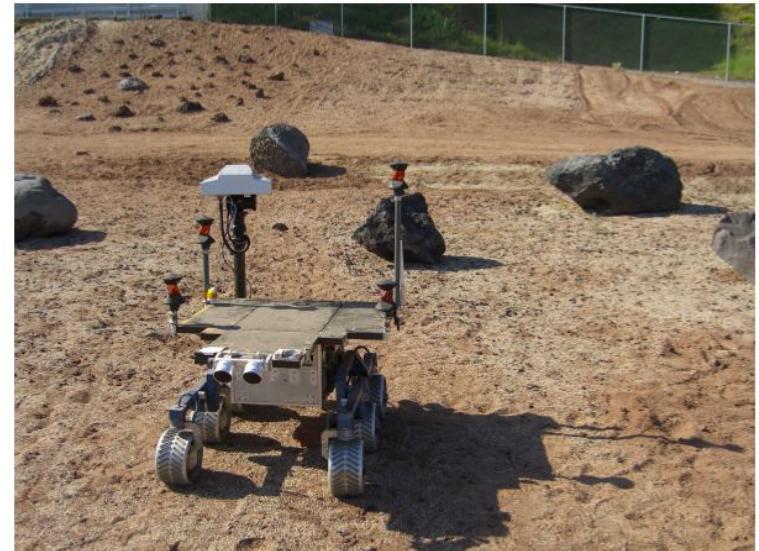


Image taken from McClelland,2013 [5]

QSR related work

- Spatial Qualitative Reasoning (QSR) approaches:

Padgett 2016+2017

- Probabilistic
- Passive + Active planning
- Not a full SLAM framework

Zilberman & Indelman 2022

- Composition in qualitative approaches (RA-L + ICRA 2022)
- Active planning (ongoing)

Padgett, 2016, Probabilistic qualitative mapping for robots
Zilberman, 2022, Incorporating Compositions in Qualitative

Contributions

Our approach: – probabilistic time and spatial dependent QSR:

- Full probabilistic SLAM framework:
 - Localization
 - mapping
- Incorporating Motion model
- Factor graph propagation

publications:

- IROS 2020
- Journal paper(in progress)
- Open-source repo (in progress)

Contributions

Benefits Vs previous QSR work:

- improve accuracy
- improve performance complexity
- estimate sets of landmarks that weren't seen together

Benefits Vs metric SLAM:

- Low computation
- Robustness to noise / sensor quality
- Simpler computational process

Single Triplet Qualitative Estimation

Our Approach – single triplet

Estimate each triplet separately:

- Landmark relative coordinate frames
- Small 3 landmark – multiple view SLAM problems

Fusing data:

- Build qualitative map and propagate data

Formulation

2D navigation:

- Metric state

$X_{1:n}$

- Camera pose at times 1:n

$L^{AB:C}$

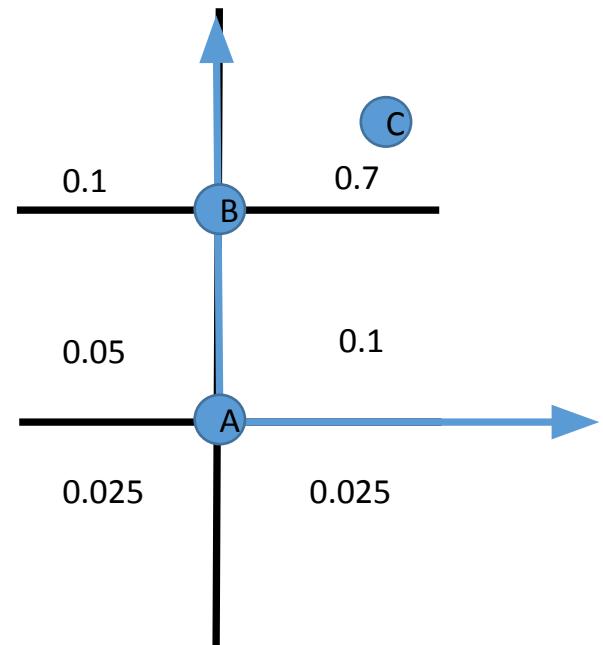
- Metric location of landmark C in AB frame

$H_n^{AB:C} = \{Z_1, \dots, Z_n\}$

- All A,B,C measurements up to time n

- qualitative state probability: $\mathbb{P}(S^{AB:C} | H_n)$

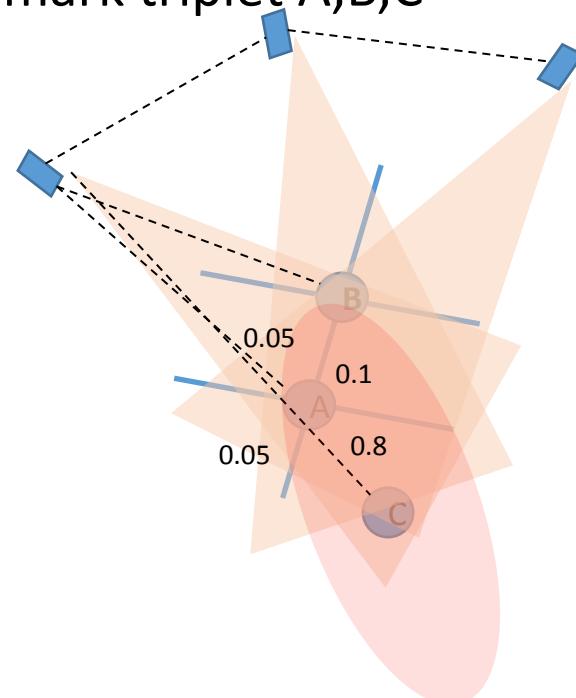
$S^{AB:C}$ - Qualitative state of landmark C in AB frame



Our Approach – single triplet

Estimation of a single landmark triplet:

- Measurements:
 - Azimuth to landmark triplet A,B:C
 - Heading between camera poses
- Metric SLAM For camera poses and landmark triplet A,B,C
 - Uses several separate camera poses
 - Incremental
- Integrate qualitative state probability



Our Approach – single triplet

- Probabilistic formulation

$$\mathbb{P}(S^{AB:C} = i | H_n^{AB:C}) = \underbrace{\int_{X_{1:n}, L^{AB:C}} \dots \int \mathbb{P}(L^{AB:C} | S^{AB:C} = i) \mathbb{P}(X_{1:n}, L_C | H_n^{AB:C}) dL^{AB:C} dX_{1:n}}$$

Integrate over metric states

Metric SLAM
For landmark triplet
A,B,C

$$\mathbb{P}(X_{1:n} L^{AB:C} | H_n) = \frac{\mathbb{P}(Z_1 | X_1, L^{AB:C}) \mathbb{P}(X_1 L^{AB:C})}{\mathbb{P}(Z_1)} \prod_{i=2}^n \underbrace{\frac{1}{\zeta_i} \mathbb{P}(Z_i | X_i, L^{AB:C})}_{\text{Measurement model}} \underbrace{\mathbb{P}(X_i | X_{i-1}, a_{i-1})}_{\text{Motion model}}$$

Measurement model Motion model

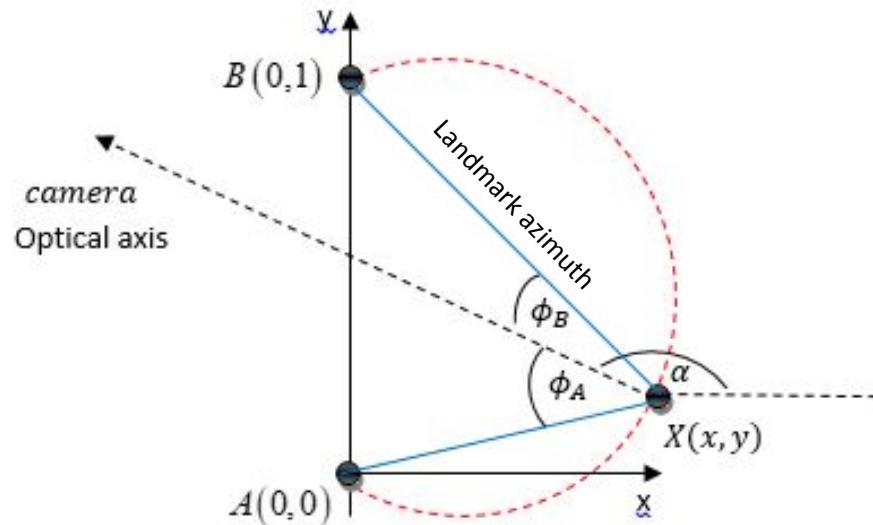
Our Approach – single triplet

Solving the 3 landmark SLAM problem:

- Non linear sample based SLAM approach
- Measurements
 - Measurements – azimuth to landmarks (ϕ)
 - Motion model – heading to next pose (Ψ)

Single view:

Camera on locus circle

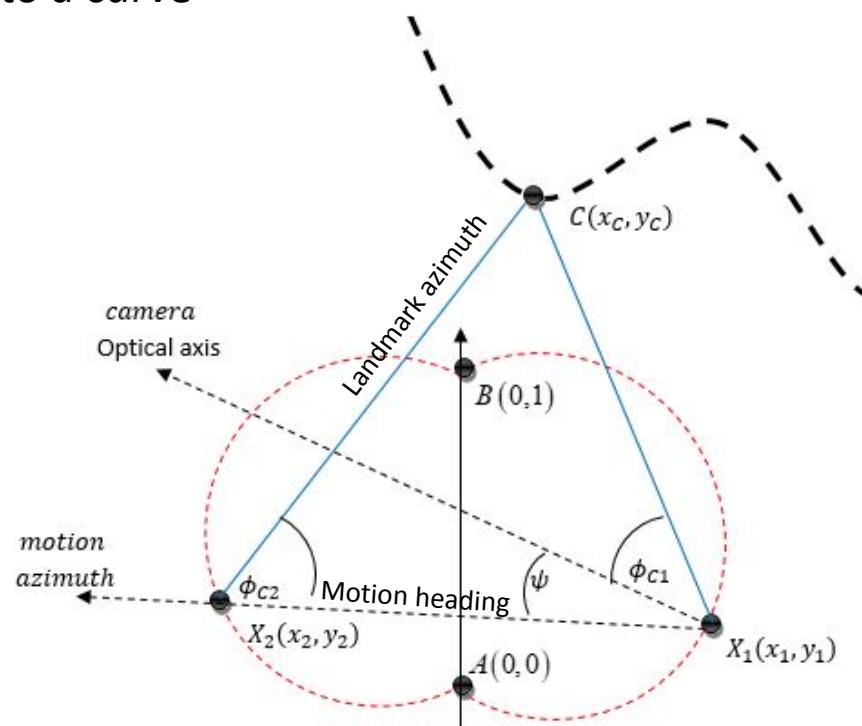


Our Approach – single triplet

Solving the 3 landmark SLAM problem:

Two views:

- Cameras on locus circles
- Landmark C can be triangulated to a curve

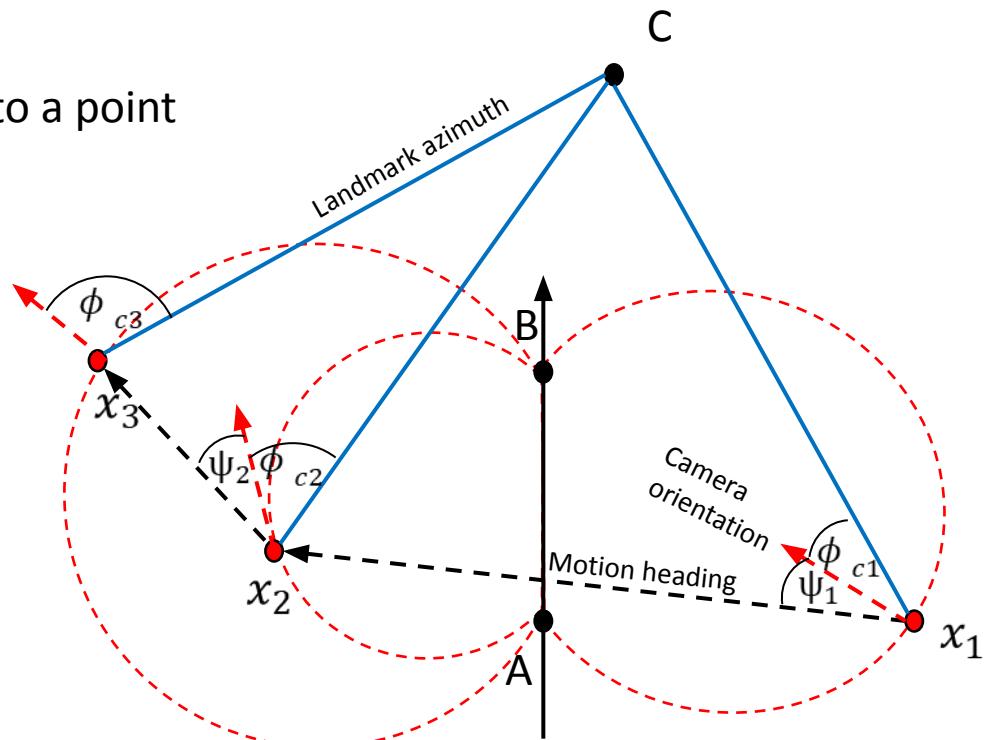


Our Approach – single triplet

Solving the 3 landmark SLAM problem:

Three views or more:

- Cameras on locus circles
- Landmark C can be triangulated to a point



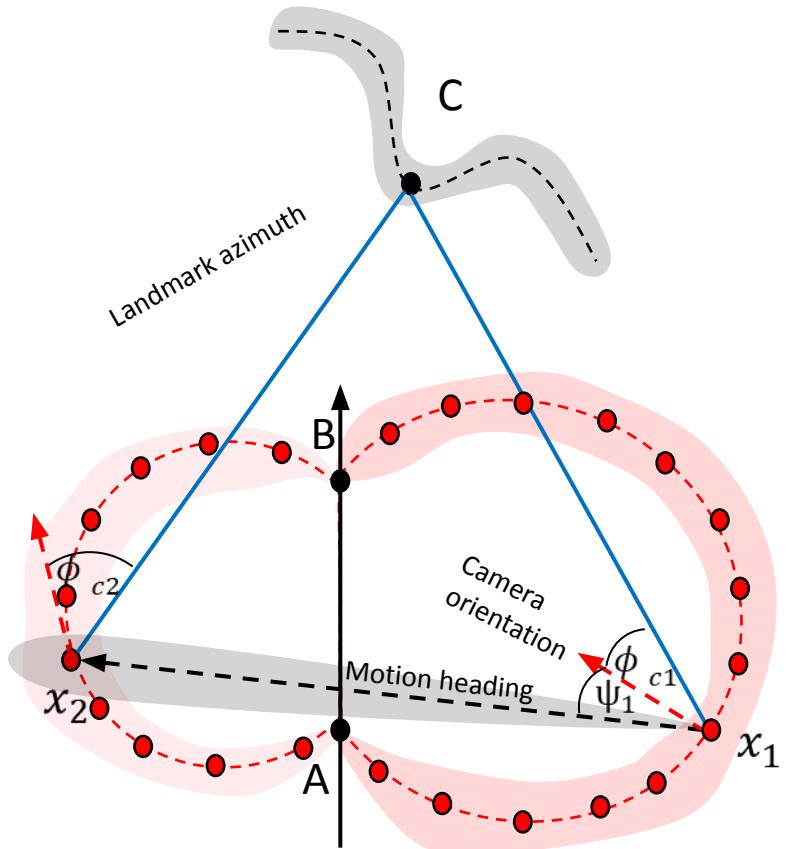
Our Approach – single triplet

Solving the 3 landmark SLAM problem:

- Non linear sample based SLAM approach
- A,B locus circle
- A,B azimuth measurements noise
- Motion heading noise

Number of samples:

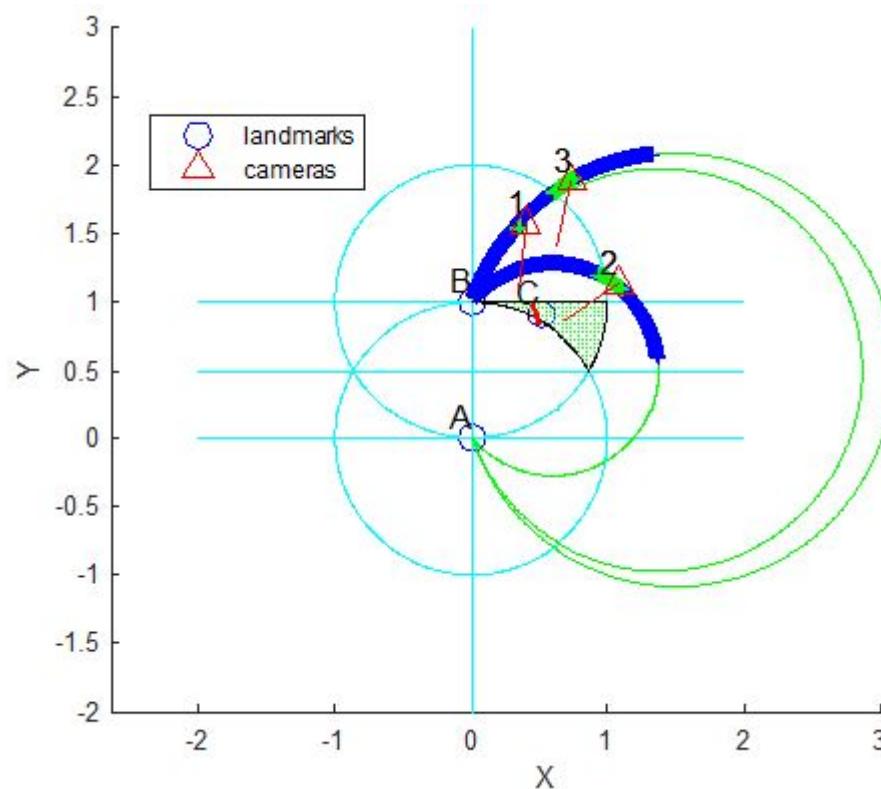
- Exponential in camera poses
- Practically reduces fast by consistency tests
- Very small for 3 camera poses or more
- Good for incremental algorithm



Our Approach – single triplet

Solving the 3 landmark SLAM problem:

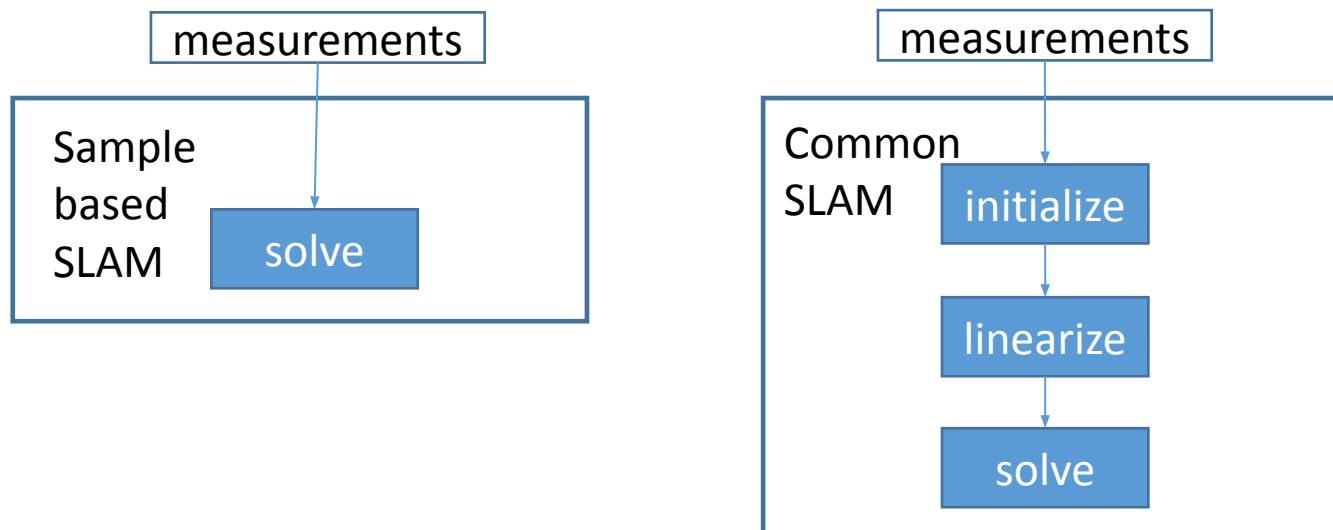
- 3 view Simulation example



Our Approach – single triplet

Our approach Vs regular SLAM

- Non linear
 - no linearization errors
 - No need for linearization
 - No initialization process
- General - variables are not assumed to be Gaussian



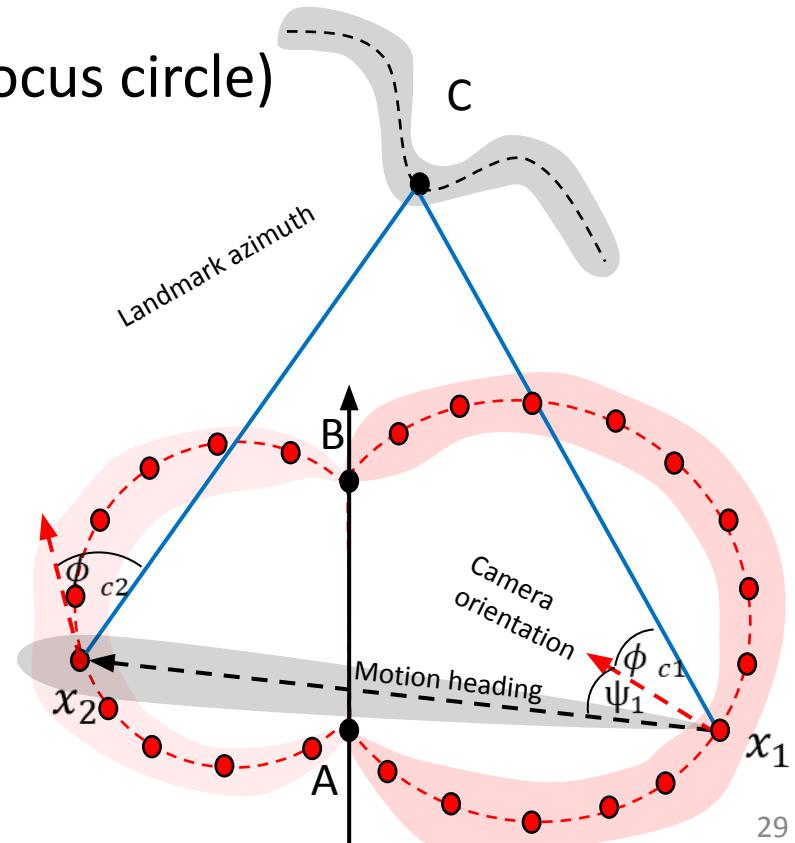
Our Approach – single triplet

Solving the 3 landmark SLAM problem – Fast approximation

Trying to capitalize on QSR coarse spatial partition

Fast solver variant:

- Sample only geometry (camera locus circle)
- No noise samples



Our Approach – single triplet

Single triplet results

single triplet EDC estimation results			
	baseline	ours	ours-fast
DMSE	0.39, 0.63, 0.71	0, 0.16, 0.63	0, 0.21, 0.62
geometric distance	0.28, 1.10, 2.30	0, 0.25, 1.15	0, 0.27, 1.16
Entropy	0.28, 0.66, 0.87	0, 5e-3, 0.58	0, 0.07, 0.64
time[sec]	26	18	0.05

Metrics:

- DMSE – probabilistic correctness
- Geometric distance – geometric correctness
- Entropy – distribution steepness
- GT rating – the position of the GT qualitative state when states are ordered by probability (1 – most probable)

Motion Model makes a difference!

$$DMSE = \sqrt{\sum_{i=1}^m (\mathbb{P}(s_i) - \mathbb{P}(s_{GT}))^2}$$

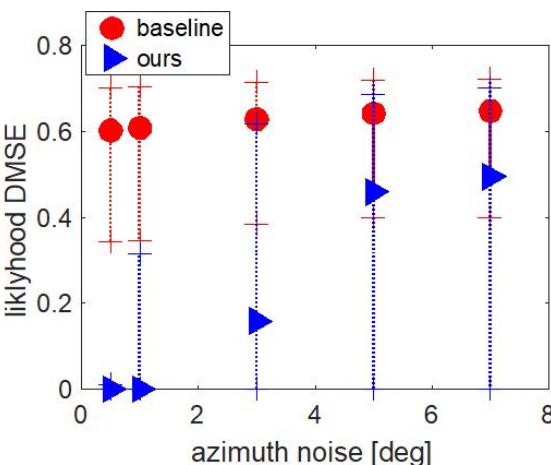
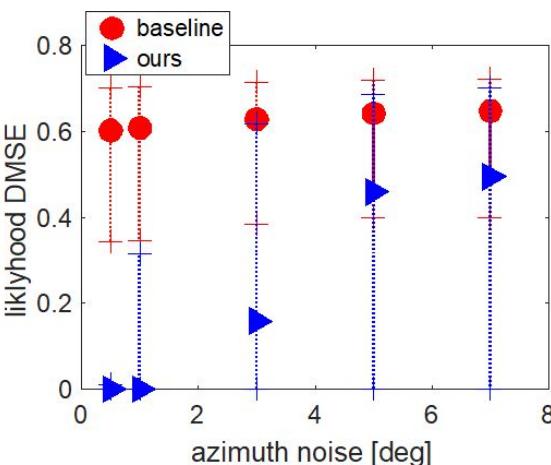
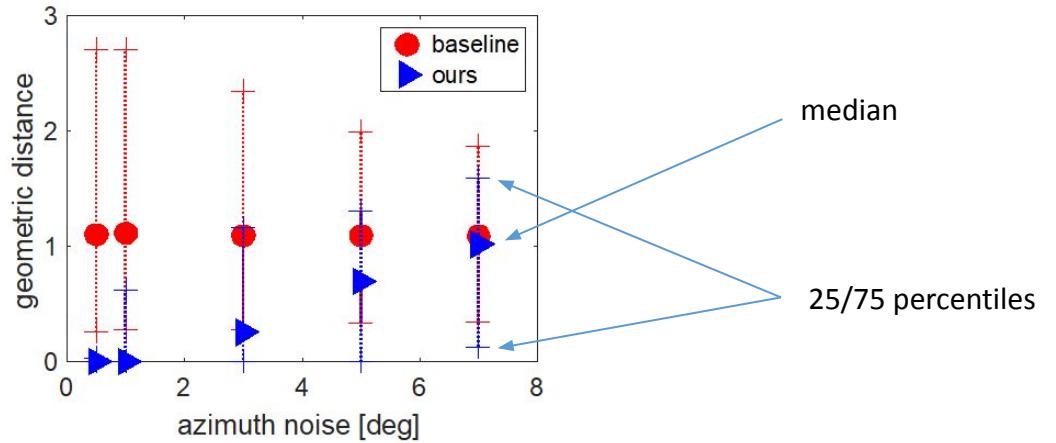
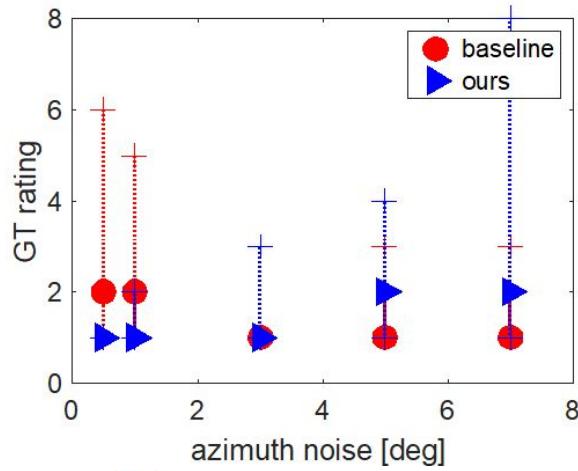
$$gmd = \sum_{i=1}^m \mathbb{P}(s_i) \|c_i - c_{GT}\|_2$$

$$e = -\sum_{i=1}^m \mathbb{P}(s_i) \log(\mathbb{P}(s_i))$$

Baseline = padget

Results

Single triplet results



Motion
Model
makes a
difference!

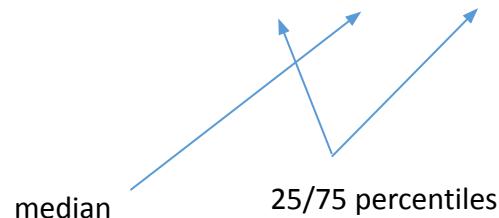
Results

MRCLAM dataset

- Autonomous Space Robotics Lab (ASRL) at the University of Toronto
- Cylindrical landmarks
- Occlusions
- Sensors
 - Camera azimuth measurements
 - Odometry



MRCLAM dataset EDC estimation		
	ours-fast	uniform
DMSE	0.03, 0.45, 0.69	0.97
gmd	5e-3, 0.27, 0.71	2.2
Entropy	4e-3, 0.38, 0.69	3
GT rating	1, 1, 2	-



[Autonomous Space Robotics Lab: MR.CLAM Dataset
\(utoronto.ca\)](http://www.utoronto.ca)

Our Approach – single triplet

Conclusions

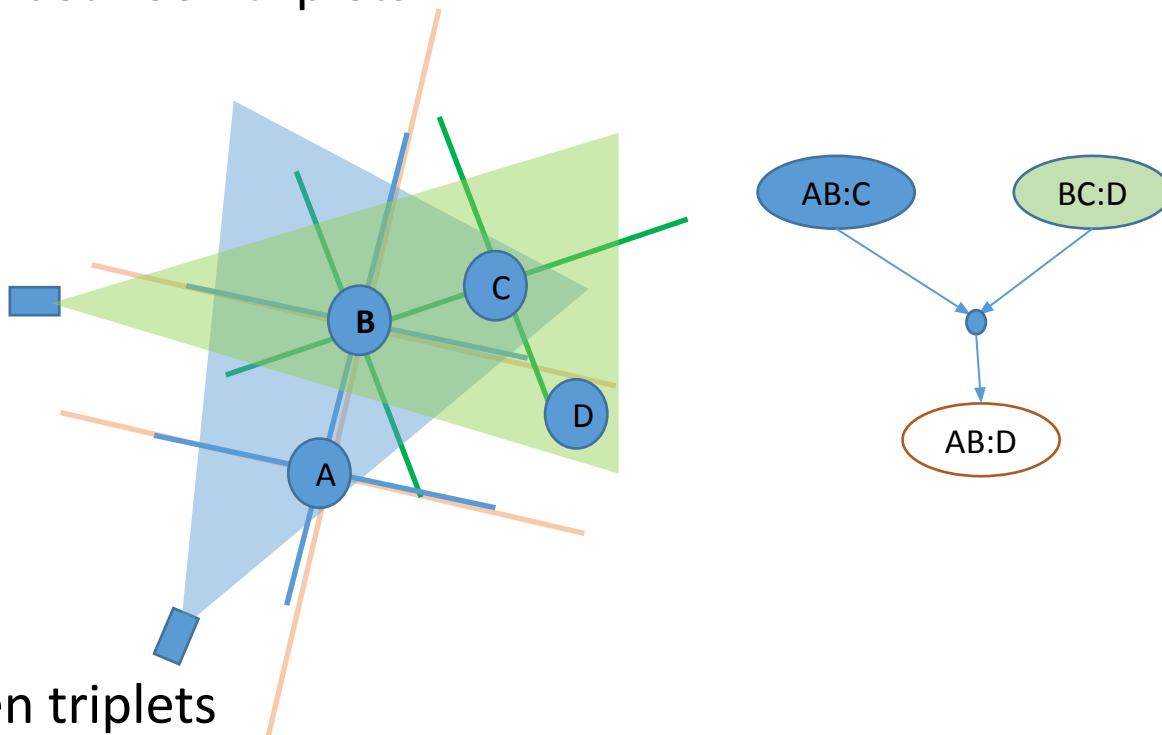
- Adding motion model:
 - Better performance
 - Better complexity (feasibility tests reduce samples faster)
- fast approximation
 - Much faster
 - Performance very close to full algorithm
 - uses qualitative inherent course spatial partition
- General performance
 - Up-to azimuth measurement noise of 3deg – very close to GT
- (Published in IROS 2020)

Qualitative Composition

Our Approach - composition

Novel probabilistic Composition:

- Propagate data between triplets



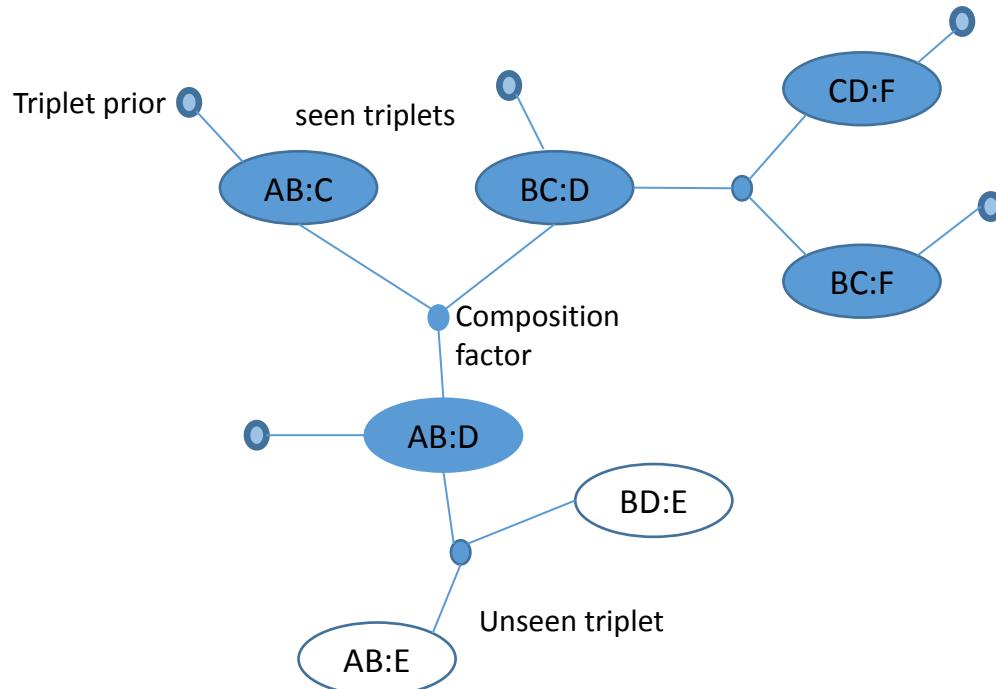
- Estimate unseen triplets
- Improve estimation

$$\mathbb{P}(S^t | H^t, H^{p1}, H^{p2}) = \sum_{s_i^{p1}} \sum_{s_j^t} p(S^t, s_i^{p1}, s_j^t | H^t, H^{p1}, H^{p2})$$

Concept - Intuition

Composition:

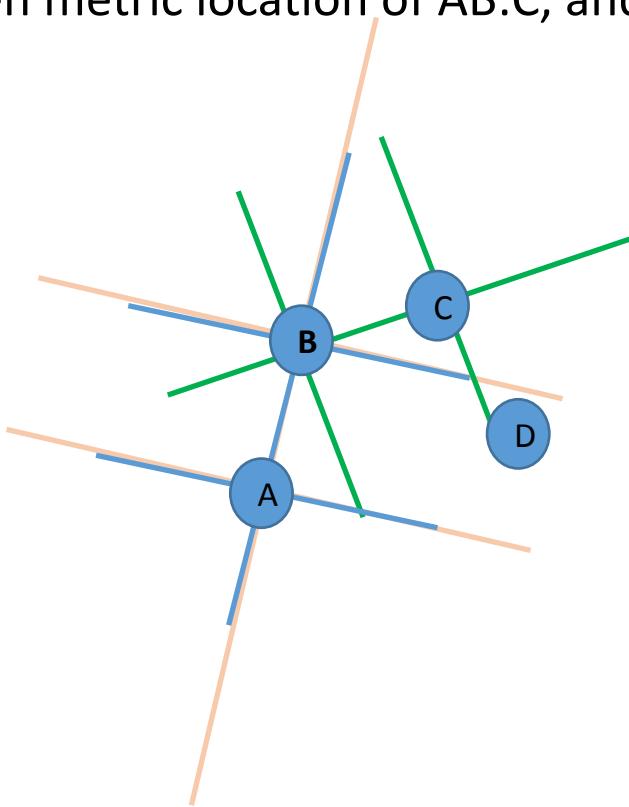
Qualitative map propagation by composition factor graph



Our Approach - composition

Composition:

- Calculate AB:D given metric location of AB:C, and BC:D



Our Approach - composition

Composition:

- Composition factor – pure qualitative approximation
 - Remember only qualitative state $\mathbb{P}(L|s_i, H) \approx \mathbb{P}(L|s_i)$
 - Forget metric data

• Formulation:

$$AB:C = p_1$$

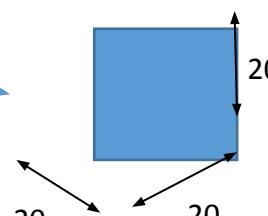
$$BC:D = p_2$$

$$AN:D = t$$

Single triplet estimation

$$\mathbb{P}(S^t | H^{p1}, H^{p2}) \approx \sum_{s_i^{p1}} \sum_{s_j^t} \mathbb{P}(s_i^{p1} | H^{p1}) \mathbb{P}(s_j^t | H^{p2}).$$
$$\iint_{\substack{L^{p1} \in s_i^{p1}, L^{p2} \in s_j^{p2}}} \mathbb{P}(S^t | L^{p1}, L^{p2}) dL^{p1} dL^{p2}.$$

**Calculate offline
Same for all factors**



Our Approach - composition

Composition:

- Composition factor pure qualitative approximation:
 - Fast graph propagation
 - Very efficient in HW accelerators
 - Low memory consumption
- (Published in IROS 2020)

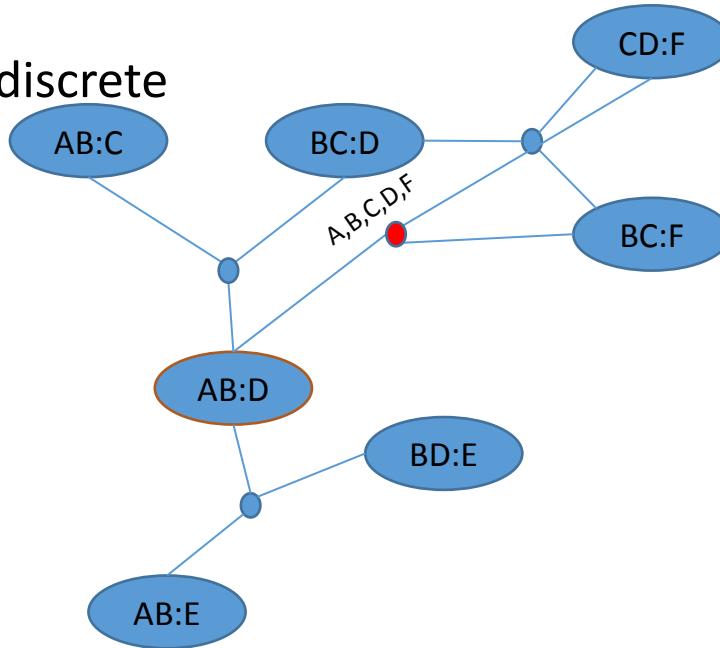
Factor Graph Propagation

Our Approach - composition

Factor graph propagation algorithm:

Accurate method :

- Elimination - trinary factors -> multiple node factor
- Calculation is exponential in number of nodes
- Runtime Not feasible
- Implemented in GTSAM-discrete

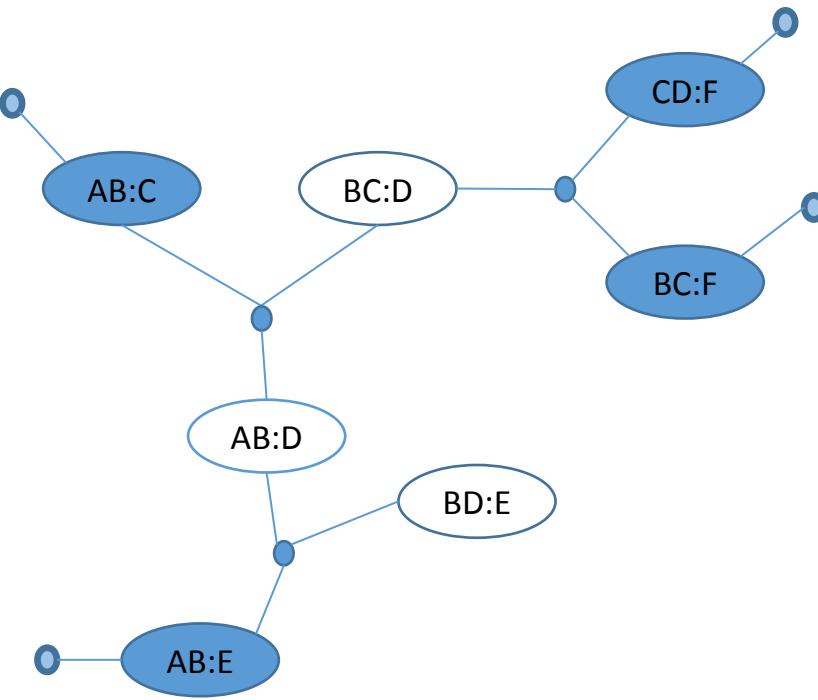


Our Approach - composition

Factor graph propagation:

Fast Approximated algorithm:

- Greedy – one most informative step
- Single best path
- One pass over each node



Our Approach - composition

Factor graph propagation:

Information score (ISC):

A metric to measure how informative is the probability distribution for a specific landmark triplet qualitative state:

- $0 < ISC < 1$
- Higher is better (more informative)

$$ISC = \frac{H_{max} - H_n}{H_{max} - H_{min}}$$

H_n = node entropy

H_{max} = uniform (max) entropy

H_{min} = perfect (min) entropy

Our Approach - composition

Factor graph propagation:

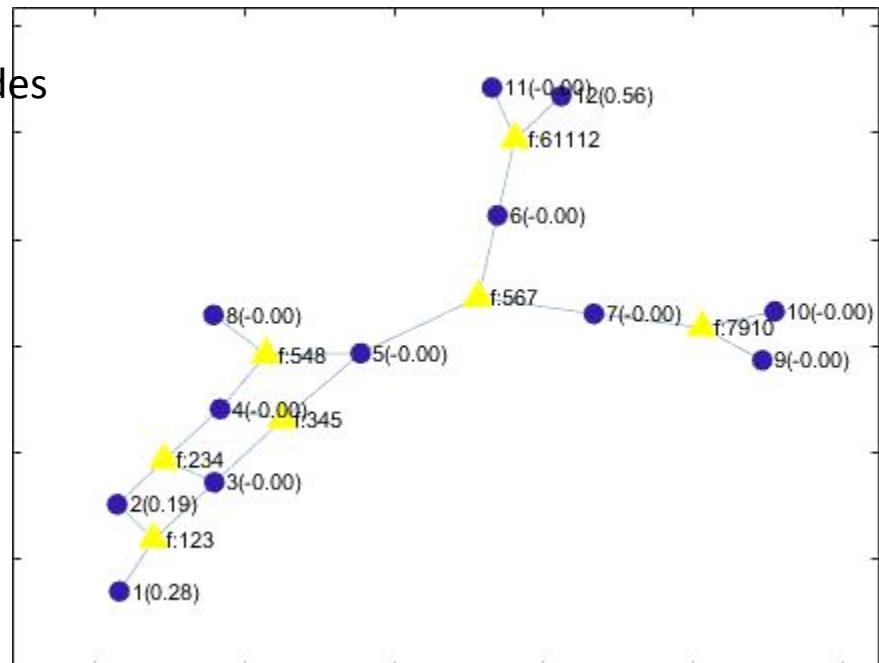
Fast Approximated algorithm:

- Observed nodes = Source nodes
- Loop:
 - Propagate any factor that has 1 or 2 ‘done’ nodes
 - Calculate ISC for all newly calculated nodes
 - Keep best ISC node, and mark as ‘done’
- Break when no factor has 1 or 2 ‘done’ nodes

Example:

Fast Approximated algorithm:

- Node text: id (ISC)
- Priors on nodes 1,2,12
- Update order: 6,5,3,4,11,7,9,10,8



Our Approach - composition

Factor graph propagation:

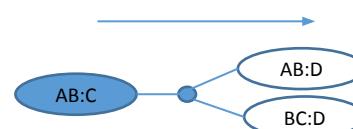
Composition level (CL):

- A tool to study composition behavior in correlation to:
 - Graph topology
 - prior information
- Propagation in graph:
 - CL = ISC for observed nodes
 - Same graph propagation algorithm
 - ISC decay Factor:

$$ISC_{AB:D} = (1-\alpha^2) \frac{ISC_{AB:C} + ISC_{BC:D}}{2}$$



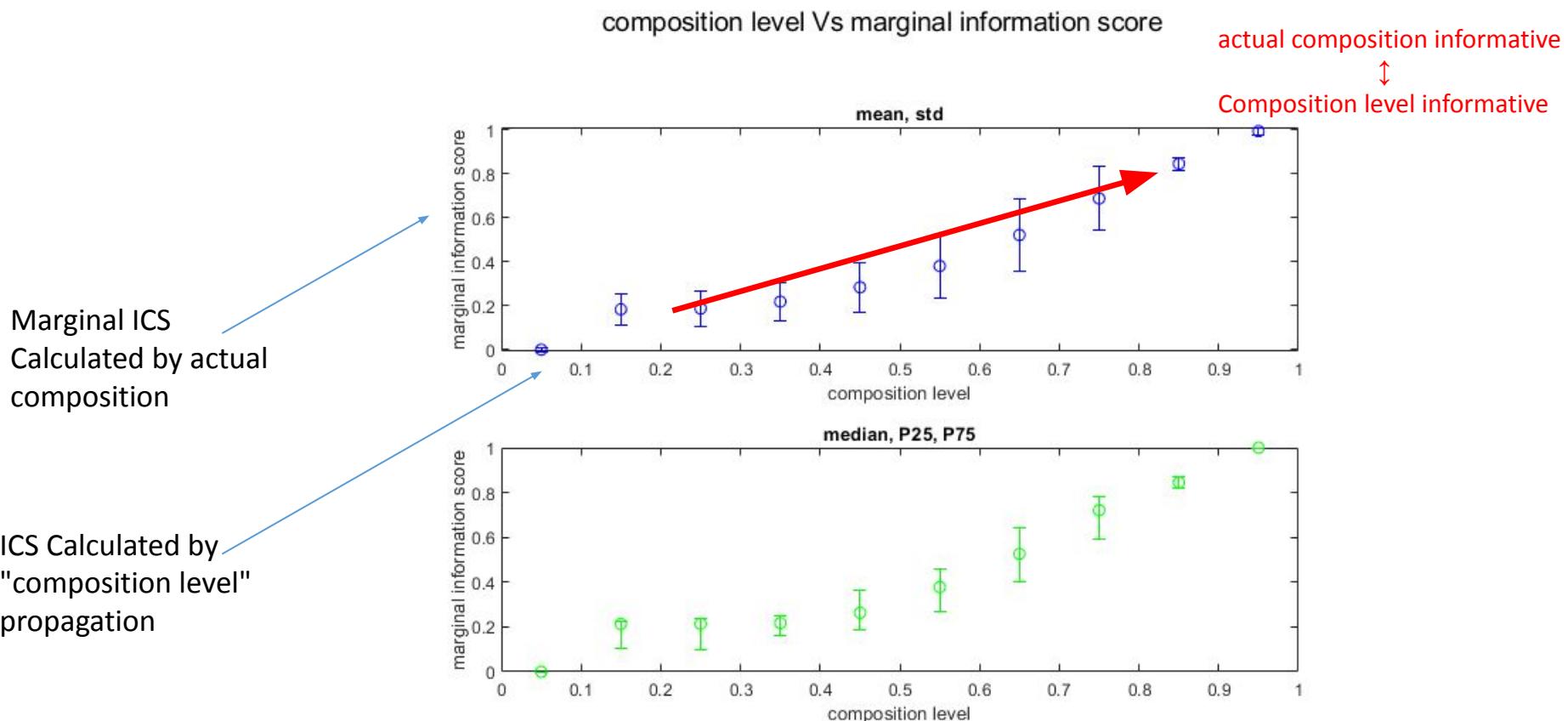
$$ISC_{AB:D} = ISC_{BC:D} = (1-\alpha) ISC_{AB:C}$$



$\alpha=0.5$ - information decay factor

Our Approach - composition

Composition results:



Our Approach – composition

Conclusions

- Composition propagates significant information
- Information propagated is correlated to graph topography (composition level)
- Might be practical for:
 - Estimating unseen nodes (for planning / landmark recognition)
 - Improving existing estimation
- (will be published soon)

Conclusions

Conclusions

- Good Performance
 - Low measurement noise -> almost perfect results (up to 3°)
 - High measurement noise -> Better performance than state of the art (up to 7°)
- Low complexity (practical for low compute systems)
 - Good performance for fast approximation
- Good for fast active planning
 - Composition is fast and informative

