A Research Assignment

MRP CALCULATIONS WHEN LEAD TIME IS FUZZY

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LIST OF ABBREVIATIONS

CP	: Cost performance
CTO	: Configure to order
DFA	: Design for assembly
ECR	: Efficient consumer response
ERP	: Enterprise resource planning
FGP	: Fuzzy goal programming
FP	: Finite planning
HH	: Hand held
HR	: Human resources
LCA	: Life cycle assessment.
LT	: Lead time
M&S	: Modellind and simulation.
MRP	: Material Requirements Planning (MRP)
MRP-II	: Manufacturing resource planning
NPI	: New Product Introduction
WIP	: Work in process

1. INTRODUCTION

1.1 What is MRP (Material Requirements Planning)

“Material requirements planning (MRP) is a production planning and inventory control system which integrates data from production schedules with that from inventory and the bill of materials (BOM) to calculate purchasing and shipping schedules for the parts or components required to build a product”.

MRP was developed by engineer **Joseph Orlicky** as a response to the **Toyota Production System**, the famous model for lean production. The first computerized MRP system was tested successfully by Black & Decker in 1964.

1.2 Primary functions of MRP

The primary functions of MRP are as follows:

1. To ensure the availability of the raw material for production and availability of the finished products to avoid shortage.
2. To reduce wastes by maintaining lowest possible materials and product levels in stock.
3. To help plan manufacturing functions, delivery schedules and purchasing.

1.3 Forms of uncertainties affecting performance of MRP systems:

They are as follows:

1. **Environmental uncertainties** – Includes uncertainty in demand and supply.
2. **System uncertainties** – Includes Fuzzy lead times, Quality uncertainty, Production system uncertainty and change of product structure.

When manufacturer wants to begin production, it is important to estimate lead time accurately. Also, estimated planned lead time will effect on the MRP and MPS respectively. After a proper estimation of lead time, the manufacturer is able to improve delivery performance into the customer.

So our objective is to Provide MRP calculations under fuzzy lead time conditions.

So here we will consider some approaches available to model a Material requirement planning problem with fuzzy lead times.

2. METHOD USED

1. Multi-objective integer linear programming approach.
2. Lead time estimation based on generating fuzzy rules basis and some linguistic rules.
3. Lead time estimation by Monte Carlo simulation (Less superior).

3. DISCUSSIONS

3.1 Multi-objective integer linear programming approach

Following are some assumptions which have followed during the formulation of given problem by above method:

1. A multi-level production system.
2. A multi-product manufacturing environment, Means multiple finished goods, raw material, sub-assemblies structure in bill material etc.
3. Backlog of the demand is defined as the non-negative difference between the cumulated demand and the volume of available product.
4. Constraints on production capacity and programmed reception.
5. Fuzzy lead times for finished goods and raw materials represented by using different values associated with different degree of possibility of each one.
6. Inventory of the each product is the available volume at the end of a given time period.
7. Lead time of the product is the number of consecutive and integer periods that are required for their finalization.
8. Master production schedule (MPS) and MRP are solved jointly.
9. Multi-period planning horizon
10. There are some overtime limits.

For the formulation of our model for MRP,

We will consider some notations that we will be using throughout our formulation, they are given by-

$P_{i,t}$ = Production quantity for product ‘i’ at period ‘t’.

$I_{i,t}$ = Size of inventory for product ‘i’ at time ‘t’.

$CP_{i,t}$ = Production cost.

$CI_{i,t}$ = Inventory holding cost for product ‘i’ at period ‘t’ .

$CTO_{r,t}$ = Overtime cost at facility ‘r’ at time ‘t’.

$CTU_{r,t}$ = Under-time cost of facility 'r' at time 't'.

$TO_{r,t}$ = Overtime at facility 'r' at time 't'.

$TU_{r,t}$ = Under-time of facility 'r' at time 't'.

$B_{i,t}$ = Backorder quantity of product 'i' at time 't'.

$P_{i,t-LT}$ = Production before lead time.

$C_{r,t}$ = Capacity of resource 'r' at time 't'.

We will take following objective functions:-

1. $Min.\ z1 \cong \sum_{i=1}^{I}\sum_{t=1}^{T}(\mathrm{CP}i,t\ \mathrm{P}i,t + \mathrm{CI}i,t\ \mathrm{I}i,t) + \sum_{r=1}^{R}\sum_{t=1}^{T}\mathrm{CTO}r,t\ \mathrm{TO}r,t$

   This objective function represents total cost calculated for all products over entire time period.

2. $Min.\ z2 \cong \sum_{i-1}^{I}\sum_{t=1}^{T}\mathrm{B}i,t$

   This minimizes the backorder quantity over entire planning horizon.

3. $Min.\ z3 \cong \sum_{r=1}^{P}\sum_{t=1}^{T}\mathrm{TU}r,t$

   This objective function minimizes the idle time of the productive resources.

Following constraints are imposed on this model

1. $I_{i,t-1} + P_{i,t-LT} + SR_{it} - I_{i,t} - B_{i,t-1} - \sum_{j=1}^{I}\alpha_{i,j}\left(P_{j,t} + SR_{j,t}\right) + B_{i,t} = d_{i,t}\ \ \forall i,t$
   This is the inventory balance equation for all the products.

2. $\sum_{i=1}^{I} P_{i,t}AR_{i,r} + TU_{r,t} - TO_{r,t} = C_{r,t} \quad \forall i,t$

   This constraint establishes the available capacity for normal, overtime and subcontracted production.

3. $B_{i,j} = 0 \quad \forall i$

   This finishes with delays in last period T of planning horizon.

4. $P_{i,t}, I_{i,t}, B_{i,t}, TU_{i,t}, TO_{i,t} \geq 0 \quad \forall i, r, t$

It contemplates the non-negativity for the decision variables and constraint.

5. $P_{i,t}, I_{i,t}, B_{i,t} \in \mathbb{Z} \quad \forall i, t$

Establishes integrity condition for some of decision variables.

3.2 Solution Methodology

An approach to transform the fuzzy goal programming (FGP) into an equivalent auxiliary crisp mathematical programming model for MRP problems is provided. This approach considers non increasing linear membership functions for each fuzzy objective function as follows-

6. $$M_k = \begin{cases} 1 & z_k < z_k^l \\ \frac{z_k^4 - z_k}{z_k^4 - z_k^l} & z_k^l < z_k < z_k^4 \\ 0 & z_k > z_k^4 \end{cases}$$

Where:

M_k = Membership function of z_k.

Zkl & Zku = Lower and upper bound on objective function Zk.

FGP programming method proposes that a multi-objective model could be transformed in a single objective model as follows:

7. $$\max_{x \in f(x)} \lambda(x) = \gamma\lambda_0 + (1-\gamma)\sum_k Q_k \mu_k(x)$$

Subjected to following constraints-

8. $\lambda_0 \leq M_k(x), \quad k = 1, .., n$
9. $x \in f(x)$

Where:

M_k represents the satisfaction degree of kth objective function.

$\lambda_0 = \min \{M_k(x)\}$ is the minimum satisfaction degree of objectives.

Q_k= Relative importance of kth objective.

γ = coefficient of compensation.

So the equivalent auxiliary crisp mathematical programming model is formulated as follows:

10. $\lambda(x) = \gamma\lambda_0 + (1-\gamma)\left[\theta_1\left(\frac{z_1^4 - z_1}{z_1^4 - z_1^l}\right) + \theta_2\left(\frac{z_2^4 - z_2}{z_2^4 - z_2^l}\right) + \theta_1\left(\frac{z_3^4 - z_3}{z_3^4 - z_3^l}\right)\right]$

Subjected to

11. $\lambda_0 \leq M_1, M_2, M_3$
12. $0 \leq \lambda_0, M_1, M_2, M_3 \leq 1$

Now all possible combinations of lead times are considered along with a possibility degree to the minimum possibility degree of all products in each combination.

Following solution procedure is adopted:-

1. Formulate original Fuzzy Goal Programming (FGP) model for the MRP problem.

2. Specify the corresponding linear membership functions (LMF) for all fuzzy objectives including upper and lower limits.

3. Determine corresponding relative importance of the objective functions (θk) and the coefficient of compensation (γ).

4. Transform the original FGP problem into an equivalent single-objective mixed-integer linear programming (MILP) form using the Torabi and Hassini (2008) fuzzy programming method.

5. Generate problem all possible instances of lead times.

6. Solve the proposed auxiliary crisp single-objective model by using a MILP solver for each problem instance and obtain a fuzzy set of solutions.

7. Defuzzify the obtained solution by applying **the center of gravity method**.

8. Determine the Manhattan and/or the Euclidean distance of each solution to crisp solution.

9. Select the solution with minimum distance to the defuzzified crisp solution.

For better understandings of this procedure, I am giving a numerical example as follows-

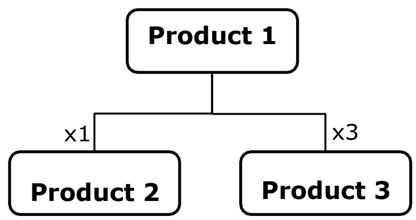
3.3 Example

Source: Díaz-Madroñero, M., Mula, J., & Jiménez, M. (2014). Fuzzy goal programming for material requirements planning under uncertainty and integrity conditions. International Journal of Production Research, 52(23), 6971-6988.

Now The proposed model we have to implement in the MPL language V4.2. The resolution is carried out with CPLEX 12.1.0 solver. The input data and the model solution values should be processed with the Microsoft Access database.

In this numerical example we are considering 21 instances.

Let we have a final product with following product structure:



1. A 30 period planning horizon is considered.
2. Let finished goods have only external demands.
3. The decision variables Pi,t , Ii,t , Bi,t are considered integer.
4. Let firm orders can't be rejected but backlog for finished product is allowed.
5. Fuzzy lead times of component are always higher than or equal to finished good lead times.

Now let we take possible values of lead times for all these three products as-

Product 1: {0/1, 1/0.5, 2/0.2}

Product 2: {1/1, 2/0.7, 3/0.3}

Product 3: {3/1, 4/0.8, 5/0.4}

Using these values following the instances were generated-

Instance	**Lead times**	**Possibility**
I1	{0,1,3}	1
I2	{0,1,4}	0.8
I3	{0,1,5}	0.4
I4	{0,2,3}	0.7
I5	{0,2,4}	0.7

I6	{0,2,5}	0.4
I7	{0,3,3}	0.3
I8	{0,3,4}	0.3
I9	{0,3,5}	0.3
I10	{1,1,3}	0.5
I11	{1,1,4}	0.5
I12	{1,1,5}	0.4
I13	{1,2,3}	0.5
I14	{1,2,4}	0.5
I15	{1,2,5}	0.4
I16	{1,3,3}	0.3
I17	{1,3,4}	0.3
I18	{1,3,5}	0.3
I19	{2,2,3}	0.2
I20	{2,2,4}	0.2
I21	{2,2,5}	0.2
I22	{2,3,3}	0.2
I23	{2,3,4}	0.2
I24	{2,3,5}	0.2

Table:-1

The obtained numerical results are as follows:-

Instance	z_1	z_2	z_3
I1	88716.42	967	1047.06
I2	88706.40	1971	1047.06
I3	88697.99	3512	1047.06
I4	88716.42	967	1047.06
I5	88706.40	1971	1047.06
I6	88697.99	3512	1047.06
I7	88716.42	967	1047.06
I8	88706.40	1971	1047.06
I9	88697.99	3512	1047.06
I10	88716.42	967	1047.06
I11	88706.40	1971	1047.06
I12	88697.99	3512	1047.06
I13	88716.42	967	1047.06
I14	88706.40	1971	1047.06
I15	88697.99	3512	1047.06
I16	88716.42	967	1047.06
I17	88706.40	1971	1047.06

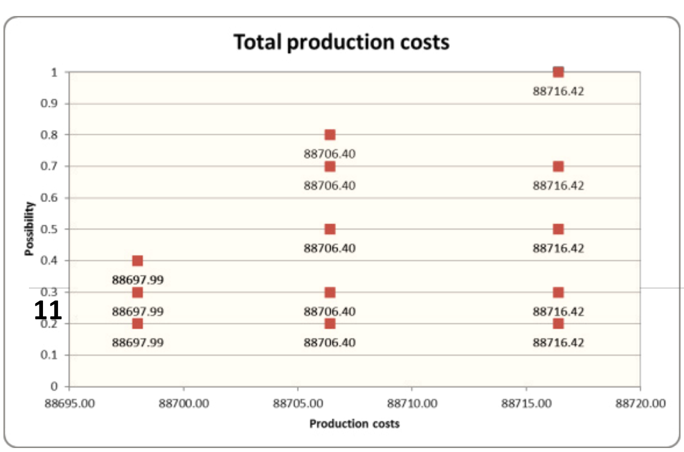
I18	88697.99	3512	1047.06
I19	88716.42	967	1047.06
I20	88706.40	1971	1047.06
I21	88697.99	3512	1047.06
I22	88716.42	967	1047.06
I23	88706.40	1971	1047.06
I24	88697.99	3512	1047.06

Table:-2 Objective function by instance (I1 to I12)

Instance	***µ1***	***µ2***	***µ3***
I1	0.9677	0.9033	0.8547
I2	0.9678	0.8029	0.8547
I3	0.9678	0.6488	0.8547
I4	0.9677	0.9033	0.8547
I5	0.9678	0.8029	0.8547
I6	0.9678	0.6488	0.8547
I7	0.9677	0.9033	0.8547
I8	0.9678	0.8029	0.8547
I9	0.9678	0.6488	0.8547
I10	0.9677	0.9033	0.8547
I11	0.9678	0.8029	0.8547
I12	0.9678	0.6488	0.8547
I13	0.9677	0.9033	0.8547
I14	0.9678	0.8029	0.8547
I15	0.9678	0.6488	0.8547
I16	0.9677	0.9033	0.8547
I17	0.9678	0.8029	0.8547
I18	0.9678	0.6488	0.8547
I19	0.9677	0.9033	0.8547
I20	0.9678	0.8029	0.8547
I21	0.9678	0.6488	0.8547
I22	0.9677	0.9033	0.8547
I23	0.9678	0.8029	0.8547
I24	0.9678	0.6488	0.8547

Table:-2 Objective function by instance (I13 to I24)

These results can be shown graphically as below:-



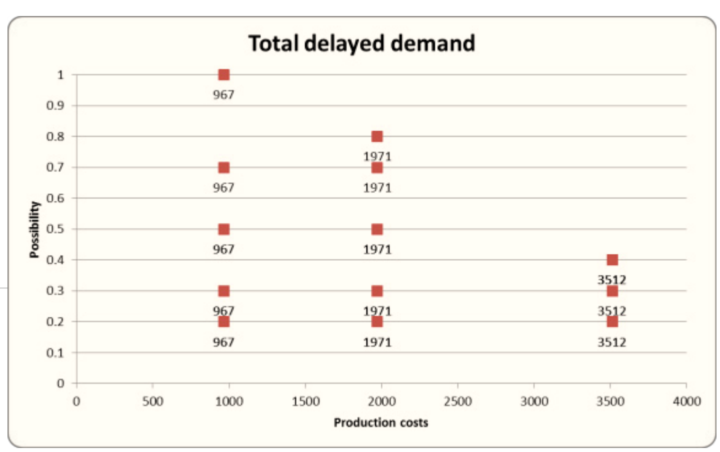


Fig. 1. Graphical results for total production costs. Fig. 2. Graphical results for total delayed demand

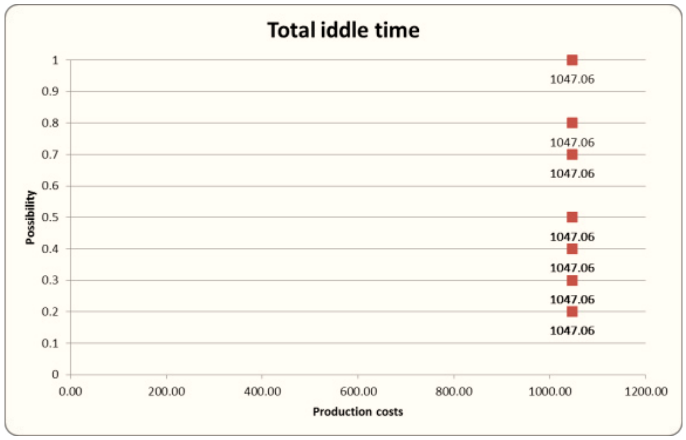


Fig. 3. Graphical results for total idle time.

	Solution 1	Solution 2	Solution 3
$z1$	88716.42	88706.40	88697.99
$z2$	967	1971	3512
$z3$	1047.06	1047.06	1047.06
Manhattan distance	286.30	60.09	595.23

Table 3: Fuzzy objective functions solution.

	Solution 1	**Solution 2**	**Solution 3**
$z1$	0.9677	0.9678	0.9678
$z2$	0.9033	0.8029	0.6488
$z3$	0.8547	0.8547	0.8547
Manhattan distance	0.0289	0.0061	0.0600
Euclidean distance	7.05E-07	1.44E-09	1.30E-05

Table 4: Membership objective fun. values.

So table-3 and Table-4 shows Manhatten and Euclidean distances with respect to center of gravity of each. So we have obtained all possible set of solutions for each objective functions, each membership function and their corresponding center of gravity.

3.4 Method of Lead time estimation based on generating fuzzy rules basis and some linguistic rules:-

In this method there are mainly three steps which provides solution of a given problem in fuzzy environment. They are as follows:-

1. Fuzzification review:- In this step all meteorological events having ambiguous characteristics and therefore their domain of change are divided into many fuzzy subsets that are complete, normal, and consistent with each other.

2. Fuzzy Inference System:- This step relates systematically pair wise all the factors taking place in the solution, which depends on the purpose of the problem. This part includes many fuzzy conditional statements to describe a certain situation.

For example if N is dependent on M and 'A' and 'B' are their two physical sets respectively. Then we can write as:
IF M is A (1) THEN N is B (1)
ALSO
IF M is A (2) THEN N is B (2) etc.

3. **Defuzzification:-**

In this method to calculate the deterministic value of linguistic variable N following method must be applied-

$$Nj = \left\{ \sum_{i-1}^{L} Ni \right\} / L$$

$$Pj(m) = \prod_{i=1}^{m} \mu(mi) \Big/ \sum_{j=1}^{L} \prod \mu(mi)$$

$$N = f(m) = \sum_{j=1}^{L} Pj(m)Nj$$

Where
P(m) = A Fuzzy basis function.
N = particular value of the linguistic variable.
Nj = Support value, in which the membership function reaches its maximum grade of membership.
Then the **center-average method** is selected and applied to defuzzify the proposed problem.

3.5 Monte Carlo simulation

Monte Carlo methods are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results.

Each observation of lead time has obtained through multiple simulation runs, based on obtained data from given values of lead times.

However Results have shown that the results show that the fuzzy lead times are smaller than simulated lead times in all cases. When the number of orders in the MRP system increases, it seems that the existing difference between simulated and fuzzy lead time becomes bigger.

4.0 CONCLUSION

This report addressed that how MRP calculations are done when lead time is fuzzy. For this we used three method namely, Multi-objective integer linear programming approach, Lead time estimation based on generating fuzzy rules basis and some linguistic rules and Lead time estimation by Monte Carlo simulation (Less superior).

Every method has its own limitations and assumptions and applicable under specific environment.

The advantages of these methods are related to: The modeling and establishment of the priorities for production objectives that traditionally are measured through costs estimated with difficulty by companies; and considering different values for product lead times associated to different possibility degrees which provide the decision maker with a broad decision spectrum with different risks levels.

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