`iX-BSP`: Incremental Belief Space Planning with Selective Resampling
Supplementary Material

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This document provides supplementary material to the paper [1]. Therefore, it should not be considered a self-contained document, but instead regarded as an appendix of [1].Throughout this report, all notations and definitions are with compliance to the ones presented in [1]. This report covers a toy example for our novel paradigm `iX-BSP`-Appendix A.

Appendix A: Toy example for `iX-BSP`

To better understand the problem let us consider a toy example. We have access to all calculations from planning time k, in-which we performed `X-BSP` (or `iX-BSP`) for a horizon of 3 steps, with $n_x = 2$ and $n_z = 1$, while considering 2 candidate actions u_1 and u_2. Figure 4a illustrates a specific action sequence, $u_1 \rightarrow u_2 \rightarrow u_1$, considered as part of planning at time k. We are currently at time k+1, performing planning using `iX-BSP` with the same horizon length and number of samples per action, for the action sequence $u_2 \rightarrow u_1 \rightarrow u_2$, see Figure 4b.

Following Alg. 1 line 1, out of the two available beliefs from planning time k, $\{b[X_{k+1|k}]\}_1^2$, the left one is closer to $b[X_{k+1|k+1}]$, so we consider all its descendants as the set $\mathcal{B}_{k+1|k}$, and denote the difference between $b[X_{k+1|k}]$ and $b[X_{k+1|k+1}]$ as $\{\delta\}$. We continue with re-using the beliefs in the set $\mathcal{B}_{k+1|k}$ (Alg. 1 line 3). First we check whether the two available samples from planning time k constitute an adequate representation for $b^-[X_{k+2|k+1}]$; since they are, we flag them both for re-use, and update $\{b[X_{k+2|k}]\}_1^2$ into $\{b[X_{k+2|k+1}]\}_1^2$. Now for the next future time step, we propagate $\{b[X_{k+2|k+1}]\}_1^2$ with action u_1 to obtain $\{b^-[X_{k+3|k+1}]\}_1^2$, and check whether the four available samples from planning time k constitute an adequate representation for $\{b^-[X_{k+3|k+1}]\}_1^2$; since only three of them are, we flag them for re-use and sample the forth one (black colored belief at $k+3|k+1$ in Figure 4b) from $b^-[X_{k+3|k+1}]$. We then update $\{b[X_{k+3|k}]\}_1^3$ into $\{b[X_{k+3|k+1}]\}_1^3$, and $b^-[X_{k+3|k+1}]$ into $b^4[X_{k+3|k+1}]$ using the newly sampled measurement. The last step of the horizon $k+4|k+1$ is calculated using `X-BSP` (Alg. 1 line 4).

At this point we have all inference results for all beliefs along the action sequence $u_2 \rightarrow u_1 \rightarrow u_2$, so we can calculate all reward(cost) values for this action sequence for planning at time k+1. For the look ahead at time $k+2$ of planning session at time $k+1$, i.e. $k+2|k+1$, we have two reward(cost) values, $\{r_{k+2|k+1}(b[X_{k+2|k+1}], u_2)\}_1^2$, each calculated with a different belief $b[X_{k+2|k+1}]$ considering a different sample $z_{k+2|k}$. Calculating the expected reward(cost) value for future time step $k+2|k+1$ would mean in this case, using measurements sampled from $\mathbb{P}(z_{k+2|k}|H_{k+1|k}, u_2)$ rather then from $\mathbb{P}(z_{k+2|k+1}|H_{k+1|k+1}, u_2)$. This problem, of performing estimation using forced samples is called importance sampling. Since for a single time step we might have samples from multiple different distributions, e.g. future time $k+3|k+1$ in Figure 4b, our problem falls within the special case of Multiple Importance Sampling. Using the formulation of multiple importance sampling using the balance heuristic (8) we can write down the estimation for the expected reward value at planning time $k+2|k+1$,

$$\mathbb{E}\left[r_{k+2|k+1}(.)\right] \sim \frac{1}{2} \frac{p_1(z_{k+2|k}^1)}{\frac{2}{2}q_1(z_{k+2|k}^1)} \cdot r_{k+2|k+1}^1(.) + \frac{1}{2} \frac{p_1(z_{k+2|k}^2)}{\frac{2}{2}q_1(z_{k+2|k}^2)} \cdot r_{k+2|k+1}^2(.), \tag{13}$$

where $p_1(.) \doteq \mathbb{P}(z_{k+2|k+1}|H_{k+1|k+1}, u_2)$ and $q_1(.) \doteq \mathbb{P}(z_{k+2|k}|H_{k+1|k}, u_2)$. In the same manner, following (9), we can also write down the estimation for the expected reward(cost) value at look ahead step $k+3$ from planning session

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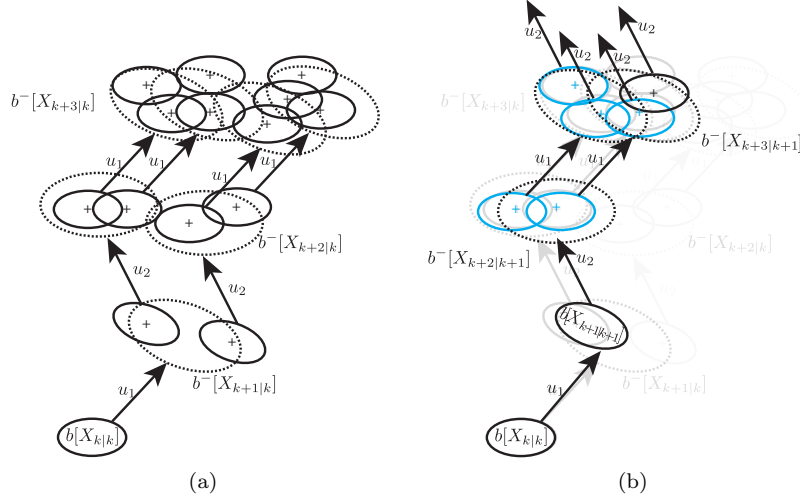
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Figure 4: Consider BSP presented in a spatial belief propagation diagram, with propagated beliefs denoted by dashed line ellipse, samples denoted by $+$, actions denoted by arrows, beliefs denoted by solid line ellipse and with $n_x = 2$ and $n_z = 1$. (a) Presents X-BSP at planning time k for the action sequence $u_1 \rightarrow u_2 \rightarrow u_1$, as such all measurements were sampled from their original distributions. Assuming action u_1 have been executed, (b) presents iX-BSP for the succeeding planning time $k + 1$, for the action sequence $u_2 \rightarrow u_1 \rightarrow u_2$, where re-used samples and corresponding re-used beliefs are denoted in blue, and freshly sampled measurements and corresponding freshly calculated beliefs are denoted in black.

at time $k + 1$, i.e. $k + 3|k + 1$,

$$\mathbb{E}\left[r_{k+3|k+1}(.)\right] \sim \frac{1}{4} \frac{p_2(z^1_{k+2:k+3|k})}{\frac{3}{4}q_2(z^1_{k+2:k+3|k}) + \frac{1}{4}p_2(z^1_{k+2:k+3|k})} r^1_{k+3|k+1}(.) + \frac{1}{4} \frac{p_2(z^2_{k+2:k+3|k})}{\frac{3}{4}q_2(z^2_{k+2:k+3|k}) + \frac{1}{4}p_2(z^2_{k+2:k+3|k})} r^2_{k+3|k+1}(.) +$$

$$\frac{1}{4} \frac{p_2(z^3_{k+2:k+3|k})}{\frac{3}{4}q_2(z^3_{k+2:k+3|k}) + \frac{1}{4}p_2(z^3_{k+2:k+3|k})} r^3_{k+3|k+1}(.) + \frac{1}{4} \frac{p_2(z^4_{k+2:k+3|k+1})}{\frac{3}{4}q_2(z^4_{k+2:k+3|k+1}) + \frac{1}{4}p_2(z^4_{k+2:+3|k+1})} r^4_{k+3|k+1}(.), \quad (14)$$

where $p_2(.) \doteq \mathbb{P}(z_{k+2:k+3|k+1}|H_{k+1|k+1}, u_2, u_1)$ and $q_2(.) \doteq \mathbb{P}(z_{k+2:k+3|k}|H_{k+1|k}, u_2, u_1)$. When considering

$$\mathbb{P}(z_{k+1:k+L|k}|H_{k|k}, u_{k:k+L-1}) = \prod_{i=k+1}^{k+L} \mathbb{P}(z_{i|k}|H^-_{i|k}) \quad (15)$$

we can re-write the measurement likelihood from (14) into a product of measurement likelihoods per look ahead step, e.g. $p_2(z^1_{k+2:k+3|k}) = p_1(z^1_{k+2|k})\tilde{p}_2(z^1_{k+3|k})$, when $p_1(.)$ need not be calculated at look ahead step $k + 3$, since it is given from (13).

References

[1] E. Farhi and V. Indelman. ix-bsp: Incremental belief space planning with selective resampling. In *IEEE Intl. Conf. on Robotics and Automation (ICRA)*, May 2020. Submitted.