Quantum Computing Basics

Presented by: Inderpreet Singh

A Refresher

- Dirac Notation: Dirac notation, also known as "bra-ket" notation, is a standard mathematical notation used to describe quantum states. There are two types of vectors in Dirac notation: the bra vector and the ket vector, so named because when put together they form a braket or inner product.
- If ψ is a column vector, then you can write it in Dirac notation as $|\psi\rangle$

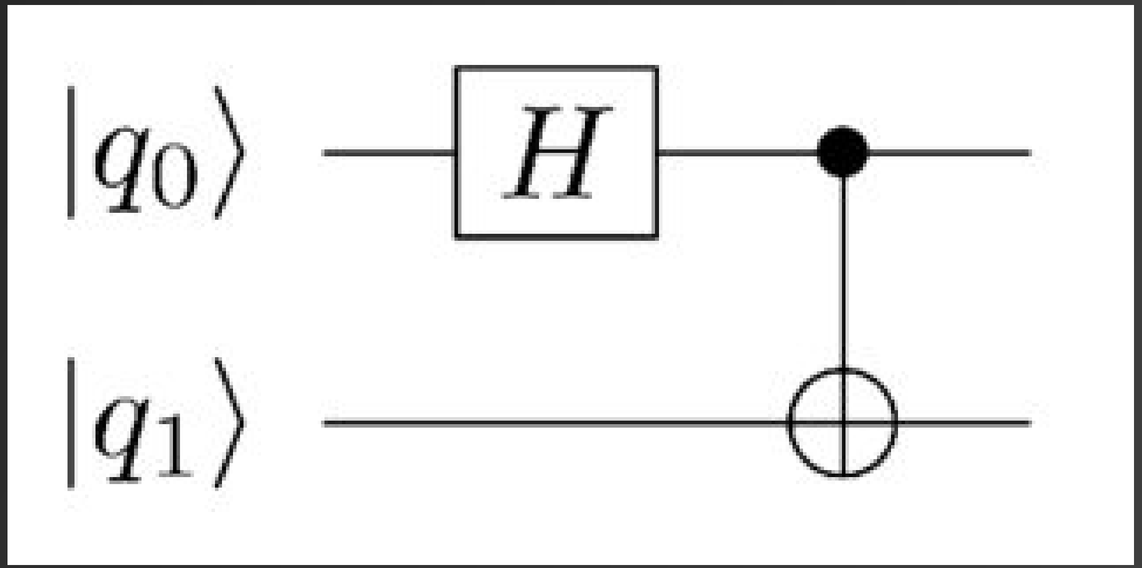
A qubit state in Dirac notation, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, can be represented in matrix form as a column vector:

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

The basis states $|0\rangle$ and $|1\rangle$ themselves can be represented in matrix form as:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Quantum Circuit



- Wires: In circuit diagrams, wires represent qubits over time. They carry the quantum information through the circuit from input to output.
- Quantum Gates: Represented as boxes or symbols along the wires in circuit diagrams, quantum gates manipulate qubits. They are described by unitary matrices, ensuring that all operations are reversible and preserve the overall quantum state's norm.

- Quantum gates, which are operations applied to qubits, are represented by matrices.
- When a quantum gate is applied to a qubit, the operation is represented by the matrix multiplication of the gate matrix with the qubit state vector.

For example if our input state is $|0\rangle$:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

And the Pauli-X gate is represented by the following unitary matrix:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- To see the effects of Pauli-X gate on basis state 0, we perform the matrix multiplication

$$X|0\rangle$$

- To see the effects of Pauli-X gate on basis state 0, we perform the matrix multiplication

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- To see the effects of Pauli-X gate on basis state 0, we perform the matrix multiplication

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \cdot 1 + 1 \cdot 0 \\ 1 \cdot 1 + 0 \cdot 0 \end{bmatrix}$$

- To see the effects of Pauli-X gate on basis state 0, we perform the matrix multiplication

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \cdot 1 + 1 \cdot 0 \\ 1 \cdot 1 + 0 \cdot 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

This vector represents the quantum state $|1\rangle$. Thus, applying the Pauli-X gate to a qubit initially in the state $|0\rangle$ results in the qubit being in the state $|1\rangle$.

Similarly, other quantum gates can be represented in matrix form as follows:

- **Pauli-X Gate** (Quantum NOT Gate):

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- **Pauli-Y Gate:**

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

- **Pauli-Z Gate:**

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

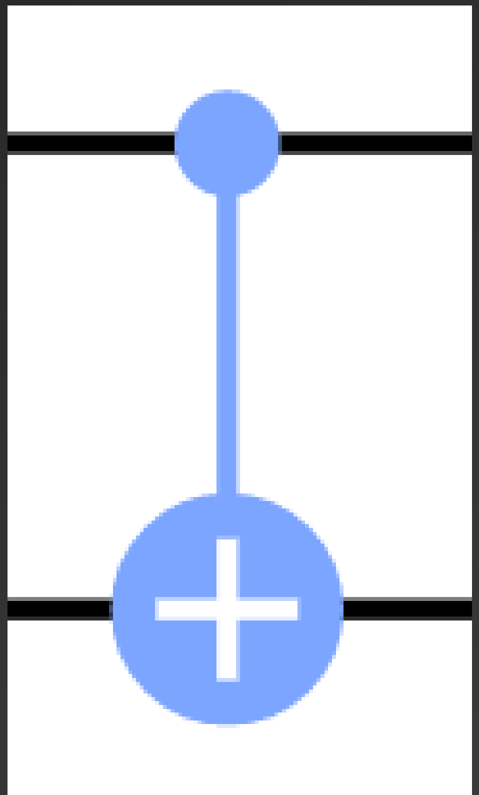
- **Hadamard Gate:**

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

CNOT Gate

The CNOT gate is two-qubit operation, where the first qubit is usually referred to as the control qubit and the second qubit as the target qubit. Expressed in basis states, the CNOT gate:

- leaves the control qubit unchanged and performs a Pauli-X gate on the target qubit when the control qubit is in state $|1\rangle\ |1\rangle$;
- leaves the target qubit unchanged when the control qubit is in state $|0\rangle\ |0\rangle$.



$$\mathrm{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

CNOT Gate

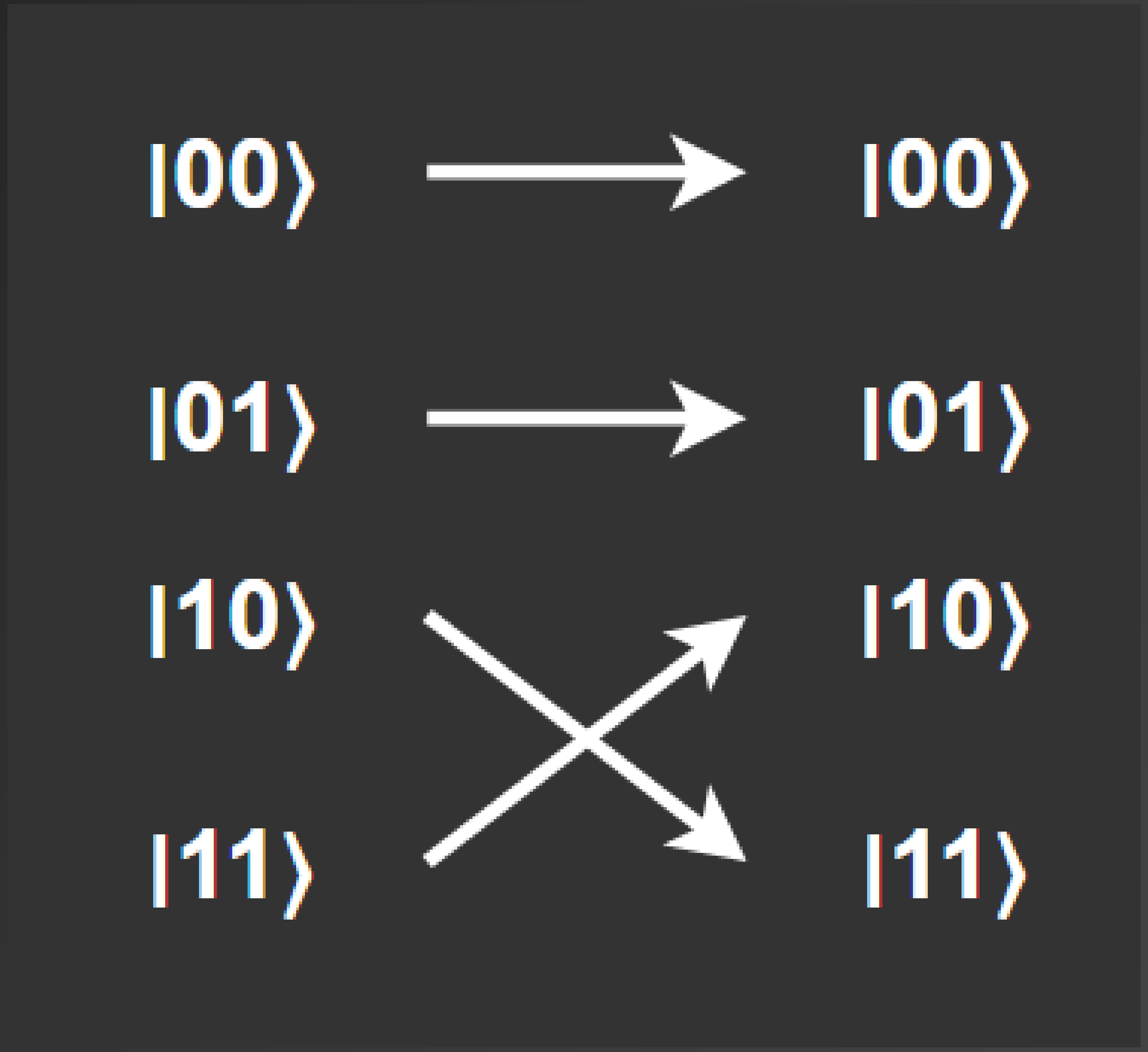
$|00\rangle$

$|01\rangle$

$|10\rangle$

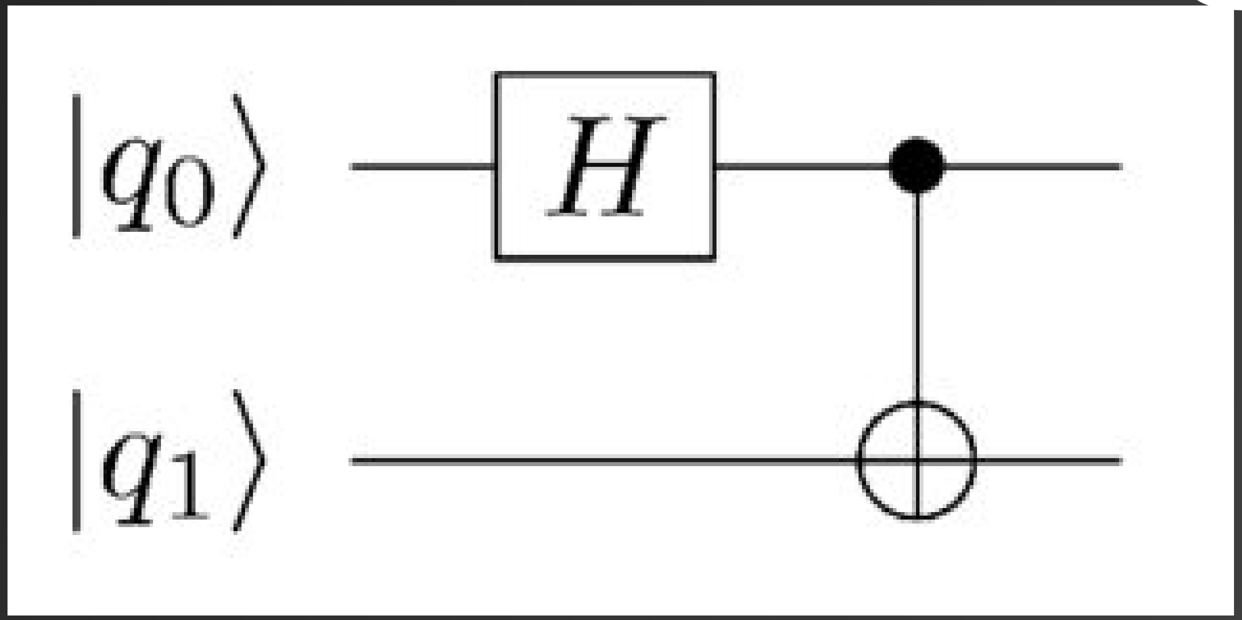
$|11\rangle$

CNOT Gate

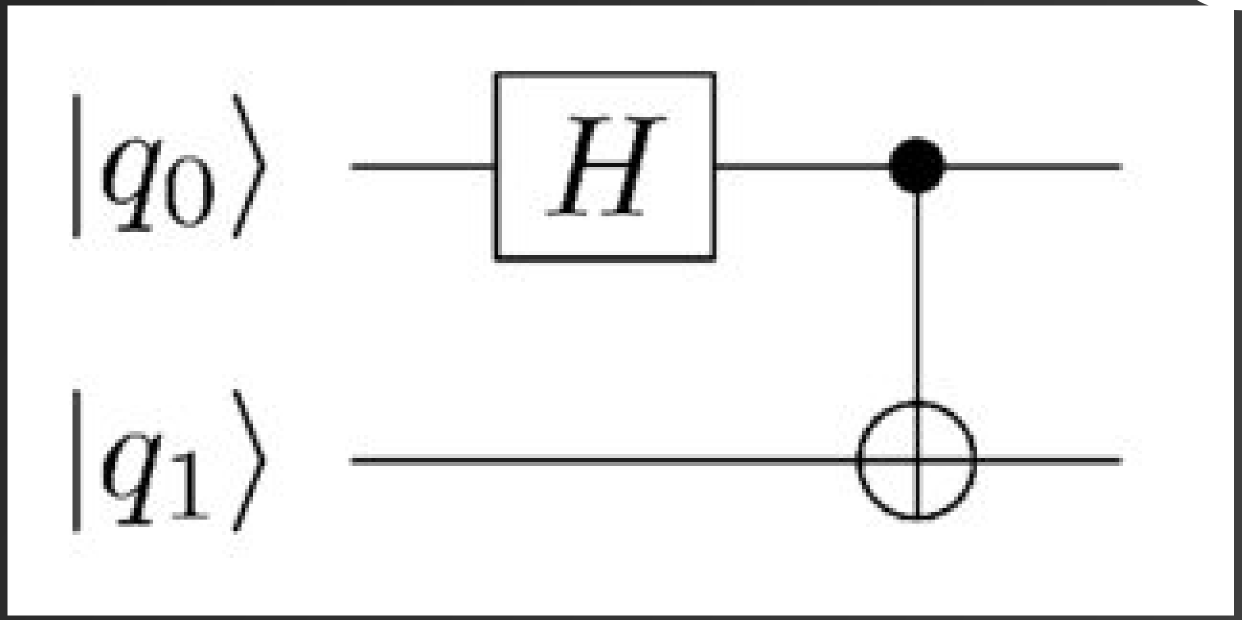


$$|x\rangle|y\rangle \longrightarrow |x\rangle|x\oplus y\rangle$$

Quantum Entanglement



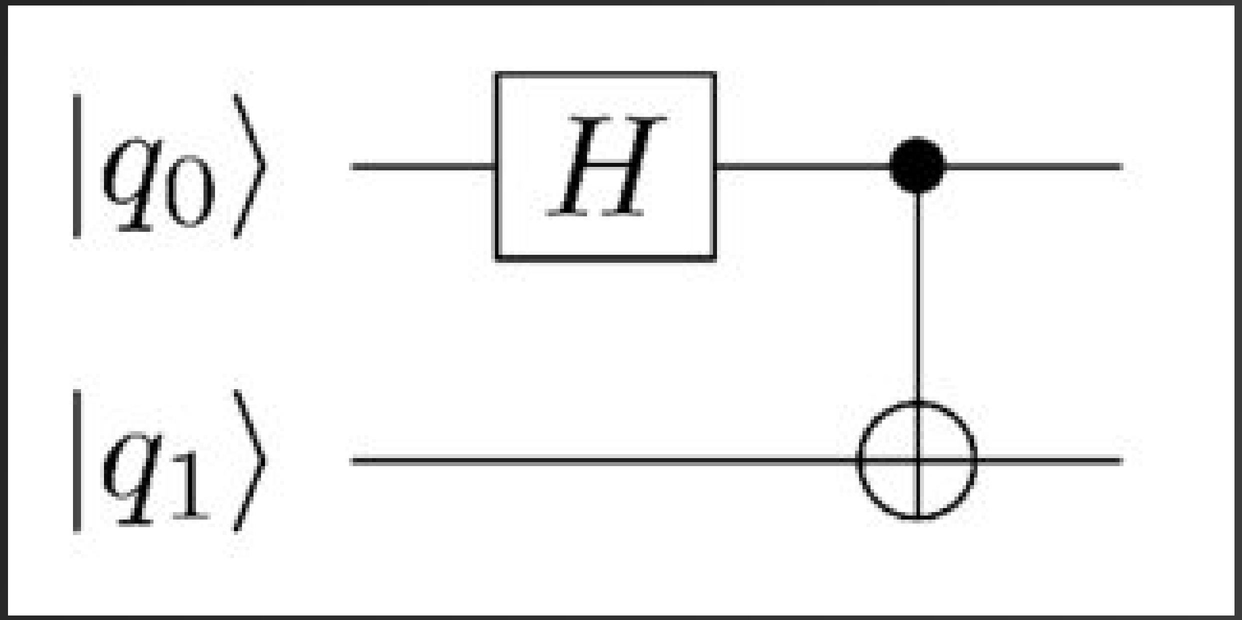
Quantum Entanglement



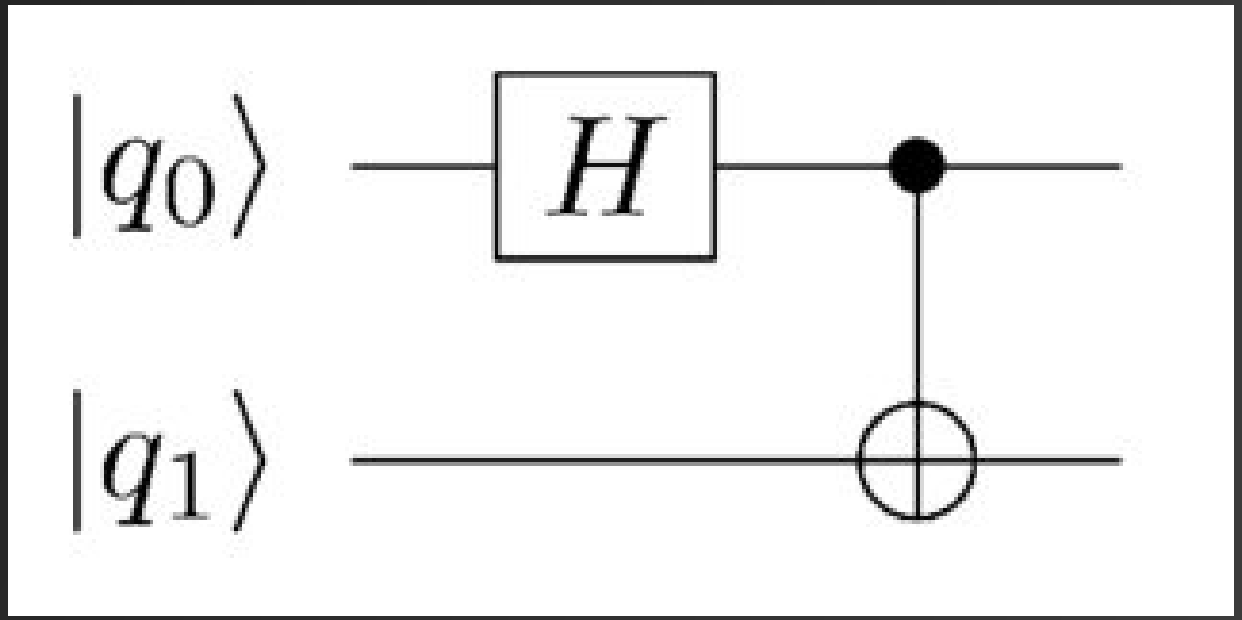
for input state $|00\rangle$ $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$$|00\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\,|0\rangle$$

$$\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$



Now apply the CNOT gate to the state $\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$:



Now apply the CNOT gate to the state $\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$:

$$= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

This Bell state represents maximally entangled qubits and is used extensively in quantum information processing, such as in quantum teleportation

- Bell states, are specific quantum states of two qubits that represent the simplest and most perfect examples of quantum entanglement.
- These states are maximally entangled, meaning that the measurement outcome of one qubit instantaneously determines the state of the other, regardless of the distance between them.

Similarly, for other input states;

$$|00\rangle \longrightarrow |\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|10\rangle \longrightarrow |\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|01\rangle \longrightarrow |\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|11\rangle \longrightarrow |\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Quantum Teleportation Protocol

Scenario

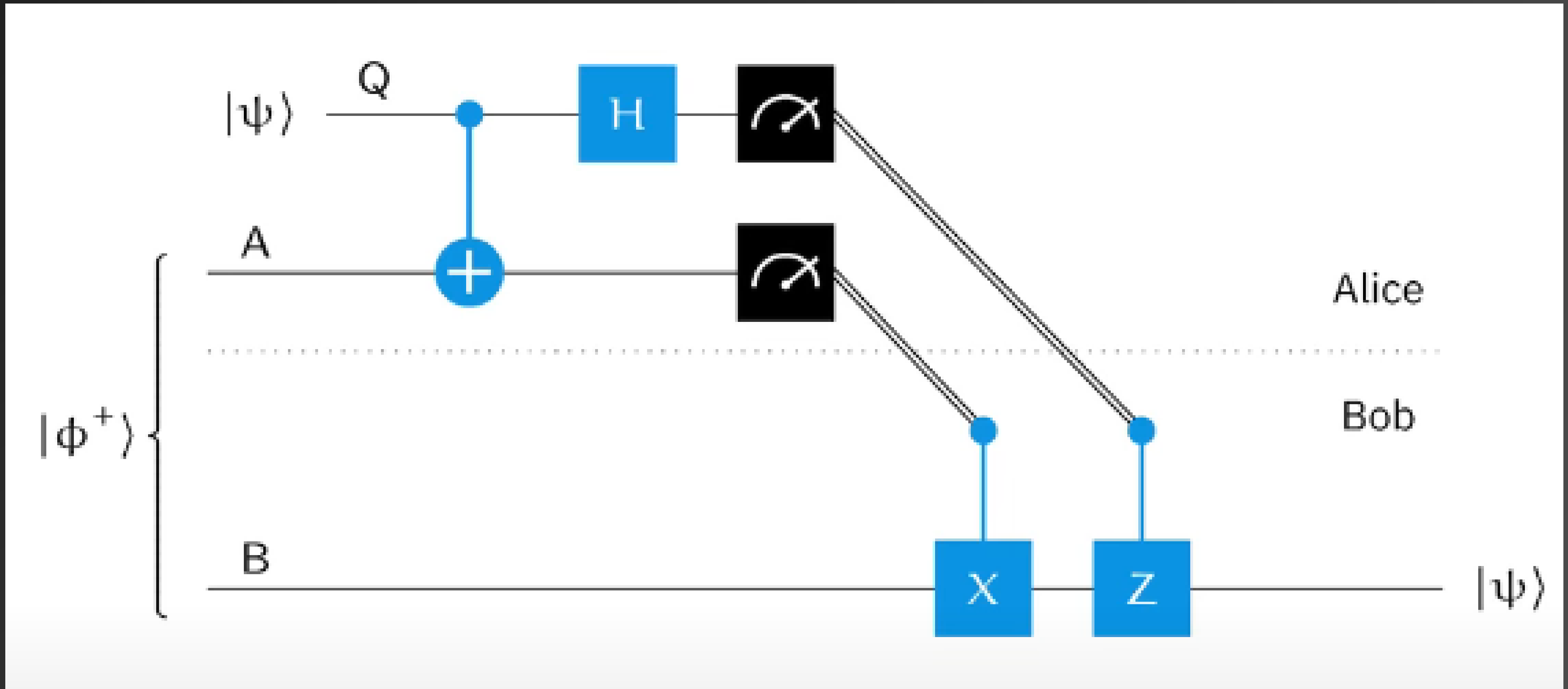
Alice has a *qubit* Q that she wishes to transmit to Bob.

- Alice is unable to physically send Q to Bob — she is only able to senc *classical information.*
- Alice and Bob *share an e-bit.*

Remarks

- The state of Q is "unknown" to both Alice and Bob.
- Correlations (including entanglement) between Q and other systems must be preserved by the transmission.
- The *no-cloning theorem* implies that if Bob receives the transmission, Alice must no longer have the qubit in its original state.

Quantum Teleportation Protocol



Quantum Teleportation Protocol

Protocol

1. Alice performs a controlled-NOT operation, where Q is the control and A is the target.
2. Alice performs a Hadamard operation on Q.
3. Alice measures A and Q, obtaining binary outcomes a and b, respectively.
4. Alice sends a and b to Bob.
5. Bob performs these two steps:

    5.1 If $a = 1$, then Bob applies an X operation to the qubit B.

    5.2 If $b = 1$, then Bob applies a Z operation to the qubit B.

Quantum Teleportation Protocol implementation on Qiskit