Modeling with a differential equation - IS609 HW-9

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The following data were obtained for the growth of a sheep population introduced into a new environment on the island of Tasmania (adapted from J. Davidson, “On the Growth of the Sheep Population in Tasmania,” Trans. R. Soc. S. Australia 62(1938): 342–346).

t(year)	1814	1824	1834	1844	1854	1864
P(t)	125	275	830	1200	1750	1650

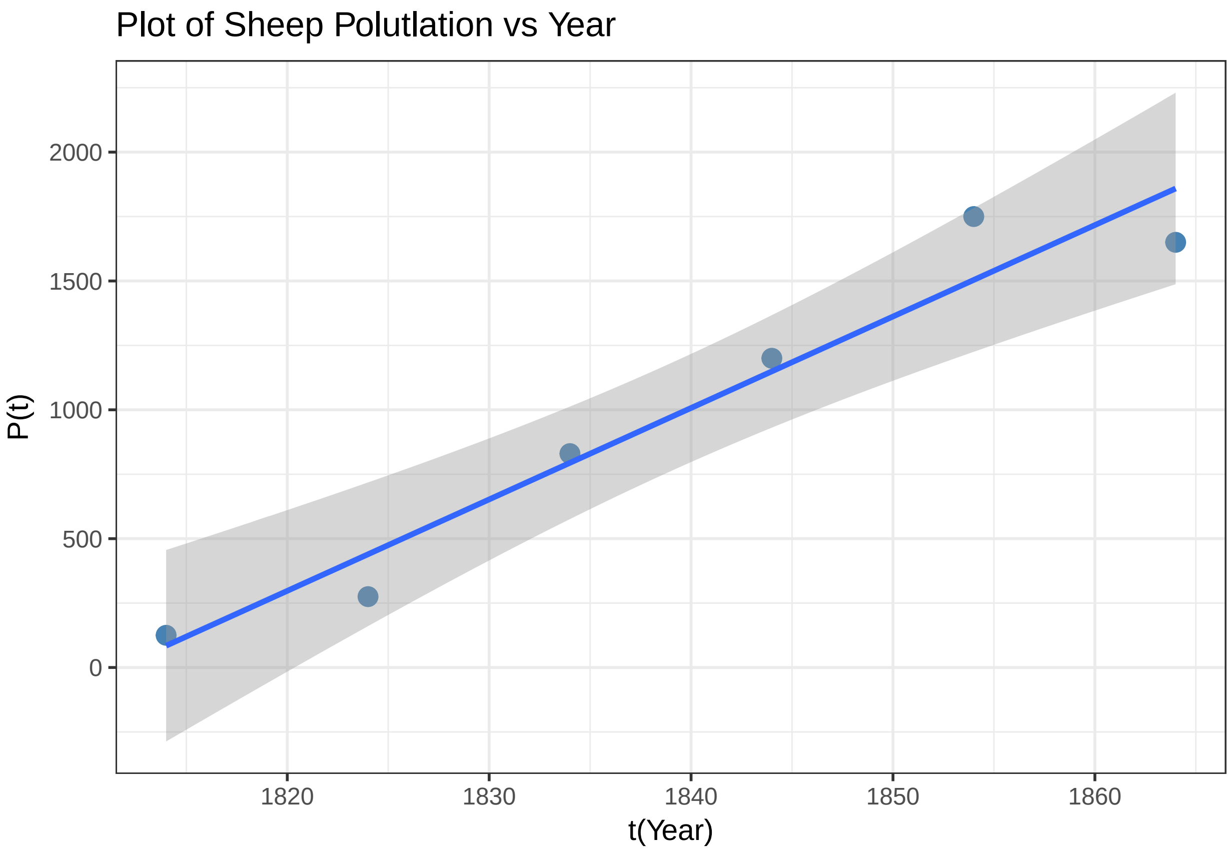
a. Make an estimate of M by graphing $P(t)$.

```
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 3.3.2
```

```
t <- c(1814, 1824, 1834, 1844, 1854, 1864)
p <- c(125, 275, 830, 1200, 1750, 1650)

ggplot(data.frame(x=t, y=p), aes(x=x, y=y), fill=p) +
    geom_point(color='steelblue', size=3) +
    geom_smooth(method = 'lm') +
    theme_bw() +
    xlab("t(Year)") +
    ylab("P(t)") +
    ggtitle("Plot of Sheep Polutlation vs Year")
```

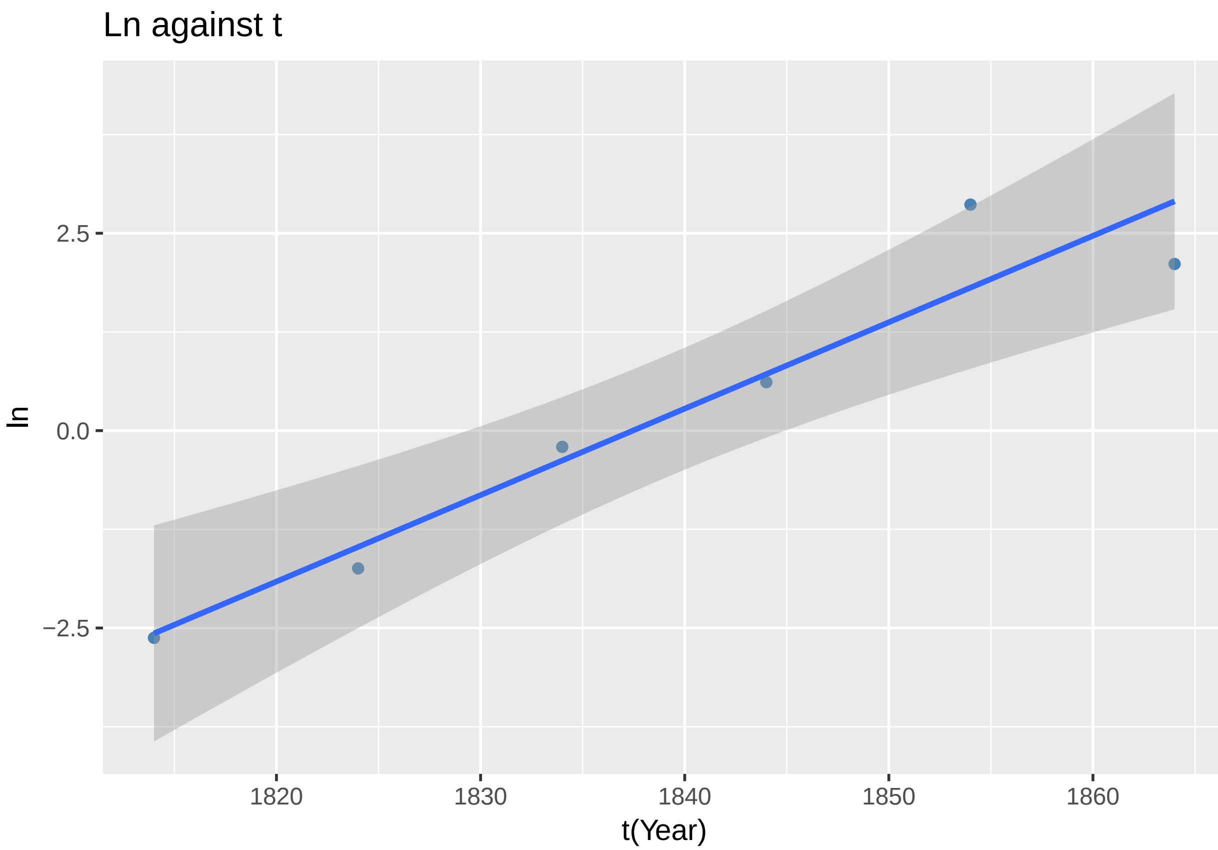


The limimitling population $M \triangleq 1850$ – the values does not exceed 1750

b. Plot ln[$P/(M-P)]$ against t. If a logistic curve seems reasonable, estimate rM and $t*$

```
m <- 1850
ln <- log(p/(m-p))

ggplot(data.frame(x=t, y=ln), aes(x=x, y=y), fill=ln) +
    geom_point(color='steelblue') +
    geom_smooth(method = 'lm') +
    xlab("t(Year)") +
    ylab(("ln")) +
    ggtitle("Ln against t")
```



The above plot shows an approximate linear relationship

Estimate rM and $t*$

$$ln[\frac{P}{M-P}]$$

```
(rm <- (ln <- lm(log(p/(m-p)) ~ t)$coefficients[[2]]))
```

```
## [1] 0.1094742
```

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Suggest other phenomena for which the model described in the text might be used.

The equation of concentration comes out to be with initial concentration. The model can be used in the following phenomena other than medicine a. The model can estimate the concentration of radioactive substances used in astronomy which decays over time b. The model can be used to estimate the age of rocks and the life of trees and plants

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a. Using the estimate that $d_b = 0.054v^2$, where 0.054 has dimension $ft.hr^2/mi^2$, show that the constant k in Equation (11.29) has the value 19.9 ft/sec^2.
b. Using the data in Table 2.4, plotd_b in ft versus $v^2/2$ in ft^2/sec^2 to estimate $1/k$ directly

$$d_b = \frac{-v_0^2}{2k} + \frac{v_0^2}{k} = \frac{v_0^2}{2k}$$

$d_b = 0.054v^2 = \frac{v^2}{2k}$ $k = \frac{1}{2X0.054} \approx 9.259\frac{mi^2}{ft.hr^2}$

$9.259\frac{mi^2}{ft.hr^2}$ x $\frac{5280^2}{1mi^2}$ x $\frac{1hr^2}{3600^2s^2} \approx 19.918\frac{ft}{s^2}$

b. Using the data in Table 2.4, plot d_b in ft versus $v^2/2$ in ft^2/sec^2 to estimate $\frac{1}{k}$ directly.

```
db <- c(20, 28, 40.5, 52.5, 72, 92.4, 118, 148.5, 182, 220.5, 266, 318, 376)
v <- c(29.33, 36.667, 44, 51.33, 58.667, 66, 73.33, 80.667, 88, 95.333, 102.667, 110.0, 117.3)

ggplot(data.frame(x=v, y=db), aes(x=x, y=y)) +
geom_smooth(method = 'lm') +
geom_point(size=3) +
xlab("V(ft per sec)") +
ylab("db")
```