D2A-BSP: Distilled Data Association Belief Space Planning with Performance Guarantees

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Abstract— **Unresolved data association in ambiguous and perceptually aliased environments leads to multi-modal hypotheses on both the robot's and the environment state. To avoid catastrophic results, when operating in such ambiguous environments, it is crucial to reason about data association within Belief Space Planning (BSP). However, explicitly considering all possible data associations, the number of hypotheses grows exponentially with the planning horizon and determining the optimal action sequence quickly becomes intractable. Moreover, with hard budget constraints where some non-negligible hypotheses must be pruned, achieving performance guarantees is crucial. In this work we present a computationally efficient novel approach that utilizes only a distilled subset of hypotheses to solve BSP problems while reasoning about data association. Furthermore, to provide performance guarantees, we derive error bounds with respect to the optimal solution. We formulate how to efficiently maintain action selection in planning by using these bounds which can be incrementally updated, if necessary, by adding hypotheses to the distilled subset. We then demonstrate our approach in an extremely aliased environment, where we manage to significantly reduce computation time without compromising on the quality of the solution.**

I. INTRODUCTION

Decision making under uncertainty is at the core operation of intelligent autonomous agents and robots. Autonomous navigation, robotic surgeries and automated warehousing are only a few examples where agents must autonomously plan and execute their actions while reasoning about uncertainty. Such uncertainty might be due to noisy or limited observations; imprecise delivery of actions; or dynamic environments, where unpredictable events might take place. As the true state of the agent and the environment is unknown, it is represented by a probability density function (belief) over the corresponding random variables. To autonomously determine which actions to take, planning and decision making are performed over that distribution of possible states (the belief space). This problem is also known as Belief Space Planning (BSP) and is an instantiation of a Partially Observable Markov Decision Problem (POMDP) [13].

In ambiguous and perceptually aliased environments, BSP is even more challenging . In such scenarios, data association cannot be considered to be given and perfect, i.e. considering that the agent properly perceives the environment by its sensors, as it can lead to incorrect posterior beliefs and catastrophic results. As such, BSP should reason about data association while also considering other sources of uncertainty. However, reasoning about data association, the number of hypotheses grows exponentially with the planning horizon and determining the optimal action quickly becomes intractable. Moreover, when considering real time operation, using inexpensive hardware, hard computational budget constraints are often required. As such, some non-negligible hypotheses might be pruned and achieving some performance guarantees becomes vital.

Given a set of candidate actions, the main goal of BSP is to retrieve the optimal action with respect to a user defined objective function. Specifically, in the case of a multi modal belief (corresponding to different hypotheses), a traditional solution requires evaluating the objective function with respect to each hypothesis. Instead, we suggest to solve a simplified problem using only a distilled subset of hypotheses where the loss in solution quality can be bounded to provide performance guarantees.

Our contributions in this paper are as follows: (a) We introduce a novel approach, D2A-BSP, that utilizes a distilled subset of hypotheses in planning to reduce computational complexity; (b) We rigorously develop the connection between our approach and the true analytical solution, owing to every possible data association, for the myopic case; (c) We derive bounds over the true analytical solution and prove they converge to the true analytical solution and can be incrementally adapted by adding additional hypotheses; (d) We formulate how to use these bounds in planning, under a given budget, with performance guarantees; (e) We demonstrate the merits of our approach in a highly ambiguous scenario containing identical landmarks.

II. RELATED WORK

In an attempt to ensure reliable and efficient operation in ambiguous environments, different approaches were recently proposed. These approaches, often referred to as robust perception, usually maintain probabilistic data association and hypothesis tracking, given accessible data.

With respect to the state inference problem, and specifically in the context of Simultaneous Localization And Mapping (SLAM), the inference mechanism should be resilient to false data association overlooked by front-end algorithms, e.g. laser scans and image matching. A common approach to handle this problem is utilizing efficient graph representations. In [3] the authors propose using a pose graph model, where graph optimization is efficient in finding the maximal subset of measurements that is internally coherent to discard false data associations. The authors of [12] and [11] utilized

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the same graphical model with an expectation-maximization (EM) approach to efficiently infer robot initial relative poses and solve a multi robot data association problem. In [22] the authors modified parts of the topological structure of the graph during optimization to discard false positive loop closures. In [18] the authors utilized factor graph [17] representations to perform inference on networks of mixtures while in [7] the authors extended the Bayes tree [14] algorithm to explicitly incorporate multi-modal measurements within the graph and generate multi-hypothesis outputs. Yet, all of these works were developed for the passive case only, i.e. no planning is involved.

Only recently, ambiguous data association was considered in different BSP approaches for active disambiguation. In [1] the authors considered data association with the prior belief, modeling it as a mixture of Gaussians, and assumed that at least one action can lead to complete disambiguation. However, their work does not reason about ambiguous data association within future beliefs (owing to future observations). The authors of [20] incorporated, for the first time, reasoning about future data association hypotheses within a belief space planning framework, terming the corresponding approach DA-BSP. Another related work in this context is [8], that also reasons about ambiguous data association in future beliefs while utilizing the graphical model presented in [7]. To handle the exponential growth in the number of hypotheses, these approaches suggested to use different heuristics, e.g. pruning and merging. However, none of them developed any analytical bounds on the loss in quality of the solution, with respect to the original problem, and cannot provide performance guarantees.

While finding the optimal solution of a POMDP was proven to be computationally intractable [19], several approaches were developed over the years to reduce computational complexity and allow online operation while planning under uncertainty. Some methods rely on approximated solutions via direct trajectory optimization, e.g. [10] and [25], while others approximate the state or the objective function to reduce the planning complexity, e.g. [2]. Belief sparsification in planning was first introduced in [9] to limit the state size and allow long-term operation. The author utilized a diagonal covariance approximation, in a myopic setting with one-row unary Jacobians, to maintain a similar action selection while significantly reducing the complexity of the objective calculation. The authors of [6] presented a sparsification approach to handle BSP problems. They suggested to identify uninvolved variables and sparsify the posterior information matrix for each candidate action to reduce computation time. Other recent approaches suggest utilizing structural properties of different graphical models in decision making under uncertainty, e.g. in [15], [16] and [21] different topological signatures were used to approximate the solution to BSP problems. Yet, in all of these approaches data association is assumed the be known and perfect.

To the best of our knowledge, in-spite of aforementioned research efforts, simplifying the BSP problem while reasoning about data association and maintaining performance guarantees is a novel concept.

III. NOTATIONS AND PROBLEM FORMULATION

Consider an autonomous agent operating in a known environment, where different objects or scenes can possibly be perceptually similar or identical. The agent aims to decide its future actions, while reasoning about ambiguous data association, based on information accumulated thus far and a user defined objective function.

A. Belief Propagation

Let x_k denote the agent's state at time instant k. For simplicity, in this work we assume the environment, represented by landmarks, to be given. We note that extending our approach to a full SLAM scenario is straightforward, using similar notations as in [24].

We denote the data association realization vector at time k as β_k. Given n_k observations at time k, $\beta_k \in \mathbb{N}^{n_k}$. Elements in β_k are associated according to the given observation model and each element, i.e. landmark, is given a unique label. A specific data association hypothesis at time k is thus given by a specific set j of associations up to and including time k and is denoted as $\beta^j_{1:k}$.

Let $Z_k \triangleq \{z_{k,1}, ..., z_{k,n_k}\}$ denote the set of all n_k measurements at time k and let u_k denote the agent's action at time k. $Z_{1:k}$ and $u_{0:k-1}$ denote all observations and actions up to time k, respectively. The motion and observation models are given by

$$x_{k+1} = f\left(x_k, u_k, w_k\right) \quad , \quad z_k = h\left(x_k, l, v_k\right), \tag{1}$$

where l is a landmark pose, and w_k and v_k are noise terms, sampled from known motion and measurement distributions, respectively.

The posterior probability density function (pdf) over the state x_k, denoted as the *belief*, is given by

$$b\left[x_k\right] \triangleq \mathbb{P}\left(x_k|z_{0:k}, u_{0:k-1}\right) = \mathbb{P}\left(x_k|H_k\right), \tag{2}$$

where $H_k \triangleq \{Z_{1:k}, u_{0:k-1}\}$ represents history at time k. We define $H^-_{k+1} \triangleq H_k \cup \{u_k\}$ and $b^-_{k+1} \triangleq \mathbb{P}\left(x_{k+1}|H^-_{k+1}\right)$ for notational convenience. The belief at time k is denoted from hereon as b_k.

As data association is not given, and different observations might be attributed to different but similar-in-appearance landmarks, the belief at time k includes different hypotheses. In particular, marginalizing over $M_k \in \mathbb{N}$ hypotheses and using the chain rule, we rewrite the belief at time k as a linear combination

$$b_k = \sum_{j=1}^{M_k} \underbrace{\mathbb{P}\left(x_k|\beta^j_{1:k}, H_k\right)}_{b^j_k} \underbrace{\mathbb{P}\left(\beta^j_{1:k}|H_k\right)}_{w^j_k}, \tag{3}$$

where b^j_k is a conditional belief, with some general distribution, that corresponds to the jth hypothesis, and w^j_k is the associated weight.

Updating the belief, after performing control u_{k+1} and taking an observation Z_{k+1}, also requires reasoning about data association. Given M_k hypotheses from time k, marginalizing over all landmarks at time $k+1$ and using the chain rule we explicitly write it as

$$b_{k+1} = \sum_{i=1}^{|L|} \sum_{j=1}^{M_k} \underbrace{\mathbb{P}\left(x_{k+1}|H_{k+1}, \beta_{k+1}^i, \beta_{1:k}^j\right)}_{b_{k+1}^{i,j}} \underbrace{\mathbb{P}\left(\beta_{k+1}^i, \beta_{1:k}^j|H_{k+1}\right)}_{w_{k+1}^{i,j}}, \tag{4}$$

where $|L|$ represents the number of different data association realizations considered at time $k+1$. The first term $b_{k+1}^{i,j}$ represents a conditional belief at time $k+1$ which originated from the jth hypothesis at time k and a specific data association realization β_{k+1}^i. The second term $w_{k+1}^{i,j}$ is the associated belief component weight.

Corollary 1: Each posterior belief component weight $w_{k+1}^{i,j}$ can be written as

$$w_{k+1}^{i,j} = \eta_{k+1}^{-1} \tilde{\zeta}_{k+1}^{i,j} w_k^j, \tag{5}$$

where w_k^j is the weight of the jth component from time k; η_{k+1} is a normalization term; and $\tilde{\zeta}_{k+1}^{i,j}$ is the probability for the ith data association at time $k+1$ given the jth hypothesis from time k,

$$\tilde{\zeta}_{k+1}^{i,j} \triangleq \mathbb{E}_{x_{k+1}}[\mathbb{P}\left(Z_{k+1}|\beta_{k+1}^i, x_{k+1}\right) \mathbb{P}\left(\beta_{k+1}^i|x_{k+1}\right)], \tag{6}$$

where the expectation is with respect to $\mathbb{P}\left(x_{k+1}|H_{k+1}^-, \beta_{1:k}^j\right)$.
The term $\mathbb{P}\left(Z_{k+1}|\beta_{k+1}^i, x_{k+1}\right)$ in (6) is the joint measurement likelihood for all observations obtained at time $k+1$ given the ith data association and state x_{k+1}. It can be explicitly written as

$$\mathbb{P}\left(Z_{k+1}|\beta_{k+1}^i, x_{k+1}\right) = \prod_{r=1}^{n_{k+1}} \mathbb{P}\Big(z_{k+1,r}|l_{\beta_{k+1}^i(r)}, x_{k+1}\Big), \tag{7}$$

where $l_{\beta_{k+1}^i(r)}$ denotes the landmark pose, corresponding to the rth measurement in the given data association realization vector β_{k+1}^i.

To allow fluid reading, proofs for all corollaries and theorems are given in the appendix.

B. Belief Space Planning

Let J denote a user defined objective function given by

$$J\left(b_k, u_{k:k+N-1}\right) = \mathbb{E}\left[\sum_{n=1}^{N} c_{k+n}\left(b_{k+n}, u_{k+n-1}\right)\right], \tag{8}$$

where c_{k+n} represents the cost function associated with the nth look-ahead step and where the expectation is taken with respect to future observations $Z_{k+1:k+N}$.

Given a set of candidate action sequences $\mathcal{U}$ and a belief b_k, the goal of BSP is to find the optimal action sequence given by

$$u_{k:k+N-1}^* = \underset{\mathcal{U}}{\operatorname{argmin}}\, J\left(b_k, u_{k:k+N-1}\right). \tag{9}$$

Evaluating (9) at each planning session for every candidate action sequence is known to be computationally intractable even without reasoning about data association. Using (4) it is not hard to see that explicitly reasoning about data association in planning adds an additional complexity as the number of belief components $|M_k|\,|L|^n$ grows exponentially with the planning horizon.

To relax the computational complexity, one could consider solving a computationally easier, simplified problem, with respect to the same set of candidate actions. If the solution can be mathematically related to the solution of the original problem, one can provide performance guarantees. This can be achieved by simplifying and bounding any of the objective function terms.

IV. APPROACH

We propose a method that reduces the computational complexity of BSP problems in which ambiguous data association is explicitly considered while providing performance guarantees on the quality of the solution.

As a first step towards applying our method for the general BSP problem (9), in this work we consider a myopic setting, i.e. one look-ahead step, which by itself can be computationally challenging in highly ambiguous scenarios. Writing the expectation operator in (9) explicitly, the objective function for the myopic setting is defined as

$$J\left(b_k, u_k\right) = \int_{Z_{k+1}} \eta_{k+1} c\left(b_{k+1}\right) dz_{k+1}, \tag{10}$$

where $\eta_{k+1} \triangleq \mathbb{P}(Z_{k+1}|H_{k+1}^-)$ is the joint measurement likelihood, denoted from hereon simply as η. In this work we interchangeably refer to η as the normalization term and the measurement likelihood.

In our approach we suggest using only a distilled subset of belief components $M_k^s \subseteq M_k$ from time k. We avoid calculating the posterior belief at time $k+1$ for components we do not consider in M_k^s. As such, the number of belief components at time $k+1$ reduces from $|M_k|\,|L|$ to $|M_k^s|\,|L|$ which also lowers the computational complexity of the considered cost function. Crucially, we analytically bound the loss in solution quality for every considered action with respect to (10).

We formally define a simplified belief at time k as

$$b_k^s \triangleq \sum_{j=1}^{M_k^s} w_k^{s,j} b_k^j \quad , \quad w_k^{s,j} \triangleq \frac{w_k^j}{w_k^{m,s}}, \tag{11}$$

where weights are re-normalized with $w_k^{m,s} \triangleq \sum_{m \in M_k^s} w_k^m$.

To provide performance guarantees, we wish to bound (10), for each candidate action u_k, using b_k^s

$$\underline{J}\left(b_k, b_k^s, u_k\right) \leq J\left(b_k, u_k\right) \leq \bar{J}\left(b_k, b_k^s, u_k\right). \tag{12}$$

To efficiently evaluate these bounds, in this work we consider simplifying and analytically bounding both η and the cost function terms in (10). Thus, we rewrite (12) as

$$\int_{Z_{k+1}} \mathcal{LB}\left[\eta\right] \mathcal{LB}\left[c\left(b_{k+1}\right)\right] dZ_{k+1} \leq J\left(b_k, u_k\right) \leq \int_{Z_{k+1}} \mathcal{UB}\left[\eta\right] \mathcal{UB}\left[c\left(b_{k+1}\right)\right] dZ_{k+1}, \tag{13}$$

where $\mathcal{LB}$, $\mathcal{UB}$ denote lower and upper bounds, respectively.

A. Bounding the cost function

While the cost function in (10) can generally include a number of different terms, e.g. distance to goal, energy spent and information measures of future beliefs, in this work we only consider an information theoretic term over data association hypotheses weights that can be used for autonomous active disambiguation of hypotheses. We believe that conceptually similar derivations can also support other terms, e.g. distance to goal, and leave that for future research.

Specifically, to disambiguate between hypotheses, we utilize the Shannon entropy, defined as $\mathcal{H} \triangleq -\sum_{i=1}^{n} w^i log\left(w^i\right)$, where each w^i corresponds to a belief component weight and $\sum_{i=1}^{n} w^i = 1$. Using Corollary 1, we rewrite $\mathcal{H}$ as

$$c\left(b_{k+1}\right) \triangleq \mathcal{H} = -\sum_{i}^{|L|}\sum_{j}^{M_k} \frac{\tilde{\zeta}_{k+1}^{i,j} w_k^j}{\eta} log\left(\frac{\tilde{\zeta}_{k+1}^{i,j} w_k^j}{\eta}\right). \quad (14)$$

To bound this cost function given a belief b_{k+1} using the same cost function given a simplified belief b_{k+1}^s, we first rigorously derive the analytic connection between the two.

Theorem 1: Given a simplified belief b_k^s at time k, for every action u_k and considered future observation Z_{k+1}, the cost due to ambiguity (14) *can be expressed by*

$$\mathcal{H} = \frac{w_k^{m,s}}{\eta}\left[\eta^s\left[\mathcal{H}^s - log(\eta^s)\right] - \sum_{i}^{|L|}\sum_{j}^{M_k^s} \tilde{\zeta}_{k+1}^{i,j} w_k^{s,j} log\left(\frac{w_k^{m,s}}{\eta}\right)\right] - \sum_{i}^{|L|}\sum_{j}^{\neg M_k^s} \frac{\tilde{\zeta}_{k+1}^{i,j} w_k^j}{\eta} log\left(\frac{\tilde{\zeta}_{k+1}^{i,j} w_k^j}{\eta}\right), \quad (15)$$

where $\neg M_k^s \triangleq M_k \setminus M_k^s$; $\mathcal{H}^s \triangleq c\left(b_{k+1}^s\right)$; *and* $\eta^s \triangleq \mathbb{P}\left(Z_{k+1}|b_k^s, u_k\right)$.

We now use Theorem 1 to derive bounds for $\mathcal{H}$ which are computationally more efficient to calculate as we only consider a subset of hypotheses. As can be seen in (27) and Section 4.1 in [20], evaluating η requires evaluating all posterior components weights $w_{k+1}^{i,j}$. As our considered cost is a function of these weights, simplifying and bounding $\mathcal{H}$ has no computational merits without simplifying and bounding η (denoted below by η^s, $\mathcal{LB}[\eta]$ and $\mathcal{UB}[\eta]$).

Theorem 2: Given a simplified belief b_k^s at time k, the cost due to ambiguity term in (10) *is bounded by*

$$\mathcal{LB}\left[c\left(b_{k+1}\right)\right] \triangleq \mathcal{LB}\left[\mathcal{H}\right] = \frac{\eta^s w_k^{m,s}}{\mathcal{UB}[\eta]}\left[\mathcal{H}^s - log(\eta^s)\right] - \frac{w_k^{m,s}}{\mathcal{UB}[\eta]}\sum_{i}^{|L|}\sum_{j}^{M_k^s} \tilde{\zeta}_{k+1}^{i,j} w_k^{s,j} log\left(\frac{w_k^{m,s}}{\mathcal{LB}[\eta]}\right), \quad (16)$$

$$\mathcal{UB}\left[c\left(b_{k+1}\right)\right] \triangleq \mathcal{UB}\left[\mathcal{H}\right] = \frac{\eta^s w_k^{m,s}}{\mathcal{LB}[\eta]}\left[\mathcal{H}^s - log(\eta^s)\right] - \frac{w_k^{m,s}}{\mathcal{LB}[\eta]}\sum_{i}^{|L|}\sum_{j}^{M_k^s} \tilde{\zeta}_{k+1}^{i,j} w_k^{s,j} log\left(\frac{w_k^{m,s}}{\mathcal{UB}[\eta]}\right) - \gamma log\left(\frac{\gamma}{|L|\left|\neg M_k^s\right|}\right), \quad (17)$$

where $\gamma \triangleq 1 - \frac{\eta^s w_k^{m,s}}{\mathcal{LB}[\eta]}$.

Furthermore, considering different levels of simplifications, i.e. adding belief components to M_k^s, these bounds become tighter.

Corollary 2: Given a simplified belief b_k^s, the bounds developed in Theorem 2 converge to $\mathcal{H}$ when $M_k^s = M_k$

$$\lim_{M_k^s \to M_k} \mathcal{LB}\left[\mathcal{H}\right] = \mathcal{H} = \mathcal{UB}\left[\mathcal{H}\right]. \quad (18)$$

Using basic log properties, it is not hard to show that these bounds can be incrementally adapted if one chooses to add additional components to M_k^s. [supplementary]

B. Bounding η

In this section we derive the bounds $\mathcal{LB}[\eta]$ and $\mathcal{UB}[\eta]$ over η. We start by expressing η using η^s.

Theorem 3: Given a simplified belief b_k^s at time k, for every action u_k and considered future observation z_{k+1}, the normalization term η in (10) *can be expressed by*

$$\eta = w_k^{m,s}\eta^s + \sum_{i}^{|L|}\sum_{j}^{\neg M_k^s} \tilde{\zeta}_{k+1}^{i,j} w_k^j. \quad (19)$$

We can now use Theorem 3 to derive bounds for η.

Theorem 4: Given a simplified belief b_k^s at time k, the measurement likelihood term η in (10) *is bounded by*

$$\mathcal{LB}[\eta] = \eta^s w_k^{m,s}, \quad (20)$$

$$\mathcal{UB}[\eta] = \eta^s w_k^{m,s} + \left(1 - w_k^{m,s}\right)\sigma\sum_{i}^{|L|}\alpha^i, \quad (21)$$

where $\sigma \triangleq max\left(\mathbb{P}\left(Z_{k+1}|\beta_{k+1}^i, x_{k+1}\right)\right)$ *and* $\alpha^i \triangleq \mathcal{UB}\left[\mathbb{P}\left(\beta_{k+1}^i|x_{k+1}\right)\right]$ *is an indicator function.*

As in Theorem 2, since we only consider a subset of hypotheses these bounds are also computationally more efficient to calculate and become tighter when adding belief components to M_k^s.

Corollary 3: Given a simplified belief b_k^s, the bounds developed in Theorem 4 converge to η when $M_k^s = M_k$

$$\lim_{M_k^s \to M_k} \mathcal{LB}[\eta] = \eta = \mathcal{UB}[\eta]. \quad (22)$$

Furthermore, these bounds can be incrementally adapted if one chooses to add additional components to M_k^s. This can easily be done by first incrementally updating both $w_k^{m,s}$ (using (11)) and η^s (see (28)), and then by updating the indicator function.

C. Simulating future observations Z_{k+1}

Evaluating the objective function (10), as explained in [20], is usually performed in two steps: we first simulate future observations Z_{k+1} by sampling from the measurement likelihood η using the generative model (1), and then calculate the measurement likelihood η for each such observation.

While previous works, either with a Maximum Likelihood (ML) assumption, e.g. [6], [16], [21], or without ML assumption, e.g. [23], [26], all consider the likelihood terms η and η^s to be equal, to the best of our knowledge, we are the first to consider impact of simplification on the normalization term in the myopic case.

Recall that in our proposed approach we only evaluate the bounds over η for each future observation. However, for the bounds in (13) to hold, we have to make sure we integrate over the same set of observations as in (10).

To handle this issue, we propose propagating and sampling from the original belief rather than from the simplified belief. We note that, as explained in [20], the concept of simulating future observations is computationally not the same as calculating the measurement likelihood which requires marginalizing over all possible data associations realizations and states. As such, using the original belief to simulate future observations does not affect the computational complexity of our proposed approach. However, we also note that this is only because we consider a myopic setting. A different approach should be considered in a non-myopic setting, which is beyond the scope of this work.

V. RESULTS

We evaluate the performance of our approach in a highly ambiguous environment comprising perceptually identical landmarks in different locations. Our prototype implementation uses the GTSAM library [5] with a python wrapper; all experiments were run on an Intel i7-7850 CPU running at 2200 GHz with 32GB RAM.

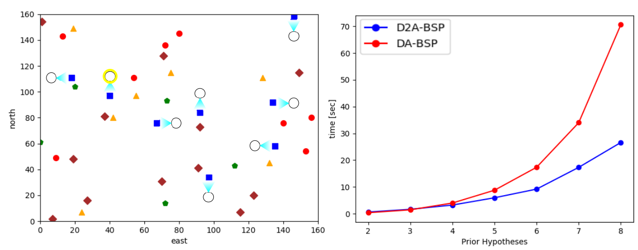

Fig. 1: Run-time comparison between our approach and DA-BSP, as a function of the number of prior hypotheses. Given a multi-modal initial belief, the agent's goal is to fully disambiguate between all hypotheses. The scenario presented contains 8 prior hypotheses, each initialized in front of a blue square and denoted by a black ellipse. Headings are denoted with cyan triangles. The true state, unknown to the agent, is highlighted in yellow.

In our experiment we specifically consider six different landmark types represented by Squares, Circles, Diamonds, Pentagons, Stars and Triangles, randomly placed within the environment. The agent is initially placed in front of a blue square. With no other prior information, the initial belief is multi-modal with the number of hypotheses M_0 equals to the number of randomly placed blue squares. This scenario can be considered as a version of the kidnapped robot problem.

The agent's goal is to fully disambiguate between hypotheses given a user defined budget constraint over the number of prior hypotheses that can be used in each planning session. As such, the agent has to solve the corresponding BSP problem for one look-ahead step, at each planning session, considering entropy over posterior belief components weights as a cost function. The considered actions set at each planning session contains predefined motion primitives in all four cardinal directions.

The distilled subset of belief components is chosen greedily at each planning session based on prior components weights, starting with a single component. If D2A-BSP cannot provide performance guarantees using the current distilled mixture M_k^s, i.e. the bounds (13) overlap, the mixture and the bounds are incrementally adapted, up to the given budget, by adding the component with the highest associated weight from $\neg M_k^s$.

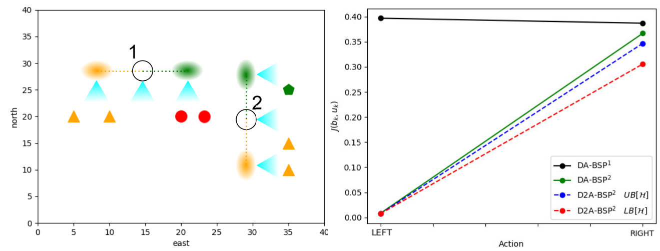

Fig. 2: A simple BSP problem where the agents can go either left or right. Each prior hypothesis is denoted by a black ellipse and heading is denoted with a cyan triangle. Green and Orange ellipses denote the propagated belief after moving left or right respectively. complete description

We compare our approach with a modified version of DA-BSP [20] where the number of belief components are chosen greedily, at each planning session, based on prior components weights up to the given budget.

We consider data association in both inference and planning. For a fair comparison, as the number of belief components grows exponentially in time in inference, for all approaches we utilize the same pruning heuristics where we prune belief components based on a user defined weight threshold.

In Fig. 1, we see the computational merits of D2A-BSP. The higher the level of ambiguity within the environment, i.e. more hypotheses to reason about, the more prominent D2A-BSP becomes. Fig. 2 presents a scenario in which under hard budget constraints, DA-BSP is unable identify the best action while D2A-BSP can provide performance guarantees.

VI. CONCLUSIONS

In this work, we introduced a novel approach that utilizes a distilled subset of hypotheses to reduce the computational complexity in data association aware BSP with performance guarantees for the myopic case. While existing approaches handle the exponential growth of the number of hypotheses within planning using different heuristics which cannot provide performance guarantees, we rigorously developed analytical bounds on the loss in quality of our proposed method solution and showed how to use them in planning to obtain performance guarantees. We demonstrated our approach in an extremely aliased scenario where we were able to significantly reduce the computational complexity compared to existing approaches.

Future work will consider a non-myopic case. Simplifying with respect to the number of considered landmarks in planning is another lucrative research direction as it directly affects the exponential growth in the number of hypotheses with the planning horizon.

VII. APPENDIX

A. *Proof of Corollary 1*

We follow a similar derivation to the one presented in [20] and factorize $w_{k+1}^{i,j}$ by first marginalizing over x_{k+1} and then by applying the Bayes rule

$$w^{i,j}_{k+1} = \int_{x_{k+1}} \frac{\mathbb{P}(Z_{k+1}|\beta^i_{k+1}, \beta^j_{1:k}, x_{k+1}, H^-_{k+1})\mathbb{P}(\beta^i_{k+1}, \beta^j_{1:k}, x_{k+1}|H^-_{k+1})}{\mathbb{P}(Z_{k+1}|H^-_{k+1})}.$$

Using the chain rule multiple times over the second term in the numerator completes the proof. ■

B. Proof of Theorem 1

We split (14) based on belief components from M^s_k and use (11) to rewrite $\mathcal{H}$ as

$$\mathcal{H} = -\sum_i^{|L|}\sum_j^{M^s_k} \frac{\tilde{\zeta}^{i,j}_{k+1} w^{s,j}_k w^{m,s}_k}{\eta} log\left(\frac{\tilde{\zeta}^{i,j}_{k+1} w^{s,j}_k w^{m,s}_k}{\eta}\right) - \sum_i^{|L|}\sum_j^{\neg M^s_k} \frac{\tilde{\zeta}^{i,j}_{k+1} w^{j}_k}{\eta} log\left(\frac{\tilde{\zeta}^{i,j}_{k+1} w^{j}_k}{\eta}\right). \quad (23)$$

Using the key observation that $\tilde{\zeta}^{s,ij}_{k+1} = \tilde{\zeta}^{i,j}_{k+1}$, basic log properties and that by definition all posterior weights sum to 1, we write the cost for a simplified belief as

$$\mathcal{H}^s = -\frac{1}{\eta^s}\sum_i^{|L|}\sum_j^{M^s_k}\left[\tilde{\zeta}^{i,j}_{k+1} w^{s,j}_k log\left(\tilde{\zeta}^{i,j}_{k+1} w^{s,j}_k\right)\right] + log\left(\eta^s\right). \quad (24)$$

Replacing (24) back into (23) and using basic log properties completes the proof. ■

C. Proof of Theorem 2

The last term in (15) is non negative as all posterior weights are at most 1 by definition. Thus, removing this term and using theorem 4 we immediately get the lower bound. For the upper bound, we revisit the last term in (15). We first define γ using (11) and (28)

$$\gamma \triangleq \sum_i^{|L|}\sum_j^{\neg M^s_k} \frac{\tilde{\zeta}^{i,j}_{k+1} w^{j}_k}{\eta} = 1 - \sum_i^{|L|}\sum_j^{M^s_k} \frac{\tilde{\zeta}^{i,j}_{k+1} w^{j}_k}{\eta} = 1 - \frac{\eta^s w^{m,s}_k}{\eta}.$$

Using the log sum inequality [4]

$$\sum_i^n a_i \cdot log\left(\frac{a_i}{b_i}\right) \geq a \cdot log\left(\frac{a}{b}\right) \text{ where} \sum_i^n a_i = a, \sum_i^n b_i = b,$$

with $a_i = \frac{\tilde{\zeta}^{i,j}_{k+1} w^{j}_k}{\eta}$, $\sum_i^{|L|}\sum_j^{\neg M^s_k} \frac{\tilde{\zeta}^{i,j}_{k+1} w^{j}_k}{\eta} = \gamma$ and $b_i = 1$, we bound the last term in (15)

$$\sum_i^{|L|}\sum_j^{\neg M^s_k} \frac{\tilde{\zeta}^{i,j}_{k+1} w^{j}_k}{\eta} log\left(\frac{\tilde{\zeta}^{i,j}_{k+1} w^{j}_k}{\eta}\right) \geq \gamma log\left(\frac{\gamma}{|L|\,|\neg M^s_k|}\right). \quad (25)$$

Substituting (25) into (15) and using Theorem 4 completes the proof. ■

D. Proof of Corollary 2

Given $M^s_k = M_k$ it holds by definition that $w^{m,s}_k = 1$ and $\mathcal{H} = \mathcal{H}^s$ as $b_{k+1} = b^s_{k+1}$. Substituting these back into (16) and using (11) and Corollary 3, the lower bound becomes

$$\mathcal{LB}\left[\mathcal{H}\right] = \mathcal{H} - log(\eta) - \sum_i^{|L|}\sum_j^{M_k} \frac{\tilde{\zeta}^{i,j}_{k+1} w^{s,j}_k}{\eta} log\left(\frac{1}{\eta}\right) = \mathcal{H}. \quad (26)$$

Given $M^s_k = M_k$ it is also straightforward by Corollary 3 that $\gamma = 0$. As such, using exactly the same derivations as for the lower bound, it immediately holds that $\mathcal{H} = \mathcal{UB}\left[\mathcal{H}\right]$. This completes the proof. ■

E. Proof of Theorem 3

We write η explicitly and first marginalize over all data association realizations and states at time $k+1$. We then marginalize over all hypotheses from time k and apply the chain rule multiple times

$$\eta \triangleq \mathbb{P}\left(Z_{k+1}|H^-_{k+1}\right) = \sum_i^{|L|}\sum_j^{M_k}\int_{x_{k+1}} \mathbb{P}\left(Z_{k+1}|x_{k+1}\right)\mathbb{P}\left(\beta^i_{k+1}|x_{k+1}\right) \cdot \mathbb{P}\left(x_{k+1}|\beta^j_{1:k}, H^-_{k+1}\right)\mathbb{P}\left(\beta^j_{1:k}|H^-_{k+1}\right) = \sum_i^{|L|}\sum_j^{M_k} \tilde{\zeta}^{i,j}_{k+1} w^j_k. \quad (27)$$

Using similar derivations and the key observation that $\tilde{\zeta}^{s,ij}_{k+1} = \tilde{\zeta}^{i,j}_{k+1}$, we also write η^s as

$$\eta^s = \sum_i^{|L|}\sum_j^{M^s_k} \tilde{\zeta}^{i,j}_{k+1} w^{s,j}_k. \quad (28)$$

Splitting (27) based on belief components from M^s_k and using (11) and (28) completes the proof. ■

F. Proof of Theorem 4

As all weights are positive by definition, removing the last term in (19) we immediately get the lower bound. For the upper bound, we rewrite the second term in (19) using $\tilde{\zeta}^{i,j}_{k+1}$

$$\sum_i^{|L|}\sum_j^{\neg M^s_k} \tilde{\zeta}^{i,j}_{k+1} w^j_k = \sum_j^{\neg M^s_k} w^j_k \sum_i^{|L|}\int_{x_{k+1}} \mathbb{P}\left(Z_{k+1}|\beta^i_{k+1}, x_{k+1}\right) \cdot \mathbb{P}\left(\beta^i_{k+1}|x_{k+1}\right)\mathbb{P}\left(x_{k+1}|H^-_{k+1}, \beta^j_{1:k}\right). \quad (29)$$

The joint measurement likelihood term, given in (7), is a product of probability distribution functions, all given by (1) and can thus be bounded using an a priori known maximum value σ. The term $\mathbb{P}\left(\beta^i_{k+1}|x_{k+1}\right)$ represents the probability for the ith data association realization given x_{k+1}, i.e. the probability of observing a specific set of landmarks. As we assume the map to be given, it can be bounded using some constant α^i, e.g. in the case of a camera, it can be an indicator function for landmarks that are within the field of view. Finally, for every hypothesis j it holds that $\int_{x_{k+1}} \mathbb{P}\left(x_{k+1}|H^-_{k+1}, \beta^j_{1:k}\right) = 1$. Substituting these and $w^{m,s}_k$ back into (19) completes the proof. ■

G. Proof of Corollary 3

Given $M^s_k = M_k$ it holds by definition that $w^{m,s}_k = 1$ and $\eta = \eta^s$ as $b_{k+1} = b^s_{k+1}$. Replacing these back into (20), (21) immediately completes the proof. ■

References

[1] Saurav Agarwal, Amirhossein Tamjidi, and Suman Chakravorty. Motion planning for active data association and localization in non-gaussian belief spaces. In *Intl. Workshop on the Algorithmic Foundations of Robotics (WAFR)*, 2020.

[2] Shaunak D Bopardikar, Brendan Englot, Alberto Speranzon, and Jur van den Berg. Robust belief space planning under intermittent sensing via a maximum eigenvalue-based bound. *IJRR*, 35(13):1609–1626, 2016.

[3] L. Carlone, A. Censi, and F. Dellaert. Selecting good measurements via l1 relaxation: A convex approach for robust estimation over graphs. In *IEEE/RSJ Intl. Conf. on Intelligent Robots and Systems (IROS)*, pages 2667–2674. IEEE, 2014.

[4] T.M. Cover and J.A. Thomas. *Elements of Information Theory*. John Wiley & Sons, New York, NY, 1991.

[5] F. Dellaert. Factor graphs and GTSAM: A hands-on introduction. Technical Report GT-RIM-CP&R-2012-002, Georgia Institute of Technology, September 2012.

[6] Khen Elimelech and Vadim Indelman. Simplified decision making in the belief space using belief sparsification. *Intl. J. of Robotics Research*, 2021. Accepted.

[7] M. Hsiao and M. Kaess. Mh-isam2: Multi-hypothesis isam using bayes tree and hypo-tree. In *IEEE Intl. Conf. on Robotics and Automation (ICRA)*, May 2019.

[8] Ming Hsiao, Joshua G Mangelson, Sudharshan Suresh, Christian Debrunner, and Michael Kaess. Aras: Ambiguity-aware robust active slam based on multi-hypothesis state and map estimations. In *IEEE/RSJ Intl. Conf. on Intelligent Robots and Systems (IROS)*, pages 5037–5044. IEEE, 2020.

[9] V. Indelman. No correlations involved: Decision making under uncertainty in a conservative sparse information space. *IEEE Robotics and Automation Letters (RA-L)*, 1(1):407–414, 2016.

[10] V. Indelman, L. Carlone, and F. Dellaert. Planning in the continuous domain: a generalized belief space approach for autonomous navigation in unknown environments. *Intl. J. of Robotics Research*, 34(7):849–882, 2015.

[11] V. Indelman, E. Nelson, J. Dong, N. Michael, and F. Dellaert. Incremental distributed inference from arbitrary poses and unknown data association: Using collaborating robots to establish a common reference. *IEEE Control Systems Magazine (CSM), Special Issue on Distributed Control and Estimation for Robotic Vehicle Networks*, 36(2):41–74, 2016.

[12] V. Indelman, E. Nelson, N. Michael, and F. Dellaert. Multi-robot pose graph localization and data association from unknown initial relative poses via expectation maximization. In *IEEE Intl. Conf. on Robotics and Automation (ICRA)*, 2014.

[13] L. P. Kaelbling, M. L. Littman, and A. R. Cassandra. Planning and acting in partially observable stochastic domains. *Artificial intelligence*, 101(1):99–134, 1998.

[14] M. Kaess, H. Johannsson, R. Roberts, V. Ila, J. Leonard, and F. Dellaert. iSAM2: Incremental smoothing and mapping using the Bayes tree. *Intl. J. of Robotics Research*, 31(2):217–236, Feb 2012.

[15] A. Kitanov and V. Indelman. Topological multi-robot belief space planning in unknown environments. In *IEEE Intl. Conf. on Robotics and Automation (ICRA)*, 2018.

[16] A. Kitanov and V. Indelman. Topological information-theoretic belief space planning with optimality guarantees. *arXiv preprint arXiv:1903.00927*, 2019.

[17] F.R. Kschischang, B.J. Frey, and H-A. Loeliger. Factor graphs and the sum-product algorithm, February 2001.

[18] E. Olson and P. Agarwal. Inference on networks of mixtures for robust robot mapping. *Intl. J. of Robotics Research*, 32(7):826–840, 2013.

[19] C. Papadimitriou and J. Tsitsiklis. The complexity of markov decision processes. *Mathematics of operations research*, 12(3):441–450, 1987.

[20] S. Pathak, A. Thomas, and V. Indelman. A unified framework for data association aware robust belief space planning and perception. *Intl. J. of Robotics Research*, 32(2-3):287–315, 2018.

[21] M. Shienman, A. Kitanov, and V. Indelman. Ft-bsp: Focused topological belief space planning. *IEEE Robotics and Automation Letters (RA-L)*, 6(3):4744–4751, July 2021.

[22] N. Sunderhauf and P. Protzel. Towards a robust back-end for pose graph slam. In *IEEE Intl. Conf. on Robotics and Automation (ICRA)*, pages 1254–1261. IEEE, 2012.

[23] Ori Sztyglic and Vadim Indelman. Online pomdp planning via simplification. *arXiv preprint arXiv:2105.05296*, 2021.

[24] V. Tchuiev, Y. Feldman, and V. Indelman. Data association aware semantic mapping and localization via a viewpoint-dependent classifier model. In *IEEE/RSJ Intl. Conf. on Intelligent Robots and Systems (IROS)*, 2019.

[25] J. Van Den Berg, S. Patil, and R. Alterovitz. Motion planning under uncertainty using iterative local optimization in belief space. *Intl. J. of Robotics Research*, 31(11):1263–1278, 2012.

[26] Andrey Zhitnikov and Vadim Indelman. Probabilistic loss and its online characterization for simplified decision making under uncertainty. *arXiv preprint arXiv:2105.05789*, 2021.