Semantic Distributed Multi-Robot Classification, Localization, and Mapping With a Viewpoint Dependent Classifier Model

Supplementary Material

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This document provides supplementary material to [1]. Therefore, it should not be considered a self-contained document, but instead regarded as an appendix of [1]. Throughout this report, all notations and definitions are with compliance to the ones presented in [1].

1 Supplementary Derivation

We present a relation that is used in equation derivation; Let A be a random variable conditioned on the set $\{B_i\}$ of random variables B_i that are independent from each other. By using Bayes Law, we can split the conditional probability $\mathbb{P}(A|\{B_i\})$ to a product of conditional probabilities:

$$\mathbb{P}(A|\{B_i\}) = \frac{\mathbb{P}(\{B_i\}|A)\mathbb{P}(A)}{\mathbb{P}(\{B_i\})} = \frac{\prod_i \mathbb{P}(B_i|A)}{\prod_i \mathbb{P}(B_i)} \mathbb{P}(A). \tag{1}$$

Using Bayes Law again on each element in the product, we reach the following expression:

$$\mathbb{P}(A|\{B_i\}) = \prod_i \left(\frac{\mathbb{P}(A|B_i)}{\mathbb{P}(A)}\right) \mathbb{P}(A). \tag{2}$$

This allow to express a random variable as a multiplication of conditionals, which will be useful to separate local and external measurements.

2 Derivation of $\mathbb{P}(\mathcal{X}_k^R \backslash x_k^r | C_k^R, \mathcal{H}_k^{R-})$

Recall splitting \mathcal{H}_k^{R-} into prior history \mathcal{H}_{k-1}^{R} and non-local measurements & actions:

$$\mathcal{H}_k^{R-} = \mathcal{H}_{k-1}^R \cup \Delta \mathcal{H}_k^{R-}. \tag{3}$$

We then use the above definition and relation (2) to split $\mathbb{P}(\mathcal{X}_k^R \setminus x_k^r | C_k^R, \mathcal{H}_k^{R-})$ into a product of two beliefs, one that depends on prior history, and one that depends on external new measurements:

$$\mathbb{P}(\mathcal{X}_k^R \backslash x_k^r | C_k^R, \mathcal{H}_k^{R-}) = \mathbb{P}(\mathcal{X}_k^R \backslash x_k^r | C_k^R) l \frac{\mathbb{P}(\mathcal{X}_k^R \backslash x_k^r | C_k^R, \Delta \mathcal{H}_k^{R-})}{\mathbb{P}(\mathcal{X}_k^R \backslash x_k^r | C_k^R)} \frac{\mathbb{P}(\mathcal{X}_k^R \backslash x_k^r | C_k^R, \mathcal{H}_{k-1}^R)}{\mathbb{P}(\mathcal{X}_k^R \backslash x_k^r | C_k^R)}. \tag{4}$$

This formulation allows us to isolate the new information sent by other robots at time k, from information already used for inference at previous times. Next, we have to address that not all known objects are present in the sent local beliefs. Because the priors are assumed independent between poses and classes, $\mathbb{P}(\mathcal{X}_k^R \setminus x_k^r | C_k^R) = \mathbb{P}(\mathcal{X}_k^R \setminus x_k^r)$. From $\mathbb{P}(\mathcal{X}_k^R \setminus x_k^r | C_k^R, \mathcal{H}_{k-1}^R)$ we can split $\mathcal{X}_k^R \setminus x_k^r$ into poses of objects that are involved in \mathcal{H}_{k-1}^R and ones that do not as:

$$\mathbb{P}(\mathcal{X}_k^R \setminus x_k^r | C_k^R, \mathcal{H}_{k-1}^R) = \mathbb{P}(\mathcal{X}_{\text{new},k}^{o,R} | C_{\text{new},k}^R, \mathcal{H}_{k-1}^R) \mathbb{P}(\mathcal{X}_{k-1}^R | C_k^R, \mathcal{H}_{k-1}^R). \tag{5}$$

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Poses of objects that r wasn't aware of at time k-1 are independent of \mathcal{H}_{k-1}^R , and without measurements, $\mathcal{X}_{\text{new},k}^{o,R}$ are independent of $C_{\text{new},k}^R$ as well. In addition, \mathcal{X}_{k-1}^R is independent of classes of objects that are observed only at time k, thus we can write:

$$\mathbb{P}(\mathcal{X}_k^R \setminus x_k^r | C_k^R, \mathcal{H}_{k-1}^R) = \mathbb{P}(\mathcal{X}_{\text{new},k}^{o,R}) \cdot \mathbb{P}(\mathcal{X}_{k-1}^R | C_{k-1}^R, \mathcal{H}_{k-1}^R), \tag{6}$$

which is the prior for poses of newly known objects at time step k, multiplied by the conditional continuous belief for objects already known. Similarly to Eq. (6), the prior for $\mathcal{X}_k^R \setminus x_k^r$ is separated to previously known and new objects:

$$\mathbb{P}(\mathcal{X}_k^R \backslash x_k^r) = \mathbb{P}(\mathcal{X}_{\text{new } k}^{o,R}) \mathbb{P}(\mathcal{X}_{k-1}^R), \tag{7}$$

therefore we can write:

$$\frac{\mathbb{P}(\mathcal{X}_k^R \setminus x_k^r | C_k^R, \mathcal{H}_{k-1}^R)}{\mathbb{P}(\mathcal{X}_k^R \setminus x_k^r | C_k^R)} = \frac{b_{k-1}^R}{\mathbb{P}(\mathcal{X}_{k-1}^R)},\tag{8}$$

and substitute it into the rightmost fracture in Eq. (4). Then we cancel out $\mathbb{P}(\mathcal{X}_k^R \backslash x_k^r | C_k^R)$ and proceed to remove r's poses from the prior by:

$$\frac{\mathbb{P}(\mathcal{X}_k^R \backslash x_k^r | C_k^R, \Delta \mathcal{H}_k^{R-})}{\mathbb{P}(\mathcal{X}_{k-1}^R)} = \frac{\mathbb{P}(\mathcal{X}_k^{o,R} | C_k^R, \Delta \mathcal{H}_k^{R-})}{\mathbb{P}(\mathcal{X}_{k-1}^{o,R})},\tag{9}$$

as robot r's poses up until time k-1 are independent from the new external measurements. Finally, after factoring out $\mathbb{P}(\mathcal{X}_k^R \setminus x_k^r | C_k^R)$, and Eq. (8) and (9) with Eq. (4) we reach the following expression that is used in the paper:

$$\mathbb{P}(\mathcal{X}_k^R \backslash x_k^r | C_k^R, \mathcal{H}_k^{R-}) = b_{k-1}^R \cdot \frac{\mathbb{P}(\mathcal{X}_k^{o,R} | C_k^{o,R}, \Delta \mathcal{H}_k^{R-})}{\mathbb{P}(\mathcal{X}_{k-1}^{o,R})}$$

$$(10)$$

3 Derivation of $\frac{\mathbb{P}(\mathcal{X}_k^{o,R}|C_k^R,\Delta\mathcal{H}_k^{R-})}{\mathbb{P}(\mathcal{X}_k^{o,R})}$ (Continuous Belief Update)

Using the relation (2) we can split the blue part by separating the new measurements and actions per robot, excluding r itself as it is not present in $\Delta \mathcal{H}_k^{R-}$:

$$\mathbb{P}(\mathcal{X}_k^{o,R}|C_k^R, \Delta\mathcal{H}_k^{R-}) = \prod_{k',r'\in R\setminus r} \left(\frac{\mathbb{P}(\mathcal{X}_{k'}^{o,R}|C_{k'}^{r'}, \Delta\mathcal{H}_{k'}^{r'})}{\mathbb{P}(\mathcal{X}_{k'}^{o,R})}\right) \mathbb{P}(\mathcal{X}_k^{o,R}). \tag{11}$$

From that, we will address every element in the product. Poses of objects that r' doesn't observe locally can be canceled out as follows, leaving only the object poses that r' observed:

$$\frac{\mathbb{P}(\mathcal{X}_{k'}^{o,R}|C_{k'}^{r'}, \Delta\mathcal{H}_{k'}^{r'})}{\mathbb{P}(\mathcal{X}_{k'}^{o,R})} = \frac{\mathbb{P}(\mathcal{X}_{k'}^{o,r'}|C_{k'}^{r'}, \Delta\mathcal{H}_{k'}^{r'})}{\mathbb{P}(\mathcal{X}_{k'}^{o,r'})}.$$
(12)

Then, using relation (2) again, we can expand $\mathbb{P}(\mathcal{X}_{k'}^{o,r'}|C_{k'}^{r'},\mathcal{H}_{k'}^{r'})$ to separate between known prior and new measurements:

$$\mathbb{P}(\mathcal{X}_{k'}^{o,r'}|C_{k'}^{r'},\mathcal{H}_{k'}^{r'}) = \mathbb{P}(\mathcal{X}_{k'}^{o,r'}) \frac{\mathbb{P}(\mathcal{X}_{k'}^{o,r'}|C_{k'}^{r'},\mathcal{H}_{k'-l'}^{r'})}{\mathbb{P}(\mathcal{X}_{k'}^{o,r'})} \frac{\mathbb{P}(\mathcal{X}_{k'}^{o,r'}|C_{k'}^{r'},\Delta\mathcal{H}_{k'}^{r'})}{\mathbb{P}(\mathcal{X}_{k'}^{o,r'})},$$
(13)

with l' being the time difference between subsequent slots of r (that can be 0 if the slot isn't updated). Then we take out all the poses that aren't dependent on the prior information, and we reach the definition of $\xi_{k-1}^{r,r'}$, i.e. the marginal object poses at the previous time.

$$\frac{\mathbb{P}(\mathcal{X}_{k'}^{o,r'}|C_{k'}^{r'},\mathcal{H}_{k'-l'}^{r'})}{\mathbb{P}(\mathcal{X}_{k'}^{o,r'})} = \frac{\mathbb{P}(\mathcal{X}_{k'-l'}^{o,r'}|C_{k'-l'}^{r'},\mathcal{H}_{k'-l'}^{r'})}{\mathbb{P}(\mathcal{X}_{k'-l'}^{o,r'})} \doteq \xi_{k-1}^{r,r'}.$$
(14)

With the definition of $\xi_k^{r,r'}$, and by substituting Eq. (14) into Eq. (13) we reach the expression for a single element of the product in Eq. (11):

$$\frac{\mathbb{P}(\mathcal{X}_{k'}^{o,r'}|C_{k'}^{r'}, \Delta\mathcal{H}_{k'}^{r'})}{\mathbb{P}(\mathcal{X}_{k'}^{o,r'})} = \frac{\xi_k^{r,r'}}{\xi_{k-1}^{r,r'}}.$$
(15)

Finally, substituting Eq. (15) we reach the expression for the external continuous update belief:

$$\frac{\mathbb{P}(\mathcal{X}_k^{o,R}|C_k^R, \Delta\mathcal{H}_k^{R-})}{\mathbb{P}(\mathcal{X}_k^{o,R})} = \prod_{r' \in R} \frac{\xi_k^{r,r'}}{\xi_{k-1}^{r,r'}}.$$
(16)

4 Derivation of $\frac{\mathbb{P}(C_k^R|\Delta\mathcal{H}_k^{R-})}{\mathbb{P}(C_k^R)}$ (Discrete Belief Update)

The discrete belief update is similar to the continuous in its derivation, with probability over class realization, rather than object poses. Again, using the relation (2) we can split the red part by separating the new measurements and actions per robot, excluding r itself as it is not present in $\Delta \mathcal{H}_k^{R-}$:

$$\mathbb{P}(C_k^R | \Delta \mathcal{H}_k^{R-}) = \prod_{k', r' \in R \setminus r} \left(\frac{\mathbb{P}(C_{k'}^R | \Delta \mathcal{H}_{k'}^{r'})}{\mathbb{P}(C_{k'}^R)} \right) \mathbb{P}(C_{k'}^R)$$
(17)

From that, we will address every element in the product. Classes of objects that r' doesn't observe locally can be canceled out as follows, leaving only the classes of object that r' observed:

$$\frac{\mathbb{P}(C_{k'}^R|\Delta\mathcal{H}_{k'}^{r'})}{\mathbb{P}(C_{k'}^R)} = \frac{\mathbb{P}(C_{k'}^{r'}|\Delta\mathcal{H}_{k'}^{r'})}{\mathbb{P}(C_{k'}^{r'})}.$$
(18)

Then, using relation (2) again, we can expand $\mathbb{P}(C_{k'}^{r'}|\mathcal{H}_{k'}^{r'})$ to separate between known prior and new measurements:

$$\mathbb{P}(C_{k'}^{r'}|\mathcal{H}_{k'}^{r'}) = \mathbb{P}(C_{k'}^{r'}) \frac{\mathbb{P}(C_{k'}^{r'}|\mathcal{H}_{k'-l'}^{r'})}{\mathbb{P}(C_{k'}^{r'})} \frac{\mathbb{P}(C_{k'}^{r'}|\Delta\mathcal{H}_{k'}^{r'})}{\mathbb{P}(C_{k'}^{r'})}, \tag{19}$$

with l' being the time difference between subsequent slots of r (that can be 0 if the slot isn't updated). Then we take out all the classes of objects that not appear in prior information, and we reach the definition of $\phi_{k-1}^{r,r'}$, i.e. the marginal object poses at the previous time.

$$\frac{\mathbb{P}(C_{k'}^{r'}|\mathcal{H}_{k'-l'}^{r'})}{\mathbb{P}(C_{k'}^{r'})} = \frac{\mathbb{P}(C_{k'-l'}^{r'}|\mathcal{H}_{k'-l'}^{r'})}{\mathbb{P}(C_{k'-l'}^{r'})} \doteq \phi_{k-1}^{r,r'}$$
(20)

With the definition of $\phi_k^{r,r'}$, and by substituting Eq. (20) into Eq. (19) we reach the expression for a single element of the product in Eq. (17):

$$\frac{\mathbb{P}(C_{k'}^{r'}|\Delta\mathcal{H}_{k'}^{r'})}{\mathbb{P}(C_{k'}^{r'})} = \frac{\phi_k^{r,r'}}{\phi_{k-1}^{r,r'}}$$
(21)

Finally, substituting Eq. (21) we reach the expression for the external continuous update belief:

$$\frac{\mathbb{P}(C_k^R | \Delta \mathcal{H}_k^{R-})}{\mathbb{P}(C_k^R)} = \prod_{r' \in R} \frac{\phi_k^{r,r'}}{\phi_{k-1}^{r,r'}}.$$
 (22)

5 Dependency Between Object Classes

In this section we show that the classes of two objects are not independent. We present a simple example where c_1 and c_2 be the underlying classes of objects 1 and 2 respectively. Let \mathcal{H}^R be the total measurement history, including semantic measurements z_1^{sem} and z_2^{sem} for objects 1 and 2 respectively. Recall that measurements are assumed independent from each other. Using the Bayes Law:

$$\mathbb{P}(c_1, c_2 | \mathcal{H}^R) \propto \mathbb{P}(c_1, c_2 | \mathcal{H}^R \setminus z_1^{sem}, z_2^{sem}) \mathbb{P}(z_1^{sem} | c_1) \mathbb{P}(z_2^{sem} | c_2). \tag{23}$$

We use a viewpoint dependent classifier model, so we must marginalize $\mathbb{P}(z_1^{sem}|c_1)\mathbb{P}(z_2^{sem}|c_2)$ by the corresponding relative viewpoints, denoted x_1^{rel} and x_2^{rel} respectively:

$$\mathbb{P}(z_1^{sem}|c_1)\mathbb{P}(z_2^{sem}|c_2) = \int_{x_1^{rel}, x_1^{rel}} \mathbb{P}(z_1^{sem}|c_1, x_1^{rel})\mathbb{P}(z_2^{sem}|c_2, x_2^{rel})\mathbb{P}(x_1^{rel}, x_2^{rel}|\mathcal{H}^R) dx_1^{rel} dx_2^{rel}. \tag{24}$$

From the above equation, the condition for c_1 and c_2 to be independent is that x_1^{rel} and x_2^{rel} must be independent, which is not true in the general case, thus c_1 and c_2 are dependent.

6 Simulation: Table of Stack Time Stamps

In this section we present a table of stack time stamps that indicates direct and indirect communication between robots in our scenario. Recall that the maximal communication radius is 5 meters, thus robots r_2 and r_3 communicate between times k=4 to k=24, robots r_1 and r_2 communicate between times k=21 and k=26.

Time step	Stack of r_1	Stack of r_2	Stack of r_3
k = 1	$t(r_1)$: 1	$t(r_1): 0$	$t(r_1): 0$
	$t(r_2): 0$	$t(r_2)$: 1	$t(r_2): 0$
	$t(r_3): 0$	$t(r_3): 0$	$t(r_3)$: 1
k=2	$t(r_1): 2$	$t(r_1): 0$	$t(r_1): 0$
	$t(r_2): 0$	$t(r_2)$: 2	$t(r_2): 0$
	$t(r_3): 0$	$t(r_3): 0$	$t(r_3): 2$
	$t(r_1)$: 3	$t(r_1): 0$	$t(r_1): 0$
k=3	$t(r_2): 0$	$t(r_2)$: 3	$t(r_2): 0$
	$t(r_3): 0$	$t(r_3): 0$	$t(r_3): 3$
k=4	$t(r_1): 4$	$t(r_1): 0$	$t(r_1): 0$
	$t(r_2): 0$	$t(r_2)$: 4	$t(r_2): 3$
	$t(r_3): 0$	$t(r_3): 3$	$t(r_3): 4$
k=5	$t(r_1)$: 5	$t(r_1): 0$	$t(r_1): 0$
	$t(r_2): 0$	$t(r_2)$: 5	$t(r_2)$: 4
	$t(r_3): 0$	$t(r_3): 4$	$t(r_3): 5$
k=6	$t(r_1)$: 6	$t(r_1): 0$	$t(r_1)$: 0
	$t(r_2): 0$	$t(r_2)$: 6	$t(r_2)$: 5
	$t(r_3): 0$	$t(r_3)$: 5	$t(r_3)$: 6
	$t(r_1)$: 7	$t(r_1): 0$	$t(r_1): 0$
k = 7	$t(r_2): 0$	$t(r_2)$: 7	$t(r_2)$: 6
	$t(r_3): 0$	$t(r_3)$: 6	$t(r_3): 7$
k = 8	$t(r_1): 8$	$t(r_1): 0$	$t(r_1)$: 0
	$t(r_2): 0$	$t(r_2)$: 8	$t(r_2)$: 7
	$t(r_3): 0$	$t(r_3)$: 7	$t(r_3): 8$
1 0	$t(r_1): 9$	$t(r_1): 0$	$t(r_1): 0$
k=9	$t(r_2)$: 0	$t(r_2)$: 9	$t(r_2)$: 8
	$t(r_3): 0$	$t(r_3): 8$	$t(r_3): 9$
1 10	$t(r_1): 10$	$t(r_1): 0$	$t(r_1): 0$
k = 10	$t(r_2): 0$	$t(r_2)$: 10	$t(r_2)$: 9
	$t(r_3): 0$	$t(r_3): 9$	$t(r_3): 10$
k = 11	$t(r_1): 11$	$t(r_1): 0$	$t(r_1): 0$
	$t(r_2): 0$	$t(r_2)$: 11	$t(r_2)$: 10
	$t(r_3): 0$	$t(r_3)$: 10	$t(r_3)$: 11
$l_{b} = 19$	$t(r_1): 12$	$t(r_1): 0$ $t(r_2): 12$	$t(r_1): 0$
k = 12	$t(r_2)$: 0	, ,	$t(r_2)$: 11 $t(r_3)$: 12
	$t(r_3): 0$	$t(r_3)$: 11 $t(r_1)$: 0	$t(r_3)$: 12 $t(r_1)$: 0
k = 13	$\begin{array}{c c} t(r_1) \colon 13 \\ t(r_2) \colon 0 \end{array}$	$t(r_1)$: 0 $t(r_2)$: 13	$t(r_1)$: 0 $t(r_2)$: 12
	$t(r_3): 0$	$t(r_2)$: 13 $t(r_3)$: 12	$t(r_3)$: 13
k = 14	$t(r_3)$: 14	$t(r_3)$: 12 $t(r_1)$: 0	$t(r_3)$: 13 $t(r_1)$: 0
	$t(r_1): 14$ $t(r_2): 0$	$t(r_1)$: 14	$t(r_2)$: 13
	$t(r_3): 0$	$t(r_3)$: 13	$t(r_3)$: 14
	$t(r_1)$: 15	$t(r_1): 0$	$t(r_1): 0$
k = 15	$t(r_2): 0$	$t(r_2)$: 15	$t(r_2)$: 14
	$t(r_3): 0$	$t(r_3)$: 14	$t(r_3)$: 15
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$k = 16 \qquad \begin{array}{c} & t(r_1): 16 \\ & t(r_2): 0 \\ & t(r_2): 16 \\ & t(r_3): 15 \\ & t(r_3): 15 \\ & t(r_3): 16 \\ & t(r_3): 15 \\ & t(r_3): 16 \\ & t(r_2): 17 \\ & t(r_2): 16 \\ & t(r_3): 16 \\ & t(r_2): 17 \\ & t(r_2): 16 \\ & t(r_3): 16 \\ & t(r_3): 16 \\ & t(r_3): 17 \\ & t(r_3): 17 \\ & t(r_3): 17 \\ & t(r_3): 18 \\ & t(r_2): 19 \\ & t(r_3): 17 \\ & t(r_3): 18 \\ & t(r_3): 17 \\ & t(r_3): 18 \\ & t(r_3): 19 \\ & t(r_3): 10 \\ & t(r_3): 19 \\ & t(r_3): 10 \\ & t(r_3): 19 \\ & t(r_3): 10 \\ & t(r_$	Time step	Stack of r_1	Stack of r_2	Stack of r_3
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$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			l ' '	
$k = 17 \qquad \begin{array}{c} t(r_1) \colon 17 \\ t(r_2) \colon 0 \\ t(r_3) \colon 0 \\ t(r_3) \colon 16 \\ t(r_3) \colon 17 \\ t(r_1) \colon 18 \\ t(r_1) \colon 18 \\ t(r_1) \colon 19 \\ t(r_2) \colon 17 \\ t(r_3) \colon 18 \\ t(r_2) \colon 17 \\ t(r_3) \colon 10 \\ t(r_3) \colon 17 \\ t(r_3) \colon 18 \\ t(r_2) \colon 18 \\ t(r_2) \colon 17 \\ t(r_3) \colon 18 \\ t(r_1) \colon 19 \\ t(r_1) \colon 19 \\ t(r_1) \colon 19 \\ t(r_1) \colon 19 \\ t(r_2) \colon 19 \\ t(r_2) \colon 19 \\ t(r_2) \colon 19 \\ t(r_2) \colon 19 \\ t(r_3) \colon 18 \\ t(r_3) \colon 19 \\ t(r_1) \colon 19 \\ t(r_1) \colon 19 \\ t(r_1) \colon 19 \\ t(r_1) \colon 19 \\ t(r_2) \colon 19 \\ t(r_2) \colon 19 \\ t(r_2) \colon 19 \\ t(r_3) \colon 19 \\ t(r_3) \colon 19 \\ t(r_3) \colon 19 \\ t(r_1) \colon 20 \\ t(r_1) \colon 21 \\ t(r_1) \colon 20 \\ t(r_1) \colon 21 \\ t(r_1) \colon 22 \\ t(r_1) \colon 21 \\ t(r_1) \colon 20 \\ t(r_1) \colon 21 \\ t(r_1) \colon 22 \\ t(r_1) \colon 21 \\ t(r_2) \colon 22 \\ t(r_2) \colon 23 \\ t(r_2) \colon 22 \\ t(r_3) \colon 23 \\ t(r_1) \colon 22 \\ t(r_1) \colon 21 \\ t(r_1) \colon 22 \\ t(r_2) \colon 23 \\ t(r_2) \colon 24 \\ t(r_2) \colon 23 \\ t(r_2) \colon 24 \\ t(r_1) \colon 23 \\ t(r_1) \colon 24 \\ t(r_1) \colon 24 \\ t(r_1) \colon 25 \\ t(r_1) \colon 24 \\ t(r_1) \colon 24 \\ t(r_1) \colon 26 \\ t(r_1) \colon 24 \\ t(r_3) \colon 23 \\ t(r_1) \colon 24 \\ t(r_3) \colon 23 \\ t(r_1) \colon 26 \\ t(r_1) \colon 24 \\ t(r_3) \colon 23 \\ t(r_1) \colon 26 \\ t(r_1) \colon 24 \\ t(r_3) \colon 23 \\ t(r_1) \colon 26 \\ t(r_1) \colon 24 \\ t(r_3) \colon 23 \\ t(r_1) \colon 26 \\ t(r_1) \colon 24 \\ t(r_3) \colon 23 \\ t(r_1) \colon 26 \\ t(r_1) \colon 24 \\ t(r_3) \colon 23 \\ t(r_1) \colon 26 \\ t(r_1) \colon 24 \\ t(r_3) \colon 23 \\ t(r_1) \colon 26 \\ t(r_1) \colon 24 \\ t(r_3) \colon 23 \\ t(r_1) \colon 26 \\ t(r_1) \colon 24 \\ t(r_3) \colon 24 \\ t(r_3) \colon 29 \\ t(r_1) \colon 24 \\ t(r_3) \colon 24 \\ t(r_3) \colon 29 \\ t(r_1) \colon 24 \\ t(r_3) \colon 24 \\ t($				\ /
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$k = 18 \qquad t(r_1): 18 \qquad t(r_2): 0 \qquad t(r_2): 18 \qquad t(r_2): 17 \qquad t(r_3): 0 \qquad t(r_3): 17 \qquad t(r_3): 18 \qquad t(r_1): 0 \qquad t(r_2): 18 \qquad t(r_2): 19 \qquad t(r_2): 18 \qquad t(r_3): 0 \qquad t(r_3): 18 \qquad t(r_3): 19 \qquad t(r_3): 18 \qquad t(r_3): 19 \qquad t(r_1): 0 \qquad t(r_1): 0 \qquad t(r_1): 0 \qquad t(r_1): 0 \qquad t(r_2): 0 \qquad t(r_2): 20 \qquad t(r_2): 19 \qquad t(r_3): 0 \qquad t(r_3): 19 \qquad t(r_3): 20 \qquad t(r_3): 19 \qquad t(r_3): 20 \qquad t(r_1): 19 \qquad t(r_3): 20 \qquad t(r_3): 21 \qquad t(r_2): 20 \qquad t(r_3): 21 \qquad t(r_3): 0 \qquad t(r_3): 21 \qquad t(r_3): 21 \qquad t(r_3): 0 \qquad t(r_3): 21 \qquad t(r_1): 22 \qquad t(r_3): 21 \qquad t(r_1): 22 \qquad t(r_2): 21 \qquad t(r_3): 22 \qquad t(r_3): 23 \qquad t(r_3): 21 \qquad t(r_3): 22 \qquad t(r_3): 23 \qquad t(r_3): 22 \qquad t(r_3): 23 \qquad t(r_1): 22 \qquad t(r_1): 21 \qquad t(r_3): 22 \qquad t(r_3): 23 \qquad t(r_1): 22 \qquad t(r_2): 23 \qquad t(r_3): 23 \qquad t(r_3): 24 \qquad t(r_1): 24 \qquad t(r_1): 23 \qquad t(r_1): 22 \qquad t(r_3): 23 \qquad t(r_3): 24 \qquad t(r_1): 25 \qquad t(r_1): 24 \qquad t(r_1): 23 \qquad t(r_1): 24 \qquad t(r_1): 24 \qquad t(r_1): 23 \qquad t(r_1): 24 \qquad t(r_1): 24 \qquad t(r_1): 24 \qquad t(r_1): 23 \qquad t(r_1): 24 \qquad t(r_1): 25 \qquad t(r_1): 24 \qquad t(r_1): 24 \qquad t(r_1): 26 \qquad t(r_1): 24 \qquad t(r_1): 26 \qquad t(r_1): 24 \qquad t(r_1): 28 \qquad t(r_1): 26 \qquad t(r_1): 24 \qquad t(r_1): 28 \qquad t(r_1): 26 \qquad t(r_1): 24 \qquad t(r_1): 29 \qquad t(r_1): 26 \qquad t(r_1): 24 \qquad t(r_1): 29 \qquad t(r_1): 26 \qquad t(r_1): 24 \qquad t(r_1): 29 \qquad t(r_1): 26 \qquad t(r_1): 24 \qquad t(r_1): 29 \qquad t(r_1): 26 \qquad t(r_1): 24 \qquad t(r_1): 29 \qquad t(r_1): 26 \qquad t(r_1): 24 \qquad t(r_1): 29 \qquad t(r_1): 26 \qquad t(r_1): 24 \qquad t(r_1): 29 \qquad t(r_1): 26 \qquad t(r_1): 24 \qquad t(r_2): 26 \qquad t(r_2): 29 \qquad t(r_2): 24 \qquad t(r_1): 20 \qquad t(r_1): 20 \qquad t(r_1): 24 \qquad t(r_2): 26 \qquad t(r_2): 29 \qquad t(r_2): 24 \qquad t(r_2): 24 $				
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$k = 19 \qquad \begin{array}{c} \mathbf{t}(r_1) \colon 19 & \mathbf{t}(r_2) \colon 0 & \mathbf{t}(r_2) \colon 19 & \mathbf{t}(r_2) \colon 18 \\ \mathbf{t}(r_3) \colon 0 & \mathbf{t}(r_3) \colon 18 & \mathbf{t}(r_3) \colon 19 \\ \end{array}$ $\mathbf{t}(r_1) \colon 20 & \mathbf{t}(r_1) \colon 0 & \mathbf{t}(r_1) \colon 0 \\ \mathbf{t}(r_2) \colon 0 & \mathbf{t}(r_2) \colon 20 & \mathbf{t}(r_2) \colon 19 \\ \mathbf{t}(r_3) \colon 0 & \mathbf{t}(r_3) \colon 19 & \mathbf{t}(r_3) \colon 20 \\ \end{array}$ $\mathbf{t}(r_1) \colon 21 & \mathbf{t}(r_1) \colon 20 & \mathbf{t}(r_1) \colon 19 \\ \mathbf{t}(r_3) \colon 0 & \mathbf{t}(r_3) \colon 19 & \mathbf{t}(r_3) \colon 20 \\ \end{array}$ $\mathbf{t}(r_1) \colon 21 & \mathbf{t}(r_1) \colon 20 & \mathbf{t}(r_1) \colon 19 \\ \mathbf{t}(r_3) \colon 0 & \mathbf{t}(r_3) \colon 20 & \mathbf{t}(r_3) \colon 21 \\ \end{array}$ $\mathbf{t}(r_1) \colon 22 & \mathbf{t}(r_1) \colon 21 & \mathbf{t}(r_1) \colon 20 \\ \mathbf{t}(r_3) \colon 0 & \mathbf{t}(r_3) \colon 20 & \mathbf{t}(r_3) \colon 21 \\ \end{array}$ $\mathbf{t}(r_1) \colon 22 & \mathbf{t}(r_1) \colon 21 & \mathbf{t}(r_1) \colon 20 \\ \mathbf{t}(r_3) \colon 20 & \mathbf{t}(r_3) \colon 21 & \mathbf{t}(r_3) \colon 22 \\ \end{array}$ $\mathbf{t}(r_1) \colon 23 & \mathbf{t}(r_1) \colon 22 & \mathbf{t}(r_1) \colon 21 \\ \mathbf{t}(r_3) \colon 20 & \mathbf{t}(r_3) \colon 21 & \mathbf{t}(r_3) \colon 22 \\ \end{array}$ $\mathbf{t}(r_1) \colon 23 & \mathbf{t}(r_1) \colon 22 & \mathbf{t}(r_1) \colon 21 \\ \mathbf{t}(r_3) \colon 20 & \mathbf{t}(r_3) \colon 21 & \mathbf{t}(r_3) \colon 22 \\ \end{array}$ $\mathbf{t}(r_1) \colon 23 & \mathbf{t}(r_1) \colon 22 & \mathbf{t}(r_1) \colon 21 \\ \mathbf{t}(r_3) \colon 21 & \mathbf{t}(r_3) \colon 22 & \mathbf{t}(r_3) \colon 23 \\ \end{array}$ $\mathbf{t}(r_1) \colon 23 & \mathbf{t}(r_1) \colon 23 & \mathbf{t}(r_1) \colon 24 \\ \mathbf{t}(r_3) \colon 24 & \mathbf{t}(r_1) \colon 23 \\ \mathbf{t}(r_3) \colon 22 & \mathbf{t}(r_3) \colon 23 & \mathbf{t}(r_3) \colon 24 \\ \end{array}$ $\mathbf{t}(r_1) \colon 24 & \mathbf{t}(r_1) \colon 23 & \mathbf{t}(r_3) \colon 24 \\ \mathbf{t}(r_3) \colon 23 & \mathbf{t}(r_3) \colon 24 & \mathbf{t}(r_3) \colon 25 \\ \end{array}$ $\mathbf{t}(r_1) \colon 26 & \mathbf{t}(r_1) \colon 24 & \mathbf{t}(r_1) \colon 24 \\ \mathbf{t}(r_3) \colon 23 & \mathbf{t}(r_3) \colon 24 & \mathbf{t}(r_3) \colon 26 \\ \end{array}$ $\mathbf{t}(r_1) \colon 26 & \mathbf{t}(r_1) \colon 26 & \mathbf{t}(r_1) \colon 24 \\ \mathbf{t}(r_3) \colon 23 & \mathbf{t}(r_3) \colon 24 & \mathbf{t}(r_3) \colon 26 \\ \end{array}$ $\mathbf{t}(r_1) \colon 28 & \mathbf{t}(r_1) \colon 26 & \mathbf{t}(r_1) \colon 24 \\ \mathbf{t}(r_3) \colon 23 & \mathbf{t}(r_3) \colon 24 & \mathbf{t}(r_3) \colon 27 \\ \end{array}$ $\mathbf{t}(r_1) \colon 28 & \mathbf{t}(r_1) \colon 26 & \mathbf{t}(r_1) \colon 24 \\ \mathbf{t}(r_3) \colon 23 & \mathbf{t}(r_3) \colon 24 & \mathbf{t}(r_3) \colon 28 \\ $ $\mathbf{t}(r_1) \colon 29 & \mathbf{t}(r_1) \colon 26 & \mathbf{t}(r_1) \colon 24 \\ \mathbf{t}(r_3) \colon 23 & \mathbf{t}(r_3) \colon 24 & \mathbf{t}(r_3) \colon 28 \\ $ $\mathbf{t}(r_1) \colon 29 & \mathbf{t}(r_1) \colon 26 & \mathbf{t}(r_1) \colon 24 \\ \mathbf{t}(r_3) \colon 23 & \mathbf{t}(r_3) \colon 24 & \mathbf{t}(r_3) \colon 29 \\ \mathbf{t}(r_1) \colon 30 & \mathbf{t}(r_1) \colon 26 & \mathbf{t}(r_1) \colon 24 \\ \mathbf{t}(r_3) \colon 23 & \mathbf{t}(r_3) \colon 24 \\ \mathbf{t}(r_3) \colon 23 & \mathbf{t}(r_3) \colon 24 \\ \mathbf{t}(r_3) \colon 24 \\ \mathbf{t}($				
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$k = 20 \qquad \begin{array}{ccccccccccccccccccccccccccccccccccc$				` ′
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	k = 20	, ,	1 ' '	1 1
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$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	k = 21			
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		` '		
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$k = 23 \qquad \begin{array}{ccccccccccccccccccccccccccccccccccc$	$\kappa = 22$			
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$k = 24 \qquad \begin{array}{ccccccccccccccccccccccccccccccccccc$	k = 23			
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$k = 25 \qquad \begin{array}{ccccccccccccccccccccccccccccccccccc$	k = 24			
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$k = 26 \qquad \begin{array}{ccccccccccccccccccccccccccccccccccc$	k=25			
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$k = 27 \qquad \begin{array}{ccccccccccccccccccccccccccccccccccc$	k = 26			
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$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			$t(r_3)$: 24	$t(r_3)$: 27
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	k = 28	$t(r_1)$: 28	$t(r_1)$: 26	$t(r_1)$: 24
$k = 29$ $t(r_1): 29$ $t(r_1): 26$ $t(r_2): 26$ $t(r_2): 29$ $t(r_2): 24$ $t(r_3): 23$ $t(r_3): 24$ $t(r_3): 29$ $t(r_1): 30$ $t(r_1): 26$ $t(r_1): 24$ $k = 30$ $t(r_2): 26$ $t(r_2): 30$ $t(r_2): 24$				
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			$t(r_3)$: 24	
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$k = 30$ $t(r_1)$: 30 $t(r_1)$: 26 $t(r_1)$: 24 $t(r_2)$: 26 $t(r_2)$: 30 $t(r_2)$: 24			$t(r_2)$: 29	
$k = 30$ $t(r_2)$: 26 $t(r_2)$: 30 $t(r_2)$: 24		$t(r_3)$: 23	$t(r_3)$: 24	$t(r_3)$: 29
$k = 30$ $t(r_2)$: 26 $t(r_2)$: 30 $t(r_2)$: 24	k = 30		$t(r_1)$: 26	$t(r_1)$: 24
		$t(r_2)$: 26	$t(r_2)$: 30	$t(r_2)$: 24

7 Simulation: Additional Results

In this section we present additional results for the simulation. At time step k = 4, we show results for local and distributed cases of robots r_2 and r_3 that start communicating at that time. Similarly we present results at time k = 21 for robot r_1 , as it communicates with r_2 that in turn relays information from r_3 , and r_3 itself. The results for r_2 are presented in the paper.

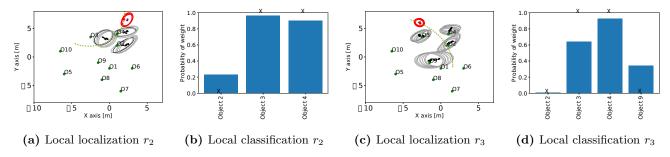


Figure 1: Figures for robot r_2 and r_3 , local beliefs for time k = 4. (a) and (b) show results for r_2 , (c) and (d) for r_3 . (a) and (c) present localization results, (b) and (d) present classification results.

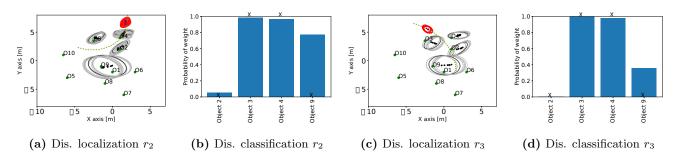


Figure 2: Figures for robot r_2 and r_3 , distributed beliefs for time k = 4. (a) and (b) show results for r_2 , (c) and (d) for r_3 . (a) and (c) present localization results, (b) and (d) present classification results.

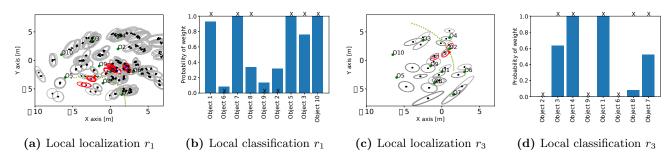


Figure 3: Figures for robot r_1 and r_3 , local beliefs for time k = 21. (a) and (b) show results for r_1 , (c) and (d) for r_3 . (a) and (c) present localization results, (b) and (d) present classification results.

The results of all the graphs support the paper results, where both classification and localization in general are more accurate for the distributed belief. In addition, the robots inferring the distributed belief take into account objects that they didn't observe directly.

References

[1] Vladimir Tchuiev and Vadim Indelman. Semantic distributed multi-robot classification, localization, and mapping with a viewpoint dependent classifier model. In *IEEE Intl. Conf. on Robotics and Automation (ICRA)*, 2020. Submitted.

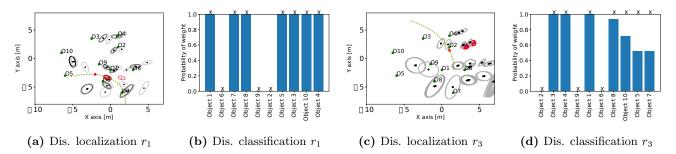


Figure 4: Figures for robot r_1 and r_3 , distributed beliefs for time k = 21. (a) and (b) show results for r_1 , (c) and (d) for r_3 . (a) and (c) present localization results, (b) and (d) present classification results.