

Taylor Series - Assignment 14

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For each function, only consider its valid ranges as indicated in the notes when you are computing the Taylor Series expansion.

1) $f(x) = \frac{1}{1-x}$

$$f'(x) = (-1)(-1)(1-x)^{-2} = 1 \quad f''(x) = (-1^2)(-1)(-2)(1-x)^{-3} = 2 \quad f'''(x) = (-1^3)(-1)(-2)(-3)(1-x)^{-4} = 6$$
$$f^n(x) = (n!)(1-x)^{-n}$$

Taylor Series expansion $\frac{1}{(1-x)} \quad f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!} (x-a)^n \quad f(x) = 1 + x + x^2 + x^3 + x^4 + \dots$

2.) $f(x) = e^x$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!} (x-a)^n \quad f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

3) $f(x) = \ln(1+x)$

$$f'(x) = \frac{1}{(1+x)} = 1 \quad f''(x) = (-1)(1+x)^{-2} = -1 \quad f'''(x) = (-1)(-2)(1+x)^{-3} = 2$$

Taylor Series $f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!} (x-a)^n \quad f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x(n+1) \quad f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$

References <http://mathworld.wolfram.com/TaylorSeries.html>