



Vision-Aided Navigation Based on Three-View Geometry

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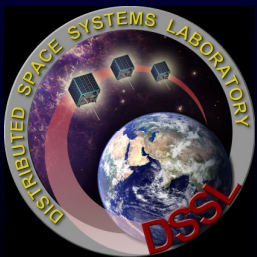
- Introduction
- Three-view geometry constraints
- Fusion with navigation system
- Results
- Summary





Introduction

- Pure inertial navigation solution diverges with time due to error integration
- Inertial solution may be compensated to eliminate or at least reduce errors, using e.g.:
 - ◆ GPS
 - ◆ DTM
 - ◆ Vision-based methods (VBM)
- GPS may be\become unavailable or unreliable
- Objective:
 - ◆ Position estimation assuming **only** INS and a **single** onboard camera
 - ★ Recover scale ambiguity
 - ◆ Real-time navigation aiding
 - ★ Handling loop scenarios





Previous Work

- SLAM: “6DoF SLAM aided GNSS/INS Navigation in GNSS Denied and Unknown Environments”, Kim J. and Sukkarieh S., 2005
- Multiple-view + Bundle adjustment: “A Dual-Layer Estimator Architecture for Long-term Localization”, Mourikis A.I. and Roumeliotis S.I., 2008
- Trifocal tensor + Motion Estimation: “Recursive Camera-Motion Estimation With the Trifocal Tensor”, Yu Y.K., et. al., 2006
- Enhanced-two-view + Constant-size state vector: “Real-Time Mosaic-Aided Aerial Navigation”, Indelman V., et. al., 2009





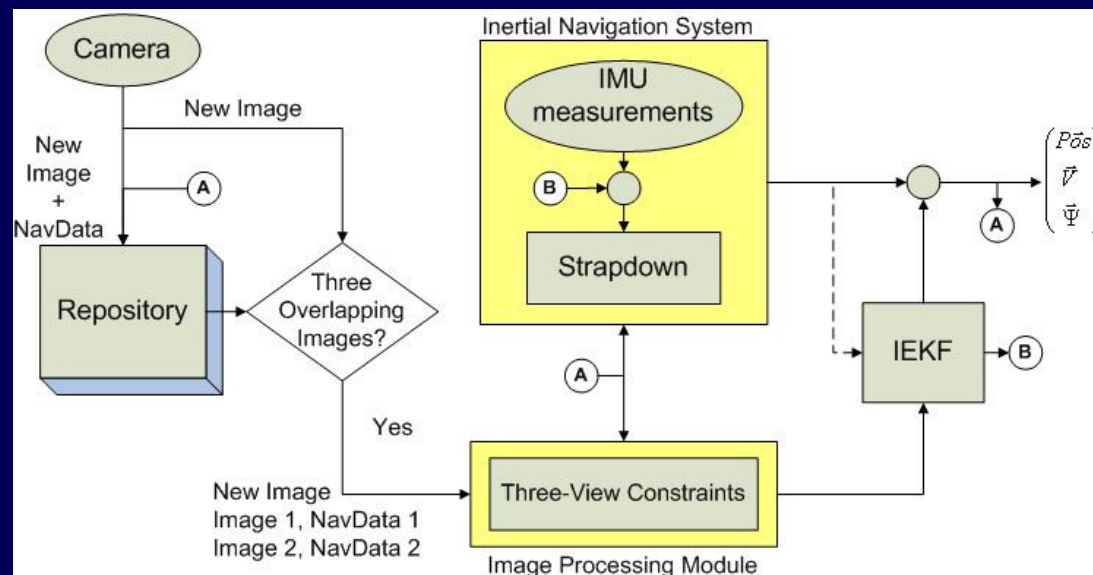
Introduction (Cont.)

Approach

- ◆ Decouple navigation aiding and environment representation construction (e.g. mosaic) – in contrast to SLAM
 - ★ Constant-size state vector
 - ★ Store and manage captured images (and some nav. data)
- ◆ Three-view geometry
- ◆ Avoid structure reconstruction

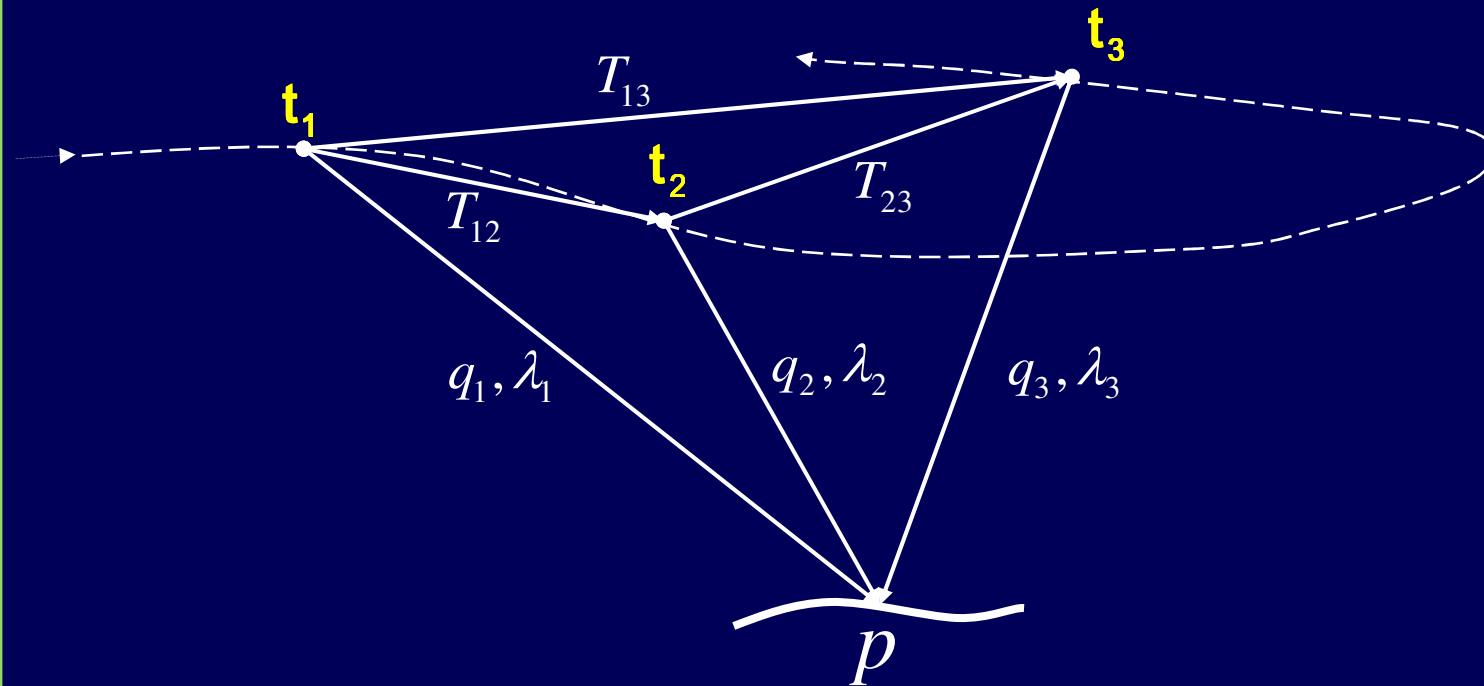
Assumptions

- ◆ INS
- ◆ Camera

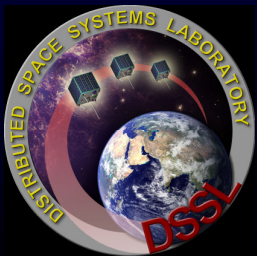




Three View Geometry



- p - ground landmark
- λ - range
- q - line of sight (LOS)
- T_{ij} - translation from i to j





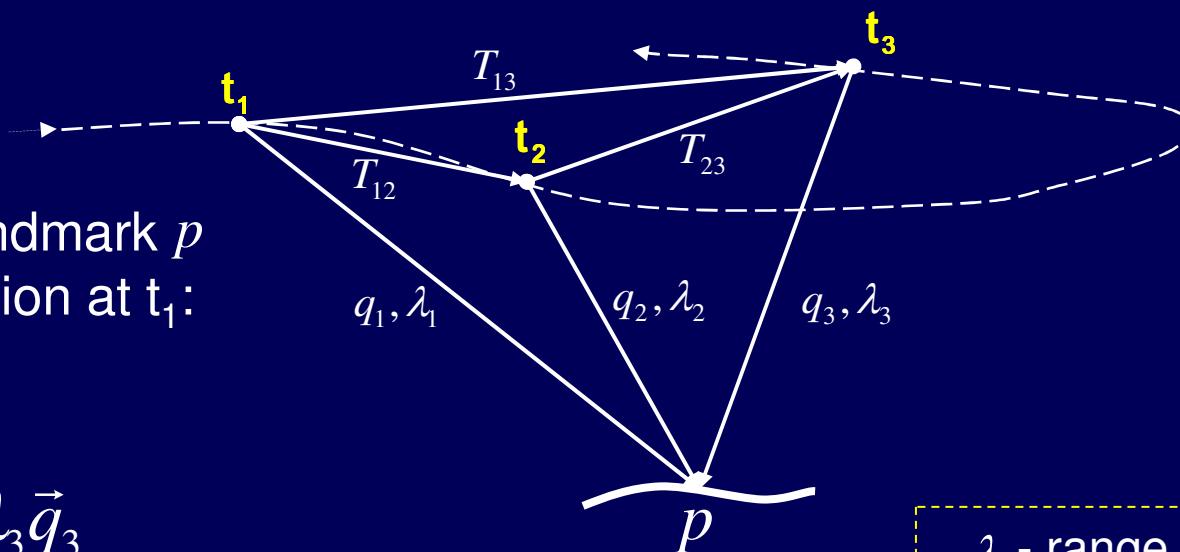
Three View Geometry (cont.)

- Coordinate systems
 - ◆ L - Local Level Local North (LLLN)
 - ◆ C - Camera

- Position of a ground landmark p relative to camera position at t_1 :

$$\lambda_1 \vec{q}_1 = \vec{T}_{12} + \lambda_2 \vec{q}_2$$

$$\lambda_1 \vec{q}_1 = \vec{T}_{12} + \vec{T}_{23} + \lambda_3 \vec{q}_3$$



λ - range
 q - LOS

- Expressing in LLLN system at t_2 :

$$\lambda_1 C_{L_2}^{C_1} \vec{q}_1^{C_1} = C_{L_2}^{C_1} \vec{T}_{12}^{C_1} + \lambda_2 C_{L_2}^{C_2} \vec{q}_2^{C_2}$$

$$\lambda_1 C_{L_2}^{C_1} \vec{q}_1^{C_1} = C_{L_2}^{C_1} \vec{T}_{12}^{C_1} + C_{L_2}^{C_2} \vec{T}_{23}^{C_2} + \lambda_3 C_{L_2}^{C_3} \vec{q}_3^{C_3}$$



Three View Geometry (cont.)

Matrix formulation

$$\underbrace{\begin{bmatrix} \vec{q}_1 & -\vec{q}_2 & \vec{0}_{3 \times 1} & -\vec{T}_{12} \\ \vec{0}_{3 \times 1} & \vec{q}_2 & -\vec{q}_3 & -\vec{T}_{23} \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ 1 \end{bmatrix}_{4 \times 1} = \vec{0}_{6 \times 1}$$

Note:

- ◆ Range parameters are unknown
- ◆ LOS and translation vectors are expressed in LLN of t_2

$$\rightarrow \text{rank}(\mathbf{A}) < 4$$

Denote

$\tilde{\mathbf{A}}$ - a matrix comprised of any 4 rows of \mathbf{A}

$$\rightarrow \det(\tilde{\mathbf{A}}) = 0$$





Three View Geometry (cont.)

- The following constraints were obtained for a single ground landmark:

$$\vec{q}_1^T \cdot (\vec{T}_{12} \times \vec{q}_2) = 0$$

$$\vec{q}_2^T \cdot (\vec{T}_{23} \times \vec{q}_3) = 0$$

$$(\vec{q}_2 \times \vec{q}_1)^T \cdot (\vec{q}_3 \times \vec{T}_{23}) = (\vec{q}_1 \times \vec{T}_{12})^T \cdot (\vec{q}_3 \times \vec{q}_2)$$

- First two equations – epipolar constraints
- Third equation – relates between the magnitudes of \vec{T}_{12} and \vec{T}_{23}



$$\begin{bmatrix} \vec{m}^T \end{bmatrix}_{1 \times 3} \vec{T}_{12} = 0$$

$$\begin{bmatrix} \vec{d}^T \end{bmatrix}_{1 \times 3} \vec{T}_{23} = 0$$

$$\begin{bmatrix} \vec{b}^T \end{bmatrix}_{1 \times 3} \vec{T}_{23} = \begin{bmatrix} \vec{c}^T \end{bmatrix}_{1 \times 3} \vec{T}_{12}$$

where

$$\vec{b} = \vec{b}(\vec{q}_1, \vec{q}_2, \vec{q}_3)$$

$$\vec{c} = \vec{c}(\vec{q}_1, \vec{q}_2, \vec{q}_3)$$

$$\vec{d} = \vec{d}(\vec{q}_2, \vec{q}_3)$$

$$\vec{m} = \vec{m}(\vec{q}_1, \vec{q}_2)$$





Three View Geometry (cont.)

■ Multiple features

- ◆ Matching pairs between 1st and 2nd view $\left\{ \vec{q}_{1_i}^{C_1}, \vec{q}_{2_i}^{C_2} \right\}_{i=1}^{N_{12}}$
- ◆ Matching pairs between 2nd and 3rd view $\left\{ \vec{q}_{2_i}^{C_2}, \vec{q}_{3_i}^{C_3} \right\}_{i=1}^{N_{23}}$
- ◆ Matching triplets between the three views $\left\{ \vec{q}_{1_i}^{C_1}, \vec{q}_{2_i}^{C_2}, \vec{q}_{3_i}^{C_3} \right\}_{i=1}^{N_{123}}$

$$\left[\vec{b}_i^T \right]_{1 \times 3} \vec{T}_{23} = \left[\vec{c}_i^T \right]_{1 \times 3} \vec{T}_{12} \quad i = 1, \dots, N_{123}$$

$$\left[\vec{d}_j^T \right]_{1 \times 3} \vec{T}_{23} = 0 \quad j = 1, \dots, N_{23}$$

$$\left[\vec{m}_r^T \right]_{1 \times 3} \vec{T}_{12} = 0 \quad r = 1, \dots, N_{12}$$



$$\begin{bmatrix} B \\ D \\ 0 \end{bmatrix}_{N \times 3} \vec{T}_{23} = \begin{bmatrix} C \\ 0 \\ M \end{bmatrix}_{N \times 3} \vec{T}_{12}$$

$$N = N_{123} + N_{12} + N_{23}$$



Fusion with Navigation

- Implicit Extended Kalman Filter (IEKF) formulation

$$\vec{z} \equiv \begin{bmatrix} B \\ D \\ 0 \end{bmatrix}_{N \times 3} \vec{T}_{23} - \begin{bmatrix} C \\ 0 \\ M \end{bmatrix}_{N \times 3} \vec{T}_{12}$$

- Recall
 - ◆ All original LOS vectors are expressed in camera system of the appropriate view
 - ◆ $\vec{T}_{23}, \vec{T}_{12}$ are functions of $Pos(t_3), Pos(t_2), Pos(t_1)$
 - ◆ t_3 is the current time

$$\vec{z} = h\left(\textcolor{yellow}{Pos}(t_3), \textcolor{yellow}{\Psi}(t_3), Pos(t_2), \Psi(t_2), Pos(t_1), \Psi(t_1), \left\{ \vec{q}_{1_i}^{C_1}, \vec{q}_{2_i}^{C_2}, \vec{q}_{3_i}^{C_3} \right\}\right)$$



Fusion with Navigation (cont.)

- State vector definition

$$\vec{X} = \begin{bmatrix} \Delta \vec{P}^T & \Delta \vec{V}^T & \Delta \vec{\Psi}^T & \vec{d}^T & \vec{b}^T \end{bmatrix}^T \in \mathbb{R}^{15 \times 1}$$

- Continuous system matrix

$$\Phi_c = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & A_s & 0_{3 \times 3} & C_L^B \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & -C_L^B & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} \in \mathbb{R}^{15 \times 15}$$

- ◆ A_s - a skew-matrix constructed based on accelerometer sensors readings
- ◆ C_L^B - DCM from Body to Local Level Local North systems





Fusion with Navigation (cont.)

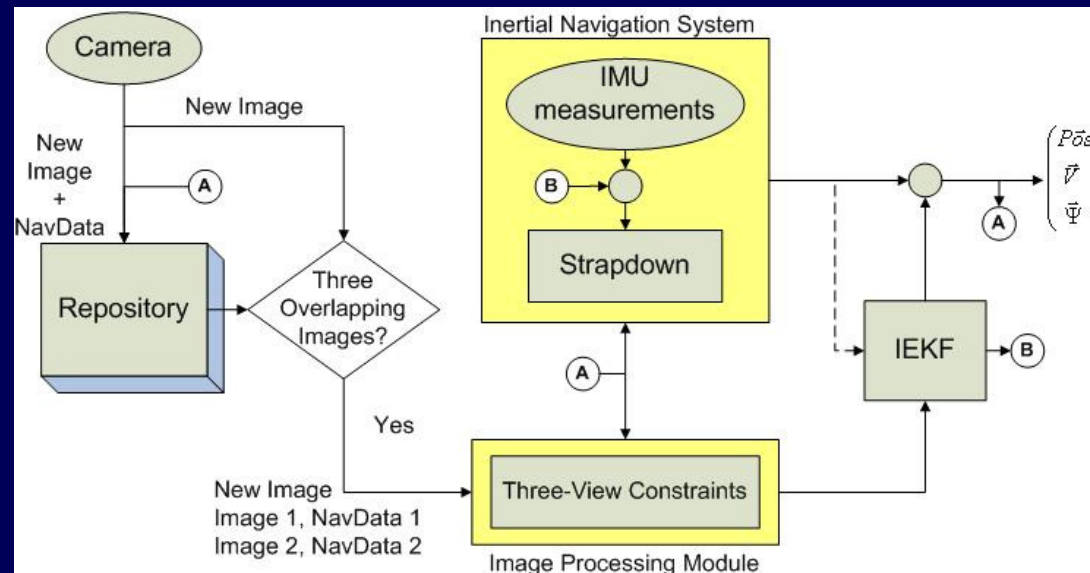
- Standard equations of IEKF update step

$$K_{k+1} = P_{k+1|k} L_{k+1}^T \left[L_{k+1} P_{k+1|k} L_{k+1}^T + D_{k+1} R_{k+1} D_{k+1}^T \right]^{-1}$$

$$\hat{X}_{k+1|k+1} = \hat{X}_{k+1|k} + K_{k+1} \left(\bar{z}_{k+1} - L_{k+1} \hat{X}_{k+1|k} \right)$$

$$P_{k+1|k+1} = \left[I - K_{k+1} L_{k+1} \right] P_{k+1|k} \left[\cdot \right]^T + K_{k+1} D_{k+1} R_{k+1} D_{k+1}^T K_{k+1}^T$$

- Simplified vision-aided navigation scheme





Experiment Results

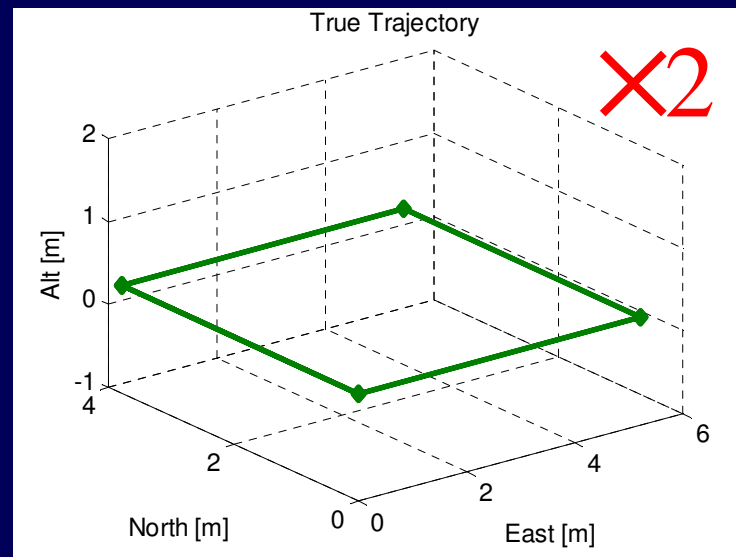
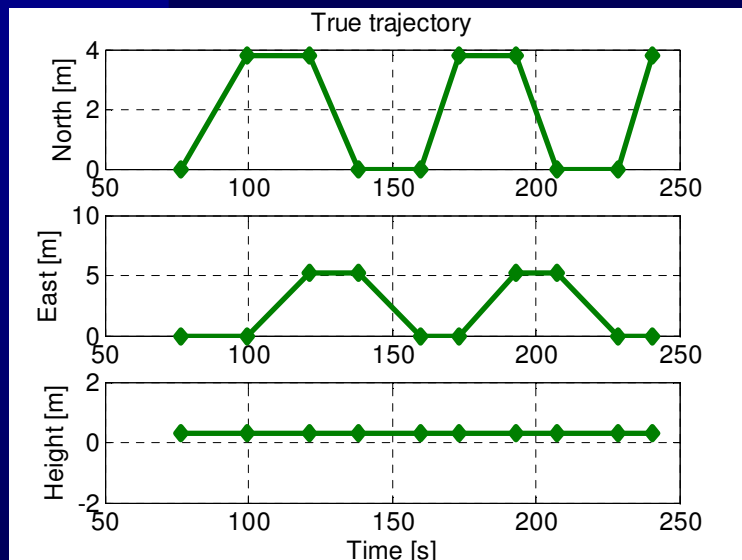
- Experiment Setup
 - ◆ An IMU and a camera were mounted on top of a ground vehicle
 - ◆ IMU\INS: Xsens MTi-G
 - ◆ Camera: Axis 207MW
- IMU data and captured images were stored and synchronized
 - ◆ IMU data @ 100Hz
 - ◆ Imagery data @ 15Hz
- The method was applied in two scenarios
 - ◆ Sequential updates
 - ◆ Loop updates





Experiment Results (Cont.)

True trajectory



Recorded imagery





Experiment Results (Cont.)

Example



Image 1



Image 2



Image 3

Implementation (Matching Triplets)



over the Fundamental matrix model
Image 1 Image 2

★ The Fundamental matrix is not required

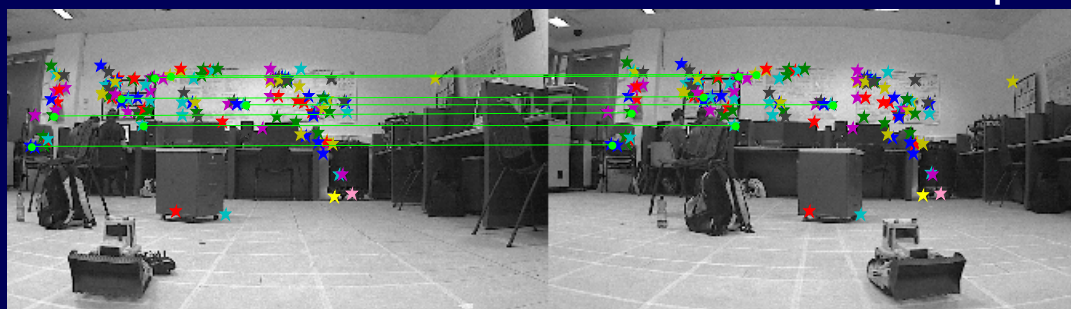


Image 2

Image 3



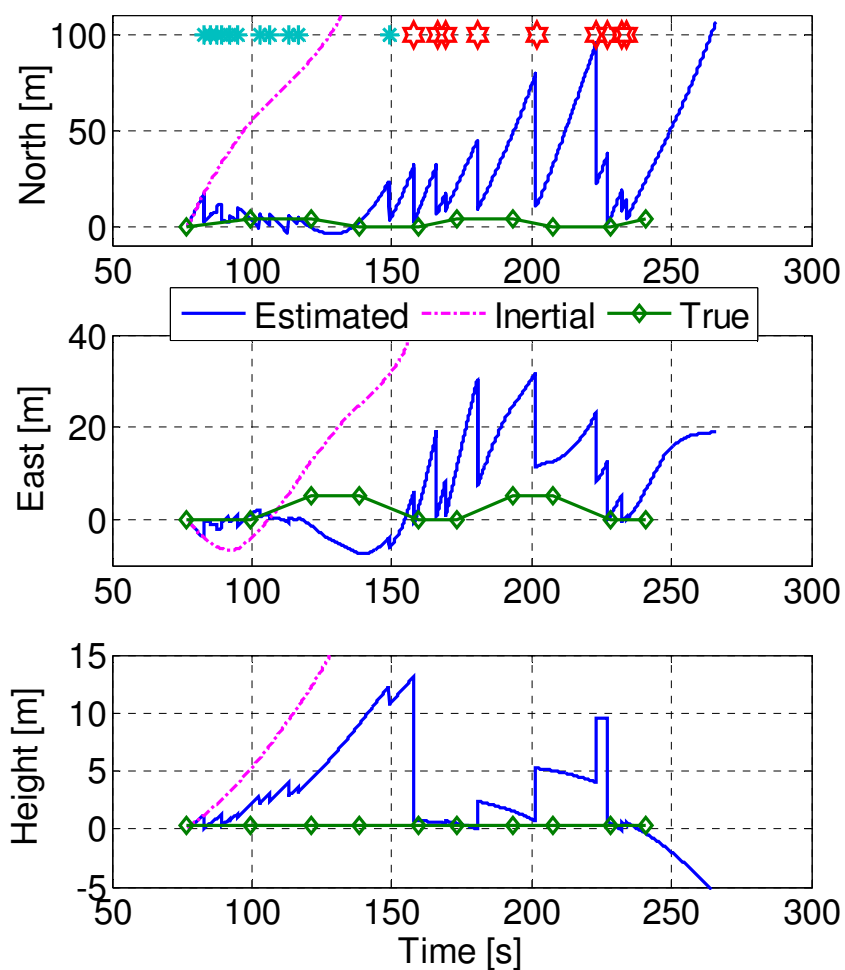
$$\left\{ \vec{q}_{1_i}^{C_1}, \vec{q}_{2_i}^{C_2}, \vec{q}_{3_i}^{C_3} \right\}_{i=1}^{N_{123}}, \left\{ \vec{q}_{1_i}^{C_1}, \vec{q}_{2_i}^{C_2} \right\}_{i=1}^{N_{12}}, \left\{ \vec{q}_{2_i}^{C_2}, \vec{q}_{3_i}^{C_3} \right\}_{i=1}^{N_{23}}$$



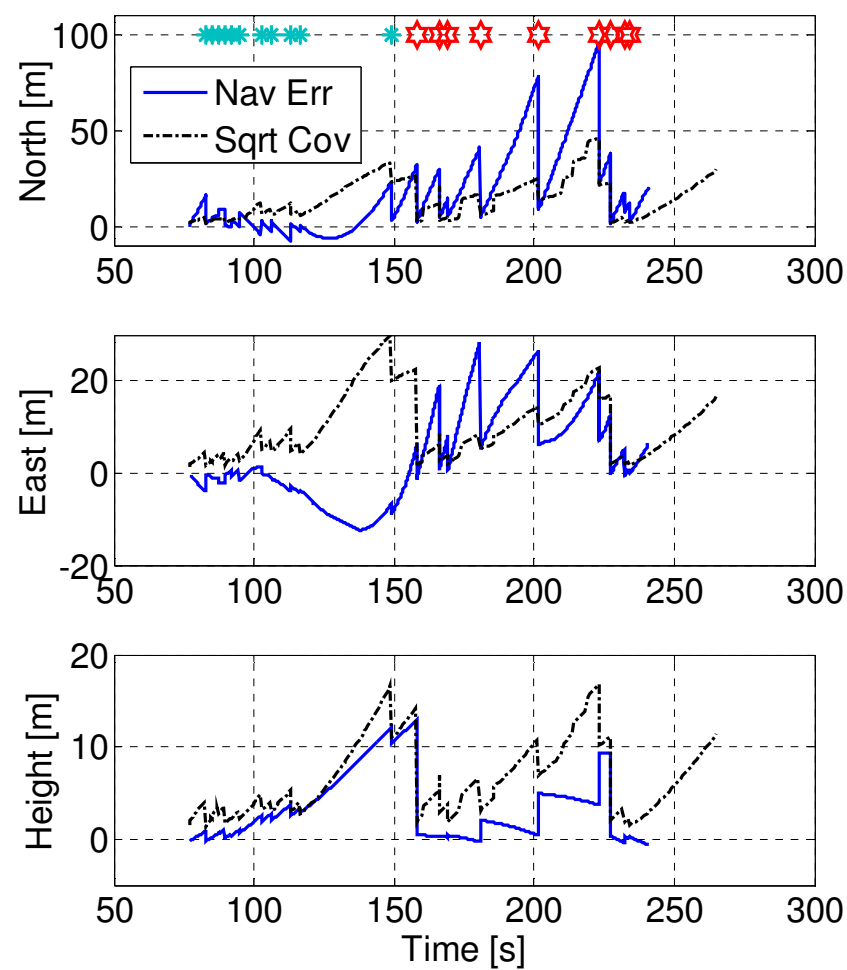
Experiment Results (Cont.)

- * Sequential upd
- * Loop upd

Position in NED



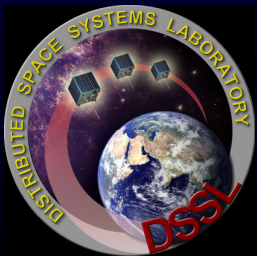
Position Error





Potential Applications

- Navigation aiding in loop scenarios
 - ◆ Additional scenarios with overlapping three images
- Satellite navigation
- Formation flying navigation
- The developed constraints may be used in SLAM framework





Summary

- A new method for vision-aided navigation based on three-view geometry was presented
 - ◆ The method does not require any a-priori information or external sensors, apart from the camera sensor
- The associated navigation data for each of the three images allowed to determine the scale ambiguity
- The method allows to reduce position errors in all axes to the levels of errors present while the first two images were captured
- Reduced computational requirements for vision-aided navigation phase
 - ◆ Environment representation construction (e.g. mosaic) may be executed in a background process
- Various potential applications

