

Topological Belief Space Planning (t-BSP)

Belief Space Planning (BSP)

- BSP determines optimal non-myopic control action $\mathcal{U}_{k:k+L-1}^* = \arg \min_{\mathcal{U}} J_k(\mathcal{U})$ over the prediction horizon L at planning time k with respect to a given objective function J_k related to the design task

$$J_k(\mathcal{U}) = \mathbb{E}_{\mathcal{Z}} \left\{ \sum_{l=0}^{L-1} c_l [b(X_{k+l}), \mathcal{U}_{k+l}] + c_L [b(X_{k+L})] \right\}.$$

- instantiation of a Partially-Observable Markov Decision Process (POMDP)
- finding optimal solution to POMDP in the most general form is computationally intractable
- in information-theoretic BSP, J_k is a function of state uncertainty

Key insight: Any topological representation \mathcal{T} and derived metric which preserve action ordering/the best action can be used to solve BSP. Exact value of the objective function is not necessary.

Topology of a belief in active SLAM

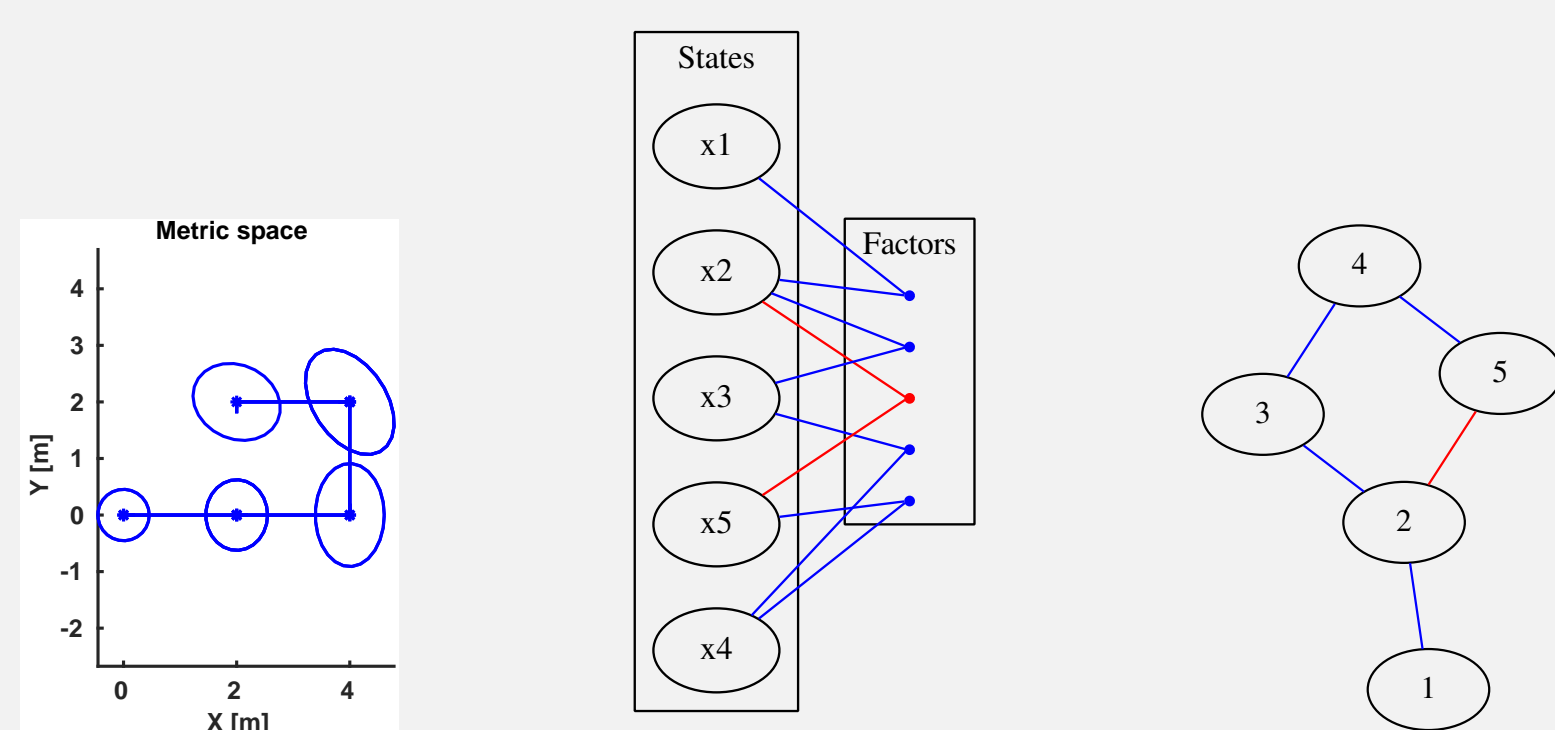
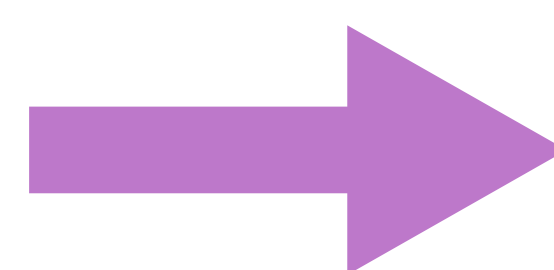


Figure 1: Each candidate action corresponds to posterior belief, i.e. trajectory uncertainty (a) which can be represented with a factor graph (b) and assigned a topology. In the case of active pose SLAM, topology can be defined with a simple undirected graph such that graph nodes represent robot's poses and edges pose constraints between them (c). Topological BSP aims to determine a graph invariant topological metric which is highly correlated with the information-theoretic cost and maintains action consistent decision making.

Topological metric



$$s : \mathcal{T} \rightarrow \mathbb{R}$$

Topological BSP

- we introduce a novel concept, topological belief space planning (t-BSP), that uses topological properties of the underlying factor graph representation of future posterior beliefs to direct a search for an optimal BSP solution
- no explicit inference required in optimization nor partial state covariance recovery
- enabling planning in high dimensional state spaces

Action consistency and t-BSP error

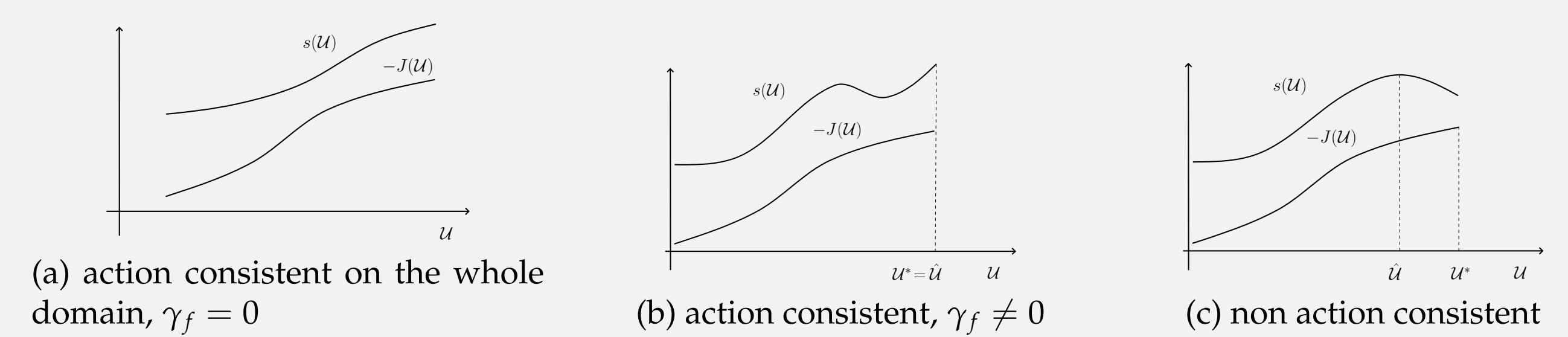


Figure 2: Different topological metrics and their influence to decision making.

Decision making via t-BSP

$$\hat{\mathcal{U}} = \arg \max_{\mathcal{U}} s(\mathcal{U}).$$

The error of t-BSP is

$$\epsilon(J, s) \doteq |J(\mathcal{U}^*) - f[s(\hat{\mathcal{U}})]|$$

where $\hat{\mathcal{U}} = \arg \max_{\mathcal{U}} s(\mathcal{U})$ and f is a monotonic function such that $f[s(\hat{\mathcal{U}})] = J(\hat{\mathcal{U}})$ and $f = \arg \min_f \gamma_f$, where $\gamma_f = \max_{\mathcal{U}} |J(\mathcal{U}) - f[s(\mathcal{U})]|$.

Information-theoretic objective

Minimizing Shannon joint entropy of the posterior Gaussian belief

$$J(\mathcal{U}) = N/2 \ln(2\pi e) + 1/2 \ln |\Sigma(X_{k+L})|,$$

where $\Sigma(X_{k+L})$ denotes the estimated covariance of the robot's belief $b[X_{k+L}]$, and N dimension of the state X_{k+L} .

Proposed topological metrics

- Von Neuman graph entropy and its approximation by a function of node degrees d (see [1]) \Rightarrow **faster to calculate, effectively $O(1)$, worst $O(n)$**

$$s_{VN}(G) = - \sum_{i=1}^n \hat{\lambda}_i / 2 \ln(\hat{\lambda}_i / 2),$$

$$\hat{s}_{VN}(G) \approx n/2 \ln 2 - 1/2 \sum_{(i,j) \in E} 1/[d(i)d(j)]$$

- Function of the number of spanning trees $t(G)$ of a graph motivated by [2] \Rightarrow **more accurate, computational complexity depends on the graph sparsity and the number of states**

$$s_{ST}(G) = 3/2 \ln t(G) + n/2 [\ln |\Omega_{vij}| - \ln(2\pi e)^K]$$

t-BSP error $\epsilon(J, s_{ST})$ can be calculated from topological properties and prior maximum likelihood estimate[3]

$$\epsilon(J, s_{ST}) \leq \Delta J_{max},$$

where $\Delta J_{max} = \mathcal{UB}[J(\hat{\mathcal{U}})] - \min_{\mathcal{U}} \mathcal{LB}[J(\mathcal{U})]$, and

$$\mathcal{UB}[J(\hat{\mathcal{U}})] = -s_{ST}(\hat{\mathcal{U}}) \text{ and } \mathcal{LB}[J(\mathcal{U})] = -s_{ST}(\mathcal{U}) + 1/2(\tau(\mathcal{U}) - \prod_{i=2}^n [d_{\mathcal{U}}(i) + \Psi(\mathcal{U})])$$

Results

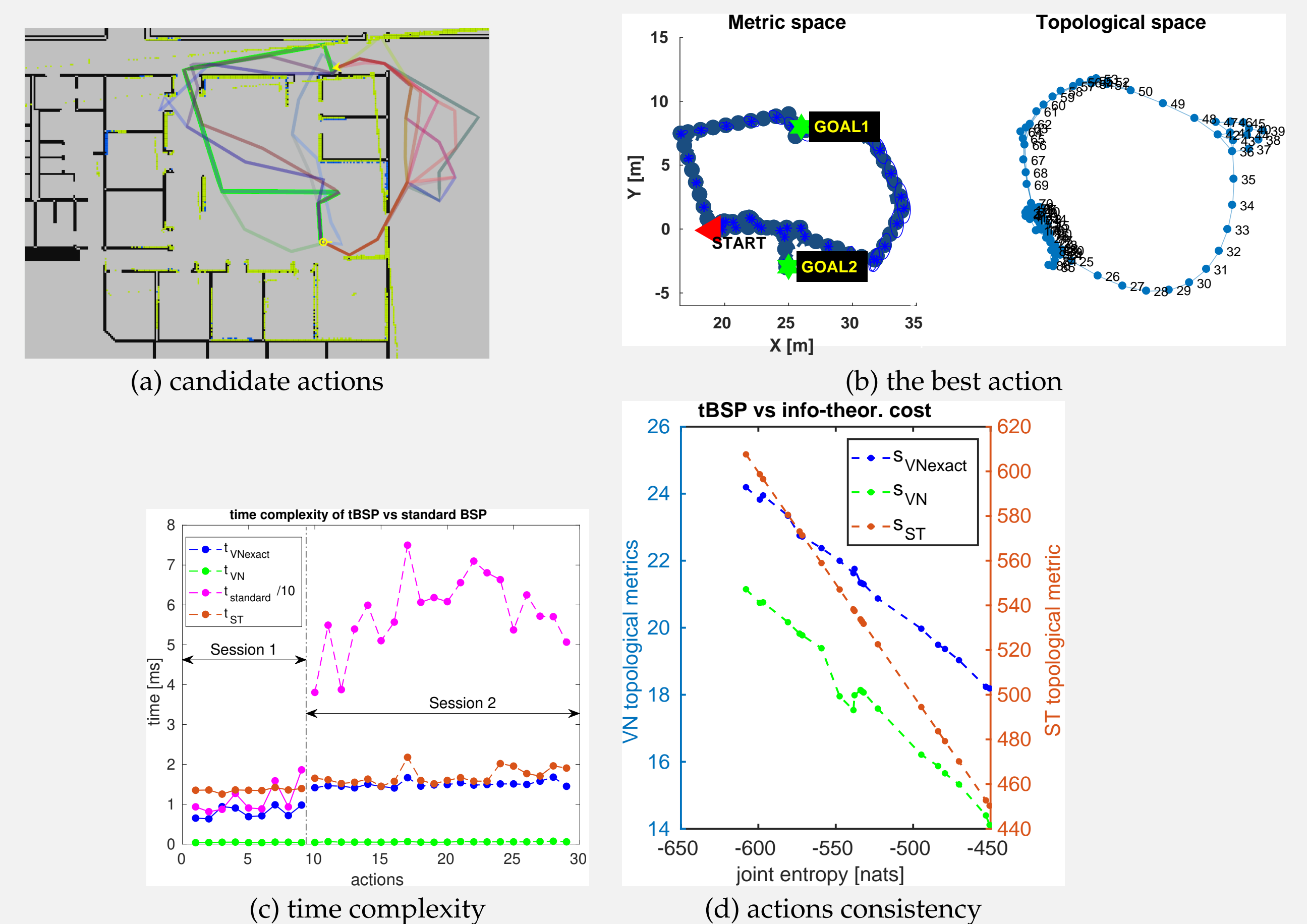


Figure 3: Second planning session in Gazebo/ROS simulation of active pose SLAM after which both exploration and exploitation actions are available (a). Topological BSP is able to determine the least uncertain path/action, corresponding to a big loop closure (b), and requires much less computation time than standard BSP. s_{ST} and s_{VN} (exact and approximated) are highly correlated with the joint entropy.

Conclusion

- topological properties of factor graphs dominantly determine estimation accuracy and enable efficient information-theoretic BSP
- decision making under some conditions (e.g. linear observation models, large diversity among candidate actions, certain noise properties) is action consistent
- in other cases, enables eliminating suboptimal actions

References

- [1] A. Kitanov and V. Indelman. Topological multi-robot belief space planning in unknown environments. In *IEEE Intl. Conf. on Robotics and Automation (ICRA)*, 2018.
- [2] Kasra Khosoussi, Matthew Giamou, Gaurav S Sukhatme, Shoudong Huang, Gamini Dissanayake, and Jonathan P How. Reliable graph topologies for SLAM. *Intl. J. of Robotics Research*, 2018.
- [3] Andrej Kitanov and Vadim Indelman. Topological information-theoretic belief space planning with optimality guarantees. *ArXiv*, March 2019.

