Numerical Integration AND Differentiation

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Write a program to compute the derivative of $f(x) = x^3 + 2x^2$ at any value of x. Your function should take in a value of x and return back an approximation to the derivative of $f(x)$ evaluated at that value. You should not use the analytical form of the derivative to compute it. Instead, you should compute this approximation using limits.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

```
#define the function f(x)
f <- function(x) {x^3 + 2*x^2}

#limit
d <- function(x) {
    h = 1e-6
    d = (f(x + h) - f(x)) / (h)
    return(d)
}

d(605)
```

```
## [1] 1100495
```

Now, write a program to compute the area under the curve for the function $3x2 + 4x$ in the range $x = [1, 3]$. You should first split the range into many small intervals using some really small x value (say 1e-6) and then compute the approximation to the area under the curve.

```
x <- seq(1, 3, by=1e-6)
func <- 3*(x^2) + 4*x
area <- func*(1e-6)
sum(area)
```

```
## [1] 42.00002
```

Please solve these problems analytically (i.e. by working out the math) and submit your answers.

Use integration by parts to solve for $\int sin(x)cos(x)dx$ $d(uv) = udv + vdu$ $u = sin(x), du = cos(x)dx$ $\frac{u^{1+1}}{1+1}$ $u = sin(x)$ $\frac{sin^{1+1}(x)}{1+1}$

$$\int sin(x)cos(x)dx = \frac{sin^2(x)}{2} + C$$

Use integration by parts to solve for $\int x^2 e^x dx$ $\int uv' = uv - \int u'v$ $u = x^2, u' = 2x, v' = e^x, v = e^x$ $= x^2e^x - \int 2xe^x dx = \int 2xe^x dx = 2(e^x x - e^x) = x^2e^x - 2(e^x x - e^x)$

$$\int x^2 e^x dx = x^2 e^x - 2(e^x x - e^x) + C$$

What is $\frac{d}{dx}(xcos(x))$? $\frac{d}{dx}(x) = 1$ $\frac{d}{dx}(cos(x)) = -sin(x) = 1cos(x) + (-sin(x))x$

$$\frac{d}{dx}(xcos(x)) = cos(x) - xsin(x)$$

- Whatis d(ex4)? $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$ $x^4 = u = \frac{d}{du}(e^u)\frac{d}{dx}(x^4)$ $\frac{d}{du}(e^u) = e^u$ $\frac{d}{dx}(x^4) = 4x^3$ $= e^u 4x^3$ $u = x^4$

$$\frac{d}{dx}(e^{x^4}) = e^{x^4}4x^3$$

References https://cran.r-project.org/web/packages/Deriv/Deriv.pdf https://www.mathsisfun.com/calculus/integration-by-parts.html http://tutorial.math.lamar.edu/Classes/CalcII/IntegrationByParts.aspx