Rank, Vector Spaces & Eigenvectors - IS605 Assignment 3

Sharon Morris

2/15/2017

1. Problem set 1 What is the rank of the matrix A?

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

The rank of a matrix is the number of pivots

```
library(matrixcalc)
# define the matrix
M <- matrix(data = c(1,2,3,4,-1,0,1,3,0,1,-2,1,5,4,-2,-3), nrow = 4, ncol = 4, byrow = TRUE)
M
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3
```

```
matrix.rank(M)
```

```
## [1] 4
```

```
#http://stackoverflow.com/questions/10881392/rank-of-a-matrix-in-r
```

(2) Given an

$$mxn$$

matrix where

$$m > n$$

, what can be the maximum rank? The mini- mum rank, assuming that the matrix is non-zero?

Maximum rank = m (rows) Minimum rank = 1 (all other rows could be linearly dependent)

What is the rank of matrix B?

$$\mathbf{B} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

```
Mb <- matrix(data = c(1,2,1,3,6,3,2,4,2), nrow = 3, ncol = 3, byrow = TRUE)
Mb
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    3    6    3
## [3,]    2    4    2
```

```
matrix.rank(Mb)
```

```
## [1] 1
```

Problem set 2 Compute the eigenvalues and eigenvectors of the matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

λ is an eigenvalue of $A \iff A\vec{v} = \lambda\vec{v}$ for some non zero $\vec{v} \iff \vec{0} = \lambda I_n\vec{v} - A\vec{v}$ is ture $\iff \vec{0} = (\lambda I_n - A)\vec{v}$ $\iff determinant(\lambda I_u - A) = 0$

$$\lambda I_3 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$\lambda I_3 - A = \begin{bmatrix} \lambda - 1 & -2 & -3 \\ 0 & \lambda - 4 & -5 \\ 0 & 0 & \lambda - 6 \end{bmatrix} \begin{bmatrix} \lambda - 1 & -2 \\ 0 & \lambda - 4 \\ 0 & 0 \end{bmatrix}$$

$(\lambda - 1)(\lambda - 4)(\lambda - 6) - (\lambda - 6) - (\lambda - 1) - (\lambda - 4) = (\lambda - 1)(\lambda^2 - 10\lambda + 24) - \lambda + 6 - \lambda + 1 - \lambda + 4$ $= (\lambda - 1)(\lambda^2 - 10\lambda + 24) - 3\lambda + 11 = \lambda^3 - 11\lambda^2 + 34\lambda - 24 - 3\lambda + 11$ $p(\lambda) = \lambda^3 - 11\lambda^2 + 31\lambda - 13 = 0$ This polynomial does not have any zeros or root – there are no possible eigenvalues

https://www.khanacademy.org/math/linear-algebra/alternate-bases/eigen-everything/v/linear-algebra-eigenvalues-of-a-3x3-n