

Belief Space Planning for Autonomous Navigation while Modeling Landmark Identification

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Introduction - Applications

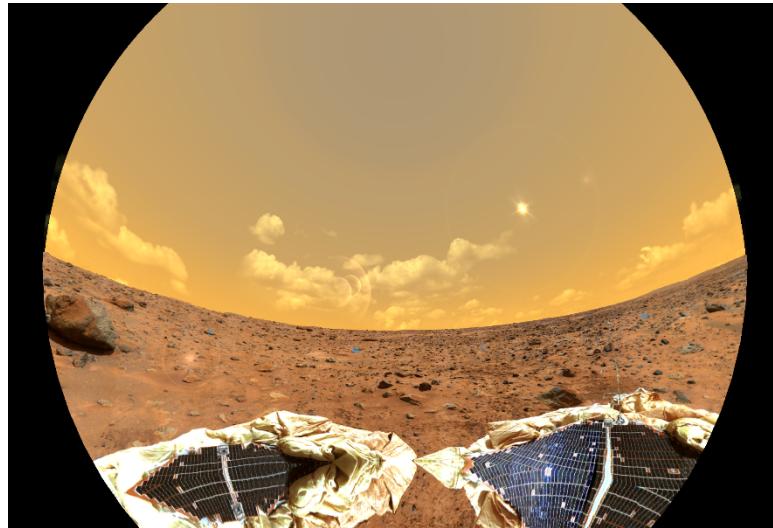
Autonomous navigation in unknown environment

Under sea exploration



[listverse.com]

Space exploration



[Nasa.gov]

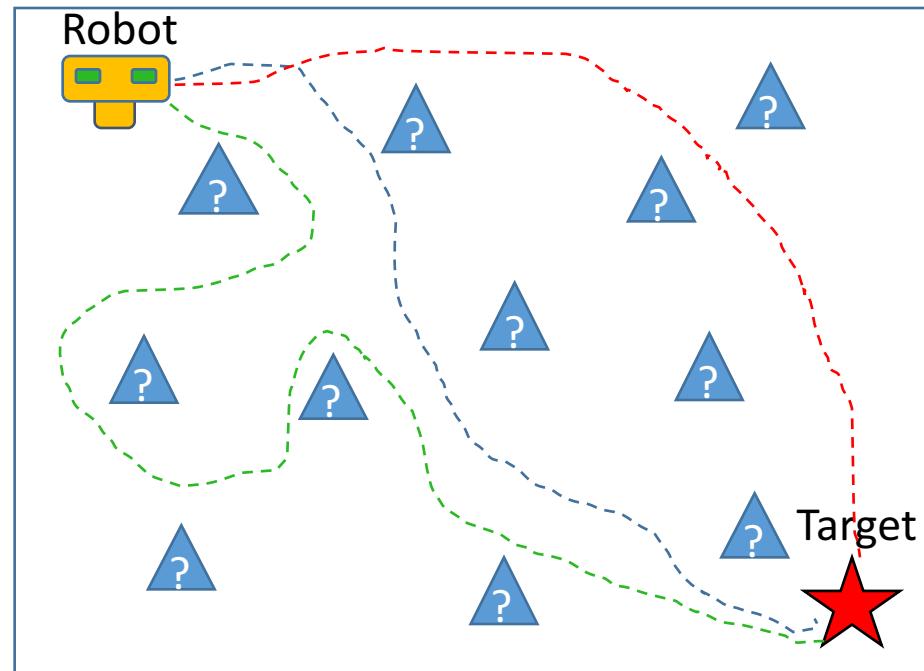
Navigation in GPS-deprived environments



[Nasa.gov]

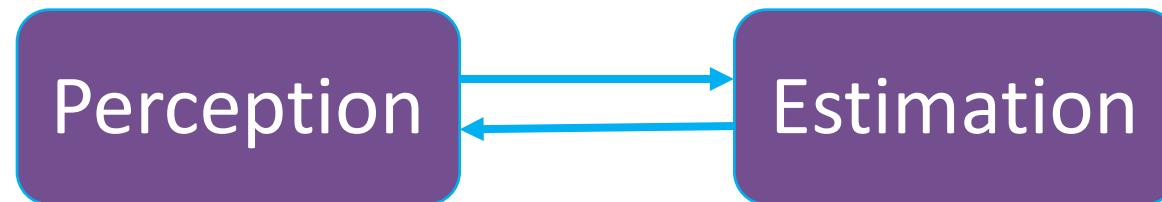
Introduction - Problem

- Autonomous navigation in unknown environment
- Planning a suitable control strategy to accomplish a given task
- Reaching a goal with highest estimation accuracy

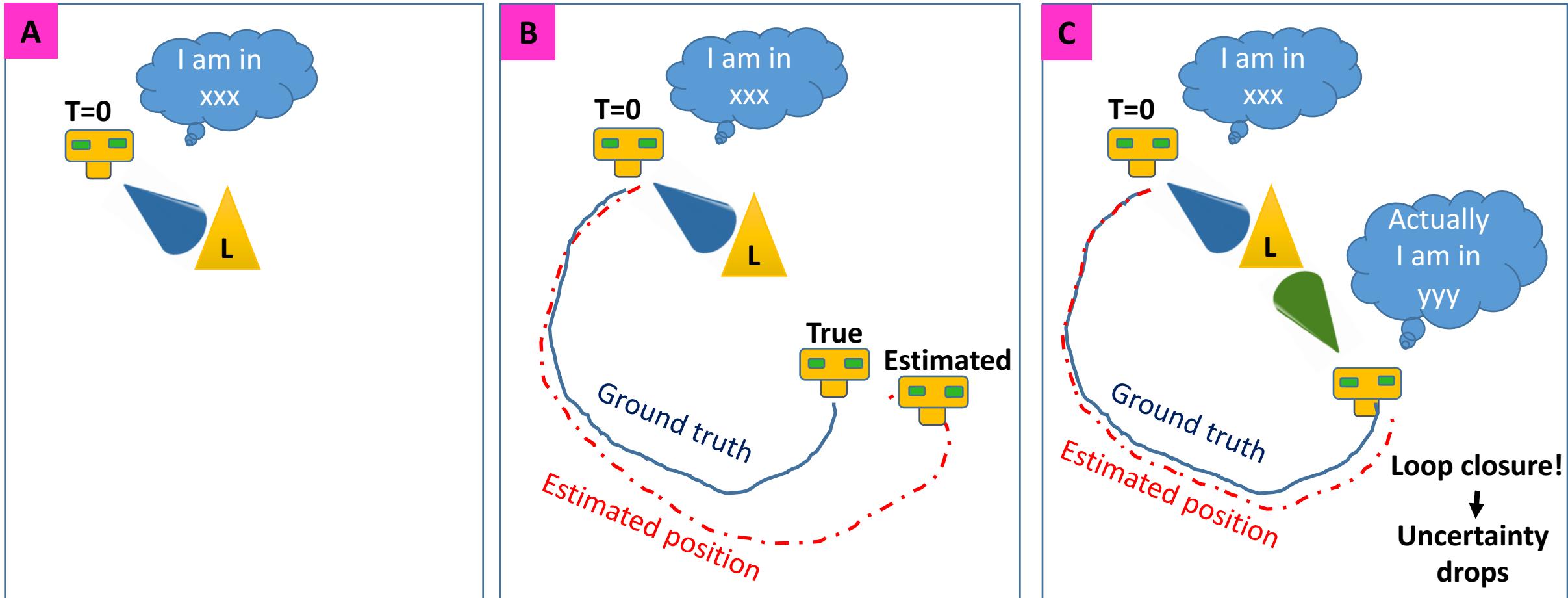


Introduction - SLAM

- **SLAM - simultaneous localization and mapping**
- Based on sensor observations, the robot :
 - Infers its own state
 - Creates a model of the environment

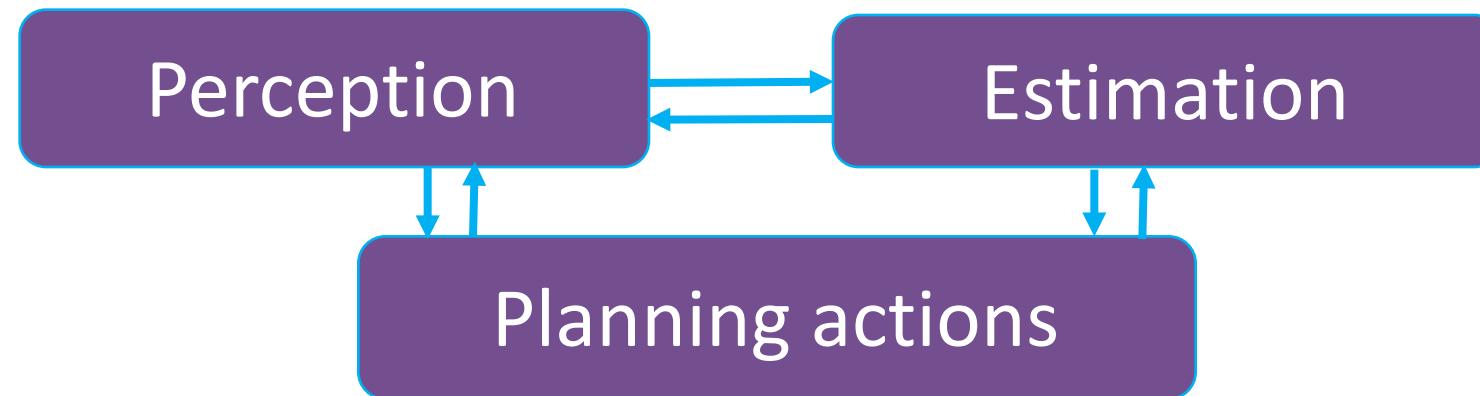


SLAM – Loop closure



Introduction - Belief space planning

Belief space planning (BSP) - Planning actions while taking into account different sources of uncertainty



- Optimizing an objective function, composed for an example by the objectives:
 - Minimum uncertainty
 - Path length
 - Reaching a specific goal

Related Work

- Many approaches assume environment/map is known
- Recent work relaxes this assumption and enables operation in unknown environments
- **BSP approaches typically consider perfect ability to re-identify an object**

In this work we:

- **Enable operation in unknown environments**
- **Not assuming perfect ability to re-identify an object**

How is a landmark being re-identified?

- It can be challenging!
- Depends on: camera viewpoint, sensor capabilities and image processing capabilities
- Different view angles may cause the landmark to look completely different

Same landmark – different view direction

- Looks completely different!
- Challenging to identify even for human



Computer vision algorithms

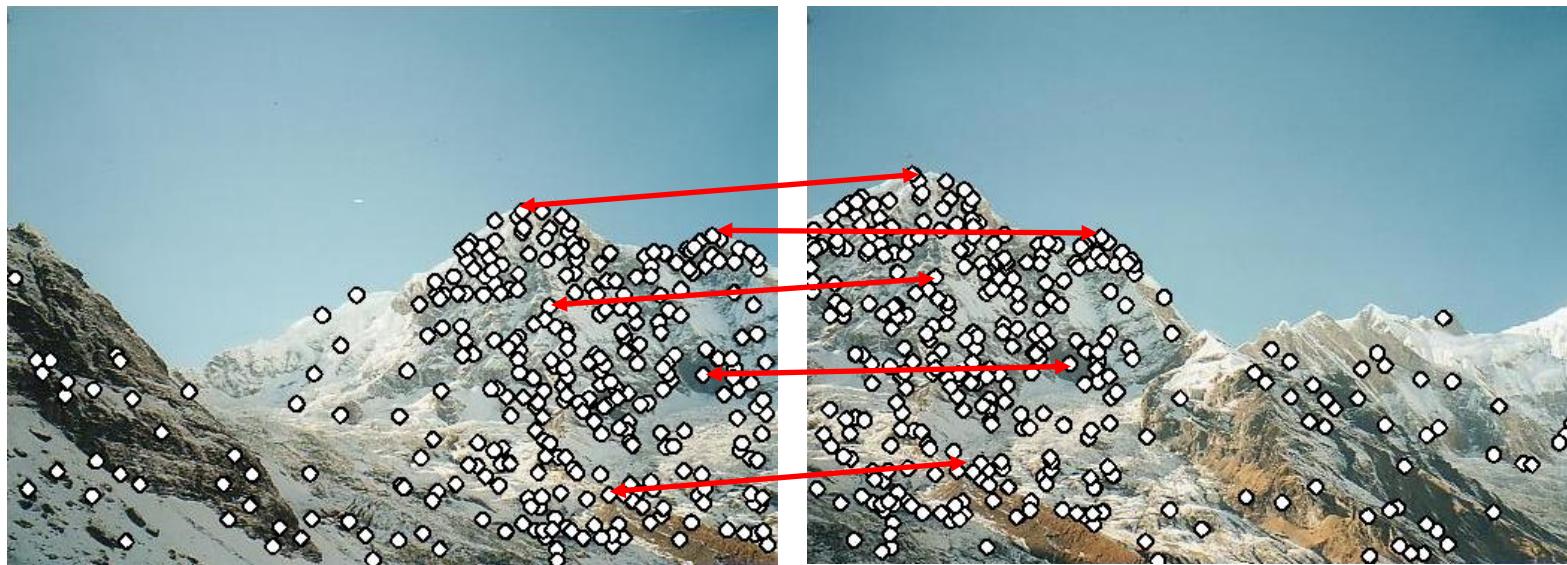
The decision on landmark identification depends on the computer vision algorithm

For an example, SIFT algorithm

Detects features in the image

Compares between features

Determines correspondence between features in two views



Images adapted from Steve Seitz and Rick Szeliski

Computer vision algorithms

- Limited in their identification ability
- Defines the conditions in which two views of the same scene will be identified as same object
- In SIFT algorithm , an object will be identified when viewpoint direction is changing in up to $30^\circ - 40^\circ$

Contribution

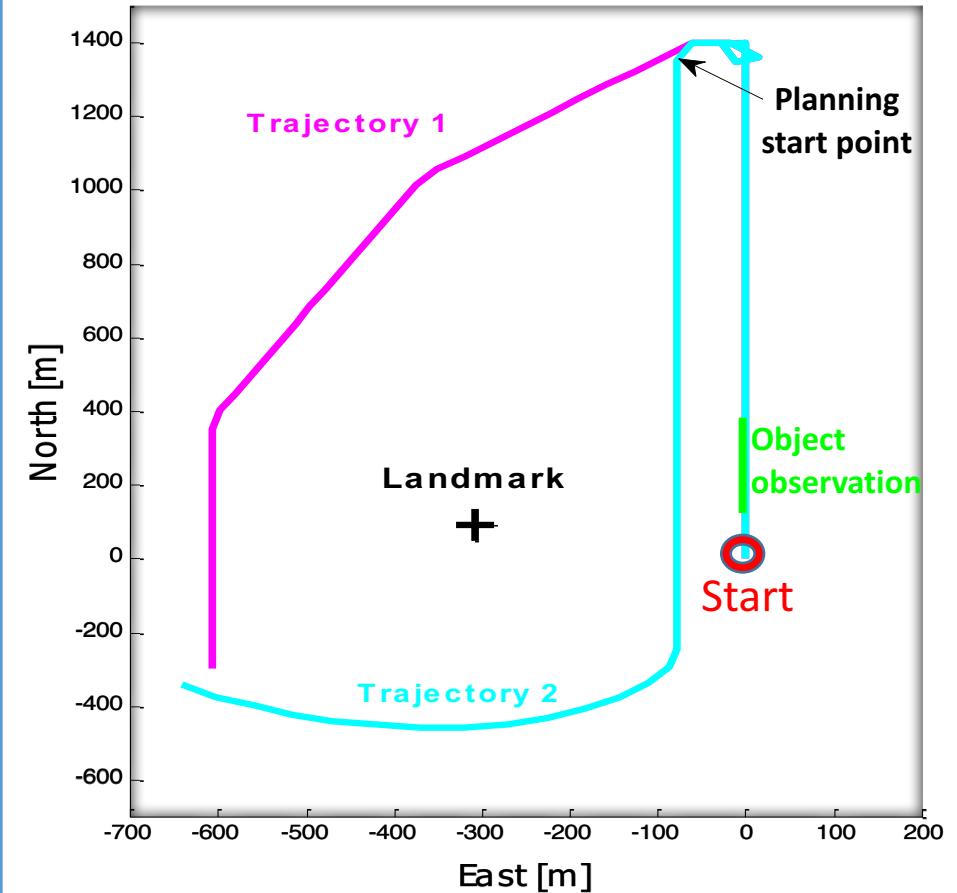
BSP approaches typically consider perfect ability to re-identify an object

inconsistent uncertainty prediction with reality (inference)

Incorrect planning and path choosing

Correct identification of landmarks is critical

Which trajectory is better?



Contribution

Develop a viewpoint aware BSP approach

Modeling object re-identification

Considering both SLAM and Planning aspects

- **Focus on object re-identification from different viewpoint when the object is known**

Concept – Modeling Object Re-Identification

LOS (Line of sight) = Straight line between the robot's camera and observed scene

We define Cone of identification

- In it, the landmark can be identified using image processing algorithms

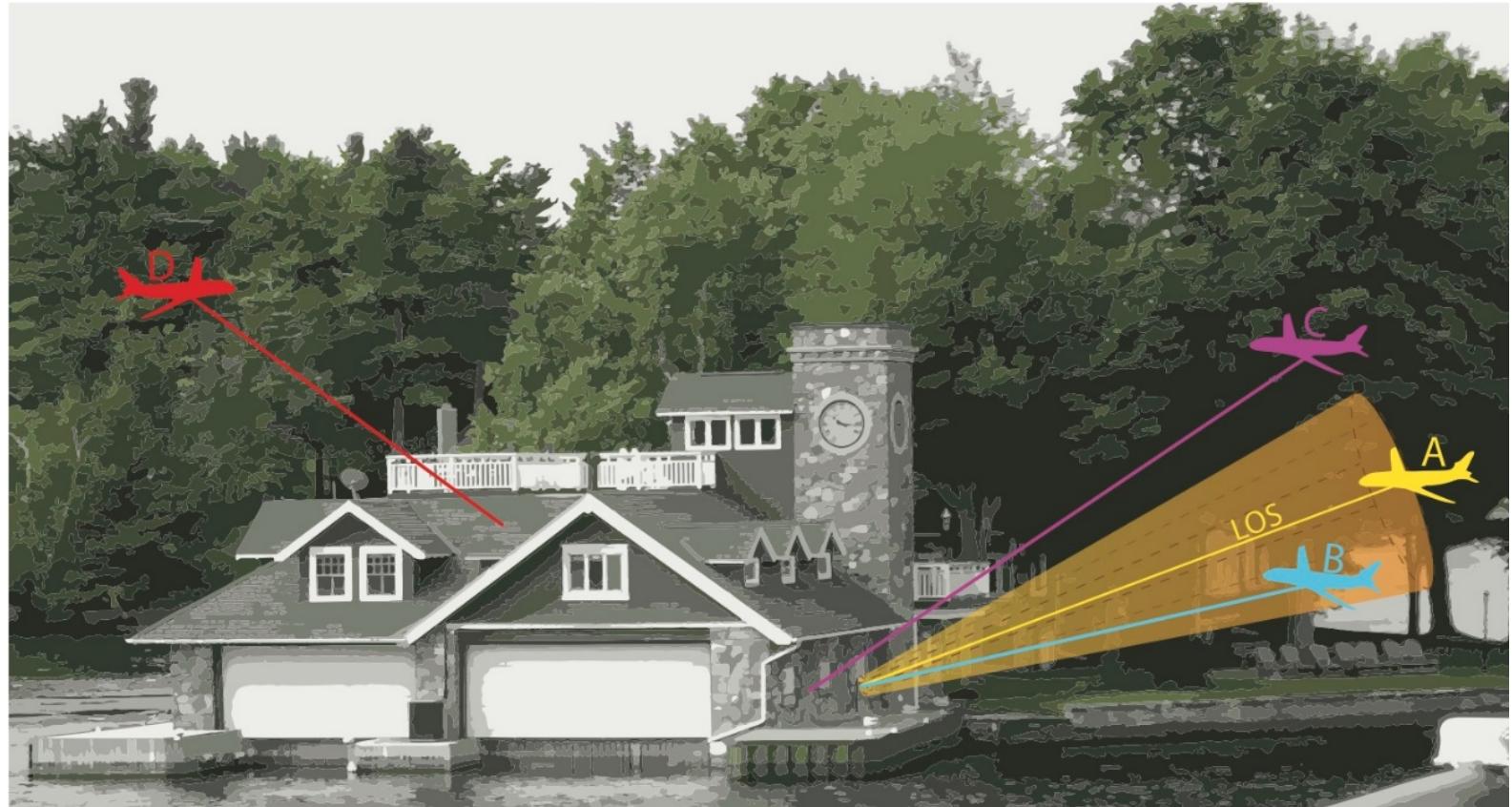
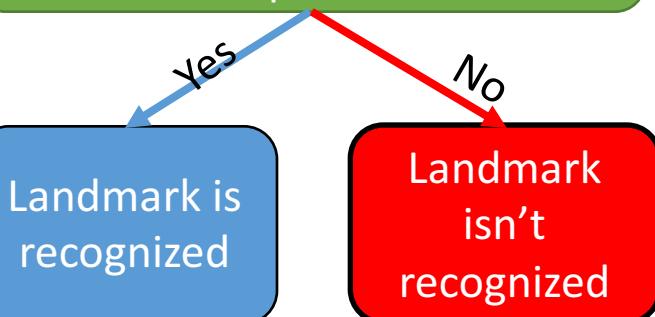


Concept – Modeling Object Re-Identification

Preserving all LOS from the past
→ LOS are calculated using information from estimation

Calculating LOS for a future view point

Check if current LOS is inside a cone of identification from the past



Formulation -SLAM

x_i - Robot state at time i

u_i - Control action applied at time i

$z_{i,j}$ -measurement of the jth landmark at time i

l_j – Coordinates of landmark j

Notations

- The motion model is :

$$x_{i+1} = f(x_i, u_i) + w_i \quad w_i : N(0, \Sigma_w) \quad p(x_{i+1} | x_i, u_i)$$

- The observation model is :

$$z_{i,j} = h(x_i, l_j) + v_{i,j} \quad v_{i,j} : N(0, \Sigma_v) \quad p(z_{i,j} | x_i, l_j)$$

Formulation -SLAM

X_k – All robot and world states until time k
 Z_k – All available observations at time k
 u_k – Control action at time k

Notations

- The problem to be solved in the SLAM part:

$$p(X_k | Z_{0:k}, u_{0:k-1})$$

Joint state vector $X_k \in \{x_0, \dots, x_k, L_k\}$

Past & current Mapped
robot states environment

We use maximum a posteriori (MAP) estimation in order to estimate X_k^*

$$p(X_k | Z_{0:k}, u_{0:k-1}) \sim N(X_k^*, \Sigma_k) \quad X_k^* = \arg \max_{X_k} (p(X_k | Z_{0:k}, u_{0:k-1}))$$

Formulation - SLAM

X_k – All robot and world states until time k
 Z_k – All available observations at time k
 u_k – Control action at time k
 n_i – Number of observations at time i
 l_j – Coordinates of landmark j [Notations](#)

- Mathematical development will lead to:

$$b(X_k) \propto p(X_k | Z_{0:k}, u_{0:k-1}) = priors \cdot \prod_{i=1}^k \left[\frac{p(x_i | x_{i-1}, u_{i-1})}{\text{Motion model}} \prod_{j=1}^{n_i} \frac{p(z_{i,j} | x_i, l_j)}{\text{Measurement model}} \right]$$

Data Association and landmark identification

Formulation

Belief Space Planning

X_k – All robot and world states until time k
 Z_k – All available observations at time k
 u_k – Control action at time k
 l_j – Coordinates of landmark j

[Notations](#)

$$b(X_{k+l}) \propto p(\underline{X_{k+l}} \mid \underline{Z_{0:k}, u_{0:k-1}}, \underline{Z_{k+1:k+l}, u_{k:k+l-1}})$$

Joint state at the k -th look ahead step Past controls & measurements Controls & measurements at the first l look-ahead steps

- This belief is represented by a Gaussian:

$$b(X_{k+l}) : N\left(X_{k+l}^*, \Sigma_{k+l}\right)$$

Formulation Belief Space Planning

X_k – All robot and world states until time k
 Z_k – All available observations at time k
 u_k – Control action at time k
 L – Number of planning steps

[Notations](#)

We want to find the planning actions
Optimizing an objective function:

$$J(u_{k:k+L-1}) @ \underset{Z_{k+1:k+L}}{\text{E}} \left\{ \sum_{l=0}^{L-1} c_l \left(b(X_{k+l}), u_{k+l} \right) \right\}$$

Expectation on all future unknown measurements $Z_{k+1:k+L}$

Sum of all L future planning steps

General cost function depends on the belief $b(X_{k+l})$ and on the control action u_{k+l}

- Composed, for example, by:
- minimum uncertainty
 - path length
 - reaching a specific goal

Formulation Belief Space Planning

X_k – All robot ($x_{1:k}$) until time k and world states ($l_{1:j}$)
 Z_k – All available observations at time k
 u_k – Control action at time k
 l_j – Coordinates of landmark j
 n_i – Number of observations at time i
 $H_k \triangleq \{Z_{0:k}, u_{0:k-1}\}$ Past measurements and controls

Notations

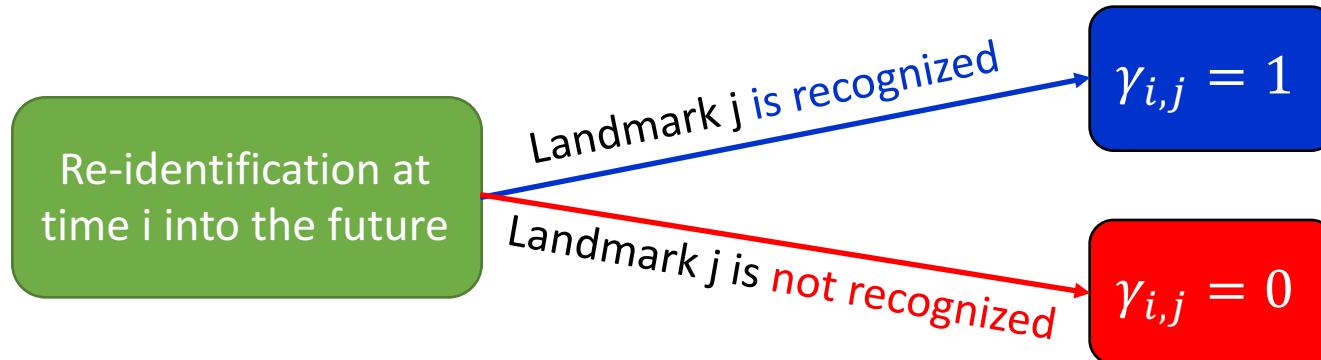
Existing BSP approaches are solving the problem while considering ideal data association and ideal ability of object re-identification

In this case, develop of the belief space leads to:

$$p(X_{k+l} | H_{k+l}) \propto p(X_k | H_k) \cdot \underbrace{\prod_{i=1}^l p(x_{k+i} | x_{k+i-1}, u_{k+i-1})}_{\text{Inference until planning time k (SLAM)}} \cdot \underbrace{\prod_{j=1}^{n_i} p(z_{k+i,j} | x_{k+i}, l_j)}_{\substack{\text{Motion model for future states from planning time k} \\ \text{Measurement model for future measurements from planning time k} \\ \text{Assuming ideal data association}}}$$

Formulation Belief Space Planning

In reality – re-identification is not perfect → Define a binary random variable $\gamma_{i,j}$



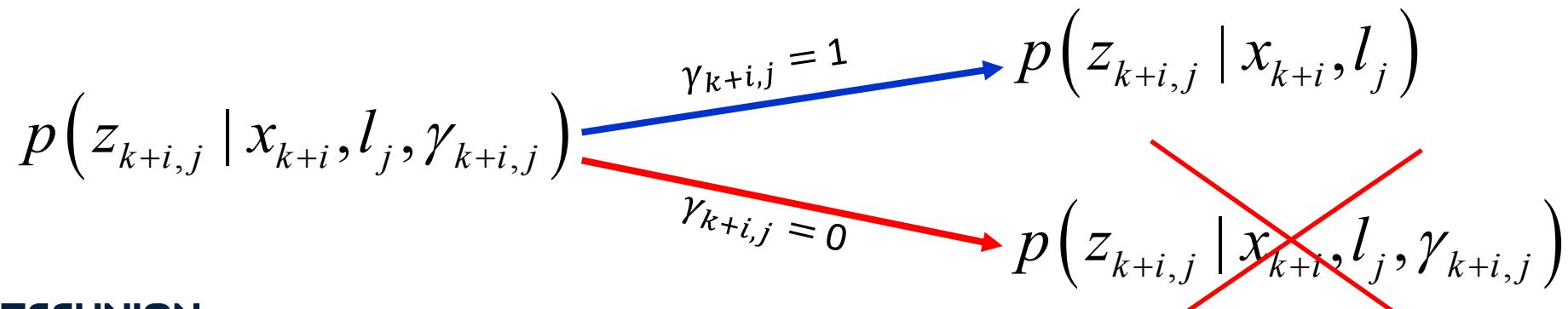
$$\Gamma_i \models \left\{ \gamma_{i,j} \right\}_{j=1}^{n_i} \quad n_i \text{ is the number of possible observations at time i}$$

Formulation Belief Space Planning

X_k	All robot ($x_{1:k}$) until time k and world states ($l_{1:j}$)
Z_k	All available observations at time k
u_k	Control action at time k
l_j	Coordinates of landmark j
$H_k \triangleq \{Z_{0:k}, u_{0:k-1}\}$	Past measurements and controls
$\gamma_{i,j}$	Event of acquiring measurement j at time l
$\Gamma_i \triangleq \{\gamma_{i,j}\}_{j=1}^{n_i}$	

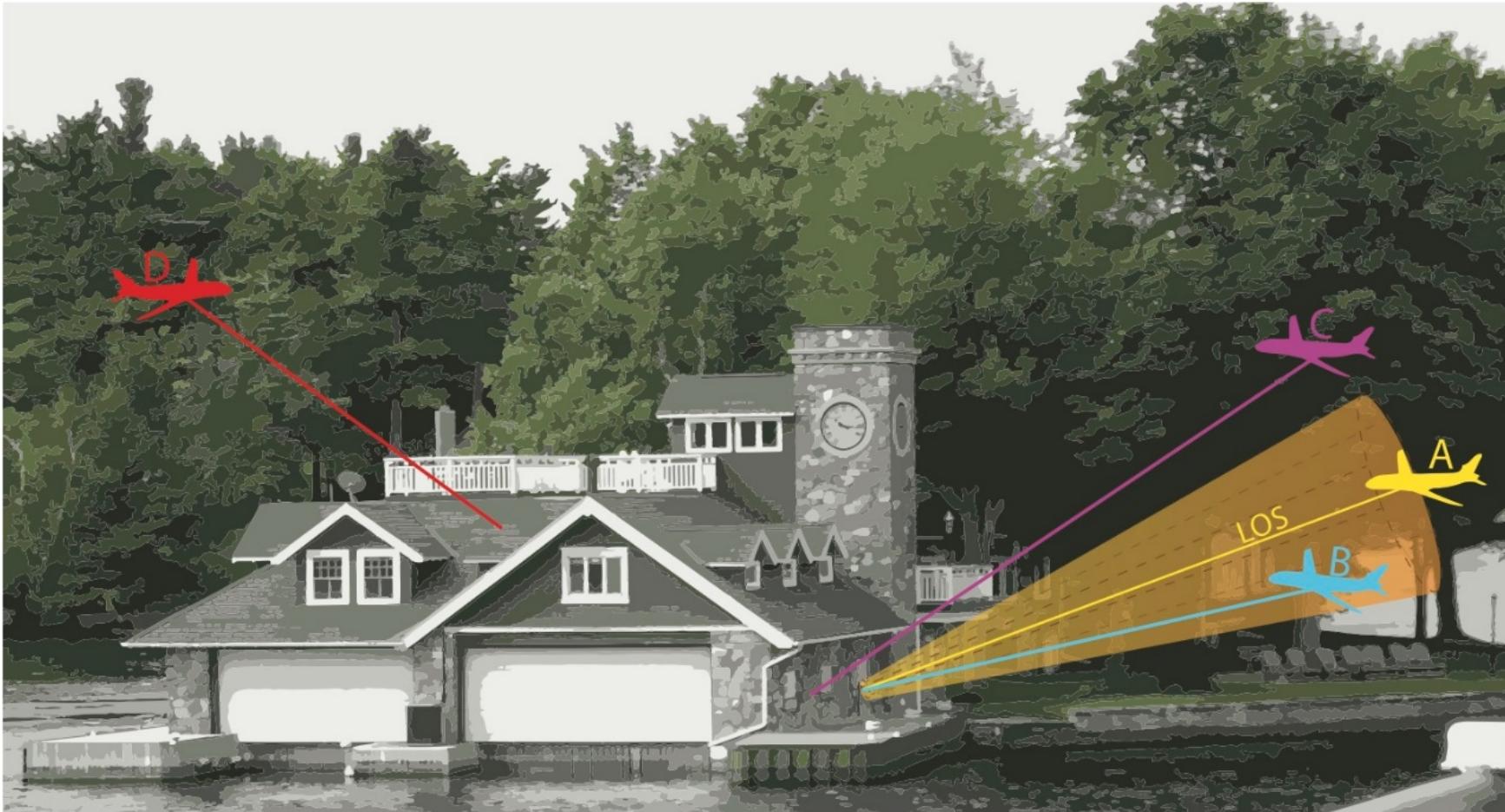
Notations

$$p(X_{k+l}, \Gamma_{k+1:k+l} | H_{k+l}) \propto p(X_k | H_k) \cdot \underbrace{\prod_{i=1}^l p(x_{k+i} | x_{k+i-1}, u_{k+i-1})}_{\text{Inference until planning time k (SLAM)}} \cdot \underbrace{\prod_{j=1}^{n_i} p(z_{k+i,j} | x_{k+i}, l_j, \gamma_{k+i,j})}_{\text{Motion model for future states from planning time k}} p(\gamma_{k+i,j} | H_{k+i-1}, x_{k+i}, l_j) \underbrace{\quad}_{\text{Measurement model for future measurements from planning time k taking into account the event of acquiring a measurement}}$$



Formulation – Belief Space Planning

Recall - Modeling Object Re-Identification

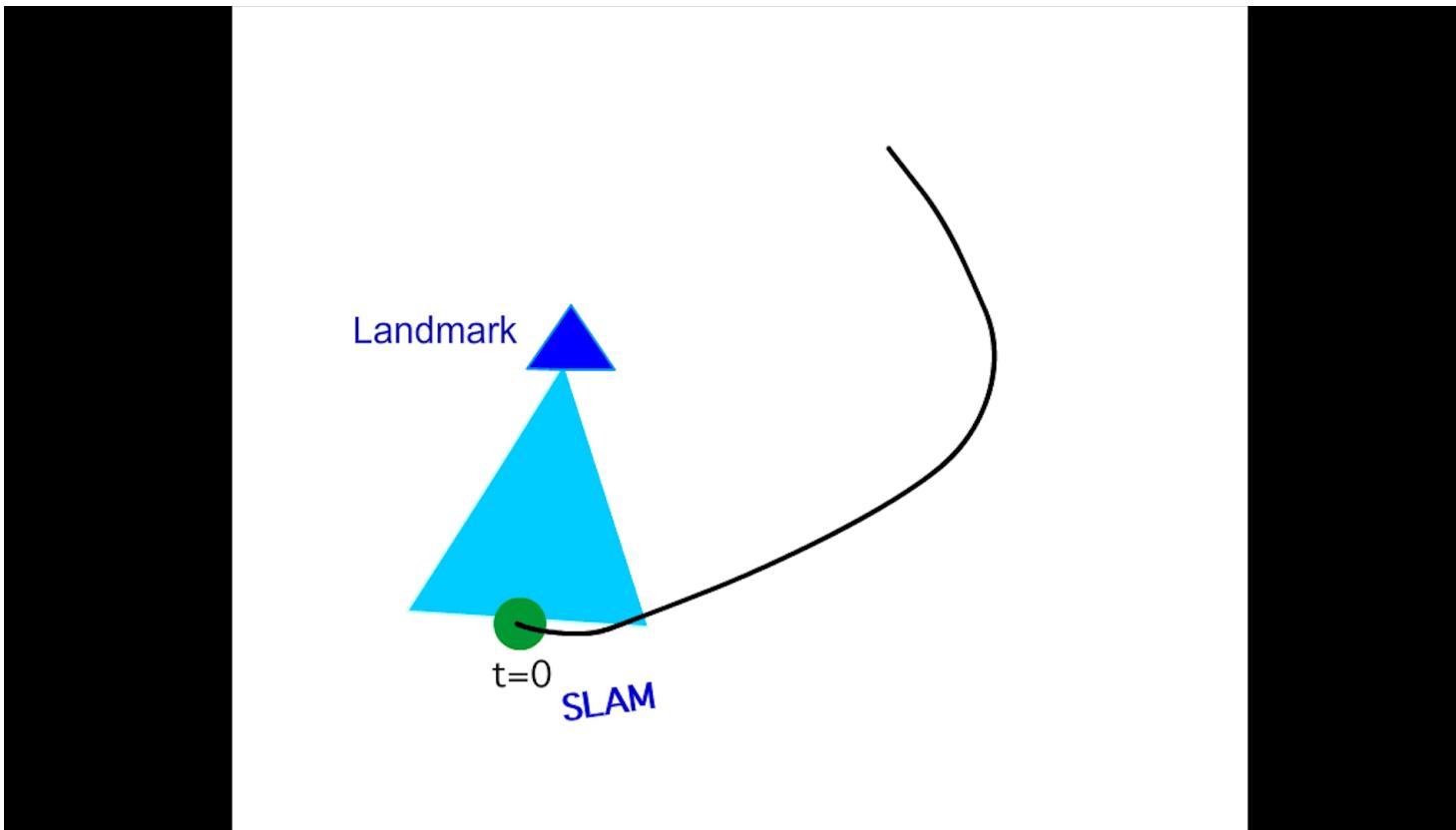


Formulation – Belief Space Planning

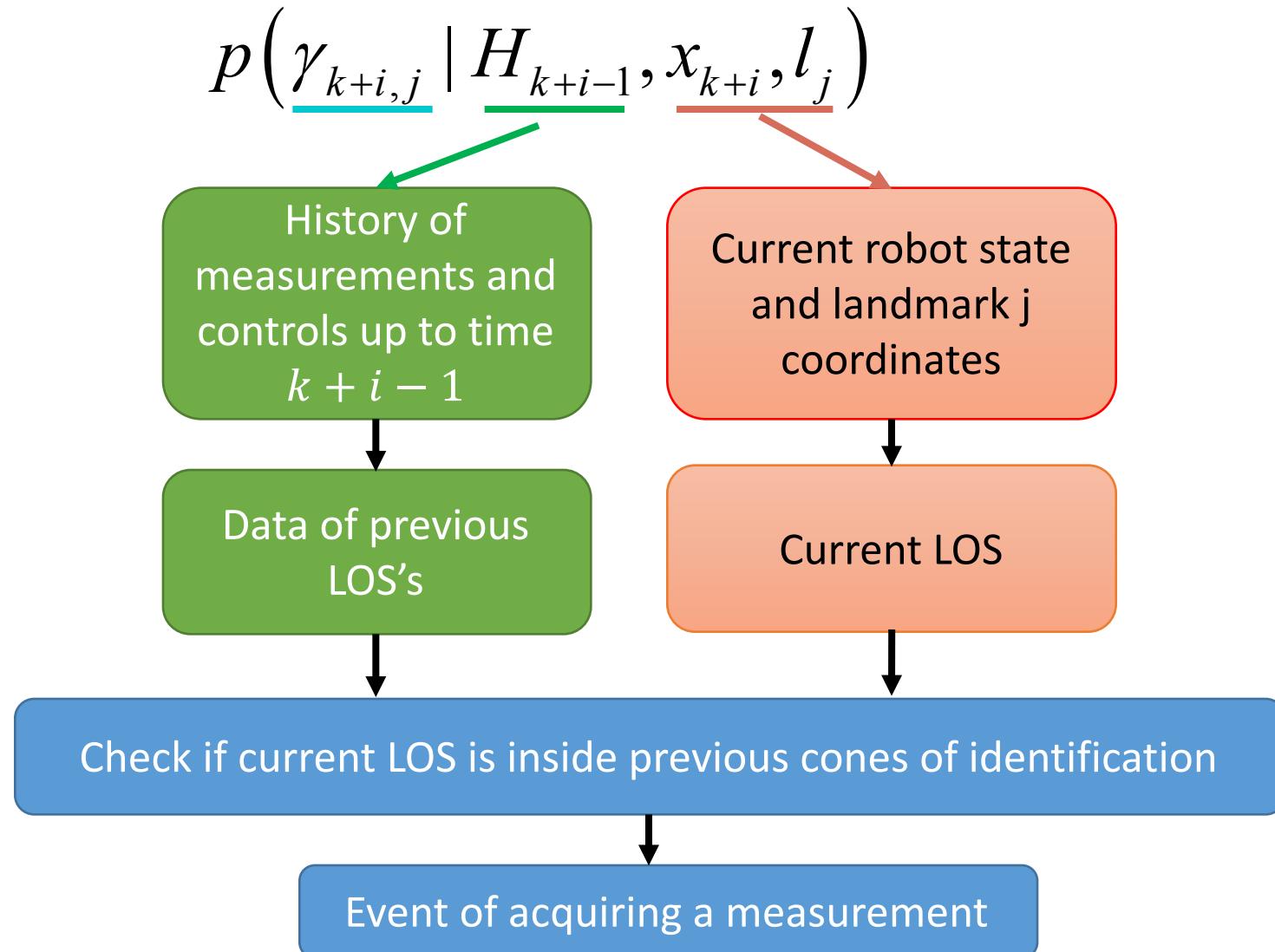
$$p(\gamma_{k+i,j} | H_{k+i-1}, x_{k+i}, l_j)$$

$H_k \triangleq \{Z_{0:k}, u_{0:k-1}\}$ Past measurements and controls

$H_{k+i-1} = \{H_k, u_{k:k+i-2}, z_{k+1:k+i-1}\}$



Formulation – Belief Space Planning



Formulation Belief Space Planning

X_k – All robot ($x_{1:k}$) until time k and world states ($l_{1:j}$)
 $H_k \triangleq \{Z_{0:k}, u_{0:k-1}\}$ Past measurements and controls
 $\gamma_{i,j}$ – Event of acquiring measurement j at time I
 $\Gamma_i \triangleq \{\gamma_{i,j}\}_{j=1}^{n_i}$

Notations

$$p(X_{k+l}, \Gamma_{k+1:k+l} | H_{k+l}) \propto p(X_k | H_k) \cdot \prod_{i=1}^l p(x_{k+i} | x_{k+i-1}, u_{k+i-1}) \cdot \prod_{j=1}^{n_i} p(z_{k+i,j} | x_{k+i}, l_j, \gamma_{k+i,j}) p(\gamma_{k+i,j} | H_{k+i-1}, x_{k+i}, l_j)$$

The event of acquiring a measurement in the future is unknown $\rightarrow \Gamma_i$ is a random variable

Joint probability function:

$$p(X_{k+l}, \Gamma_{k+1:k+l} | H_{k+l})$$

Formulation Belief Space Planning

X_k – All robot ($x_{1:k}$) until time k and world states ($l_{1:j}$)
 $H_k \triangleq \{Z_{0:k}, u_{0:k-1}\}$ Past measurements and controls
 $\gamma_{i,j}$ – Event of acquiring measurement j at time l
 $\Gamma_i \triangleq \{\gamma_{i,j}\}_{j=1}^{n_i}$

Notations

Recall, in order to calculate the objective function J, we are using the belief $b(X_{k+l})$:

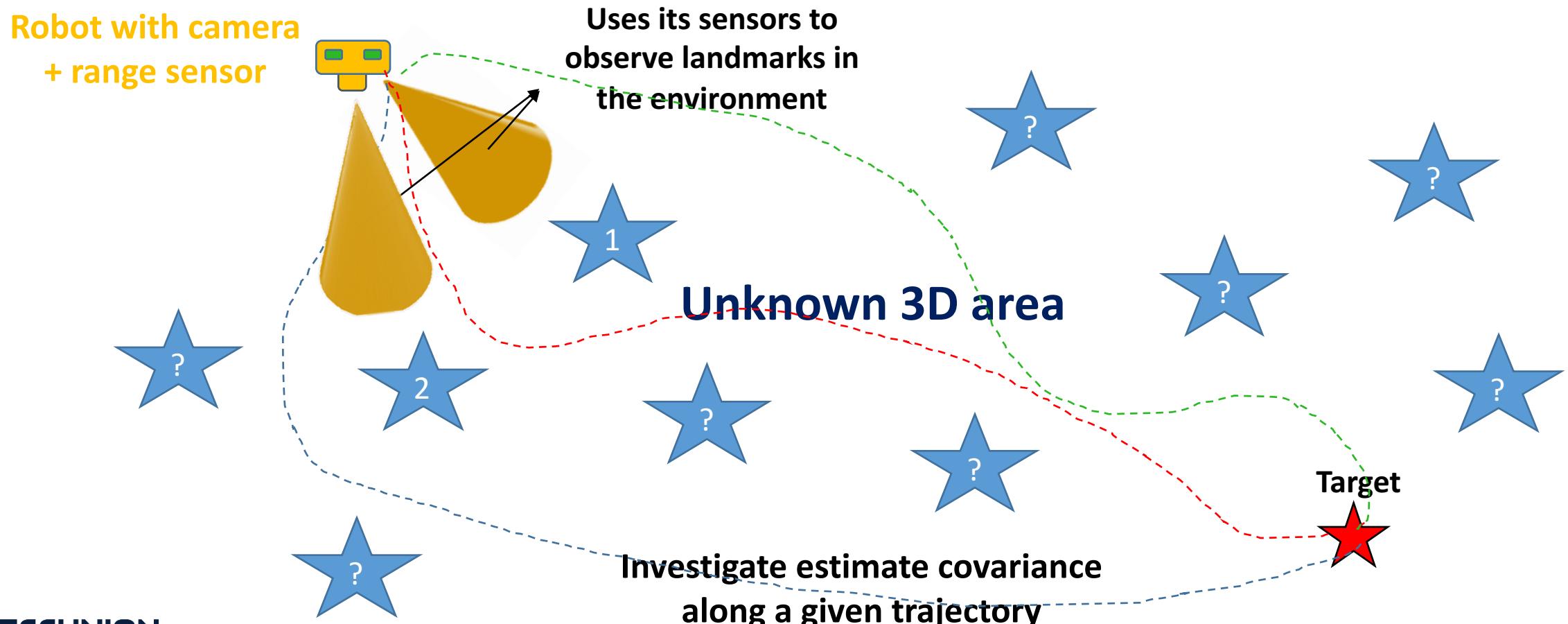
$$J(u_{k:k+L-1}) @_E \left\{ \sum_{l=0}^{L-1} c_l \left(b(X_{k+l}), u_{k+l} \right) \right\}$$

Therefore we do Marginalization:

$$b(X_{k+l}) = p(X_{k+l} | H_{k+l}) = \sum_{\Gamma_{k+1:k+l}} p(X_{k+l}, \Gamma_{k+1:k+l} | H_{k+l})$$

Results – Simulation overview

Using a simulation to check the influence of modeling object re-identification

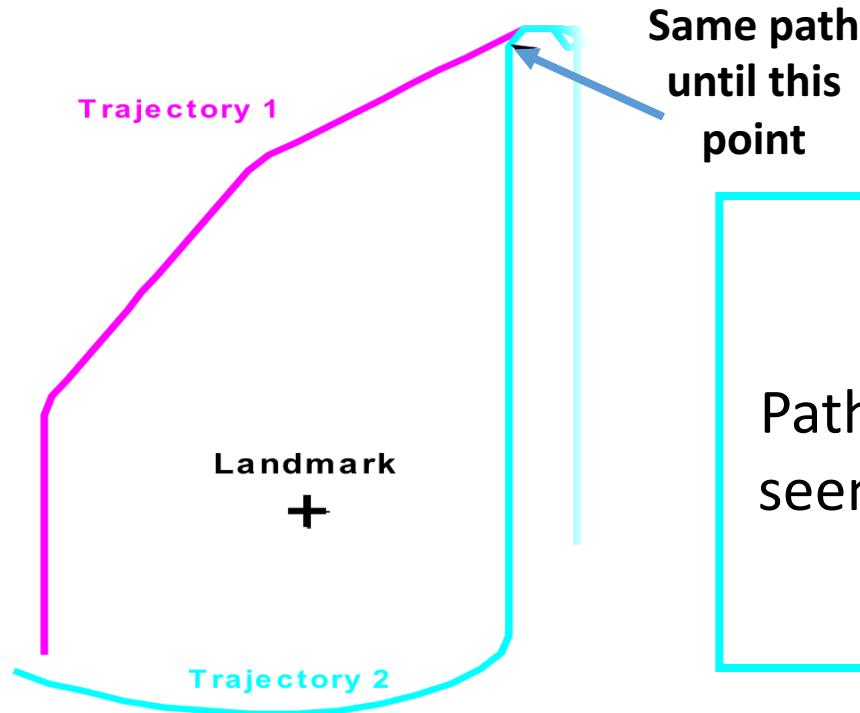


Results – problem definition

- Checking two predefined trajectories that differ in:
 - Landmark's view directions
 - Trajectory length

The objectives are:
Minimum uncertainty
Path length
Reaching a specific target

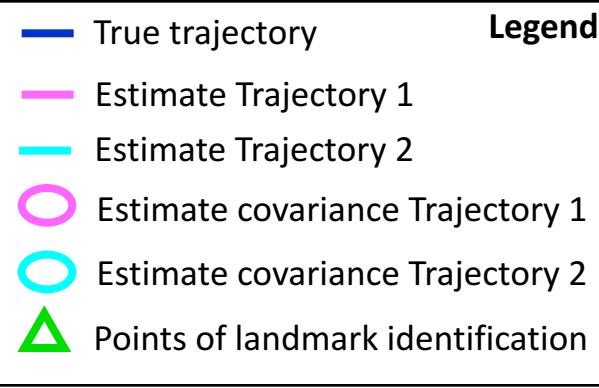
Trajectory 1:
Shorter,
Path includes new
landmark's view point



Trajectory 2:
Longer,
Path includes already
seen landmark's view
point

Results -SLAM

Using only SLAM , Represents true results in real world

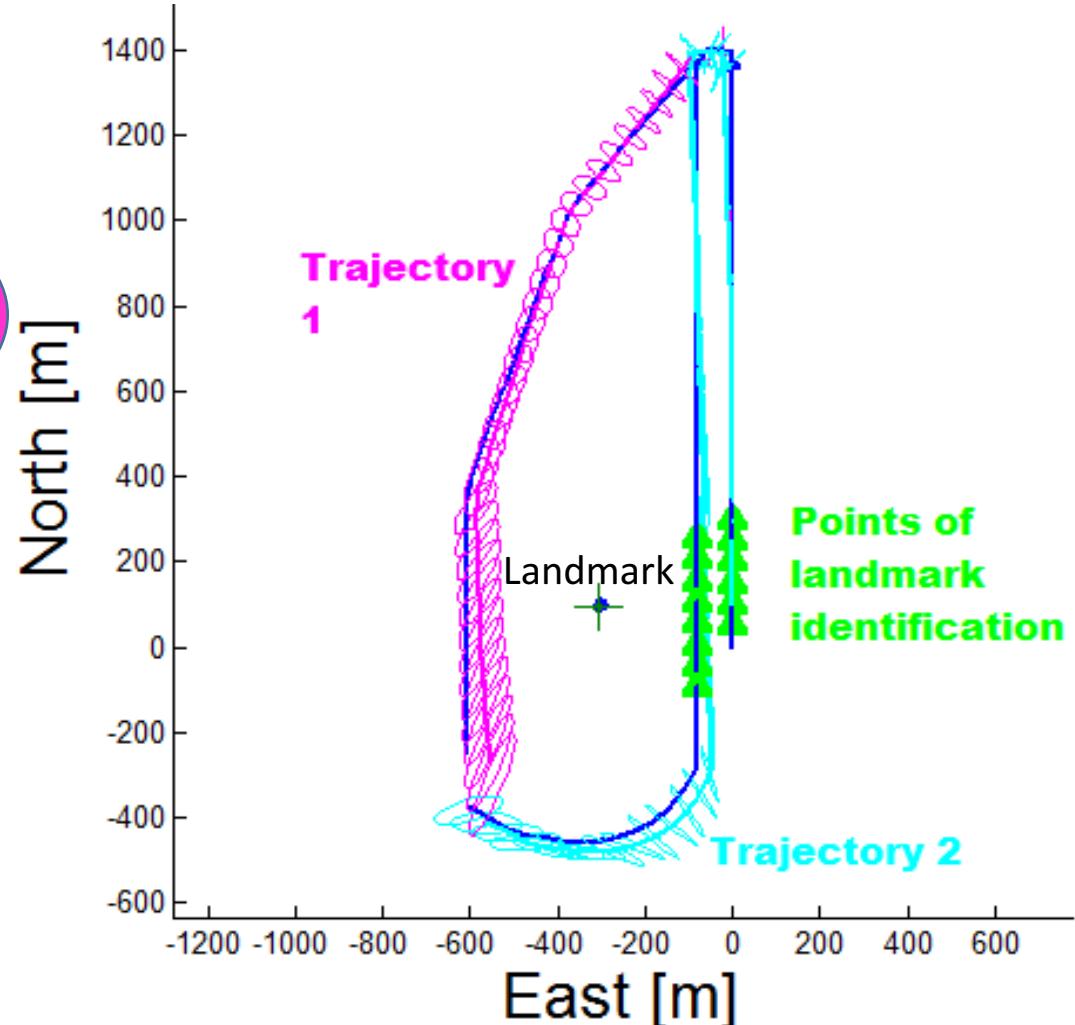


Trajectory 1

Completely different view directions of the landmark

Landmark isn't identified

Estimate covariance keep growing



Trajectory 2

Similar view directions of the landmark

Landmark is identified

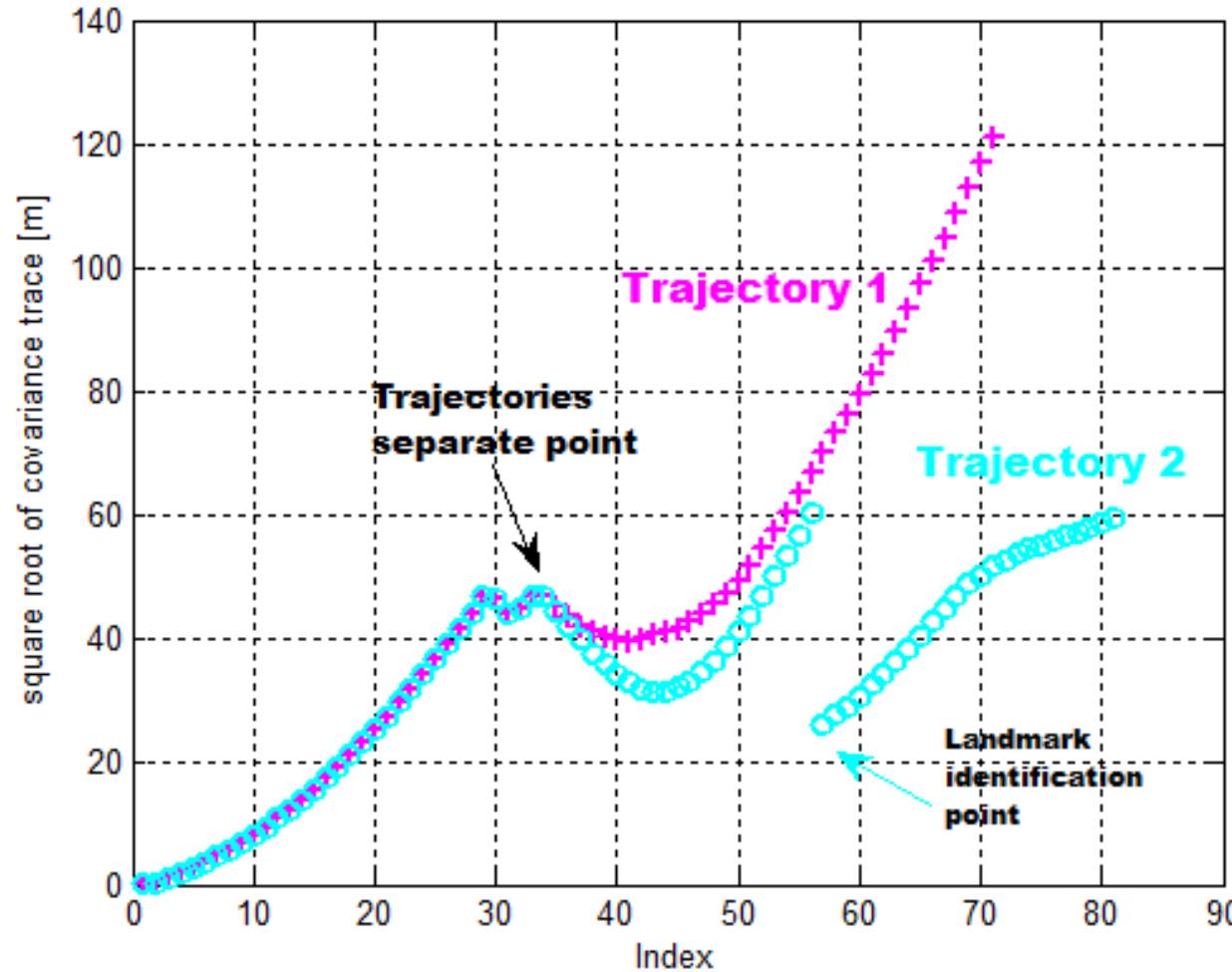
Estimate covariance drops at identification point

Results -SLAM

Trajectory 1

Landmark isn't identified

Estimate covariance keep growing



Trajectory 2

Landmark is identified

Estimate covariance drops at identification point

Trajectory 2 has lower estimate covariance though it is longer → preferred

Results - Planning Without applying object identification

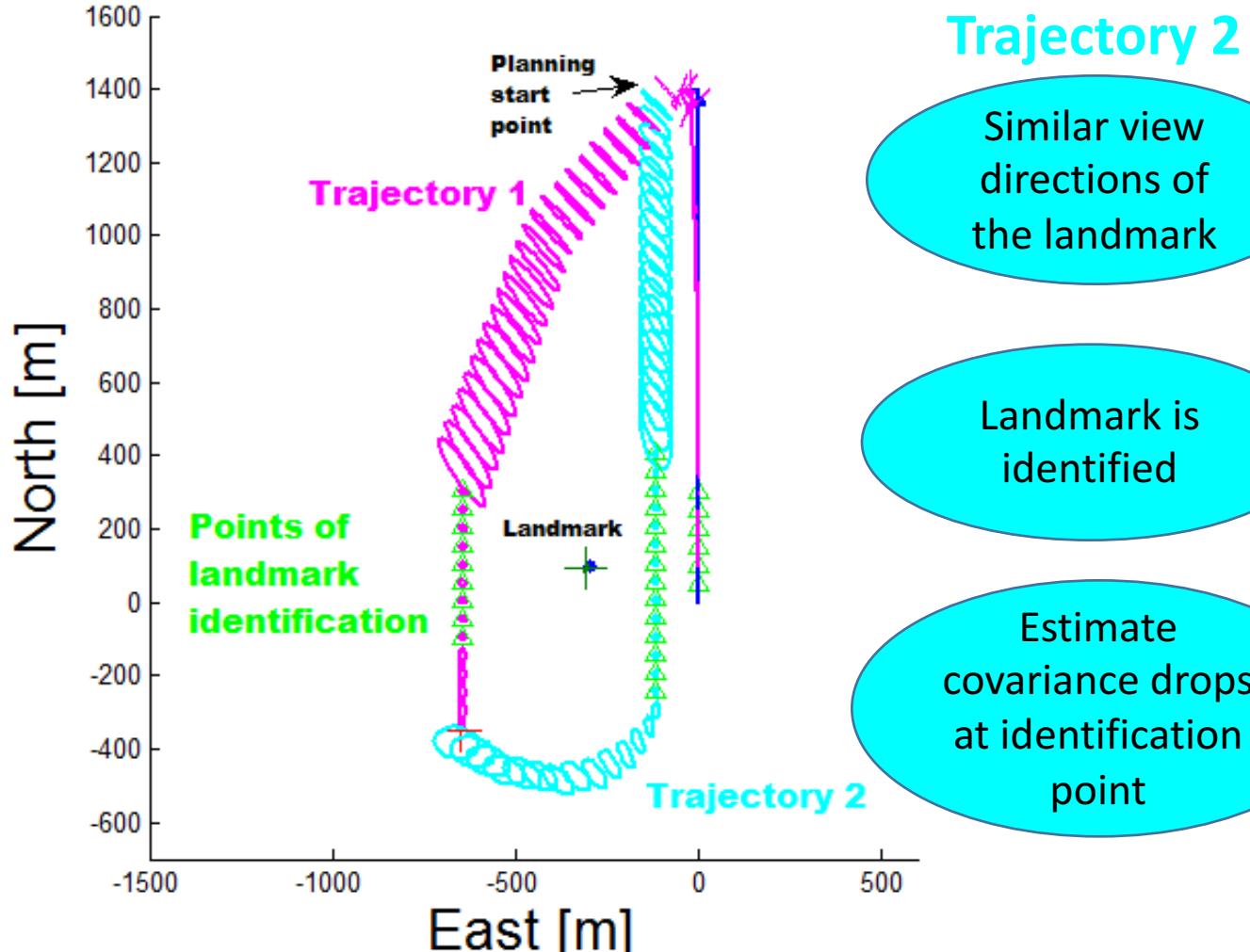
- Legend
- True trajectory
 - Estimate Trajectory 1
 - Estimate Trajectory 2
 - Estimate covariance Trajectory 1
 - Estimate covariance Trajectory 2
 - Points of landmark identification

Trajectory 1

New view directions of the landmark

Landmark is incorrectly identified
(Differently than SLAM)

Estimate covariance incorrectly drops



Trajectory 2

Similar view directions of the landmark

Landmark is identified

Estimate covariance drops at identification point

Results - Planning Without applying object identification

Trajectory 1

Landmark is
incorrectly
identified

covariance
incorrectly drops
at identification
point

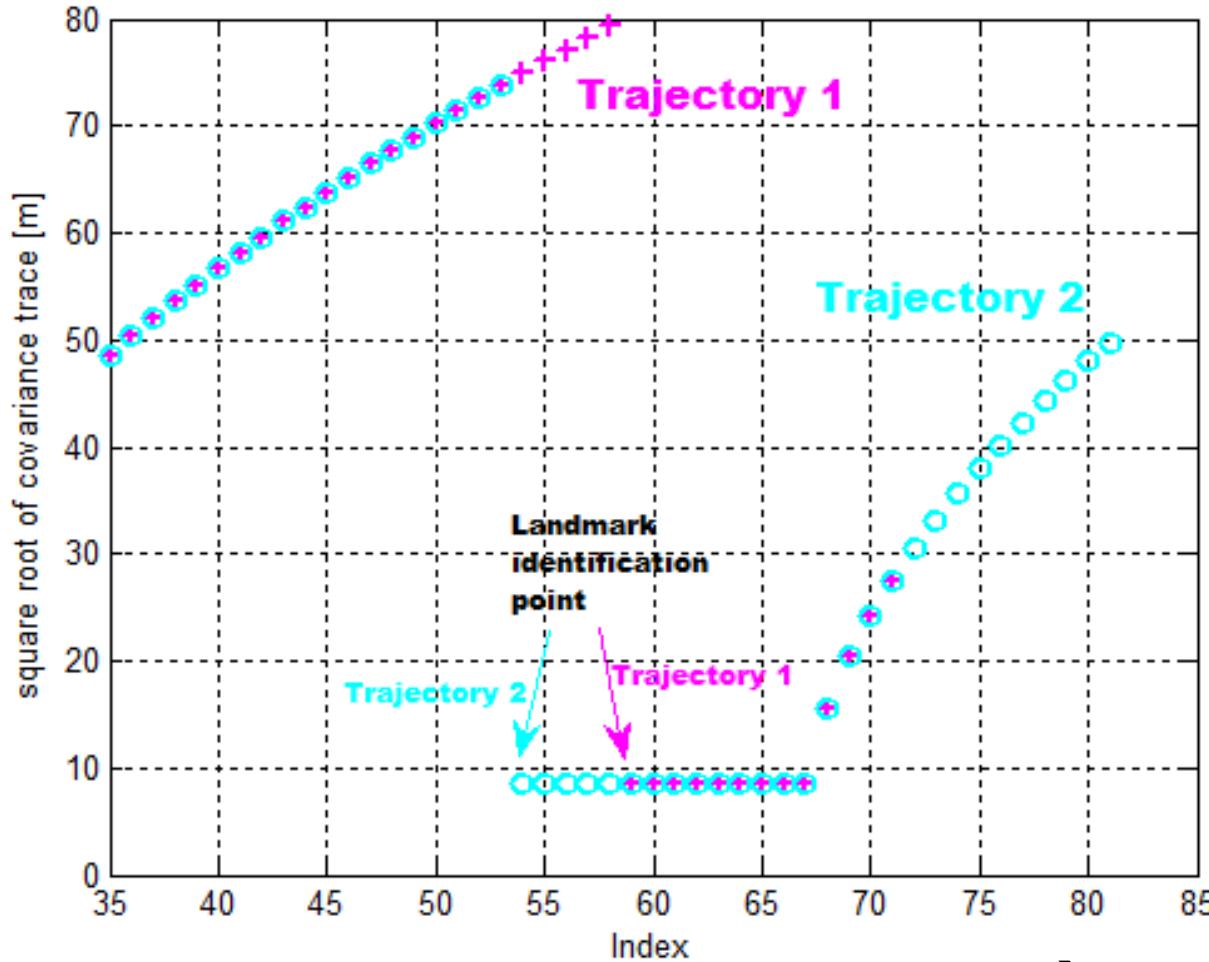


Inconsistent with SLAM

Trajectory 2

Landmark is
identified

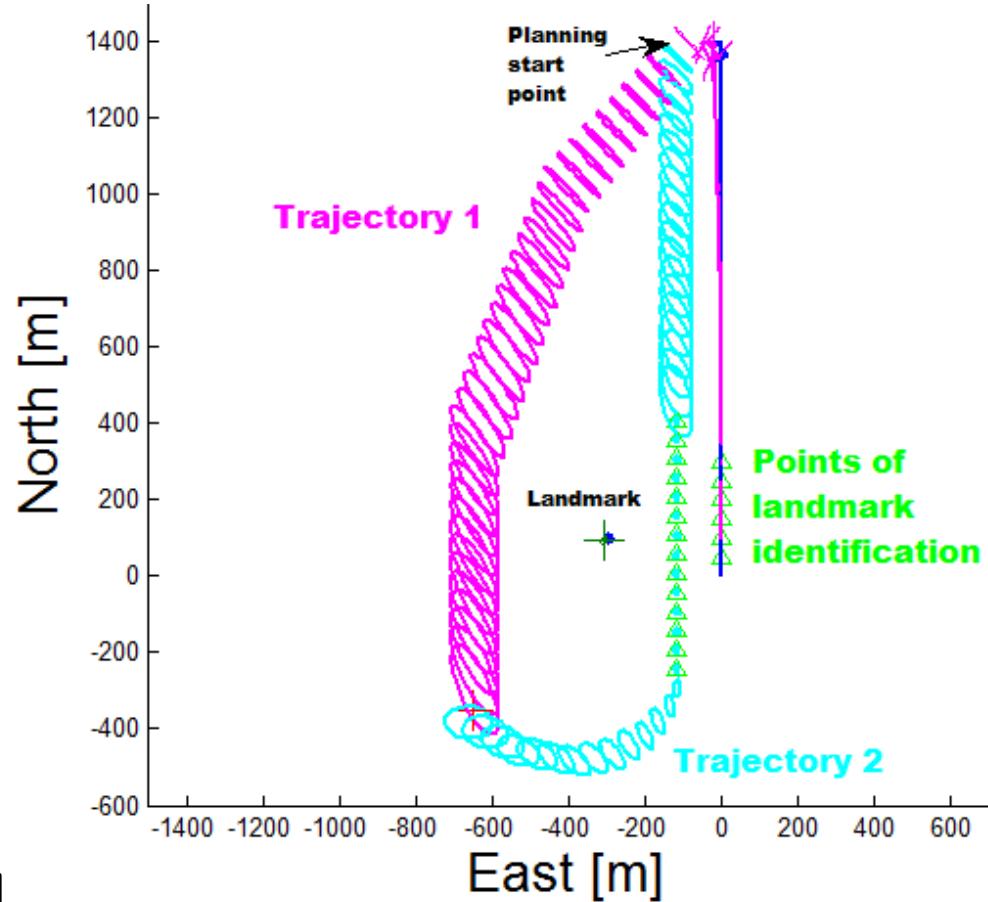
Estimate
covariance drops
at identification
point



Trajectory 1 has lower estimate covariance → incorrectly preferred

Results - Planning With applying object identification

- Trajectory 1**
- New view directions of the landmark
 - Landmark isn't identified
 - Estimate covariance keep growing
- ↓
- Consistent with SLAM**



- Trajectory 2**
- Similar view directions of the landmark
 - Landmark is identified
 - Estimate covariance drops at identification point

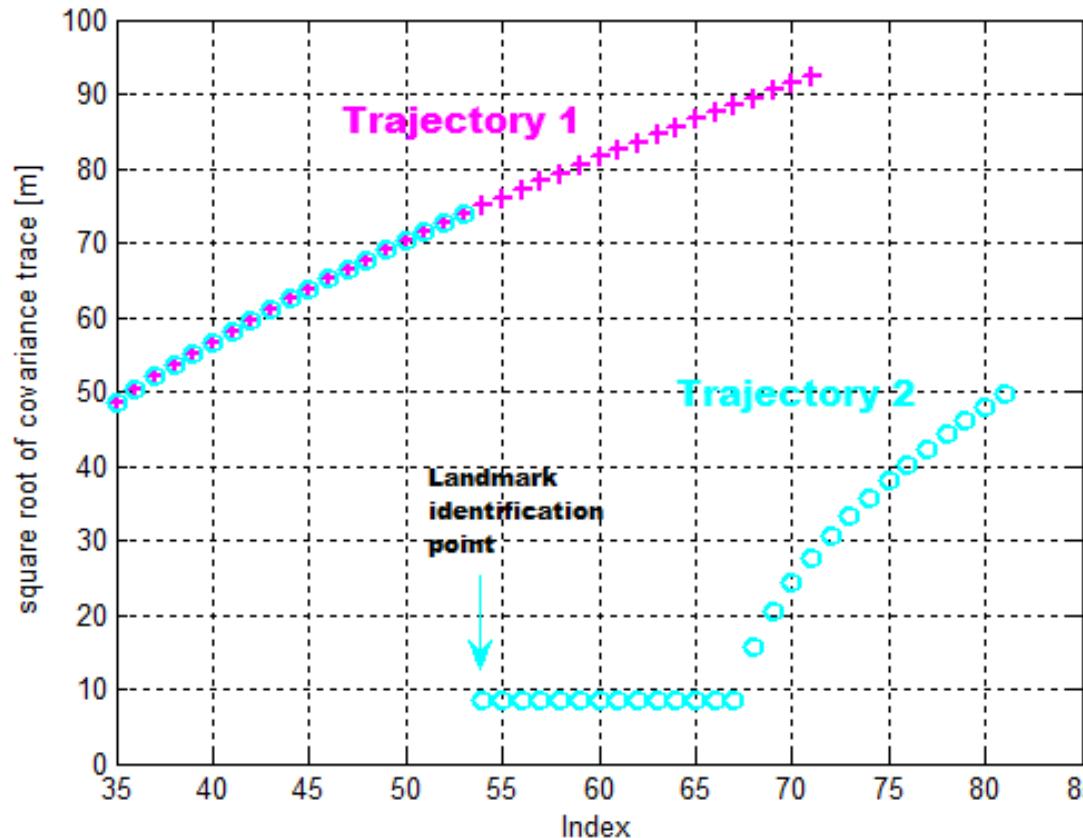
Results - Planning With applying object identification

Trajectory 1

Landmark isn't identified

Estimate covariance keep growing

Consistent with SLAM



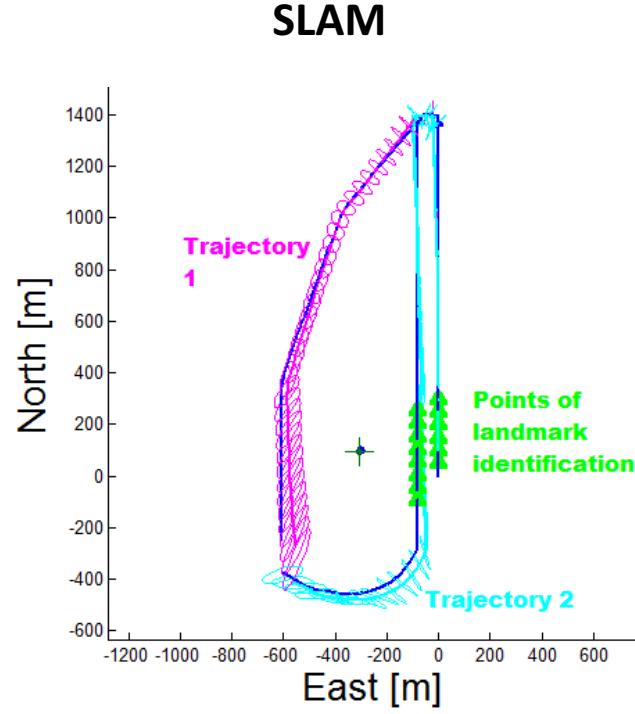
Trajectory 2

Landmark is identified

Estimate covariance drops at identification point

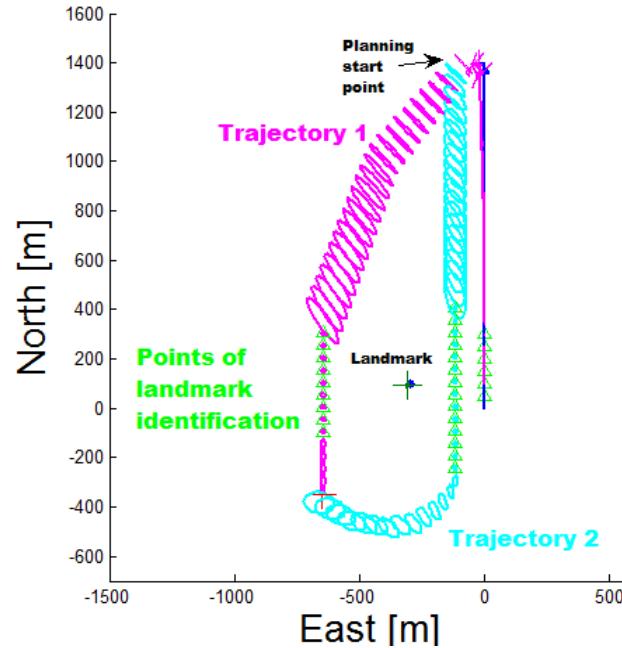
Trajectory 2 has lower estimate covariance though it is longer → preferred
Consistent with SLAM

Results - summary



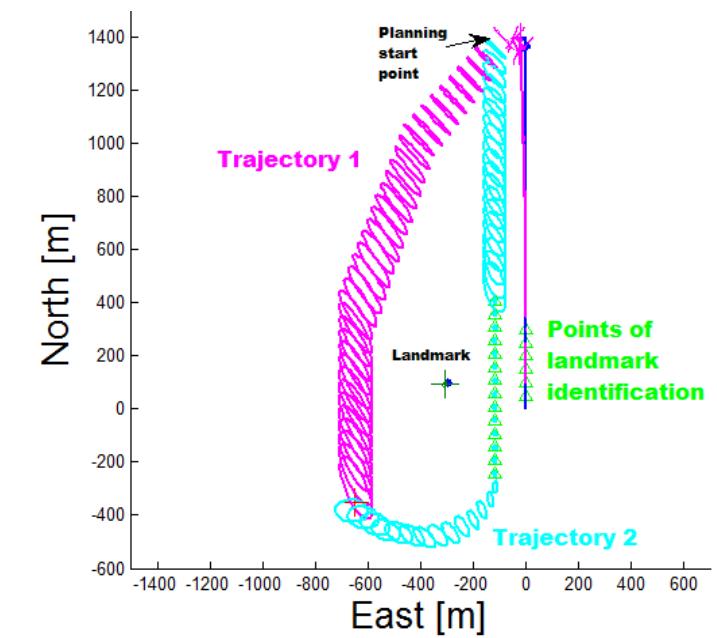
In reality – the landmark is re-identified only in trajectory 2

Planning - Without applying object identification



When not applying object identification – the landmark is re-identified incorrectly also in trajectory 1

Planning – With applying object identification



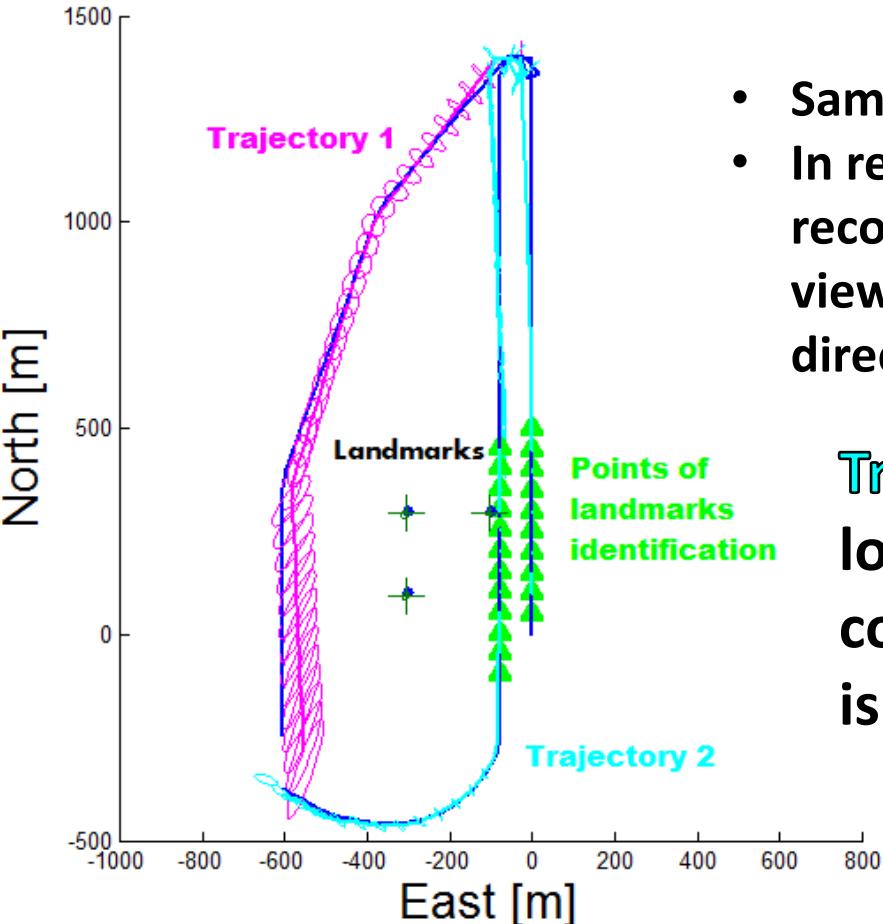
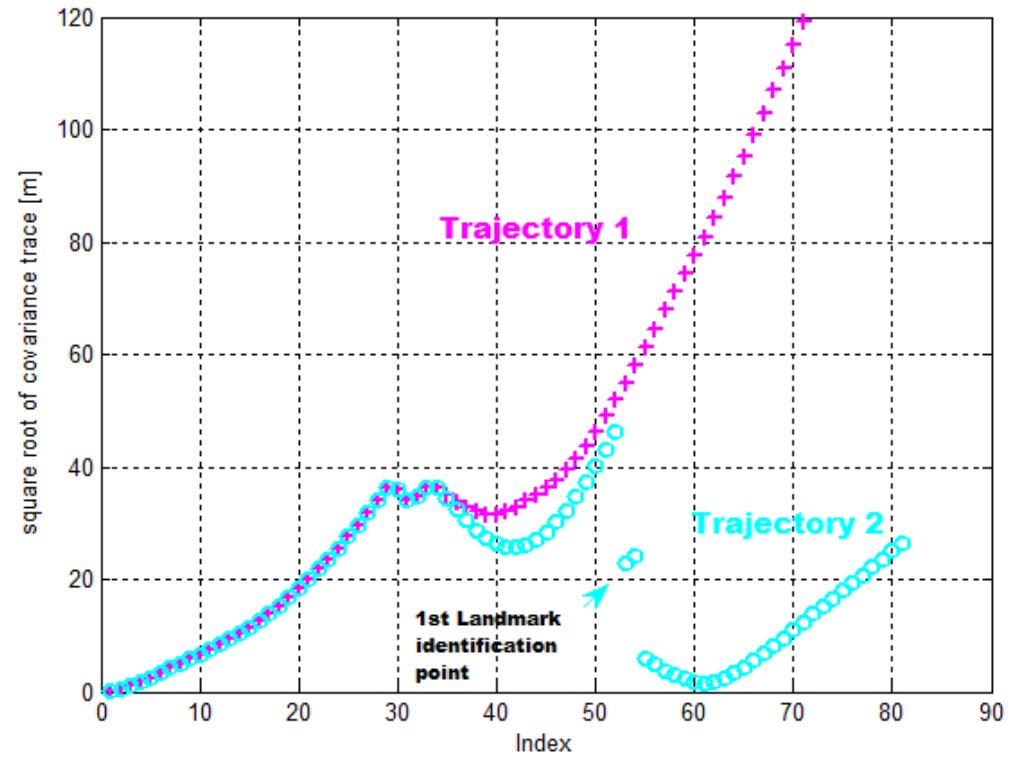
When applying object identification – the landmark is re-identified only in trajectory 2, similarly to reality

Results – SLAM

Multiple landmarks

Legend

- True trajectory
- Estimate Trajectory 1
- Estimate Trajectory 2
- Estimate covariance Trajectory 1
- Estimate covariance Trajectory 2
- Points of landmark identification



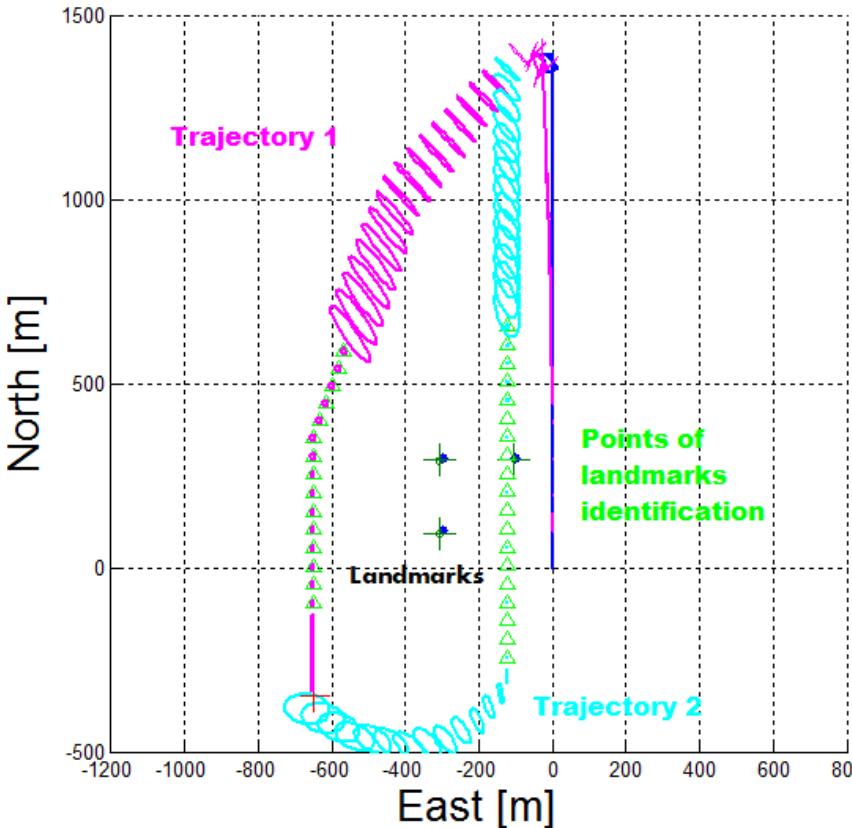
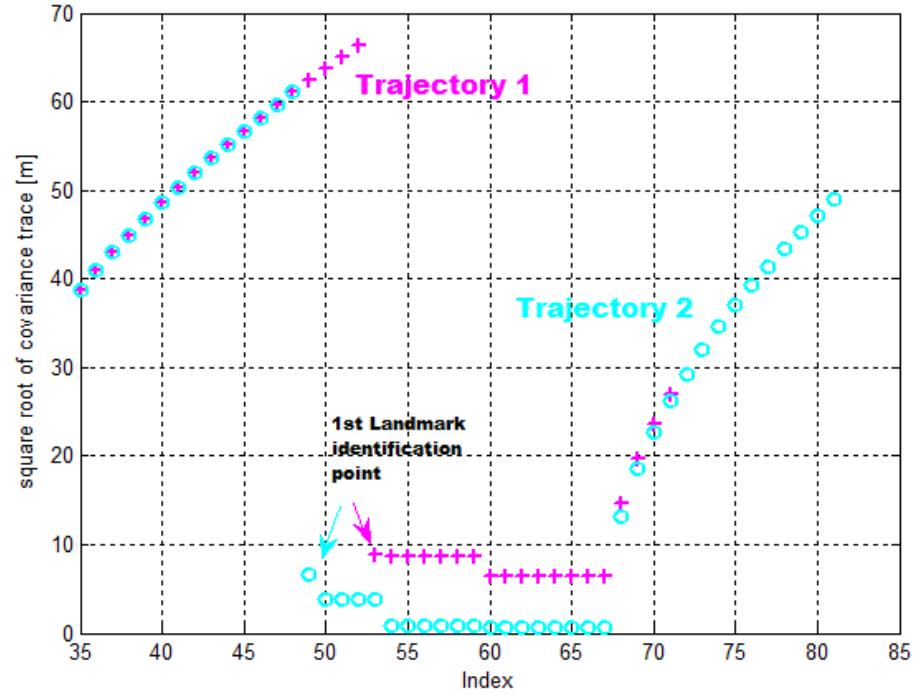
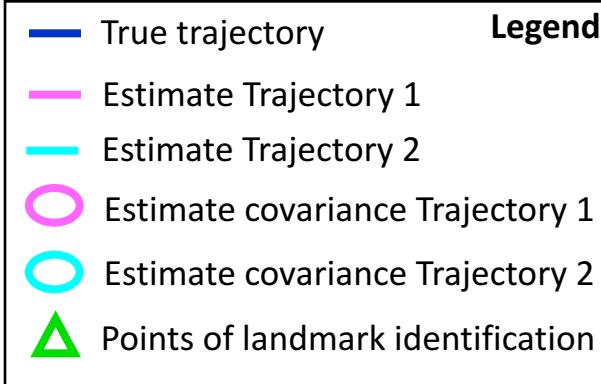
- Same as one landmark
- In reality: Landmark is recognized only where already viewed from similar view directions

Trajectory 2 has lower estimate covariance though it is longer → preferred

Results – Planning

Multiple landmarks

Without applying object identification



- Same as one landmark
- When not applying object identification: landmark is recognized incorrectly from new view directions

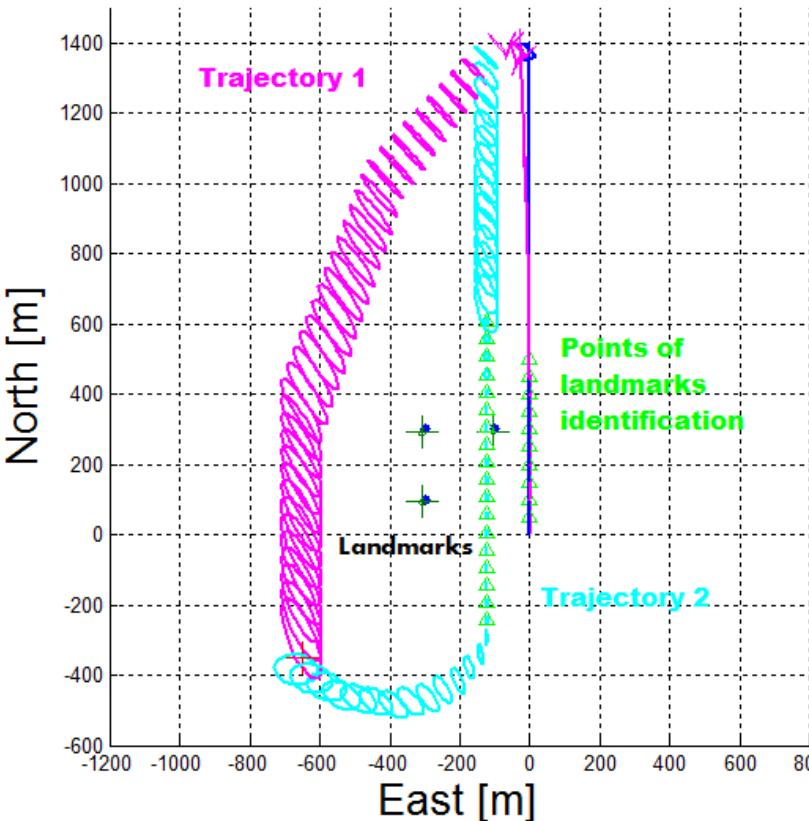
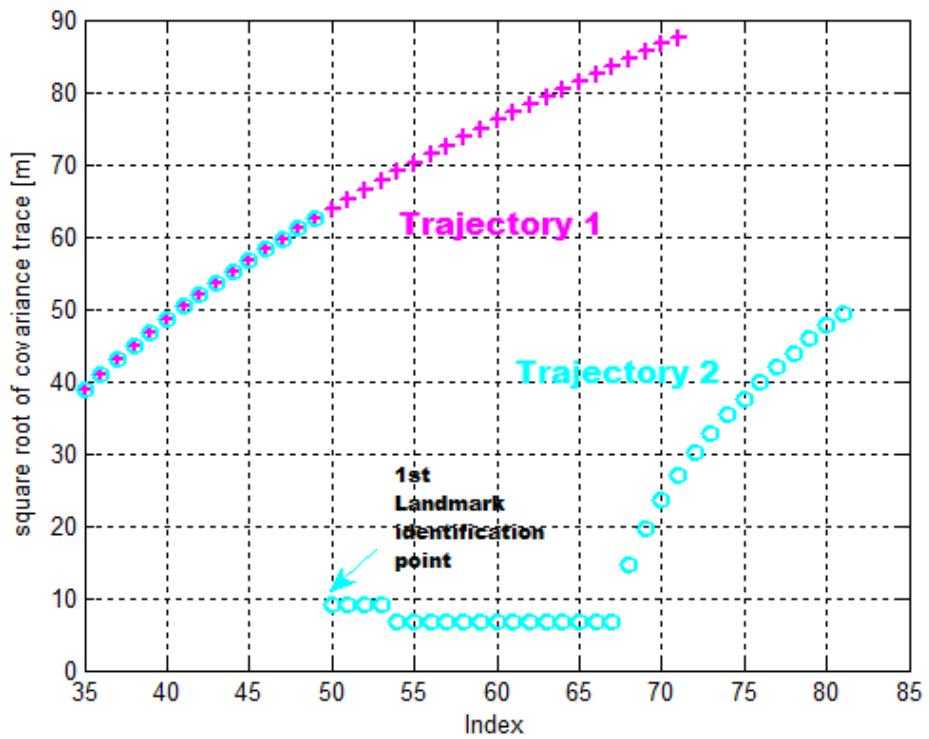
Trajectory 1 has lower estimate covariance → incorrectly preferred

Results – Planning

Multiple landmarks

With applying object identification

- Legend
- True trajectory
 - Estimate Trajectory 1
 - Estimate Trajectory 2
 - Estimate covariance Trajectory 1
 - Estimate covariance Trajectory 2
 - △ Points of landmark identification



Conclusions

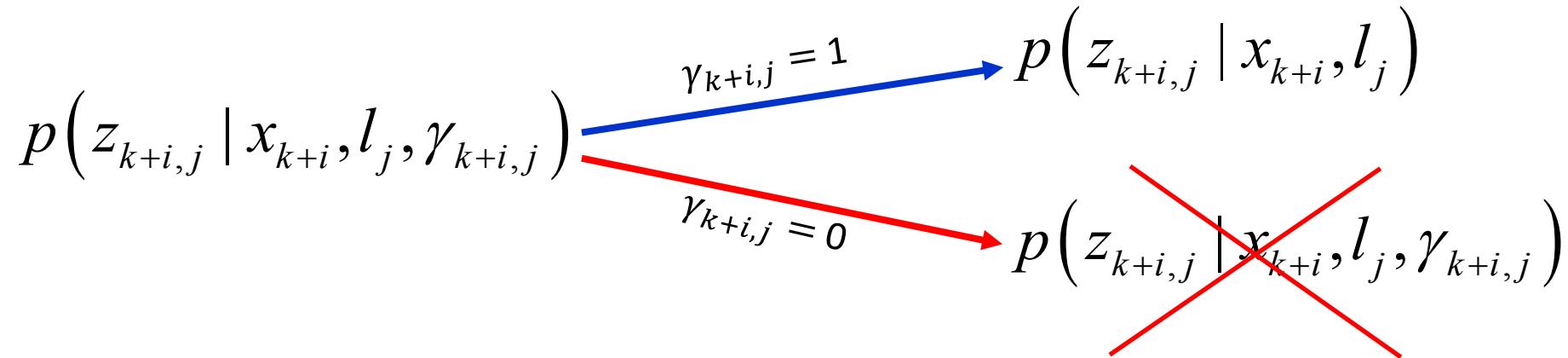
We developed a viewpoint aware BSP approach
and modeled object re-identification



Correct identification of landmarks is critical
Uncertainty prediction consistent with reality (inference)
Correct planning and path choosing

Thank you

Why $p(z_{k+i,j} | x_{k+i}, l_j, \gamma_{k+i,j})$ is uninformative?



Observation model:

$$z_{i,j} = h(x_i, l_j) + v_{i,j}(\gamma_{i,j}) \quad v_{i,j} : N(0, \Sigma_v)$$

$$\Sigma_v = \begin{cases} \Sigma_v & \gamma_{i,j} = 1 \\ \rightarrow \infty & \gamma_{i,j} = 0 \end{cases}$$

How we use the belief $b(X_{k+l})$ when the future measurements are unknown?

$$b(X_{k+l}) : N\left(X_{k+l}^*, \Sigma_{k+l}\right)$$

- We solve $X_{k+l}^* = \underset{X_{k+l}}{\operatorname{argmax}}(b(X_{k+l}))$ using optimization method Non linear least squares
- In order to find the covariance Σ_{k+l} we do not need to know the measurements, only the fact that they were acquired or not
- We assume Maximum likelihood assumption:

$$z = h(\bar{x}) \rightarrow x^* = \bar{x}$$

- Where \bar{x} is the predicted value of x , according to motion model

Marginalization

X_k – All robot ($x_{1:k}$) until time k and world states ($l_{1:j}$)
$H_k \triangleq \{Z_{0:k}, u_{0:k-1}\}$ Past measurements and controls
$\gamma_{i,j}$ – Event of acquiring measurement j at time I
$\Gamma_i \triangleq \{\gamma_{i,j}\}_{j=1}^{n_i}$

Notations

$$p(X_{k+l}|H_{k+l}) = \sum_{\Gamma_{k+1:k+l}} p(X_{k+l}, \Gamma_{k+1:k+l} | H_{k+l})$$

For example, for only one observation (Γ_{k+1}):

$$p(X_{k+1}|H_{k+1}) = p(X_{k+1}, \gamma_{k+1,1} = 1 | H_{k+1}) + p(X_{k+1}, \gamma_{k+1,1} = 0 | H_{k+1})$$